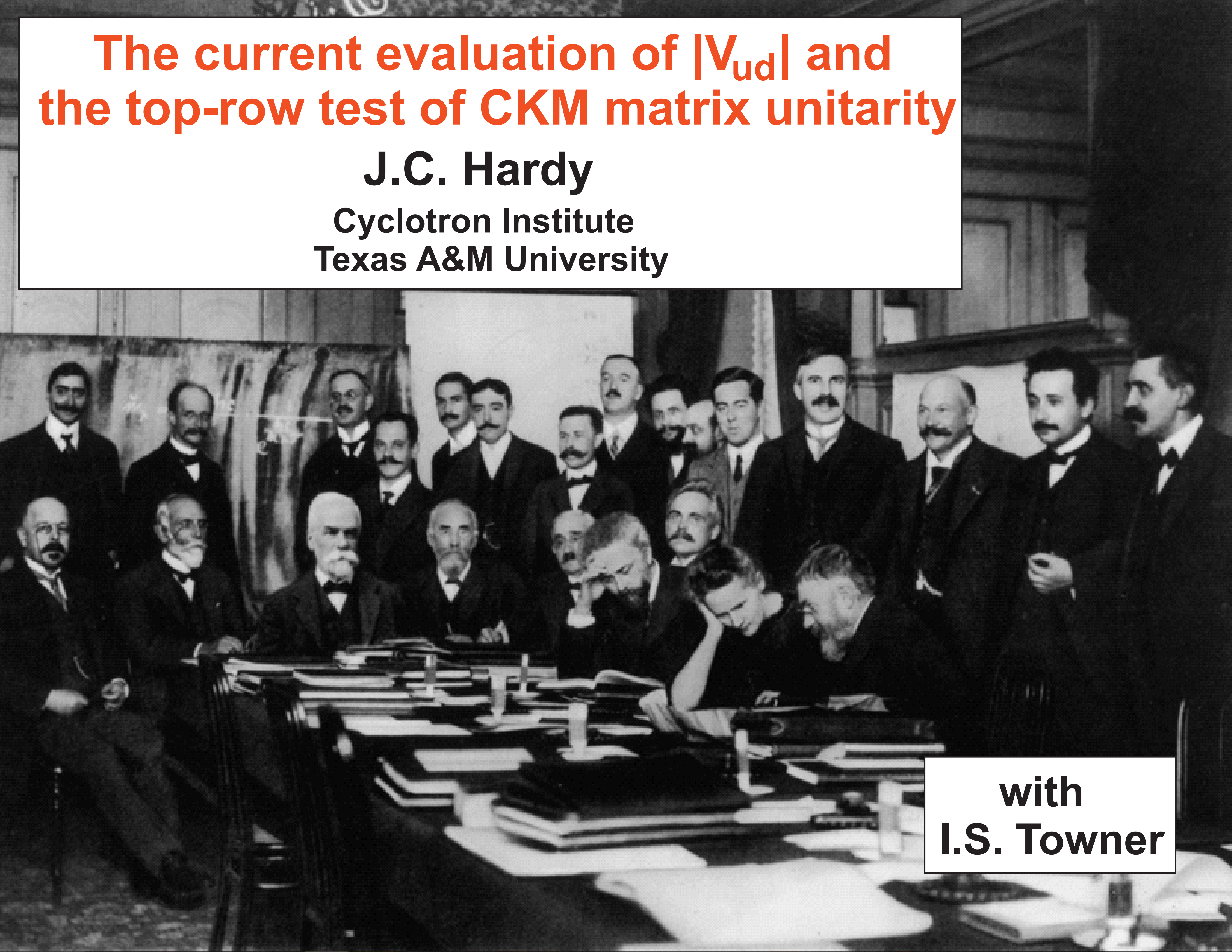
The current evaluation of $|V_{ud}|$ and the top-row test of CKM matrix unitarity

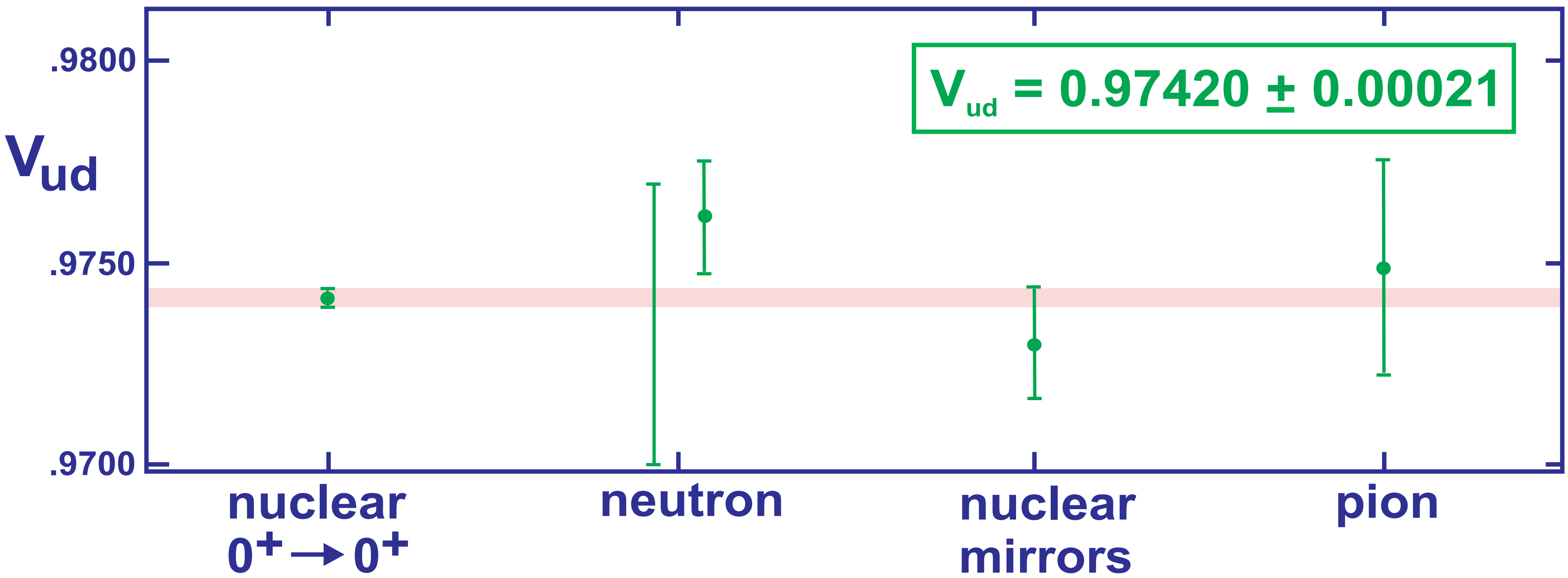
J.C. Hardy

Cyclotron Institute
Texas A&M University



with
I.S. Towner

CURRENT STATUS OF V_{ud}

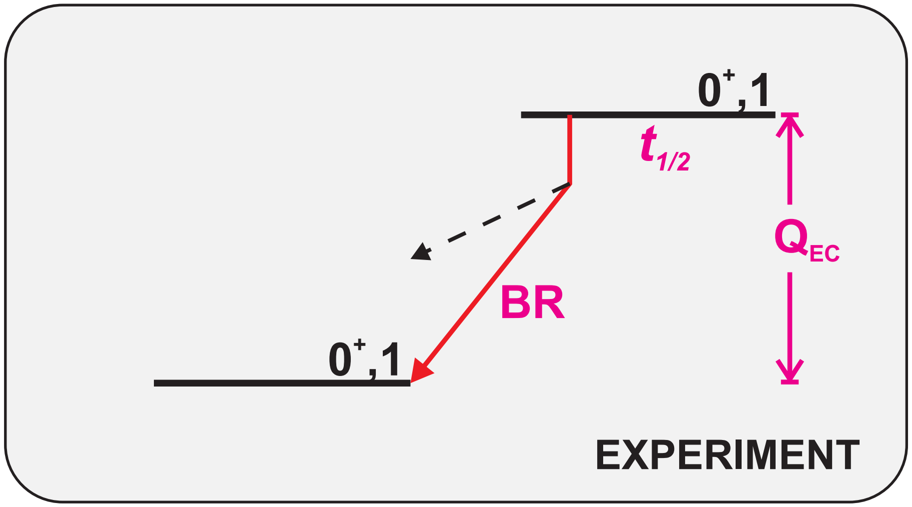


SUPERALLOWED $0^+ \rightarrow 0^+$ BETA DECAY

BASIC WEAK-DECAY EQUATION

$$ft = \frac{K}{G_V^2 \langle \tau \rangle^2}$$

f = statistical rate function: $f(Z, Q_{EC})$
t = partial half-life: $f(t_{1/2}, BR)$
G_V = vector coupling constant
$\langle \tau \rangle$ = Fermi matrix element



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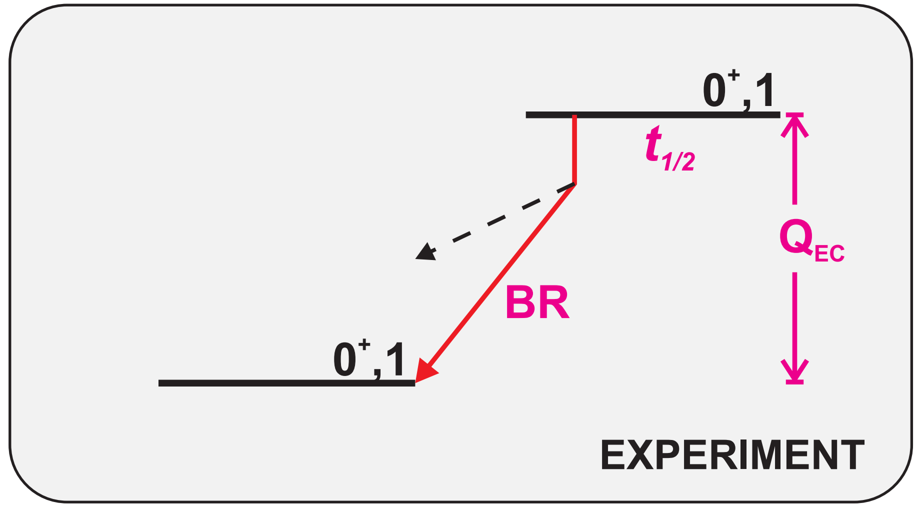

INCLUDING RADIATIVE AND ISOSPIN-SYMMETRY-BREAKING CORRECTIONS

$$\mathcal{F}t = ft\,(1 + \delta_R')[1 - (\delta_C - \delta_{NS})] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

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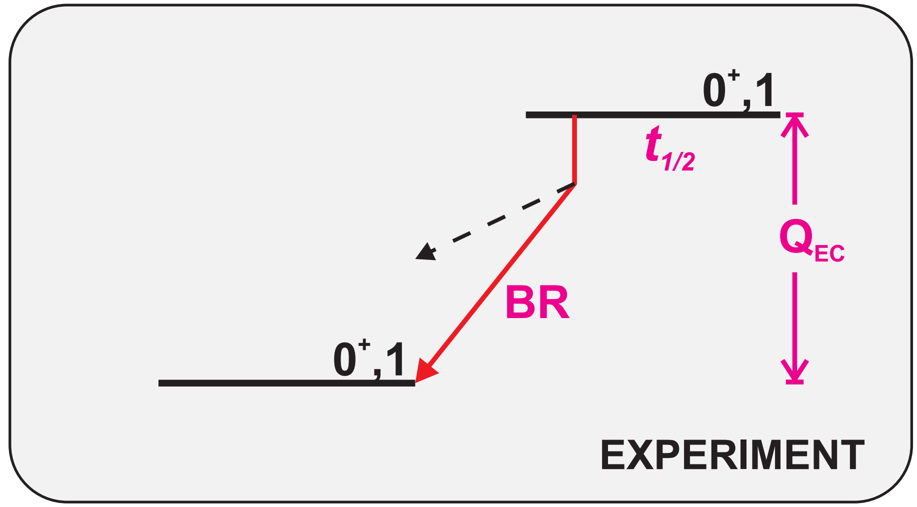

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$f(Z, Q_{EC})$
~1.5%

f(nuclear structure)
0.3-1.5%

f(interaction)
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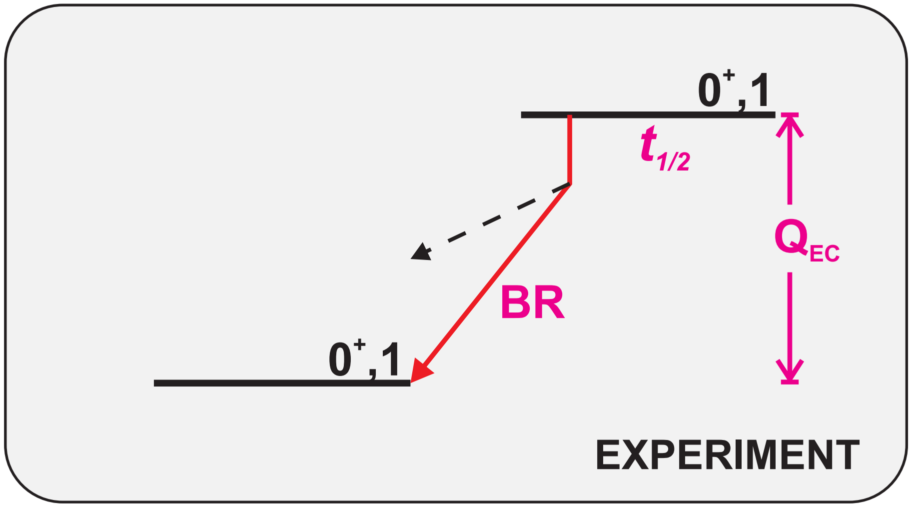

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THEORETICAL UNCERTAINTIES

0.05 – 0.10%

THE PATH TO V_{ud}

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \Delta_R)$

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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate the correction terms

Test for presence of a Scalar current

$\mathcal{F}t$ values constant

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$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates

Cabibbo Kobayashi Maskawa (CKM) matrix

mass eigenstates

Obtain precise value of $G_V^2(1 + \Delta_R)$

Determine V_{ud}^2

$$V_{ud}^2 = G_V^2/G_\mu^2$$

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weak eigenstates — Cabibbo Kobayashi Maskawa (CKM) matrix — mass eigenstates

Obtain precise value of $G_V^2(1 + \Delta_R)$

Determine V_{ud}^2

$$V_{ud}^2 = G_V^2/G_\mu^2$$

Test CKM unitarity

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1$$

THE PATH TO V_{ud}

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weak eigenstates — Cabibbo Kobayashi Maskawa (CKM) matrix — mass eigenstates

Obtain precis... $(1 + \Delta_R)$

Determin...

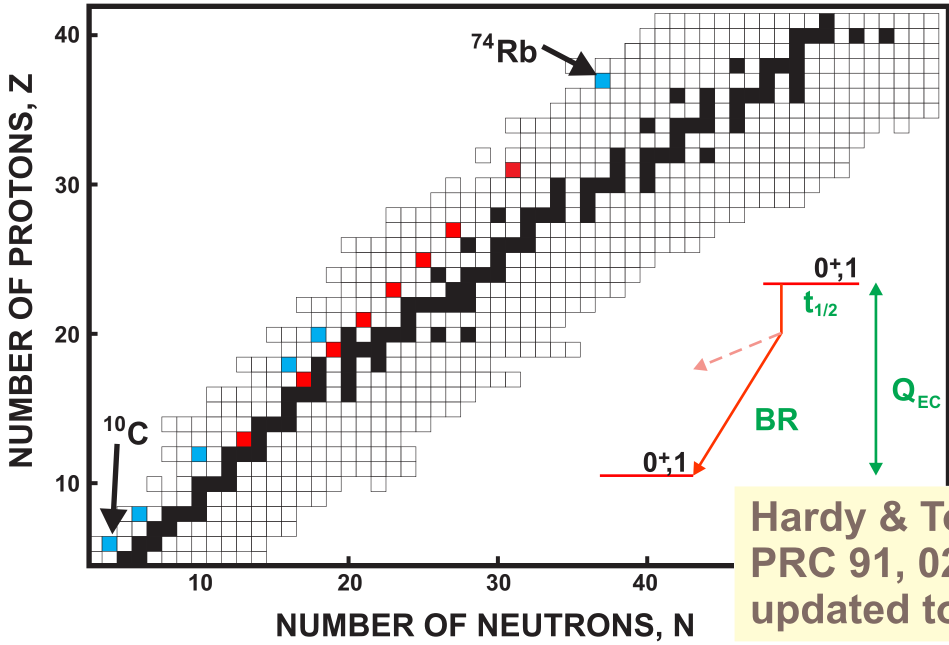
T... unitarity

ONLY POSSIBLE IF PRIOR CONDITIONS SATISFIED

$$V_{ud}^2 = G_V^2/G_\mu^2$$

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 1$$

WORLD DATA FOR $0^+ \rightarrow 0^+$ DECAY, 2017



Hardy & Towner
PRC 91, 025501 (2015)
updated to 2017

- 8 cases with *ft*-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.

