Chiara Toldo | Milano U.

Thermodynamics of spinning black holes near extremality

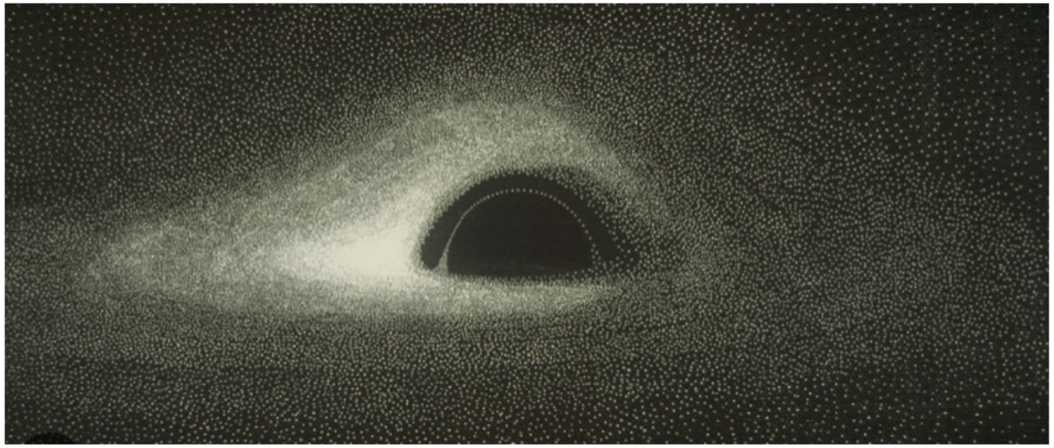

Image by Jean-Pierre Luminet, 1979

Solvay workshop on "Near-extremal black holes",

ULB, September 2–4, 2024

Based on work in collaboration with D. Kapec, A. Sheta, A. Strominger & M. Heydeman

Black hole thermodynamics

The correspondence between laws of black hole mechanics and laws of thermodynamics dates back 50 years

$$\delta E = \frac{\kappa}{8\pi}\delta A + \phi\delta Q + \Omega_H \delta J$$

Not just an analogy, but deep lesson that drives the progress in quantum gravity

According to General Relativity: black hole is simple object, characterized by M, Q, J. However S_{BH} is huge

Black hole entropy scales as area, not volume: it shows a "holographic" behavior.

Extremal black holes

The existence of a horizon imposes bounds on the charges

$$J \leqslant M^2 \qquad\qquad |Q| \leqslant M$$

Violation of this bound results in a naked singularity.

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Extremal black holes saturate the bound. They have zero temperature $T = 0$, while $S_{\rm BH}$ is nonzero. Inner and outer horizons coincide.

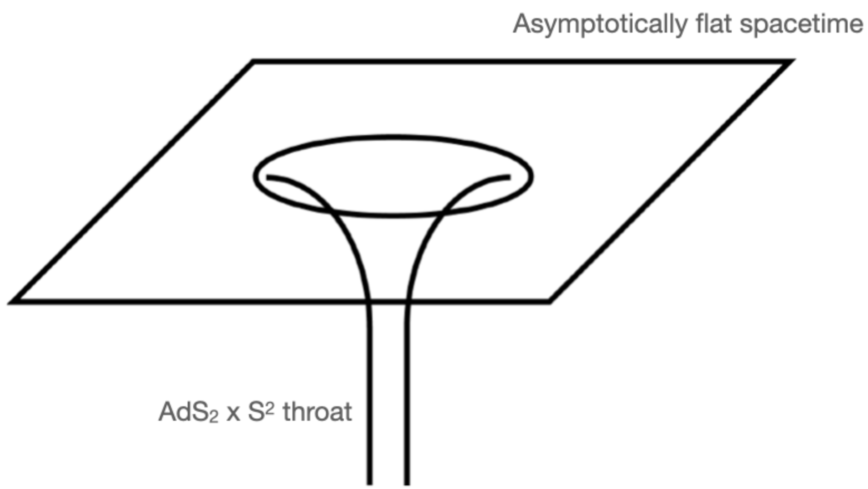
- huge degeneracy of ground states at $T = 0$
- Absence of symmetry protecting it (unless i.e. there is SUSY)

Extremal black holes

Symmetry enhancement near horizon: Geometry near the horizon develops an AdS_2 throat

e.g. for static black holes near horizon geometry is $AdS_2 \times S^2$. This happens also in spaces with cosmological constant

The region outside the horizon seems infinitely far away



Puzzle: Kerr black holes near extremality

Close to extremality, the energy accessible to system is [Preskill, Schwarz, Shapere, Trivedi, Wilczek, '91]

$$E_{BH} = 4\pi^2 J^{3/2} T^2 = \frac{T^2}{E_{gap}}$$

Typical energy of Hawking quantum is

$$E_{Hawk} \sim T$$

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Typical energy of Hawking quantum is

$$E_{Hawk} \sim T$$

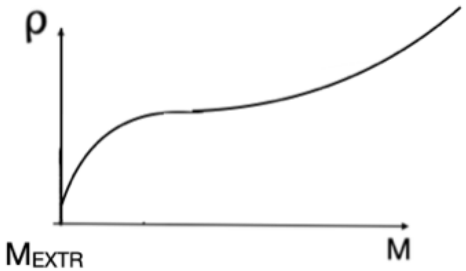
Below E_{gap} the energy available to the BH is not sufficient for the emission of a single Hawking quantum

