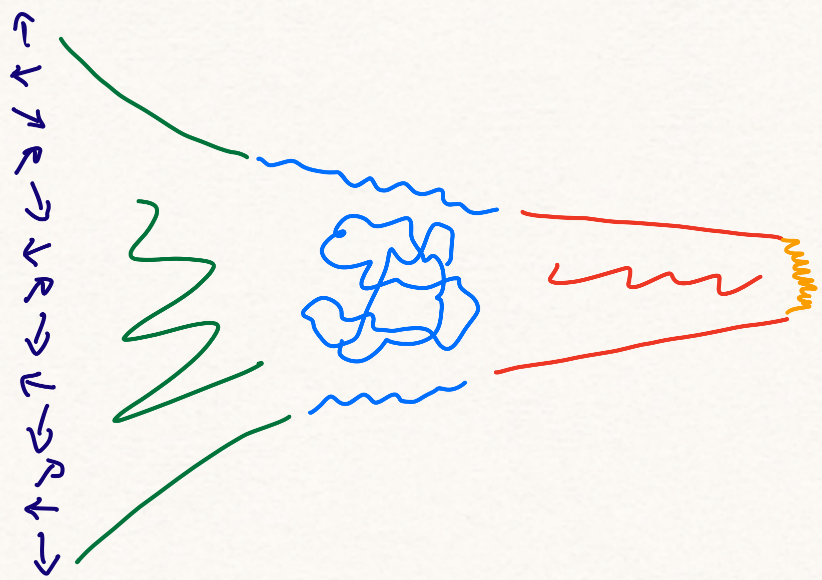
FLOW GEOMETRIES



Solvay Near-Extreme Black Hole Workshop, ULB, 9/2024

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OUTLINE

* MOTIVATION
* MACROSCOPIC EXPLORATIONS
* MICROSCOPIC EXPLORATIONS
* OUTLOOK.

MOTIVATION

* WE ARE ACCUSTOMED TO THE IDEA THAT A HORIZON IS UNIVERSAL & FEATURELESS.

* NONETHELESS, THERE ARE EXCEPTIONS TO THIS "RULE"

e.g.
- FIELD CONDENSATES NEAR HORIZON
- MULTICENTERED/FRAGMENTED HORIZONS
- NO HAIR VIOLATIONS IN 5^+ DIMS
- TOPOLOGICAL HORIZONS FOR $\Lambda < 0$

etc.

[Majumdar-Papapetrou '40s]

* THE SPIRIT OF FEATURELESSNESS HOWEVER TIES TO THERMODYNAMIC PROPERTIES AND IS HENCE SOMEHOW "GENERIC"

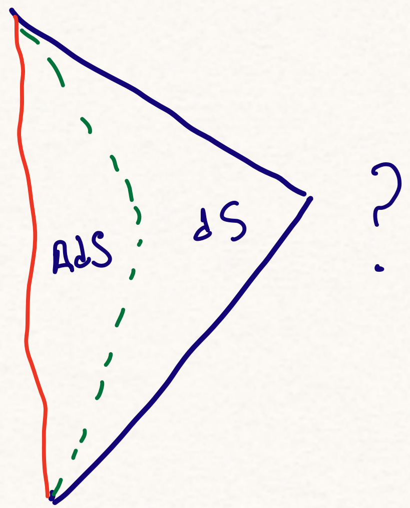
  i.e. LARGE AdS BLACK HOLES.

* EXTREME OR NEAR EXTREME HORIZONS MAY BE OF A LESS "GENERIC" NATURE.