- ~220 individual measurements with compatible precision

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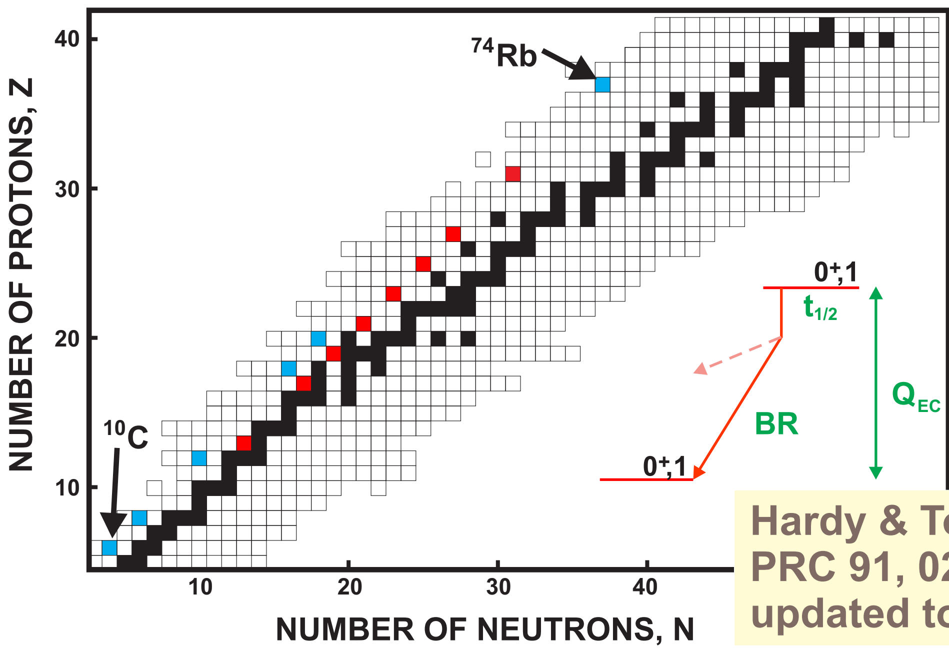

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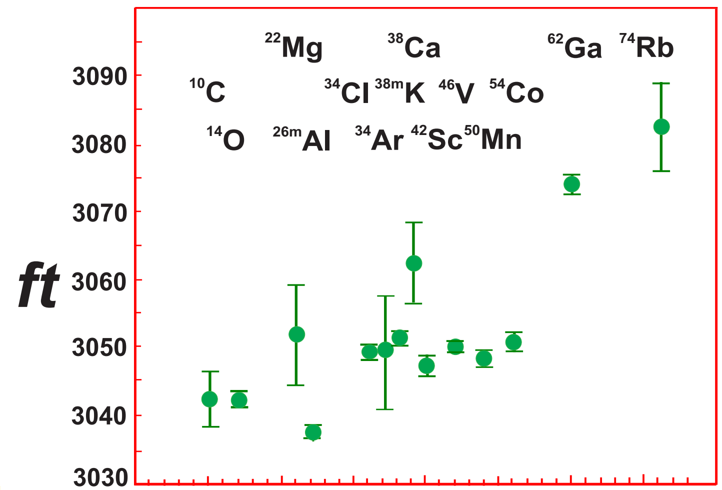

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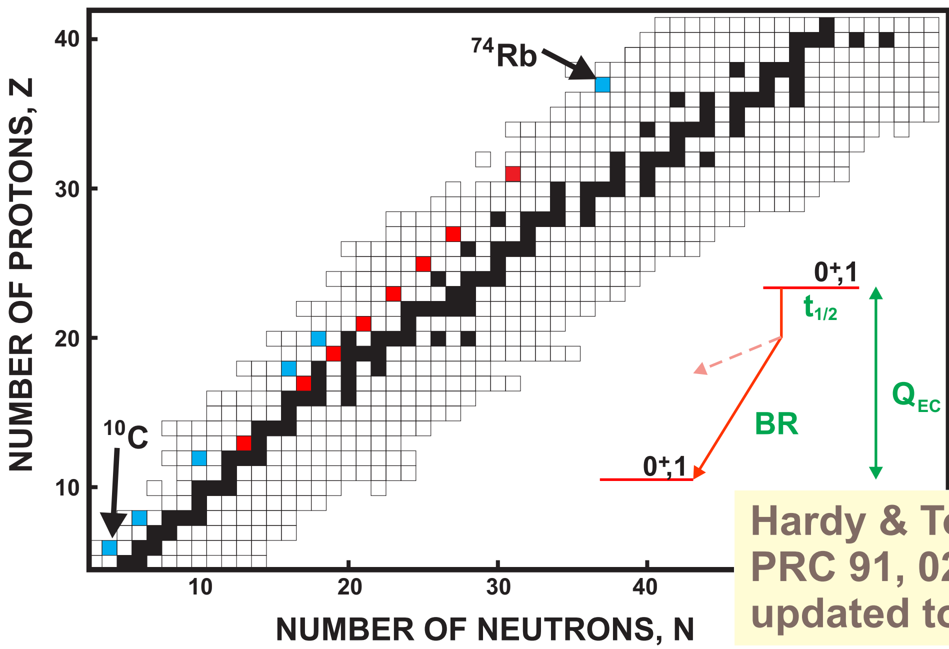

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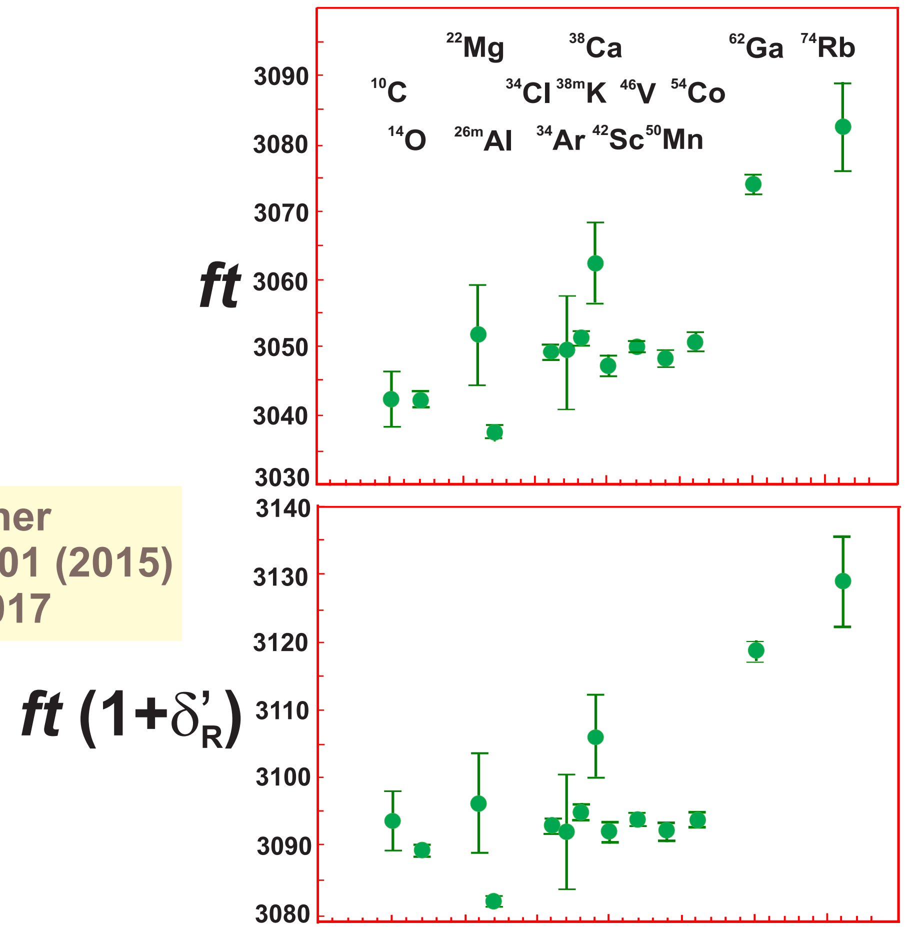

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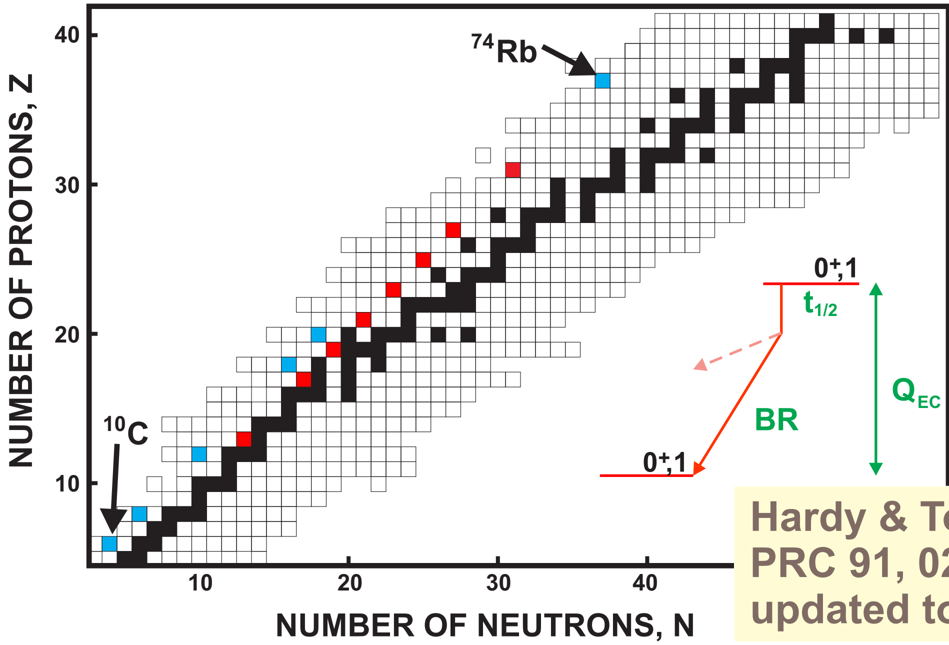

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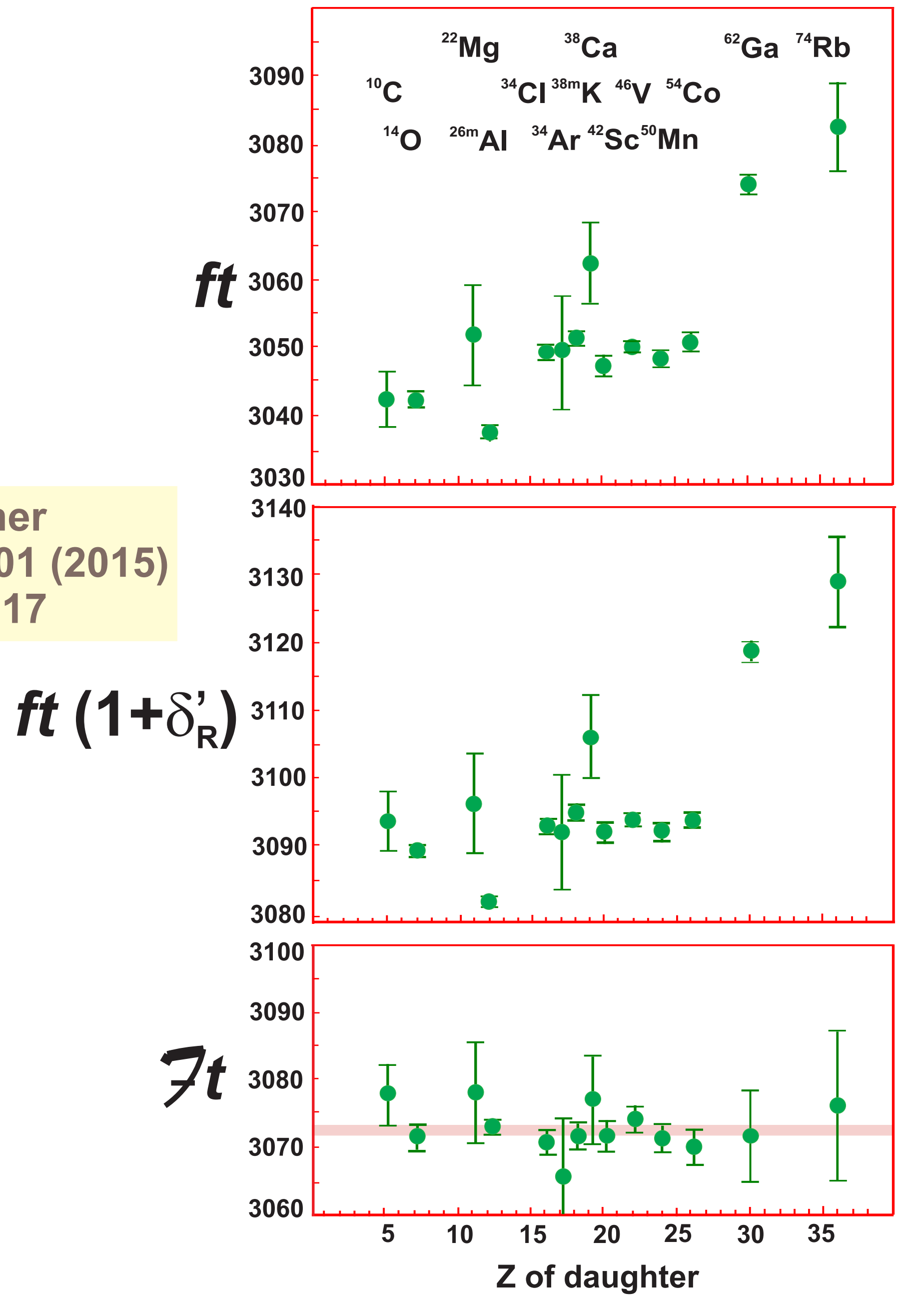

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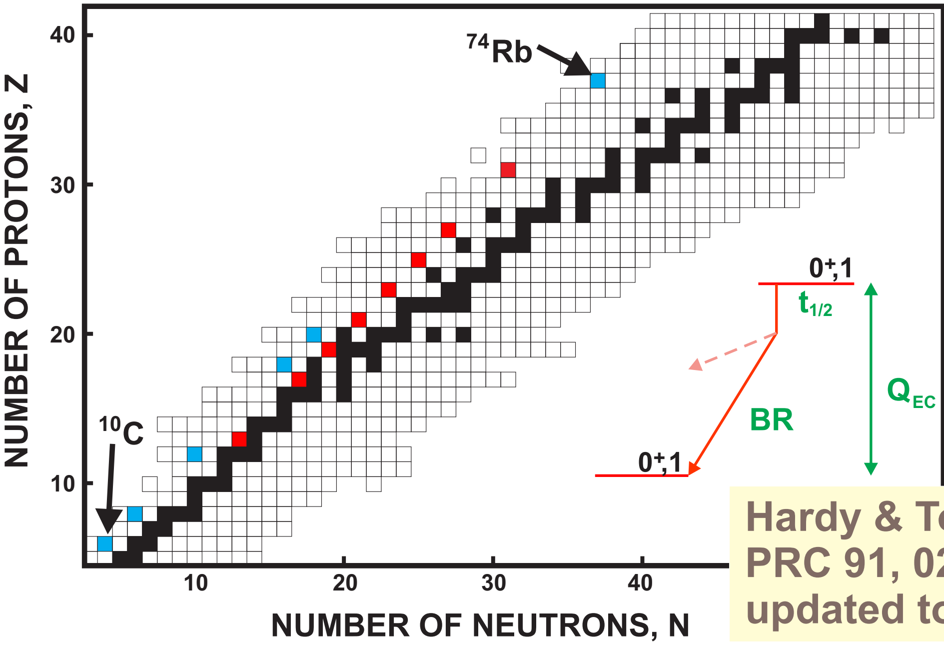

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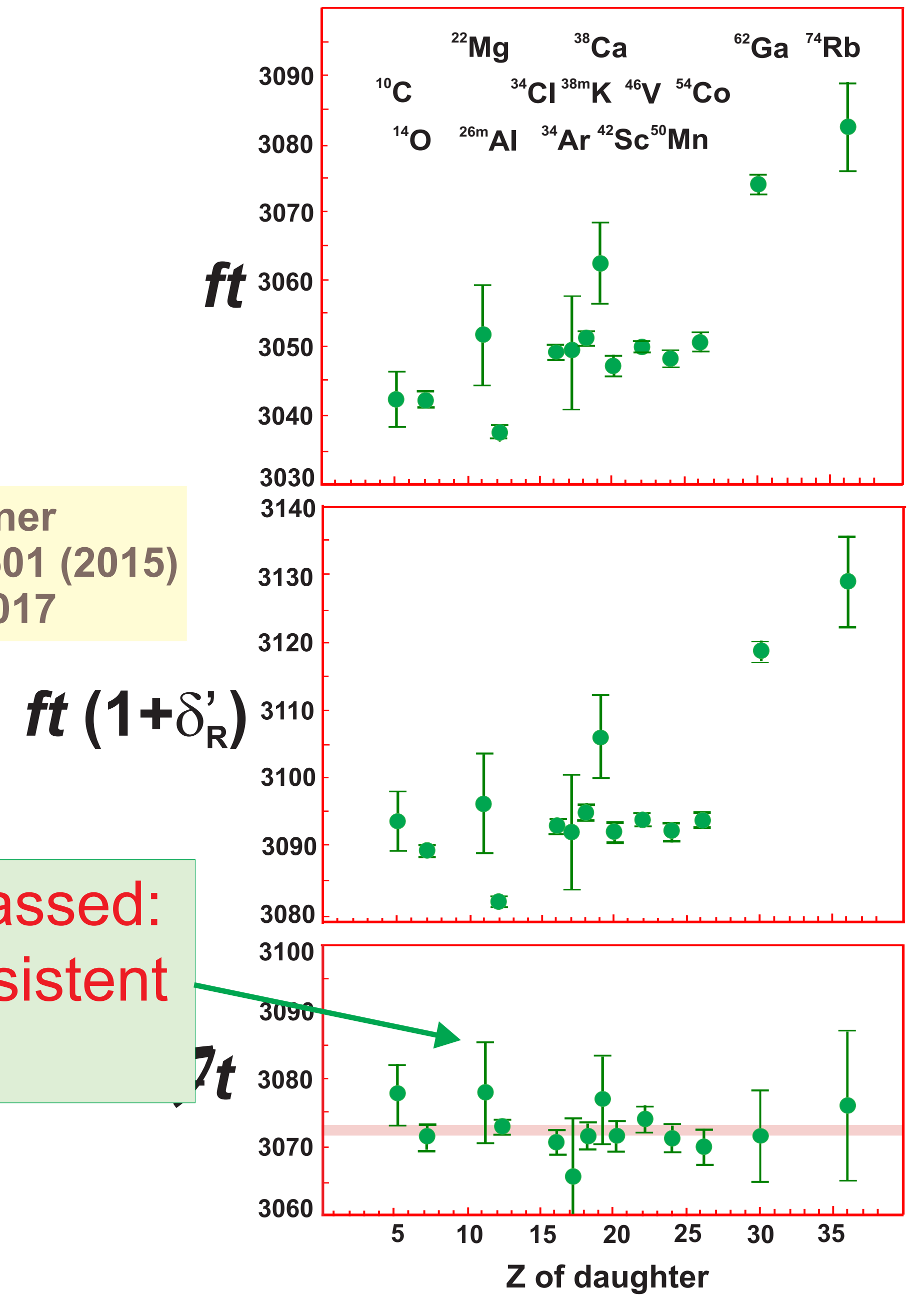

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Critical test passed:
$\mathcal{F}t$ values consistent
$\chi^2/n = 0.6$

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CALCULATED CORRECTIONS TO $0^+ \rightarrow 0^+$ DECAYS

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1. Radiative corrections

$$\delta'_R = \frac{\alpha}{2\pi}\left[g(E_m) + \delta_2 + \delta_3 + \dots\right]$$

One-photon brem. + low-energy γW-box

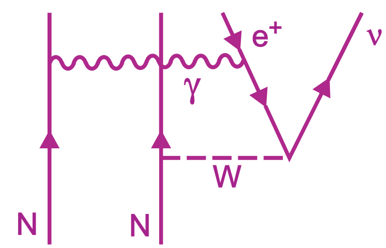
(terms of order: $g(E_m)$ ← α; δ_2 ← $Z\alpha^2$; δ_3 ← $Z^2\alpha^3$)

$$\Delta_R = \frac{\alpha}{2\pi}\left[4\ln(m_Z/m_p) + \ln(m_p/m_A) + 2C_{Born} + \dots\right]$$

High-energy γW-box
+ZW-box

universal ($\rightarrow 2C_{Born}$)

δ_{NS} Order-α axial-vector photonic contributions



Dependent on nuclear structure

2. Isospin symmetry-breaking corrections

δ_C Charge-dependent mismatch between parent and daughter analog states (members of the same isospin triplet).

