Effective Field Theory Breakdown Near Cool Black Holes

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NYU

Introduction

Effective field theory

How do we encode hints of ultraviolet physics in the infrared?

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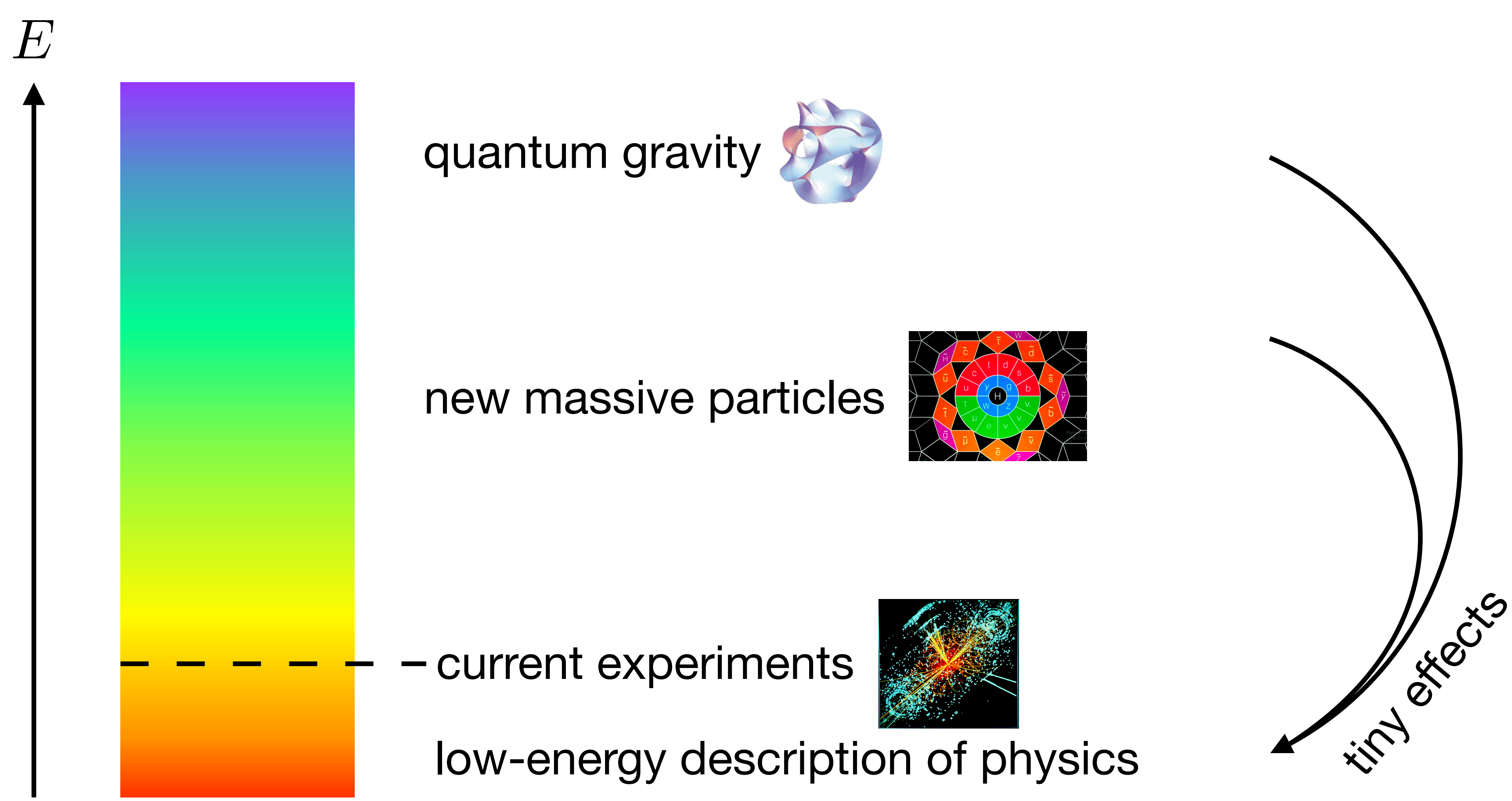


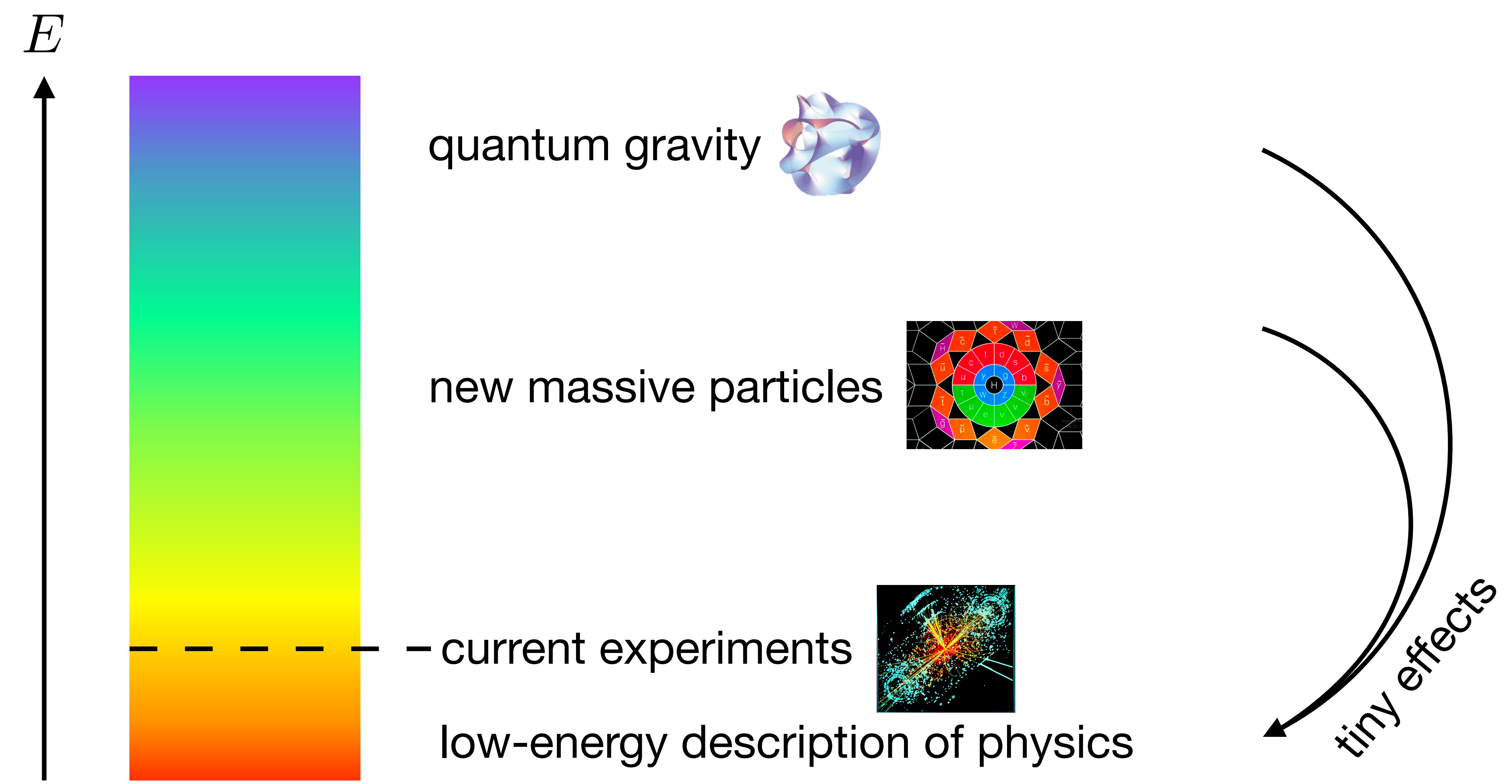
Effective field theory

How do we encode hints of ultraviolet physics in the infrared?

- Write down a Lagrangian, built out of operators $\mathcal{O}_i$, with couplings c_i:

$$\mathcal{L} = \overline{\mathcal{L}} + \sum_i c_i \mathcal{O}_i$$

including all possible operators consistent with the fields and symmetries.



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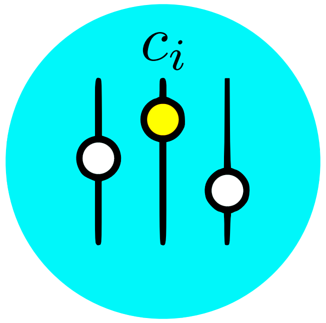
Einstein-Hilbert term   Higher-derivative operators



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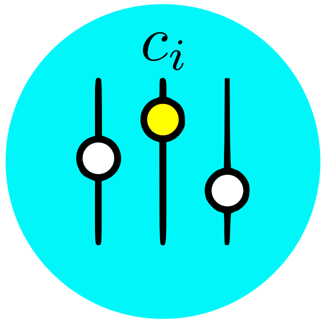
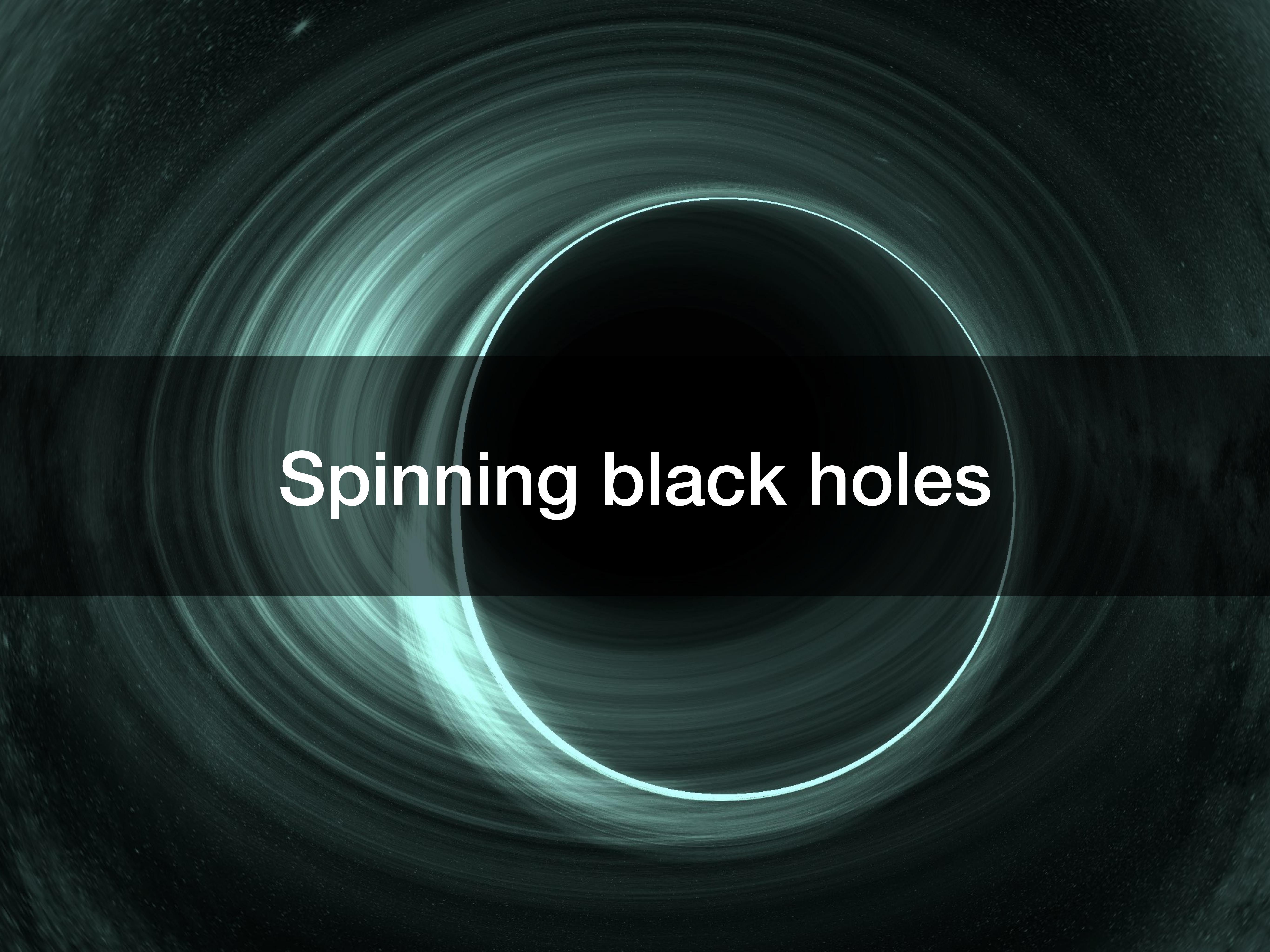


- The famous nonrenormalizability of perturbative quantum gravity is now well understood as a manifestation of the fact that GR is itself an effective field theory.

- Higher-derivative terms are generated by the UV completion, and are suppressed by either the Planck scale or the scale of massive degrees of freedom. Run from loops of light states (gravitons, photons, etc.).

Arkani-Hamed, Huang, Liu, **GR** [2109.13937]


Spinning black holes

Kerr solution

- The Kerr geometry solves the vacuum Einstein equations (i.e., it is Ricci-flat) and describes a spinning black hole:

$$
\begin{aligned}
\mathrm{d}s_{\mathrm{K}}^2 = & -\frac{\Delta(r)}{\Sigma(r,\theta)}(\mathrm{d}t - a\,\sin^2\theta\,\mathrm{d}\phi)^2 + \Sigma(r,\theta)\left(\frac{\mathrm{d}r^2}{\Delta(r)} + \mathrm{d}\theta^2\right) \\
& + \frac{\sin^2\theta}{\Sigma(r,\theta)}\left[a\,\mathrm{d}t - (r^2 + a^2)\mathrm{d}\phi\right]^2 \\
\Sigma(r,\theta) = & \; r^2 + a^2\cos^2\theta \\
\Delta(r) = & \; r^2 + a^2 - 2Mr \qquad\qquad a = GJ/M
\end{aligned}
$$

- Event horizon at $r_+ = M + \sqrt{M^2 - a^2}$

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&+ \frac{\sin^2\theta}{\Sigma(r,\theta)}\left[a\,\mathrm{d}t - (r^2+a^2)\mathrm{d}\phi\right]^2 \\
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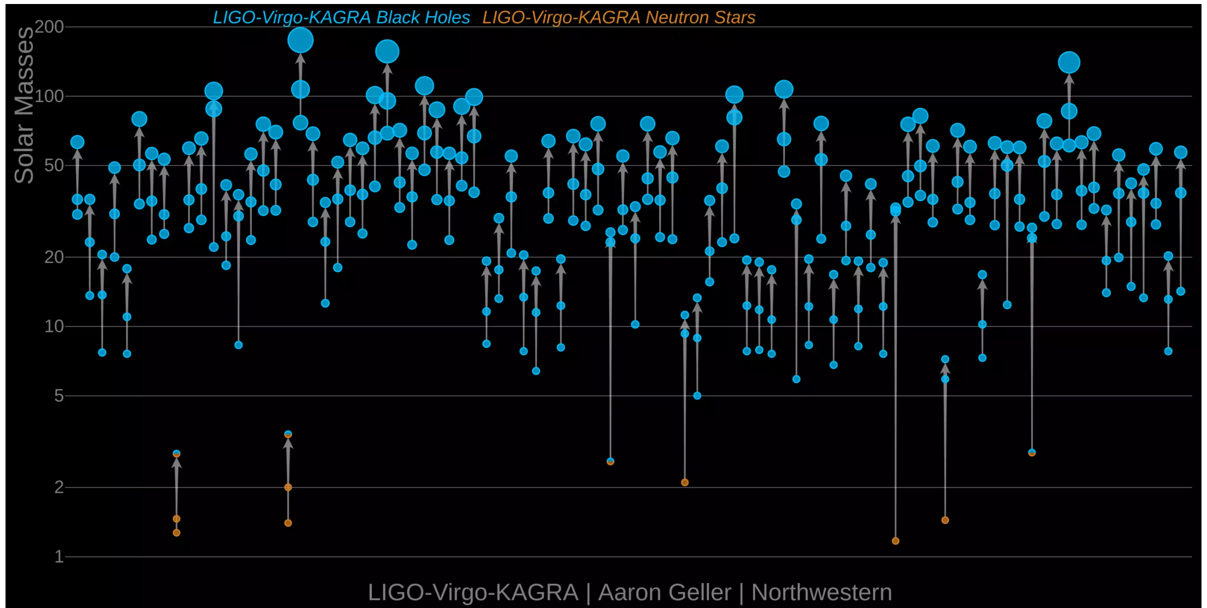
- Event horizon at $r_+ = M + \sqrt{M^2 - a^2}$

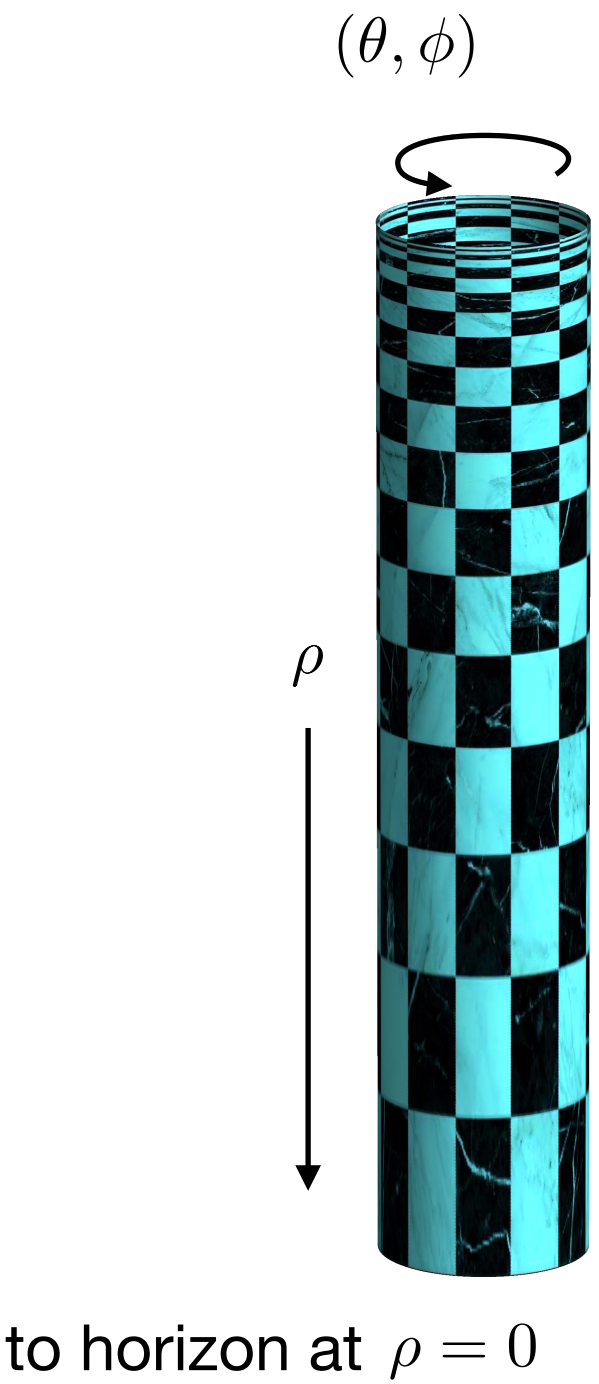
- No Birkhoff theorem: exterior spacetime rotating matter is generally not Kerr.