At $E = E_{gap}$ temperature fluctuations become large compared to T itself

Puzzle: Kerr black holes near extremality

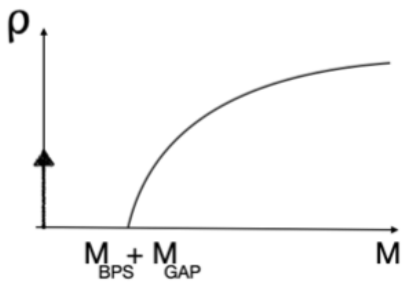
This issue was tackled in [Iliesiu, Turiaci '19][Heydeman, Iliesiu, Turiaci, Zhao '20] for static charged black holes (and rotating AdS_5), showing that the apparent degeneracy at $T = 0$ is lifted if there is no susy.

Aim here: compute quantum correction to S_{BH} for rotating black holes, including Kerr BH



Kerr (non-susy):

Quantum corrections: NO ground state denenenacy



Charged AdS Kerr (susy)

Presence of mass gap [Heydeman, CT, to appear]

Quantum corrections

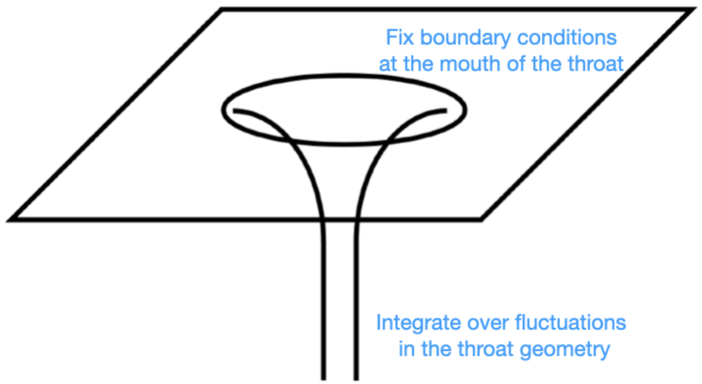
We will make use of the throat geometry and its approximate decoupling from far region.

Assumes that relevant part of the BH Hilbert space can be captured by gravitational dynamics near the throat

$$Z_{grav} = \int [Dg] e^{-S[g]} \qquad g \to \overline{g} \text{ at boundary}$$

Integrate over metrics subject to some boundary conditions fixed by the ensemble.

$\rightarrow$ Use saddle point approximation



Quantum corrections

At zero temperature $Z_{grav} \sim e^{2\pi J} = e^{S_0}$ reproduce BH entropy.

The first correction comes from integrating over the quantum fluctuations about the saddle. This is where subtleties lie: divergencies due to zero modes appearing in the 1-loop computation [Sen '08]

Quantum corrections

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We need to compute a functional determinant. Schematically

$$\int_{-\infty}^{+\infty} e^{-\lambda x^2} dx = \sqrt{\frac{\pi}{\lambda}} \qquad \int \prod_i e^{-\lambda_i x_i^2} dx_i = \sqrt{\prod_i \frac{\pi}{\lambda_i}}$$

Quantum corrections

$\rightarrow$ Responsible for lifting the degeneracy of ground states [Iliesiu, Murthy, Turiaci '22]

Avatar of the fact that backreaction effects in AdS_2 spacetimes are very strong [Maldacena, Stanford, Yang '16]. Studied in Kerr in [Castro, Godet '20] [Castro, Godet, Simon, Yu '21].

Approach: making use of the near-extremal configuration, which will act as a regulator for these divergencies

Outline

- Kerr black holes
  - NHEK and near-extremal limit
  - Zero Modes for NHEK
  - Lifting of Extremal Zero Modes and log T Corrections to the Entropy
- AdS_4 black holes
  - extremal and supersymmetric limit
  - Corrections to density of states (w inclusion of ϑ-term)
- Conclusions and outlook

Kerr black hole: near-extremal limit

Describes a black hole with angular momentum $J = aM$. Metric is

$$ds^2 = -\frac{\Delta}{\Sigma}\left(d\hat{t} - a\sin^2\theta\, d\hat{\phi}\right)^2 + \frac{\Sigma}{\Delta}d\hat{r}^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}\left((\hat{r}^2 + a^2)d\hat{\phi} - a d\hat{t}\right)^2$$

$$\Delta(\hat{r}) = \hat{r}^2 - 2M\hat{r} + a^2, \qquad \Sigma(\hat{r},\theta) = \hat{r}^2 + a^2\cos^2\theta.$$

Singularity cloaked by horizons at

$$r_\pm = M \pm \sqrt{M^2 - a^2}$$

Entropy and temperature given by

$$A = 4\pi(r_+^2 + a^2) \qquad T = \frac{1}{4\pi M}\frac{\sqrt{M^2 - (J/M)^2}}{M + \sqrt{M^2 - (J/M)^2}}$$

Kerr black hole: near-extremal limit

Extremal limit obtained by

$$M^2 = J = M_0^2$$

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Expansion close to extremality:

$$\begin{aligned} M(T, J) &= J^{1/2} + 4\pi^2 J^{3/2} T^2 + O(T^3) \\ S(T, J) &= 2\pi J + 8\pi^2 J^{3/2} T + O(T^2) \end{aligned}$$