ISOSPIN SYMMETRY BREAKING CORRECTIONS

$$\delta_C = \delta_{C1} + \delta_{C2}$$

δ_{C1}: Difference in configuration mixing between parent and daughter.

- Shell-model calculation with well-established 2-body matrix elements.
- Charge dependence tuned to known single-particle energies and to measured IMME coefficients.
- Results also adjusted to measured non-analog 0^+ state energies.

δ_{C2}: Mismatch in radial wave function between parent and daughter.

- Full-parentage Saxon-Woods wave functions for parent and daughter.
- Matched to known binding energies and charge radii as obtained from electron scattering.
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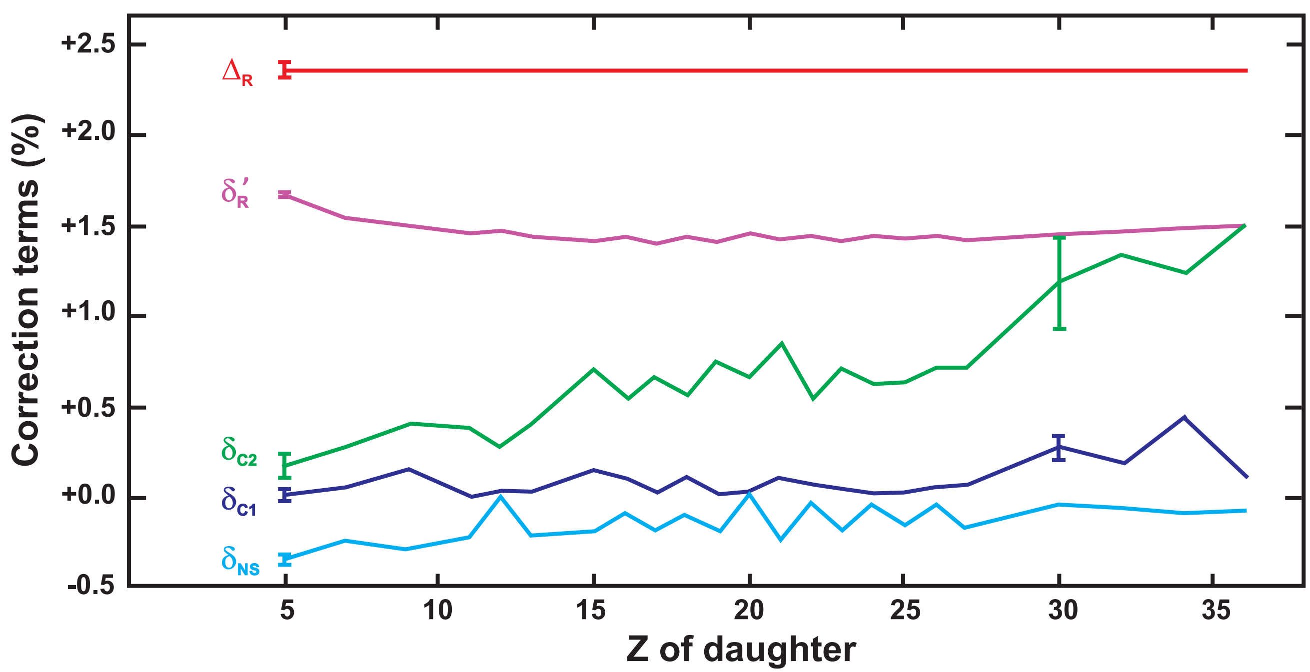

TESTS OF STRUCTURE-DEPENDENT CORRECTION TERMS

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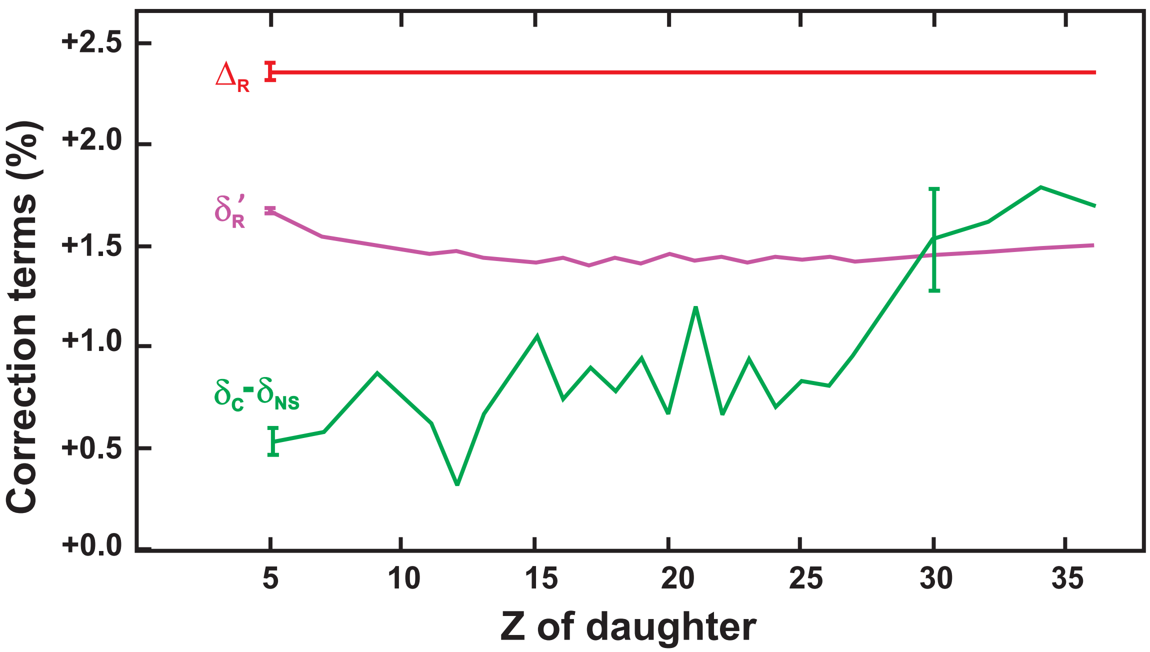

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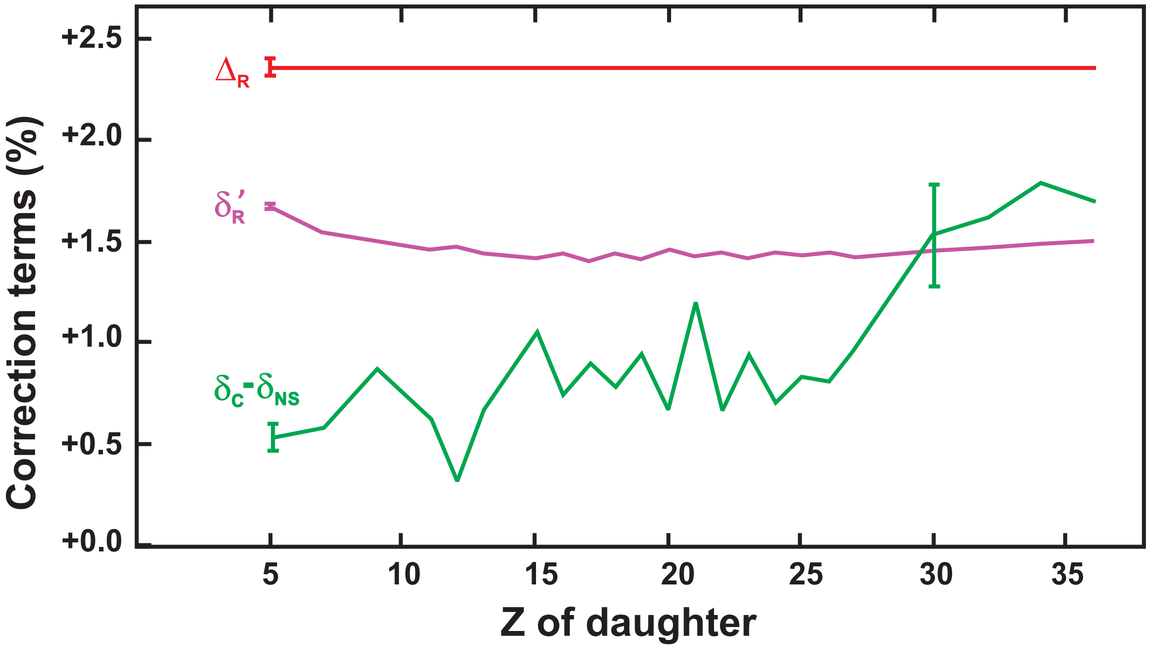

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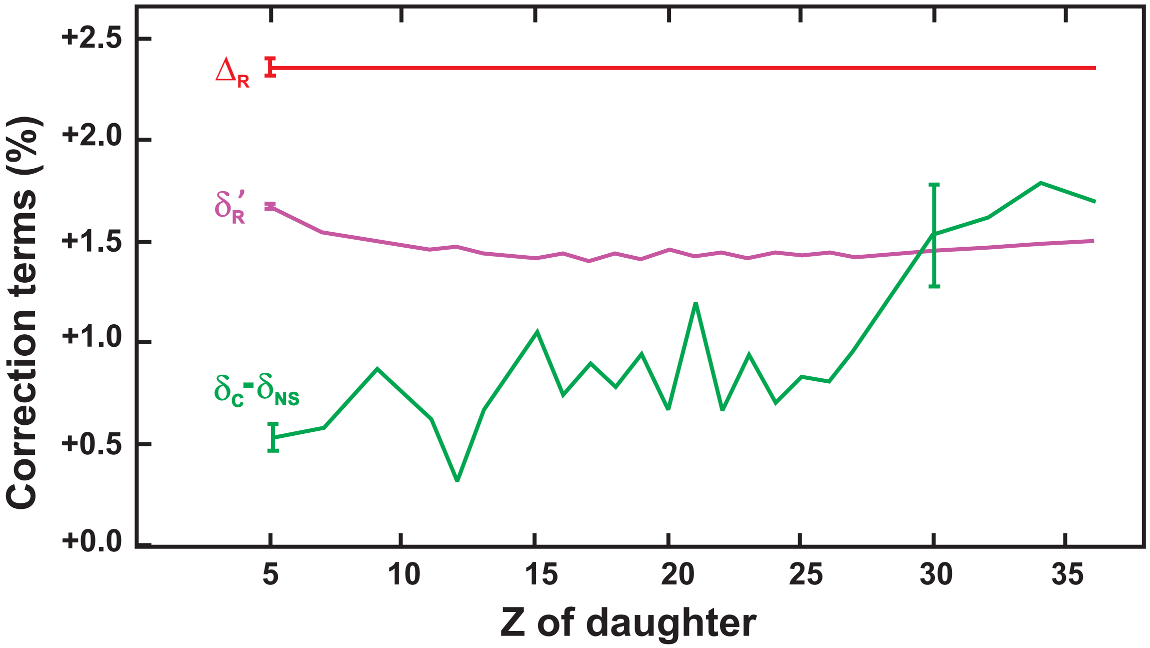

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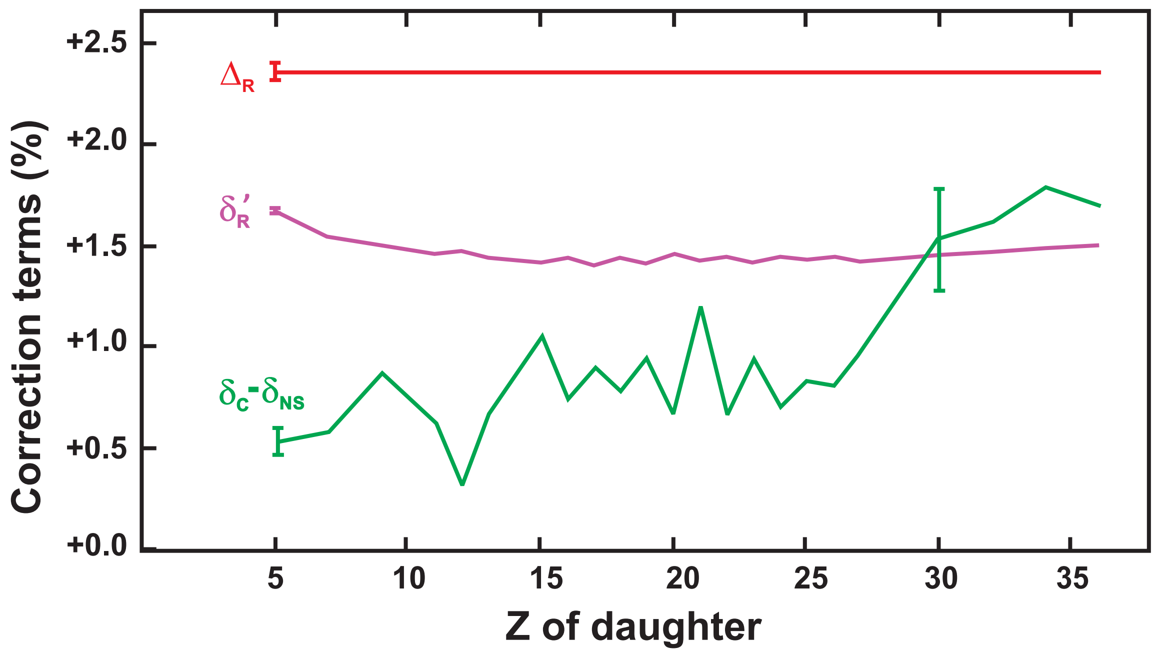

