Instability of Extremal Black Holes in AdS Supergravity

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Introduction: Extremal Black Holes are Unstable

- Black holes are parametrized by their **conserved charges**.

  Examples: angular momentum J and electric charge Q.

- **Extremal** black holes: **minimal mass** for given charges.

- Lowest possible energy $\Rightarrow$ **ground state** of the system.

  Intuition on extremal BH's: **particularly stable**. **Misleading**

- **Better** intuition for extremal black holes:

  Constituents **repel** maximally: BH's **fall apart** easily.

Supersymmetry and Unitarity

- A **theory** with **supersymmetry**: charges form an algebra.
- Unitarity of quantum **states** $\Rightarrow$ **lower bound** on energy.

$$M \geq \sum_i Q_i + \sum_a J_a$$

- **Supersymmetric** states $\Leftrightarrow$ unitarity bound **saturated**.
- **Fact**: Supersymmetric black holes are exceedingly **rare**.

  They **do not exist**, unless charges are **fine-tuned**.
- Moreover: **non**-SUSY extremal BHs are **classically** unstable.

Example: 4D Black Holes in Flat Space

- Entropy of Kerr-Newman black holes:

$$S = \frac{A}{4G_4} = \frac{\pi}{G_4}\left[\frac{J^2}{M^2} + \left(MG_4 + \sqrt{(MG_4)^2 - Q^2 - \frac{J^2}{M^2}}\right)^2\right]$$

- **Extremality** bound:

$$G_4 M^2 \geq \frac{1}{2}Q^2 + \sqrt{\frac{1}{4}Q^4 + J^2}$$

  Black holes solutions exist **only** for these masses.

- **Supersymmetry**: condition **in addition** to extremality.

$$G_4 M^2 = Q^2 \ , \quad J = 0$$

Instability: Superradiance

- Extremal black holes with $J \neq 0$ are **unstable**.
- Naïve emission rate of Hawking quanta:

$$\Gamma(\omega) = \frac{\sigma_{\rm abs}(\omega)}{e^{\beta(\omega - m\Omega)} - 1} \frac{d^3k}{(2\pi)^3}$$

  Ω is the **rotational velocity**.

- The rate **diverges** at sufficiently low energy $\omega \leq m\Omega$.
- Physical interpretation: **superradiant instability**.

Outline of Talk

- Part A: Thermodynamic Instability of **BTZ Black Holes**.

  Spontaneous emission of **chiral primaries**

- Part B: **Phase Diagram** of of Extremal AdS_5 Black Holes.

  Basic thermodynamics and its consequences.

- Part C: **Superfluidity**: the **Spectrum** of Scalar Fields.

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Part A

Thermodynamic Instability of BTZ Black Holes

Black Holes in AdS_3: SUSY or Not

- CFT_2 with $\mathcal{N} = (4,4)$ supersymmetry: $c_{L,R} = 6k_{L,R}$.

- The **unitarity bound** due to supersymmetry:

$$E \geq P + J_L - \frac{1}{2}k_L = E_{\mathrm{BPS}}$$

  "Angular" momenta: P within AdS_3, $J_{L,R}$ on S^3.

- **Black hole** solutions exist only when

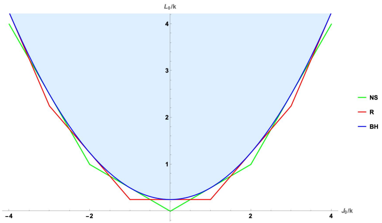
$$E \geq E_{\mathrm{ext}} = P + \frac{1}{2k_L}J_L^2 \geq E_{\mathrm{BPS}}$$

  Extremal black holes have $E = E_{\mathrm{ext}}$.

- **Supersymmetry** requires: $E_{\mathrm{ext}} = E_{\mathrm{BPS}} \quad \Leftrightarrow \quad J_L = k_L$.

The Unitarity Bound in AdS_3

- Black holes (blue) possible above extremality: dark blue curve.
- BPS limit (green): linear in J with range $0 \leq J \leq 2k_L$.
- Red curve and J in broader range: **spectral flow**.
- Black holes **impossible** between straight lines and blue region.

