https://web.uniroma1.it/gmunu/team

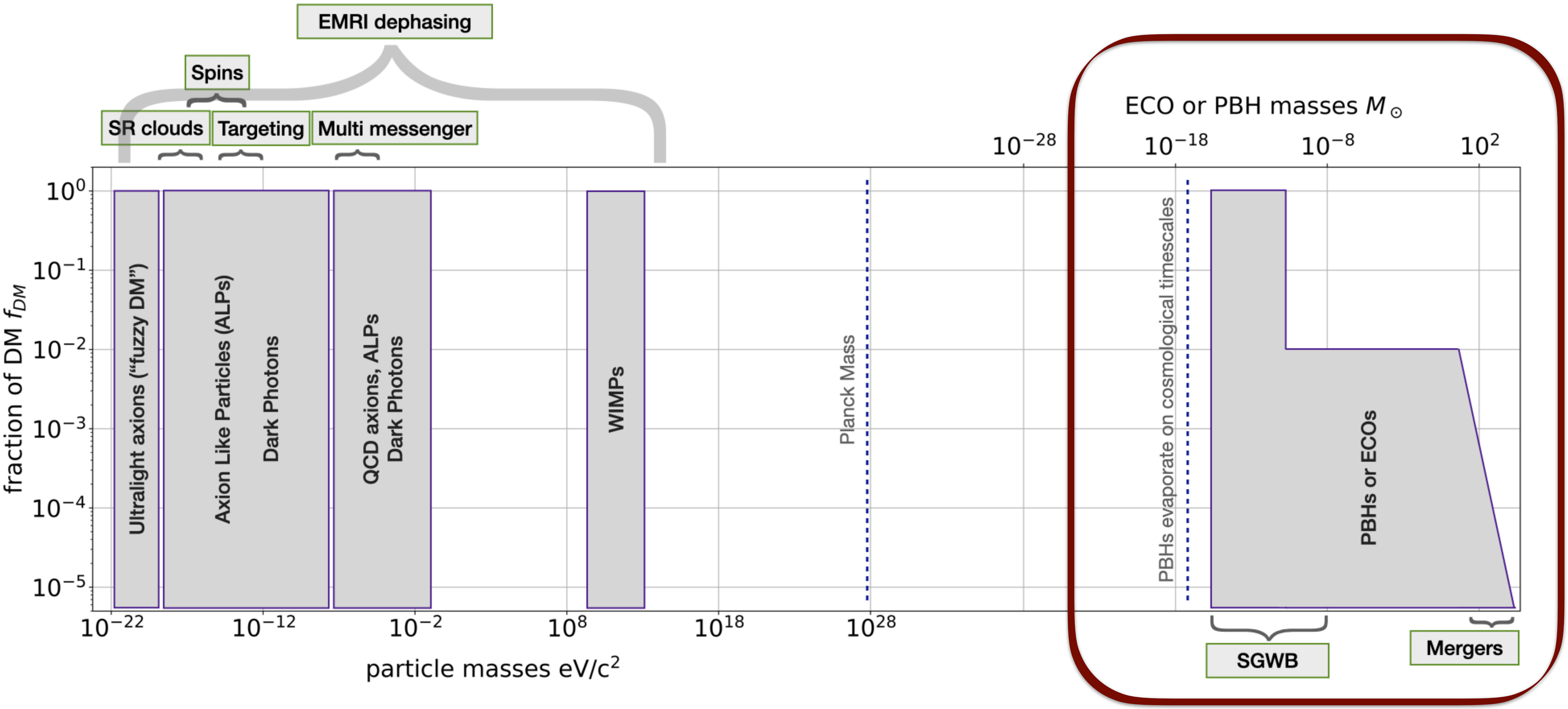
Gabriele Franciolini

Testing Primordial Black Hole as Dark matter with LISA

Fundamental physics with LISA

Brussels, 26-28 April 2022

Dark Matter and Primordial Black Holes



Thanks Katy!

Outline

- *Introduction on Primordial Black Holes (PBHs)*
- *How can LISA search for and constrain PBH?*

part I

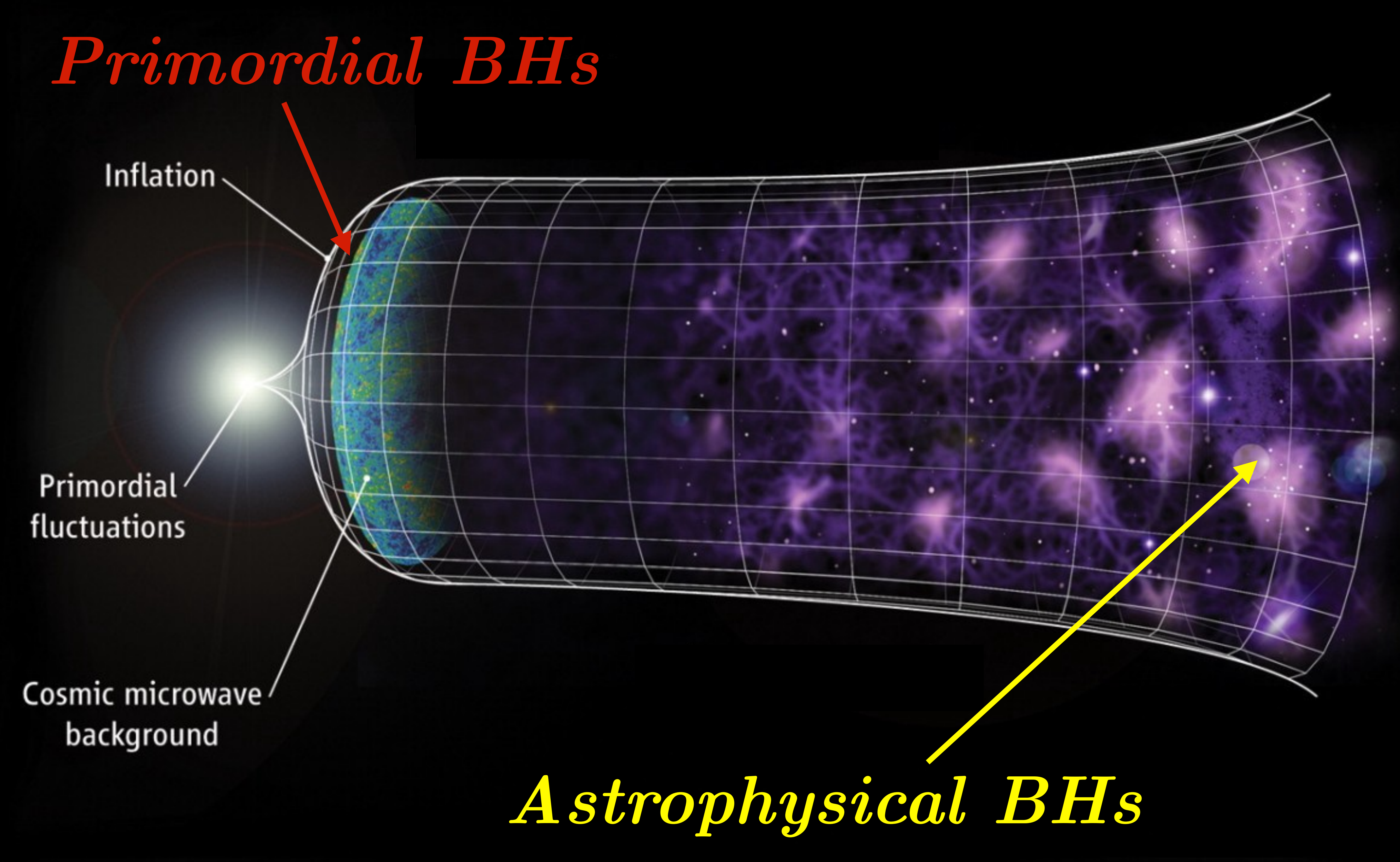
GWs and PBH formation

part II

PBH mergers

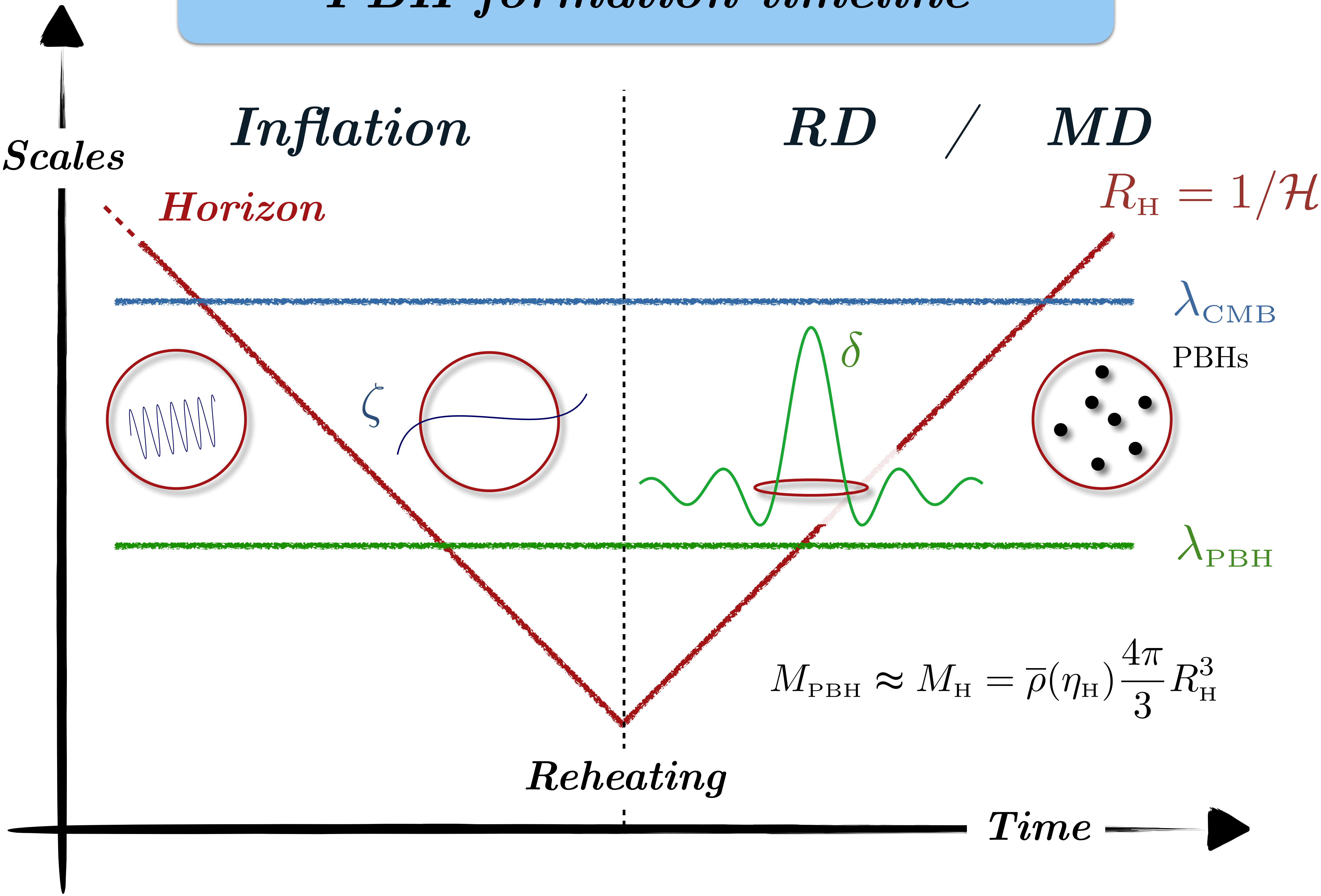
- *Open questions?*

Introduction



Primordial BHs
Inflation
Primordial fluctuations
Cosmic microwave background
Astrophysical BHs

PBH formation timeline

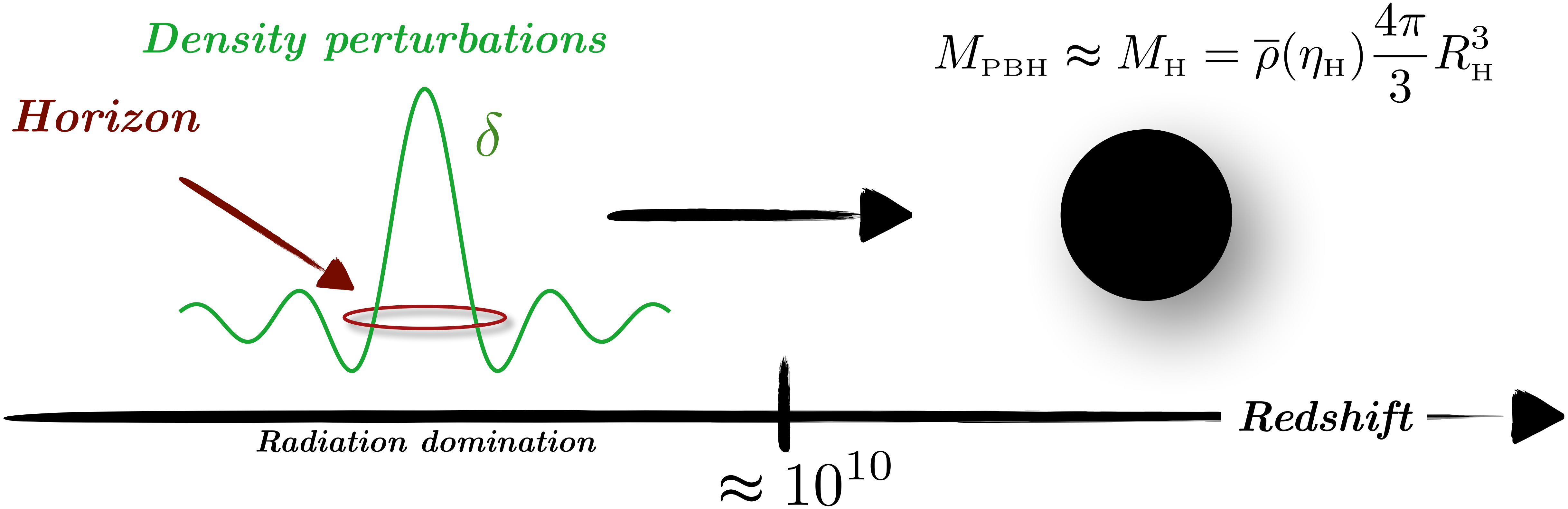


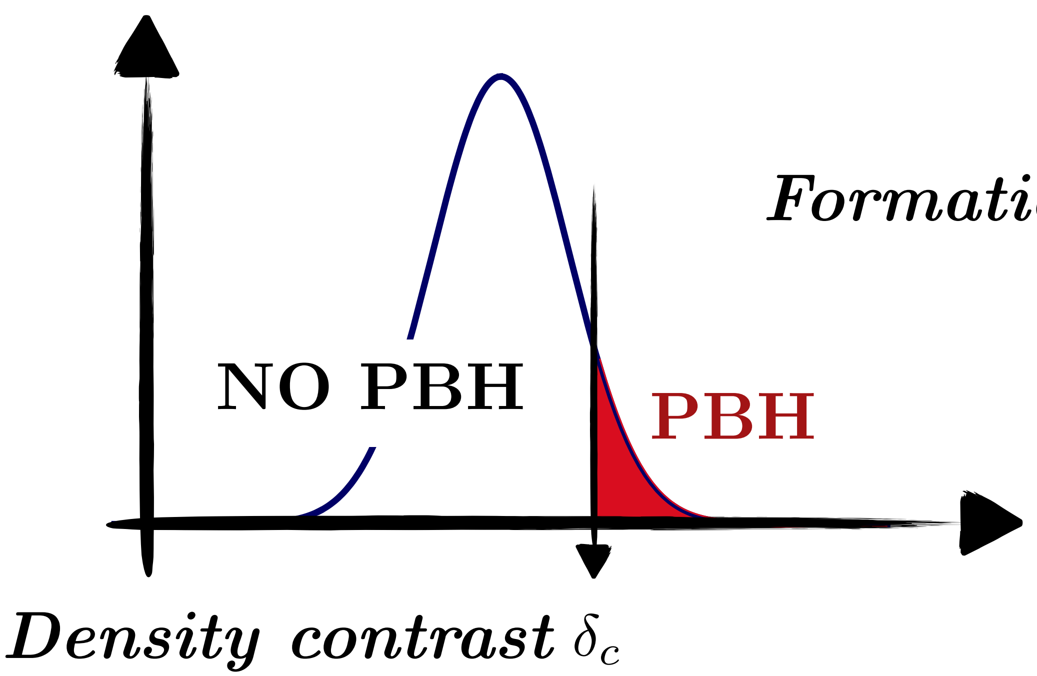
(+ many alternative scenarios)

e.g. review: Sasaki et al [1801.05235]

PBH formation

$$M_{\rm PBH} \approx M_{\rm H} = \overline{\rho}(\eta_{\rm H}) \frac{4\pi}{3} R_{\rm H}^3$$





Formation criterion for horizon-crossing perturbations:

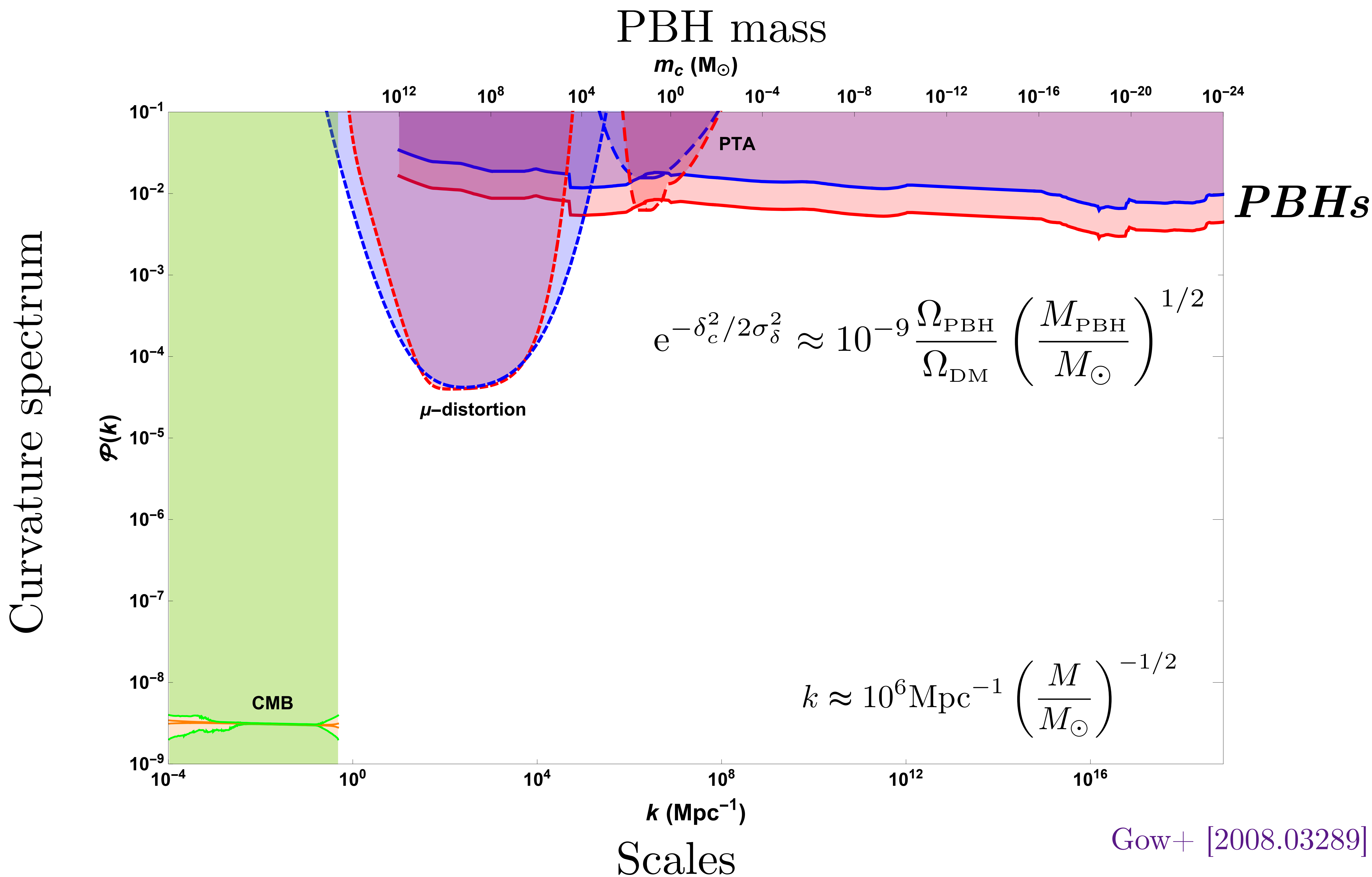
$$\delta \geq \delta_c \; (\approx 0.5)$$

Shibata+ (1999), Musco (2018), ...

Threshold dependence on the statistical properties of curvature perturbations

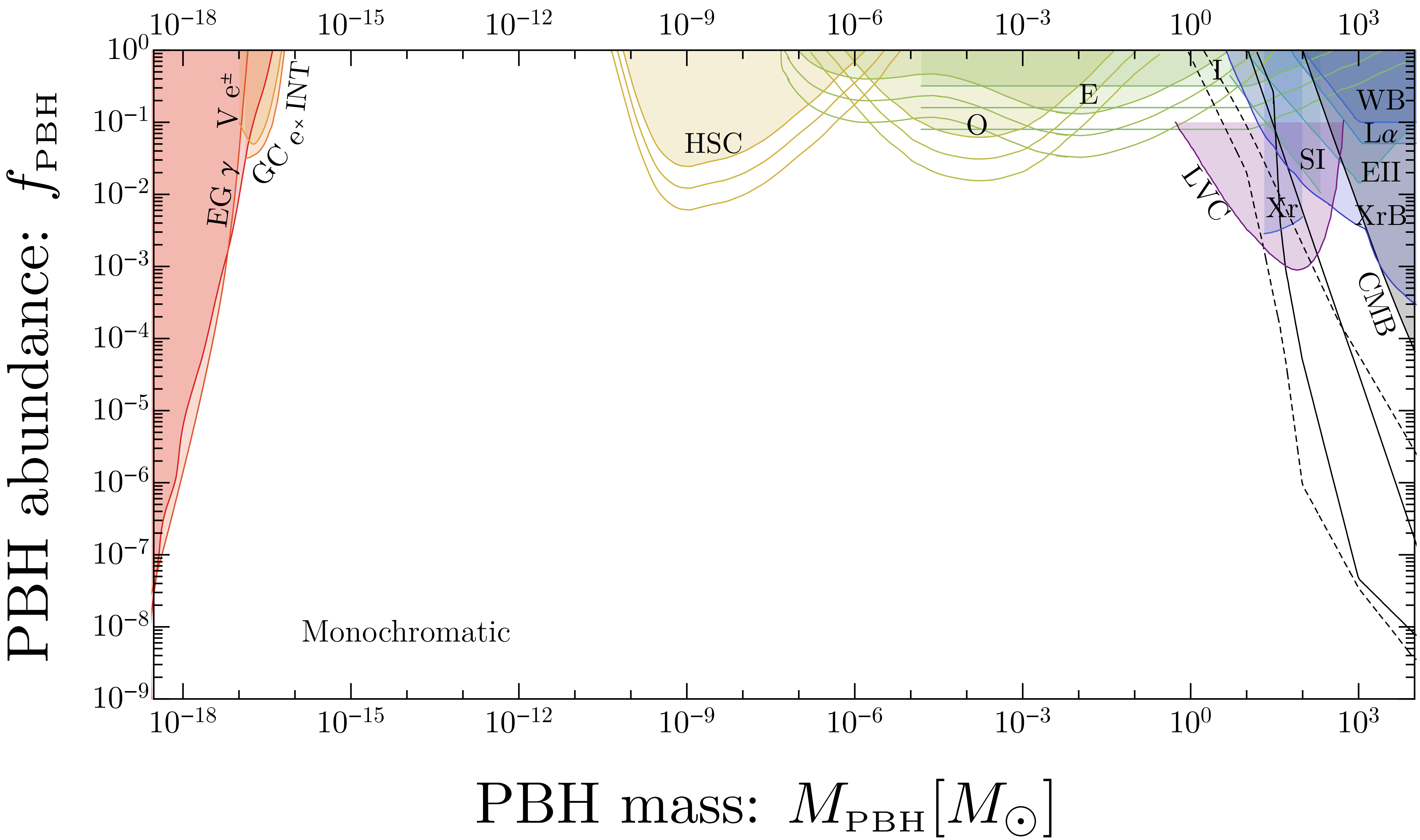
Musco, De Luca, **GF**, Riotto PRD (2020), ...

Small scales - Large amplitude



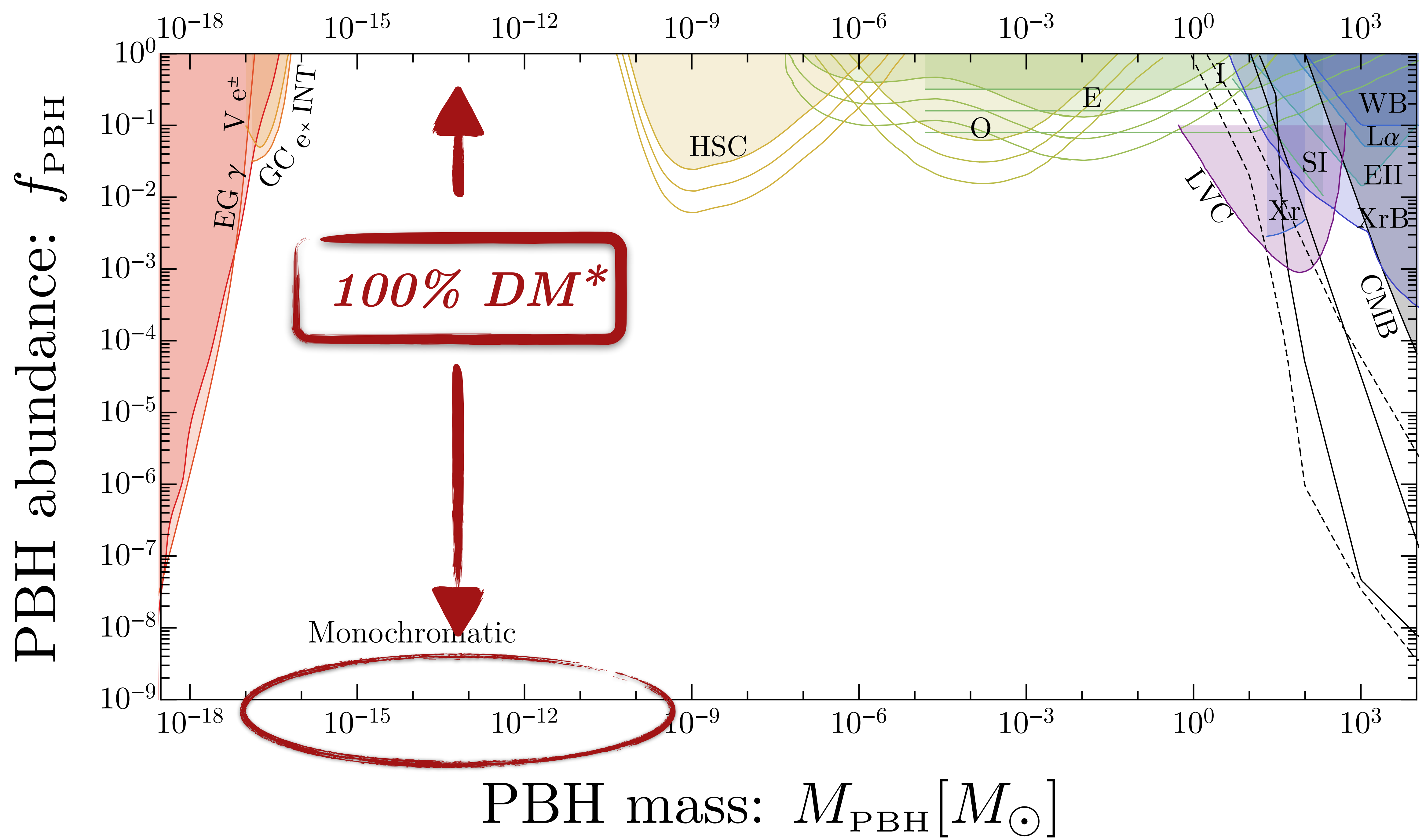
Gow+ [2008.03289]

Constraints on the PBH abundance



e.g. review: Carr et al [2002.12778]

Constraints on the PBH abundance



GW: Formation signatures Mergers

part I: Induced GWs

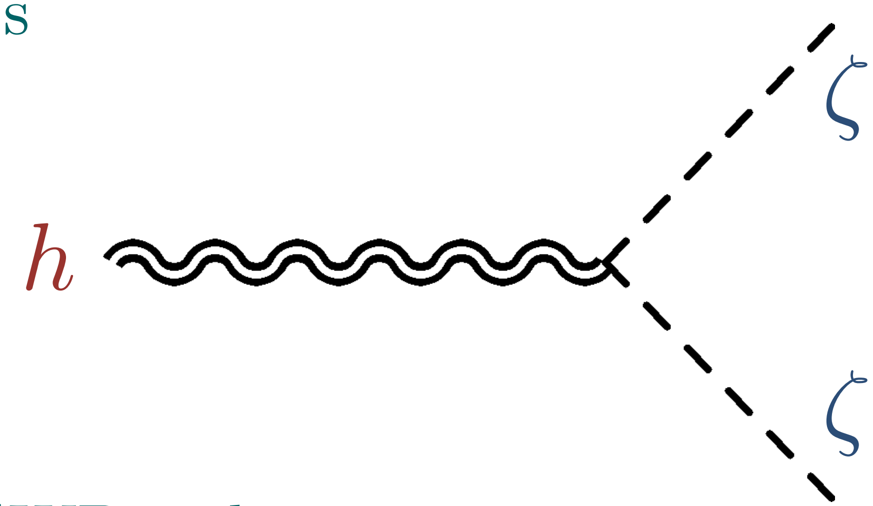
GWs at second order

Acquaviva et al. (2002), Mollerach et al. (2003), Ananda et al (2006), Baumann et al. (2007),

Ando et al (2018), Espinosa et al (2018), Khori et al (2018), ...

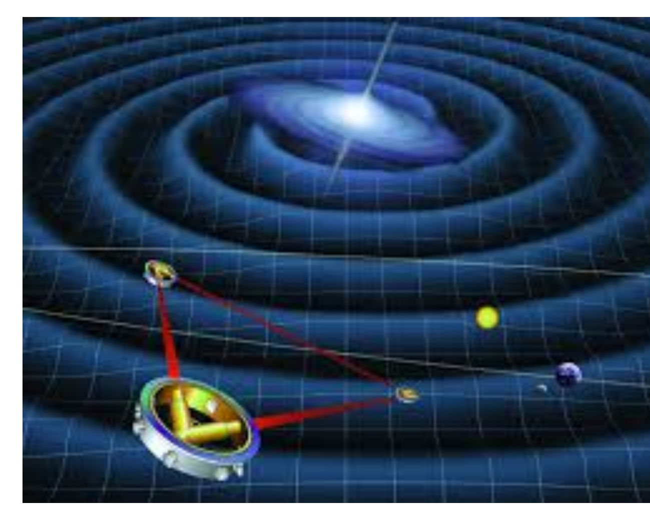
The same curvature perturbations giving rise to PBHs are unavoidably a source for GWs

$$\boxed{h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} \approx \mathcal{S}_{ij}\left(\zeta\zeta\right)}$$



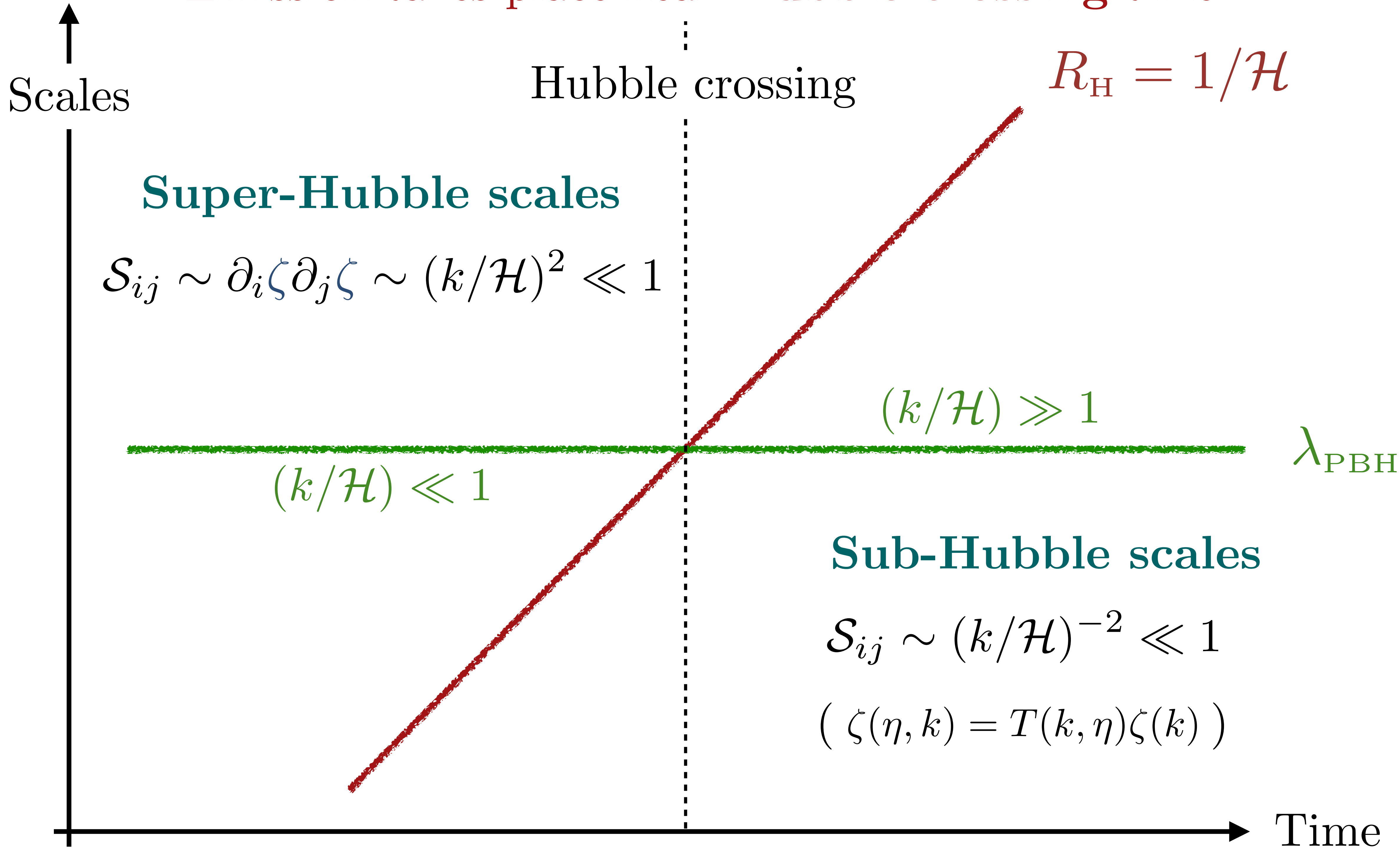
Since perturbations are large, they generate a sizeable SGWB at horizon re-entry

Potentially observable at current and future GW observatories (LIGO, Virgo, LISA,...)



