

BRUSSELS

- Thank the organisers
- Waste with Sasha Haco, Stephen Hawking, Andy Strominger.

Can soft hair account for black hole entropy —

Why do believe in black hole entropy

- (i) 2nd law $dA \geq 0$
- (ii) $dM = \kappa dA/8\pi + \oint dQ + \Omega dJ$ together with $T = \kappa/2\pi \Rightarrow S = A/4$

(iii) The Euclidean calculation of Gibbons + Hawking.
 Here the horizon has been turned into an S^2 effectively hiding over anything inside the horizon.

Will come back to other approaches at the end

lets look at the covariant phase space

(Peierls, Ashtekar + ..., Witten + ..., Zuckerman, Wald + ...)

Start from the action

$$I = \frac{1}{16\pi} \int_m R + \frac{1}{8\pi} \int_m K$$

Integrate by parts $\rightarrow \int \theta^a d\Sigma_a$
 This is $\approx \sum_i p_i \theta_i$

Variation $\delta I = \int_m (\text{Einstein})^{ab} h_{ab} + \int_m \nabla_a \theta^a$
 $g_{ab} \rightarrow g_{ab} + h_{ab}$

θ^a is the presymplectic potential

Explicitly $\theta^a = \frac{1}{16\pi} (\nabla_b h^{ab} - \nabla^a h) = \theta^a(g, h)$

Next construct the presymplectic current

$$\omega^a = \delta' \theta^a(g, h) - \delta \theta^a(g, h')$$

$$= \omega^a(g, h, h')$$

[Think ~~W~~ $\delta p_i \delta q_i$]

This is a 2-form on the phase space labelled by tangent vectors h , which will obey the linearised Einstein equations.

The symplectic form is then

$$\Omega(g, h, h') = \int_{\Sigma} w_a d\varepsilon^a \quad \leftarrow \text{Think } \Omega \sim \sum dp_i \wedge dq_i$$

But if g obeys Einstein and h, h' obey linearised this is independent of Σ and becomes h' is pure gauge then

$$\approx \int_{\partial\Sigma} g_{ab} ds^{ab} \quad \text{~~more complicated~~$$

Suppose that $h'_{ab} = \nabla_a \xi_b + \nabla_b \xi_a =$ pure gauge then

$$Q_{\Sigma'}(g, h) = \int_{\partial\Sigma} g_{ab} ds^{ab} \quad \text{is the change enclosed in } \Sigma, \text{ between } g \text{ and } g+h \text{ and conjugate to } \Sigma.$$

This should depend on h and not on how one gets to h , in other words, this 1-form on phase space should be exact, Q might therefore need correction.

Diffeomorphism invariance requires $h_{\Sigma_1, \Sigma_2} - h_{\Sigma_2, \Sigma_1} = K[\Sigma_1, \Sigma_2]$ which is reflected in required

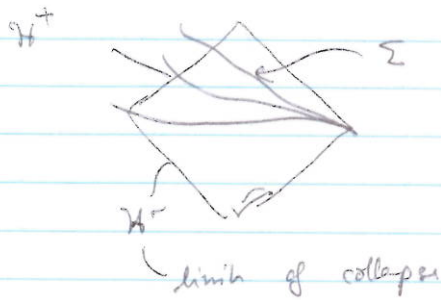
$$Q_{\Sigma} Q_{\Sigma'} - Q_{\Sigma'} Q_{\Sigma} = Q[\Sigma, \Sigma'] + K_{\Sigma, \Sigma'} \quad \text{1-cycle}$$

It will be a boundary integral, and unless it vanishes there will be a diffeomorphism anomaly. Note this is classical and comes from the Chern-Simons - Bittenberg cohomology

Explicitly

$$K = \frac{1}{16\pi} \int ds^{ab} \left[4 \nabla_c \xi_a \nabla^c \xi^b - 2 \nabla_c \xi^c (\nabla^a \xi^b) + 2 \nabla_c \xi^{bc} (\nabla^a \xi^d) - 2 R^{abcd} \xi_c \xi'_d \right]$$

Now think about an observer in the domain of outer communication



In each case, H^+ is the boundary of what such an observer can see.

So the question is does K vanish.

Let's look at K on H_+ .

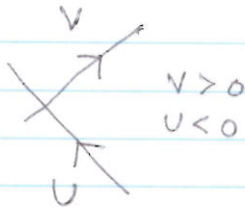
Easiest to do in Kruskal u -coordinates, lets do this in Kerr -

$$ds^2 = \tilde{\rho}_+^2 dUdV + \rho_+^2 d\theta^2 + \frac{(r_+^2 + a^2) \sin^2 \theta}{\tilde{\rho}_+^2} d\phi^2$$

Horizon at $r_{\pm} = m \pm \sqrt{m^2 - a^2}$

$$+ \frac{(UdV - VdU) a \sin^2 \theta}{2r_+ \rho_+^2} (3r_+^4 + a^2 r_+^2 + a^2 r_+^2 \cos^2 \theta - a^4 \cos^2 \theta)$$

$$+ O(2)$$



[Carter, Boyer, Lindquist]

Choose

$$S_{(n)}^U = - (2\pi T_R h_n + h'_n) U / T_L + T_R$$

$$S_{(n)}^V = (2\pi T_L h_n - h'_n) V / T_L + T_R$$

$$S_{(n)}^\phi = 2\pi T_R h_n + h'_n / 2\pi(T_L + T_R)$$

$$S_{(n)}^\theta = 0$$

$$h_n = e^{in(\phi + \ln V / 2\pi T_R)}$$

$$T_R = \frac{r_+ - r_-}{4\pi a}$$

$$T_L = \frac{r_+ + r_-}{4\pi a}$$

$n=0$ —
linear combinations
of the Killing vectors

These are horizon superrotations

This vector field obeys

$$[h_n, h_m] = (n-m)h_{n+m} \quad \text{ie Witt algebra}$$

And

$$K_{n,m}^{(R)} = 2\pi J \frac{T_R}{T_L + T_R} - i\kappa^3 \delta_{n+m,0}$$

Although this was worked out at the bifurcation surface, it is OK all the way up H^+

There is a second vector field with $T_L \leftrightarrow T_R$,
Hence plus

$$K_{n,m}^{(L)} = 2\pi J \frac{T_L}{T_L + T_R} - i\kappa^3 \delta_{n+m,0}$$

so the charge algebra is anomalous.

