Near-extremal charged and rotating black holes in 3+1d de Sitter space

Francesca Mariani

Ghent University

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Based on 2212.14356 with A. Castro & C. Toldo and on work in progress with C. Toldo

Outline & Motivations

- (Near-)Extremal black holes in de Sitter spacetime
- Study deviations away from extremality for $RNdS_4$ and Kerr-dS_4 BHs
- Characterise the differences between BHs in de Sitter ($\Lambda > 0$) versus AdS ($\Lambda < 0$) and Minkowski ($\Lambda = 0$) $\rightarrow$ more extremal limits & richer phase space of solutions
- Charged and rotating dS BH; dimensional reduction and gravitational perturbations. Are they described by JT?
- Review of Reissner-Nordström dS and presentation of Kerr-dS

De Sitter black holes

Solutions of Einstein's equations with a positive Cosmological Constant ($\Lambda > 0$).

Presence of Λ has qualitative and quantitative repercussions on our understanding of black holes:

- adds a **cosmological horizon**, r_c
- Thermodynamics at the cosmological horizon ['77 Gibbons, Hawking]['22 Banihashemi, Jacobson, Svesko, Visser]

  $$dM = -T_c dS + \Phi_c dQ + \Omega_c dJ$$

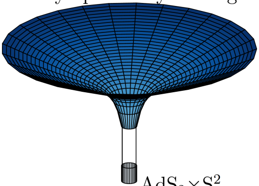
  $\rightarrow$ to what extent can we treat the cosmological horizon as a thermal entity?
- New (near-)extremal limits & near horizon geometries

BHs suffer modifications due to the surroundings, de Sitter BHs ideal lab to epxlore and quantify these differences.

(Near-)Extremality

Extremality: two or three horizons coincide

- Temperature at the extremal horizon vanishes $\rightarrow T_{r_h} = 0$
- in Minkowski, Extremal BHs have minimum value of mass, given a fixed value of charge: $M = Q$
- Geometry develops a throat, near horizon region completely decouples from far away region:
  AdS_2 factor NH geometry $\Rightarrow$ enhancement of symmetry



Near-extremality: the horizons are slightly separated from each other

- The system acquires a little temperature: $T_{r_h} \neq 0$
- Mass increases, $\delta M \sim T_{r_h}^2$, $\delta S \sim T_{r_h}$
- Finite distance separates the NH region from the far away region

Reissner-Nordström black holes in de Sitter

Charged BHs with spherical symmetry:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R - 2\Lambda - F_{\mu\nu}F^{\mu\nu})\,,$$

$$ds^2 = -V(r)dt^2 + \frac{dr^2}{V(r)} + r^2 d\Omega_2^2, \quad V(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}r^2$$

- Three horizons as solutions of $V(r) = 0$ at $r = \{r_-, r_+, r_c\}$

RNdS$_4$

$\downarrow$ **Decoupling limit**

- Three different extremal limits: Cold, Nariai, Ultracold
- Near horizon geometries are of the form $\mathcal{M}_2 \times S^2$

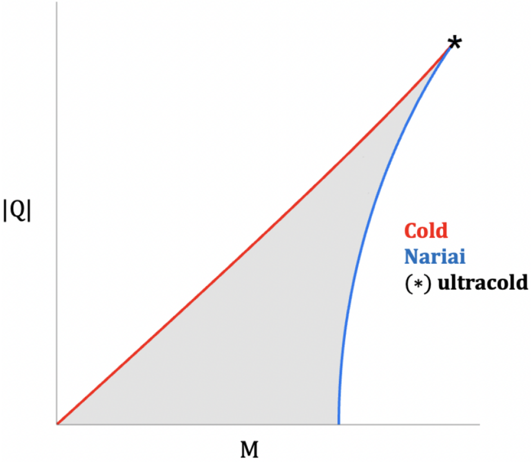
$$\mathcal{M}_2 = \{\mathrm{AdS}_2, \mathrm{dS}_2, \mathrm{Mink}_2\}$$