Emission

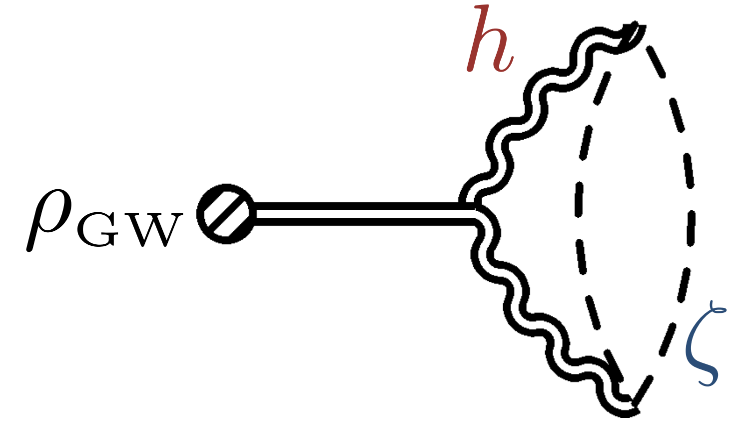
Emission takes place near **Hubble crossing** time



The emitted **GWs** reach us **redshifted** as radiation: $\rho_{\mathrm{GW}} \sim a^{-4}$

Energy abundance

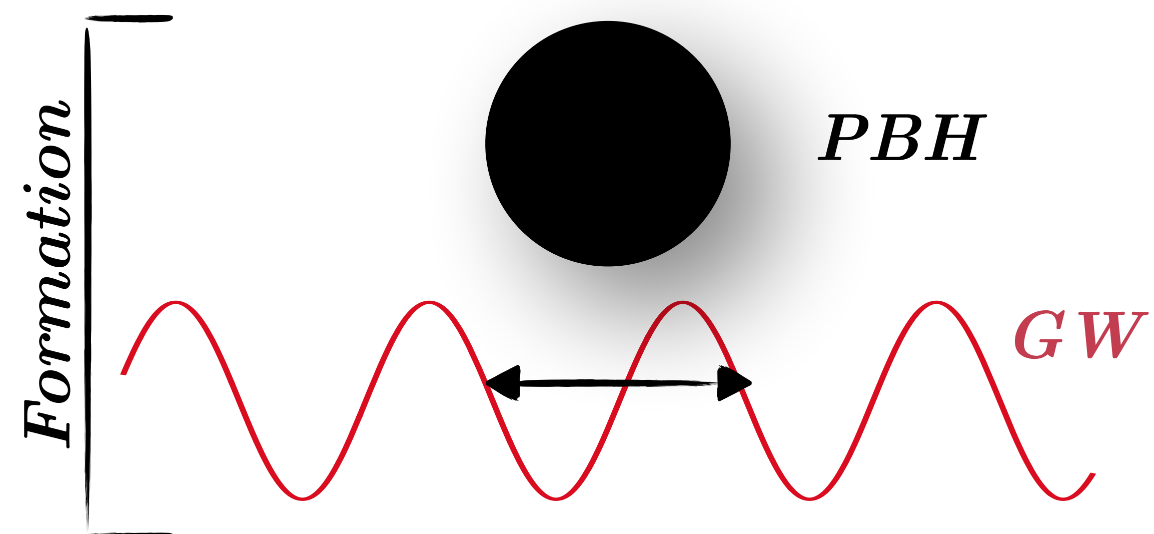
The energy density of GWs is given by the time average over several cycles



$$\Omega_{\rm GW}(\eta,\vec{x}) = \frac{\rho_{\rm GW}(\eta,\vec{x})}{\overline{\rho}(\eta)} = \frac{M_p^2}{4a^2\overline{\rho}(\eta)} \left\langle h'_{ab}\left(\eta,\, \vec{x}\right) h'_{ab}\left(\eta,\, \vec{x}\right)\right\rangle_{\rm t.a.} \approx \mathcal{P}_\zeta \mathcal{P}_\zeta$$

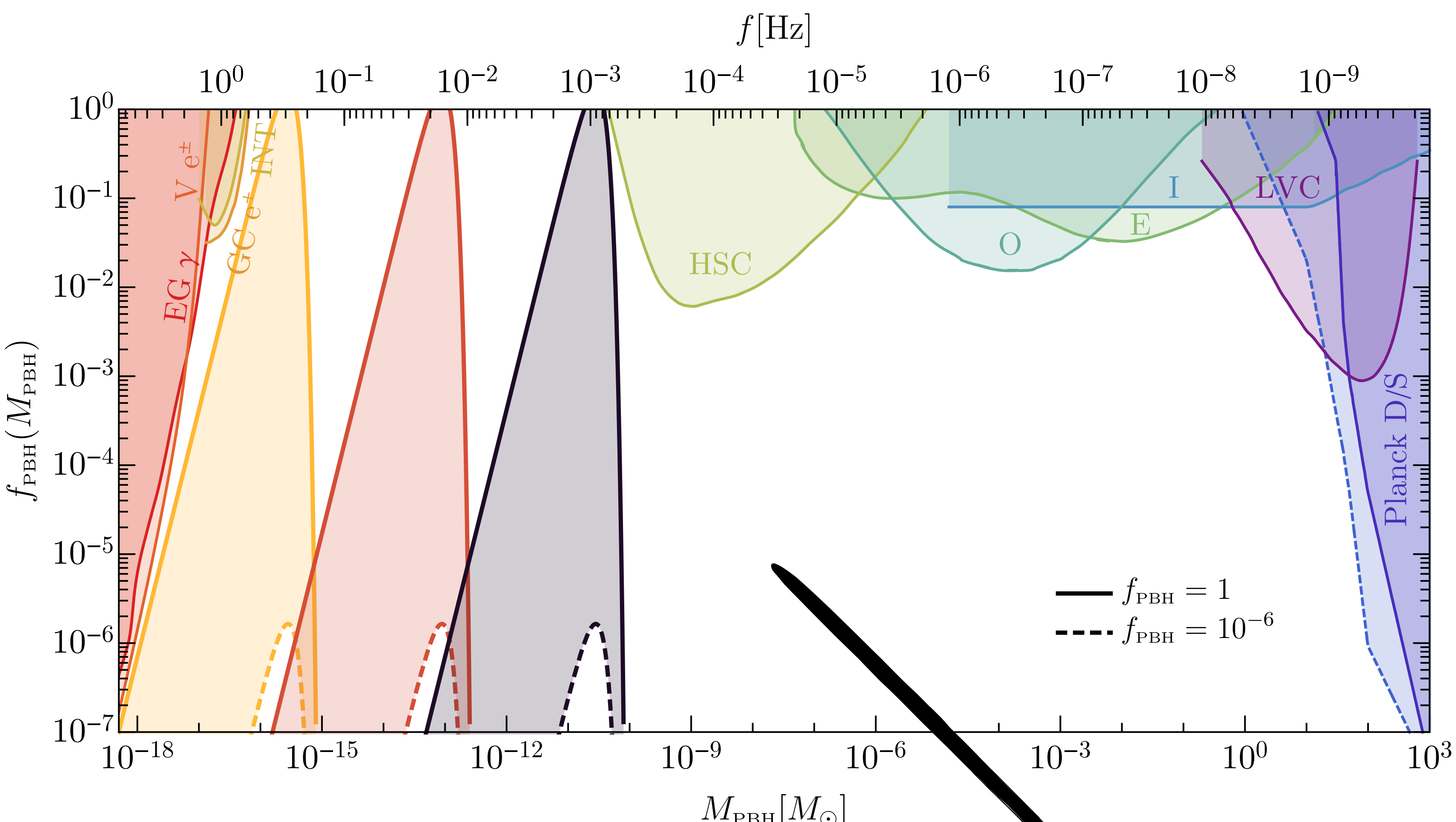
Frequency

The characteristic frequency of the GWs is similar to the frequency of the scalar perturbations, related to the PBH mass



$$\boxed{f \approx 3 \cdot 10^{-9}\mathrm{Hz} \left(\frac{\gamma}{0.2}\right)^{1/2} \left(\frac{M}{M_\odot}\right)^{-1/2}}$$

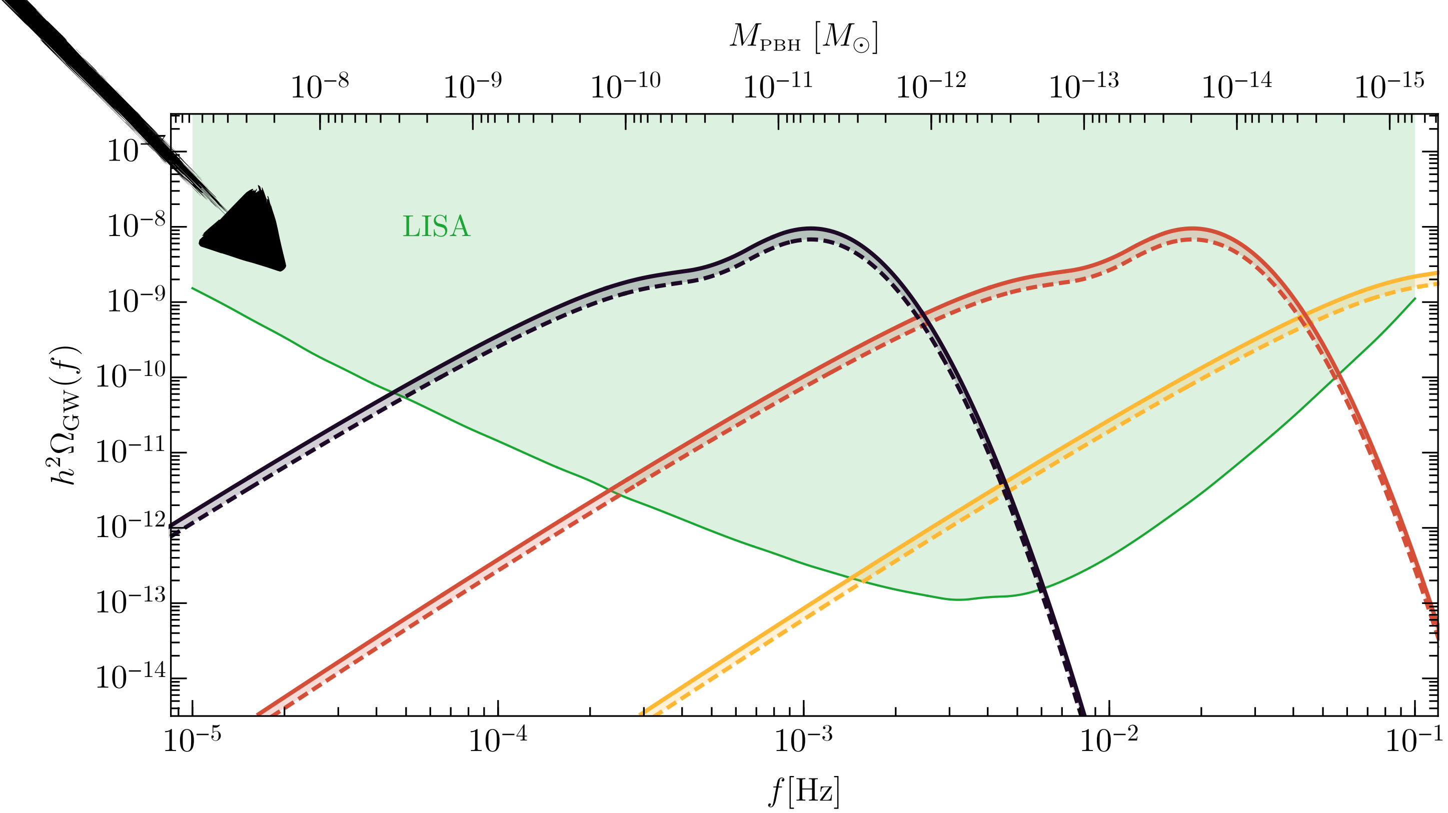
Test the unconstrained mass range with LISA



Bartolo, **GF** et al PRL (2019)

$$M_{\rm PBH} \approx 10^{-12} M_\odot$$

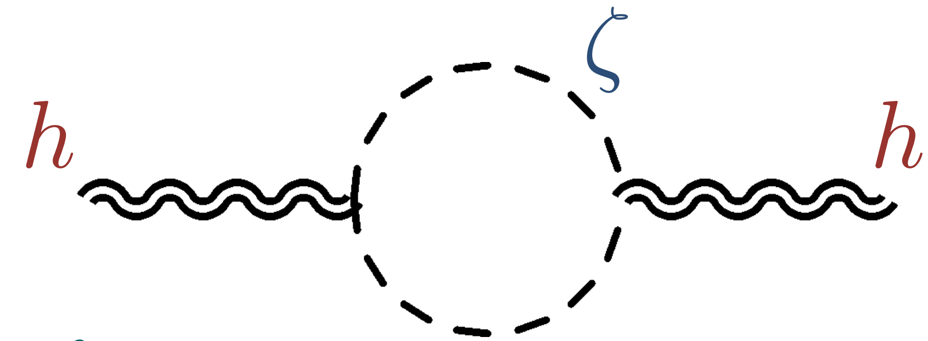
$$f_{\rm LISA} = 3.4 \text{ mHz}$$



Characterisation of the SGWB: spectrum

Power spectrum of GWs:

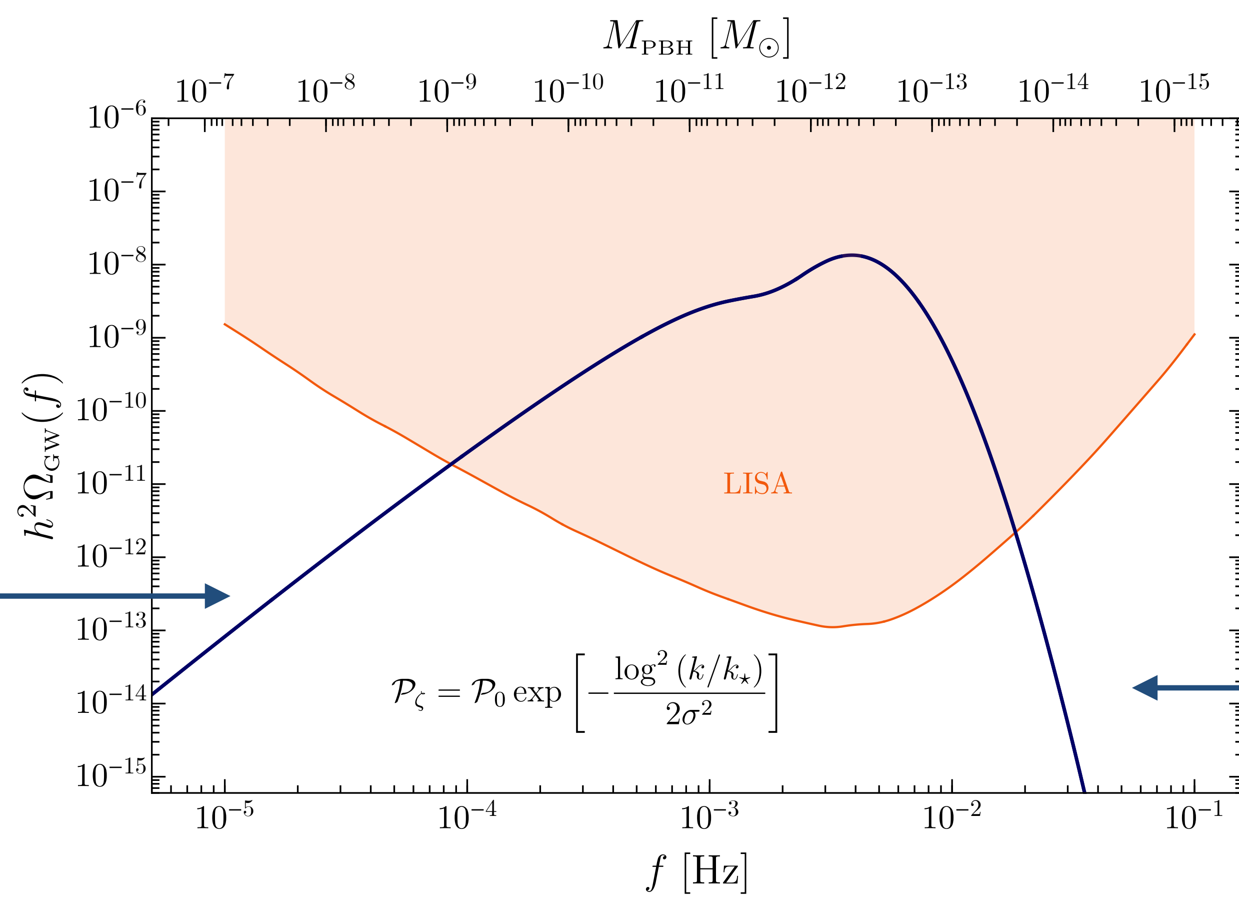
$$\left\langle h^{\lambda_1}(\eta,\vec{k}_1)h^{\lambda_2}(\eta,\vec{k}_2)\right\rangle' \approx \mathcal{P}_\zeta \mathcal{P}_\zeta$$