* IN WHAT FOLLOWS WE WILL TRY TO ELABORATE ON THIS, AT LEAST SUPERFICIALLY, BY STUDYING A SIMPLE EXAMPLE

ANOTHER MOTIVATION (THAT INITIATED THESE WORKS) CAME FROM ATTEMPTS* TO EMBED DS HORIZON IN ASYMPTOTICALLY AdS_2

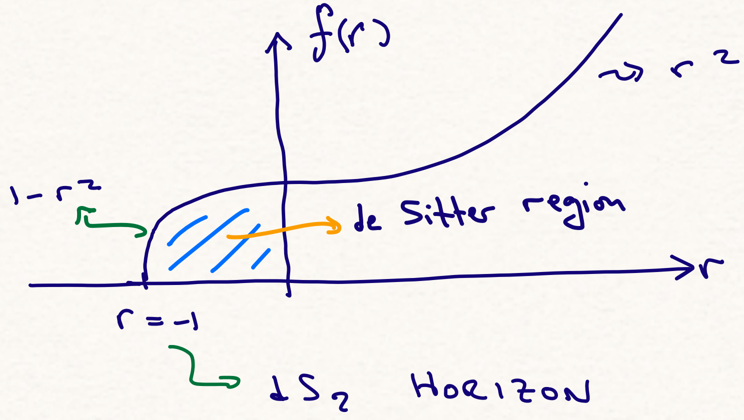
[D.A., Hofman, Galante...]



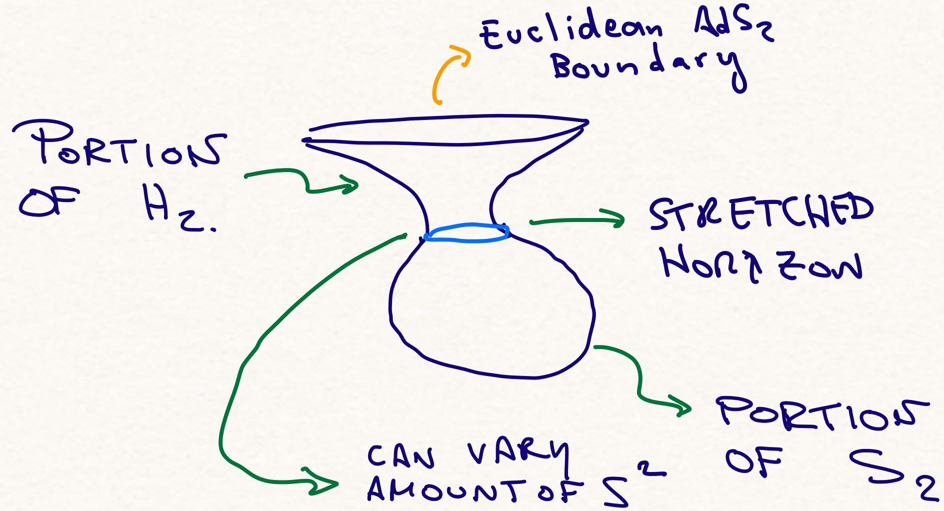
* Previous attempts include "building cosmology in the lab" [Guth-Farhi, ..., Freivogel-Hubeny-Myers-Maloney-Rangamani-Shenker...]

LORENTZIAN INTERPOLATING SPACETIME:

$$\frac{ds^2}{\ell^2} = -dt^2 f(r) + \frac{dr^2}{f(r)}$$



EUCLIDEAN INTERPOLATING SPACETIME:



THIS BRINGS FORTH A GENERAL QUESTION IN NEAR AdS_2 HOLOGRAPHY

$\boxed{\text{WHAT IS THE EFFECT OF “RELEVANT” DEFORMATIONS ON NEAR-}AdS_2\text{?}}$

MACROSCOPIC CONSIDERATIONS

FOUR-DIMENSIONS:
$$\boxed{ds^2 = -dt^2 f(r) + \frac{dr^2}{f(r)} + g(r)^2 d\Omega_2^2}$$

FURTHER ASSUME THERE EXISTS r_h:

$$f(r_h) = 0 \quad \& \quad g(r_h) \neq 0.$$

NULL ENERGY CONDITION:

$$\Longrightarrow \quad g'' \leq 0 \quad \& \quad (g^4 F')' \geq -2$$

with $F \equiv f/g^2$ $\rightsquigarrow$ FLOW OF TIME NORMALIZED BY SPHERE SIZE!

ABOVE INEQUALITY ADMITS:

$$\boxed{(A)dS_2 \times S^2 \quad \text{to} \quad AdS_2 \times S^2}$$ RADIAL INTERPOLATIONS

S^2 SIZE CAN BE $\left\{\begin{array}{l}\text{CONSTANT}\\ \text{DECREASE}\\ \text{INCREASE}\end{array}\right\}$ TOWARD AdS_2 BOUNDARY.

N.B. Non-spherical cases also: [e.g. Horowitz-Santos-Kolanowski-Remmen-...]