$\rightarrow$ We build the effective gravitational theory on S^2

['22 A. Castro, FM, C. Toldo]

Phase space of $RNdS_4$

- Main difference with AdS and Minkowski BHs: finite region of admitted physical solutions & naked singularities outside of it



Thermodynamics of RNdS_4

- Thermodynamics of Cold and Nariai at fixed charge, $\delta Q = 0$:

$$T_+ \sim \mathcal{O}(\lambda), \quad M = M_{ext,c} + \frac{T_+^2}{M_{gap}} + \cdots, \quad S_+ = S_c + \frac{2T_+}{M_{gap}}, \quad M_{gap}^{cold} > 0$$

$$T_c \sim \mathcal{O}(\lambda), \quad M = M_{ext,n} + \frac{T_c^2}{M_{gap}} + \cdots, \quad S_c = S_n - \frac{2T_c}{M_{gap}}, \quad M_{gap}^{Nariai} < 0$$

- Thermodynamics of Ultracold is different and present some subtleties

$$\delta Q \neq 0, \quad \delta S_c \sim \delta \Phi_c$$

Change in entropy driven by a change in chemical potential rather than a change in temperature → infinite specific heat!

$$C_S^{-1} = \frac{1}{T}\left(\frac{dT}{dS}\right)\Bigg|_{Q=const}$$

→ reminiscent of flat 2D gravity ['19 Afshar, Gonzalez, Grumiller]['19 Vassilevich]

Effective two-dimensional theory

Dimensional reduction of Einstein-Maxwell theory on $\mathbf{S}^2$:

$$I_{4D} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(\mathcal{R}^{(4)} - 2\Lambda_4 - F_{\mu\nu}F^{\mu\nu}\right)$$

$$ds_4^2 = g_{\mu\nu}^{(4)}x^\mu x^\nu = \frac{\Phi_0}{\Phi}g_{ab}dx^a dx^b + \Phi^2\left(d\theta^2 \sin^2\theta d\phi^2\right)$$

$$F = F_{ab}x^a \wedge x^b$$

$$I_{2D} = \frac{1}{4G_4}\int d^2x\sqrt{g^{(2)}}\Phi^2\left(\mathcal{R}^{(2)} + \frac{2\Phi_0}{\Phi^3} - 2\Lambda_4\frac{\Phi_0}{\Phi} - \frac{\Phi}{\Phi_0}F_{ab}F^{ab}\right)$$

- Φ = dilaton, scalar field parametrizing the size of the 2-sphere
- $F_{\mu\nu}$ purely electric
- The 2D system that we obtain shares many features with JT gravity

Effective two-dimensional theory

Link between 4D and 2D language:
Extremal NH background = constant IR background:

$$\Phi(x) = \Phi_0 \ , \quad g_{ab} = \bar{g}_{ab} \ , \quad A_a = \bar{A}_a \ .$$

$\Phi(x) = \Phi_0$ means:

- Constant radius of the 2D sphere
- Constant curvature of the 2D manifold $\mathcal{M}_2$:

$$\mathcal{R}_0^{(2)} = -\frac{2}{\ell_2^2} = -\frac{2}{\Phi_0^2}(1 - 2\Lambda_4 \Phi_0^2) \begin{cases} \Phi_0^2 < \frac{1}{2\Lambda_4} \Rightarrow \mathrm{AdS}_2 \\ \Phi_0^2 > \frac{1}{2\Lambda_4} \Rightarrow \mathrm{dS}_2 \\ \Phi_0^2 = \frac{1}{2\Lambda_4} \Rightarrow \mathrm{Mink}_2 \end{cases}$$

Near-Extremal NH configuration = perturbations IR background

$$\Phi(x) = \Phi_0 + \lambda \mathcal{Y}(x) \ , \quad g_{ab} = \bar{g}_{ab} + \lambda h_{ab} \ , \quad A_a = \bar{A}_a + \lambda \mathcal{A}_a \ .$$