- Many likely examples observed by LIGO:



NHEK

In the extreme limit, the Kerr geometry has an infinitely long throat near the horizon, the near-horizon extremal Kerr (NHEK) geometry:



$$ds^2_{\mathrm{NHEK}} = J(1+x^2)\left[-\rho^2 \mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{\rho^2} + \frac{\mathrm{d}x^2}{1-x^2} + \frac{2(1-x^2)}{(1+x^2)^2}(\mathrm{d}\phi + \rho\, \mathrm{d}t)^2\right]$$

Bardeen, Horowitz [hep-th/9905099]

- Similar to $\mathrm{AdS}_2 \times S^2$, with $\mathrm{O}(2,1) \times \mathrm{U}(1)$ symmetry

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to horizon at $\rho = 0$

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- Similar to $\mathrm{AdS}_2 \times S^2$, with $\mathrm{O}(2,1) \times \mathrm{U}(1)$ symmetry
- First correction around NHEK patching onto asymptotically flat Kerr geometry can be understood as coming from certain modes of the linearized Einstein equations in the NHEK background cf. Hadar, Lupsasca, Porfyriadis [2012.06562]
- Of great interest to Kerr-CFT proposals Guica, Hartman, Song, Strominger [0809.4266]

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In the extremal limit, the Kerr geometry has an infinitely long throat near the horizon, the near-horizon extremal Kerr (NHEK) geometry:

$$\mathrm{d}s^2_{\mathrm{NHEK}} = J(1+x^2)\left[-\rho^2\mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{\rho^2} + \frac{\mathrm{d}x^2}{1-x^2} + \frac{2(1-x^2)}{(1+x^2)^2}(\mathrm{d}\phi + \rho\,\mathrm{d}t)^2\right]$$

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- Of great interest to Kerr-CFT proposals
  Guica, Hartman, Song, Strominger [0809.4266]
- We will determine the near-horizon behavior of the EFT-corrected Kerr geometry by computing EFT corrections to the NHEK.

EFT in vacuum

- Let us first consider the gravitational EFT where no matter is present.
- We must build the EFT out of the Riemann tensor, with no contractions. Any appearance of the Ricci tensor can be removed via field redefinitions.

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- In four spacetime dimensions, the Gauss-Bonnet term is topological:

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\begin{aligned}
&\int \mathrm{d}^4x\,\sqrt{-g}(R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2) \\
&= \int \mathrm{d}^4x\,\partial_a\left[\sqrt{-g}\epsilon^{abcd}\epsilon_{ef}{}^{gh}\Gamma^e_{gb}\left(\frac{1}{2}R^f{}_{hcd} - \frac{1}{3}\Gamma^f_{ci}\Gamma^i_{hd}\right)\right] \\
&= 8\pi^2\,\chi(\mathcal{M})
\end{aligned}
$$

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- First interesting terms at cubic and quartic order in the Riemann tensor:

$$\mathcal{L} = \frac{1}{2\kappa^2}\left(R + \eta\kappa^4\mathcal{R}^3 + \lambda\,\kappa^6\mathcal{C}^2 + \tilde{\lambda}\,\kappa^6\tilde{\mathcal{C}}^2\right)$$

$$\begin{aligned}\mathcal{R}^3 &= R_{ab}{}^{cd}R_{cd}{}^{ef}R_{ef}{}^{ab}\\ \mathcal{C} &= R_{abcd}R^{abcd}\\ \tilde{\mathcal{C}} &= \tilde{R}_{abcd}R^{abcd}\\ \tilde{R}_{abcd} &= \epsilon_{ab}{}^{ef}R_{efcd}\end{aligned}$$

EFT in vacuum

- Classical equations of motion:

$$R_{ab} - \frac{1}{2} R g_{ab} = T_{ab}^{\text{cubic}} + T_{ab}^{\text{quartic}}$$

$$T_{ab}^{\text{cubic}} = \eta\, \kappa^4 \, \left[3\, R_a{}^{cde} R_{de}{}^{gh} R_{ghcb} + \frac{1}{2} g_{ab} R_{gh}{}^{cd} R_{cd}{}^{ef} R_{ef}{}^{gh} - 6 \nabla^c \nabla^d \left(R_{acgh} R_{bd}{}^{gh} \right) \right]$$

$$T_{ab}^{\text{quartic}} = -\lambda\, \kappa^6 \, \left(8 R_{acbd} \nabla^c \nabla^d \mathcal{C} + \frac{g_{ab}}{2} \mathcal{C}^2 \right) - \tilde{\lambda}\, \kappa^6 \, \left(8 \tilde{R}_{acbd} \nabla^c \nabla^d \tilde{\mathcal{C}} + \frac{g_{ab}}{2} \tilde{\mathcal{C}}^2 \right)$$

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$$T^{\text{quartic}}_{ab} = -\lambda\, \kappa^6 \, \left(8 R_{acbd} \nabla^c \nabla^d \mathcal{C} + \frac{g_{ab}}{2} \mathcal{C}^2 \right) - \tilde{\lambda}\, \kappa^6 \, \left(8 \tilde{R}_{acbd} \nabla^c \nabla^d \tilde{\mathcal{C}} + \frac{g_{ab}}{2} \tilde{\mathcal{C}}^2 \right)$$

- Solve at linear order in the Wilson coefficients,

$$g_{ab} = g^{(0)}_{ab} + \eta\, h^{(6)}_{ab} + \lambda\, h^{(8)}_{ab} + \tilde{\lambda}\, \tilde{h}^{(8)}_{ab}$$

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- Solve at linear order in the Wilson coefficients,

$$g_{ab} = g_{ab}^{(0)} + \eta\, h_{ab}^{(6)} + \lambda\, h_{ab}^{(8)} + \tilde{\lambda}\, \tilde{h}_{ab}^{(8)}$$

- $O(\eta^2)$ contribution $\delta h_{ab}^{(6)}$ to $h_{ab}^{(6)}$ is smaller than $h_{ab}^{(8)}$:
  By power counting and the form of the equations of motion,

$$(\delta h_{ab}^{(6)})/(h_{ab}^{(8)}) \sim (\eta^2/\lambda)(\kappa^2/J)$$

  Indeed, by Caron-Huot, Li, Parra-Martinez, Simmons-Duffin [2201.06602], we expect $\eta^2/\lambda \lesssim m_{\text{Pl}}^2/\Lambda_{\text{UV}}^2$

EFT-corrected NHEK geometry

- Stationary, axisymmetric ansatz:

$$\mathrm{d}s^2 = 2J\Omega^2 \left[-\rho^2 \mathrm{d}t^2 + \frac{F_1}{\rho^2}(\mathrm{d}\rho + \rho F_2 \mathrm{d}x)^2 + \frac{\mathrm{d}x^2}{A} + B^2(\mathrm{d}\phi + \rho\omega\,\mathrm{d}t)^2 \right]$$

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- Impose $\mathrm{O}(2,1)\times\mathrm{U}(1)$ symmetry,

$$\begin{aligned}\Omega &= \Omega_{\mathrm{NH}}(x)\\ B &= B_{\mathrm{NH}}(x)\\ A &= \frac{1-x^2}{\Gamma_{\mathrm{NH}}^2}\\ \Gamma_{\mathrm{NH}} &= \mathrm{const.}\\ \omega &= \omega_{\mathrm{NH}} = \mathrm{const.}\end{aligned}$$

so that the ansatz becomes:

$$\mathrm{d}s^2 = 2J\Omega_{\mathrm{NH}}^2\left[-\rho^2\mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{\rho^2} + \frac{\Gamma_{\mathrm{NH}}^2\mathrm{d}x^2}{1-x^2} + B_{\mathrm{NH}}^2(\mathrm{d}\phi + \rho\omega_{\mathrm{NH}}\mathrm{d}t)^2\right]$$

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- Expand around NHEK solution:

$$\Omega_{\mathrm{NH}} = \sqrt{\frac{1+x^2}{2}}\left[1 + \eta\Omega^{(6)}(x) + \lambda\Omega^{(8)}(x) + \tilde{\lambda}\tilde{\Omega}^{(8)}(x)\right]$$

$$B_{\mathrm{NH}} = \frac{2\sqrt{1-x^2}}{1+x^2}\left[1 + \eta B^{(6)}(x) + \lambda B^{(8)}(x) + \tilde{\lambda}\tilde{B}^{(8)}(x)\right]$$

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- Solving the equations of motion, we find:

$$\Gamma^{(6)} = -\frac{15\kappa^4}{32\sqrt{2}J^2} \qquad \omega^{(6)} = \frac{\kappa^4}{7J^2}$$

$$\Gamma^{(8)} = -\frac{366435\kappa^6}{256\sqrt{2}J^3} \qquad \omega^{(8)} = \frac{(4864+1575\pi)\kappa^6}{20J^3}$$

$$\tilde{\Gamma}^{(8)} = -\frac{368829\kappa^6}{64\sqrt{2}J^3} \qquad \tilde{\omega}^{(8)} = \frac{(4736+1575\pi)\kappa^6}{5J^3}$$

EFT-corrected NHEK geometry

$$\mathrm{d}s^2 = 2J\Omega_{\mathrm{NH}}^2\left[-\rho^2\mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{\rho^2} + \frac{\Gamma_{\mathrm{NH}}^2\mathrm{d}x^2}{1-x^2} + B_{\mathrm{NH}}^2(\mathrm{d}\phi + \rho\omega_{\mathrm{NH}}\mathrm{d}t)^2\right]$$

- Solving the equations of motion, we find:

$$B^{(6)}(x) = \frac{\kappa^4}{J^2}\left[\frac{2656 - 42885x^2 + 45895x^4 - 8130x^6 - 1218x^8 + 183x^{10} + 139x^{12}}{224(1+x^2)^6}\right.$$
$$\left. - \frac{15\sqrt{2}x(3-x^2)}{64(1+x^2)\sqrt{1-x^2}}\left(\arcsin\frac{\sqrt{2}x}{\sqrt{1+x^2}} - \arcsin x\right)\right]$$

$$\Omega^{(6)}(x) = \frac{\kappa^4}{J^2}\left[C^{(6)} - \frac{3285 - 55449x^2 + 54210x^4 - 7058x^6 - 1527x^8 - 309x^{10}}{224(1+x^2)^6}\right.$$
$$\left. + \frac{15x\sqrt{2}\sqrt{1-x^2}}{64(1+x^2)}\left(\arcsin\frac{\sqrt{2}x}{\sqrt{1+x^2}} - \arcsin x\right)\right]$$

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- Solving the equations of motion, we find:

$$B^{(8)}(x) = \frac{\kappa^6}{J^3}\Bigg[\frac{832989}{1280} - \frac{315\pi}{4} - \frac{407005 + 32887800x^2 + 38302380x^4 + 227158536x^6}{1280(1+x^2)^9}$$
$$- \frac{244951182x^8 + 207667400x^{10} + 108083820x^{12} + 31954360x^{14} + 4114685x^{16}}{1280(1+x^2)^9}$$
$$+ \frac{630x}{(1+x^2)}\arctan x - \frac{366435x(3-x^2)}{256\sqrt{2}\sqrt{1-x^2}(1+x^2)}\left(\arcsin\frac{\sqrt{2}x}{\sqrt{1+x^2}} - \arcsin x\right)\Bigg]$$

$$\Omega^{(8)}(x) = \frac{\kappa^6}{J^3}\Bigg[C^{(8)} + \frac{783837 + 16684758x^2 + 33602022x^4 + 119986542x^6}{1280(1+x^2)^9}$$
$$+ \frac{27639936x^8 + 23049562x^{10} + 11880370x^{12} + 3484978x^{14} + 445863x^{16}}{256(1+x^2)^9}$$
$$- \frac{315x}{1+x^2}\arctan x + \frac{366435x\sqrt{1-x^2}}{256\sqrt{2}(1+x^2)}\left(\arcsin\frac{\sqrt{2}x}{\sqrt{1+x^2}} - \arcsin x\right)\Bigg]$$

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$$\mathrm{d}s^2 = 2J\Omega_{\mathrm{NH}}^2\left[-\rho^2\mathrm{d}t^2 + \frac{\mathrm{d}\rho^2}{\rho^2} + \frac{\Gamma_{\mathrm{NH}}^2\mathrm{d}x^2}{1-x^2} + B_{\mathrm{NH}}^2(\mathrm{d}\phi + \rho\omega_{\mathrm{NH}}\mathrm{d}t)^2\right]$$

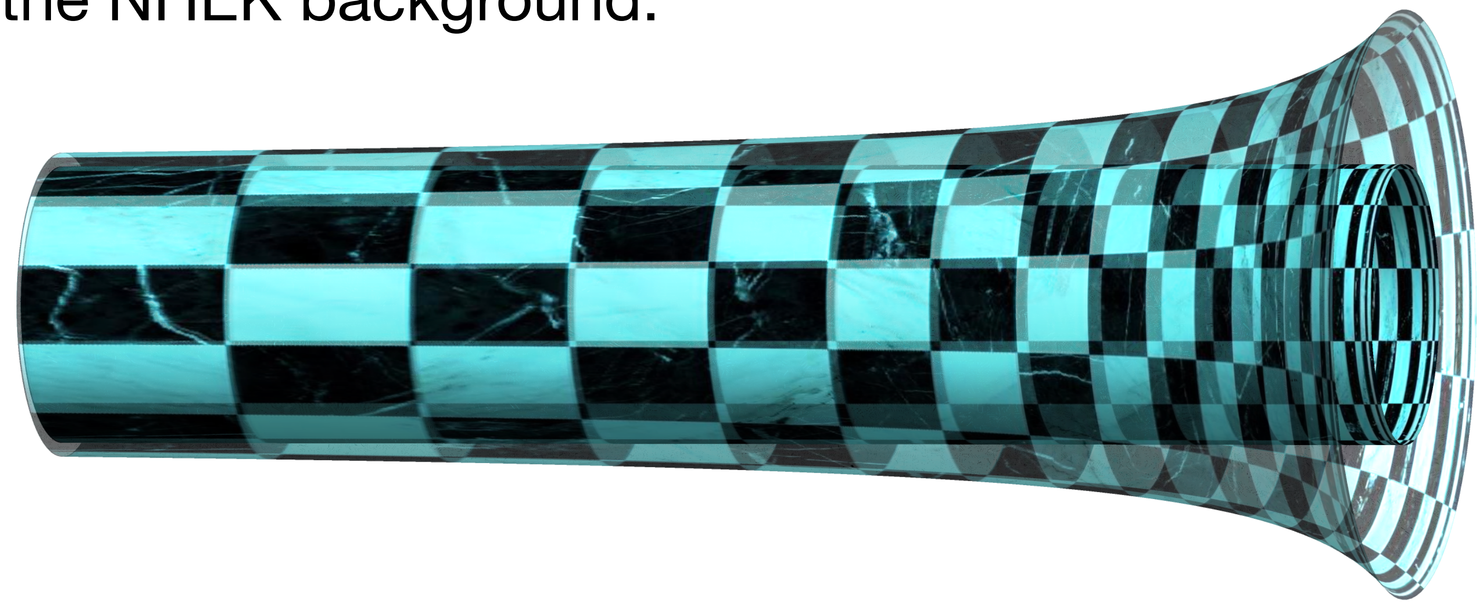
- Solving the equations of motion, we find:

$$\tilde{B}^{(8)}(x) = \frac{\kappa^6}{J^3}\Bigg[\frac{846339}{320} - 315\pi - \frac{1149443+5618952x^2+136013268x^4+154320120x^6+254641842x^8}{320(1+x^2)^9}$$
$$- \frac{208733752x^{10}+108674580x^{12}+32136008x^{14}+4138723x^{16}}{320(1+x^2)^9}$$
$$+ \frac{2520x}{1+x^2}\arctan x - \frac{368829x(3-x^2)}{64\sqrt{2}\sqrt{1-x^2}(1+x^2)}\left(\arcsin\frac{\sqrt{2}x}{\sqrt{1+x^2}} - \arcsin x\right)\Bigg]$$

$$\tilde{\Omega}^{(8)}(x) = \frac{\kappa^6}{J^3}\Bigg[\tilde{C}^{(8)} + \frac{1018371+7724394x^2+67516506x^4+96062418x^6+141833088x^8}{320(1+x^2)^9}$$
$$+ \frac{115923454x^{10}+59757382x^{12}+17530822x^{14}+2243037x^{16}}{320(1+x^2)^9}$$
$$- \frac{1260x}{1+x^2}\arctan x + \frac{368829x\sqrt{1-x^2}}{64\sqrt{2}(1+x^2)}\left(\arcsin\frac{\sqrt{2}x}{\sqrt{1+x^2}} - \arcsin x\right)\Bigg]$$

Perturbing the NHEK geometry

- Patching the near-horizon geometry to asymptotically flat Kerr solution can be thought of as inducing some perturbative mode solving the linearized Einstein equation in the NHEK background:



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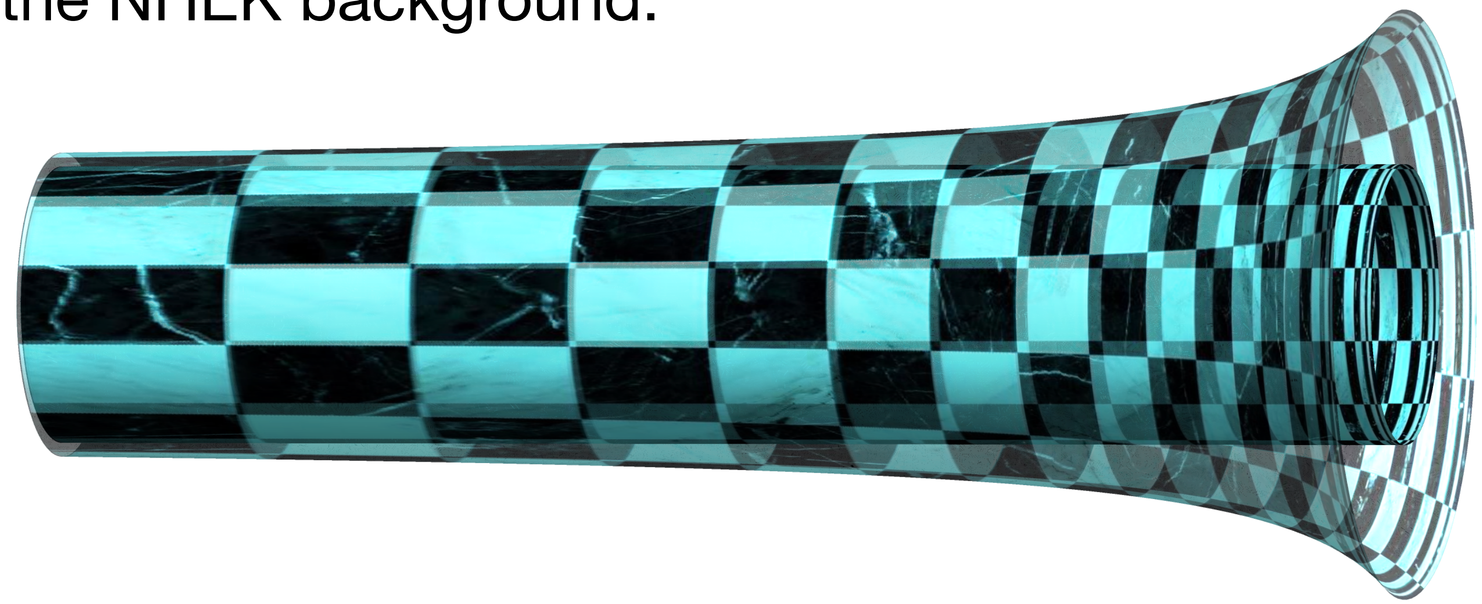

- Let us see how this works explicitly for Kerr/NHEK, before repeating the calculation with EFT corrections. We start with the ansatz,

$$\mathrm{d}s^2 = 2J\Omega^2 \left[-\rho^2 \mathrm{d}t^2 + \frac{F_1}{\rho^2}(\mathrm{d}\rho + \rho F_2 \mathrm{d}x)^2 + \frac{\mathrm{d}x^2}{A} + B^2(\mathrm{d}\phi + \rho\omega\, \mathrm{d}t)^2 \right]$$

with gauge choice $F_1 = 1$, $F_2 = 0$.