Rewritten as

$$M = M_0 + \frac{T^2}{M_{gap}} \qquad S = S_0 + \frac{2T}{M_{gap}} \qquad M_{gap} = \frac{1}{4\pi^2 J^{3/2}}$$

Near Horizon Extremal Kerr

Zoom in the near-horizon geometry with change of coords

$$\hat{t} = \frac{2r_0}{\varepsilon(T)} t, \qquad \hat{r} = r_+(T) + r_0 \varepsilon(T)(\cosh\eta - 1), \qquad \hat{\phi} = \phi + \frac{t}{\varepsilon(T)} - t \,, \qquad \varepsilon(T) = 4\pi r_0 T,$$

For $T \to 0$ the geometry is the Near Horizon Extremal Kerr (NHEK) geometry
[Bardeen, Horowitz '99]

$$ds^2 = J\left(1+\cos^2\theta\right)\left(-\sinh^2\eta\, dt^2 + d\eta^2 + d\theta^2\right) + J\frac{4\sin^2\theta}{1+\cos^2\theta}\left(d\phi + (\cosh\eta - 1)\, dt\right)^2$$

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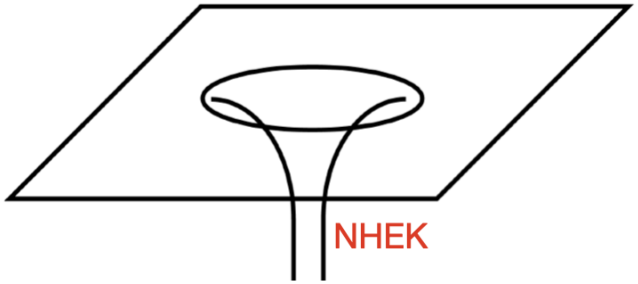

Near Horizon Extremal Kerr

The NHEK metric has $SL(2,\mathbb{R}) \times U(1)$ symmetry with generators

$$L_{\pm 1} = \frac{e^{\mp t}}{\sinh\eta}\left(\cosh\eta\,\partial_t \pm \sinh\eta\,\partial_\eta + (\cosh\eta - 1)\partial_\phi\right) ,$$

$$L_0 = \partial_t + \partial_\phi\ , \qquad W = \partial_\phi .$$

- Maximal spin allowed for black hole
- It is a solution to the Einstein's equations
- Extensively studied in the context of Kerr/CFT correspondence [Guica, Hartman, Song, Strominger '08]

Quantum corrections and zero modes

First analytically continue $t = -i\tau$. Regularity at $\eta = 0$ requires periodicity $\tau \to \tau + 2\pi$

Partition function in the NH region is given by integral over metrics, subject to boundary conds

$$Z = \int [Dg] e^{-I[g]} \qquad I[g] = -\frac{1}{16\pi}\int_M d^4x\sqrt{g}R + I_{bdary}$$

NHEK is saddle point solution

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NHEK is saddle point solution

Writing $g = \bar{g} + h$ where $\bar{g} = g_{NHEK}$ and expanding action at quadratic order

$$Z \approx \exp\left(-I[\bar{g}]\right) \int [Dh] \exp\left[-\int d^4x \sqrt{\bar{g}}\, h \mathcal{D}[\bar{g}] h\right] .$$

$\mathcal{D}$ is a 2nd-order differential operator. Path-integral computes $\int [Dh] e^{\left[-\int d^4x\, h\mathcal{D}[\bar{g}]h\right]} \sim \frac{1}{\det(\mathcal{D})}$

Quantum corrections and zero modes

We choose a gauge-fixing term

$$\mathcal{L}_{\text{GF}} = \frac{1}{32\pi}\bar{g}_{\mu\nu}\left(\bar{\nabla}_\alpha h^{\alpha\mu} - \frac{1}{2}\bar{\nabla}^\mu h^\alpha_\alpha\right)\left(\bar{\nabla}_\beta h^{\beta\nu} - \frac{1}{2}\bar{\nabla}^\nu h^\beta_\beta\right)$$

Quadratic fluctuation operator term for Einstein-Hilbert action in NHEK is [Sen '11]

$$h_{\alpha\beta} D^{\alpha\beta,\mu\nu}_{\text{NHEK}} h_{\mu\nu} = -\frac{1}{16\pi} h_{\alpha\beta}\left(\frac{1}{4}\bar{g}^{\alpha\mu}\bar{g}^{\beta\nu}\bar{\Box} - \frac{1}{8}\bar{g}^{\alpha\beta}\bar{g}^{\mu\nu}\bar{\Box} + \frac{1}{2}\bar{R}^{\alpha\mu\beta\nu}\right) h_{\mu\nu}$$