Effective two dimensional theory

- Solutions to the equations of motion of this 2D JT-like system are solutions of the 4D system as well

Strategy:

- Solve 2D equations for the three different near-extremal systems (Cold, Nariai & Ultracold) and compute 2D on-shell action (Cold & Ultracold)
- Match 2D and 4D thermodynamic behaviour

Cold

IR backaground is the locally AdS_2 solution, in radial gauge

$$\bar{g}_{ab}dx^a dx^b = d\rho^2 + \gamma_{TT}dT^2 \ , \gamma_{TT} = -\left(\alpha(T)e^{\rho/\ell_A} + \beta(T)e^{-\rho/\ell_A}\right)$$

has a horizon at $\gamma_{TT}(\rho = \rho_h) = 0 \rightarrow$ 2D black hole with associated temperature and entropy:

$$T_{2D} = \frac{1}{2\pi}\partial_\rho\sqrt{\gamma}|_{\rho=\rho_h}, \quad S_{2D} = \pi\Phi(x)^2_{horizon} = \pi\Phi_0^2 + 2\pi\Phi_0\lambda\mathcal{Y}(x)|_{horizon}$$

Contact with 4D thermodynamics:

Upon identification of $\Phi_0 = r_0$ and $M^{cold}_{gap} = 1/2\pi^2\ell_A^2\Phi_0$,

$$T_+ = \frac{\lambda}{\ell_A^2}T_{2D} \ , \quad S_+ = S_{2D}$$

Ultracold

IR background is Mink_2, in Eddington-Finkelstein coordinates

$$\bar{g}_{ab}dx^a dx^b = -2\left(\mathcal{P}(u)r + \mathcal{T}(u)\right)du^2 - 2dudr \ .$$

We specify to static solutions, along the lines of [Godet, Marteau '21][Grumiller, Ruzziconi, Zwickel '22],

$$\mathcal{P}(u) = \mathcal{P}_0 \ , \quad \mathcal{T}(u) = \mathcal{T}_0 \ .$$

- Dilaton independent from background metric at fixed charge, different from AdS_2 case. Strange interplay between deformations of dilaton and heating up Mink_2. Same solutions found from ['21 Godet, Marteau] in $\widehat{CGHS}$ models, we impose the same boundary conditions for the JT field

Holographic renormalization

2D renormalized on-shell action for constant $\mathcal{P}_0$ and $\mathcal{T}_0$:

$$I_{2D,UC} = -2\pi\Phi_0 b_0 \lambda + I_{global}{}^1$$

We extract the entropy

$$S_{2D} = \beta \left(\frac{\partial I}{\partial \beta} \right) - I \quad \Rightarrow S_{2D} = -I_{2D,UC}$$

⇒ The temperature does not affect the on-shell action! In agreement with the 4D behaviour found for ultracold.

[1] I_{global} is the value of the integral evaluated at the horizon.

Kerr black holes

- Characterized by a more complicated metric, no spherical symmetry
- Kerr BHs in 3+1d Minkowski: JT gravity description of near-extremal dynamics

['19 U. Moitra, S. K. Sake, S. P. Trivedi, V. Vishal]
['20 V. Godet, C. Marteau]
['20 A. Castro, V. Godet]

['21 A. Castro, V. Godet, J. Simón, W .Song, B. Yu]

- Can we extend the analysis also to Kerr-de Sitter?
- Anti de Sitter: NH geometry, perturbations above extremality and holographic renormalization

Kerr black holes in de Sitter

Rotating BHs in a spacetime with a positive cosmological constant ($\Lambda > 0$):

$$S = \frac{1}{16\pi G} \int d^4x\sqrt{-g}(R - 2\Lambda),$$

$$ds^2 = -\frac{\Delta_r}{\rho^2}\left(du - \frac{a}{\Xi}\sin^2\theta d\phi\right)^2 + 2dudr - \frac{2a\sin^2\theta}{\Xi}drd\phi$$
$$+ \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta}{\rho^2}sin^2\theta\left(adu - \frac{r^2 + a^2}{\Xi}d\phi\right)^2$$
$$\Delta_r = (r^2 + a^2)(1 - \frac{r^2}{\ell^2}) - 2mr, \quad \Delta_\theta = 1 + \frac{a^2}{\ell^2}cos^2\theta,$$
$$\rho^2 = r^2 + a^2cos^2\theta, \quad \Xi = 1 + \frac{a^2}{\ell^2}$$