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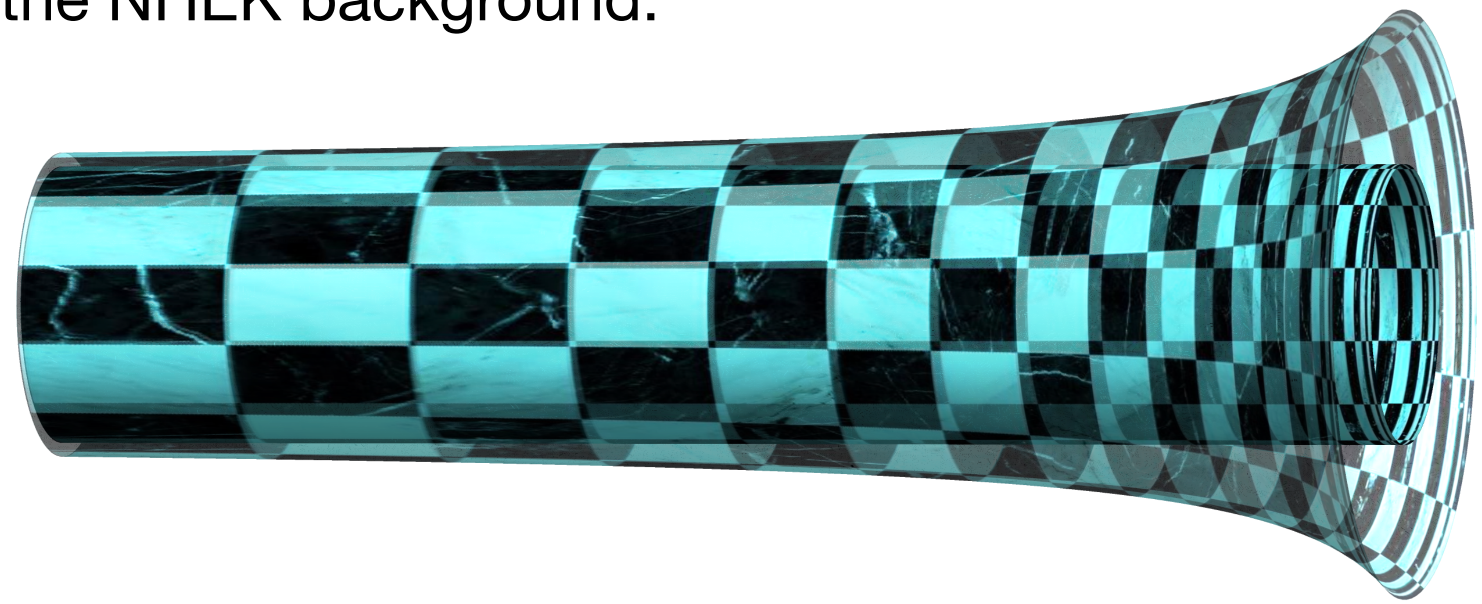

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with gauge choice $F_1 = 1$, $F_2 = 0$.

Decompose stationary, axisymmetric perturbations into **AdS$_2$ harmonics** $\sim \rho^\gamma$:

$$\begin{aligned}
A(\rho, x) &= A_{\mathrm{NH}}(x)\,[1 + \varepsilon\, \rho^\gamma\, Q_1(x)] \\
B(\rho, x) &= B_{\mathrm{NH}}(x)\,[1 + \varepsilon\, \rho^\gamma\, Q_2(x)] \\
\Omega(\rho, x) &= \Omega_{\mathrm{NH}}(x)\,[1 + \varepsilon\, \rho^\gamma\, Q_3(x)] \\
\omega(\rho, x) &= \omega_{\mathrm{NH}}\,[1 + \varepsilon\, \rho^\gamma\, Q_4(x)]
\end{aligned}$$

Perturbing the NHEK geometry

- Expand in Wilson coefficients:

$$Q_i(x) = Q_i^{(0)}(x)+\eta Q_i^{(6)}(x)+\lambda Q_i^{(8)}(x)+\tilde{\lambda}\tilde{Q}_i^{(8)}(x)$$
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- Second-order equations yield two families of solutions:

$$\begin{aligned}
Q^{(0)}_{1\;+}(x) &= P'_\ell(x) \\
Q^{(0)}_{2\;+}(x) &= -\frac{1}{2(1+x^2)}\left[2\ell(\ell+1)\,x\,P_\ell(x) + (1-3x^2)P'_\ell(x)\right] \\
Q^{(0)}_{3\;+}(x) &= \frac{1}{2(1+x^2)}\left[\ell(\ell+1)\,x\,P_\ell(x) + (1-x^2)P'_\ell(x)\right] \qquad \ell \geq 2 \\
Q^{(0)}_{4\;+}(x) &= \frac{1}{2}\left[\ell\,x\,P_\ell(x) + \frac{(1-x^2)(\ell^2+\ell+2)}{2(\ell+1)}P'_\ell(x)\right]
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Q^{(0)}_{1\,-}(x) &= 0 \\
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$$\mathrm{d}s^2 = 2J\Omega^2\left[-\rho^2\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}\rho + h_{ab}(\mathrm{d}y^a + U^a\mathrm{d}v)(\mathrm{d}y^b + U^b\mathrm{d}v)\right]$$

Mapping to our other coordinates, we have:

$$C_{\rho a\rho b} = J\,(1-\gamma)\,\rho^{\gamma-2}\,\varepsilon \begin{bmatrix} -\dfrac{\gamma\,\Gamma_{\mathrm{NH}}^2(Q_1+2Q_2)\Omega_{\mathrm{NH}}^2}{2(1-x^2)} & B_{\mathrm{NH}}^2\Omega_{\mathrm{NH}}^2 Q_4' \\ B_{\mathrm{NH}}^2\Omega_{\mathrm{NH}}^2 Q_4' & -\dfrac{\gamma\,B_{\mathrm{NH}}^2(Q_1+2Q_2)\Omega_{\mathrm{NH}}^2}{2} \end{bmatrix}$$

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Away from extremality, $C \propto \rho^{\gamma-2} \implies C \propto T^{\gamma-2}$

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- EFT corrections to the $Q_i(x)$ functions can be obtained (too long to give here).

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- Important results are the corrections to the scaling dimensions. For example,

$$\gamma_+^{(6)}(2) = +\frac{24\,\kappa^4}{7J^2}$$
$$\gamma_-^{(6)}(1) = +\frac{24\,\kappa^4}{7J^2}$$
$$\gamma_+^{(8)}(2) = -\frac{21(32+45\pi)\kappa^6}{5J^3}$$
$$\gamma_-^{(8)}(1) = -\frac{9(8576+3045\pi)\kappa^6}{20J^3}$$
$$\tilde{\gamma}_+^{(8)}(2) = -\frac{12(736+315\pi)\kappa^6}{5J^3}$$
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If $\eta < 0$ or $\lambda, \tilde{\lambda} > 0$, the *horizons* of extremal Kerr black holes will be singular.

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  - Examples:
    - Einstein-Maxwell theory Cheung, **GR** [1407.7865]; Cheung, Liu, **GR** [1801.08546, 1903.09156]
    - Higher-curvature gravity (R^2, R^4 terms) Bellazzini, Cheung, **GR** [1509.00851]; Cheung, **GR** [1608.02942]
    - Massive gravity Cheung, **GR** [1601.04068]
    - $(\partial\phi)^4$ and F^4 couplings Arkani-Hamed, Huang, Liu, **GR** [2109.13937]
    - Higher-point couplings Chandrasekaran, **GR**, Shahbazi-Moghaddam [1804.03153]; Arkani-Hamed, Cheung, Figueiredo, **GR** [2312.07652]
    - Cosmic inflation Kumar, Freytsis, **GR**, Rodd [2210.10791]
    - SMEFT **GR**, Rodd [1908.09845, 2004.02885, 2010.04723, 2206.13524]

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- But **precisely the opposite holds**:
  - Causality and unitarity imply that $\lambda, \tilde{\lambda} > 0$ — Bellazzini, Cheung, **GR** [1509.00851]; Gruzinov, Kleban [hep-th/0612015]
  - String values:

$$(\lambda, 4\tilde{\lambda}) = \frac{\alpha'^3}{256\kappa^6} \times \left\{ \underbrace{\Big(13+\zeta(3), 1+\zeta(3)\Big)}_{\text{bosonic}}, \quad \underbrace{\Big(\zeta(3), \zeta(3)\Big)}_{\text{type II}}, \quad \underbrace{\left(\tfrac{1}{2}+\zeta(3), \tfrac{1}{2}+\zeta(3)\right)}_{\text{type I / heterotic}} \right\}$$

  - Quartic Riemann terms induce singularities.

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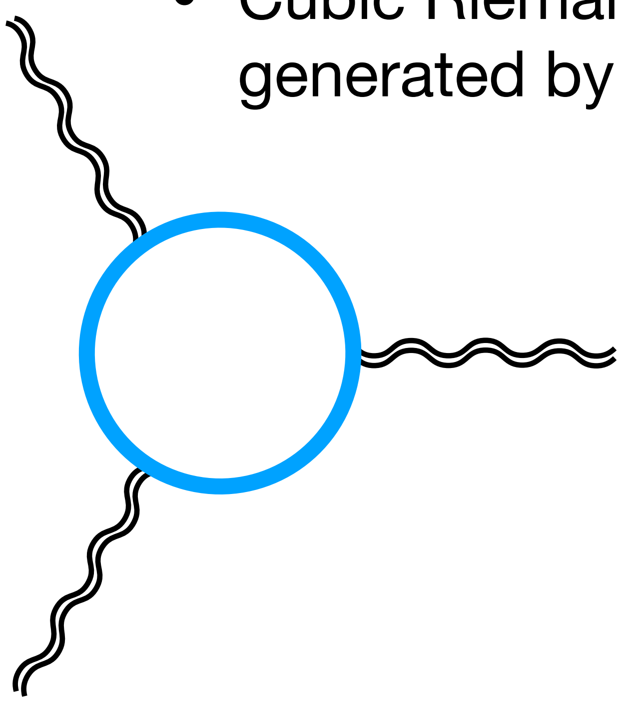

$$\eta = \frac{1}{15120(4\pi)^2\kappa^2}\sum\left(\frac{1}{m_{\rm s}^2} - \frac{4}{m_{\rm f}^2} + \frac{3}{m_{\rm v}^2}\right)$$

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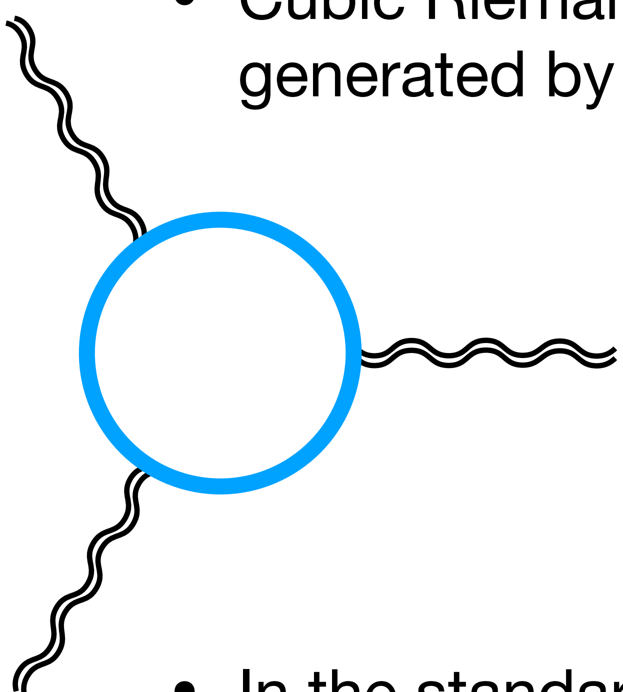

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    - In the standard model, the neutrinos are the lightest massive state, and $\eta < 0$.

$\Longrightarrow$ singular horizons for extremal Kerr or new ultralight hidden-sector bosons


Adding charge

Adding a photon

- Let us now add a U(1) gauge field.

- Good reasons for doing this: first higher-derivative terms show up at fourth order in derivatives, rather than sixth $\Longrightarrow$ much larger effects

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- Let us now add a U(1) gauge field.
- Good reasons for doing this: first higher-derivative terms show up at fourth order in derivatives, rather than sixth $\Longrightarrow$ much larger effects
- Mode with $\gamma = 1$ will be physical for charged black holes. Defining the vectors

$$\ell = \partial/\partial\rho$$
$$m = \partial/\partial\phi$$

we will find the Weyl tensor near the horizon goes like

$$C_{abcd}\ell^a m^b \ell^c m^d \sim \gamma(\gamma-1)\rho^{\gamma-2}$$

so there is a mode for which $C_{abcd}\ell^a m^b \ell^c m^d|_H \sim \delta\gamma/T$

- We will not need to be exponentially close to extremality to see the effect.

Einstein-Maxwell EFT

- Leading (four-derivative) contributions to the Einstein-Maxwell EFT:

$$
\begin{aligned}
\mathcal{L} = \frac{1}{2\kappa^2} R - \frac{1}{4} F_{ab}\, F^{ab} &+ c_1\, R^2 + c_2\, R^{ab}\, R_{ab} + c_3\, R_{abcd}\, R^{abcd} \\
&+ c_4\, R\, F^{ab}\, F_{ab} + c_5\, R^{ab}\, F_a{}^c\, F_{bc} + c_6\, R^{abcd}\, F_{ab}\, F_{cd} \\
&+ c_7\, F_{ab}\, F^{ab}\, F_{cd}\, F^{cd} + c_8\, F_{ab}\, F^{bc}\, F_{cd}\, F^{da}
\end{aligned}
$$

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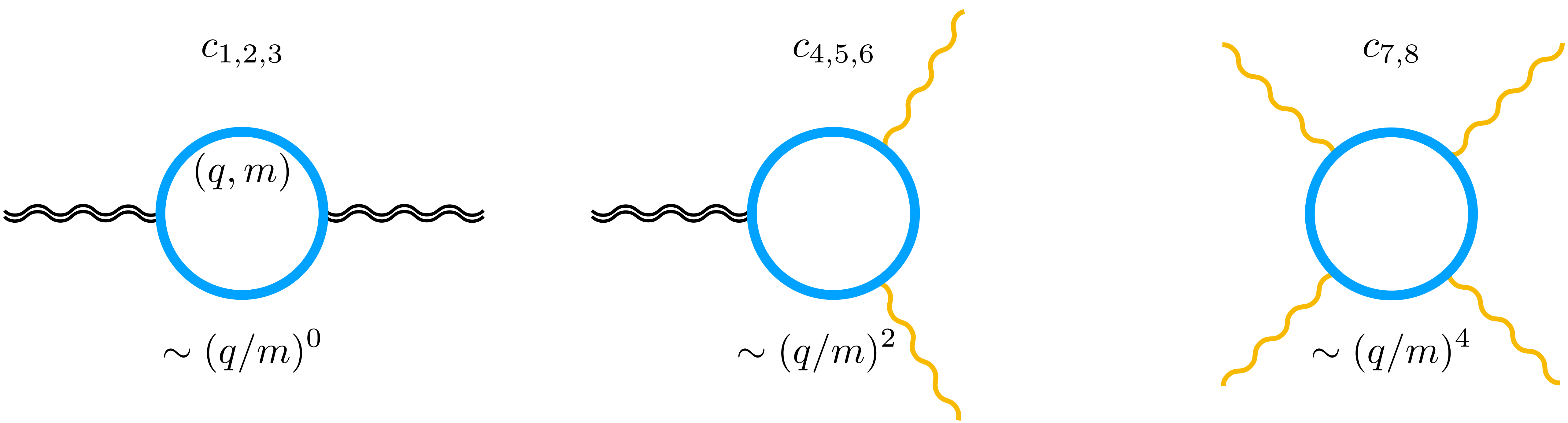
Gravitational generalization of Euler-Heisenberg:



Convenient to rescale, so that $d_i \sim 1/\Lambda_{\mathrm{UV}}^2$:

$d_{1,2,3} = \kappa^2 c_{1,2,3}$ $\quad$ $d_{4,5,6} = c_{4,5,6}$ $\quad$ $d_{7,8} = \kappa^{-2} c_{7,8}$

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Under field redefinitions, only four combinations are invariant:

$$\begin{cases} d_0 = d_2 + 4d_3 + d_5 + d_6 + 4d_7 + 2d_8 \\ d_3 \\ d_6 \\ d_9 = d_2 + 4d_3 + d_5 + 2d_6 + d_8 \end{cases}$$

Equivalently, since we can drop Gauss-Bonnet in $D = 4$, we can take $d_{6,7,8}$ as an operator basis.