Quantum corrections and zero modes

The operator D_{NHEK} supports an infinite family of normalizable zero modes on NHEK

$$h^{(n)}_{\mu\nu}dx^\mu dx^\nu = \frac{1}{4\pi}\sqrt{\frac{3}{2}}\sqrt{|n|(n^2-1)}(1+\cos^2\theta)e^{in\tau}\frac{(\sinh\eta)^{|n|-2}}{(1+\cosh\eta)^{|n|}}(d\eta^2+2i\frac{n}{|n|}\sinh\eta\, d\eta\, d\tau-\sinh^2\eta\, d\tau^2)$$

for $|n| > 1$

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for $|n|>1$

$\rightarrow$ They are metric perturbations generated by large diffeomorphisms left unfixed by harmonic gauge. They obey

$$h^{(n)} \propto \mathcal{L}_{\xi^{(n)}} g_{\text{NHEK}}$$

with the vector field

$$\xi^{(n)} = e^{in\tau}\tanh^{|n|}\frac{\eta}{2}\left(\frac{in(|n|+\cosh\eta)}{\sinh\eta}\partial_\eta - \frac{|n|(|n|+\cosh\eta)+\sinh^2\eta}{\sinh^2\eta}\partial_\tau + \frac{i(\cosh\eta+1+|n|-n^2)}{\cosh\eta+1}\partial_\phi\right)$$

Relation to the Schwarzian

Repackaging these modes

$$\xi = \sum_n f_n \xi^{(n)}$$

Defining

$$f(\tau) = \sum_n f_n e^{in\tau}$$

Large η behavior

$$\xi \approx -f(\tau)\partial_\tau + f'(\tau)\partial_\eta + if(\tau)\partial_\phi \ .$$

Diffeos correspond to boundary time reparameterizations that send $\tau \to \tau - f(\tau)$, $\eta \to \eta + f'(\tau)$, and $\phi \to \phi + if(\tau)$

Modes are parameterized by element of $\mathrm{Diff}(S^1)/SL(2,\mathbb{R})$ ($n = -1, 0, 1$ are isometries hence h vanishes)

Recap

- NHEK operator admits an infinite family of zero modes

$$h^{(n)}_{\mu\nu} dx^\mu dx^\nu = \frac{1}{4\pi}\sqrt{\frac{3}{2}}\sqrt{|n|(n^2-1)}(1+\cos^2\theta)e^{in\tau}\frac{(\sinh\eta)^{|n|-2}}{(1+\cosh\eta)^{|n|}}\left(d\eta^2 + 2i\frac{n}{|n|}\sinh\eta\, d\eta\, d\tau - \sinh^2\eta\, d\tau^2\right)$$

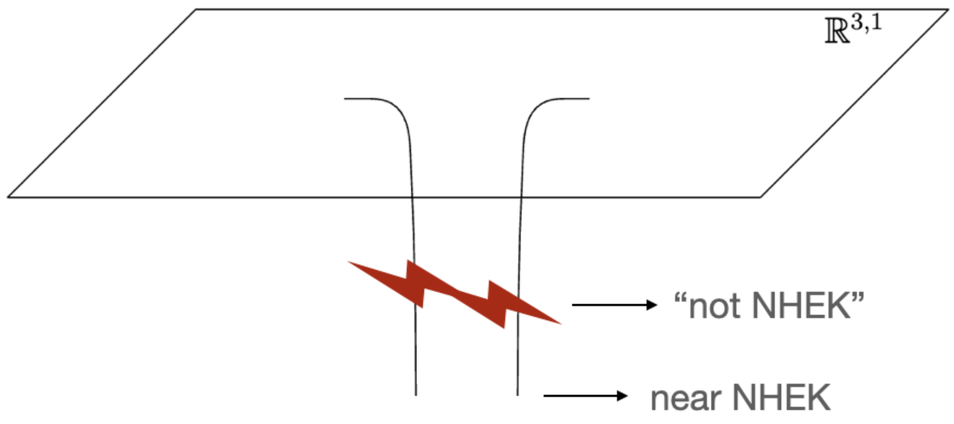
- these modes cost no action and have infinite volume, the one-loop approximation to the path integral therefore suffers from an infrared divergence

$$Z \propto \int_{\mathrm{Diff}(S^1)/SL(2,\mathbb{R})} [Df(\tau)] = \infty$$

- NHEK path integral is divergent and not well defined

How to fix this

Do not fully decouple geometry, keep O(T) term near horizon [Iliesiu, Murthy, Turiaci '22]



Use this "not-NHEK" geometry to regulate the computation

Compute the correction to eigenvalues via perturbation theory

"Not-NHEK"

Take the linear term in T in the decoupling limit

Zoom in the near-horizon geometry with change of coords

$$\hat{t} = \frac{2r_0}{\varepsilon(T)} t, \qquad \hat{r} = r_+(T) + r_0 \varepsilon(T)(\cosh\eta - 1), \qquad \hat{\phi} = \phi + \frac{t}{\varepsilon(T)} - t \, , \qquad \varepsilon(T) = 4\pi r_0 T,$$

Expansion in small T gives

$$ds^2 = g_{\mathrm{NHEK}} + T\,\delta g_{\mu\nu} dx^\mu dx^\nu$$

”Not-NHEK”