- Horizons at $\Delta_r = 0 \Rightarrow r = \{r_-, r_+, r_c\}$
- Three extremal solutions: Cold, Nariai, ultracold

[2009 T. Hartman, K. Murata, T. Nishioka, A. Strominger]
['10 D. Anninos, T. Anous]
['10 D. Anninos, T. Hartman]

Kerr-dS$_4$

Extremal solutions still have 3 different NH geometries:

$$ds^2 = \Gamma(\theta)\Big(\tilde{g}_{ab}dx^a dx^b + \alpha(\theta)d\theta^2\Big) + \gamma(\theta)\,(d\phi + krdu)^2$$

$$\tilde{g}_{ab}dx^a dx^b = \begin{cases} -r^2du^2 + 2dudr, & \text{cold} & \rightarrow & \text{AdS}_2 \\ -du^2 + 2dudr, & \text{ultracold} & \rightarrow & \text{Mink}_2 \\ r^2du^2 + 2dudr, & \text{Nariai} & \rightarrow & \text{dS}_2 \end{cases}$$

- Similar phase-space diagram (M, J) with a finite region of admitted physical solutions as RNdS$_4$
- Similar thermodynamics as RNdS$_4$, with M_{gap} for Cold and Nariai

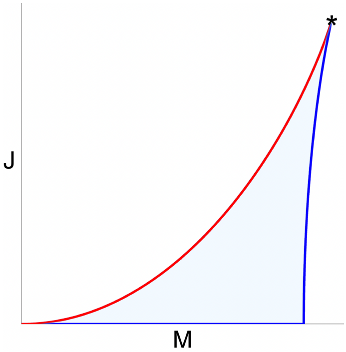
Thermodynamics and phase-space diagram

$$T_{\rm h} = \frac{|\Delta_r'(r_h)|}{4\pi r_h^2} = \frac{r_h^2 - a^2 + 2r_h^4/\ell^2}{2\pi r_h(r_h^2 + a^2)},$$

Fixed angular momentum: $\delta J = 0$:

$$M_{\rm cold} = M_0 + \frac{T_+^2}{M_{\rm gap}^{\rm cold}} + \cdots, \quad M_{\rm gap}^{\rm cold} > 0, \quad S_+ = S_0 + \frac{2T_+}{M_{\rm gap}^{\rm cold}} + \cdots$$

$$M_{\rm Nariai} = M_{\rm n} + \frac{T_{\rm n}^2}{M_{\rm gap}^{\rm n}} + \cdots, \quad M_{\rm gap}^{\rm n} < 0, \quad S_{\rm n} = S_c + \frac{2T_c}{M_{\rm gap}^{\rm n}} + \cdots$$



Gravitatational perturbations of Kerr-dS_4

Goal:

- Understand whether a JT mode is responsible for deviations away from extremality also for Kerr-dS_4 BHs

Strategy:

- Consider gravitational perturbations around extremal NHEK background (Cold, Nariai, Ultracold)
- Solve linearized Einstein's equations
- See if one of the modes perturbing the NH geometry satisfies a JT-like equation

Linearized Einstein's equations

Perturb the background extremal metric by looking at higher orders contributions in the NH geometry ['20 V. Godet, C. Marteau]:

$$ds^2_{\text{Kerr}} \xrightarrow{\text{Dec. limit}} \bar{g}_{\mu\nu,\text{NHEK}} dx^\mu dx^\nu + \lambda h_{\mu\nu} dx^\mu dx^\nu + \cdots$$