EFT-corrected Einstein-Maxwell equations

- We wish to solve the EFT-corrected equations of motion:

$$\nabla^a F_{ab} = c_4 J_b^{c_4} + c_5 J_b^{c_5} + c_6 J_b^{c_6} + c_7 J_b^{c_7} + c_8 J_b^{c_8}$$

$$R_{ab} - \frac{1}{2} R g_{ab} - \kappa^2 \left(F_a{}^c F_{bc} - \frac{1}{4} g_{ab} F_{cd} F^{cd} \right) = \kappa^2 \Big(c_1 T_{ab}^{c_1} + c_2 T_{ab}^{c_2} + c_3 T_{ab}^{c_3} + c_4 T_{ab}^{c_4} + c_5 T_{ab}^{c_5} + c_6 T_{ab}^{c_6} + c_7 T_{ab}^{c_7} + c_8 T_{ab}^{c_8} \Big)$$

$$\begin{aligned}
J_a^{c_4} &= 4 \left(R \nabla^b F_{ba} - F_{ab} \nabla^b R \right) \\
J_a^{c_5} &= -2 \left(R_a{}^c \nabla_b F_c{}^b + R^{cb} \nabla_b F_{ac} + F_a{}^c \nabla_b R_c{}^b + F^{cb} \nabla_b R_{ac} \right) \\
J_a^{c_6} &= -4 R_{adbc} \nabla^d F^{bc} - 4 F^{bc} \nabla^d R_{adbc} \\
J_a^{c_7} &= 8 \nabla^e \left(F_{ea} F^{cd} F_{cd} \right) \\
J_a^{c_8} &= -8 \nabla^b \left(F_a{}^p F_{cp} F^c{}_b \right)
\end{aligned}$$

$$\begin{aligned}
T_{ab}^{c_1} &= 4 \nabla_a \nabla_b R - 4 g_{ab} \Box R - 4 R_{ab} R + g_{ab} R^2 \\
T_{ab}^{c_2} &= 4 \nabla_c \nabla_{(a} R_{b)}{}^c - 2 \Box R_{ab} - 2 g_{ab} \nabla_d \nabla_c R^{cd} - 4 R_a{}^c R_{bc} + g_{ab} R_{cd} R^{cd} \\
T_{ab}^{c_3} &= - \left(4 R_a{}^{cde} R_{bcde} - g_{ab} R_{cdef} R^{cdef} + 8 \nabla_c \nabla_d R_{(a}{}^c{}_{b)}{}^d \right) \\
T_{ab}^{c_4} &= 4 F^{cd} \nabla_{(a} \nabla_{b)} F_{cd} + 4 \nabla_a F^{cd} \nabla_b F_{cd} - 4 g_{ab} F^{cd} \Box F_{cd} \\
&\quad - 4 g_{ab} \nabla_e F_{cd} \nabla^e F^{cd} - 2 R_{ab} F_{cd} F^{cd} - 4 F_a{}^c F_{bc} R + g_{ab} F_{cd} F^{cd} R \\
T_{ab}^{c_5} &= 4 F_{(a}{}^c R_{b)d} F_c{}^d - 2 F_a{}^c F_b{}^d R_{cd} + g_{ab} F_c{}^e F^{cd} R_{de} - 2 \nabla_{(a} F_{b)}{}^c \nabla_d F_c{}^d \\
&\quad - 2 \nabla_d \nabla_{(a} F_{b)c} F^{cd} - 2 \nabla_d \nabla_{(a} F^{cd} F_{b)c} - 2 \Box F_{(a}{}^c F_{b)c} \\
&\quad - g_{ab} F^{cd} \nabla_d \nabla^e F_{ce} - 2 \nabla^d F_{(a}{}^c \nabla_{b)} F_{cd} - 2 \nabla^d F_a{}^c \nabla_d F_{bc} \\
&\quad + g_{ab} \nabla_c F^{cd} \nabla_e F_d{}^e - g_{ab} F^{cd} \nabla_e \nabla_d F_c{}^e - g_{ab} \nabla_d F_{ce} \nabla^e F^{cd} \\
T_{ab}^{c_6} &= - \Big(6 F_{(a}{}^c F^{de} R_{b)cde} - g_{ab} F^{cd} F^{ef} R_{cdef} - 4 F_{c(a} \nabla^c \nabla^d F_{b)d} \\
&\quad - 4 F_{d(a} \nabla^d \nabla^c F_{b)d} + 4 \nabla_c F_a{}^c \nabla_d F_b{}^d + 4 \nabla_c F_{bd} \nabla^d F_a{}^c \Big) \\
T_{ab}^{c_7} &= F^{pq} F_{pq} \left(g_{ab} F^{cd} F_{cd} - 8 F_{ac} F_b{}^c \right) \\
T_{ab}^{c_8} &= g_{ab} F_c{}^p F_{dp} F^{cq} F^d{}_q - 8 F_a{}^p F_{cp} F_b{}^q F^c{}_q
\end{aligned}$$

Kerr-Newman solution

- The solution to the Einstein-Maxwell equations for a charged, rotating black hole was found via an inspired guess by Ezra Newman in 1965: a complex coordinate transformation of Reissner-Nordström.

$$\mathrm{d}s^2_{\mathrm{KN}} = -\frac{\Delta(r)}{\Sigma(r,\theta)}(\mathrm{d}t - a\,\sin^2\theta\,\mathrm{d}\phi)^2 + \Sigma(r,\theta)\left(\frac{\mathrm{d}r^2}{\Delta(r)} + \mathrm{d}\theta^2\right) + \frac{\sin^2\theta}{\Sigma(r,\theta)}\left[a\,\mathrm{d}t - (r^2+a^2)\mathrm{d}\phi\right]^2$$

$$A_{\mathrm{KN}} = -\frac{\sqrt{2}\,Q\,r}{\kappa\,\Sigma(r,\theta)}(\mathrm{d}t - a\,\sin^2\theta\,\mathrm{d}\phi) - \frac{\sqrt{2}\,P\,\cos\theta}{\kappa\,\Sigma(r,\theta)}\left[a\,\mathrm{d}t - (r^2+a^2)\mathrm{d}\phi\right]$$

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$$\begin{aligned}\Sigma(r,\theta) &= r^2 + a^2\cos^2\theta\\ \Delta(r) &= r^2 + a^2 - 2\,M\,r + Q^2 + P^2\end{aligned}$$

- Komar charges:

$$Q_{\mathrm{e}} = \lim_{r\to+\infty}\int_{S^2_r}\star F = \frac{4\pi\sqrt{2}}{\kappa}Q, \qquad J = \frac{1}{2\kappa^2}\lim_{r\to+\infty}\int_{S^2_r}\star\mathrm{d}m = aE,$$

$$Q_{\mathrm{m}} = \lim_{r\to+\infty}\int_{S^2_r} F = \frac{4\pi\sqrt{2}}{\kappa}P, \qquad E = -\frac{1}{\kappa^2}\lim_{r\to+\infty}\int_{S^2_r}\star\mathrm{d}k = \frac{8\pi M}{\kappa^2}$$

Kerr-Newman solution

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- Angular velocity: $\Omega = \dfrac{a}{a^2 + r_+^2}$
- Temperature:

$$T = \frac{1}{2\pi}\sqrt{-\frac{1}{4}\left.\frac{\nabla_a(K^c K_c)\nabla^a(K^d K_d)}{K^e K_e}\right|_H} = \frac{r_+^2 - a^2 - Q^2}{4\,\pi\, r_+\,(r_+^2 + a^2)}$$

- Chemical potential:

$$\mu = -\frac{\kappa}{\sqrt{2}}\left(K^a A_a|_H - \lim_{r\to+\infty} K^a A_a\right) = \frac{Q}{r_+^2 + a^2}$$

Kerr-Newman solution

- Horizon: $r_+ = M + \sqrt{M^2 - a^2 - Q^2}$ generated by Killing vector $K = k + \Omega m$
- Angular velocity: $\Omega = \dfrac{a}{a^2 + r_+^2}$
- Temperature:

$$T = \frac{1}{2\pi}\sqrt{-\frac{1}{4}\left.\frac{\nabla_a(K^c K_c)\nabla^a(K^d K_d)}{K^e K_e}\right|_H} = \frac{r_+^2 - a^2 - Q^2}{4\,\pi\, r_+\,(r_+^2 + a^2)}$$

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- Entropy:

$$S_{\mathrm{W}} = -2\pi \oint_{\mathcal{B}+} \mathrm{d}^2x\,\sqrt{\sigma}\,\frac{\delta\mathcal{L}}{\delta R_{abcd}}\varepsilon_{ab}\varepsilon_{cd}$$

$$S_{\mathrm{BH}} = \lim_{c_i\to 0} S_{\mathrm{W}} = \frac{2\pi\, A}{\kappa^2}$$

- First law of black hole mechanics: $\mathrm{d}M = T\,\mathrm{d}S_{\mathrm{W}} + \mu\,\mathrm{d}Q + \Omega\,\mathrm{d}J$

Near-horizon geometry

- Like the NHEK for Kerr, the Kerr-Newman solution enjoys an $\mathrm{O}(2,1)\times\mathrm{U}(1)$ symmetric near-horizon geometry, discovered fairly recently:

Hartman, Murata, Nishioka, Strominger [0811.4393]

$$\mathrm{d}s^2_{\mathrm{NH-KN}} = 2M^2[F_1^{(0)}(x)]^2\Bigg[-\rho^2\mathrm{d}\tau^2+\frac{\mathrm{d}\rho^2}{\rho^2}+\frac{\mathrm{d}x^2}{1-x^2} + [F_2^{(0)}(x)]^2(1-x^2)\left(\mathrm{d}\varphi+\rho\,\omega^{(0)}_{\mathrm{NH}}\,\mathrm{d}\tau\right)^2\Bigg]$$

$$A_{\mathrm{NH-KN}} = \frac{\sqrt{2}\,M}{\kappa}\left[Q^{(0)}_{\mathrm{NH}}\,\rho\,\mathrm{d}\tau+(1-x^2)F_2^{(0)}(x)F_3^{(0)}(x)\left(\mathrm{d}\varphi+\rho\,\omega^{(0)}_{\mathrm{NH}}\,\mathrm{d}\tau\right)\right]$$

$$F_1^{(0)}(x)=\frac{\sqrt{1+(1-Z^2)x^2}}{\sqrt{2}}\,,\quad F_2^{(0)}(x)=\frac{(2-Z^2)}{1+(1-Z^2)x^2}\,,\quad F_3^{(0)}(x)=\frac{Z\sqrt{1-Z^2}}{2-Z^2}\,,$$

$$\omega^{(0)}_{\mathrm{NH}}=\frac{2\sqrt{1-Z^2}}{2-Z^2},\quad \text{and}\quad Q^{(0)}_{\mathrm{NH}}=\frac{Z^3}{2-Z^2}\qquad\qquad Z=Q/M$$

Connecting to asymptotia

- As in the Kerr case, the modes solving the linearized equations in the near-horizon background give us the first corrections connecting this solution to full Kerr-Newman. We start with the ansatz:

$$\mathrm{d}s^2 = 2\,M^2\,[f_1^{(0)}(x,\rho)]^2\left[-\rho^2\mathrm{d}\tau^2 + \frac{\mathrm{d}\rho^2}{\rho^2} + \frac{\mathrm{d}x^2}{f_6^{(0)}(x,\rho)\,(1-x^2)} \right.$$
$$\left. + f_2^{(0)}(x,\rho)^2\,(1-x^2)\left(\mathrm{d}\varphi + \rho\,f_4^{(0)}(x,\rho)\,\mathrm{d}\tau\right)^2\right]$$

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- Modes:

$$f_i^{(0)}(x,\rho) = F_i^{(0)}(x)\left[1+\delta\hat{f}_i^{(0)}(x,\rho)\right] \quad \text{for} \quad i=1,2,3$$

$$f_4^{(0)}(x,\rho) = \omega_{\mathrm{NH}}^{(0)}\left[1+\delta\hat{f}_4^{(0)}(x,\rho)\right]$$

$$f_5^{(0)}(x,\rho) = Q_{\mathrm{NH}}^{(0)}\left[1+\delta\hat{f}_5^{(0)}(x,\rho)\right]$$

$$f_6^{(0)}(x,\rho) = 1+\delta\hat{f}_6^{(0)}(x,\rho)$$

$$\delta\hat{f}_i^{(0)}(x,\rho) = \rho^{\gamma^{(0)}}\delta f_i^{(0)}(x)$$

$\mathrm{O}(2,1)$ harmonics, for $\gamma^{(0)} \neq 1$

Connecting to asymptotia

- We expect four physical modes: two for the graviton and two for the photon. But since we are in a background with nonzero electric field, the photon and graviton kinetically mix. To disentangle them, we make the substitution:

$$
\begin{aligned}
\delta f_1^{(0)}(x) &= \frac{v_1(x)}{2} + \frac{1}{1+(1-Z^2)x^2}\left[\frac{Z^2(1-x^2)}{2+Z^2}v_4(x) - (2-Z^2)x(1-x^2)\frac{v_1'(x)}{4}\right. \\
&\quad \left. + (2+Z^2)(1-Z^2)(1-x^2)^2\frac{v_2'(x)}{x} - Z^4(2+Z^2)(1-Z^2)x(1-x^2)v_3'(x)\right] \\
\delta f_2^{(0)}(x) &= -\frac{v_1(x)}{2} - \frac{2}{1+(1-Z^2)x^2}\left[\frac{Z^2(1-x^2)}{2+Z^2}v_4(x) - (2-Z^2)x(1-x^2)\frac{v_1'(x)}{4}\right. \\
&\quad \left. + (2+Z^2)(1-Z^2)(1-x^2)^2\frac{v_2'(x)}{x} - Z^4(2+Z^2)(1-Z^2)x(1-x^2)v_3'(x)\right] \\
\delta f_3^{(0)}(x) &= \frac{v_1(x)}{2} + v_4(x) - Z^2(2+Z^2)(1-x^2)\frac{v_2'(x)}{x} - Z^2(2+Z^2)(2-3Z^2)xv_3'(x) \\
\delta f_4^{(0)}(x) &= \frac{1}{\gamma^{(0)}+1}\left[\frac{2(1+x^2)+\lambda^{(0)}(1-x^2)}{2}\frac{v_1(x)}{2} + \lambda^{(0)}(4-Z^4)v_2(x)\right. \\
&\quad + \lambda^{(0)}Z^4(2+Z^2)v_3(x) + \frac{Z^2}{2+Z^2}(1+x^2)v_4(x) \\
&\quad \left. - x(1-x^2)\frac{v_1'(x)}{2} - \frac{Z^2}{2+Z^2}x(1-x^2)v_4'(x)\right] \\
\delta f_5^{(0)}(x) &= \frac{1}{\gamma^{(0)}+1}\left[\frac{2(1+x^2)+\lambda^{(0)}(1-x^2)}{2}\frac{v_1(x)}{2} - \lambda^{(0)}(2+Z^2)^2(1-Z^2)v_3(x)\right. \\
&\quad - \frac{2-3Z^2}{2+Z^2}(1+x^2)\frac{v_4(x)}{Z^2} - x(1-x^2)\frac{v_1'(x)}{2} \\
&\quad \left. + \frac{2-3Z^2}{2+Z^2}x(1-x^2)\frac{v_4'(x)}{Z^2}\right] \\
\delta f_6^{(0)}(x) &= v_1(x)
\end{aligned}
$$

Connecting to asymptotia

- The Einstein-Maxwell equations then take the simple form:

$$
\begin{aligned}
\left[(1-x^2)^2 v_1'\right]' + [\gamma^{(0)}(\gamma^{(0)}+1)-2](1-x^2)v_1 &= 0 \\
\left[\frac{(1-x^2)^2}{x^2} v_2'\right]' + \gamma^{(0)}(\gamma^{(0)}+1)\frac{1-x^2}{x^2} v_2 &= 0 \\
\left[(1-x^2) v_3'\right]' + \gamma^{(0)}(\gamma^{(0)}+1) v_3 &= 0 \\
\left[(1-x^2)^2 v_4'\right]' + [\gamma^{(0)}(\gamma^{(0)}+1)-2](1-x^2)v_4 &= 0
\end{aligned}
$$

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$$\left[(1-x^2) v_3'\right]' + \gamma^{(0)}(\gamma^{(0)}+1) v_3 = 0$$

$$\left[(1-x^2)^2 v_4'\right]' + [\gamma^{(0)}(\gamma^{(0)}+1)-2](1-x^2)v_4 = 0$$

$$\Downarrow$$

$$v_1(x) = P'_{\ell_1}(x), \qquad \gamma_1^{(0)} = \ell_1$$

$$v_2(x) = x(\ell_2+1)\ell_2 P_{\ell_2}(x) + (1+x^2\ell_2)P'_{\ell_2}(x), \qquad \gamma_2^{(0)} = \ell_2 + 1$$

$$v_3(x) = P_{\ell_3}(x), \qquad \gamma_3^{(0)} = \ell_3$$

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$$\ell_{1,3,4} \geq 2, \quad \ell_2 \geq 1$$

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axial: $(-1)^{\ell+1}$ under parity

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$$\left[(1-x^2) v_3'\right]' + \gamma^{(0)}(\gamma^{(0)}+1)v_3 = 0$$
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$$v_4(x) = P'_{\ell_4}(x), \qquad \gamma_4^{(0)} = \ell_4$$

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axial: $(-1)^{\ell+1}$ under parity

polar: $(-1)^{\ell}$ under parity

Connecting to asymptotia

- For $\gamma^{(0)} = 1$, there are various complications: residual gauge symmetry, plus non-power law terms that we must include, shifting $\delta f_i^{(0)}$ by terms $\propto \rho$ and $\rho \log \rho$. These are coordinate artifacts and do not contribute to tidal force singularities.

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- We will focus on the even-parity $\gamma^{(0)} = 1$ modes.

$$\Longrightarrow \quad \begin{array}{l} \ell_2 = 0 \\ \ell_1 = 1 \\ \ell_4 = 1 \end{array}$$

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- We will focus on the even-parity $\gamma^{(0)} = 1$ modes.

$$\implies \begin{array}{ll} \ell_2 = 0 & \text{vanishes identically} \\ \ell_1 = 1 & \text{pure gauge} \\ \ell_4 = 1 & \end{array}$$

EFT-correcting the near-horizon modes

- As always, choose an ansatz with the appropriate near-horizon symmetry:

$$\mathrm{d}s^2_{\mathrm{NH}} = 2\,M^2\,[F_1(x)]^2\Bigg[-\rho^2\mathrm{d}\tau^2+\frac{\mathrm{d}\rho^2}{\rho^2}+\frac{\Gamma^2_{\mathrm{NH}}\mathrm{d}x^2}{1-x^2} + [F_2(x)]^2\left(1-x^2\right)\left(\mathrm{d}\varphi+\rho\,\omega_{\mathrm{NH}}\,\mathrm{d}\tau\right)^2\Bigg]$$

$$A_{\mathrm{NH}} = \frac{\sqrt{2}\,M}{\kappa}\left[Q_{\mathrm{NH}}\,\rho\,\mathrm{d}\tau+(1-x^2)F_2(x)F_3(x)\left(\mathrm{d}\varphi+\rho\,\omega_{\mathrm{NH}}\,\mathrm{d}\tau\right)\right]$$

$$F_i(x) = F_i^{(0)}(x)\left[1+\sum_{K=1}^{8}\frac{d_K}{M^2}\,\delta F_i^{(K)}(x)\right],\qquad i=1,2,3$$

$$\Gamma_{\mathrm{NH}} = 1+\sum_{K=1}^{8}\frac{d_K}{M^2}\,\delta\Gamma^{(K)}_{\mathrm{NH}}$$

$$\omega_{\mathrm{NH}} = \omega^{(0)}_{\mathrm{NH}}\left(1+\sum_{K=1}^{8}\frac{d_K}{M^2}\,\delta\omega^{(K)}_{\mathrm{NH}}\right)$$

$$Q_{\mathrm{NH}} = Q^{(0)}_{\mathrm{NH}}\left(1+\sum_{K=1}^{8}\frac{d_K}{M^2}\,\delta Q^{(K)}_{\mathrm{NH}}\right)$$

Analytically compute all quantities to first order in the Wilson coefficients.

