Anabasis and Accidental Symmetry

Achilleas P. Porfyriadis

University of Crete

Solvay workshop "Near-Extremal Black Holes," Sept 2024

H.F.R.I. Hellenic Foundation for Research & Innovation

Greece 2.0 NATIONAL RECOVERY AND RESILIENCE PLAN

Funded by the European Union NextGenerationEU

Why extreme black holes?

Two reasons:

- They are observationally relevant:

  Many accreting black holes are found to be spinning very rapidly

- They are theoretically manageable:

  Near the horizon of (near-)extreme black holes spacetime is AdS-like

Rapidly spinning black holes

Annual Review of Astronomy and Astrophysics

Observational Constraints on Black Hole Spin

Christopher S. Reynolds[1]

[1]Institute of Astronomy, University of Cambridge, Cambridge, CB3 OHA, United Kingdom; email: csr12@ast.cam.ac.uk

Annu. Rev. Astron. Astrophys. 2021. 59:117–54

The *Annual Review of Astronomy and Astrophysics* is online at astro.annualreviews.org

https://doi.org/10.1146/annurev-astro-112420-035022

Keywords

active galactic nuclei, accretion disks, general relativity, gravitational waves, jets

Abstract

The spin of a black hole is an important quantity to study, providing a window into the processes by which a black hole was born and grew. Furthermore, spin can be a potent energy source for powering relativistic jets and energetic particle acceleration. In this review, I describe the techniques currently used to detect and measure the spins of black holes. It is shown that:

- Two well-understood techniques, X-ray reflection spectroscopy and thermal continuum fitting, can be used to measure the spins of black holes that are accreting at moderate rates. There is a rich set of other electromagnetic techniques allowing us to extend spin measurements to lower accretion rates.
- Many accreting supermassive black holes are found to be rapidly spinning, although a population of more slowly spinning black holes emerges at masses above $M > 3 \times 10^7\,M_\odot$ as expected from recent structure formation models.
- Many accreting stellar-mass black holes in X-ray binary systems are rapidly spinning and must have been born in this state.
- The advent of gravitational wave astronomy has enabled the detection of spin effects in merging binary black holes. Most of the premerger

Rapidly spinning black holes

Many accreting black holes are found to be spinning very rapidly

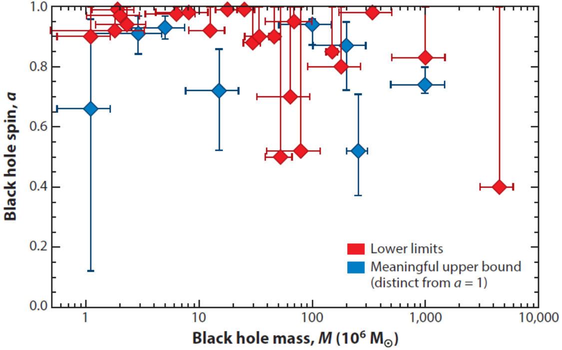


Figure 6

SMBH spins as a function of mass for the 32 objects in **Table 1** that have available mass estimators. All spin measurements reported here are from the X-ray reflection method. Lower limits are reported in red, and measurements that include a meaningful upper bound (distinct from $a = 1$) are reported in blue. Following the convention of the relevant primary literature, error bars in spin show the 90% confidence range. The error bars in mass are the 1σ errors from **Table 1** or, where that is not available, we assume a $\pm 50\%$ error. Abbreviation: SMBH, supermassive black hole.

Rapidly spinning black holes

Table 2 Measurements of black hole spin in dynamically confirmed black hole X-ray binaries from the X-ray reflection and CF methods[a]