Take the linear term in T in the decoupling limit

Full correction to the NHEK metric

$$
\begin{aligned}
\frac{\delta g_{\mu\nu} dx^\mu dx^\nu}{4\pi J^{3/2}} =& (1+\cos^2\theta)(2+\cosh\eta)\tanh^2\frac{\eta}{2}(d\eta^2-\sinh^2\eta\, d\tau^2) \\
&+\sin^2\theta\cosh\eta(d\eta^2+\sinh^2\eta\, d\tau^2)+2\cosh\eta\, d\theta^2 \\
&+2\frac{\sin^2\theta}{1+\cos^2\theta}(\cosh\eta-1)\left((\sin^2\theta\sinh^2\eta-3)-4\frac{\cos^2\theta}{1+\cos^2\theta}\cosh\eta(\cosh\eta-1)\right)d\tau^2 \\
&+2i\frac{\sin^2\theta}{1+\cos^2\theta}\left((\sin^2\theta\sinh^2\eta-3)-8\frac{\cos^2\theta}{1+\cos^2\theta}\cosh\eta(\cosh\eta-1)\right)d\tau d\phi \\
&+8\cosh\eta\frac{\sin^2\theta\cos^2\theta}{(1+\cos^2\theta)^2}d\phi^2 \ .
\end{aligned}
$$

Eigenvalue correction

Expanding everything to first order in T

$$(\overline{D} + \delta D)(h_n^0 + \delta h_n) = (\Lambda_n^0 + \delta\Lambda_n)(h_n^0 + \delta h_n)$$

$h^0 =$ extremal eigenfunctions with eigenvalues Λ^0

Isolating $O(T)$ terms, we get

$$\overline{D}\delta h_n + \delta D h_n^0 = \Lambda_n^0 \delta h_n + \delta\Lambda_n h_n^0 \,.$$

Taking the inner product with h_m^0, using orthonormality

$$\delta\Lambda_n = \int d^4x \sqrt{g} (h_n^0)_{\alpha\beta} \delta D^{\alpha\beta,\mu\nu} (h_n^0)_{\mu\nu} \,.$$

Therefore corrected one loop determinant* is $\log Z = -\frac{1}{2}\sum_n \log(\Lambda_n^0 + \delta\Lambda_n)$

*Modes with nonzero eigenvalues produce subleading corrections T-dependent terms

Eigenvalue correction

Using

$$\delta D^{\alpha\beta,\mu\nu} = \delta D^{\alpha\beta,\mu\nu}_{\mathrm{Box}} + \delta D^{\alpha\beta,\mu\nu}_{\mathrm{Riemann}}$$

where

$$\delta D^{\alpha\beta,\mu\nu}_{\mathrm{Box}} = -\frac{1}{16\pi}\delta\left(\frac{1}{4}g^{\alpha\mu}g^{\beta\nu}\Box - \frac{1}{8}g^{\mu\nu}g^{\alpha\beta}\Box\right) \qquad \delta D^{\alpha\beta,\mu\nu}_{\mathrm{Riemann}} = -\frac{1}{32\pi}\delta\left(R^{\alpha\mu\beta\nu}\right)$$

with $g_{\text{not-NHEK}} = \overline{g} + \delta g$.

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Inserting form of zero-modes

$$\int d^4x\sqrt{\bar{g}}(h^0_n)_{\alpha\beta}\delta D^{\alpha\beta,\mu\nu}_{\mathrm{Riemann}}(h^0_n)_{\mu\nu} = -\frac{3n(n^2-1)T}{128J^{1/2}}\int_0^\infty d\eta\left[16(\pi-2)\coth\eta\,\mathrm{csch}^2\eta\ \tanh^{2n}\left(\frac{\eta}{2}\right)\right]$$

$$\int d^4x\sqrt{\bar{g}}(h^0_n)_{\alpha\beta}\delta D^{\alpha\beta,\mu\nu}_{\mathrm{Box}}(h^0_n)_{\mu\nu} = -\frac{3n(n^2-1)T}{128J^{1/2}}\int_0^\infty d\eta\left[\mathrm{csch}^3\eta\,\mathrm{sech}^4\left(\frac{\eta}{2}\right)\left((\pi-2)\cosh 3\eta + \right.\right.$$

$$\left.\left.+(4(n-2)n+7\pi-30)\cosh\eta - 2(n-2\pi+4)\cosh 2\eta + 2n(4n+7)+4\pi\right)\tanh^{2n}\left(\frac{\eta}{2}\right)\right]$$

log T Corrections to the Entropy

After integration it simplifies to

$$\delta\Lambda_n = \frac{3nT}{64J^{1/2}}\,, \qquad n \geqslant 2\,.$$

hence correction is

$$\delta \log Z = 2 \cdot (-1/2) \sum_{n\geqslant 2} \log \delta\Lambda_n = \log\left(\prod_{n\geqslant 2} \frac{64J^{1/2}}{3nT}\right)$$

Using zeta function regularization $\prod_{n\geqslant 2} \frac{\alpha}{n} = \frac{1}{\sqrt{2\pi}}\frac{1}{\alpha^{3/2}}$ we obtain

$$\delta \log Z = \log\left(\frac{\sqrt{27}}{512\sqrt{2\pi}}\frac{T^{3/2}}{J^{3/4}}\right) = \frac{3}{2}\log T + \ldots\,.$$

[Kapec, Sheta, Strominger, **CT** '23] [Rakic, Rangamani, Turiaci '23]

log T Corrections to the Entropy

At low temperatures

$$Z[T]_{\text{Black Hole}} \sim T^{3/2} exp[S_0 + 8\pi^2 J^{3/2} T] + \ldots$$

Few remarks:

-
-

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Few remarks:

- Approximation is not valid when T is too small, i.e. $T < J^{\beta} e^{-\alpha S_0}$, where $\alpha, \beta \sim O(1)$. Below this temperature, the partition function is so small that other saddles will begin competing
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We find $\rho(E) \to 0$ as $E \to 0$: no exponential ground state degeneracy or thermodynamic mass gap. Ground states are spread out over a dense energy band above the vacuum.