EFT-correcting the near-horizon modes

- Compute linearized deformations *around* the EFT-corrected near-horizon background, and map onto our O(2,1) modes defined previously.

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$$\delta\gamma^{(1)} = \delta\gamma^{(4)} = 0$$

$$\delta\gamma^{(2)} = \delta\gamma^{(5)} = \frac{1}{4}\delta\gamma^{(3)}$$

$$\delta\gamma^{(5)} = 4\delta\gamma^{(8)} - \delta\gamma^{(7)}$$

$$\delta\gamma^{(6)} = \frac{3(\mathfrak{a}^2-1)}{10\mathfrak{a}^4(\mathfrak{a}^2+1)^4}\left(15+25\mathfrak{a}^2-201\mathfrak{a}^4+89\mathfrak{a}^6-187\mathfrak{a}^8+195\mathfrak{a}^{10}+245\mathfrak{a}^{12}+75\mathfrak{a}^{14}\right) + \frac{9(\mathfrak{a}^2-1)^2(\mathfrak{a}^2+1)(1-2\mathfrak{a}^2+5\mathfrak{a}^4)}{2\mathfrak{a}^5}\arctan\mathfrak{a}$$

$$\delta\gamma^{(7)} = \delta\gamma^{(6)} + \frac{16(\mathfrak{a}^2-1)}{5(\mathfrak{a}^2+1)^6}(149-522\mathfrak{a}^2+436\mathfrak{a}^4-166\mathfrak{a}^6+7\mathfrak{a}^8)$$

$$\delta\gamma^{(8)} = \frac{3}{4}\delta\gamma^{(6)} + \frac{4(\mathfrak{a}^2-1)}{5(\mathfrak{a}^2+1)^6}(167-558\mathfrak{a}^2+316\mathfrak{a}^4-226\mathfrak{a}^6+13\mathfrak{a}^8)$$

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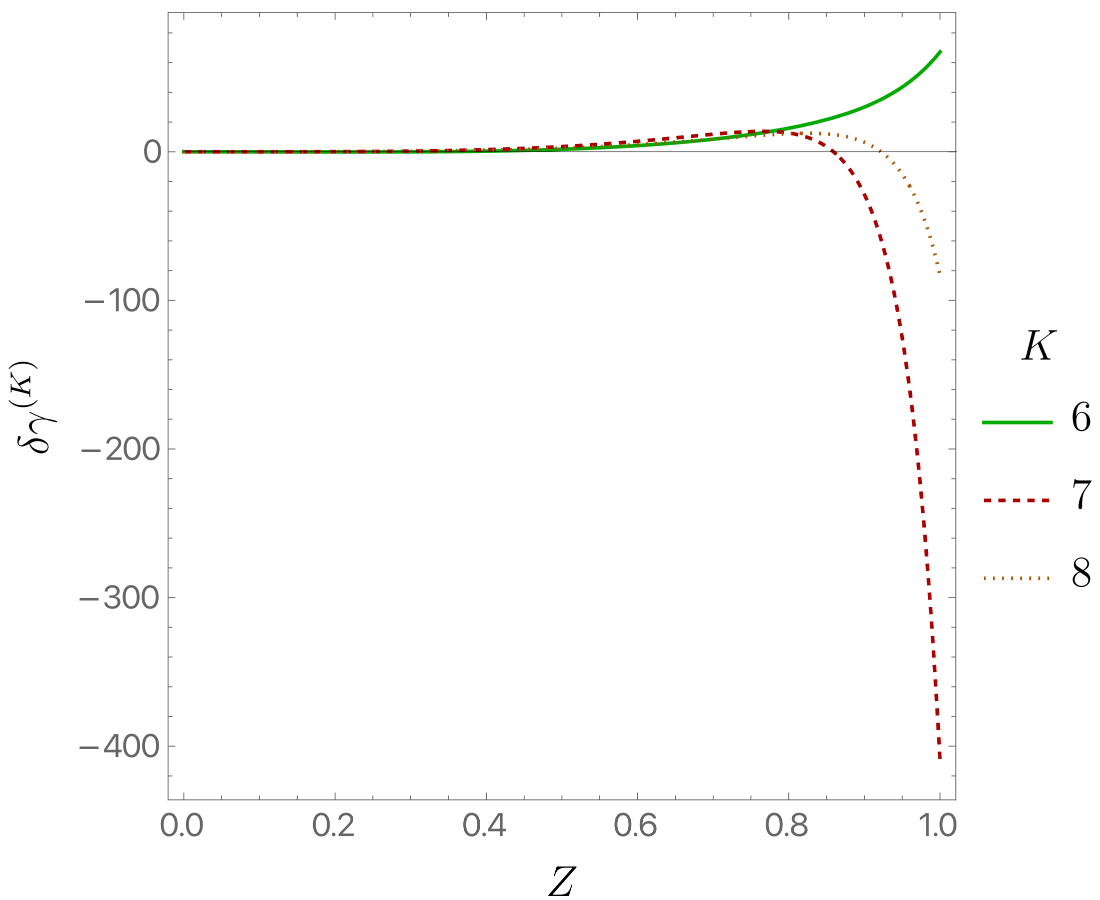
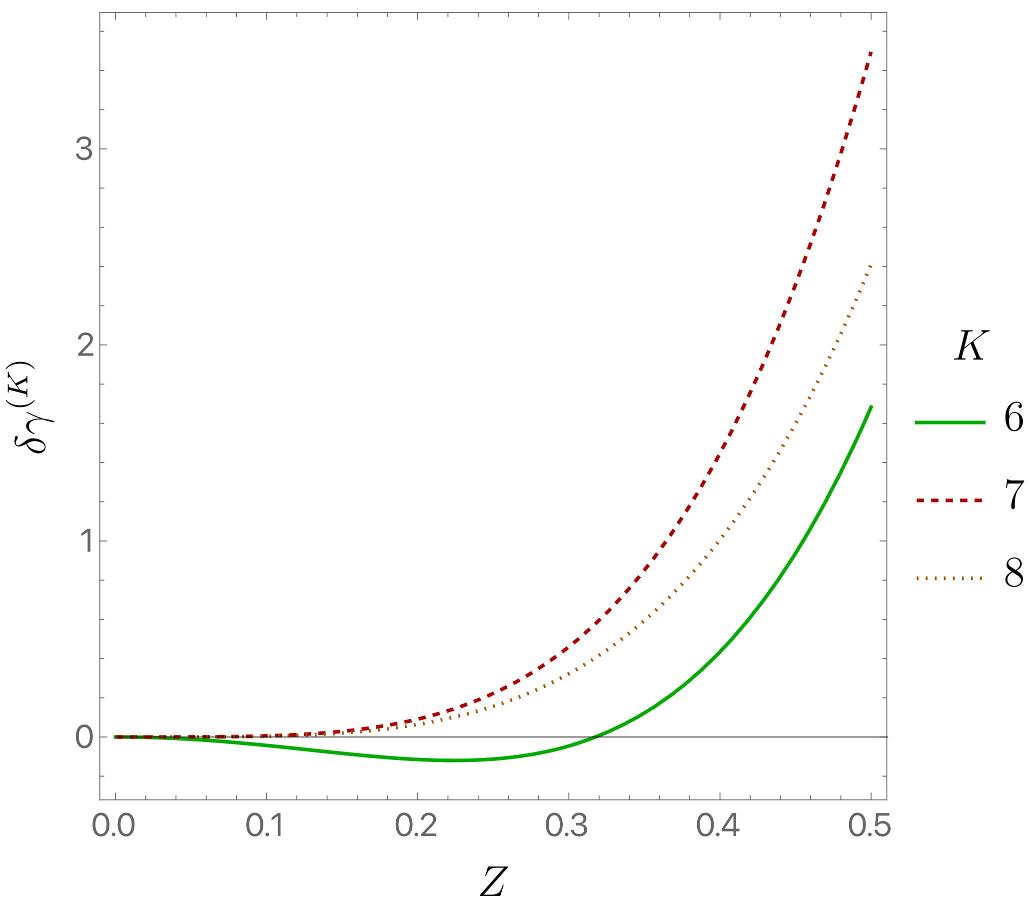
- Full shift is field redefinition invariant, representing a nontrivial check:

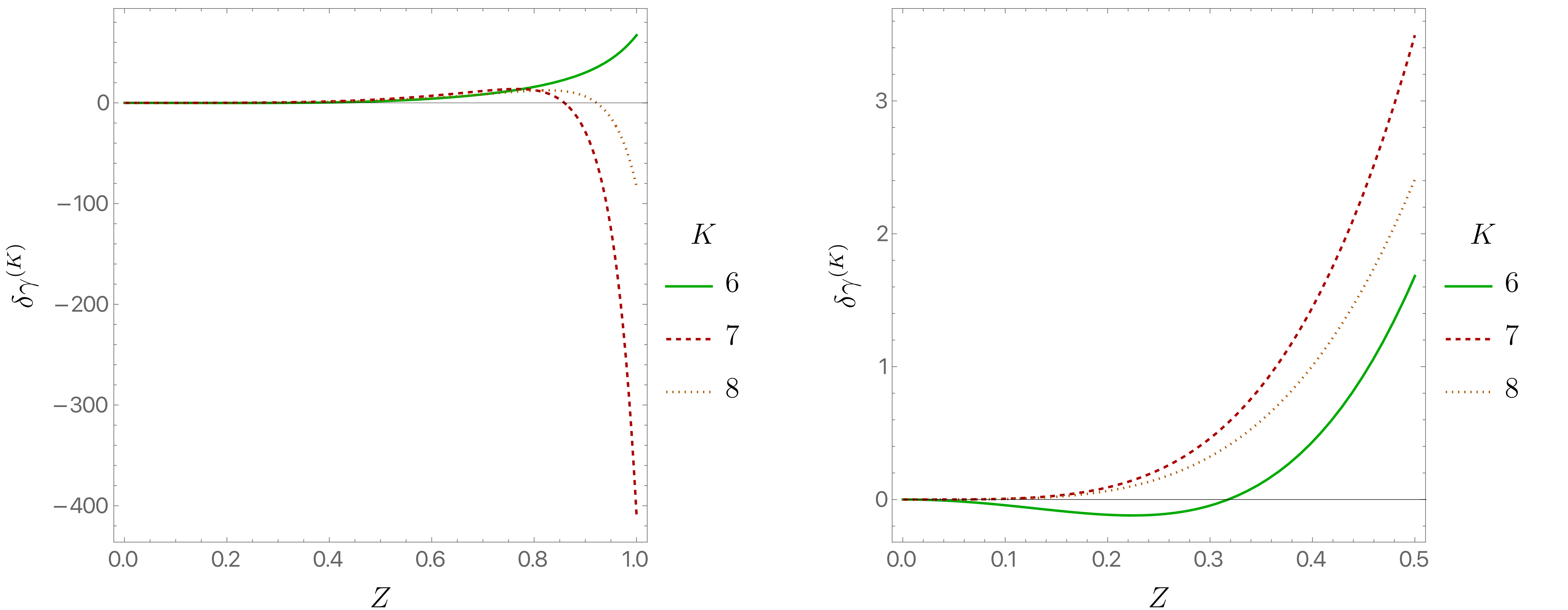
$$\sum_{K=1}^{8} d_K\delta\gamma^{(K)} = \frac{1}{4}\delta\gamma^{(7)}d_0 + \left(\delta\gamma^{(6)} + \frac{3}{4}\delta\gamma^{(7)} - 2\,\delta\gamma^{(8)}\right)d_6 + \left(\delta\gamma^{(8)} - \frac{1}{2}\delta\gamma^{(7)}\right)d_9$$

EFT-correcting the near-horizon modes





EFT-correcting the near-horizon modes



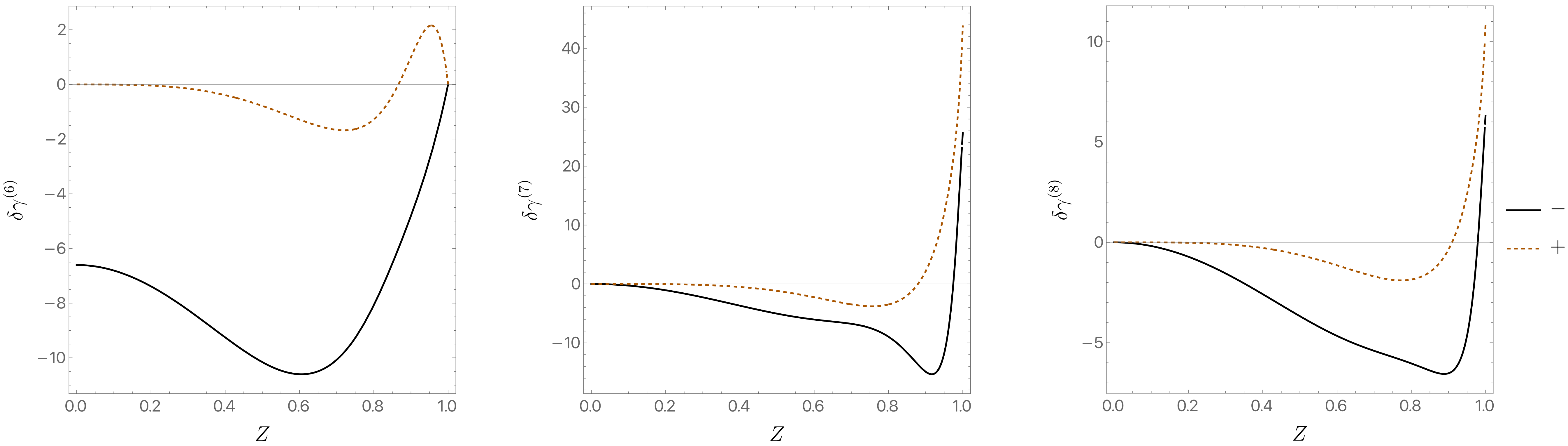
In ingoing Bondi-Sachs coordinates, we find a Weyl curvature singularity, irrespective of the sign of the Wilson coefficients:

$$C_{\rho a \rho b} = \frac{1-Z^2}{4\rho}\begin{bmatrix} -\dfrac{1}{2-Z^2} & -\dfrac{\sqrt{1-Z^2}\,x}{1+(1-Z^2)x^2} \\ -\dfrac{\sqrt{1-Z^2}\,x}{1+(1-Z^2)x^2} & \dfrac{(2-Z^2)(1-x^2)}{\left[1+(1-Z^2)x^2\right]^2} \end{bmatrix} \sum_{K=1}^{8} d_K \delta\gamma^{(K)}$$

Power-law divergence, and vanishes for extreme, non-spinning Reissner-Nordström.

EFT-correcting the near-horizon modes

Correction to scaling dimension is generically nonzero. For example, the two even-parity $\gamma^{(0)} = 2$ modes:



Finite temperature

Away from extremality

- To explicitly compute EFT-corrected black holes *away from the extremal limit*, i.e., at finite temperature, we will need to solve the equations numerically.

$$
\begin{aligned}
\mathrm{d}s^2 = &-\frac{\Delta(r)}{\Sigma(r,X)}F_1(r,X)\left[\mathrm{d}t-(1-X^2)F_4(r,X)\mathrm{d}\phi\right]^2 \\
&+\frac{1-X^2}{\Sigma(r,X)}F_3(r,X)\left[F_4(r,X)\,\mathrm{d}t-(r^2+a^2)\mathrm{d}\phi\right]^2 \\
&+\Sigma(r,X)F_2(r,X)\left[\frac{\mathrm{d}r^2}{\Delta(r)}+\frac{\mathrm{d}X^2}{1-X^2}\right] \\
A = &-\frac{\sqrt{2}\,r\,F_5(r,X)}{\kappa\,\Sigma(r,X)}\left[\mathrm{d}t-F_4(r,X)\,(1-X^2)\,\mathrm{d}\phi\right] \\
&-\frac{\sqrt{2}\,(1-X^2)\,F_6(r,X)}{\kappa\,\Sigma(r,X)}\left[F_4(r,X)\,\mathrm{d}t-(r^2+a^2)\mathrm{d}\phi\right]
\end{aligned}
$$

$$
F_i(r,X) = 1+\sum_{K=6}^{8}\frac{d_K}{M^2}f_i^{(K)}(r,X), \quad i=1,2,3 \qquad F_5(r,X)=\bar{Q}+\left(1-\frac{r_+}{r}\right)\sum_{K=6}^{8}\frac{d_K}{M^2}f_5^{(K)}(r,X)
$$

$$
F_4(r,X)=\bar{a}+\left(1-\frac{r_+}{r}\right)\sum_{K=6}^{8}\frac{d_K}{M^2}f_4^{(K)}(r,X) \qquad F_6(r,X)=\sum_{K=6}^{8}\frac{d_K}{M^2}f_6^{(K)}(r,X)
$$

Away from extremality

- To explicitly compute EFT-corrected black holes *away from the extremal limit*, i.e., at finite temperature, we will need to solve the equations numerically.

- We will numerically solve the EFT-corrected Einstein-Maxwell equations *away from the horizon* as well, out to $r = \infty$. Compactify coordinates using

$$r = \frac{r_+}{1 - Y}$$

$$Y = 0 \quad \text{horizon}$$
$$Y = 1 \quad \text{infinity}$$

- Due to large gradients near horizon, must use multiple Chebyshev-Gauss-Lobatto grids, and patch at interfaces.