Object	Spin from reflection	Spin from CF	Reference from reflection	Reference from CF
IC 10 X-1	ND	$0.85^{+0.04}_{-0.07}$	ND	Steiner et al. (2016)
M33 X-7	ND	0.84 ± 0.05	ND	Liu et al. 2008
LMC X 3	ND	$0.25^{+0.20}_{=0.29}$	ND	Steiner et al. (2014)
LMC X 1	>0.55	$0.92^{+0.05}_{-0.07}$	Steiner et al. (2012)	Tripathi et al. (2020)
AO620–00	ND	0.12 ± 0.19	ND	Gou et al. (2010)
Nova Mus 1991	ND	$0.63^{+0.16}_{-0.19}$	ND	Chen et al. (2016)
GS 1354–645	>0.98	ND	El-Batal et al. (2016)	ND
4U 1543–475	$0.67^{+0.15}_{-0.08}$	0.8 ± 0.1	Dong et al. (2020)	Shafee et al. (2006)
XTE J1550–564	0.33–0.77	$0.34^{+0.37}_{-0.45}$	Miller et al. (2009)	Steiner et al. (2011)
4U 1630–472	>0.97	ND	King et al. (2014)	ND
XTE J1650–500	0.79 ± 0.01	ND	Miller et al. (2009)	ND
XTE J1652–453	0.45 ± 0.02	ND	Hiemstra et al. (2011)	ND
GRO J1655–40	$> 0.9^{*}$	0.7 ± 0.1	Reis et al. (2009)	Shafee et al. (2006)
GX339–4	>0.95	ND	J. Jiang et al. 2019	ND
SAX J1711.6–3808	$0.6^{+0.2}_{-0.4}$	ND	Miller et al. (2009)	ND
GRS1716–249	$a > 0.92$[b]		Tao et al. (2019)[b]	
XTE J1752–223	0.92 ± 0.06	ND	García et al. (2018)	ND
Swift J1753.5–0127	$0.76^{+0.11}_{-0.15}$	ND	Reis et al. (2009)	ND
MAXI J1836–194	0.88 ± 0.03	ND	Reis et al. (2012)	ND
EXO 1846–031	>0.99	ND	Draghis et al. (2020)	ND
XTE J1908+094	0.75 ± 0.09	ND	Miller et al. (2009)	ND
Swift J1910.2–0546	<-0.32	ND	Reis et al. (2013)	ND
GRS1915+105	$0.88^{+0.06}_{-0.13}$	>0.95	Shreeram & Ingram (2020)	McClintock et al. (2006)

AdS_2 and near-extreme black holes

Near the horizon of (near-)extreme black holes spacetime is AdS_2-like

- Extreme Reissner-Nordstrom; Bertotti-Robinson: [Bertotti, Robinson (1959)]

$$ds^2 = M^2\left[-r^2dt^2 + \frac{dr^2}{r^2} + d\Omega^2\right], \qquad A_t = Mr$$

- Extreme Kerr; NHEK: [Bardeen, Horowitz (1999)]

$$ds^2 = 2M^2\Gamma(\theta)\left[-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda(\theta)^2(d\phi + r\,dt)^2\right]$$

- Applies for a wide class of theories in any D [Kunduri, Lucietti, Reall (2007)]

  Kinematics of extremal horizon $\rightarrow$ scaling symmetry
  Einstein equations $\rightarrow$ $SL(2)$

- Near-horizon approximations *and* Exact solutions

"AdS_2 has no dynamics"

Anti-de Sitter fragmentation

Juan Maldacena, Jeremy Michelson*and Andrew Strominger

Department of Physics, Harvard University
Cambridge, MA 02138
E-mail: malda@bose.harvard.edu, jeremy@bohr.harvard.edu,
andy@planck.harvard.edu

ABSTRACT: Low-energy, near-horizon scaling limits of black holes which lead to string theory on $AdS_2 \times S^2$ are described. Unlike the higher-dimensional cases, in the simplest approach all finite-energy excitations of $AdS_2 \times S^2$ are suppressed. Surviving zero-energy configurations are described. These can include tree-like structures in which the $AdS_2 \times S^2$ throat branches as the horizon is approached, as well as disconnected $AdS_2 \times S^2$ universes. In principle, the black hole entropy counts the quantum ground states on the moduli space of such configurations. In a nonsupersymmetric context AdS_D for general D can be unstable against instanton-mediated fragmentation into disconnected universes. Several examples are given.

KEYWORDS: Black Holes in String Theory, Conformal Field Models in String Theory, Supersymmetry and Duality.