Subtleties: the rotational zero mode

In addition to tensor modes there are vector zero modes in the metric arising from isometries of S^2 in the case of static BHs (and $U(1)$ for rotating BHs) [Sen '11]

$$h_{i\mu} = \frac{1}{\sqrt{2}}\epsilon_{ij}\partial^j Y_l^m(\theta,\phi)v_\mu \qquad v_\mu = \partial_\mu \Phi_n(\tau,\eta)$$

They are zero modes in a specific ensemble

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For Kerr black holes, naive guess does not satisfy the gauge condition and is not a zero mode of the operator (treated in more detail in [Rakic, Rangamani, Turiaci '23] and in Mukund's talk yesterday!).

Nevertheless, the $3/2 \log T$ correction is universal, ensemble-independent

AdS_4 black holes

[w.i.p. with M. Heydeman]

AdS$_4$ black holes: extremal and supersymmetric limits

Exist AdS$_4$ black holes that preserve susy [Romans '92] [Kostelecky, Perry '94].

Work with minimal $\mathcal{N}=2$ gauged supergravity, $S = -\frac{1}{16\pi G}\int d^4x\sqrt{g}\left(R+6-F^2\right)$, arising from M-theory on S^7 with ABJM dual

1/4 BPS rotating black holes: Electric Kerr-Newman BH, no static limit

1/4 BPS "twisted" black holes: Dyonic, magnetic charge flux fixed. Can switch off rotation.

Microstate counting achieved via computation of superconformal index [S. Kim et al, '19] and topologically twisted index [Benini, Hristov, Zaffaroni '16]

Strategy

Aim: Computing (quantum-corrected) density of near-BPS states
[Boruch, Iliesiu, Heydeman, Turiaci '22]

$\rightarrow$ Give prediction for ABJM ground state degeneracy and gap, assuming that the throat computation provides the dominant effect at low T

Find dimensional reduction to 2d in NH region, which has $SU(1,1|1)$ symmetry. Theory describing excitations over extremality is $\mathcal{N} = 2$ JT gravity. Path integral computation is one-loop exact and known from previous works [Stanford, Witten '17] [Mertens, Turiaci, Verlinde '17]

Need to input parameters coming from gravity: expansion of I_{BH} around its BPS values

Rotating, electric BHs

Work in mixed ensemble: one charge $j = J + R_4$ fixed

Chemical potential

$$\alpha = \frac{\beta}{4\pi i}(4\Phi_4 - \Omega - 1) \qquad \alpha_{susy} = \frac{1}{2}$$

Perform sum over configurations with the same boundary conditions: $\alpha \to \alpha + n$

1-loop partition function:

$$Z = e^{4\pi i \alpha R_*} \sum_{n \in \mathbb{Z}} e^{in\vartheta} \left(\frac{2\cos(\pi(\alpha+n))}{\pi(1-4(\alpha+n)^2)} \right) e^{S_* + \frac{2\pi^2}{\beta M_{GAP}}(1-4(\alpha+n)^2)} ,$$

Input from gravity: BH thermodynamics (expansion of on-shell action)

$$I_{ME}(\beta, j, \alpha) = -S_* - 4\pi i \alpha R_* - \frac{2\pi^2}{\beta M_{GAP}} \left(1 - 4\alpha^2\right)$$

Rotating, electric BHs

Work in mixed ensemble: one charge $j = J + R_4$ fixed

Chemical potential

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Corrections to entropy

Extract form for the density of states via Laplace transform

$$\rho(\alpha, j, E) = e^{4\pi i \alpha R_*} e^{S_*} \left(\delta_{Z_{Sch},0} + \sum_{Z_{Sch}\in\mathbb{Z}} (e^{2\pi i \alpha Z_{Sch}} + e^{2\pi i \alpha (Z_{Sch}-1)}) \frac{\sinh\left(2\pi\sqrt{\frac{2(E-E_{gap})}{M_{GAP}}}\right)}{2\pi E} \Theta(E - E_{gap}) \right)$$