Probing the horizon singularity

- Once we have constructed the solutions, we can measure the EFT-generated singularity at the horizon using an ingoing null geodesic: a massless probe falling into the horizon. Coordinates: $\dot{x} = (\dot{t}(\lambda), \dot{r}(\lambda), \dot{X}(\lambda), \dot{\phi}(\lambda))$

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- Choose geodesic in equatorial plane ($X = 0$), with zero angular momentum ($\partial(\dot{x}^2)/\partial\dot{\phi} = 0$), and write in ingoing Bondi-Sachs coordinates:

$$\mathrm{d}t = \mathrm{d}v - \frac{\mathrm{d}r}{\Delta(r)}(r_+^2 + \bar{a}^2)\left(1 - \frac{1}{2r_+^2}\sum_{K=6}^{8}\lambda^K d_K\right),$$

$$\mathrm{d}\phi = \mathrm{d}\varphi - \bar{a}\frac{\mathrm{d}r}{\Delta(r)}\left(1 - \frac{1}{2r_+^2}\sum_{K=6}^{8}\lambda^K d_K\right)$$

$$\dot{x} = \frac{r^2}{\Delta(r) f_1(r,0) f_3(r,0)[r^2+\bar{a}^2-f_4(r,0)^2]} \begin{bmatrix} \frac{\left(\bar{a}^2+r^2\right)^2 f_3(r,0)-\Delta(r) f_1(r,0) f_4(r,0)^2}{\bar{a}^2+r^2-f_4(r,0)^2} \\ -\frac{\Delta(r)\sqrt{f_1(r,0)f_3(r,0)}\sqrt{(\bar{a}^2+r^2)^2 f_3(r,0)-\Delta(r) f_1(r,0) f_4(r,0)^2}}{r^2\sqrt{f_2(r,0)}} \\ 0 \\ \frac{\left(\bar{a}^2+r^2\right) f_3(r,0)-\Delta(r) f_1(r,0)}{\bar{a}^2+r^2-f_4(r,0)^2} f_4(r,0) \end{bmatrix}$$

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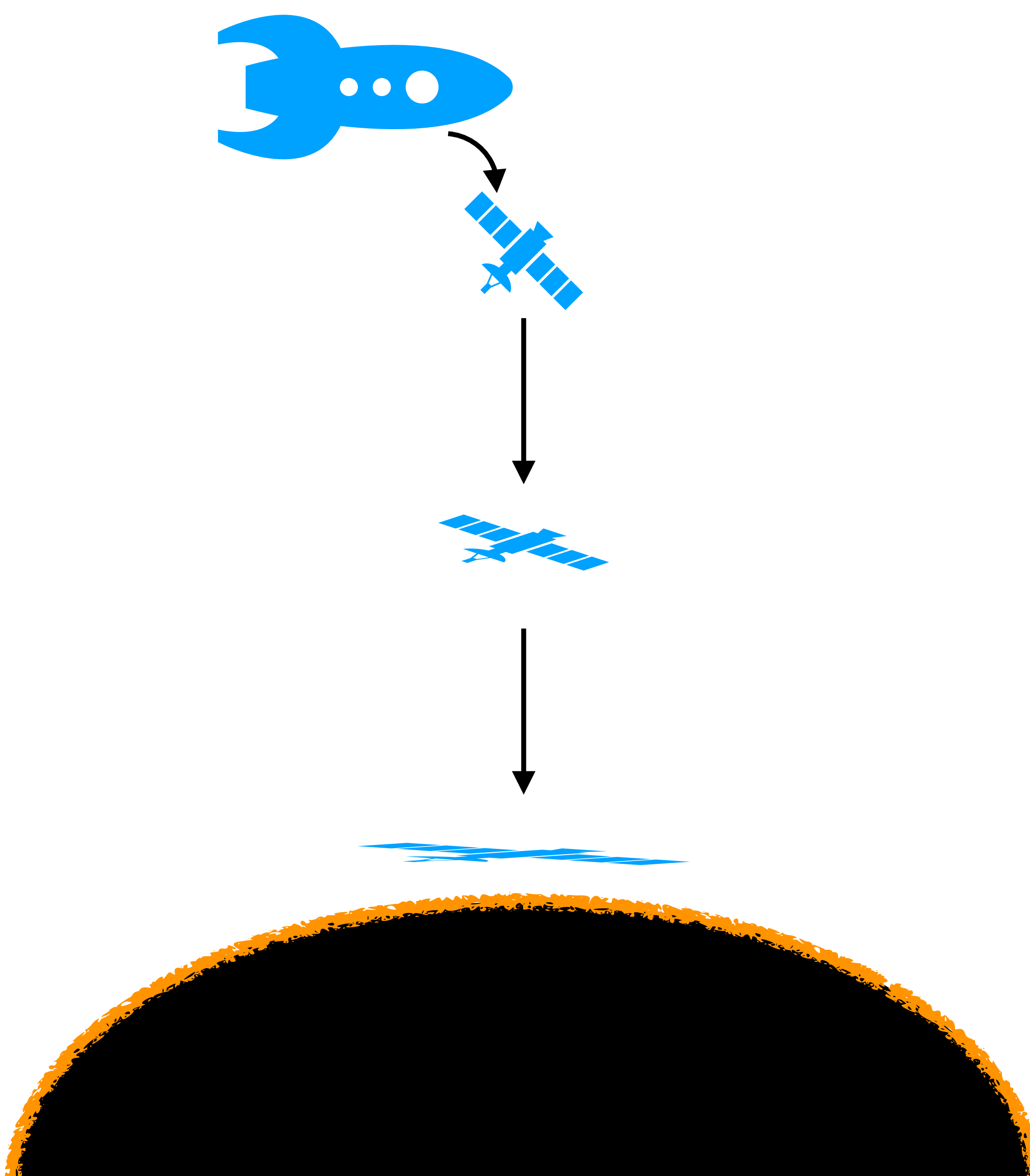
- Define a tidal force observable:

$$C_{\varphi\varphi} = \dot{x}^a \dot{x}^b C_{\varphi a \varphi b}$$

- Measure on horizon at equator: $C^{\mathcal{H}}_{\varphi\varphi}$



Probing the horizon singularity

- Once we have constructed the solutions, we can measure the EFT-generated singularity at the horizon using an ingoing null geodesic: a massless probe falling into the horizon. Coordinates: $\dot{x} = (\dot{t}(\lambda), \dot{r}(\lambda), \dot{X}(\lambda), \dot{\phi}(\lambda))$

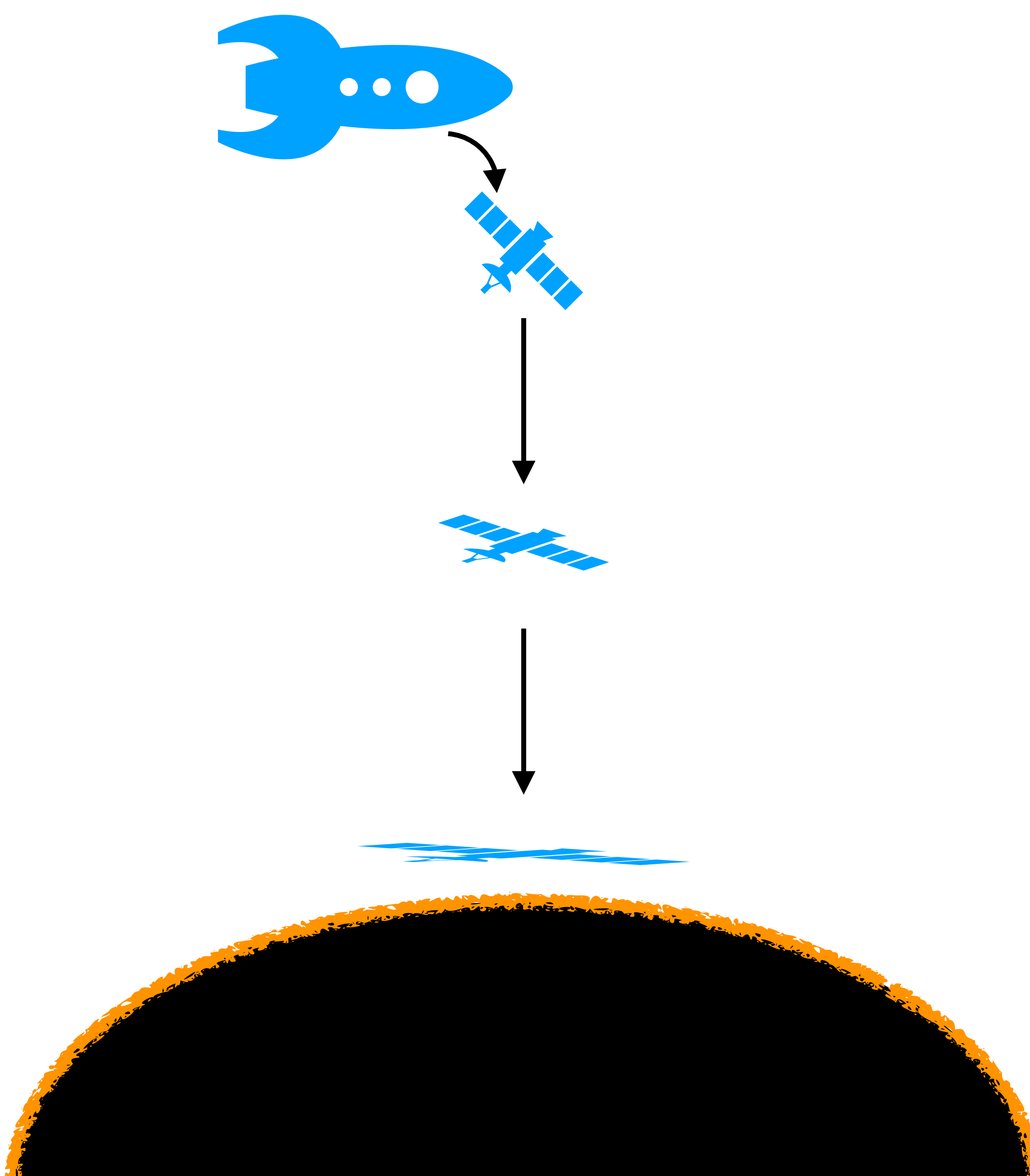
- Define a tidal force observable:

$$C_{\varphi\varphi} = \dot{x}^a \dot{x}^b C_{\varphi a \varphi b}$$

- Measure on horizon at equator: $C^{\mathcal{H}}_{\varphi\varphi}$

- Compare to un-corrected Kerr-Newman black hole with same charge, temperature, and angular momentum:

$$\delta C^{(K)} = \frac{Q^2}{d_K} \frac{C^{\mathcal{H}}_{\varphi\varphi} - \bar{C}^{\mathcal{H}}_{\varphi\varphi}}{\bar{C}^{\mathcal{H}}_{\varphi\varphi}}$$



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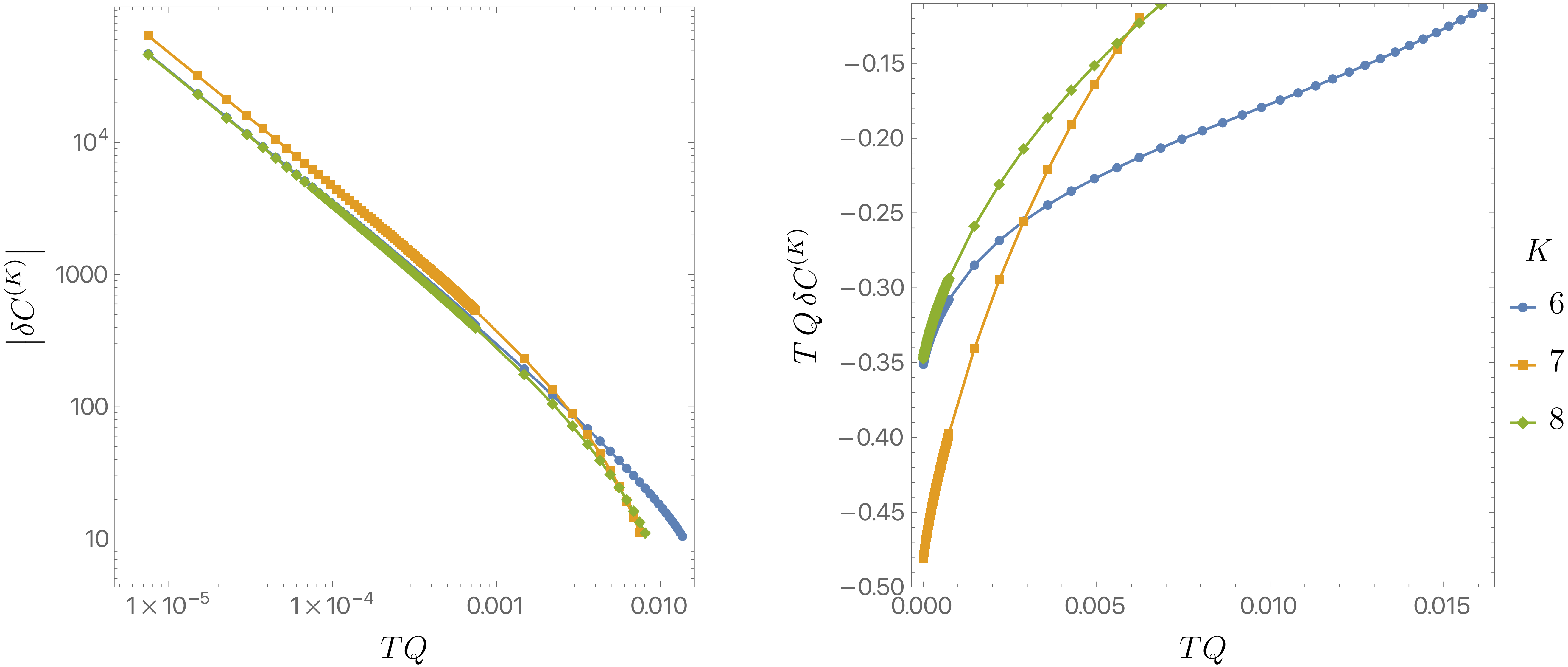

At low temperatures, we see that $\delta C^{(K)} \propto 1/T$

Defining parameters $\mathfrak{a} = \bar{a}/r_+, \ \mathfrak{q} = \bar{Q}/r_+$
Numerics plotted for $\mathfrak{a} = \mathfrak{q}$

Thermodynamics

- Let us examine the thermodynamic properties of our solutions.
- Chemical potential and angular velocity defined as before.
- Parameters $\mathfrak{a}$ and $\mathfrak{q}$ define moduli space of solutions.
- Perturbations to Komar charges,

$$E = \bar{E} - \frac{4\pi}{\kappa^2 \, r_+} \sum_{K=1}^{8} \alpha_0^K d_K$$

$$J = \bar{J} + \frac{4\pi}{\kappa^2} \sum_{K=1}^{8} \left[\omega_0^K (1 + \mathfrak{a}^2 + \mathfrak{q}^2) - 2\mathfrak{a}\alpha_0^K\right] d_K$$

$$Q_e = \frac{4\pi\sqrt{2}}{\kappa^2} \left[\bar{Q} + \frac{1}{r_+} \sum_{K=1}^{8} \rho_0^K d_K\right]$$

Thermodynamics

- Temperature: $T = \dfrac{1-\mathfrak{a}^2-\mathfrak{q}^2}{4\pi r_+(1+\mathfrak{a}^2)}\left(1+\dfrac{1}{2}\sum_{K=6}^{8}[f_1^{(K)}(X,0)-f_2^{(K)}(X,0)]\dfrac{d_K}{r_+^2}\right)$

- Wald entropy for our explicit numerical solutions:

$$S = \frac{8\pi^2}{\kappa^2}(1+\mathfrak{a}^2)r_+^2\left\{1+\frac{1}{4r_+^2}\sum_{K=6}^{8}\int_{-1}^{1}\mathrm{d}X\left[f_2^{(K)}(X,0)+f_3^{(K)}(X,0)\right]d_K\right.$$
$$\left.-\frac{2\mathfrak{q}^2}{3r_+^2}\left[\frac{9+4\mathfrak{a}^2+3\mathfrak{a}^4}{(1+\mathfrak{a}^2)^3}+\frac{3\arctan\mathfrak{a}}{\mathfrak{a}}\right]d_6\right\}$$

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- Prediction using on-shell action (comparing black holes with same Komar charges):

Cheung, Liu, **GR** [1903.09156]

$$\Delta S=\beta\int \mathrm{d}^3x\,\sqrt{-g}\,\Delta\mathcal{L}$$

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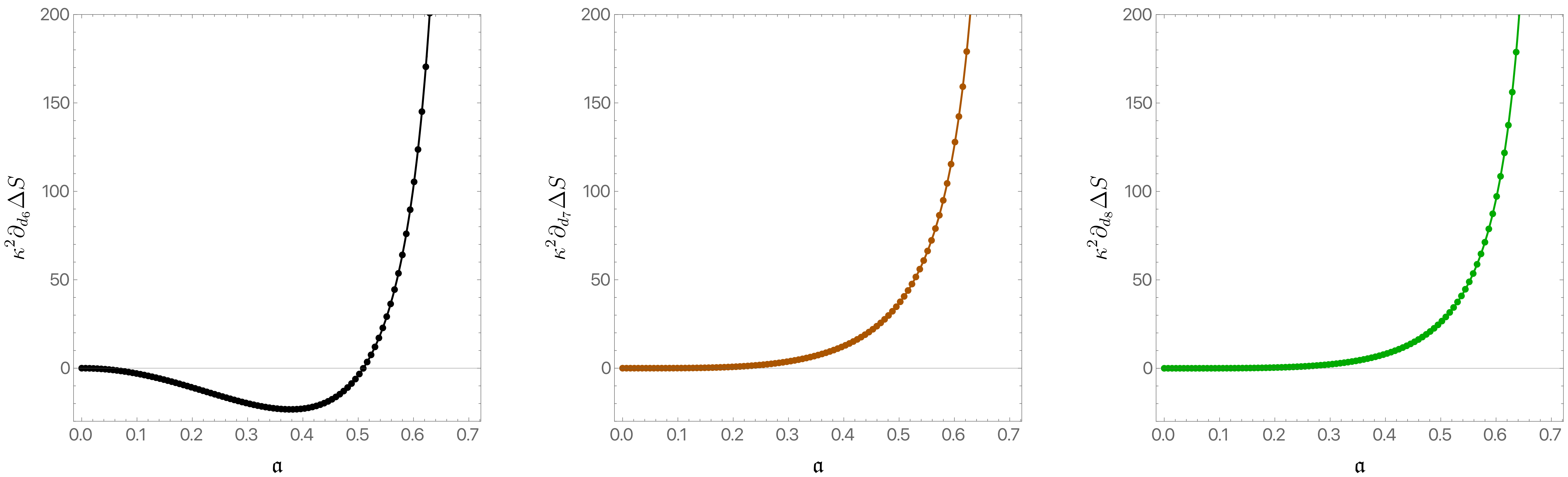
Cheung, Liu, **GR** [1903.09156]

$$\Delta S(\mathfrak{a},\mathfrak{q}) = \frac{16\pi^2(\mathfrak{a}^2-3)(3\mathfrak{a}^2-1)[1-\xi-\mathfrak{a}^2(1+\xi)]^2}{15\kappa^2\xi(1+\xi)(1+\mathfrak{a}^2)^4}(d_0+d_6-d_9)$$
$$+\frac{\pi^2\left[1-\xi-\mathfrak{a}^2(1+\xi)\right]^2\left[\mathfrak{a}(3+2\mathfrak{a}^2+3\mathfrak{a}^4)+3(\mathfrak{a}^2-1)\left(1+\mathfrak{a}^2\right)^2\arctan\mathfrak{a}\right]}{2\kappa^2\xi(1+\xi)\mathfrak{a}^5}(d_0+d_6+d_9)$$
$$+\frac{64\pi^2}{\kappa^2}d_3+\frac{32\pi^2[1-\xi-\mathfrak{a}^2(1+\xi)][\mathfrak{a}^2(3+4\xi)-1-4\xi]}{5\kappa^2\xi(1+\xi)(1+\mathfrak{a}^2)^2}d_6$$

$$\xi = \frac{\sqrt{\bar{M}^2-\bar{a}^2-\bar{Q}^2}}{\bar{M}} = \frac{1-\mathfrak{a}^2-\mathfrak{q}^2}{1+\mathfrak{a}^2+\mathfrak{q}^2} = \frac{\kappa^2}{4\pi\bar{M}}\bar{T}\bar{S}$$

Thermodynamics

Numerical results for entropy precisely match our on-shell action prediction from euclidean quantum gravity:



Numerics plotted for $\mathfrak{a} = \mathfrak{q}$

Back to extremal

- We can also use numerical methods to compute the full asymptotically flat metric for the EFT-corrected extremal black hole, and compare to our near-horizon analytical results.