JHEP02(1999)011

“AdS_2 has no dynamics”

No dynamics in the extremal Kerr throat

Aaron J. Amsel, Gary T. Horowitz, Donald Marolf and Matthew M. Roberts

Department of Physics, UCSB,
Santa Barbara, CA 93106, U.S.A.
E-mail: amsel@physics.ucsb.edu, gary@physics.ucsb.edu,
marolf@physics.ucsb.edu, matt@physics.ucsb.edu

ABSTRACT: Motivated by the Kerr/CFT conjecture, we explore solutions of vacuum general relativity whose asymptotic behavior agrees with that of the extremal Kerr throat, sometimes called the Near-Horizon Extreme Kerr (NHEK) geometry. We argue that all such solutions are diffeomorphic to the NHEK geometry itself. The logic proceeds in two steps. We first argue that certain charges must vanish at all times for any solution with NHEK asymptotics. We then analyze these charges in detail for linearized solutions. Though one can choose the relevant charges to vanish at any initial time, these charges are not conserved. As a result, requiring the charges to vanish at all times is a much stronger condition. We argue that all solutions satisfying this condition are diffeomorphic to the NHEK metric.

KEYWORDS: Gauge-gravity correspondence, Black Holes, Space-Time Symmetries

JHEP09(2009)044

Wider picture on AdS_2 dynamics

- Backreaction in *asymptotically AdS_2 spacetimes* is problematic.
  - Q: Starting with a linear solution for a scalar ϕ on $AdS_2 \times S^2$, does it extend to a non-linear solution of Einstein-Maxwell-Scalar?
  - A: Not if we insist on an asymptotically AdS_2 solution. E.g. if we impose Dirichlet boundary conditions on the AdS_2 boundary then backreaction of the scalar on the geometry destroys them.
- Backreaction in *asymptotically flat spacetimes* makes perfect sense.
  - Q: Starting with a linear solution for a scalar $\phi \sim \sqrt{\epsilon}$ on ERN, does it extend to a non-linear solution of Einstein-Maxwell-Scalar?
  - A: Yes. Generically the fully backreacted nonlinear endpoint is a near-extreme RN with $Q = M\sqrt{1 - \mathcal{O}(\epsilon)}$. [Murata, Reall, Tanahashi (2013)]

The connection of AdS_2 with the asymptotically flat region of BHs allows for consistent backreaction. How? What are the correct boundary conditions?

1. Anabasis:

Backreaction that destroys the AdS_2 boundary and builds the asymptotically flat region of (near-)extreme BHs.

[JHEP 2103, 223]	*with*	S. Hadar, A. Lupsasca
[JHEP 2303, 125]	*with*	G. Remmen
[WIP]	*with*	M. de Cesare, R. Oliveri

Perturbations of Bertotti-Robinson

- Backgound:

$$ds^2 = M^2\left[-r^2dt^2 + \frac{dr^2}{r^2} + d\Omega^2\right], \qquad A_t = Mr$$

- Spherically symmetric perturbations $(h_{\mu\nu}, a_\mu)$ fully characterized by:

$$h_{\theta\theta} = \Phi_0 + ar + brt + cr\left(t^2 - 1/r^2\right)$$

Comments:

- $h_{\theta\theta}$ is gauge invariant under $h_{\mu\nu} \to h_{\mu\nu} + \mathcal{L}_\xi g_{\mu\nu}$.
- 4-parameter (Φ_0, a, b, c) family of solutions.
- Φ_0 parameterizes overall rescaling $M \to M + \delta M$ with $\Phi_0 = 2M\,\delta M$.
- Focus on the remaining triplet:

$$\Phi = ar + brt + cr\left(t^2 - 1/r^2\right)$$

$SL(2)$ transformation properties

$$\Phi = ar + brt + cr\left(t^2 - 1/r^2\right)$$

- The background is invariant under the $SL(2)$ isometries of AdS_2:

$$\begin{aligned} H: &\quad t \to t + \alpha \\ D: &\quad t \to t/\beta\,, \quad r \to \beta r \\ K: &\quad t \to \frac{t - \gamma\left(t^2 - 1/r^2\right)}{1 - 2\gamma t + \gamma^2\left(t^2 - 1/r^2\right)}\,, \quad r \to r\left[1 - 2\gamma t + \gamma^2\left(t^2 - 1/r^2\right)\right] \end{aligned}$$