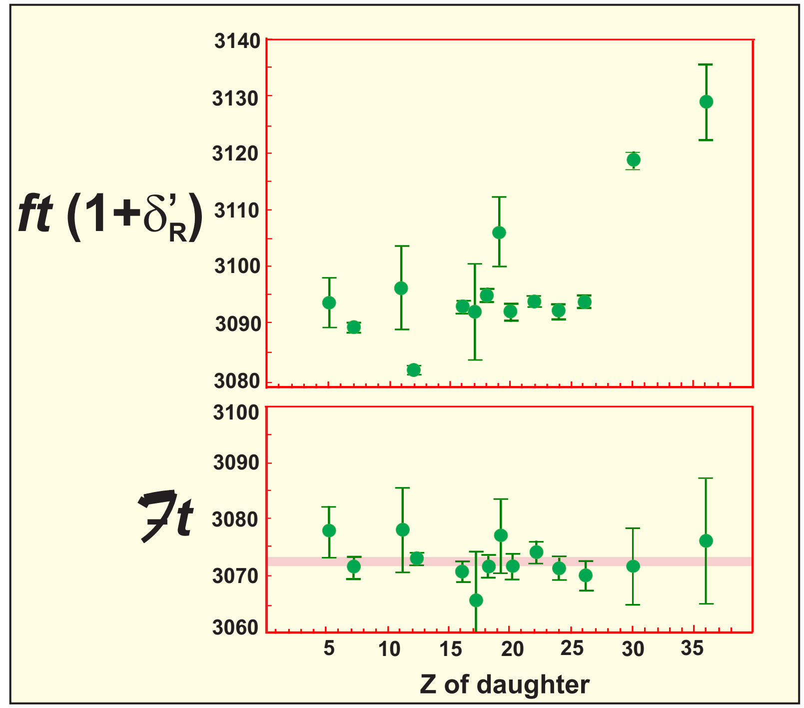

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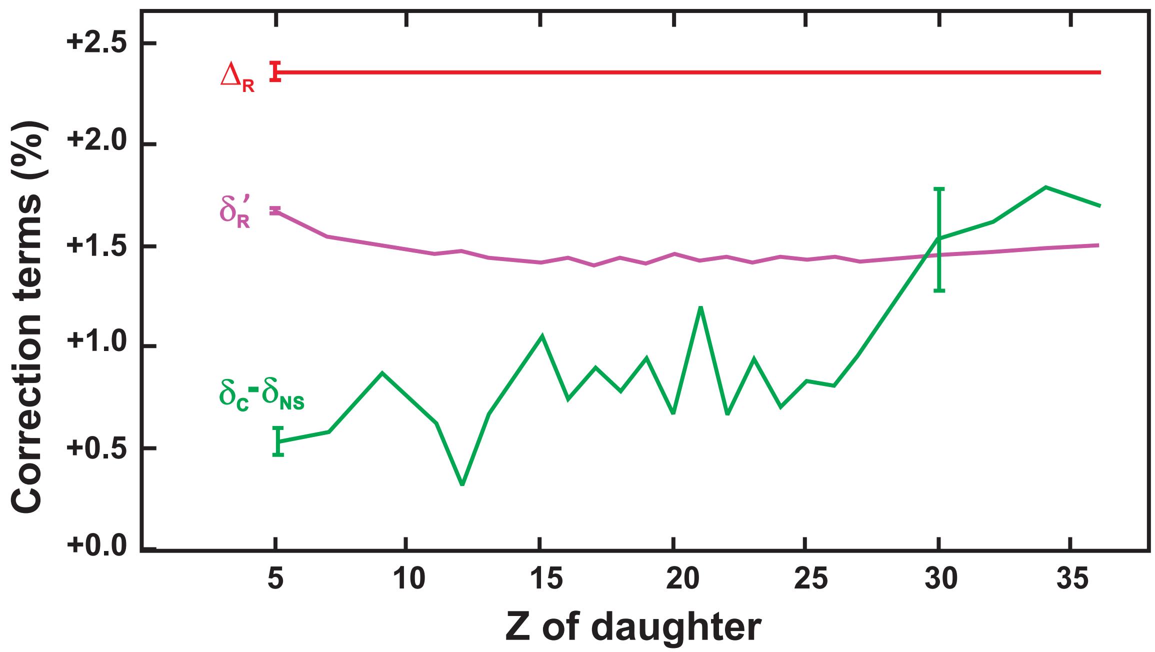

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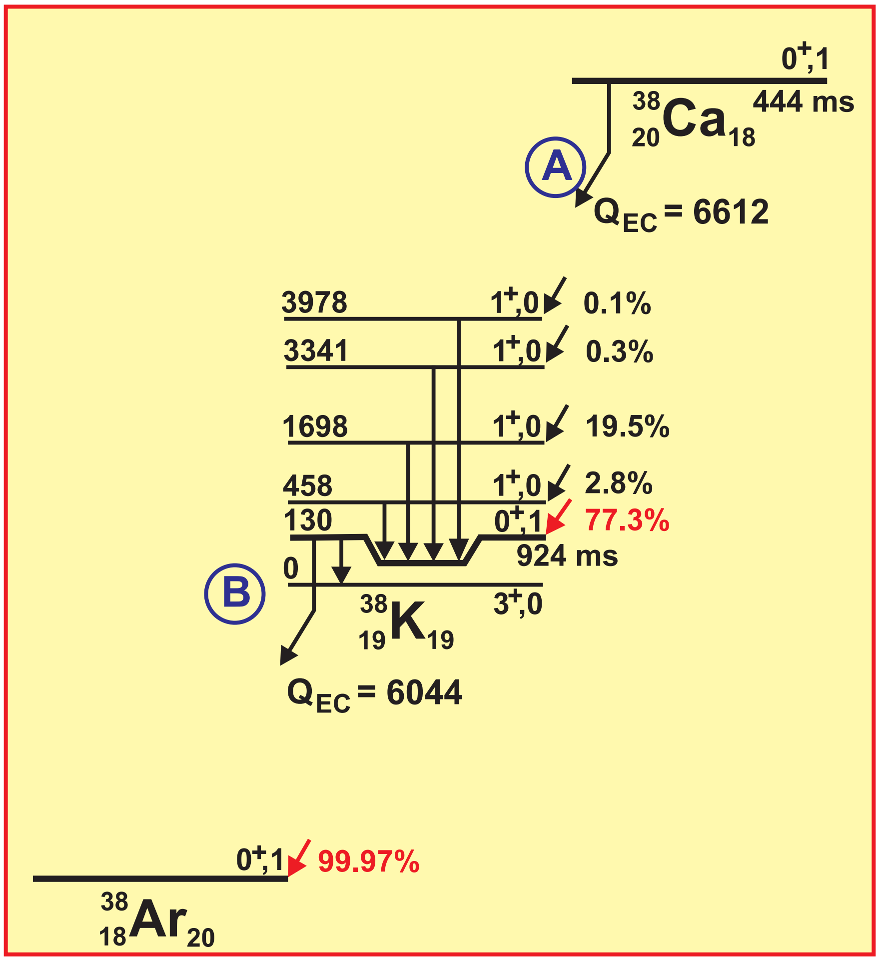
B. Measure the ratio of *ft* values for mirror $0^+ \rightarrow 0^+$ superallowed transitions and compare the results with calculations.

TESTS OF (δ_C-δ_{NS}) CALCULATIONS

B. Measurements of mirror superallowed transitions:



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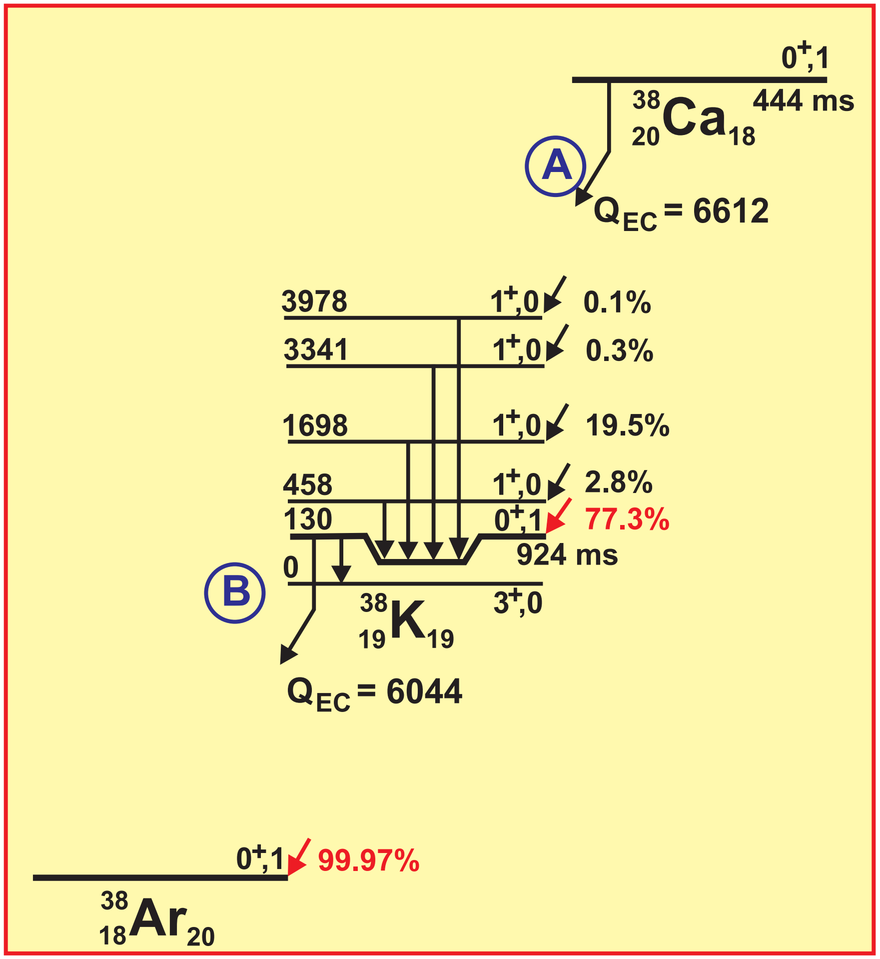

$$\frac{ft_A}{ft_B} = \frac{(1 + \delta_R'^B)[1 - (\delta_C^B - \delta_{NS}^B)]}{(1 + \delta_R'^A)[1 - (\delta_C^A - \delta_{NS}^A)]}$$

$$= 1 + (\delta_R'^B - \delta_R'^A) + (\delta_{NS}^B - \delta_{NS}^A) - (\delta_C^B - \delta_C^A)$$

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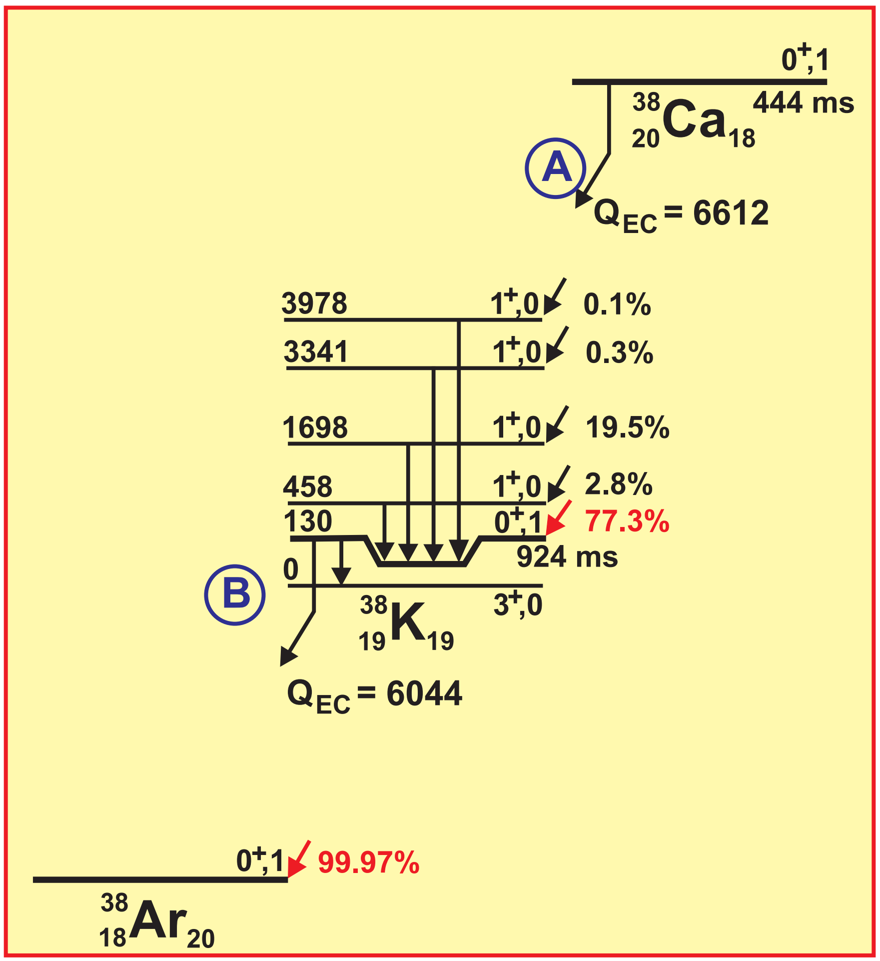

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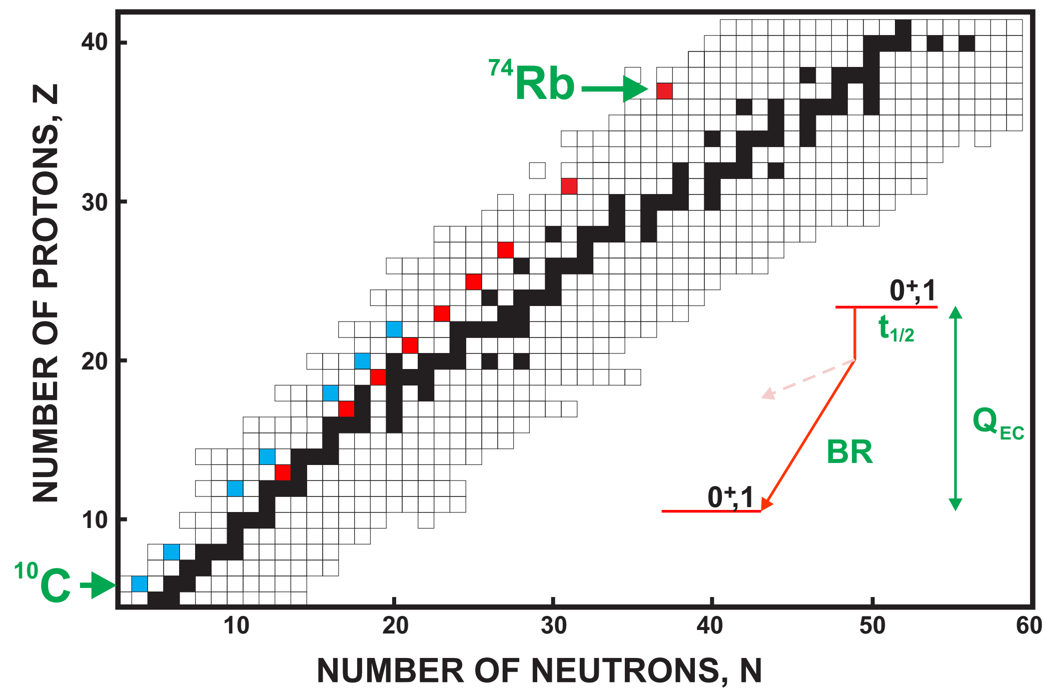

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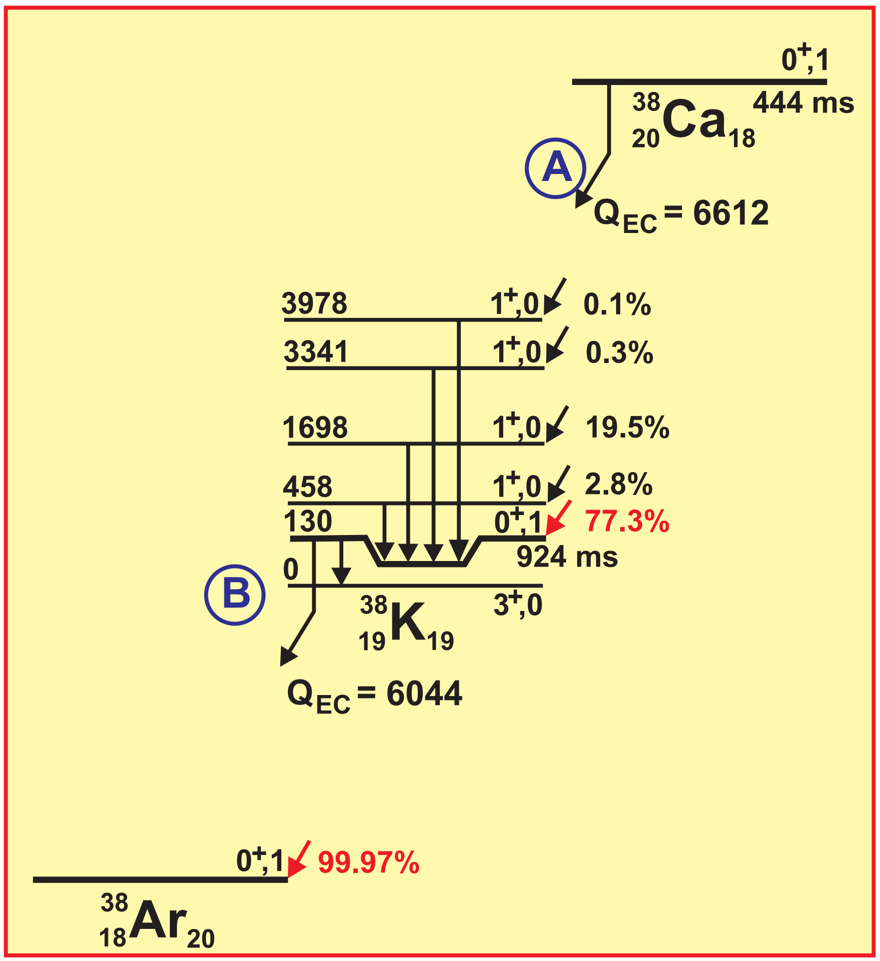