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$$
\begin{aligned}
\mathrm{d}s^2 = &-\frac{\Delta(r)}{\Sigma(r,X)}F_1(r,X)\left[\mathrm{d}t-(1-X^2)F_4(r,X)\mathrm{d}\phi\right]^2 \\
&+\frac{1-X^2}{\Sigma(r,X)}F_3(r,X)\Xi(r)\left[F_4(r,X)\,\mathrm{d}t-(r^2+\bar{a}^2)\mathrm{d}\phi\right]^2 \\
&+\Sigma(r,X)F_2(r,X)\left[\frac{\mathrm{d}r^2}{\Delta(r)}+\Xi(r)\frac{\mathrm{d}X^2}{1-X^2}\right]
\end{aligned}
$$

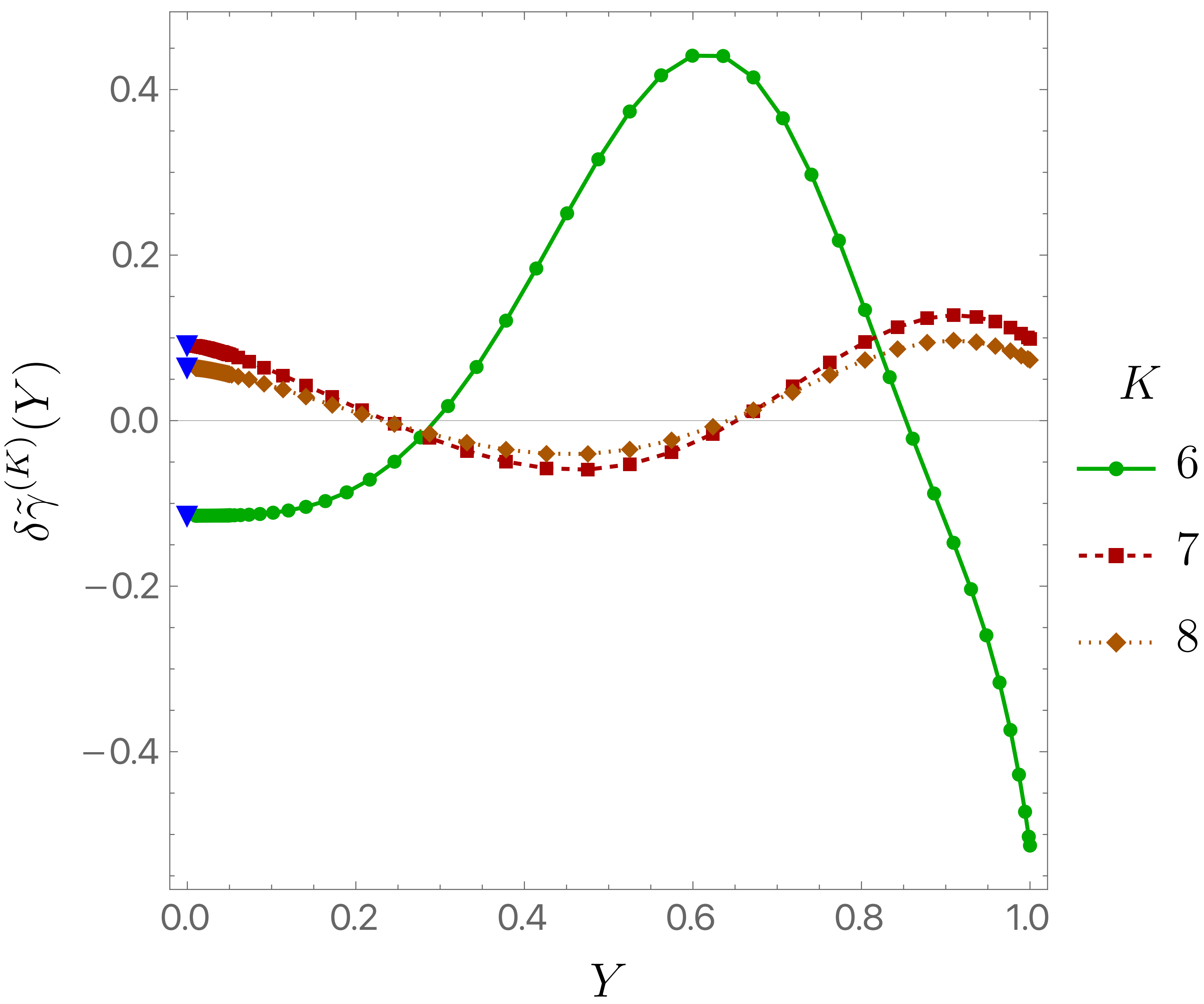
$$
\Xi(r) = 1+\sum_{K=6}^{8}\frac{r_+^4}{r^4}\frac{d_K}{M^2}a^{(K)}
$$

$$
F_i(r,X) = 1+\sum_{K=6}^{8}\frac{d_K}{M^2}f_i^{(K)}(r,X), \quad i=1,2,3 \qquad F_5(r,X) = \bar{Q}+\left(1-\frac{r_+}{r}\right)\sum_{K=6}^{8}\frac{d_K}{M^2}f_5^{(K)}(r,X)
$$

$$
F_4(r,X) = \bar{a}+\left(1-\frac{r_+}{r}\right)\sum_{K=6}^{8}\frac{d_K}{M^2}f_4^{(K)}(r,X) \qquad F_6(r,X) = \sum_{K=6}^{8}\frac{d_K}{M^2}f_6^{(K)}(r,X)
$$

Back to extremal

We can read off the shift in the scaling dimension for our mode of interest in terms our our numerical solution:



$$\delta\gamma^{(K)} = \lim_{Y\to 0} \delta\tilde{\gamma}^{(K)}(Y)$$

$$\delta\tilde{\gamma}^{(K)}(Y) = -\frac{(2-\mathfrak{q}^2)}{8(1-\mathfrak{q}^2)} Y \frac{\partial^3 f_3}{\partial Y^3}\bigg|_{X=0}$$

$\delta\tilde{\gamma}^{(K)}(Y)$ from numerics: lines

$\delta\gamma^{(K)}$ from near-horizon analytical methods: blue triangles

Numerics plotted for $\mathfrak{q} = 0.2$

Probing the horizon singularity

- We can again construct a null geodesics heading towards the horizon and measure the tidal force.

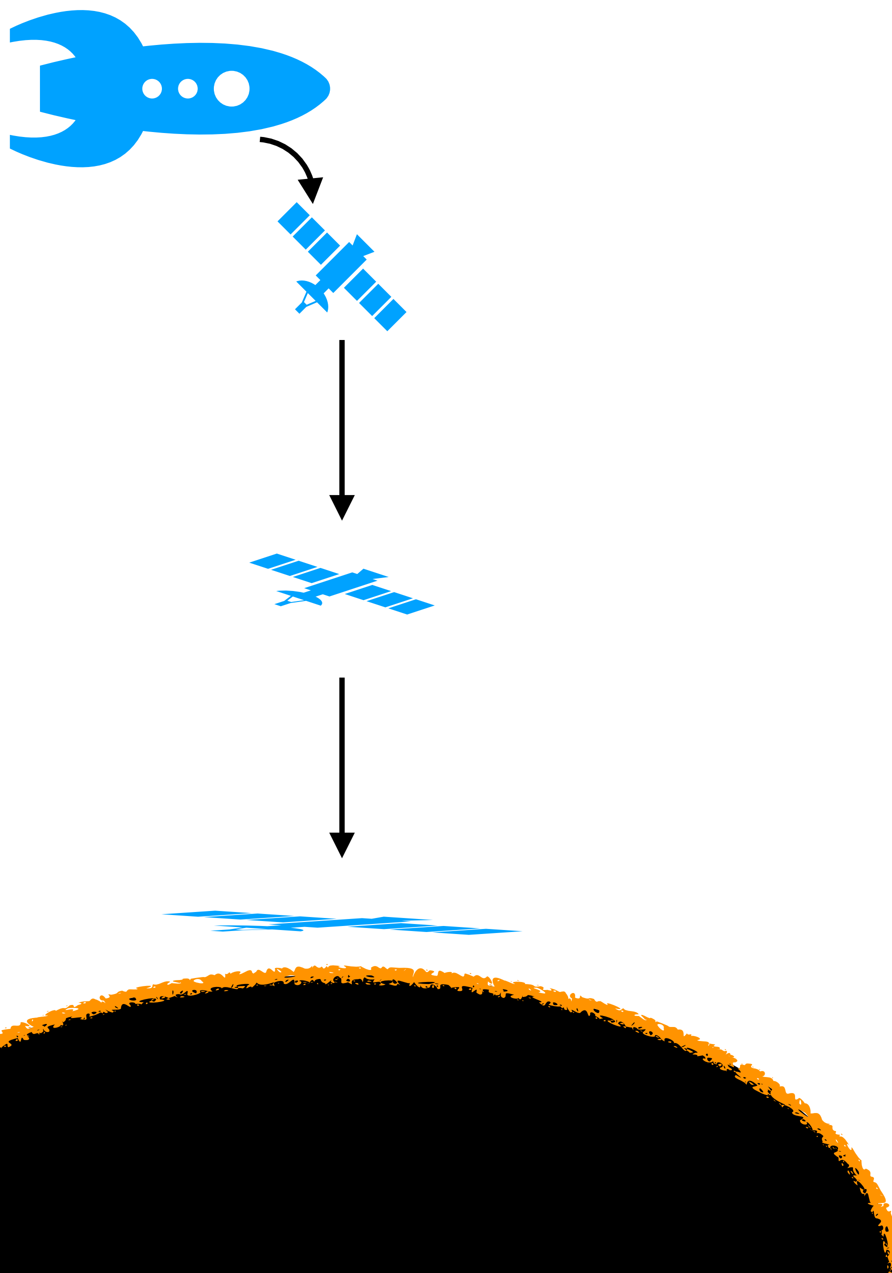
- As before, define a tidal force observable:

$$C_{\varphi\varphi} = \dot{x}^a \dot{x}^b C_{\varphi a \varphi b}$$

- Measure at equator, but do not restrict to horizon.

- Compare to un-corrected Kerr-Newman black hole with same charge, (zero) temperature, and angular momentum:

$$\delta\widetilde{C}^{(K)} = \frac{\kappa^2 J}{8\pi d_K} \frac{C_{\varphi\varphi}^{X=0} - \bar{C}_{\varphi\varphi}^{X=0}}{\bar{C}_{\varphi\varphi}^{X=0}}$$



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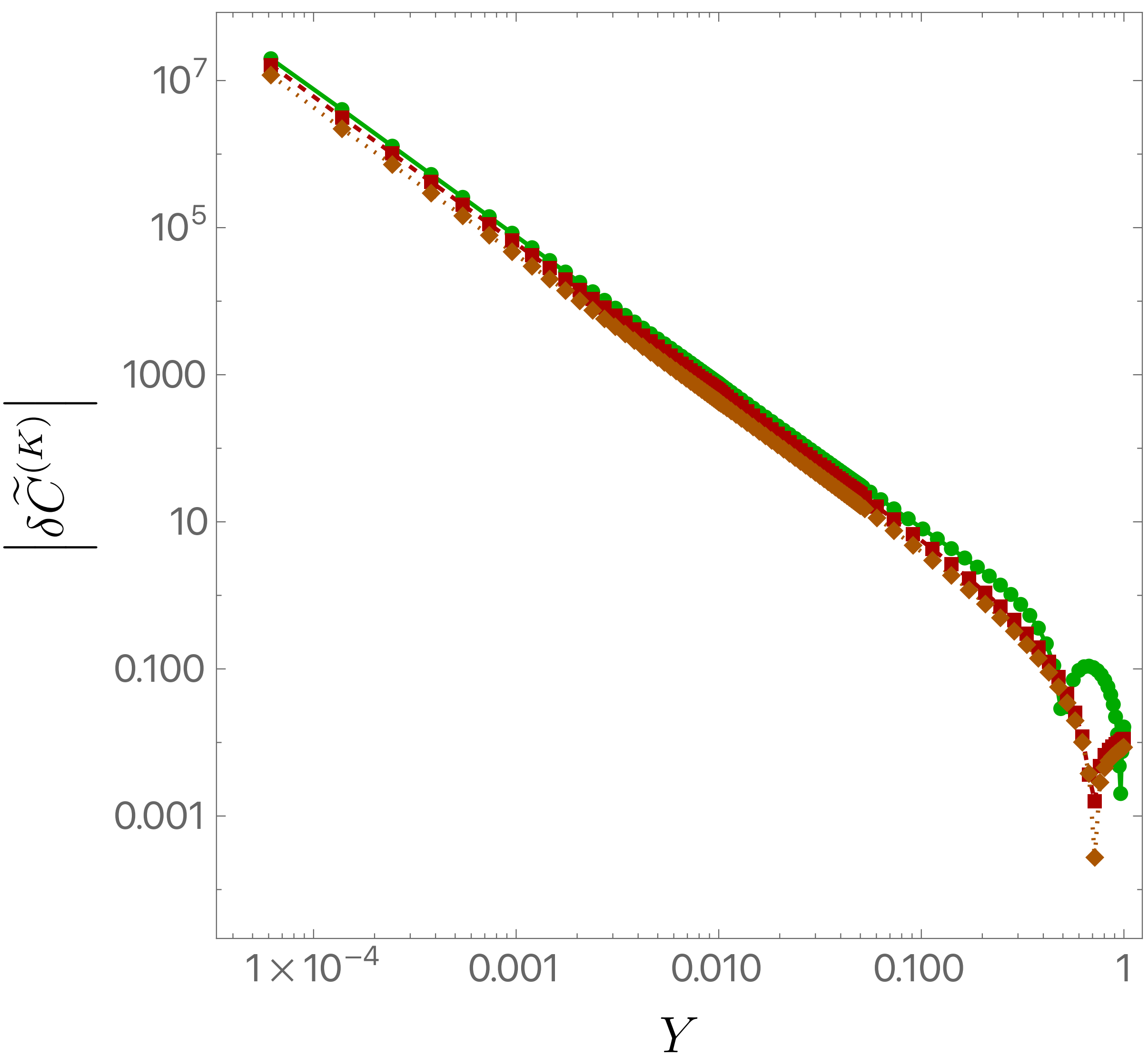
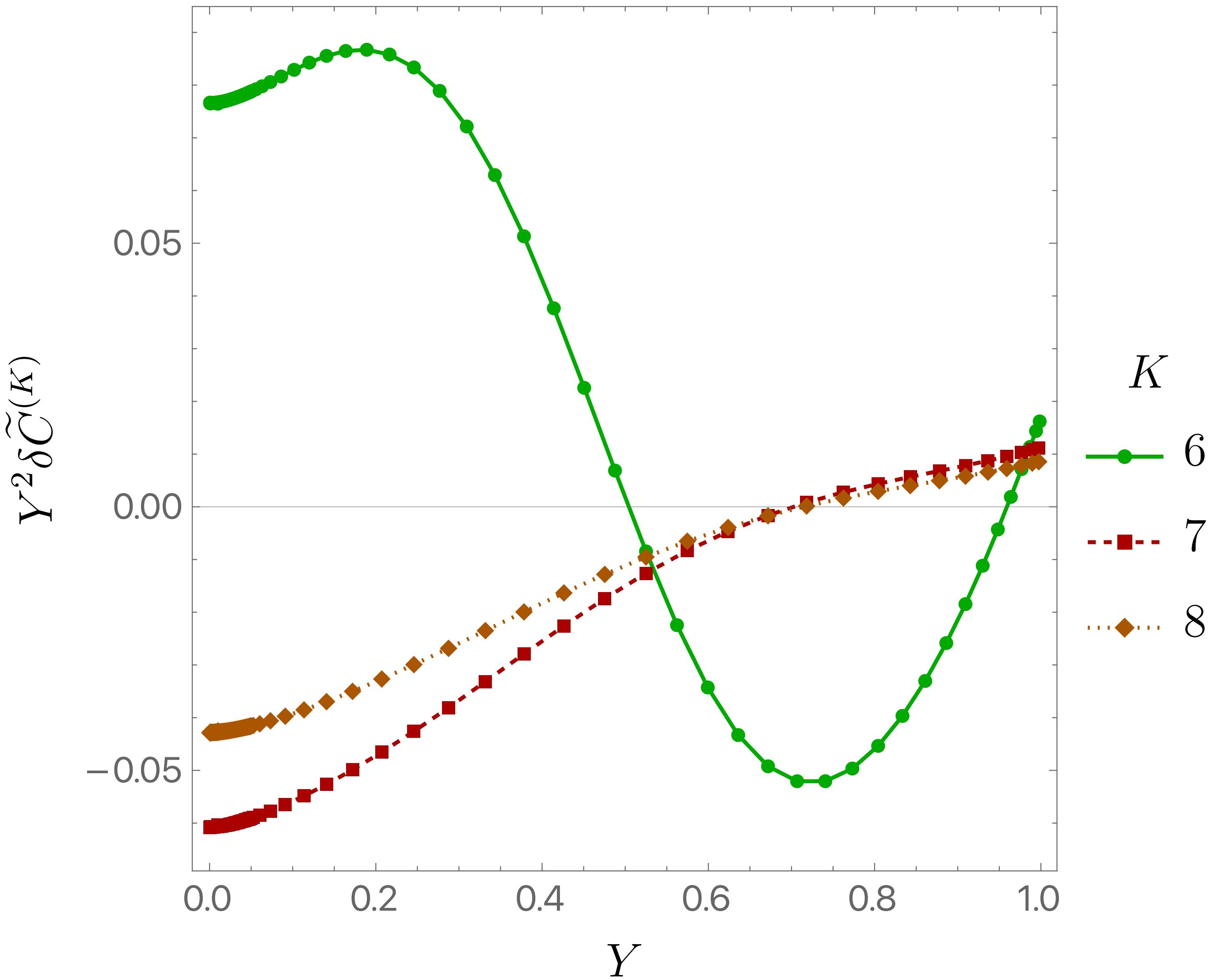
$$\delta\widetilde{C}^{(K)} = \frac{\kappa^2 J}{8\pi d_K}\frac{C^{X=0}_{\varphi\varphi} - \bar{C}^{X=0}_{\varphi\varphi}}{\bar{C}^{X=0}_{\varphi\varphi}}$$