- Φ is $SL(2)$-breaking: (a, b, c) get rotated by the above transformations.
- However,

$$\mu = b^2 - 4ac \qquad \text{is} \quad SL(2)\text{-invariant}$$

- Using $SL(2)$ transformations one may set

$$\begin{aligned} \Phi &= 2r\,, && \text{when} \quad \mu = 0\,,\ \mathrm{sgn}(a + c) = 1 \\ \Phi &= -\sqrt{\mu}\,rt\,, && \text{when} \quad \mu > 0 \end{aligned}$$

- $SL(2)$-breaking solutions Φ are *not* asymptotically $AdS_2 \times S^2$

Anabasis perturbations

Bertotti-Robinson arises from two physically distinct near-horizon near-extremality scaling limits, $\lambda \to 0$, of Reissner-Nordstrom

- Limit #1: Begin with $Q = M$ and put the BH horizon at $r = 0$ (set $M = 1$):

$$ds^2 = -\left(\frac{r}{1+\lambda r}\right)^2 dt^2 + \left(\frac{r}{1+\lambda r}\right)^{-2} dr^2 + (1+\lambda r)^2\, d\Omega^2\,, \quad A_t = \frac{r}{1+\lambda r}$$

At $\mathcal{O}(1)$ we get Bertotti-Robinson in Poincare coordinates

$$ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2} + d\Omega^2\,, \qquad A_t = r$$

At $\mathcal{O}(\lambda)$ we get, by definition, a linear solution around the above.

$$h_{\theta\theta} = 2r$$

This is the $SL(2)$-breaking $\mu = 0$ solution $\Phi = 2r$ —Poincare *anabasis solution*

Begins to build the asymptotically flat region of an extreme Reissner-Nordstrom

The nonlinear solution obtained from the $\mu = 0$ perturbation of $AdS_2 \times S^2$, when backreaction is fully taken into account in the Einstein-Maxwell theory, is the extreme Reissner-Nordström black hole.

Anabasis perturbations

- Limit #2: Begin with $Q = M\sqrt{1-\lambda^2\kappa^2}$ and put the BH horizon at $\rho = 0$:

$$ds^2 = -\frac{\rho(\rho+2\kappa+\lambda\kappa\rho)}{(1+\lambda\kappa)(1+\lambda\rho)^2}d\tau^2 + \frac{(1+\lambda\kappa)^3(1+\lambda\rho)^2}{\rho(\rho+2\kappa+\lambda\kappa\rho)}d\rho^2 + (1+\lambda\kappa)^2(1+\lambda\rho)^2 d\Omega^2$$

$$A_\tau = \frac{1}{\lambda}\left(1-\sqrt{\frac{1-\lambda\kappa}{1+\lambda\kappa}}\frac{1}{1+\lambda\rho}\right)$$

At $\mathcal{O}(1)$ we get Bertotti-Robinson in Rindler coordinates

$$ds^2 = -\rho(\rho+2\kappa)d\tau^2 + \frac{d\rho^2}{\rho(\rho+2\kappa)} + d\Omega^2\,, \quad A_\tau = M(\rho+\kappa)$$

At $\mathcal{O}(\lambda)$ we get, by definition, a linear solution around the above.

$$h_{\theta\theta} = 2(\rho+\kappa)$$

Anabasis perturbations

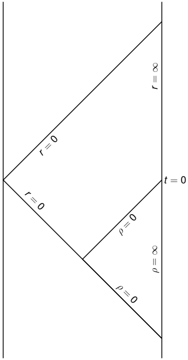
- Rindler to Poincare transformation for the Bertotti-Robinson:

$$\tau = -\frac{1}{2\kappa}\ln\left(t^2 - 1/r^2\right)$$
$$\rho = -\kappa(1 + rt)$$
$$A \to A + d\Lambda\,,\Lambda = \frac{1}{2}\ln\frac{\rho}{\rho + 2\kappa}$$

Transforms the Rindler anabasis solution to

$$h_{\theta\theta} = 2(\rho + \kappa) = -2\kappa rt$$

This is the $SL(2)$-breaking $\sqrt{\mu} = 2\kappa$ solution $\Phi = -2\kappa rt$.
Begins to build the asymptotically flat region of a near-extreme RN
In general, $\Phi = ar + brt + cr(t^2 - 1/r^2)$ with $\mu > 0$, leads to
Rindler anabasis with $\sqrt{\mu} = \sqrt{b^2 - 4ac} = 2\kappa$



The nonlinear solution obtained from the $\mu > 0$ perturbation of $AdS_2 \times S^2$, when backreaction is fully taken into account in the Einstein-Maxwell theory, is the near-extreme Reissner-Nordström black hole with $Q = M\sqrt{1 - \mu/4}$.