TWO-DIMENSIONAL PERSPECTIVE

$$\left.\begin{aligned} S_{DG} = & -\frac{1}{2\kappa}\int_{\mathcal{M}} d^2x\sqrt{g}\left(\Phi R - U(\Phi)\right) \\ & -\frac{1}{\kappa}\int_{\partial\mathcal{M}} du\sqrt{h}\,\Phi K + S_{\text{topological}} \end{aligned}\right\}$$ GENERAL CLASS OF 2D MODELS

MICROSCOPIC CONSIDERATIONS

BASIC QUESTION:

$$\boxed{\text{WHAT IS THE INFRARED OF (NEAR) CONFORMAL Q.M. SUBJECT TO RELEVANT DEFORMATIONS.}}$$

EXPLORE SOLVABLE RELAVANT DEFORMATIONS IN (NEAR) CQM.

MORE CONCRETELY:

<u>SOLVABLE</u> <u>TOY</u> <u>MODEL</u>

$$SYK_q + SYK_{\tilde{q}}$$

[Jiang, Yang, D.A., Galante, Sheorey, ...]

$$\hat{H} = \sum_{i_1 \dots i_{2q}} J_{i_1 \dots i_q} \psi_{i_1} \cdots \psi_{i_q} + s \sum_{i_1 \dots i_{\tilde{q}}} K_{i_1 \dots i_{\tilde{q}}} \psi_{i_1} \cdots \psi_{i_{\tilde{q}}}$$

NEW DIMENSIONLESS PARAMETER

TO PRESERVE DISORDER IN DEEP IR.

$\{\psi_i, \psi_j\} = \delta_{ij}$ ARE MAJORANA FERMIONS

J & K: RANDOMLY DRAWN WITH VARIANCE $\sim \mathcal{J}$

S = 0 REVIEW:

LARGE N SADDLE POINT METHOD.

LARGE TIME SEPARATIONS OR

LOW TEMPERATURE = STRONG COUPLING ≈ CONFORMAL

$$\boxed{\Delta_\psi = \frac{1}{q}}$$ + MANY CONFORMAL OPERATORS

$$\boxed{C = N\alpha T}$$ → IMPLIES CONFORMAL SYMMETRY BROKEN

WITH $\alpha \sim 1/J$

LOW TEMPERATURE THERMODYNAMIC SECTOR
HAS APPROXIMATE 2d HOLOGRAPHIC PICTURE

DILATON GRAVITY
TYPE THEORY

MACROSCOPIC

$$U(\Phi) = \Phi .$$

AdS_2 + Running Dilaton $\rightsquigarrow$ BREAKS $SL(2,\mathbb{R})$

$$C_{macro} \sim T$$

[... -Jensen - Maldacena, Stanford, Yang; ...]

OPERATORS OF THE TYPE:

$$\boxed{\mathcal{O}_m \approx \frac{1}{N}\sum_{i=1}^{N} \psi_i \, \partial_t^m \, \psi_i} \rightarrow \text{BULK MATTER FIELDS}$$

DUAL TO FERMION OPERATORS ψ_i FOR $i=1,\dots,N$

LESS CLEAR: MAYBE ALSO BULK FIELDS?

SOFT BOUNDARY MODE GOVERNED BY

SCHWARZIAN "QM"

$S \neq 0$

* LARGE-N SECTOR ADMITS SIMPLE SADDLE-POINT DESCRIPTION, SIMILAR TO SYK_q.

* DEFORMATION INVOLVES $\tilde{q}$ FERMIONS

$$\tilde{q} \times \underbrace{\frac{1}{q}}_{\Delta_\psi} < 1 \longrightarrow \text{NAÏVELY RELEVANT.}$$

EXAMPLE: $q \gg 1$ & $\tilde{q} \gg 1$ WITH $\frac{q}{\tilde{q}} = n$ fixed.