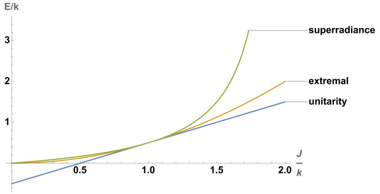


Thermodynamic Instability: Emission of Chiral Primary

- An (anti)-**chiral primary** has $\epsilon - p = j_L$ ($\epsilon + p = j_R$).
- Spontaneous emission may **increase** black hole entropy:

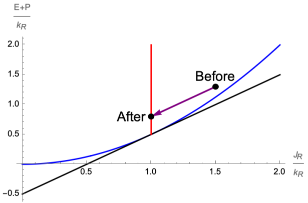
$$TdS = dM - \mu dP - \omega_R dJ_R - \omega_L dJ_L > 0 \quad \Rightarrow \quad \frac{J_L}{k_L} > 1$$

- End product: stable BH + gas of chiral primaries.



Scenario: Instability of NearExtremal BTZ BH

- Initial: BH is above extremality, but not too much.
- Final: BH has less mass and charge, but **more entropy**.
- Gas of chiral primaries carries negligible entropy.
- Interaction between two phases negligible.



Dynamics of Particle Emission

- Outward trajectory of a chiral primary: **no classical barrier**.
- **Finite** radial velocity at the horizon, particle never stops.
- Eventually: stable remnant and then emitted chiral primaries acquire circular orbits far from the central BH.
- The final gas is dilute because AdS has large volume.
- Upshot: BH shed finite mass and charge, increased entropy.

Part B

Phase Diagram of AdS_5 Black Holes

Black Holes in $\mathrm{AdS}_5 \times S^5$: Quantum Numbers

- Symmetry of theory: $SO(2,4) \times SO(6)$ + SUSY.
- $SO(2,4)$ representation of fields:
  Conformal weight E and **angular momenta** $J_{a,b}$.
- $SO(6)$ representation of fields: **R-charges** Q_I $(I = 1,2,3)$.
- So **asymptotic data** of black holes in AdS_5:
  Mass $\mathrm{M} = \mathrm{E}$, 2 Angular momenta $J_{a,b}$, and 3 R-charges Q_I.
- The mass of **supersymmetric** black holes:

$$M_{\mathrm{SUSY}} = Q_1 + Q_2 + Q_3 + J_a + J_b$$

This talk: $\frac{\pi}{4G_5}\ell_5^3 = \frac{1}{2}N^2$ and $\ell_5 = 1$.

The Constraint on Charges

- Supersymmetric black holes in AdS_5 **impossible** satisfy the quantum numbers of all **constraint**:

$$\left((Q_1Q_2 + Q_2Q_3 + Q_1Q_3) - \frac{1}{2}N^2(J_a + J_b)\right)\left(\frac{1}{2}N^2 + (Q_1 + Q_2 + Q_3)\right) + \frac{1}{2}N^2 J_a J_b - Q_1Q_2Q_3 = 0$$

- For example, **rotation is mandatory** $J_{a,b} \neq 0$.

- Extremal BHs that violate the constraint have **excess** mass.

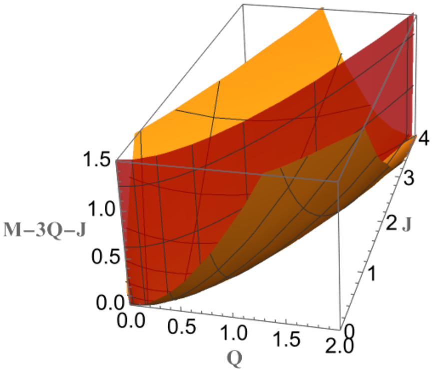
$$M - M_{\mathrm{SUSY}} \geq 0$$

The Mass Excess: Above the BPS Bound

- Extremal BHs that violate the constraint have **excess** mass.

$$M - M_{\mathrm{SUSY}} \geq 0$$

- It is favorable to shed the excess mass.



Instability Criterion of AdS_5 BH from the First Law

- Hypothesis: extremal nonSUSY BHs are **unstable**.

- The **first law** of black hole thermodynamics:

$$\begin{aligned} TdS &= dM - 2\Omega dJ - 3\Phi dQ \\ &= \underbrace{d(M - 2J - 3Q)}_{\text{mass excess}} + 2(1-\Omega)dJ + 3(1-\Phi)dQ \end{aligned}$$

- Emission of a **BPS particle**:

  $dJ \leq 0$ and $dQ \leq 0$ and mass excess preserved.