The connected $AdS_2 \times S^2$

Definition:

The *connected* $AdS_2 \times S^2$ is defined as the geometry obtained by the addition of anabasis perturbations to Bertotti-Robinson.

The addition of anabasis perturbations may be thought of as boundary condition for consistent backreaction calculation in an $AdS_2 \times S^2$ throat that maintains connection.

- Let's do a matter backreaction calculation in the connected $AdS_2 \times S^2$

$$8\pi T_{vv}^{\text{matter}} = \epsilon\, \delta(v - v_0)\,, \quad v = t - 1/r\,.$$

For $\epsilon \ll 1$ we may find an $\mathcal{O}(\epsilon)$ perturbation around the Bertotti-Robinson that solves the Einstein-Maxwell equations with this matter source:

$$h_{\theta\theta} = \Phi_0 + ar + brt + cr\left(t^2 - 1/r^2\right) - \frac{\epsilon}{2}\frac{r^2(t - v_0)^2 - 1}{r}\Theta(v - v_0)$$

What are the correct boundary conditions?

The connected $AdS_2 \times S^2$

$$h_{\theta\theta} = \Phi_0 + ar + brt + cr\left(t^2 - 1/r^2\right) - \frac{\epsilon}{2}\frac{r^2(t-v_0)^2 - 1}{r}\Theta(v - v_0)$$

- If this $AdS_2 \times S^2$ was connected to extreme RN for $v < v_0$ then we need the Poincare anabasis:

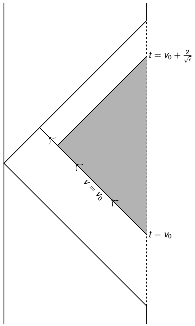
$$h_{\theta\theta} = 2r\,, \quad \text{for} \quad v < v_0\,.$$

Then after the pulse, for $v > v_0$, we get

$$h_{\theta\theta} = 2r - \frac{\epsilon}{2}\frac{r^2(t-v_0)^2 - 1}{r}\,, \quad \text{for} \quad v > v_0\,.$$

This solution after the pulse is a $\mu = 4\epsilon$ solution which may be mapped to the Rindler anabasis $h_{\theta\theta} = 2(\rho + \kappa)$ with

$$\kappa = \sqrt{\epsilon}$$



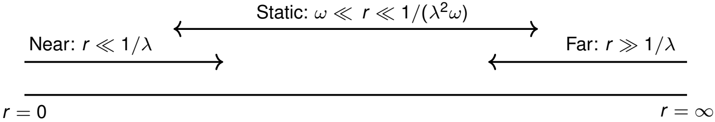
Throwing a pulse of energy $\epsilon \ll 1$ into the extreme RN, $Q = M$, "shifts the horizon" and the black hole becomes near extreme with $Q = M\sqrt{1-\epsilon}$. ✓

Deriving the connected $AdS_2 \times S^2$ via MAE [WIP]

- Consider ERN or BR in the Einstein-Maxwell-Scalar theory with $\phi = \mathcal{O}(\sqrt{\epsilon})$ and solve for the $\mathcal{O}(\epsilon)$ perturbation in both cases.
- Impose the boundary conditions in ERN, e.g.

$$h_{\theta\theta}^{\text{ERN}} \sim \int dr \left[\frac{(1+\lambda r)^3}{2r^2} \int dv \, (\partial_v \phi^{\text{ERN}})(\partial_r \phi^{\text{ERN}}) \right] \to 0 \,, \quad \text{for} \quad v \to -\infty \,.$$

- Ensure the scalar is matched, in Fourier space, using the method of Matched Asymptotic Expansions for low energy, $\lambda\omega \ll 1$, ERN modes



- Observe that this leads to

$$h_{\theta\theta}^{\text{BR}} = h_{\theta\theta}^{\text{ERN}}\big|_{\text{Near}} \to 2r \,, \quad \text{for} \quad v \to -\infty \,.$$

Summary

Anabasis: Backreaction that destroys the AdS_2 boundary and builds the asymptotically flat region of (near-)extreme BHs.