At second order in comoving curvature perturbation, after averaging over the fast oscillating pieces

$$\Omega_{\rm GW}(\eta,k) = \frac{\pi^2}{243\mathcal{H}^2\eta^2}\int \frac{d^3p}{(2\pi)^3}\frac{p^4\left[1-\mu^2\right]^2}{p^3\,|\vec{k}-\vec{p}|^3}\boxed{\mathcal{P}_\zeta(p)\,\mathcal{P}_\zeta\left(|\vec{k}-\vec{p}|\right)}\,\boxed{\mathcal{I}^2\left(\vec{k},\,\vec{p}\right)}$$

$\propto k^3$

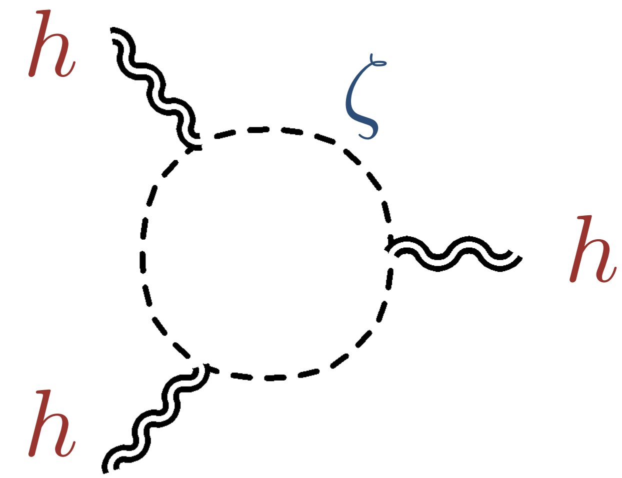


$\propto \mathcal{P}_\zeta^2(k \gg k_*)$

Characterisation of the SGWB: NGs at emission

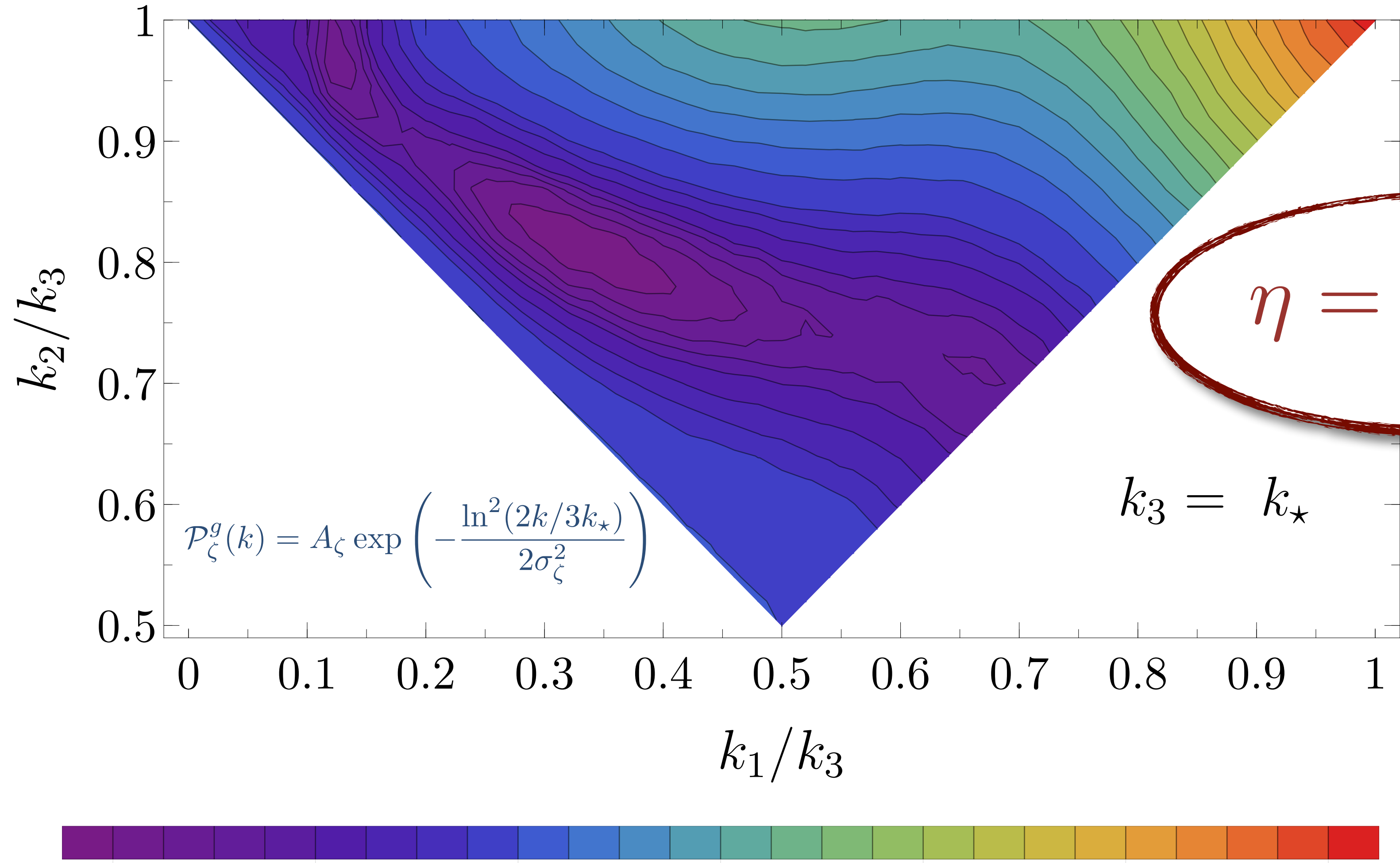
Bispectrum of GWs:

$$\left\langle h_{\lambda_1}\left(\eta_1,\vec{k}_1\right) h_{\lambda_2}\left(\eta_2,\vec{k}_2\right) h_{\lambda_3}\left(\eta_3,\vec{k}_3\right)\right\rangle' \approx \mathcal{P}_\zeta\mathcal{P}_\zeta\mathcal{P}_\zeta$$



Since sourced at second order, the emitted GWs are NG

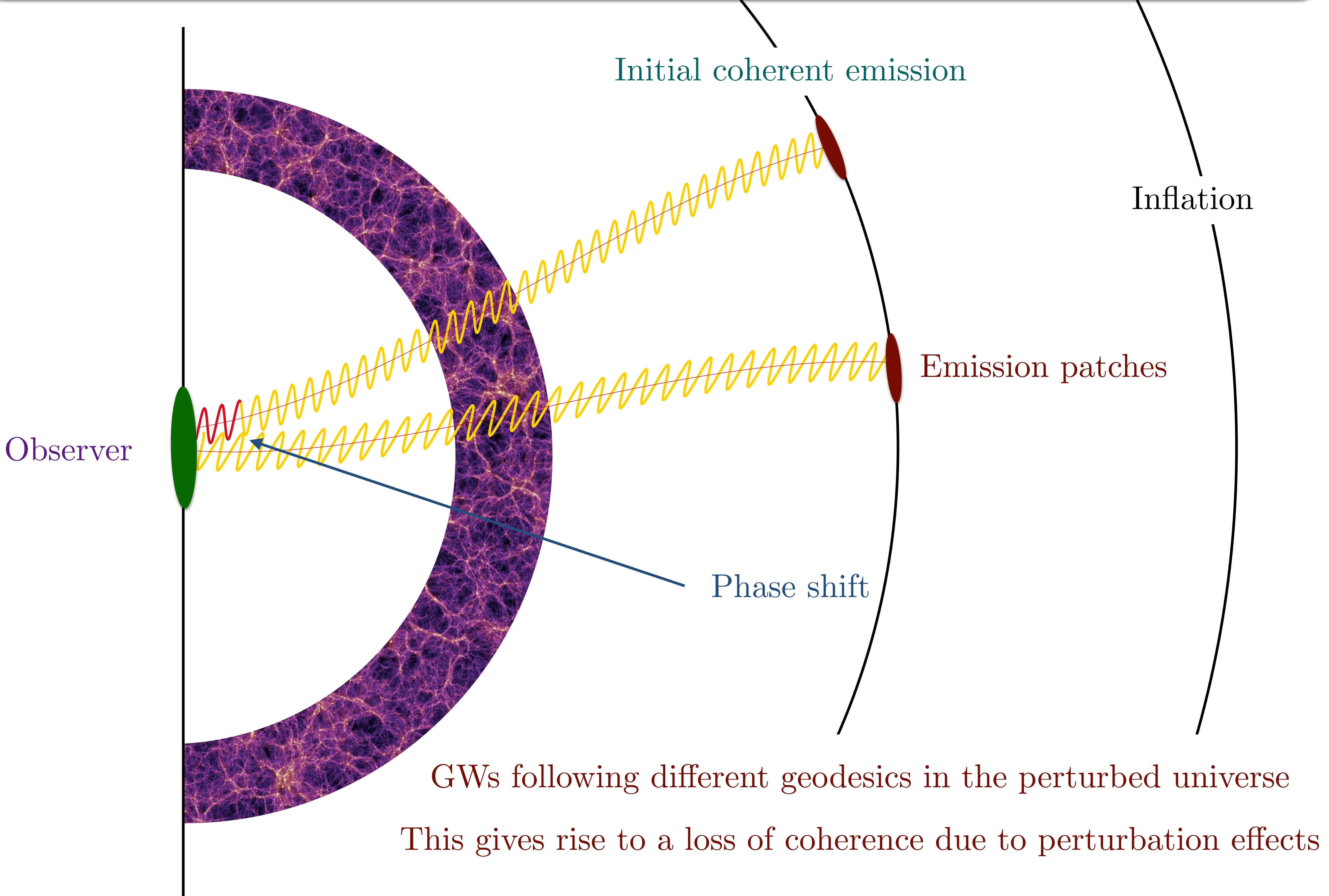
$$\mathcal{S}_h^{\lambda_1\lambda_2\lambda_3}(\vec{k}_1,\vec{k}_2,\vec{k}_3) = k_1^2k_2^2k_3^2\frac{\langle h_{\lambda_1}(\eta,\vec{k}_1)h_{\lambda_2}(\eta,\vec{k}_2)h_{\lambda_3}(\eta,\vec{k}_3)\rangle'}{\sqrt{\mathcal{P}_h(\eta,k_1)\mathcal{P}_hh(\eta,k_2)\mathcal{P}_h(\eta,k_3)}}$$



$\eta = \eta_{\rm emission}$

BUT...

Characterisation of the SGWB: propagation



GWs following different geodesics in the perturbed universe

This gives rise to a loss of coherence due to perturbation effects

Characterisation of the SGWB: NGs suppressed today

$$\boxed{h''_{ij} + 2\mathcal{H}h'_{ij} - (1+4\Phi)\, h_{ij,kk} = 0}$$

$$h_{ij} = A_{ij} \mathrm{e}^{ik\eta + i2k\int^{\eta} d\eta'\, \Phi(\eta')}$$

(similar to Shapiro time-delay)

- ***Power spectrum: unaffected***

(the PS is basically proportional to the energy density)

- ***Bispectrum: hugely suppressed***

$$B_h^{\lambda_i}\left(\eta_0,\, \vec{k}_i\right)\Big|_{\text{inhom.}} \approx \mathrm{e}^{-A_L\, k^2 (\eta_0)^2} B_h^{\lambda_i}\left(\eta_0,\, \vec{k}_i\right)\Big|_{\text{hom.}}$$

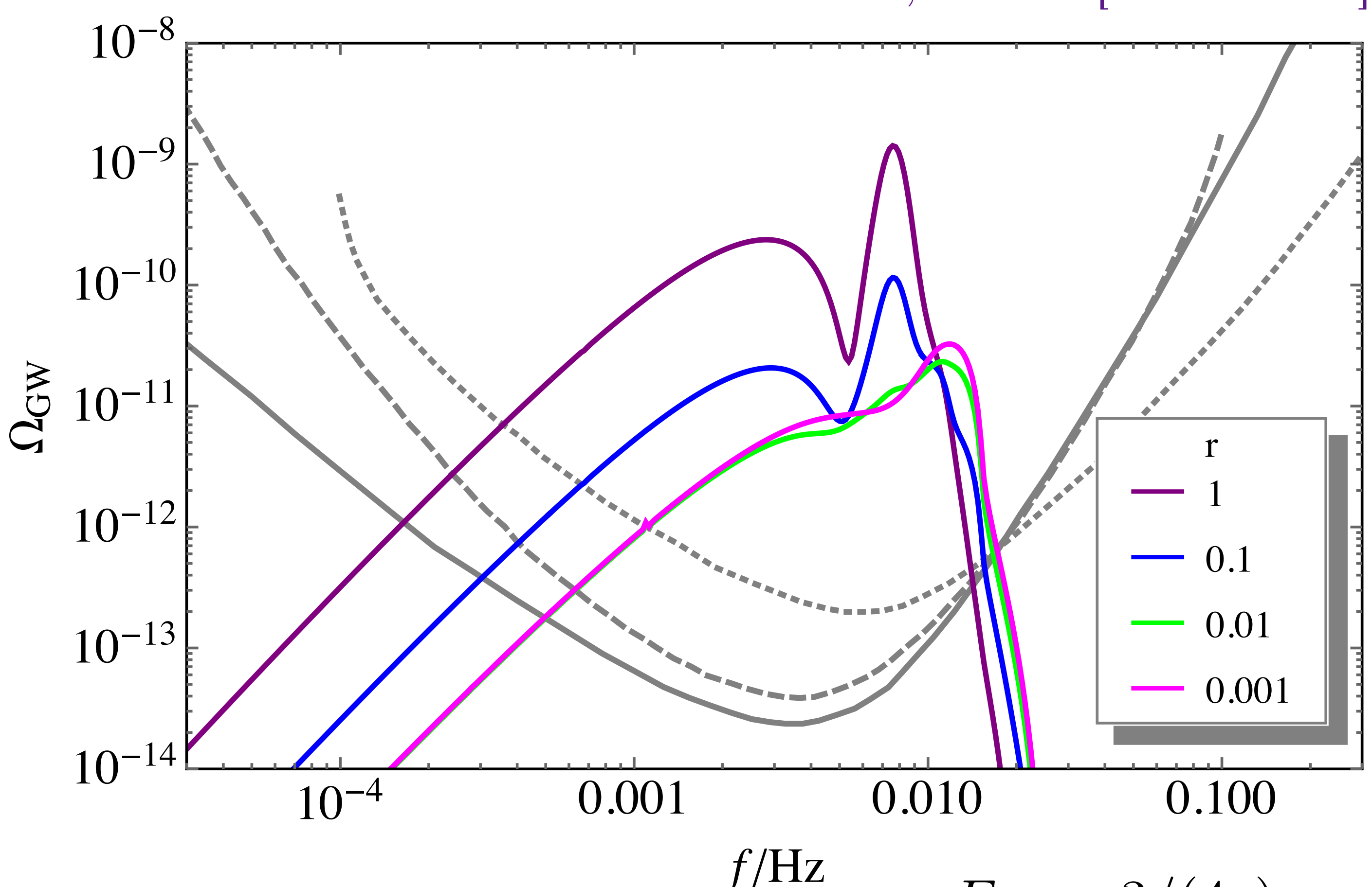
$$\left(A_L \sim 10^{-9},\;\; k\eta_0 \sim 10^{16}\right)$$