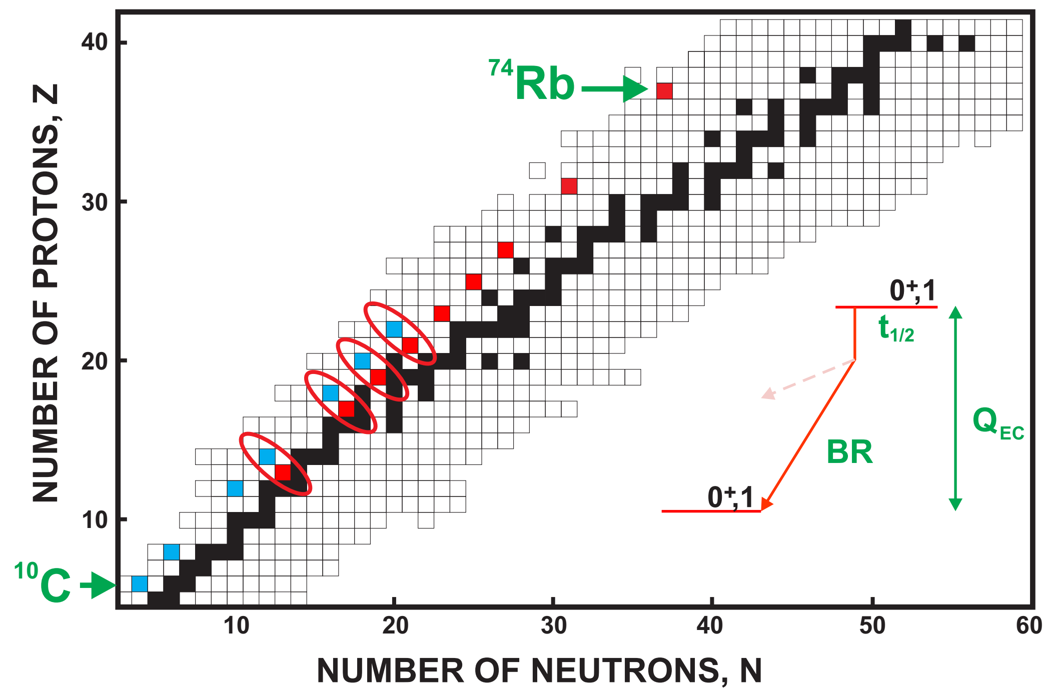

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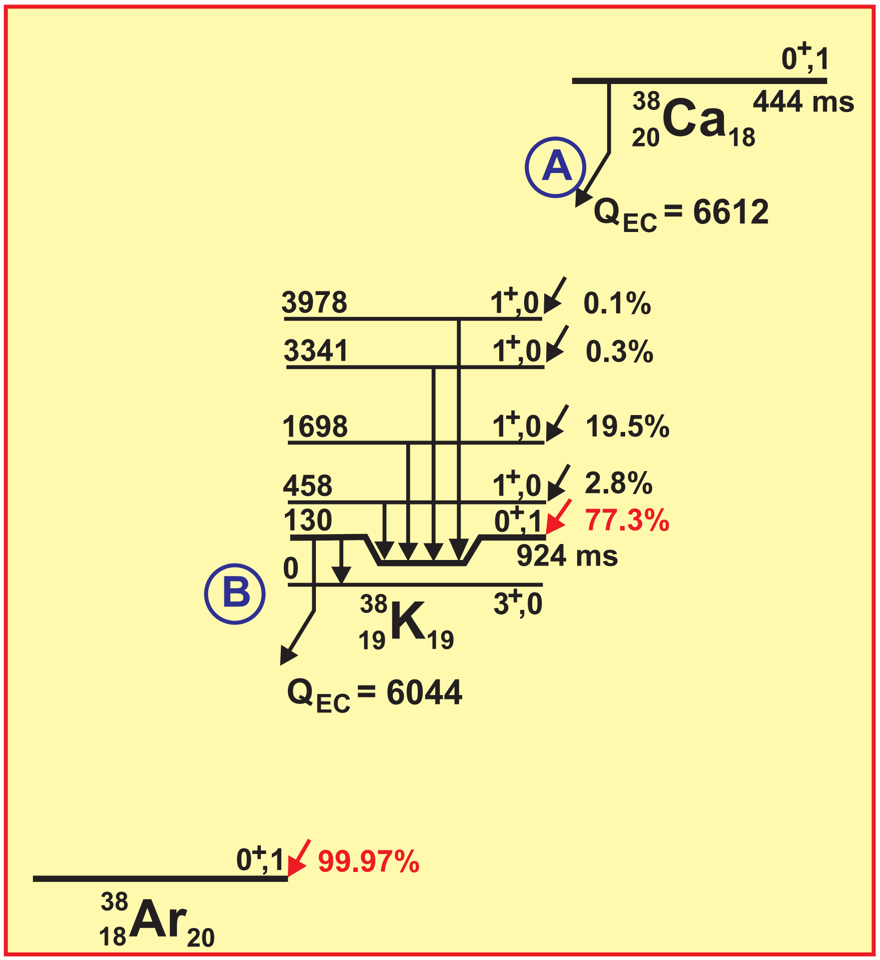

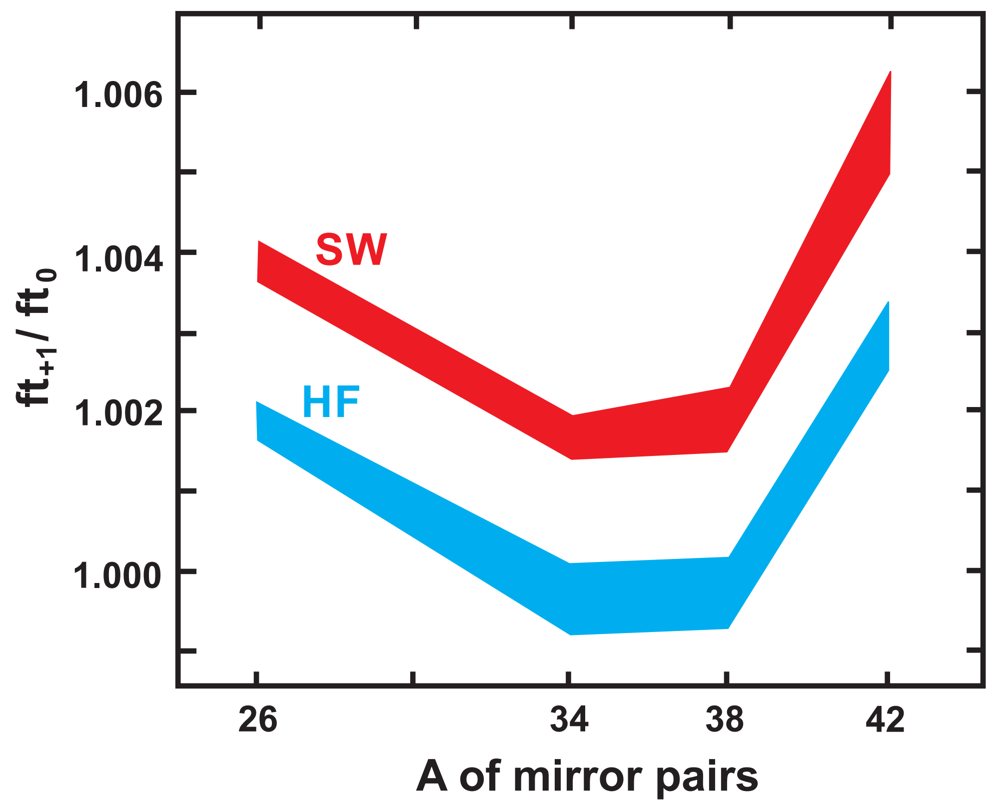

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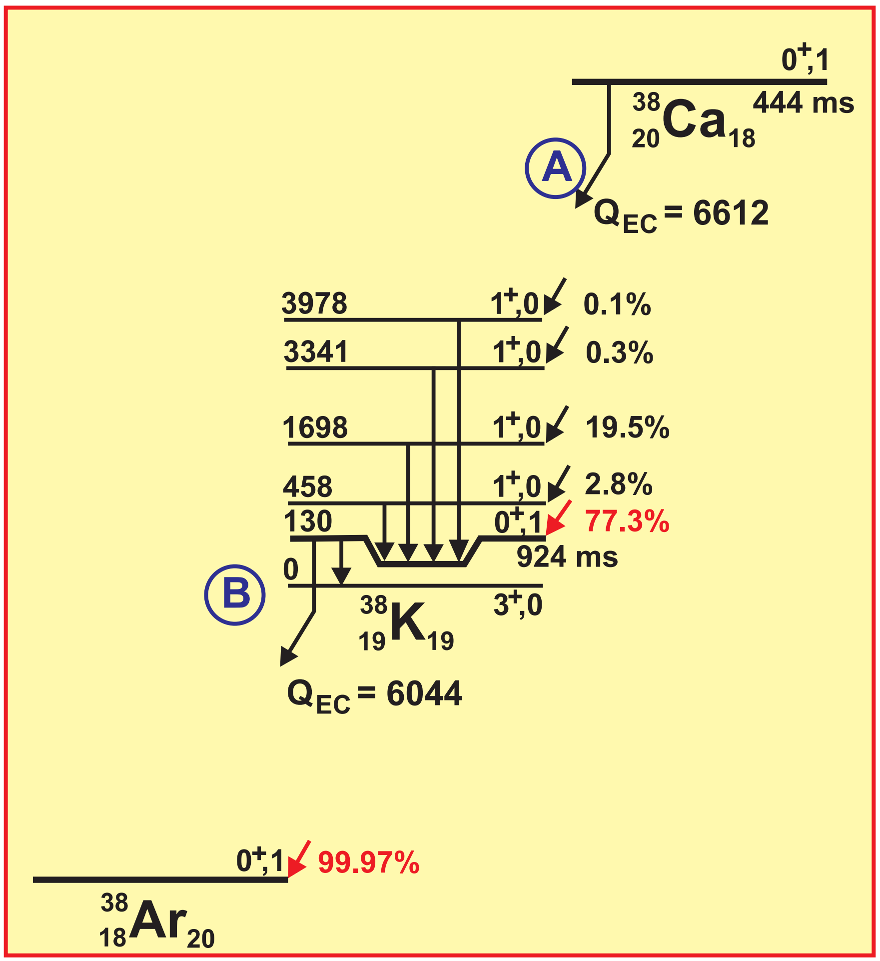
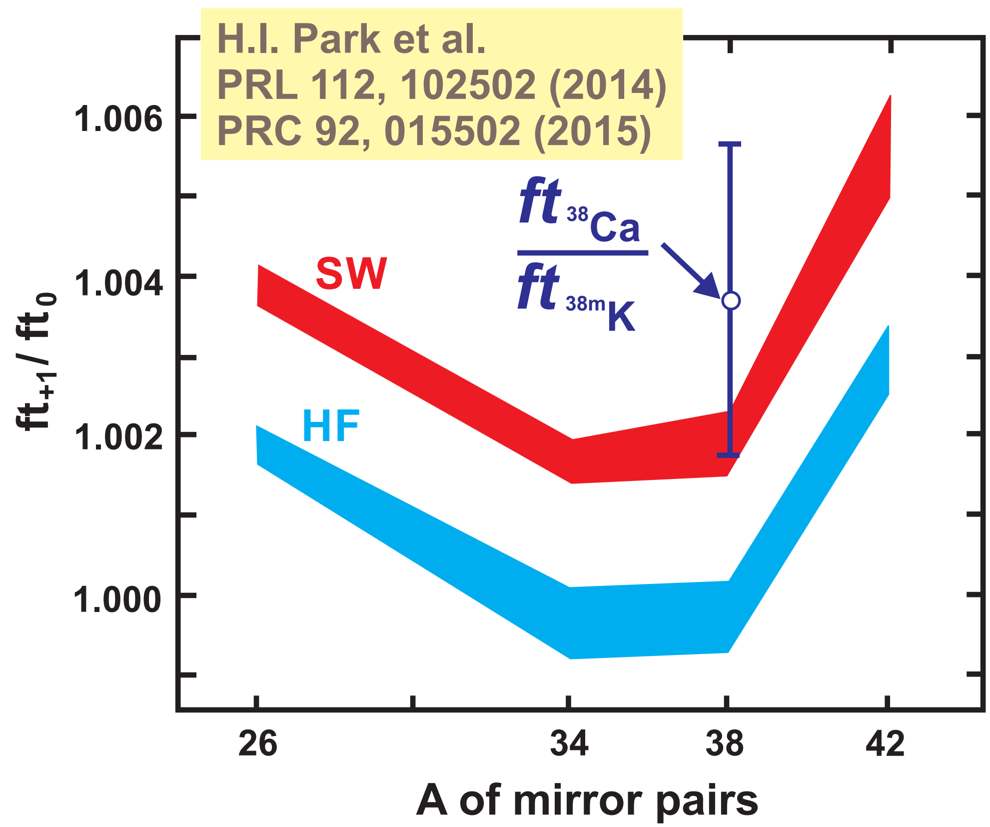
$$\frac{ft_A}{ft_B} = \frac{(1 + \delta_R'^B)[1 - (\delta_C^B - \delta_{NS}^B)]}{(1 + \delta_R'^A)[1 - (\delta_C^A - \delta_{NS}^A)]}$$

$$= 1 + (\delta_R'^B - \delta_R'^A) + (\delta_{NS}^B - \delta_{NS}^A) - (\delta_C^B - \delta_C^A)$$





TESTS OF $(\delta_C-\delta_{NS})$ CALCULATIONS

B. Measurements of mirror superallowed transitions:

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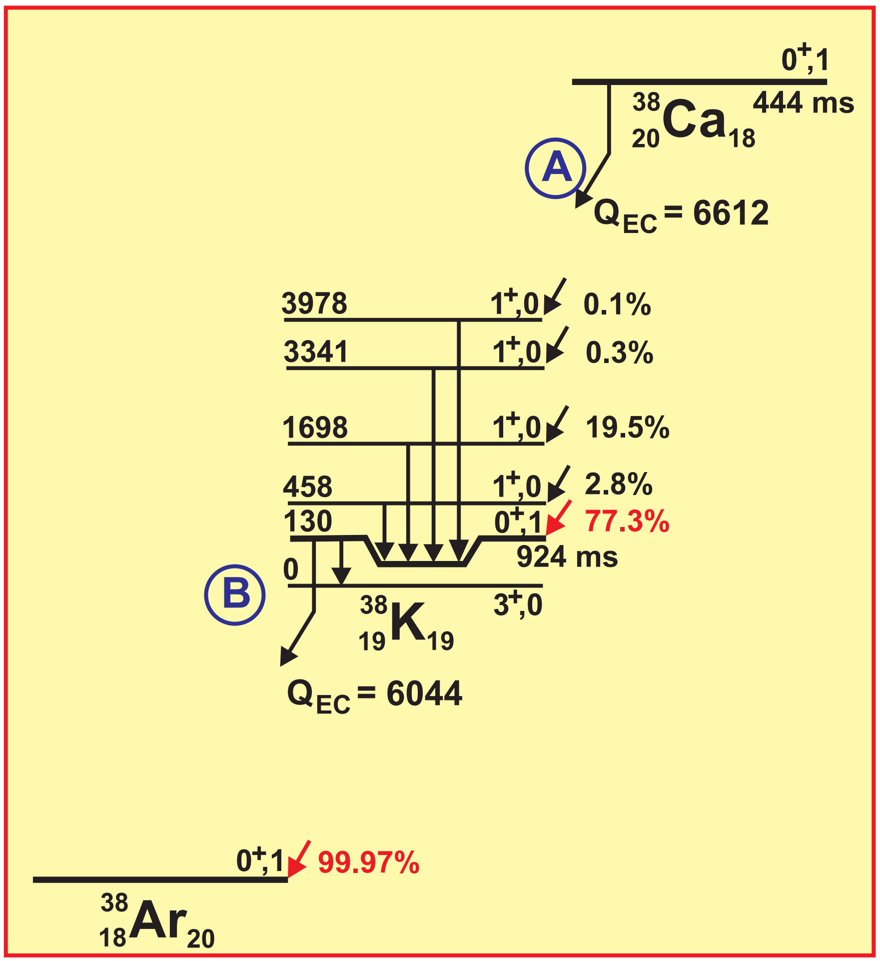

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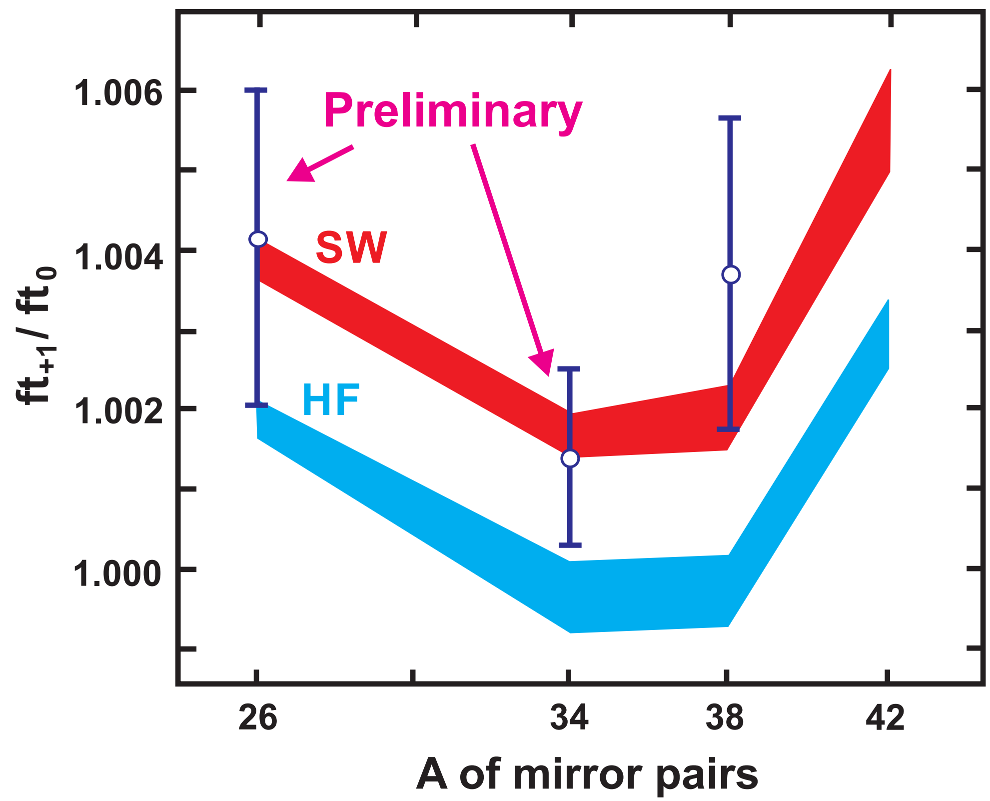


RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

Experimentally determine $G_V^2(1 + \Delta_R)$

$$\mathcal{F}t = ft\,(1 + \delta'_R)\left[1 - (\delta_C - \delta_{NS})\right] = \frac{K}{2G_V^2\,(1 + \Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

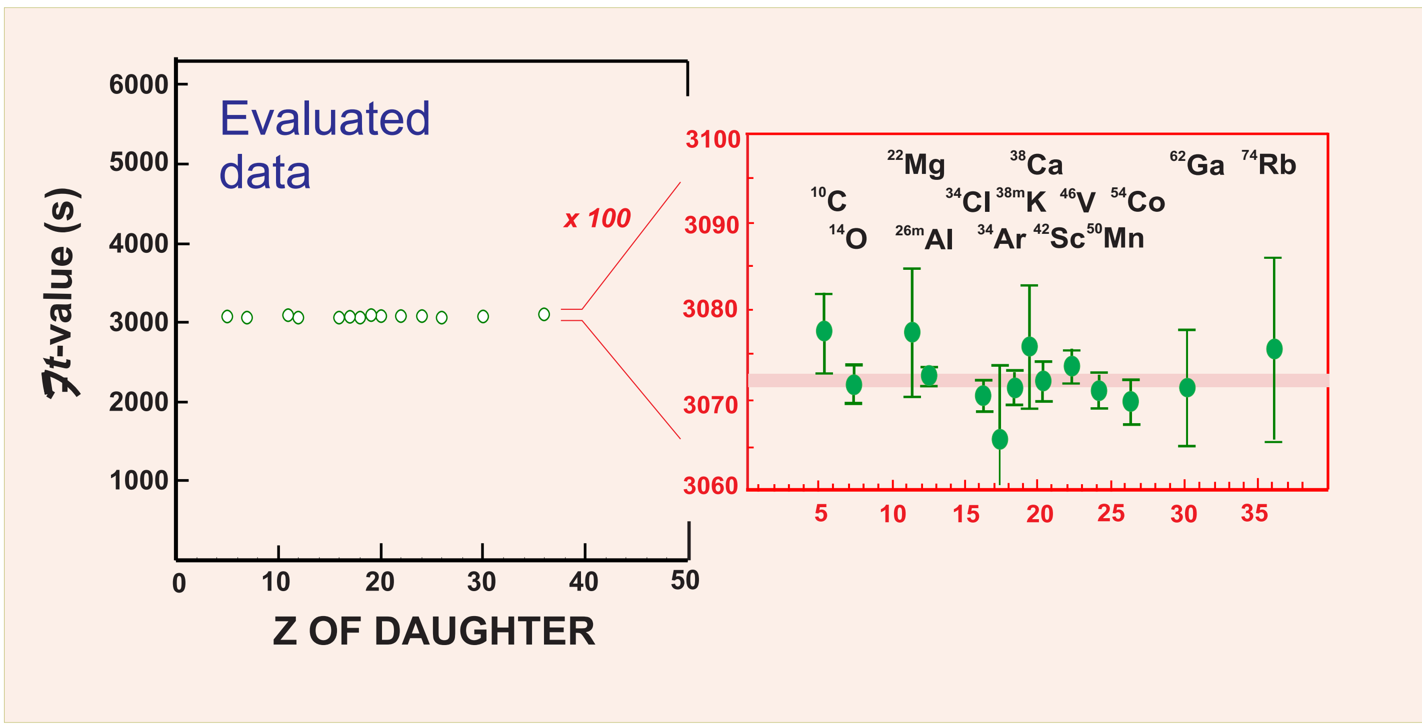
Experimentally determine $G_V^2(1+\Delta_R)$

$$\mathcal{F}t = ft\,(1+\delta_R')[1-(\delta_C-\delta_{NS})] = \frac{K}{2G_V^2\,(1+\Delta_R)}$$

FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

G_V constant to ± 0.011%



$\overline{\mathcal{F}t}$ = 3072.1(7)

$G_V(1+\Delta_R)^{1/2}/(\hbar c)^3$ = 1.14962(13) $\times 10^{-5}$ GeV^{-2}

$\chi^2/\nu = 0.6$

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

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Experimentally determine $G_V^2(1 + \Delta_R)$

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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms

G_V constant to ± 0.011%

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

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Experimentally determine $G_V^2(1 + \Delta_R)$

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Test Conservation of the Vector current (CVC)

Validate correction terms ✔

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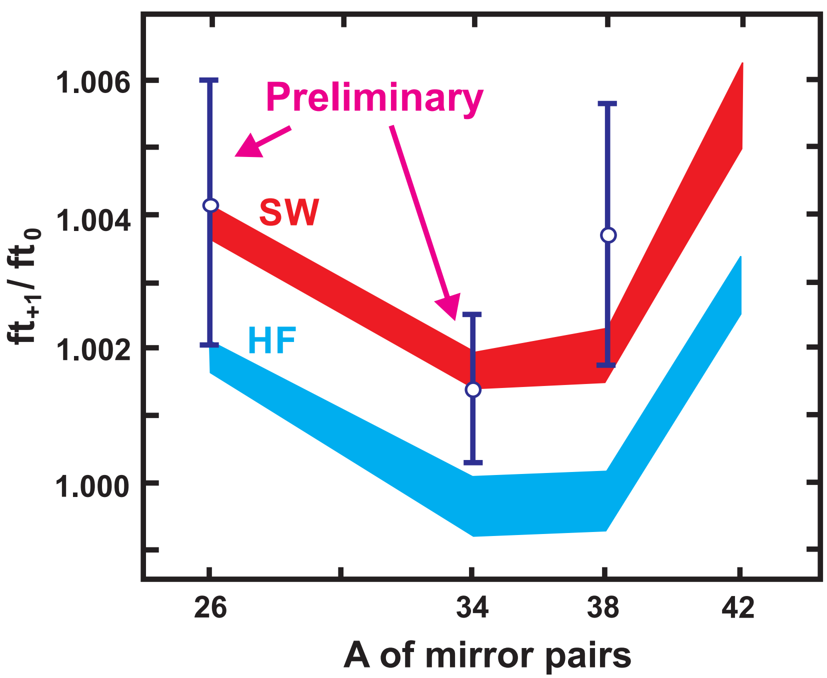

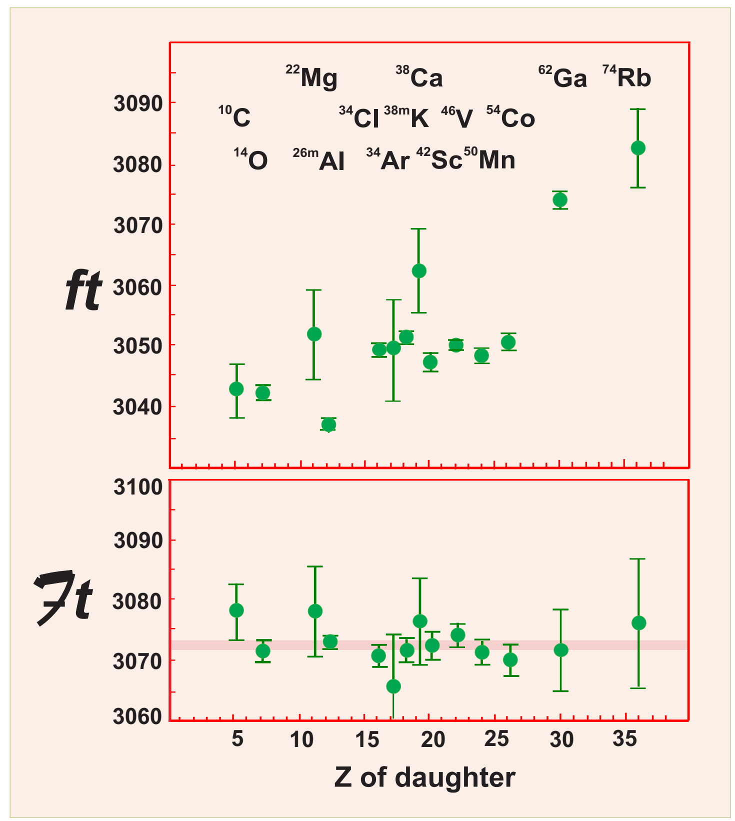

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FROM MANY TRANSITIONS

Test Conservation of the Vector current (CVC)

Validate correction terms ✔

Test for Scalar current

G_V constant to ± 0.011%

RESULTS FROM $0^+ \rightarrow 0^+$ DECAY

FROM A SINGLE TRANSITION

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FROM MANY TRANSITIONS

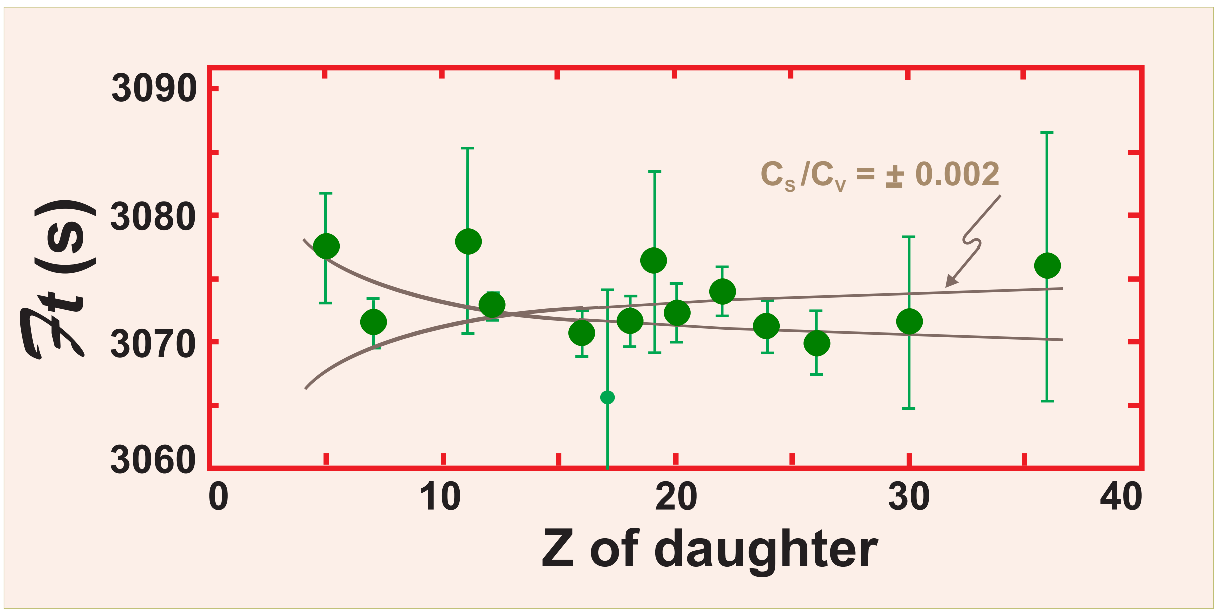
Test Conservation of the Vector current (CVC)

G_V constant to ± 0.011%

Validate correction terms ✔

Test for Scalar current

limit, C_S/C_V = 0.0012 (10)



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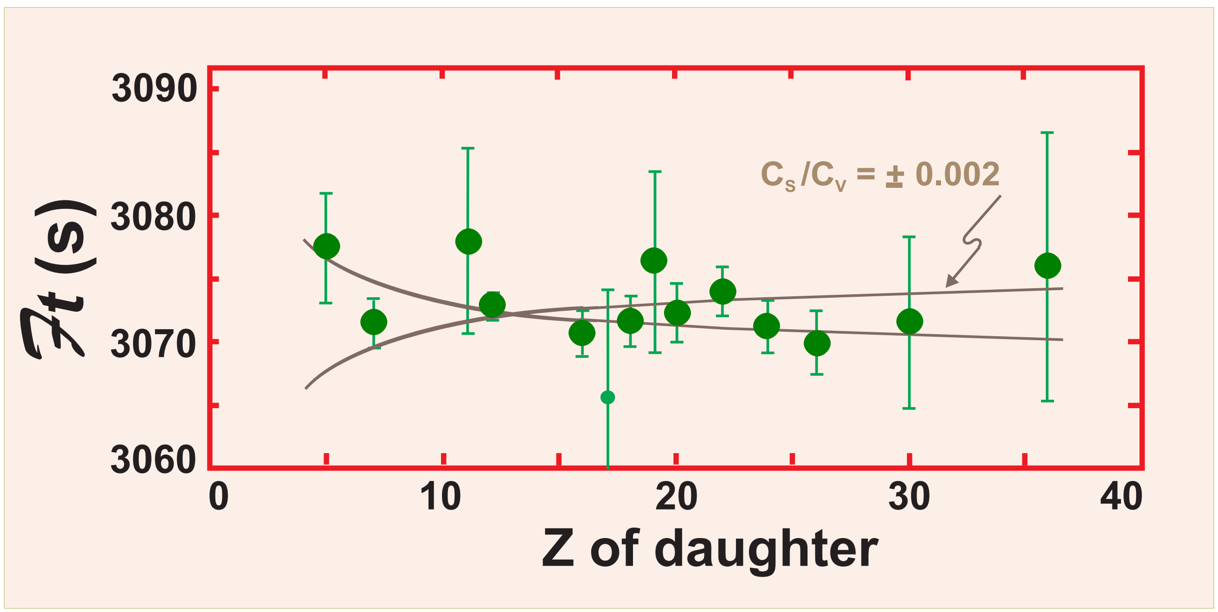

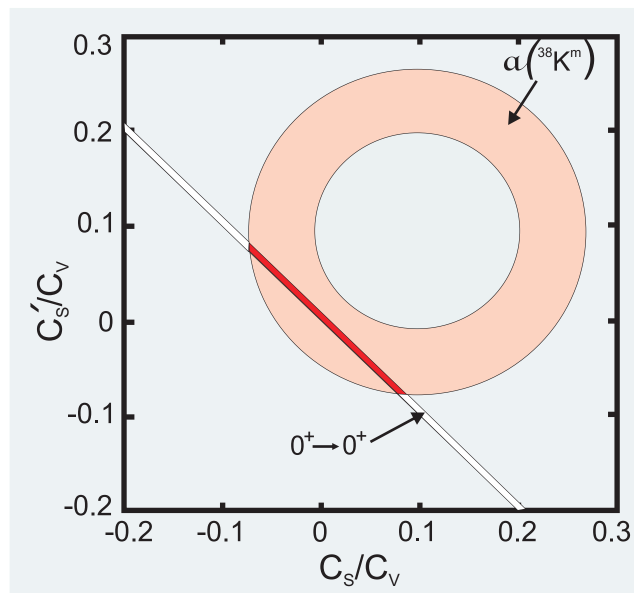

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WITH CVC VERIFIED

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates

mass eigenstates

Cabibbo-Kobayashi-Maskawa matrix

Obtain precise value of $G_V^2(1 + \Delta_R)$

Determine V_{ud}^2

$$V_{ud}^2 = G_V^2/G_\mu^2 = 0.94906 \pm 0.00041$$

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weak eigenstates (d', s', b') — mass eigenstates (d, s, b)

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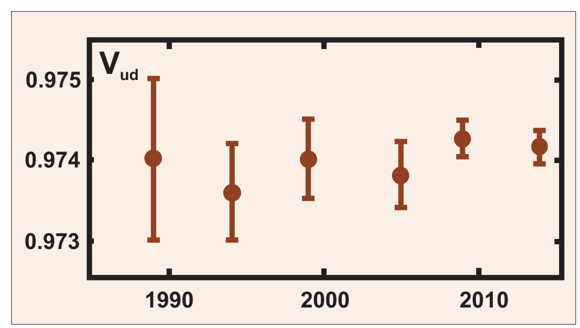

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Determine V_{ud}^2

$$V_{ud}^2 = G_V^2/G_\mu^2 = 0.94906 \pm 0.00041$$

Test CKM unitarity

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99962 \pm 0.00049$$

T=1/2 SUPERALLOWED BETA DECAY

BASIC WEAK-DECAY EQUATION

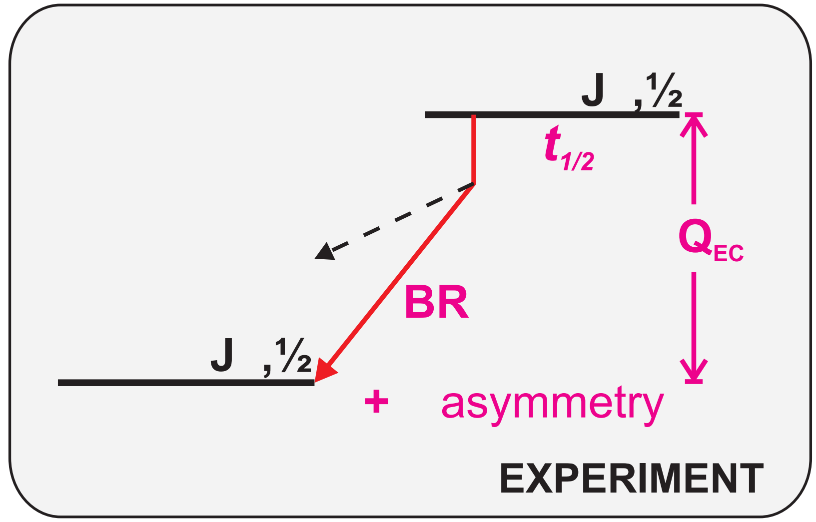
$$ft = \frac{K}{G_V^2 \langle \quad \rangle^2 + G_A^2 \langle \quad \rangle^2}$$

f = statistical rate function: f(Z, Q_{EC})

t = partial half-life: f($t_{1/2}$, BR)