Connected AdS_2: The geometry obtained by the addition of anabasis perturbations $\Leftrightarrow$ Boundary condition for consistent backreaction calculation within the throat.

Remarks

- Q: What is the dual of anabasis in AdS/CFT?
  A(?): Following inverse RG, from IR to UV, along an irrelevant deformation of the boundary field theory that does *not* respect AdS boundary conditions (e.g. the single-trace $T\overline{T}$ deformation of CFT_2 studied by [Giveon, Itzhaki, Kutasov, et al 2017–]).
  Q: What is it for AdS_2 though? A: No idea
- Q: What about JT gravity?
  A: $\Phi = \Phi_{JT}$ solves the JT eom $\nabla_\mu \nabla_\nu \Phi_{JT} - g_{\mu\nu} \nabla^2 \Phi_{JT} + g_{\mu\nu} \Phi_{JT} = 0$ on AdS_2.
  $\mu =$ ADM mass of the 2D black holes in JT gravity.
  Connected AdS_2 is a "nearly-AdS_2" with $SL(2)$ broken to maintain connection

2. Accidental Symmetry:

Coordinate transformation that acts on the perturbative solutions of Einstein equation near extreme black hole horizon

[JHEP 2203, 107] *with* G. Remmen
[WIP] *with* A. Banerjee, G. Remmen

The linearized Einstein equation

Schematic notation:

- Background geometry $\bar{g}$ —the Bertotti-Robinson spacetime
- Metric perturbation h —the Φ solution
- The linearized Einstein equation as a linear differential operator

$$\mathcal{E}(\bar{g}, h) = 0$$

Consider a finite diffeomorphism

$$(t, r) \to (t, r) + \lambda \left(\xi^t(t, r), \xi^r(t, r)\right)$$

which transforms both $\bar{g} \to \bar{g}(\lambda)$ and $h \to h(\lambda)$.

By general covariance, for *arbitrary* λ and ξ^μ, we have:

$$\mathcal{E}(\bar{g}(\lambda), h(\lambda)) = 0$$

Expanding in λ, we have

$$\mathcal{E}(\bar{g}(0), h(0)) + \lambda \frac{\delta}{\delta\lambda} \mathcal{E}(\bar{g}(\lambda), h(0)) + \lambda \frac{\delta}{\delta\lambda} \mathcal{E}(\bar{g}(0), h(\lambda)) + \mathcal{O}(\lambda^2) = 0$$

Accidental symmetry: definition

Starting with a solution to the linearized Einstein equations around the original background, $\mathcal{E}(\bar{g}(0), h(0)) = 0$, we have

$$\lim_{\lambda \to 0} [\partial_\lambda \mathcal{E}(\bar{g}(\lambda), h(0)) + \partial_\lambda \mathcal{E}(\bar{g}(0), h(\lambda))] = 0 \tag{1}$$

- 1st term: hold perturbation fixed, act with a linearized diffeo on the background
- 2nd term: on fixed background, transform perturbation using linearized diffeo

Equation (1) is valid for any diffeo, i.e. for any ξ^μ.

What if we impose the strong requirement that each term in (1) vanishes individually?

$$\lim_{\lambda \to 0} \partial_\lambda \mathcal{E}(\bar{g}(0), h(\lambda)) = 0 \tag{2}$$

- Trivial solutions: Isometries of the background $\bar{g}(\lambda) = \bar{g}(0)$
- Other solution: *accidental symmetry* —transforms solns h among themselves