Characterisation of the SGWB: local NGs

Can we detect local NGs of curvature perturbations?

$$\zeta_{\rm NG}(\vec{x}) = \zeta(\vec{x}) + F_{\rm NL}\left(\zeta^2(\vec{x}) - \left\langle \zeta^2(\vec{x}) \right\rangle\right)$$

Pi, Sasaki [2112.12680]



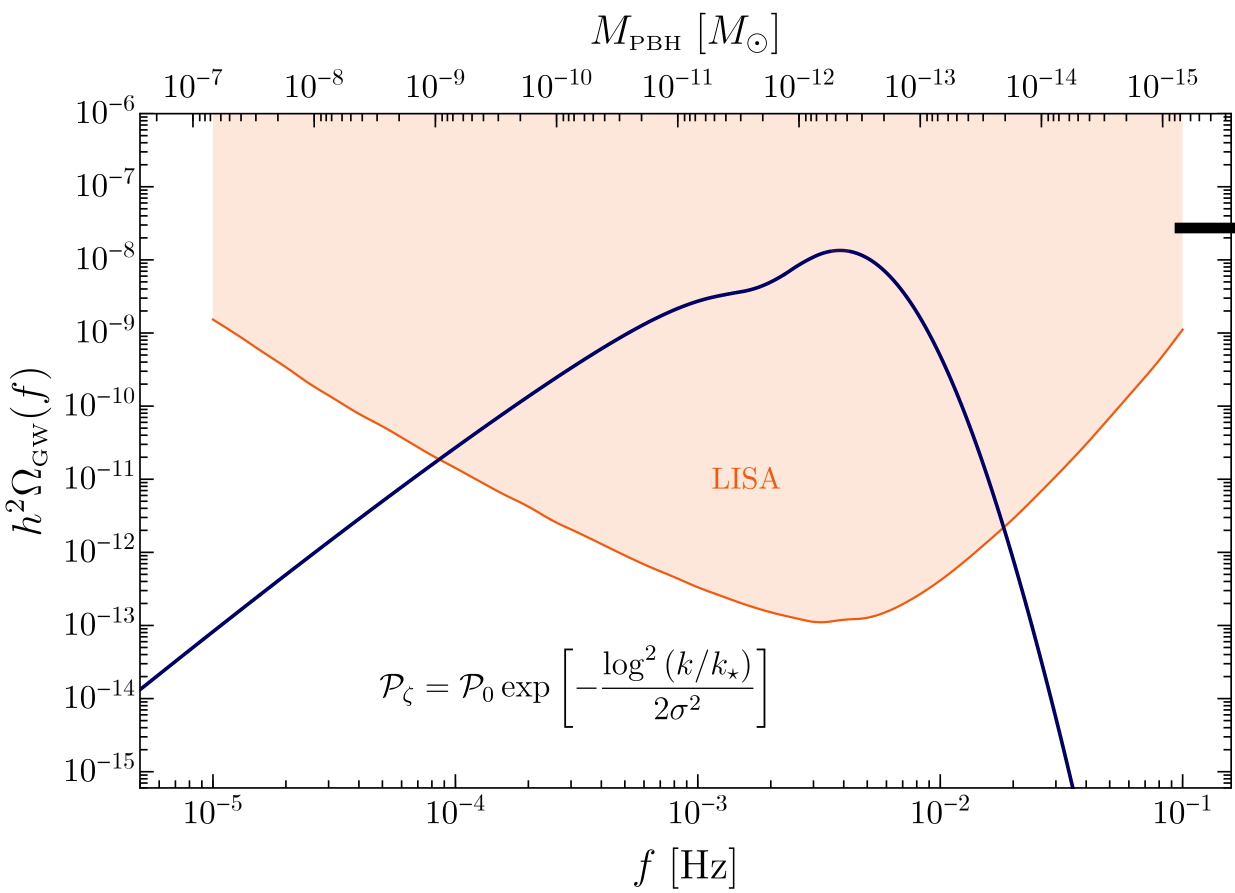
$F_{\rm NL} = 3/(4r)$

Result for very narrow PS

- ***F_{NL}>0 suppresses the signal***
- ***Smoothing of the high-peak***
- ***Additional features?***
- ***Low-k tail maintained***

see also: Sasaki+ 2018, Unal 2018, ...

Assuming LISA detected a SGWB



Constrain curvature power spectrum

↓

PBH abundance

$$\beta(M) \equiv \frac{\rho_{\rm PBH}(\eta_{\rm H})}{\rho_{\rm tot}(\eta_{\rm H})} = \int_{\delta_{\rm c}} {\rm d}\delta_{r_m} {\rm P}_{\rm G}(\delta_{r_m}) \approx e^{-\delta_{\rm c}^2/2\sigma_\delta^2(M)}$$

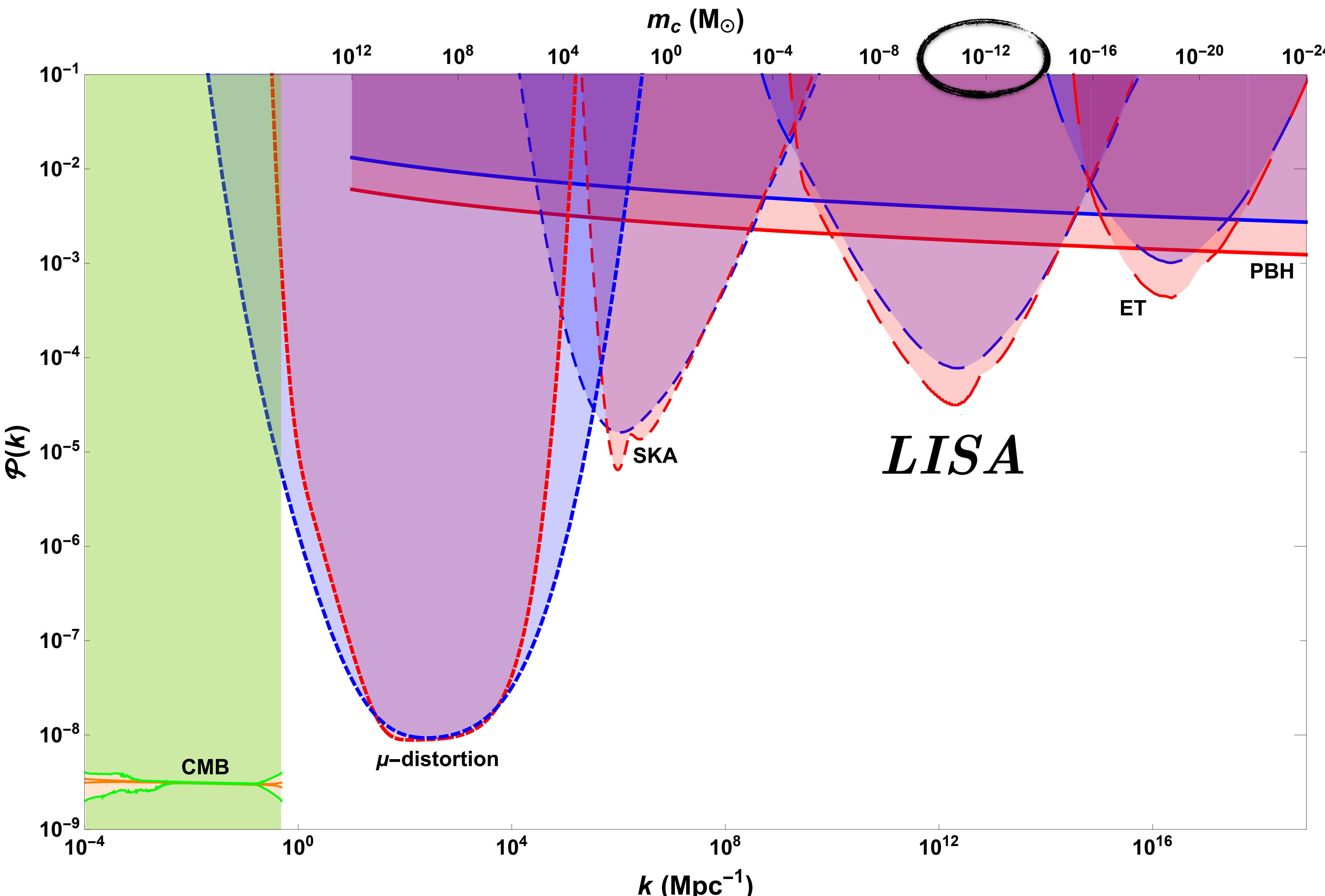
*Abundance scales exponentially with the power spectrum **

To constrain the PBH abundance $f_{\rm PBH}$ *:*

i) threshold; ii) statistics of peaks with NGs corrections;

e.g. constraints from PTAs...

Assuming LISA didn't see this SGWB



Gow+ [2008.03289]

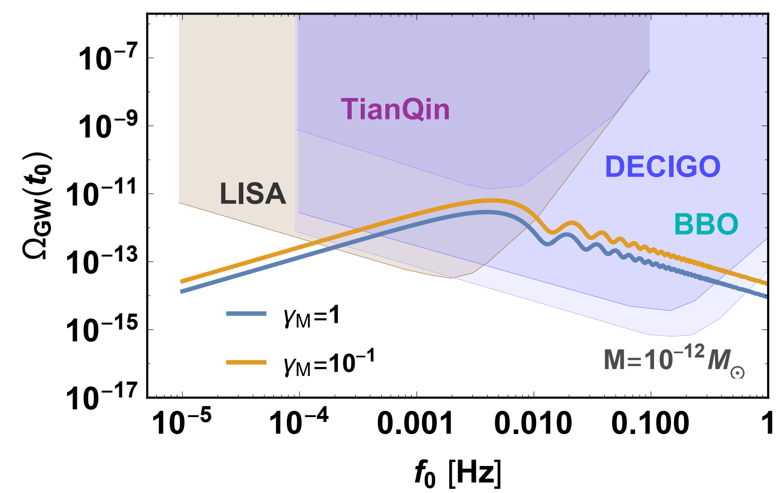
Did we rule out existence of asteroidal mass PBH?

Assuming LISA didn't see this SGWB

What about alternative formation scenarios?

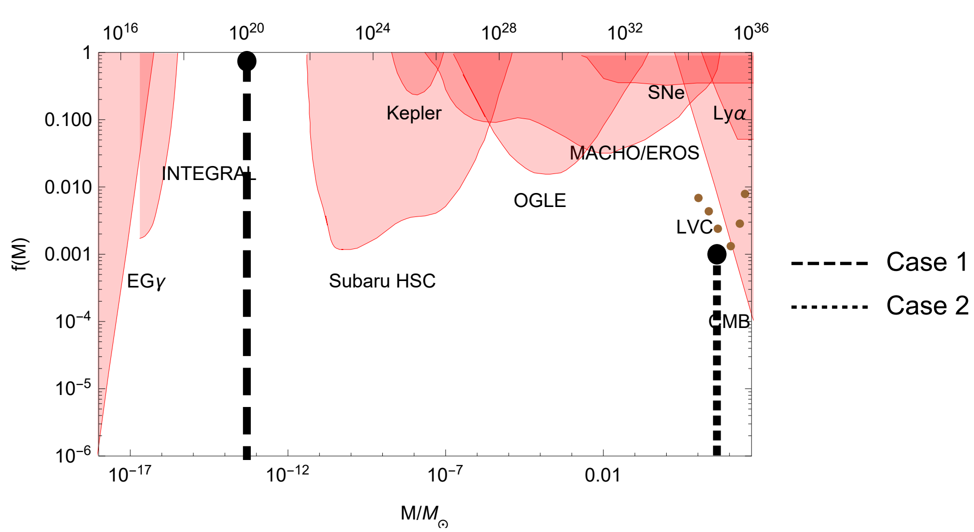
Early Matter dominated era: smaller threshold for collapse.

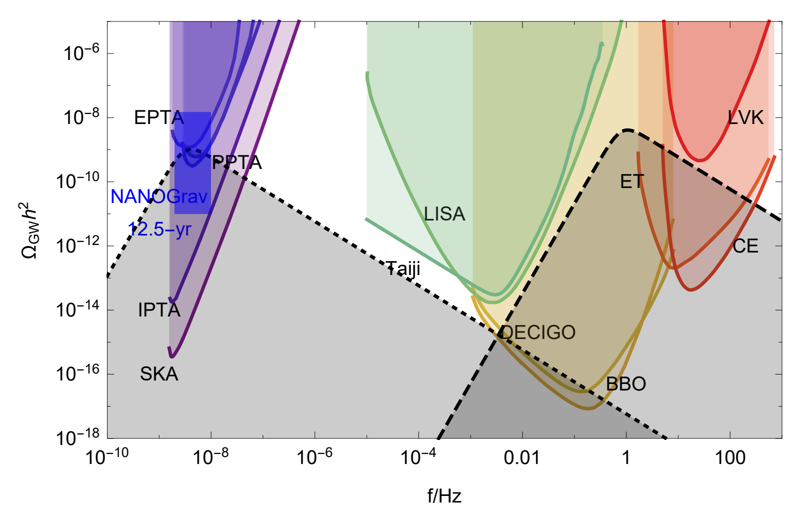
Dalianis, Kouvaris [2012.09255]



Phase transitions:

Liu+ [2106.05637]



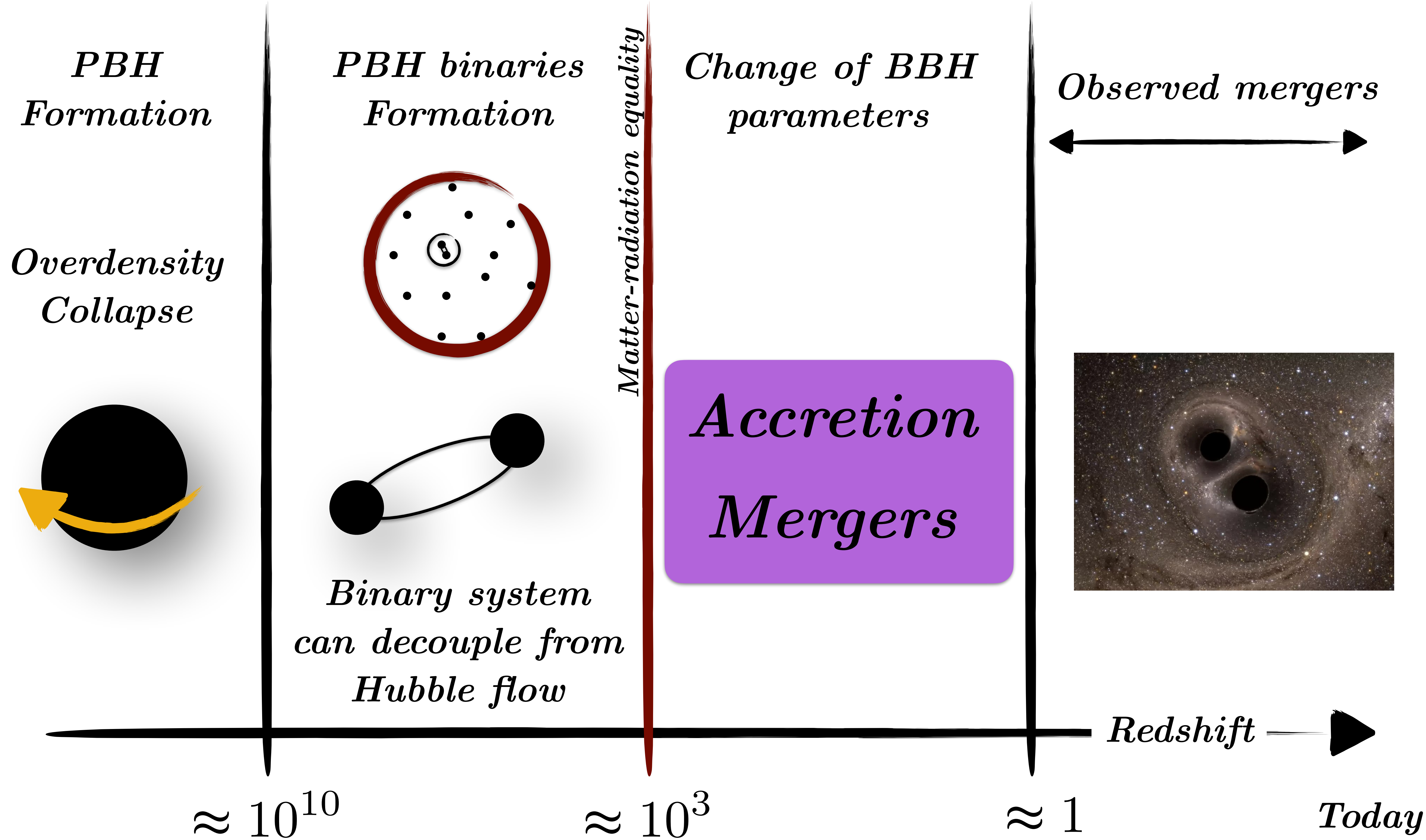


?