$$E_{gap} = \frac{M_{GAP}}{8}\left(Z_{Sch} - \frac{1}{2}\right)^2$$

Corrections to entropy

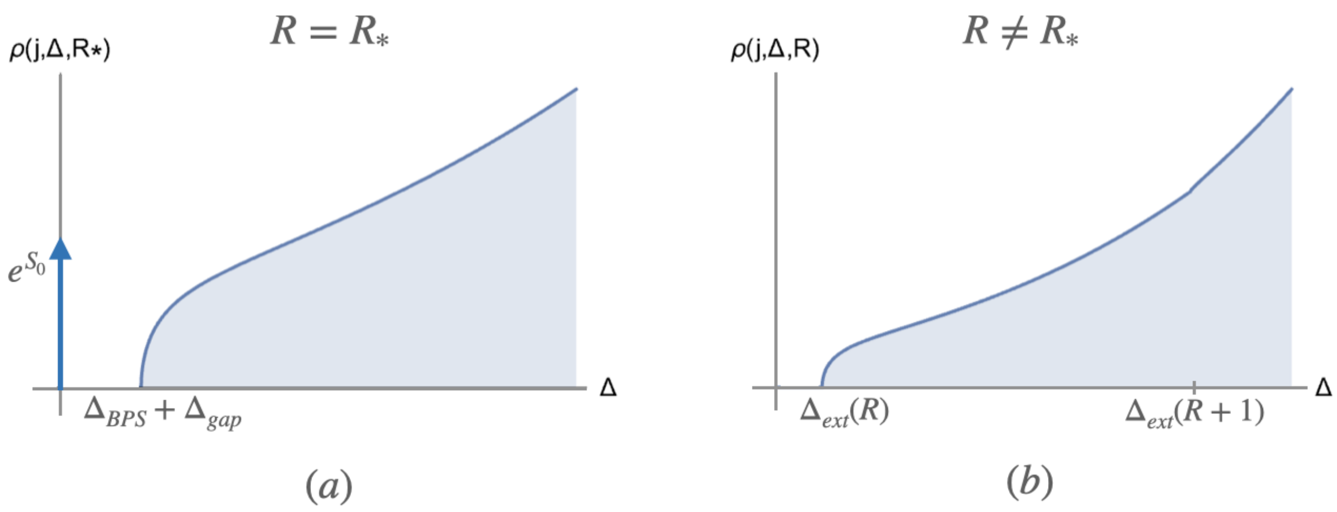
In terms of field theory variables: Schwarzian energy related to the scaling dimension Δ

$$\Delta = \Delta_{BPS} + E + (R - R_*) = \Delta_{BPS} + E + Z_{Sch}$$

and

$$R = R_* + Z_{Sch}$$

Density of states



(a) susy

mass gap present

e^{S_0} matches SCI

Corrections to entropy

In terms of field theory variables: Schwarzian energy related to the scaling dimension Δ

$$\Delta = \Delta_{BPS} + E + (R - R_*) = \Delta_{BPS} + E + Z_{Sch}$$

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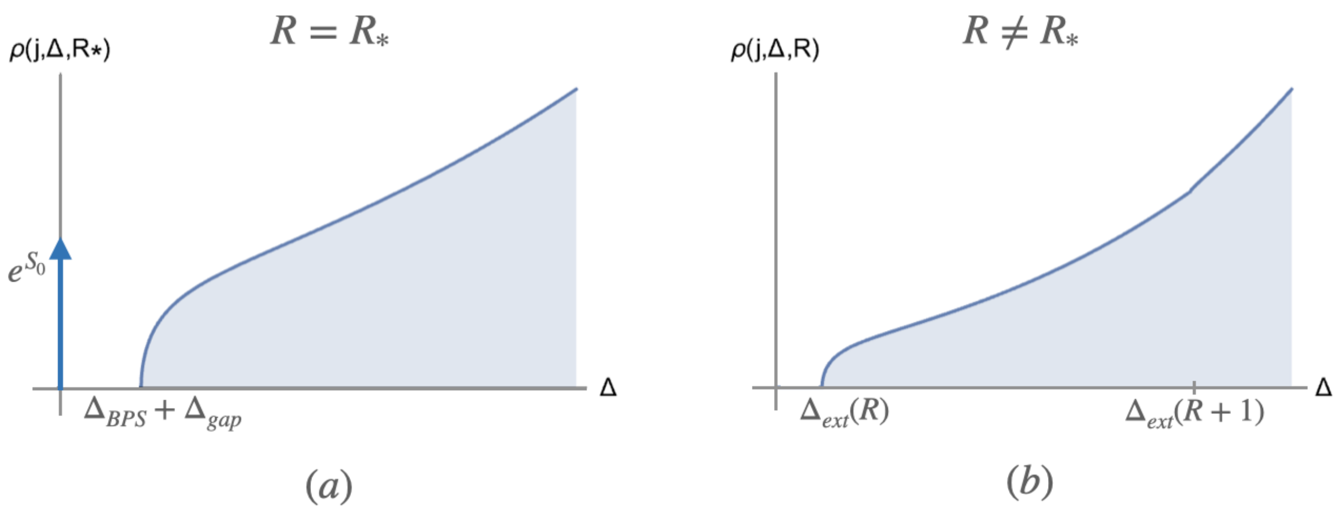
Density of states



(b) non-susy density $\to$ 0 for extremal BH

Dyonic twisted solutions

Work in truncation of M5-branes wrapping hyperbolic manifolds Σ_3. The 7d internal space is an S^4 fibered over Σ_3, and the resulting 4d sugra

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{g}\,\left(R + 6 - F^2\right) + \frac{i\vartheta}{8\pi^2}\int F \wedge F \tag{1}$$

whose duals are class R-theories. Susy solution:

$$ds^2 = V(r)d\tau^2 + \frac{dr^2}{V(r)} + r^2(d\theta^2 + \sinh^2\theta d\phi^2) \qquad V(r) = \left(r - \frac{1}{2r}\right)^2 - \frac{Q^2}{r^2}$$

$$F = \frac{Q}{r^2}d\tau \wedge dr \pm \frac{1}{2}\sinh\theta$$

Expansion around BPS limit: read off $M_{GAP} = \frac{2\sqrt{2}G_4}{\pi^2}$ and $S* = \frac{\pi}{2G_4}(\mathbf{g} - 1)$, which are input parameters for the quantum corrected action