Accidental symmetry: electrovacuum case

$\mathcal{E}$: linearized Einstein-Maxwell equations (electrovacuum)
$\bar{g}(0)$: Bertotti-Robinson
$h(0)$: $\Phi = a\, r$ ($\mu = 0$ solution)

the solution of $\lim_{\lambda\to 0} \partial_\lambda \mathcal{E}(\bar{g}(0), h(\lambda)) = 0$ is given by

$$\xi = -\left[\epsilon(t) + \frac{\epsilon''(t)}{2r^2} + \frac{t\epsilon'''(t)}{r^2}\right]\partial_t + \left[r\epsilon'(t) - \frac{\epsilon'''(t)}{2r}\right]\partial_r,$$

where $\epsilon(t)$ is an arbitrary cubic polynomial in t,

$$\epsilon(t) = e_0 + e_1 t + e_2 t^2 + e_3 t^3.$$

- $\xi_{0,1,2}$: SL(2) Killing vectors of AdS_2

$$\xi_0 = -(1, 0)\,, \quad \xi_1 = -(t, -r)\,, \quad \xi_2 = -\left(t^2 + \frac{1}{r^2}, -2rt\right)$$

- ξ_3: non-trivial accidental symmetry

$$\xi_3 = -\left(t^3 + \frac{9t}{r^2}, \frac{3}{r} - 3rt^2\right)$$

Accidental symmetry: electrovacuum equations

Question: What does ξ_3 do?

Answer: Relates $\mu = 0$ to $\mu \neq 0$. Indeed, we have

$$\Delta\mu = -4a\Delta c = -12\lambda e_3 a^2$$

Accidental symmetries enlarge the possible mappings among solutions to include those beyond the SL(2) isometries, thereby allowing to move from one μ orbit to another.

In spherical symmetry the electrovacuum solutions are constrained by Birkhoff's theorem to the non-propagating degrees of freedom that we have discussed so far.

Can accidental symmetries also turn on propagating d.o.f.?

Accidental symmetry: electrovacuum equations

Question: What does ξ_3 do?

Answer: Relates $\mu = 0$ to $\mu \neq 0$. Indeed, we have

$$\Delta\mu = -4a\Delta c = -12\lambda e_3 a^2$$

Accidental symmetries enlarge the possible mappings among solutions to include those beyond the SL(2) isometries, thereby allowing to move from one μ orbit to another.

In spherical symmetry the electrovacuum solutions are constrained by Birkhoff's theorem to the non-propagating degrees of freedom that we have discussed so far.

Can accidental symmetries also turn on propagating d.o.f.?

Accidental symmetry: adding matter

$$\lim_{\lambda\to 0} \partial_\lambda \mathcal{E}(\bar{g}(0), h(\lambda)) = T \qquad (3)$$

Source T must satisfy equations of motion. We consider Klein-Gordon scalar $\Box\phi = 0$ s.t. the most general spherically symmetric solution is ($u = t - 1/r$, $v = t + 1/r$)

$$\phi = f_+(v) + f_-(u)$$

Can get solution to (3) from the electrovacuum $\Phi = r$ using the transformation

$$\begin{aligned}\xi^t = \; & \frac{3}{2r}[F_+'(v) + F_-'(u)] - \frac{3}{2r^2}[F_+''(v) - F_-''(u)] \\ & + \frac{3}{r^3}\left[\int^v \frac{F_+(t_0)}{(t-t_0)^4}\,\mathrm{d}t_0 + \int^u \frac{F_-(t_0)}{(t-t_0)^4}\,\mathrm{d}t_0\right] \\ & - \frac{1}{r^3}\int^r \int^t \frac{f_+'\left(\hat{t} + \frac{1}{\hat{r}}\right) f_-'\left(\hat{t} - \frac{1}{\hat{r}}\right)}{\hat{r}}\,\mathrm{d}\hat{t}\,\mathrm{d}\hat{r}\end{aligned}$$

$$\xi^r = r[F_+'(v) - F_-'(u)] - [F_+''(v) + F_-''(u)],$$

where $F_+''''(v) = [f_+'(v)]^2$ and $F_-''''(u) = [f_-'(u)]^2$.