+ Cosmic strings, etc. ...

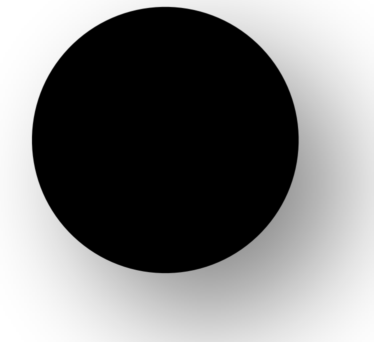
part II: PBH mergers

The PBH timeline



(One may also form binaries in the late time universe)

PBH: masses and spins



- ***Mass distribution depends on the curvature spectrum and statistical properties:***

  - ***Theoretical expectations:***

    i) No gaps in the mass distribution

    ii) Minimum width due to critical collapse

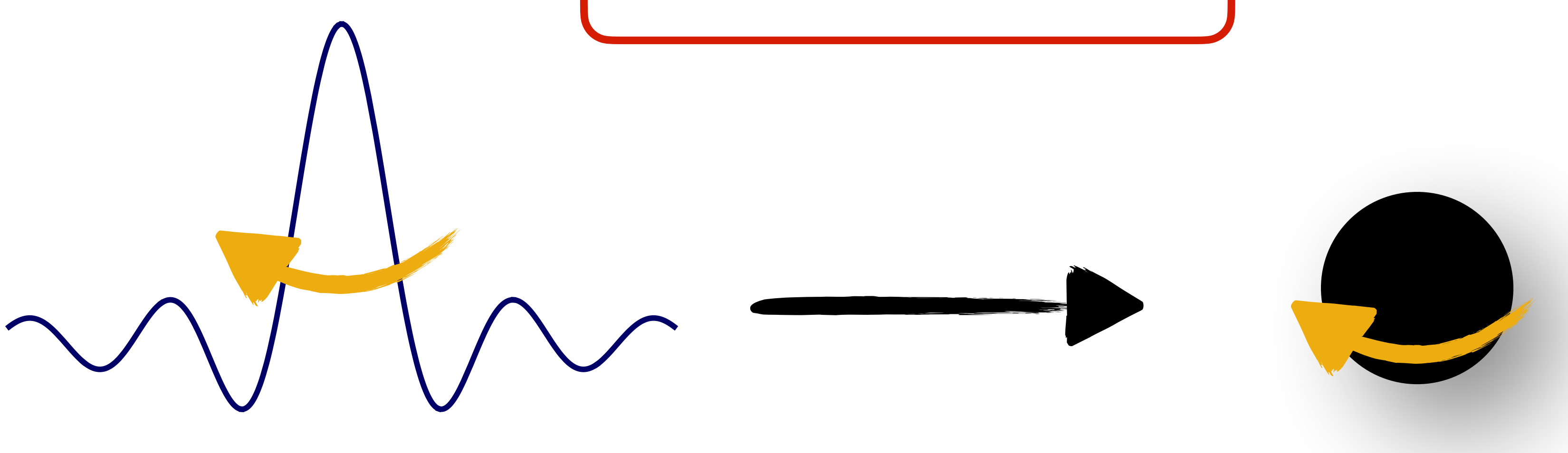
    (effect of thermal features around QCD epoch...)

Extreme peaks $\longrightarrow$ ***close to spherical symmetry*** Bardeen+ (1986)

Small torques on the collapsing radiation overdensity

De Luca, Desjacques, **GF**, Malhotra, Riotto JCAP (2019)

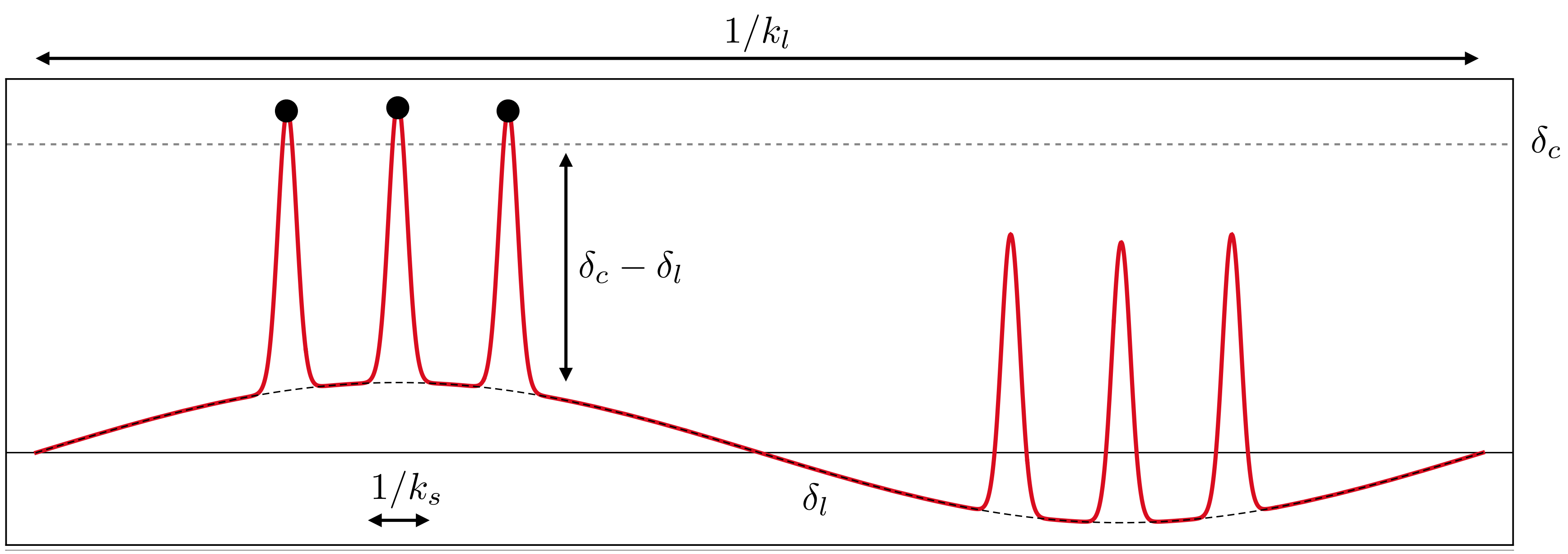
$$\boxed{|S|/m^2 \lesssim 10^{-2}}$$



PBH Clustering

In the standard scenario, PBHs are not clustered at formation

$$\left\langle \frac{\delta\rho_{\rm PBH}(\vec{x},z)}{\bar{\rho}_{\rm DM}} \frac{\delta\rho_{\rm PBH}(0,z)}{\bar{\rho}_{\rm DM}} \right\rangle = \frac{f^2_{\rm PBH}}{n_{\rm PBH}} \delta_{\rm D}(\vec{x}) + \cancel{\xi(x,z)}$$



Tada & Yokoyama (2015), Young & Byrnes (2015)...

For gaussian perturbations:

$$\delta_l \simeq \nu/\sigma (R_{\rm PBH}/R_{\rm cl})^2 \zeta(k_{\rm cl}) \ll 1$$

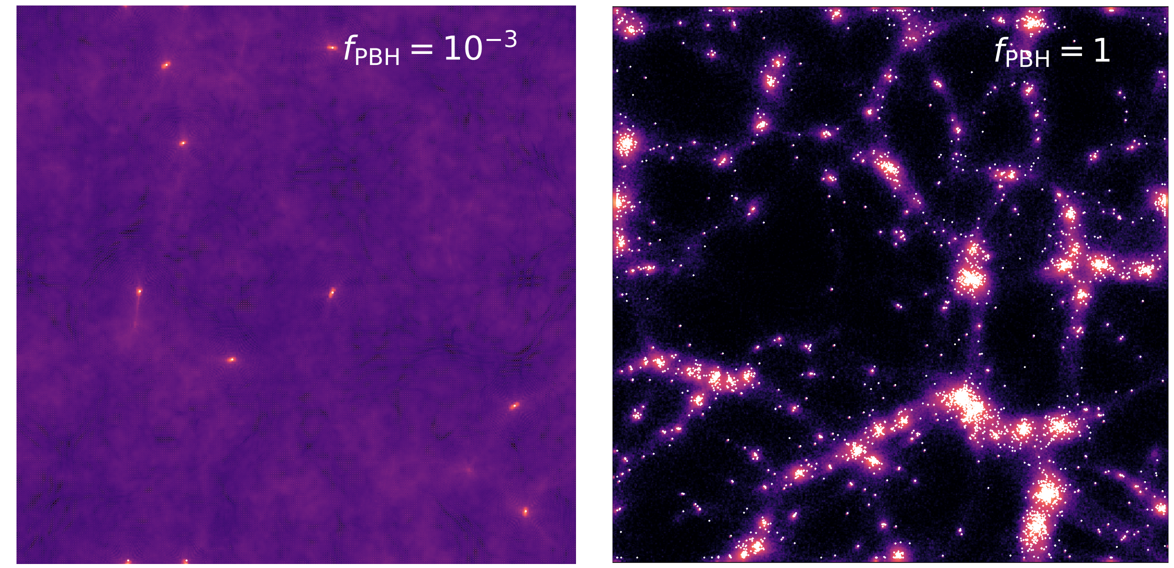
To generate initial clustering, need local NG at the charecteristic PBH distance scale

PBH Clustering evolution

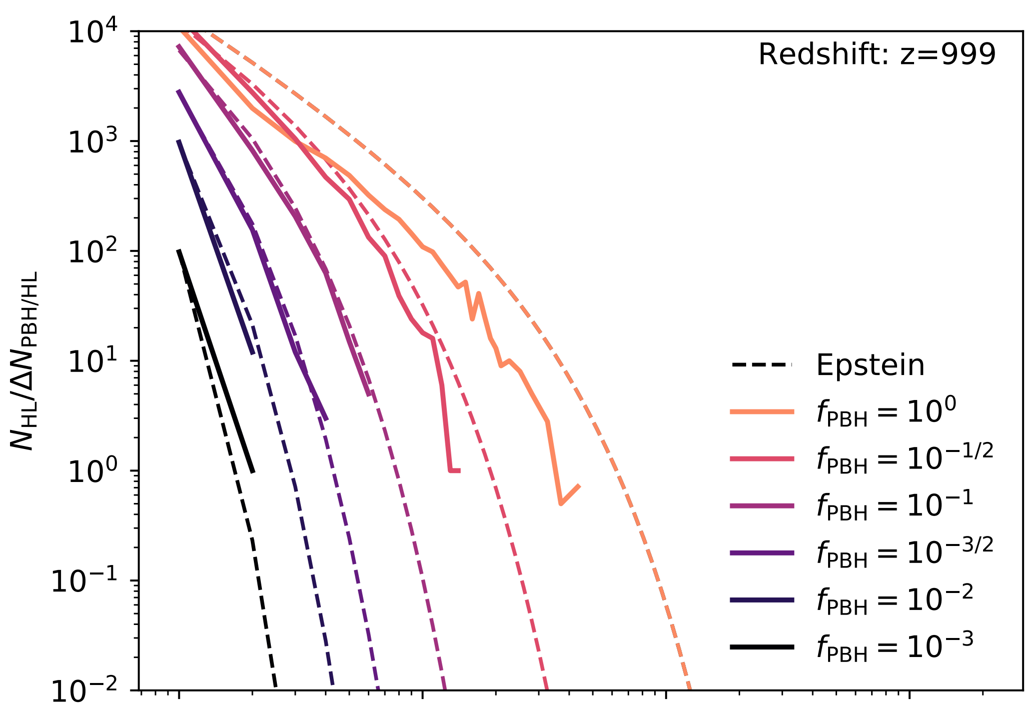
$z \approx 100$ Inman, Ali-Haïmoud PRD (2019)

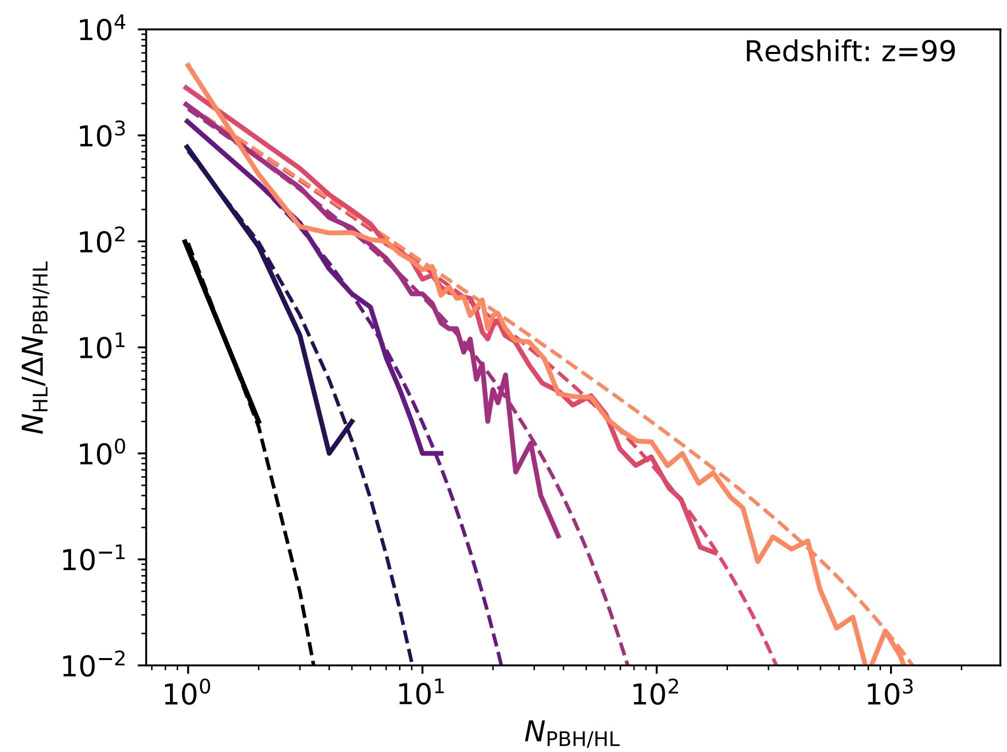
- ***Shot noise drives early structure formation, depending on $f_{\rm PBH}$***
- ***PBHs isolated for:***

$$f_{\rm PBH} \lesssim z \cdot 10^{-4}$$



PBH small scale properties well described by Press-Schechter theory



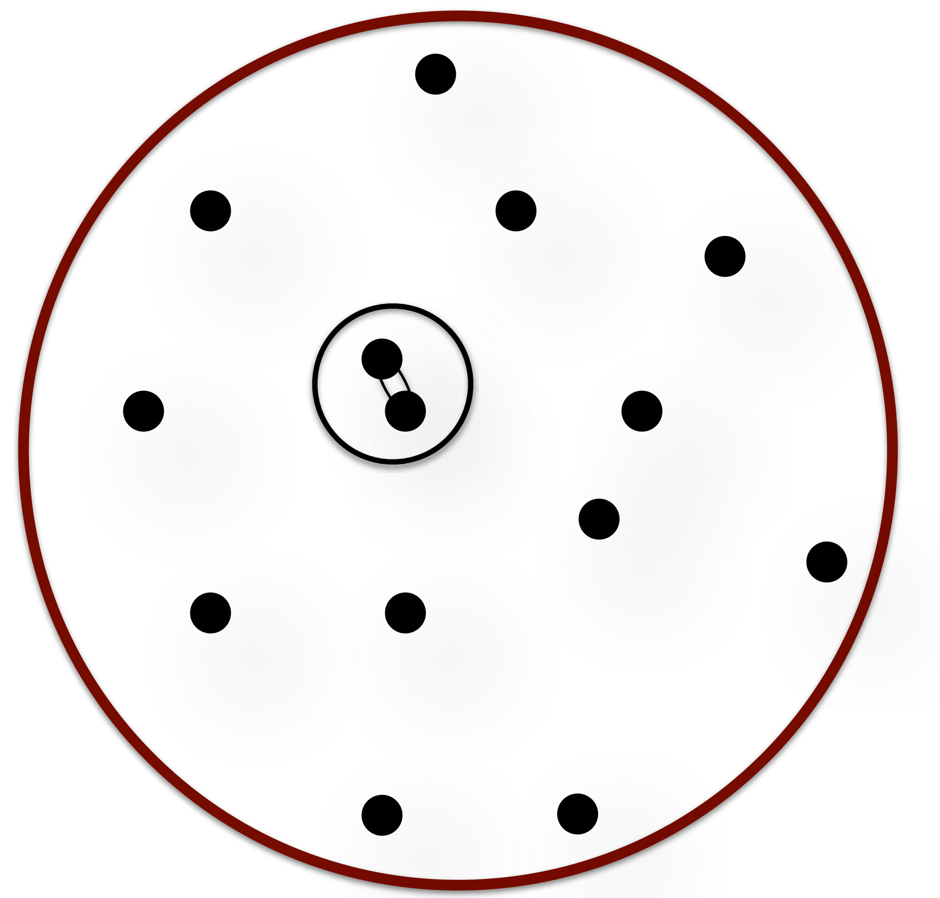


See also De Luca,**GF**+ [2009.04731]