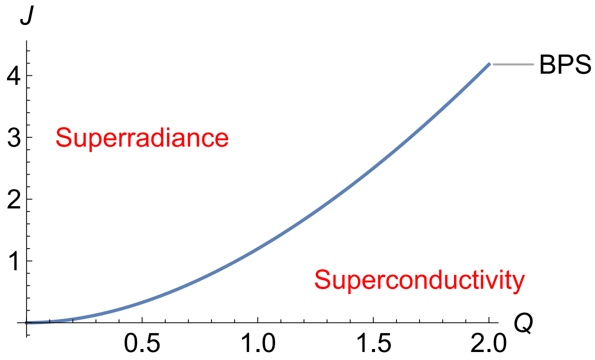
- Thermodynamically **favorable**: $dS > 0$ if $\Omega > 1$ or $\Phi > 1$.

Instability Criterion: Extremal Black Holes

- Three types of **extremal** black holes in AdS_5:
  - BPS: constraint **satisfied** and $\Phi = \Omega = 1$.
  - Reissner-Nordström-like: constraint **positive** and $\Phi > 1$.
  - Kerr-Like: constraint **negative** and $\Omega > 1$.
- Conclusion: **all** nonBPS extremal BHs are **unstable**.
- Stability bound **above** the extremality bound:

$$M_{\text{stability}} \geq M_{\text{ext}} \geq M_{\text{BPS}}$$

Phases of Extremal Black Holes in AdS_5



- Instability near axes: **superradiance/superconductivity**.

- They extend **all the way** to the BPS line.

  The BPS line is a **phase boundary**.

Pause

Part C

Superfluidity

Instability: the Breitenlohner Freedman Bound

- Model: free scalar field with mass m^2 in AdS_{d+1}.
- The wave equation reduces to the Schrödinger equation:

$$-\frac{d^2\psi}{dz^2} + \left[\vec{k}^2 + \frac{1}{z^2}(m^2\ell^2 - \frac{1-d^2}{4})\right]\psi = \omega^2\psi$$

- Bound states in $1/r^2$ potential if coefficient is too negative.

  Instability: this corresponds to exponential time dependence.
- The **Breitenlohner-Freedman bound**:

$$m^2\ell_{d+1}^2 \geq -\frac{1}{4}d^2$$

Scalars in AdS_5

- $\mathcal{N} = 4$ SYM $\underset{\text{AdS/CFT}}{\Leftrightarrow}$ $\mathcal{N} = 8$ SUGRA in bulk.

- 42 Scalars fields in **vacuum** of $\mathcal{N} = 8$ AdS_5 supergravity:
  - t in $\mathbf{20}'$ of $SU(4)_R$: $m^2 = -4 \leftrightarrow \Delta = 2$
  - φ in $\mathbf{10}_c$ of $SU(4)_R$: $m^2 = -3 \leftrightarrow \Delta = 3$
  - Λ in $\mathbf{1}_c$ of $SU(4)_R$: $m^2 = 0 \leftrightarrow \Delta = 4$

- The BF stability bound in AdS_5:

$$m^2 \ell_5^2 \geq -\frac{1}{4} d^2 \underset{d=4}{=} -4$$

The t scalars are **at** the BF bound in AdS_5.

Kerr-Newman AdS as a Supergravity Solution

- Here the environment is an AdS_5 **black hole**, not the vacuum.
- Kerr-Newman-AdS solves Einstein-Maxwell-AdS theory.

  Now: **reinterpret** it as a solution to supergravity.
- $\mathcal{N} = 8$ SUGRA has $SU(4)_R$ symmetry $\Rightarrow$ 15 vector fields.
- Pick **background** vector field as the unique linear combination that permits **constant** scalars.

Fluctuating Matter Fields in AdS_5

- Fluctuations around black hole: supergravity fields expanded to quadratic order around background.
- Symmetry breaking pattern: $SU(4)_R \to SU(3) \times U(1)$.
- Generally: matter fields are **charged** with respect to “the” gauge field in the black hole background.
- Also: **degeneracy** remains due to $SU(3)$ global symmetry.
- $\mathbf{20}'$ scalars $t \quad \Rightarrow \quad 8$ neutral t_- and 12 t_+ with charge $e = \pm 2$:

$$\mathbf{20}' \;\to \mathbf{8}_0 \;\oplus \left(\mathbf{3}_2 \oplus \bar{\mathbf{3}}_2 \oplus \mathbf{3}_{-2} \oplus \bar{\mathbf{3}}_{-2}\right)$$

Attractor Flow

- The radial dependence in BH background: **attractor flow**.
- Very well developed in **ungauged** supergravity.
- Complicating factors in current context:
  - AdS vacua in **gauged** supergravity.
  - **Rotation**, or else supersymmetry is **not possible**.
  - **General extremal** case, not necessarily supersymmetric.
- Upshot: an effective mass in near horizon AdS_2 region (with squashed S^3 fibre).