$G_{V,A}$ = coupling constants

< > = Fermi, Gamow-Teller matrix elements



INCLUDING RADIATIVE CORRECTIONS

$$\mathcal{F}t = ft\,(1 + \;'_R)[1 - (\;_C - \;_{NS})] = \frac{K}{G_V^2\,(1 + \;_R)(1 + \;^2 \langle \quad \rangle^2)}$$

$= G_A/G_V$

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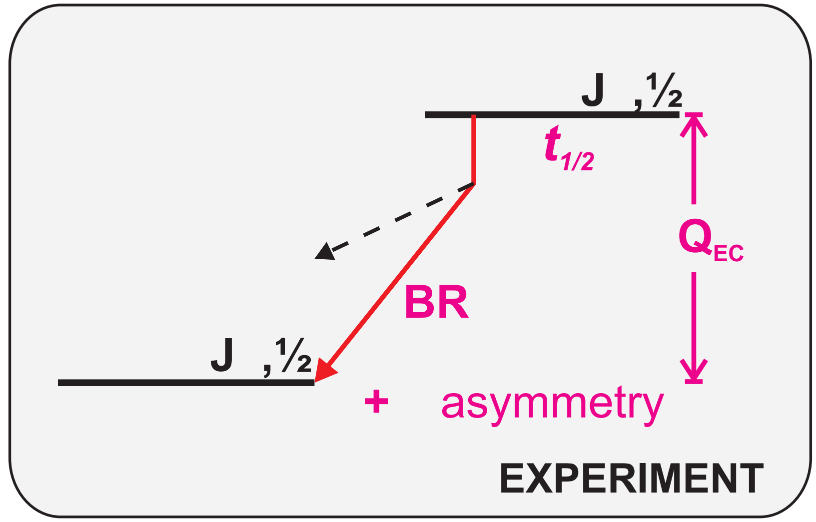

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Requires additional experiment:
for example, asymmetry (*A*)

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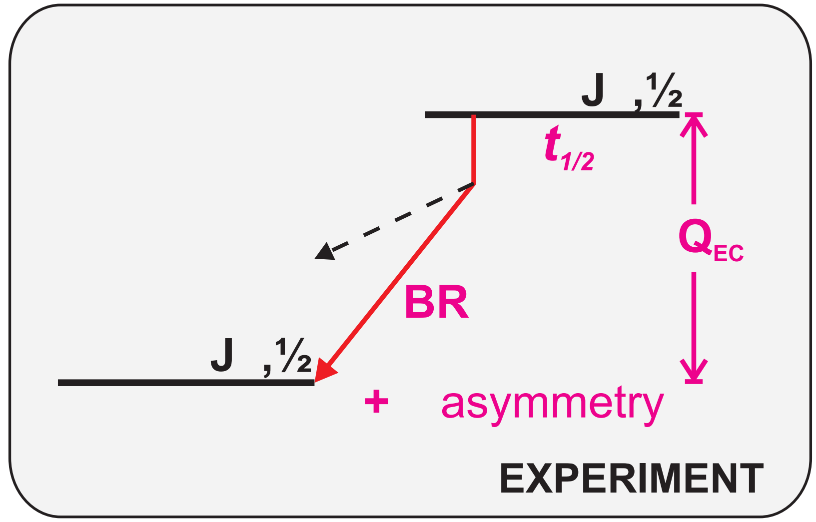

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NEUTRON DECAY

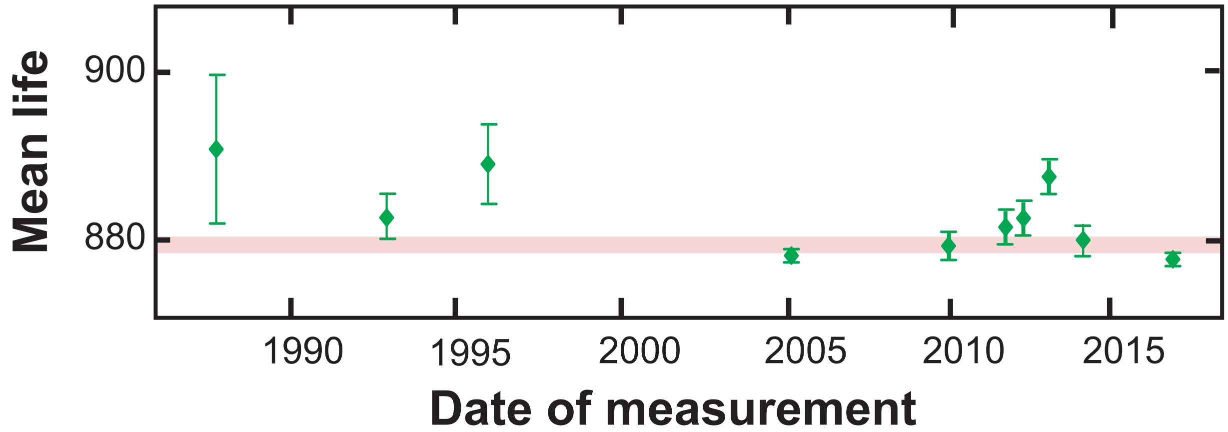
Requires additional experiment:
for example, asymmetry (*A*)

NEUTRON DECAY DATA 2017

Mean life:

τ = **879.4 ± 0.9 s**

χ^2/N = **4.2**



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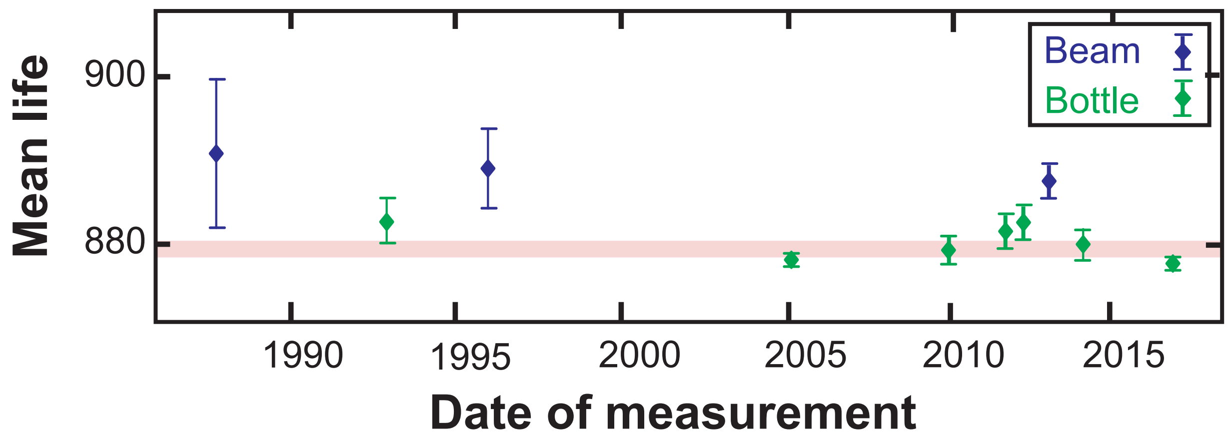

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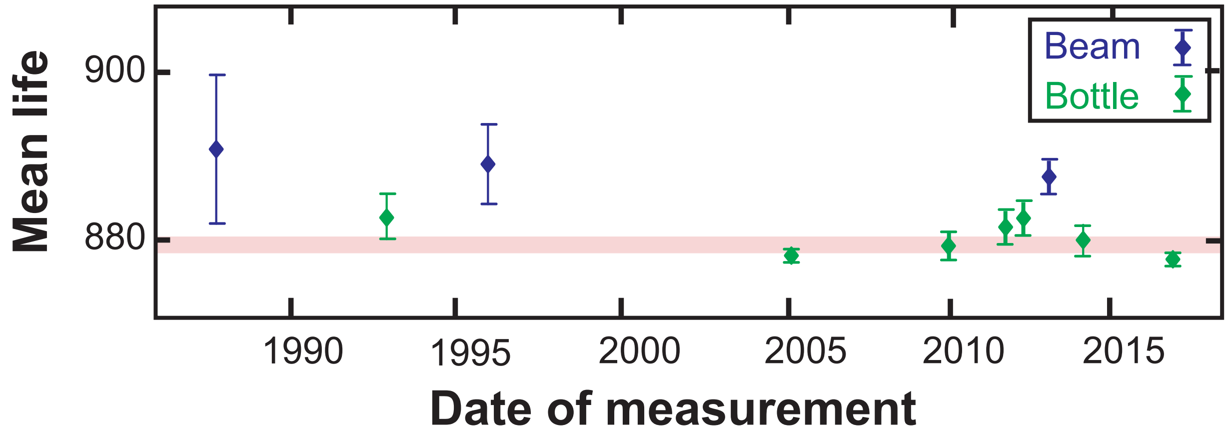
Bottle: 878.9 ± 0.6 s



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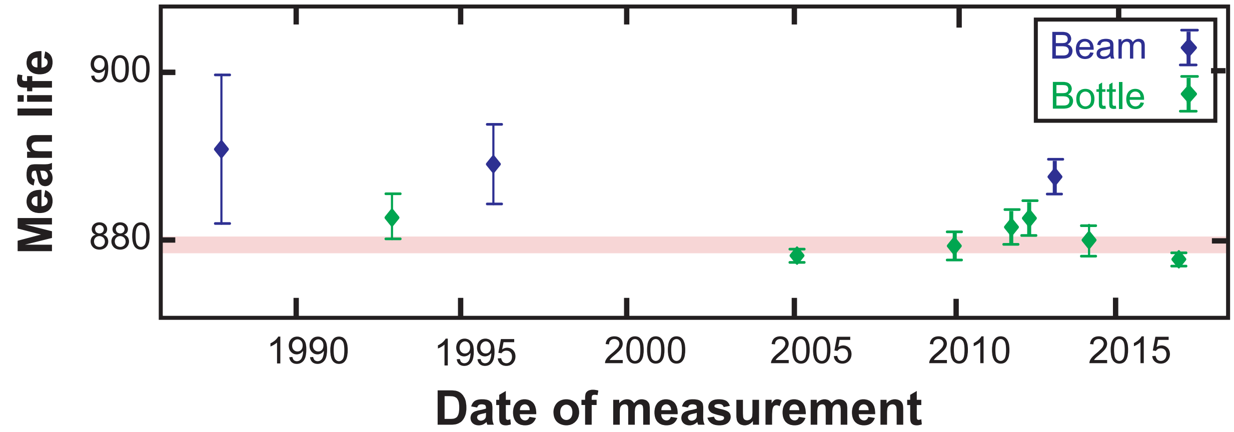
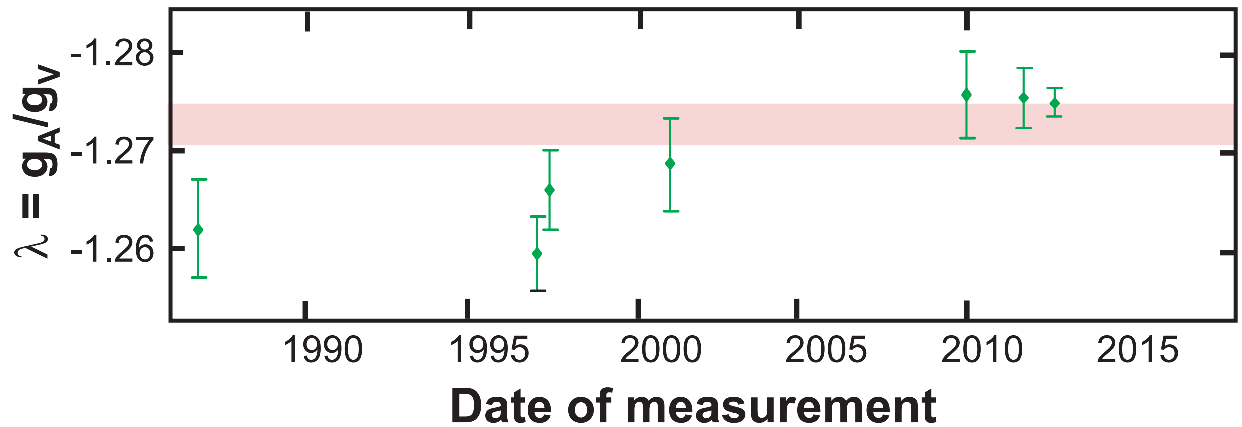


β **asymmetry:**

λ **= -1.2725 ± 0.0020**

χ^2**/N = 4.1**



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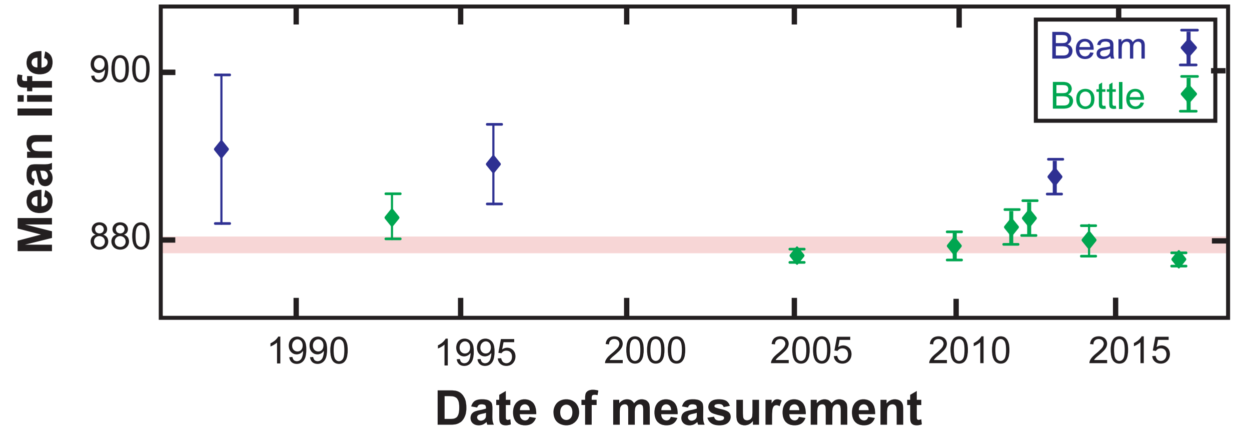

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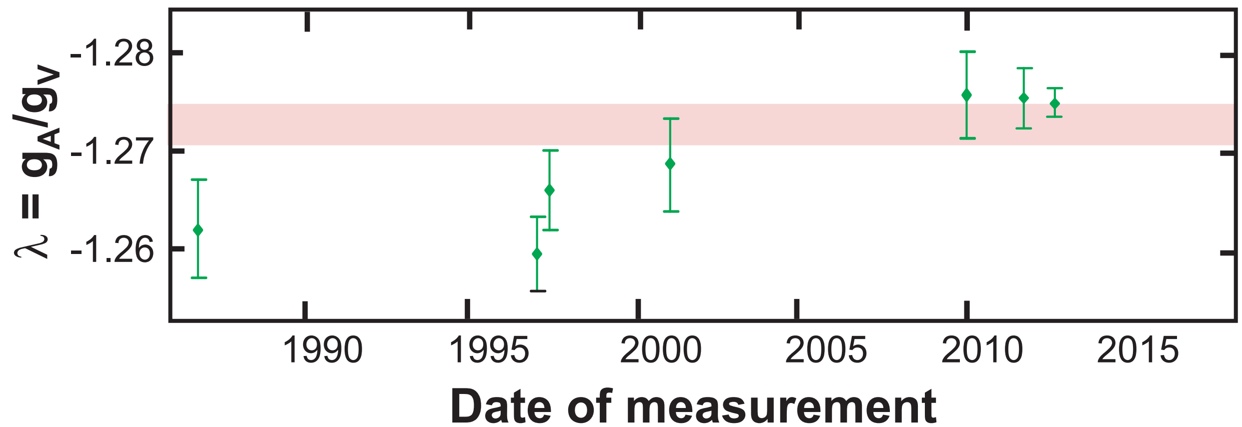