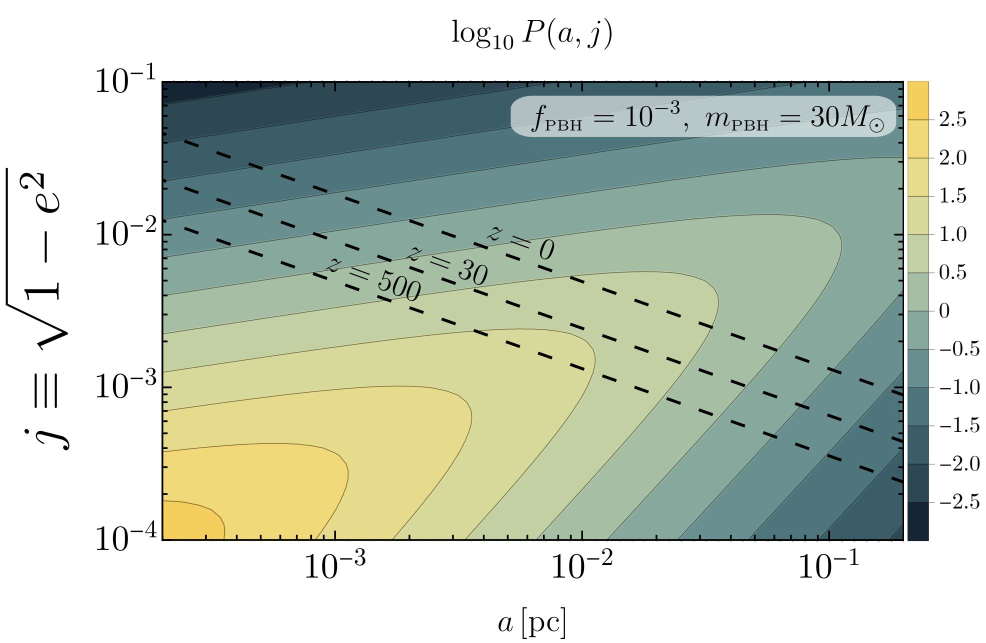
PBH binary properties

- *Initial spatial Poisson distribution*
- *Random decoupling of binary systems from the Hubble flow* Nakamura (1997), ...
- *Binary formation happening before matter-radiation equality*
- *The distribution of initial semi-major axis and eccentricity determines the merger rate*

(Peters' time-scale $t_{\rm GW} \propto a^4(1-e^2)^{7/2}$ *)*

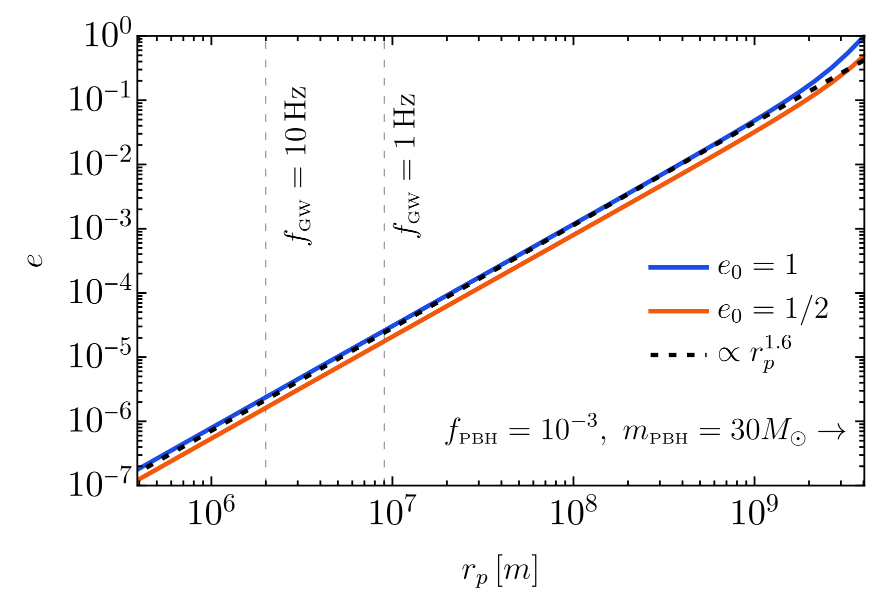


Initial large eccentricity...

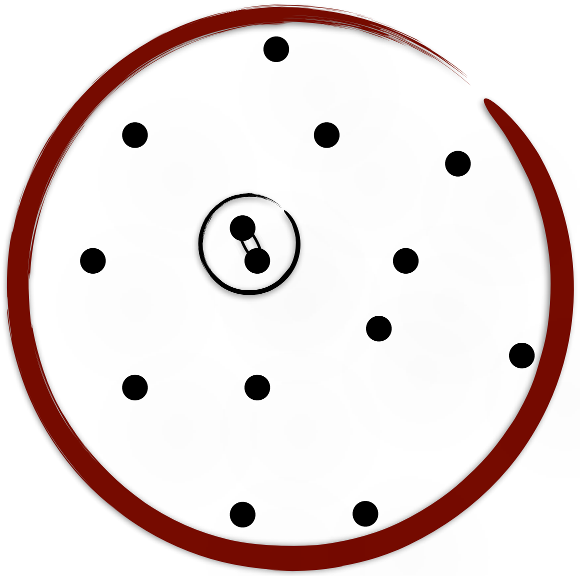


$e \propto f_{\rm GW}^{-19/18}$

...lost before entering LVK band



Merger rate



Nakamura+ (1997), ...

- ***Decoupling of binary systems from Hubble flow***

Ali-Haïmoud+(2017), Raidal+ (2018), ...

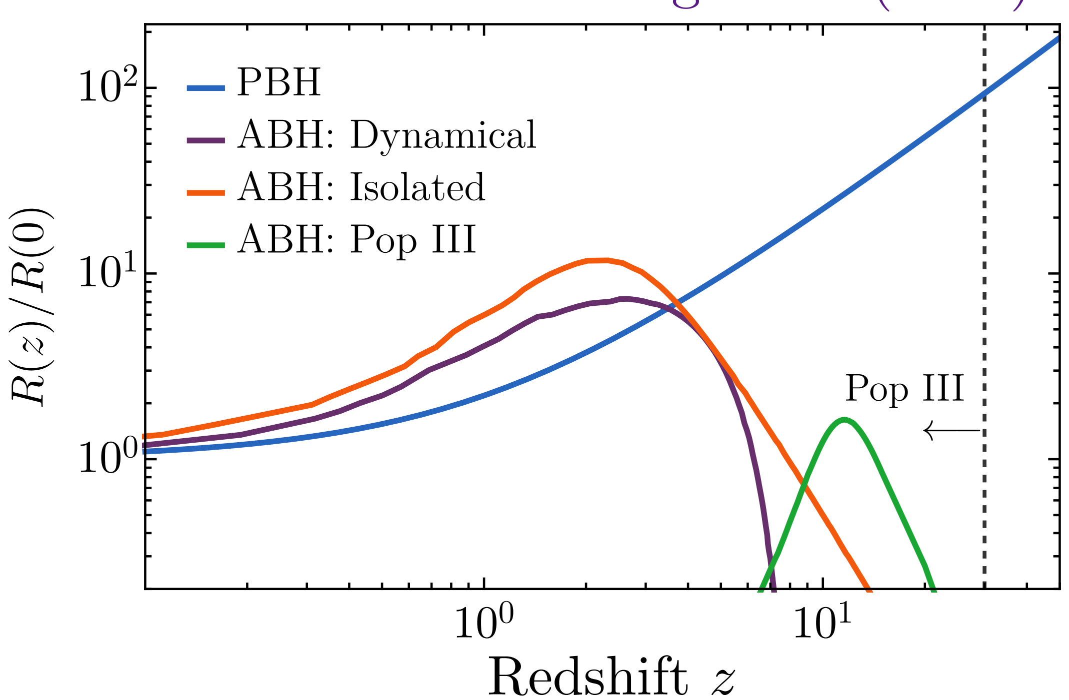
$$\frac{\mathrm{d}R}{\mathrm{d}m_1\mathrm{d}m_2} = \frac{1.6\times10^6}{\mathrm{Gpc}^3\,\mathrm{yr}} f_{\mathrm{PBH}}^{\frac{53}{37}} \eta^{-\frac{34}{37}} \left(\frac{t}{t_0}\right)^{-\frac{34}{37}} \left(\frac{M_{\mathrm{tot}}}{M_\odot}\right)^{-\frac{32}{37}} S\left(M_{\mathrm{tot}}, f_{\mathrm{PBH}}\right) \mathcal{A}_{\mathrm{acc}}(m_j)\psi(m_1)\psi(m_2)$$

- ***The abundance sets the merger rate:*** $R \propto f_{\mathrm{PBH}}^2$

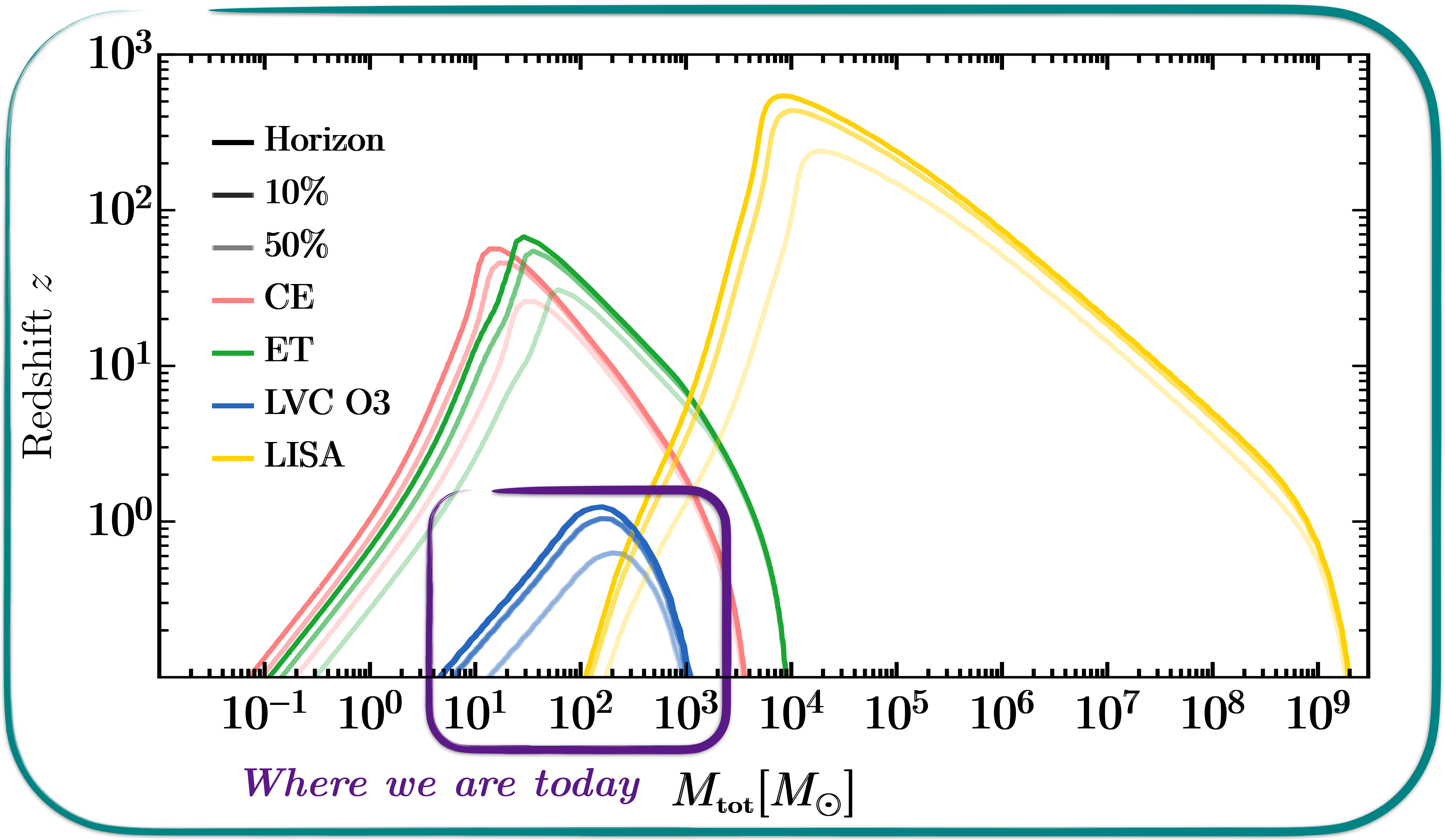
- ***Merger rate evolution with redshift:***

$$\boxed{R \approx t^{-34/37} \textit{ up to } z \gtrsim 10^3}$$

astro from Ng et al. (2020)