Now look at black hole scattering (Pres, Teukolsky + ...)

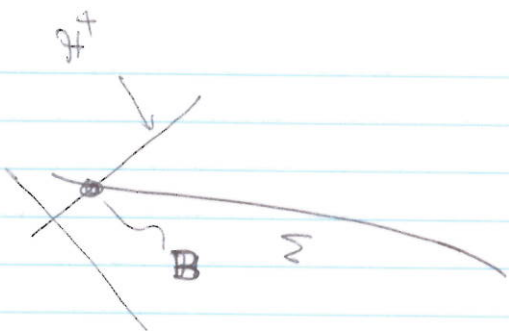
Absorption probability for particles of energy δE , δJ and angular momentum

$$\sim \left| \Gamma\left(1 + \frac{i\omega_L}{2\pi T_L}\right) \right|^2 \left| \Gamma\left(1 + \frac{i\omega_R}{2\pi T_R}\right) \right|^2$$

where	$\omega_L = 2M^3/J \delta E$	} These are the energies conjugate to $\mathcal{I}^+(a)$ and $\mathcal{I}^-(a)$ respectively
	$\omega_R = 2M^3/J \delta E - \delta J$	

This is a relation that appears elsewhere in physics

It is the absorption probability for a pair of 2-d conformal field theories with energy ω_L at temperature T_L and ω_R, T_R



If observers on Σ regard H as a boundary then they will be cancel the anomaly. They will ascribe to B a CFT that cancels the anomaly and describes the properties of the black hole.

~~Black~~ Black hole horizons then need to have such a holographic picture to account for observations of the black hole.

CFT's are described by their central charge amongst other things. Explicitly

$$K(L) + K(R) \Rightarrow 2J = \frac{C_L + C_R}{12} \Rightarrow C_L + C_R = 24J$$

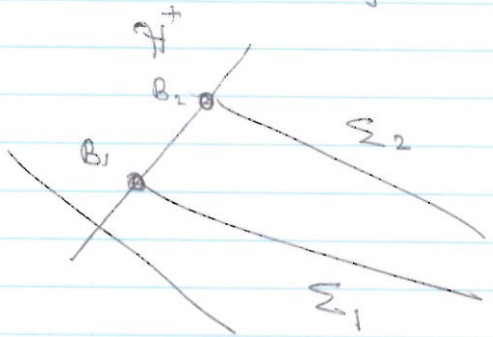
but $C_L = 12J \frac{T_L}{T_L + T_R}$ $C_R = 12J \frac{T_R}{T_L + T_R}$

Note that this cannot be continuously varied as required by the Zamolodnikov thm $J = \text{integer (or odd half-integer)}$

The problem here is that C_L, R do not obey that property so this picture is incomplete. Generally believed that $C_L = C_R$ else T not conserved.

$$\nabla_\mu T^\mu \sim C_L - C_R$$

Not certain of what follows



There could be evolution between B_1 and B_2 and so there should be something on the horizon. In Kerr

$$\frac{1}{2} \text{Eubed } R^{ab} \text{ of } R^{cd} \neq 0$$

\Rightarrow On H one could have a Chern-Simons gravity term

connection on the boundary of Γ is in the algebra of $SO_{3,1}$ (6)

$$I_{CS} = \beta \int_{\mathbb{R}^2} (\Gamma d\Gamma + \frac{2}{3} \Gamma^3) \quad (\text{Kraus + Larsen})$$

on the horizon. β gives rise to a contribution to the central charges on the boundary of

$$c_L - c_R = 96\pi\beta$$

Thus the choice $\beta = \frac{J}{4} \frac{T_L - T_R}{T_L + T_R}$

will give $c_L = c_R = 12J$.

← This comes from requiring the energy-momentum tensor to ~~be~~ non-conserved to cancel.

This allows the possibility for evolution as stuff passing through \mathcal{H} will couple to the CS and charge see things.

Cardy: The entropy of a CFT at temperature T is

$$S = \frac{\pi^2}{3} c T$$

so in our case $S_{\text{Horizon CFTs}} = \frac{\pi^2}{3} (c_L T_L + c_R T_R)$
 $= \frac{1}{4} A$

by plugging in our previous formulae

Relation to other works =

① Strominger + Vafa = + - - - -

The took configurations with some quantum numbers as extreme (or near extreme) and found the CFT of the corresponding string theory. Agreement here.

② Wald

Entropy is a Noether charge and as we found the area of A_+ , conjugate $\partial_t + \Omega_+ \partial_\phi$

Extended by Castro + Rodrigues to speculate there was entropy associated with the inner horizon $\partial_t + \Omega_- \partial_\phi$

$$S_{\pm} = CFT_L \pm CFT_R$$

but this is in complete agreement with our conjecture.

③ Ashtekar

Branched polymer description of the horizon but some of these are known to be essentially described by CFTs (Cardy, Fisher, Parisi, Brydges)