S.D. EQNS REDUCE TO:

$$\boxed{\partial_t^2 g(t) = \mathcal{J}^2 \left(2s^2 n e^{g(t)/n} + 2 e^{g(t)} \right)}$$

WHERE:

$$\langle \psi_i(t) \psi_j(0) \rangle \approx \frac{\delta_{ij}}{2} \operatorname{sgn} t \, e^{2g(t)/q}$$

FOR CONCRETENESS TAKE $\boxed{\tilde{q}/q = 1/2}$

ZERO - TEMPERATURE ANALYSIS

$$\boxed{g(t) = -\log\left(1 + \gamma|t|\sqrt{1+4s^2} + s^2\gamma^2 t^2\right)}$$

FOR $\boxed{s \ll 1}$

I $\quad g(t) \simeq 0 \qquad$ U.V. REGIME

II $\quad g(t) \simeq -\log \gamma|t|\sqrt{1+4s^2} \qquad$ INTERMEDIATE I.R.

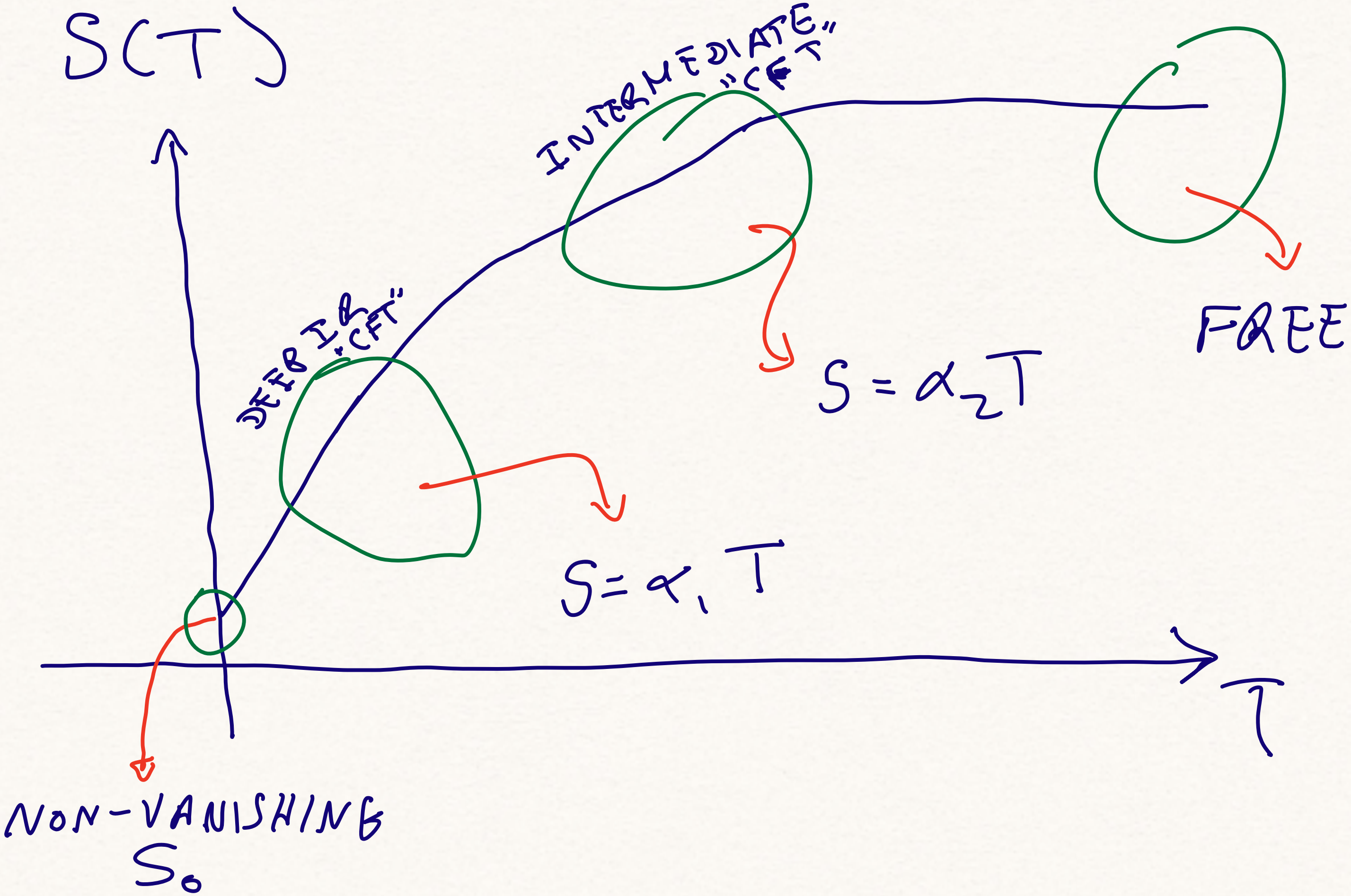
III $\quad g(t) \simeq -\log s^2\gamma^2 t^2 \qquad$ DEEP I.R.