Accidental symmetry as on-shell large diffeo of AdS_2

Putting on-shell the large diffeomorphisms of AdS_2 in JT gravity

- The large diffeomorphisms of AdS_2, in FG gauge, are given by

$$t \to f(t) + \frac{2f''(t)f'(t)^2}{4r^2 f'(t)^2 - f''(t)^2}, \qquad r \to \frac{4r^2 f'(t)^2 - f''(t)^2}{4rf'(t)^3}$$

$$\mathrm{d}s_2^2 \to -r^2\left(1 + \frac{\mathrm{Sch}(f,t)}{2r^2}\right)^2 \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{r^2} \qquad \text{and} \qquad \Phi \to \phi_0(t)r + \frac{v(t)}{r},$$

with $\phi_0(t) = [a + bf(t) + cf(t)^2]/f'(t)$ and $v(t) = -[\phi_0''(t) + \mathrm{Sch}(f,t)\phi_0(t)]/2$.

- For *arbitrary* f, this source satisfies the Schwarzian equation of motion

$$\left[\frac{1}{f'}\left(\frac{(f'\phi_0)'}{f'}\right)'\right]' = 0$$

- If one imposes that $\phi_0(t) = \text{constant}$, before as well as after acting with the large diffeo, then for infinitesimal diffeo $f(t) = t + \epsilon(t)$, the Schwarzian eom reduces to

$$\epsilon''''(t) = 0$$

with its cubic solution $\epsilon(t) = e_0 + e_1 t + e_2 t^2 + e_3 t^3$. ✓

Accidental symmetry as on-shell large diffeo of AdS_2

Putting on-shell the large diffeomorphisms of AdS_2 in JT gravity

- The large diffeomorphisms of AdS_2, in FG gauge, are given by

$$t \to f(t) + \frac{2f''(t)f'(t)^2}{4r^2f'(t)^2 - f''(t)^2}, \qquad r \to \frac{4r^2f'(t)^2 - f''(t)^2}{4rf'(t)^3}$$

$$\mathrm{d}s_2^2 \to -r^2\left(1 + \frac{\mathrm{Sch}(f,t)}{2r^2}\right)^2 \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{r^2} \qquad \text{and} \qquad \Phi \to \phi_0(t)r + \frac{v(t)}{r},$$

  with $\phi_0(t) = [a + bf(t) + cf(t)^2]/f'(t)$ and $v(t) = -[\phi_0''(t) + \mathrm{Sch}(f,t)\phi_0(t)]/2$.
- For *arbitrary* f, this source satisfies the Schwarzian equation of motion

$$\left[\frac{1}{f'}\left(\frac{(f'\phi_0)'}{f'}\right)'\right]' = 0$$

- If one imposes that $\phi_0(t) = \text{constant}$, before as well as after acting with the large diffeo, then for infinitesimal diffeo $f(t) = t + \epsilon(t)$, the Schwarzian eom reduces to

$$\epsilon''''(t) = 0$$

  with its cubic solution $\epsilon(t) = e_0 + e_1 t + e_2 t^2 + e_3 t^3$. ✓

Summary

Accidental Symmetry: Coordinate transformation that acts on the perturbative solutions of Einstein equation near extreme black hole horizon and maps them among themselves.

- Electrovacuum eqs: turn on deviation from extremality
- Adding KG matter: turn on arbitrary KG source
- May be thought of: on-shell large diffeomorphisms of AdS_2

- What's next? WIP: Work out analogous statement in pure gravity for NHEK. New feature: turn on axisymmetric gravitational waves in NHEK.

The end

Summary

Accidental Symmetry: Coordinate transformation that acts on the perturbative solutions of Einstein equation near extreme black hole horizon and maps them among themselves.

- Electrovacuum eqs: turn on deviation from extremality
- Adding KG matter: turn on arbitrary KG source
- May be thought of: on-shell large diffeomorphisms of AdS_2

- What's next? WIP: Work out analogous statement in pure gravity for NHEK. New feature: turn on axisymmetric gravitational waves in NHEK.

The end

Acknowledgments

The research project is implemented in the framework of H.F.R.I. call “Basic Research Financing (Horizontal support of all Sciences)” under the National Recovery and Resilience Plan “Greece 2.0” funded by the European Union - NextGenerationEU (HFRI Project Number: 015384)

H.F.R.I.
Hellenic Foundation for Research & Innovation

Greece 2.0
NATIONAL RECOVERY AND RESILIENCE PLAN

Funded by the European Union
NextGenerationEU