Corrections to entropy

Read off density of states

$$\rho(\alpha, E, \vartheta) = e^{S_*}\left(\sum_{Z_{Sch}\in\mathbb{Z},\, |Z_{Sch}-\frac{\vartheta}{2\pi}|<\frac{1}{2}} \cos(\frac{\vartheta}{2})(-1)^{Z_{Sch}} e^{2\pi i\alpha Z_{Sch}} \right.$$

$$\left. + \sum_{Z_{Sch}\in\mathbb{Z}} (e^{2\pi i\alpha Z_{Sch}} + e^{2\pi i\alpha(Z_{Sch}-1)}) \frac{\sinh\left(2\pi\sqrt{\frac{2(E-\tilde{E}_{gap})}{M_{GAP}}}\right)}{2\pi E} \Theta(E-\tilde{E}_{gap})\right)$$

with redefined Schwarzian charge $\tilde{E}_{gap} = \frac{M_{GAP}}{8}\left(Z_{Sch} - \frac{1}{2} - \frac{\vartheta}{2\pi}\right)^2$

Corrections to entropy

- Supersymmetric limit obtained for $\alpha = 1/2$. With ϑ angle there are cancellations due to additional phase, matching [Choi,Gang,Kim'20] [Benetti Genolini '21]

$$\mathcal{I}_{[H_3;\Sigma_g]} = Z_{[H_3;\Sigma_g]}(\beta, \alpha = \frac{1}{2}) = \frac{1}{2}\left(e^{S_* + i\frac{\vartheta}{2}} + e^{S_* - i\frac{\vartheta}{2}}\right) = e^{S_*} \cos\left(\frac{\vartheta}{2}\right)$$

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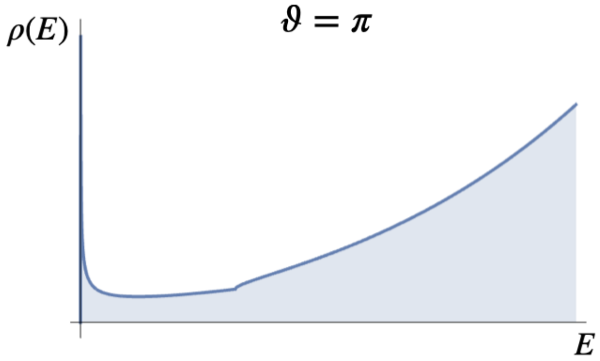
- Density of states for $\vartheta = \pi$ diverges at the origin due to a single gapless set of multiplets



Similar behaviour found in $\mathcal{N} = 2$ SYK with odd number of fermions [Stanford, Witten '17].
Same found in black holes with 11d uplift!

Corrections to entropy

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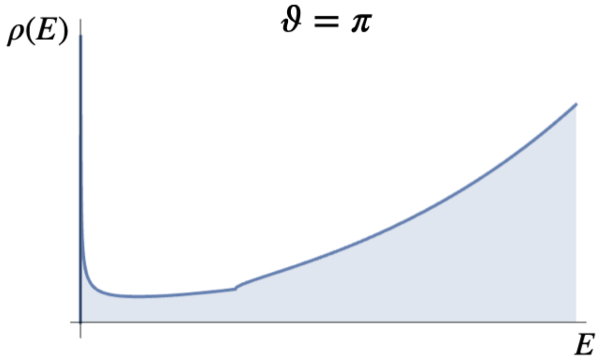

" mixed 't Hooft anomaly ": Partition function is initially invariant under $\alpha \to -\alpha$ (time reversal) and $\alpha \to \alpha + 1$. For generic ϑ, time reversal anomalous

To be continued...

Conclusions and perspectives

Provided a framework where quantum corrections to near extremal black hole entropy can be computed. They predict lifting of the ground state degeneracy of extremal non-supersymmetric black holes (such as Kerr) and a mass gap for susy ones.

Open questions & further directions:

- How to account for superradiance? In NHEK or needs full geometry?
- AdS setups, how to reproduce answer via dual field theory?
- Different asymptotics (i.e. de Sitter black holes)?

the end. Thank you!

Additional slides

Classical contribution:

$$\log Z_{\text{tree}} = S_0 + 2\pi^2 J^{3/2} T$$

1-loop contribution (T-dependent)

$$\log Z_{1-\text{loop}} = \frac{1}{180}(2n_S - 26n_V + 7n_F + 154)\log S_0 + \frac{3}{2}\log\left(\frac{T}{J^{3/2}}\right)$$

Total free energy (neglecting $\log S_0$ terms)

$$-\beta F = \log Z_{\text{tree}} + \log Z_{1-\text{loop}} = \log(Z_{\text{tree}} * Z_{1-\text{loop}}) = S_0 + 2\pi^2 J^{3/2} T + \frac{3}{2}\log\left(\frac{T}{J^{3/2}}\right)$$

Hence

$$Z_{\text{tot}} \propto T^{3/2} exp[S_0 + 2\pi^2 J^{3/2} T]$$