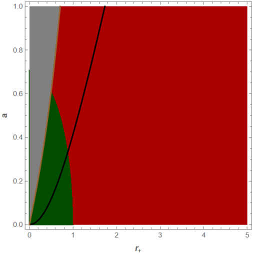
Light Scalars in KNAdS Background

- The BF-bound in AdS_2:

$$m^2\ell_2^2 = -4\frac{\ell_2^2}{\ell_5^2} \geq -\frac{1}{4}d^2 \underset{d=1}{=} -\frac{1}{4}$$

- The fate depends on the BH parameters via the AdS_2 radius.

- Large unstable region **includes** many BPS black holes.



Non-Minimal Couplings

- **All** of the $\mathbf{20}'$ scalars have **non-minimal** couplings.
- Kinetic terms for vectors in supergravity:

$$\mathcal{L} \sim -\mathcal{N}(\phi) F_{\mu\nu} F^{\mu\nu}$$

  Kinetic function $\mathcal{N}(\phi)$ depends on the scalar field.

- In AdS_2, this **Pauli Coupling** is an effective mass:

$$m^2_{\mathrm{Pauli}} = -p \cdot \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

  The coupling $p = +2$ for t_+ and $p = -2$ for t_-.

- Moreover, 8 neutral t_- mix with 8 fluctuating gauge fields a_-.

The Physics of Minimal Couplings

- **Minimal coupling** to the background vector field:

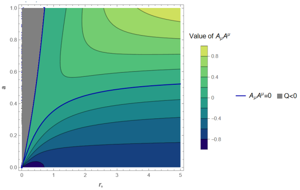
$$|(D_\mu + ieA_\mu)\phi|^2$$

For the 12 t_+ fields, the $U(1)_R$ charge is $e = 2$.

- In AdS$_2$, the coupling gives an effective mass

$$m^2_{\text{minimal}} = e^2 g^{\mu\nu} A_\mu A_\nu$$

It is **negative** in an electric background.



Superfluidity

- The background electric potential gives **charged scalars** an **expectation value**.
- This is **superfluidity**
- Holographic superconductivity was much studied, but not embedded in full supergravity.
- Superconductivity applies when the BH is **under**rotating.

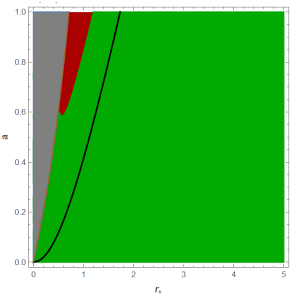
The Fate of the Lightest Scalars: Charged Sector

- 12 charged t_+ also have Pauli couplings to $\mathcal{F}_{\mu\nu}$.

  The coupling $p = 2 > 0$ compensates the minimal coupling.

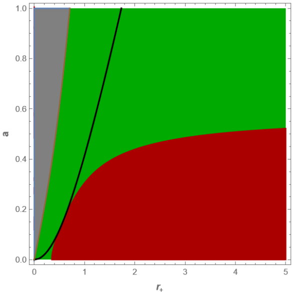
- **On balance**: the t^+ scalars are **stable** for BPS black holes.

- Also stable in Reissner-Nordström: **no superconductivity**



The Pseudo Scalars φ_1 Condense

- $\mathbf{10}_c$ fields φ have $m^2\ell_5^2 = -3$: **easily** stable in AdS_5 vacuum.
- In KN-AdS_5 background: two components φ_1 have $e = \pm 3$.
- This **large** minimal coupling drives **superconductivity**.



Summary

- Charges of **supersymmetric** BH's must satisfy a **constraint**.

- **Non**SUSY extremal black holes are **classically unstable**.

- AdS_3: **spontaneous** emission of chiral primaries.

- AdS_5:
  - overrotation $\Rightarrow$ **superradiance**
  - overcharging $\Rightarrow$ **superconductance**