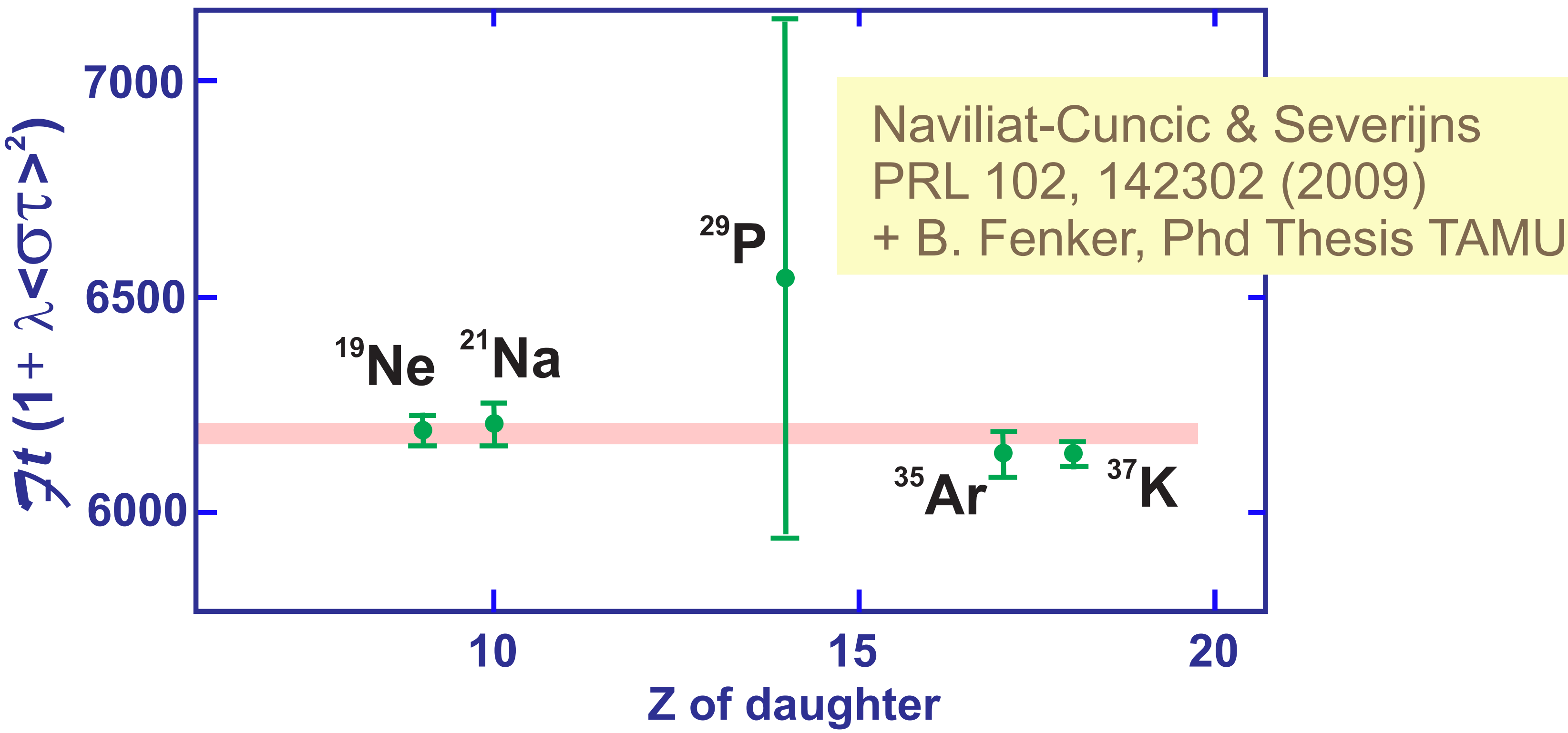
$V_{ud} = 0.9762 \pm 0.0014$

Beam-bottle span
$0.9700 \leq V_{ud} \leq 0.9770$

nuclear $0^+ \rightarrow 0^+$
$V_{ud} = 0.9742 \pm 0.0002$

NUCLEAR T=1/2 MIRROR DECAY DATA 2009

$$\mathcal{F}t = ft\,(1 + \delta_R')[1 - (\delta_C - \delta_{NS})] = \frac{K}{G_V^2\,(1 + \Delta_R)(1 + \lambda^2 \langle\sigma\tau\rangle^2)}$$



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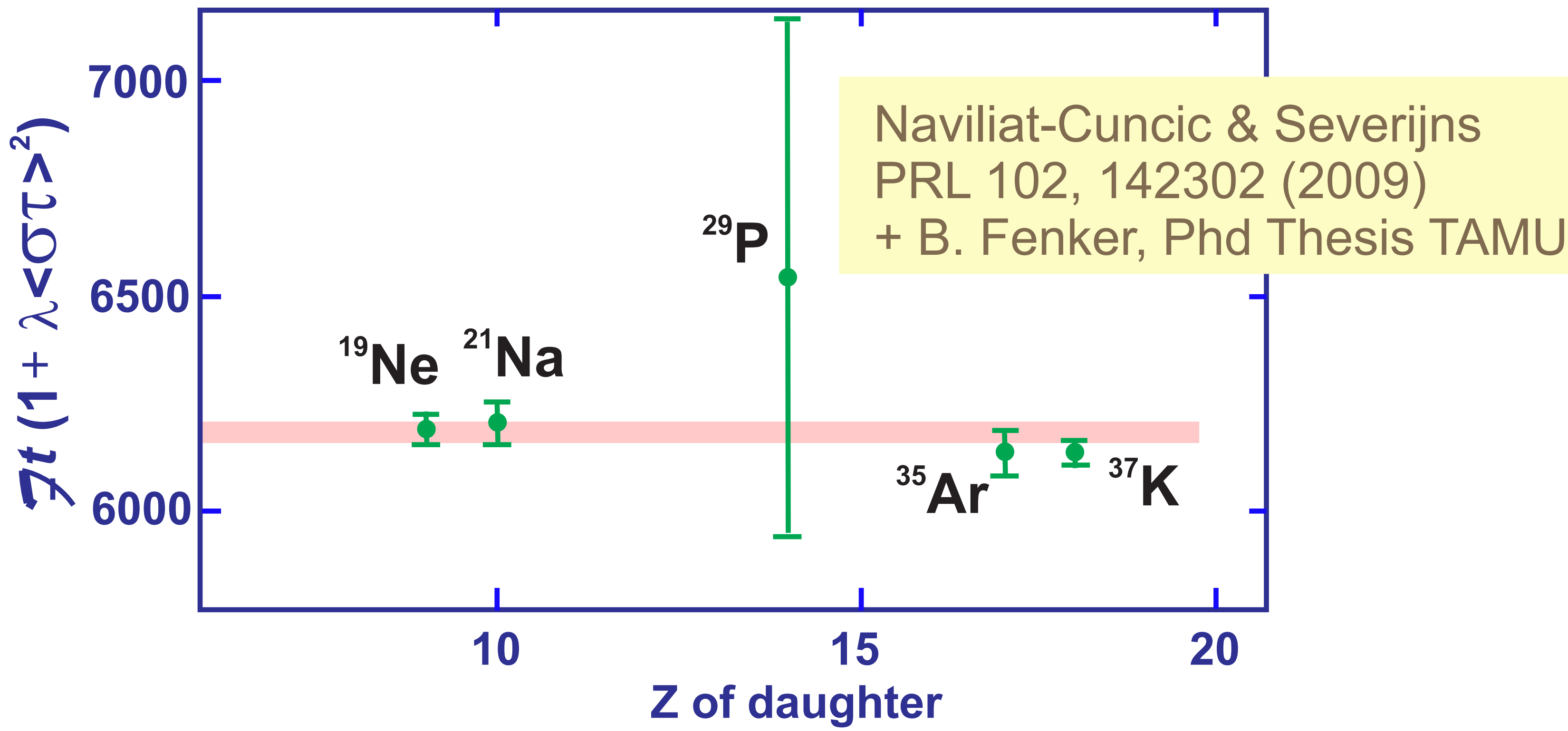


$V_{ud} = 0.9730 \pm 0.0014$

nuclear $0^+ \rightarrow 0^+$

$V_{ud} = 0.9742 \pm 0.0002$

PION BETA DECAY

Decay process:

$$\pi^+ \longrightarrow \pi^0 e^+ \nu_e$$

$$0^-,1 \longrightarrow 0^-,1$$

Experimental data:

$$\tau = 2.6033 \pm 0.0005 \times 10^{-8}\,\mathrm{s}$$ (PDG 2017)

$$BR = 1.036 \pm 0.007 \times 10^{-8}$$

Pocanic *et al*,
PRL 93, 181803 (2004)

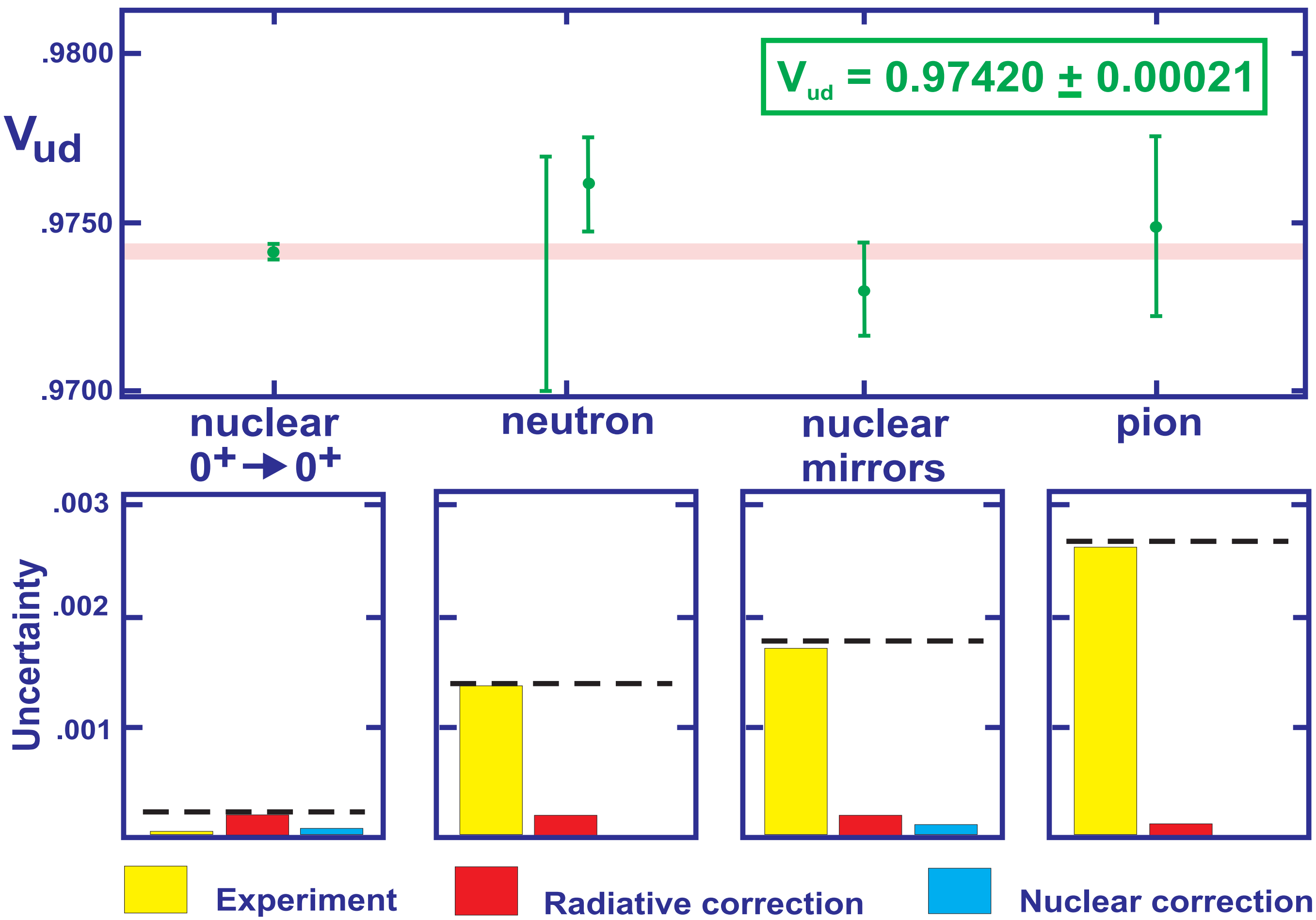
Result:

$$V_{ud} = 0.9749 \pm 0.0026$$

nuclear $0^+ \rightarrow 0^+$

$V_{ud} = 0.9742 \pm 0.0002$

CURRENT STATUS OF V_{ud} AND CKM UNITARITY



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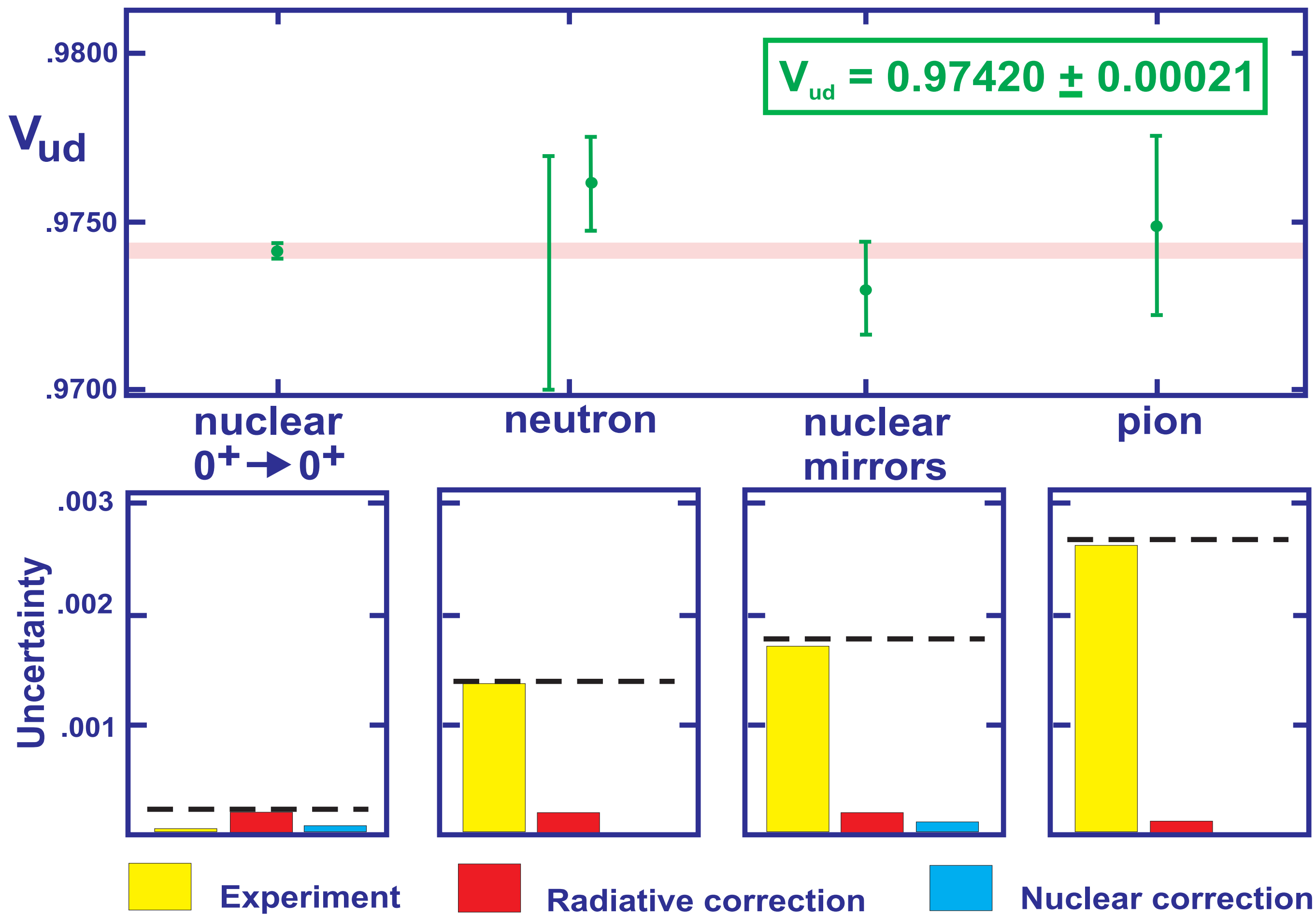

$$V_{ud}^2 + V_{us}^2 + V_{ub}^2 = 0.99962 \pm 0.00049$$

V_{ud}^2 nuclear decays, muon decay: **0.94906 ± 0.00041**

V_{us}^2 PDG, kaon decays: **0.05054 ± 0.00027**

V_{ub}^2 B decays: **0.00002**

SUMMARY AND OUTLOOK

1. Analysis of superallowed $0^+ \rightarrow 0^+$ nuclear β decay confirms CVC to ±0.011% and thus yields V_{ud} = 0.97420(21).
2. The three other experimental methods for determining V_{ud} yield consistent results, but are less precise by a factor of 7 or more.
3. The current value for V_{ud}, when combined with the PDG values for V_{us} and V_{ub}, satisfies CKM unitarity to ±0.05%.

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1. Analysis of superallowed $0^+ \rightarrow 0^+$ nuclear β decay confirms CVC to ±0.011% and thus yields V_{ud} = 0.97420(21).
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3. The current value for V_{ud}, when combined with the PDG values for V_{us} and V_{ub}, satisfies CKM unitarity to ±0.05%.

4. The largest contribution to V_{ud} uncertainty is from the inner radiative correction, Δ_R. Very little reduction in V_{ud} uncertainty is possible without improved calculation of Δ_R.
5. Isospin symmetry-breaking correction, δ_C, has been tested by requiring consistency among the 14 known transitions (CVC), and agreement with mirror-transition pairs. It contributes much less to V_{ud} uncertainty than does Δ_R.
6. With significant improvement in Δ_R uncertainty alone, the V_{ud} uncertainty could be reduced by factor of 2!

Supplementary slides

FINAL REMARK ON V_{us}

Kaon decay yields two independent determinations of V_{us}:

1) Semi-leptonic $K \rightarrow \pi \ell \nu_\ell$ decay ($K_{\ell 3}$) yields $|V_{us}|$.

2) Pure leptonic decays $K^+ \rightarrow \mu^+ \nu_\mu$ and $\pi^+ \rightarrow \mu^+ \nu_\mu$ together yield $|V_{us}| / |V_{ud}|$.

Both require lattice calculations of form factors to obtain their result.

Until March 2014 these gave highly consistent results for $|V_{us}|$.

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Until March 2014 these gave highly consistent results for $|V_{us}|$.

BUT, Bazavov et al. [PRL 112, 112001 (2014)] produced a new lattice calculation of the form factor used for $K_{\ell 3}$ decays.

Their new result for $|V_{us}|$ is inconsistent with the $|V_{us}|/|V_{ud}|$ result and, when combined with the superallowed result for $|V_{ud}|$, leads to a unitarity sum over two standard deviations below 1.

Stay tuned ...

TESTS OF δ_c CALCULATIONS

A. Agreement with CVC:

$\mathcal{F}t$ values have been calculated with different models for δ_C, then tested for consistency. No theoretical uncertainties are included. Normalized χ^2 and confidence levels are shown.

Model	χ^2/N	CL(%)
SM-SW	1.37	17
SM-HF	6.38	0
DFT	4.26	0
RHF-RPA	4.91	0
RH-RPA	3.68	0