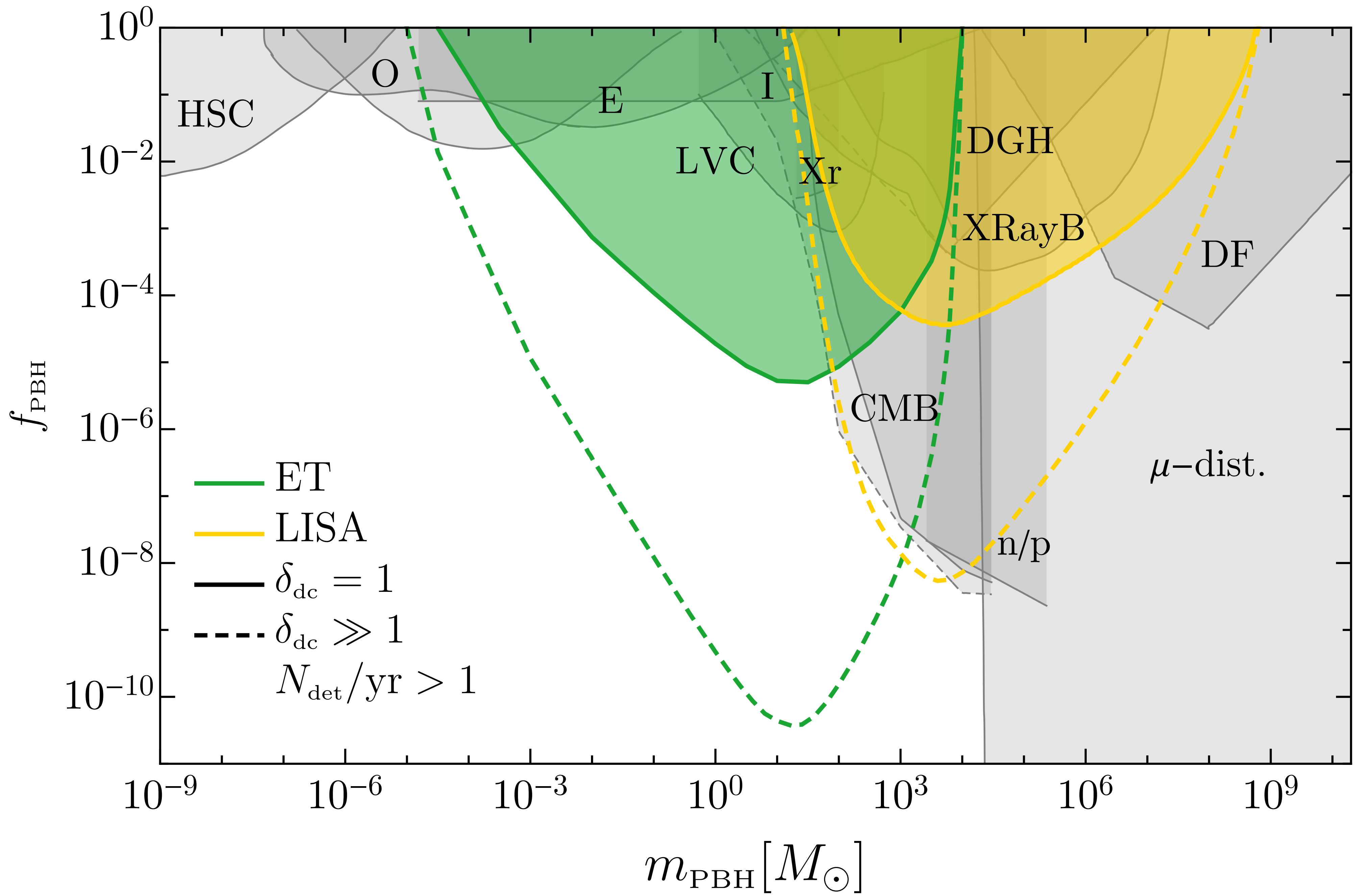
Future gravitational wave horizon



Where we'll be in 15 years

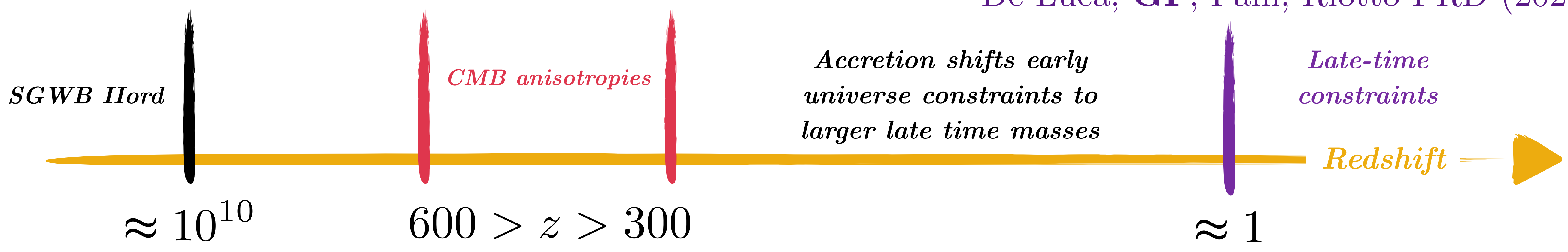
Naive comparison with constraints

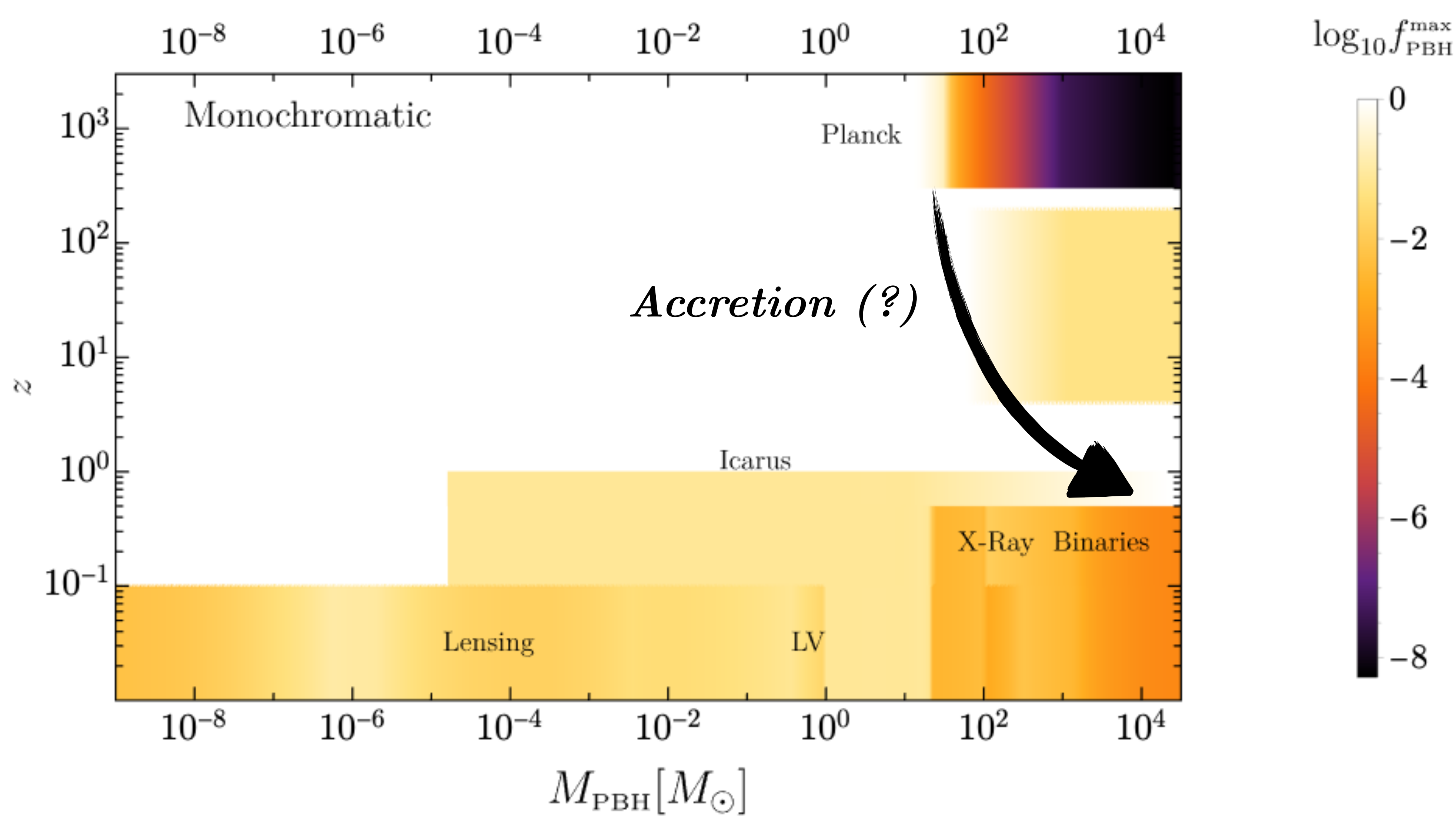
Detectable mergers from narrow mass distribution



The role of accretion

De Luca, **GF**, Pani, Riotto PRD (2020)





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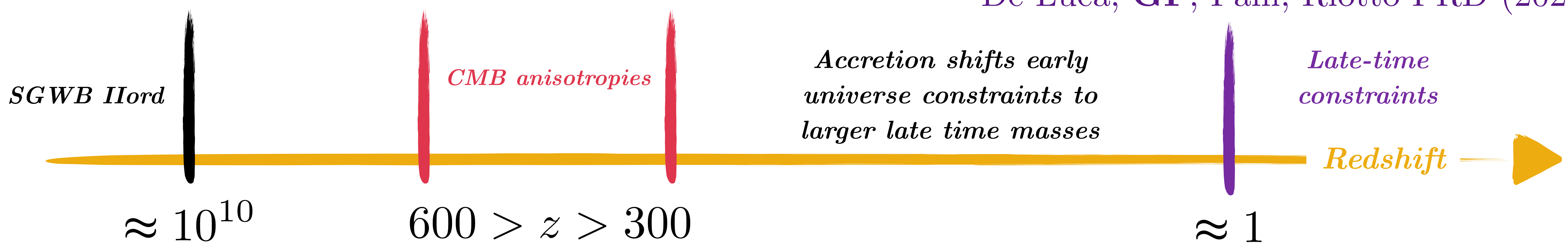
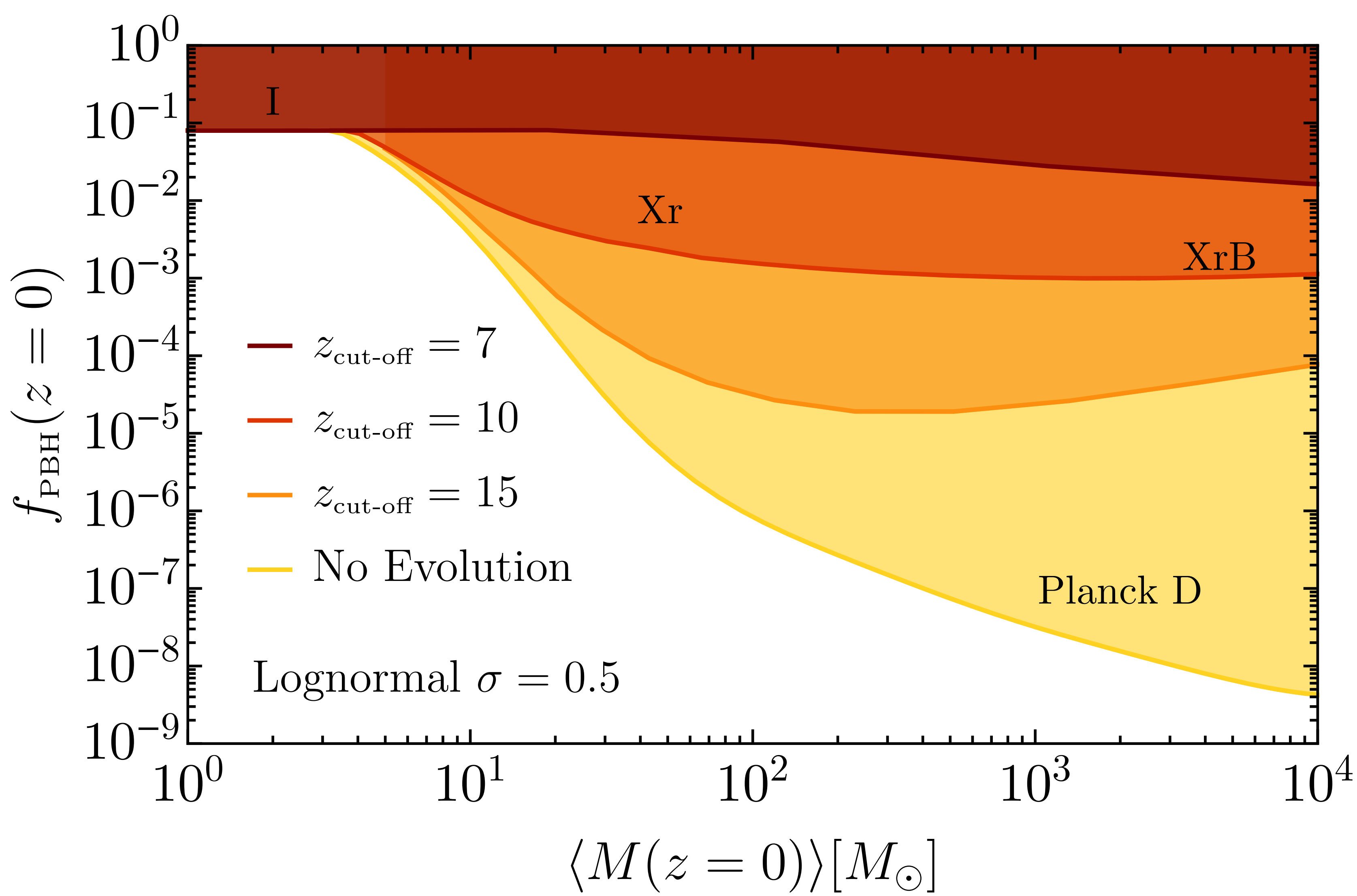


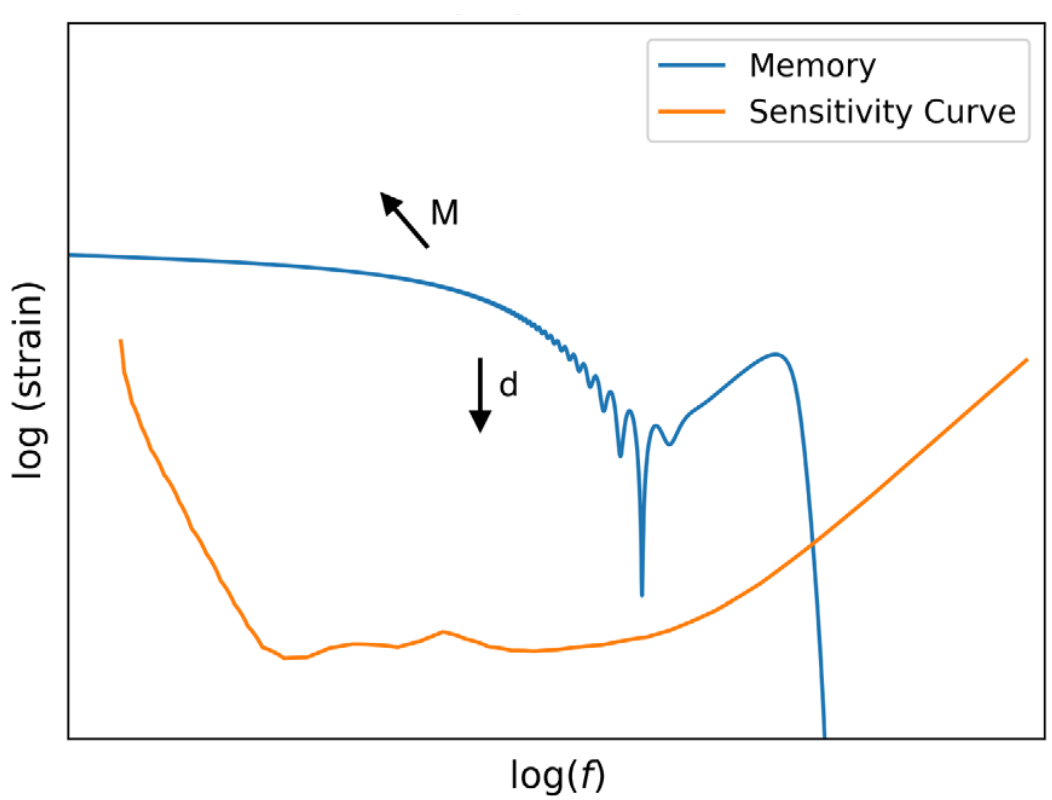


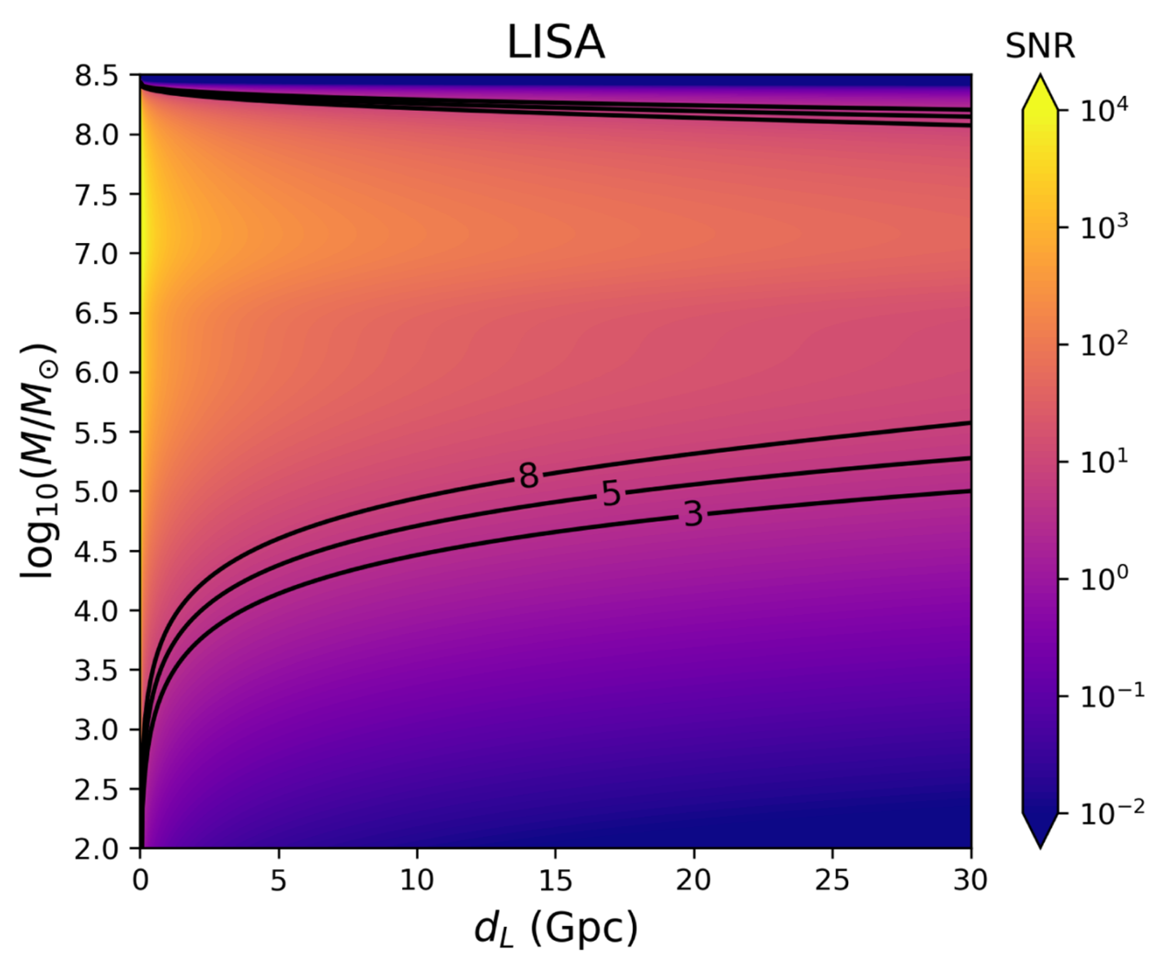
Bonus slide (straight from the google doc)

- Q2. Can the memory effect be used to detect subsolar (primordial) black holes with LISA?

No, unfortunately.

$$\kappa = \sqrt{\frac{\langle h^2_{\rm mem}\rangle}{\langle h^2_{\rm osc}\rangle}} \simeq 1/20$$ McNeill+ PRL 2017





Johnson+ [1810.09563]

Testing Primordial Black Hole as Dark matter with LISA

Burning questions

SGWB vs PBH formation

- *Do we control PBH formation systematics well enough?*
- *Can alternative formation scenario evade LISA constraint?*

PBH mergers

- *Can we reduce uncertainties in the PBH accretion model?*
- *Can PBH be the seeds for SMBHs and comply to bounds?*

(Clustering? EMRIs rates? ...)

Thanks!