- $h_{\mu\nu} dx^\mu dx^\nu$ is the $\mathcal{O}(\lambda)$ contribution to the NH geometry
- We will perturb the extremal metric with an ansatz that is motivated by the NH geometry:

$$ds^2 = \Big(\Gamma(\theta) + \epsilon\chi(u,r)\Big)\Big(\big(\kappa r^2 + r\mathcal{P}(u) + \mathcal{T}(u)\big)\, du^2 + \epsilon\psi(u,r)du^2 + \alpha(\theta)d\theta^2\Big)$$
$$+ \big(2\Gamma(\theta) + \epsilon\eta(u,r)\sin^2\theta\big)\, dudr + \Gamma(\theta)\gamma(\theta)\left(\frac{1+\epsilon\Phi(u,r)}{\Gamma(\theta)+\epsilon\chi(u,r)}\right)(d\phi + krdu + \epsilon A)^2$$

Linearized Einstein's equations

Ansatz for the perturbed metric, motivated by geometric considerations ['20 V. Godet, C. Marteau]:

$$ds^2 = \Big(\Gamma(\theta) + \epsilon\chi(u,r)\Big)\Big(\big(\kappa r^2 + r\mathcal{P}(u) + \mathcal{T}(u)\big)\, du^2 + \epsilon\psi(u,r)du^2 + \alpha(\theta)d\theta^2\Big)$$

$$+ \big(2\Gamma(\theta) + \epsilon\eta(u,r)\sin^2\theta\big)\, dudr + \Gamma(\theta)\gamma(\theta)\left(\frac{1+\epsilon\Phi(u,r)}{\Gamma(\theta)+\epsilon\chi(u,r)}\right)(d\phi + krdu + \epsilon A)^2$$

$$\kappa = \begin{cases} -1, & \text{cold} \\ \ \ 0, & \text{ultracold} \\ +1, & \text{Nariai} \end{cases}$$

- Perturbations parametrized in terms of the fields χ, η, Φ and ψ and the Gauge field A:

$$A(u,r,\theta) = A_u(u,r,\theta)du + A_r(u,r,\theta)dr$$

- Dynamics is however dictated solely by the dilaton field Φ if we impose conditions to avoid conical singularities:

$$\chi \propto \Phi, \quad \eta \propto \Phi, \quad \psi \propto r^2\Phi$$

Linearized Einstein's equations

Look for solutions of linearized Einstein's equations:

$$R_{\mu\nu} - \Lambda g_{\mu\nu} = 0, \quad R_{\mu\nu} = R^{(0)}_{\mu\nu} + R^{(\epsilon)}_{\mu\nu}, \quad g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(\epsilon)}_{\mu\nu}$$

From $R^{(\epsilon)}_{\mu\nu} - \Lambda g^{(\epsilon)}_{\mu\nu} = 0 \to$ solutions for the modes and for the Gauge field:

$$\Phi(u,r) = r\phi_1(u) + \phi_0(u)$$
$$\Box_2 \chi = -2\kappa\chi$$

$\Box_2 \equiv$ Laplacian on the 1+1d metrics

$$g_{ab}dx^a dx^b = \left(\kappa r^2 + r\mathcal{P}(u) + \mathcal{T}(u)\right) du^2 + 2dudr$$

From $R^{(\epsilon)}_{t\phi} - \Lambda g^{(\epsilon)}_{t\phi} = R^{(\epsilon)}_{tt} - \Lambda g^{(\epsilon)}_{tt} = 0 \to$ **JT equations**:

$$\boxed{\left(\nabla_a \nabla_b - g_{ab}\Box\right)\Phi - \kappa g_{ab}\Phi = 0}$$

Conclusions & future outlook

- Our work shows that the dynamics of de Sitter black holes is classically described by JT gravity
  → For $RNdS_4$ dimensional reduction and analysis of 2D system
  → For Kerr-dS_4 gravitational perturbations around extremal 4D metric
- Generalization to Kerr-AdS_4 & holographic renormalization (work in progress)
- Introduction of quantum corrections

Thank you!