REGIMES $\underline{\overline{\text{II}}}$ & $\underline{\overline{\text{III}}}$ ARE STRONGLY COUPLED NEAR-CONFORMAL PHASES!

FINITE - TEMPERATURE ANALYSIS.

STILL SOLVABLE!

AGAIN TAKE $\boxed{s << 1}$



* TWO REGIMES WITH LINEAR IN T SPECIFIC HEAT.
* INTERMEDIATE REGIME α_2 AS IN SYK_q

CONFORMAL PERTURBATION THEORY?

$$I_{CFT} + g_\Delta \int_0^{1/T} d\tau \, \mathcal{O}_\Delta(\tau) \, ,$$ LEADS TO:

$$\log Z \approx \log Z_{CFT} + \frac{g_\Delta^2}{2} \underbrace{\int_0 d\tau_1 d\tau_2 \langle \mathcal{O}_\Delta(\tau_1) \mathcal{O}_\Delta(\tau_2) \rangle}_{= T^{2(\Delta-1)}}{}_{CFT}$$

WHERE WE ASSUMED $\langle \mathcal{O}_\Delta(\tau) \rangle_{CFT} = 0$ & set $\mathcal{J} = 1$.

PREDICTS DEVIATION AWAY FROM $S \sim \alpha_2 T$ TO BE $\underline{\underline{T^{2(\frac{1}{n}-1)}}}$ WHICH AGREES WITH ANALYTICS & NUMERICS

DEEP IR

$$C(T) \simeq \frac{\pi^2}{2\mathcal{N}_s} \, \frac{N \;\; T}{\breve{g}^2 \, y}$$

$$\boxed{\mathcal{N}_s \equiv \frac{s^2}{\sqrt{1+4s^2}}}$$

$(0, \infty)$

ALONG $s \in \mathbb{R}$

OR MORE GENERALLY $\mathcal{N}_s(n)$.

GENERAL REMARKS

* FOR $s^2 \sim 1$ THERE IS NO INTERMEDIATE REGIME $s >> 1 \Rightarrow \mathcal{N}_s \sim |s|/2$ & DEEP IR COMPATIBLE WITH $SYK_{\tilde{q}}$

* FINITE q & $\tilde{q}$ ANALYSIS NUMERICALLY AMENABLE RG flows with 2 new fixed points PERSIST.

* FOR CERTAIN CHOICES OF q & $\tilde{q}$ $\left(1 < \tilde{q}/q < 2 \Rightarrow \Delta_{IR} \in (1,2)\right)$ S_0 DEPENDS NON-TRIVIALLY ON s

  $\rightsquigarrow$ DANGEROUS IRRELEVANCE?

* FINITE N ANALYSIS AMENABLE UP TO $N = 34$

"LANDSCAPE" OF RELEVANT DEFORMATIONS:

$$\boxed{H_{tot} = H_{q_f} + \sum_{i=1}^{n} \lambda_i H_{q_{f_i}}}$$

OPENS POSSIBILITY OF ENGINEERING MANY

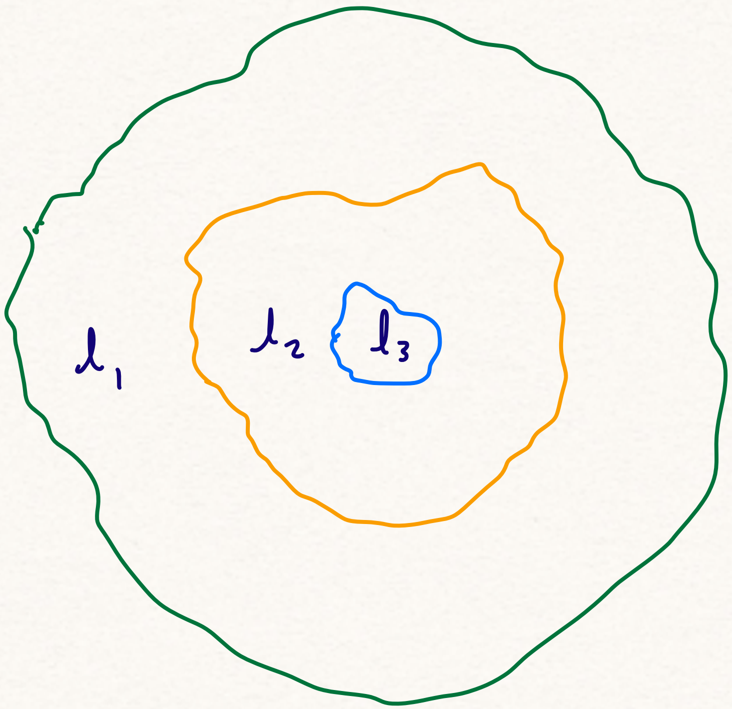
ASYMPTOTICALLY NEAR-AdS_2 SPACETIMES ...

GEOMETRIZATION OF RG FLOW?

GIVEN $S(T)$, WE CAN RECONSTRUCT

$U(\Phi)$ IN PUTATIVE DILATON-GRAVITY MODEL

e.g.



FOR THE CASE $q/\tilde{q} = 2$ & $q \gg 1$

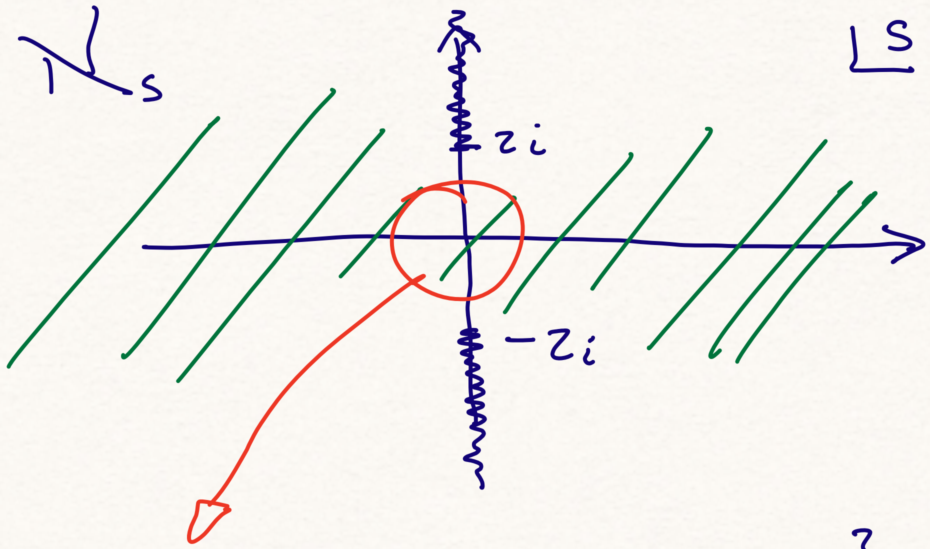
$$\boxed{\mathcal{N}_s = \frac{s^2}{\sqrt{1+4s^2}} \quad \text{SETS} \quad \frac{l_1^2}{l_2^2}}$$

MANY DIRECTIONS TO EXPLORE,

CONNECTION TO "SINGULAR" HORIZONS STUDIED BY [Horowitz-Santos-Kolanowski-Remmen-...]?

TIME & AUDIENCE PERMITTING, I WOULD LIKE TO MAKE SOME DESITTER REMARKS.

$s \in \mathbb{C}$



POTENTIALLY TAKES $\ell_2^2 \to -\ell_2^2$

PERHAPS $s \in \mathbb{C}$ GIVES US A NOVEL AVENUE TO EXPLORE AdS_2 SPACETIMES WITH dS_2 INTERIORS

RECENT WORK IN CONTEXT OF

$\boxed{\text{OPEN QUANTUM SYSTEMS}}$

HAS EXPLORED SOME FEATURES OF

$H_1 + iH_2$ TYPE HAMILTONIANS

[Zhang, Jian, Liu, Chen, Swingle, Roy, Sa, García-García...]

A MORE COMPLETE PICTURE MAY STEM FROM

LINDBLADIAN DYNAMICS.

MERCI BIEN!