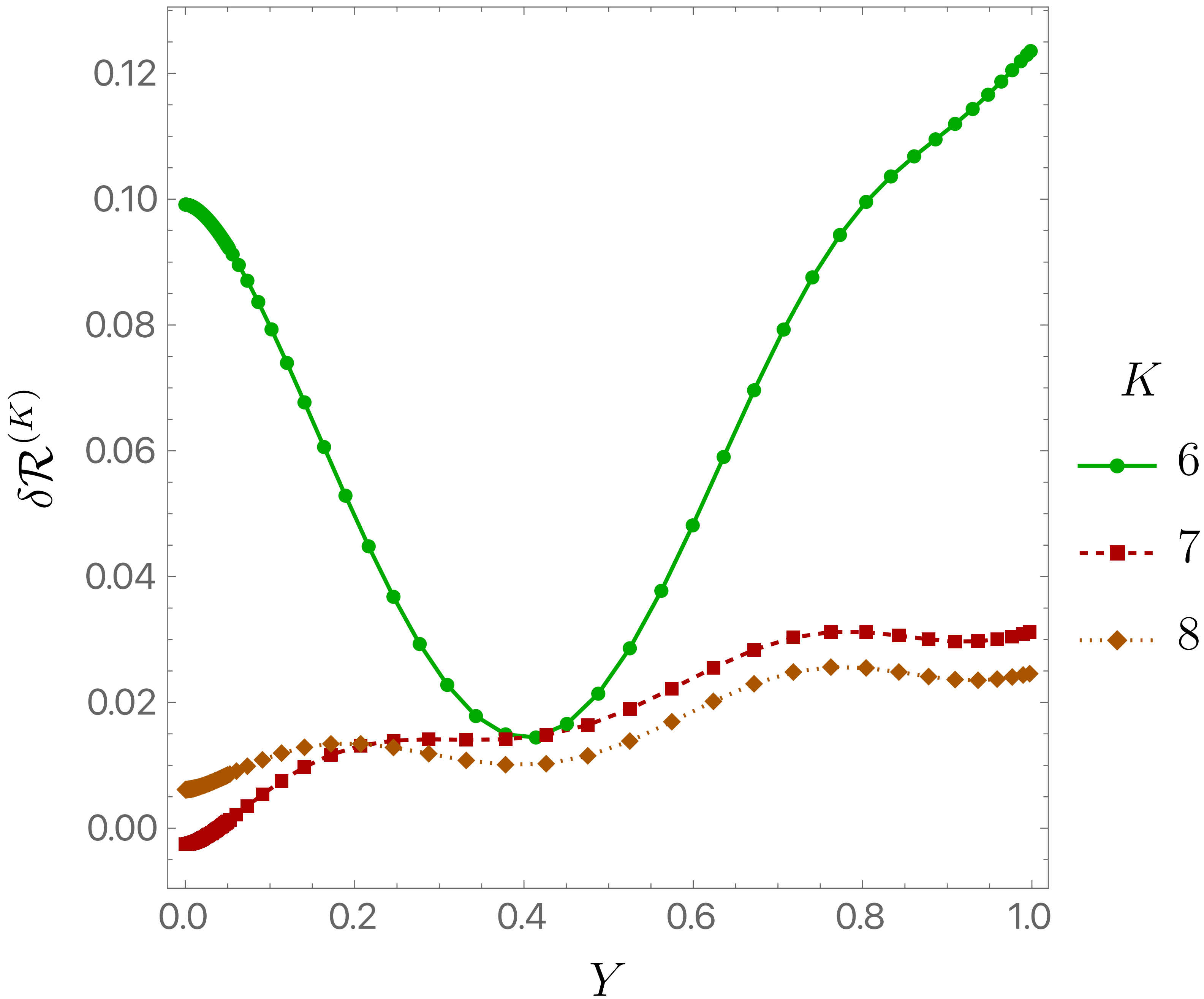


Close to the horizon, we see that $\delta\widetilde{C}^{(K)} \propto 1/Y^2 \sim 1/(r-r_+)$

Numerics plotted for $\mathfrak{q} = 0.2$

Probing the horizon singularity

- Interestingly, all curvature *invariants* remain finite on the horizon.
- Correction to KN metric $\delta g \sim \rho^{\gamma}$ $\Longrightarrow$ curvature $\delta C \sim \rho^{\gamma-2}$
- Equations of motion are second-rank tensor, and all but two ρ derivatives in any invariant are contracted with $g^{\rho\rho} = \rho^2$ $\Longrightarrow$ no higher divergence
- Example:

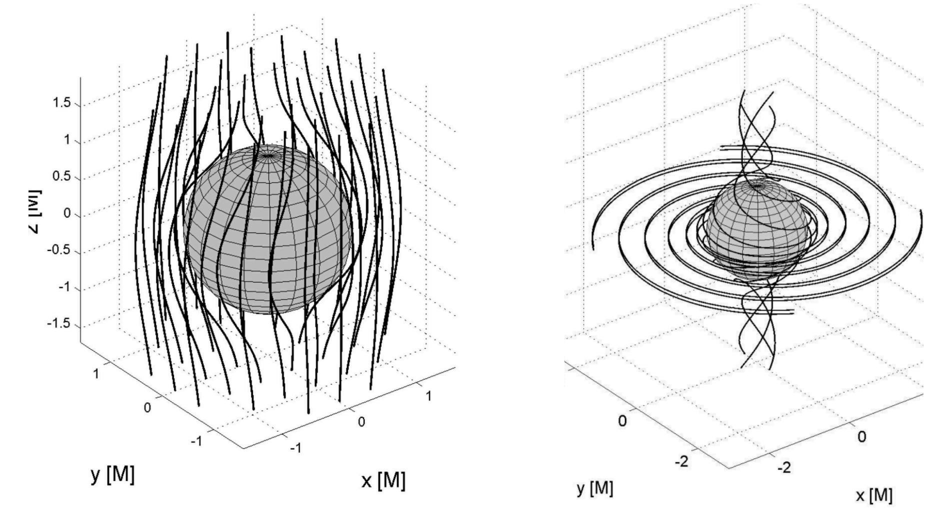


$$\delta\mathcal{R}^{(K)} = \frac{\kappa^2 J}{8\pi d_K} \left. \frac{R_{abcd}R^{abcd} - \bar{R}_{abcd}\bar{R}^{abcd}}{\bar{R}_{abcd}\bar{R}^{abcd}} \right|_{X=0}$$


Astrophysics

Charge on astrophysical black holes?

- Black hole in plasma: $Q/M \sim m_e/q_e \sim 10^{-21}$
- Wald effect: a black hole spinning in a magnetic field produces an electric field on the horizon, leading to accumulation of equilibrium charge $Q_{\mathrm{W}} = 2BaM$ Wald (1974)



Karas, Kopáček, Kunneriath [1201.0009]

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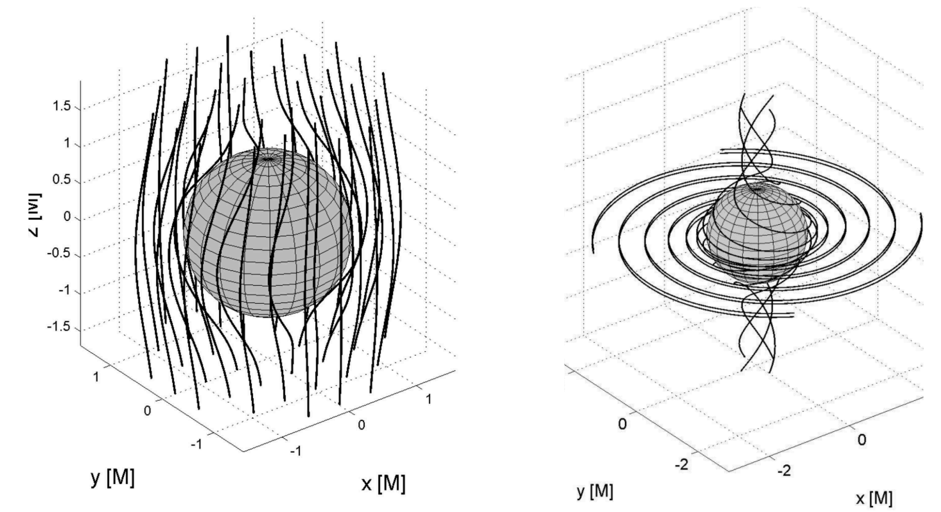
- For Sag A*, the galactic magnetic field of 10 G induces a charge-to-mass ratio of $Q/M \sim 10^{-12}$ Zajaček, Tursunov [1904.04654]



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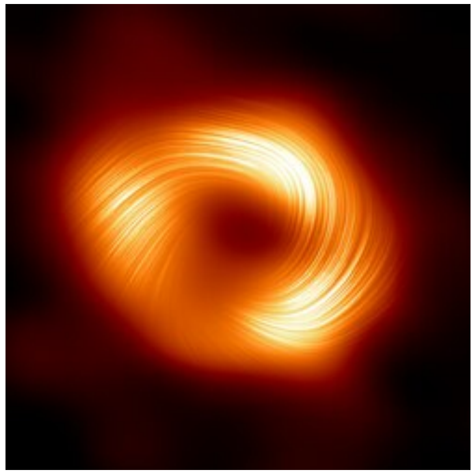

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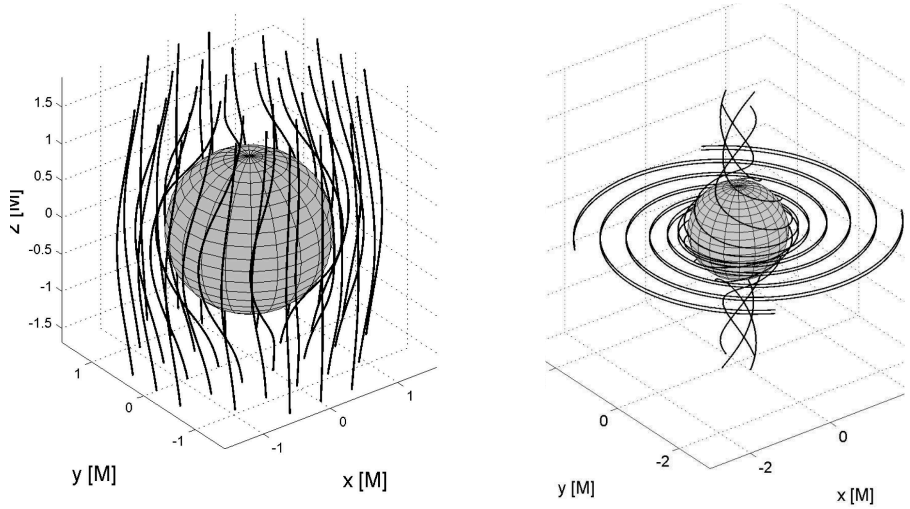

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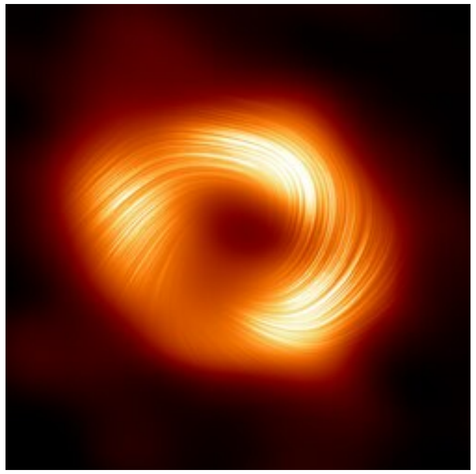
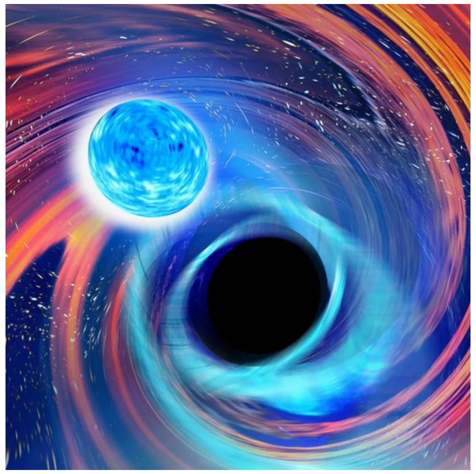


- Black hole collision with a typical pulsar (10 $M_\odot$ black hole, 10 km apart, 10^{12} G), $Q/M \sim 10^{-7}$ Levin, D'Orazio, Garcia-Saenz [1808.07887]



Rough estimate

- Shift in scaling dimension $\delta\gamma \sim G c_{7,8} F^4 (GM)^2 \sim c_{7,8} Z^4 / G^3 M^2$

- Standard model contribution to Euler-Heisenberg Lagrangian $c_{7,8} \sim 10^{-4} (q_e/m_e)^4$

$$\implies \delta\gamma \sim 10^{81} Z^4 / S$$

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- Quantum corrections? $T_q = \dfrac{\pi}{GMS} \ll T_{\mathrm{EFT}} \sim \dfrac{10^{81} Z^4}{GMS}$

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- For a typical black hole colliding with a neutron star, $T_{\mathrm{EFT}} \sim 10^{-25}/GM$, so this effect is too small to be observable in that case.

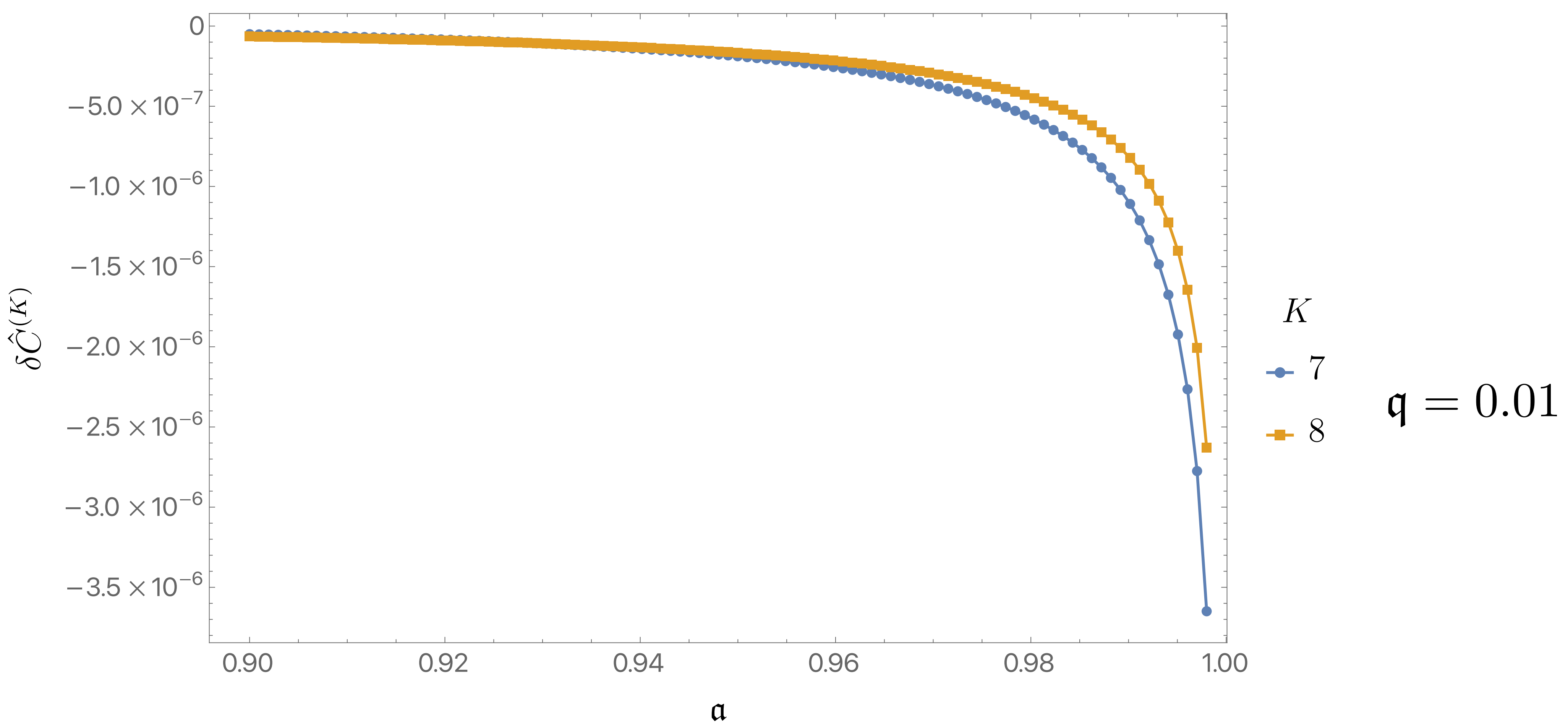
- Exotic scenarios and probes of the dark sector: new light particles or forces, e.g., $\mathrm{U}(1)_{B-L}$ charges, are a direction of future interest.

Numerical results for astrophysics

- Let us use our numerical results to assess the best-case astrophysical scenario for observation of our EFT enhancement effect.

- Maximum realistic astrophysical spin: $\mathfrak{a} \approx 0.998$ Thorne (1974)
(spin-up from hot accretion disk balanced by torque from thermal radiation)

- Numerical results: $\delta \hat{C}^{(K)} = \frac{\kappa^2 J}{8\pi d_K} \frac{C^{\mathcal{H}}_{\varphi\varphi} - \bar{C}^{\mathcal{H}}_{\varphi\varphi}}{\bar{C}^{\mathcal{H}}_{\varphi\varphi}} \sim (5 \ \text{to} \ 300) \times \mathfrak{q}^4$



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$$\frac{C^{\mathcal{H}}_{\varphi\varphi} - \bar{C}^{\mathcal{H}}_{\varphi\varphi}}{\bar{C}^{\mathcal{H}}_{\varphi\varphi}} \sim (10^5 \ \text{ to } \ 10^7) \times \left(\frac{10\, M_{\odot}}{M}\right)^2 \times \mathfrak{q}^4$$

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- Induced Wald charge largest for black hole of same size as neutron star, $M = 7\, M_\odot$

$$\frac{C^{\mathcal{H}}_{\varphi\varphi} - \bar{C}^{\mathcal{H}}_{\varphi\varphi}}{\bar{C}^{\mathcal{H}}_{\varphi\varphi}} \sim (10^{-8} \ \text{ to } \ 10^{-6}) \times (B/10^{16}\, \mathrm{G})^4$$

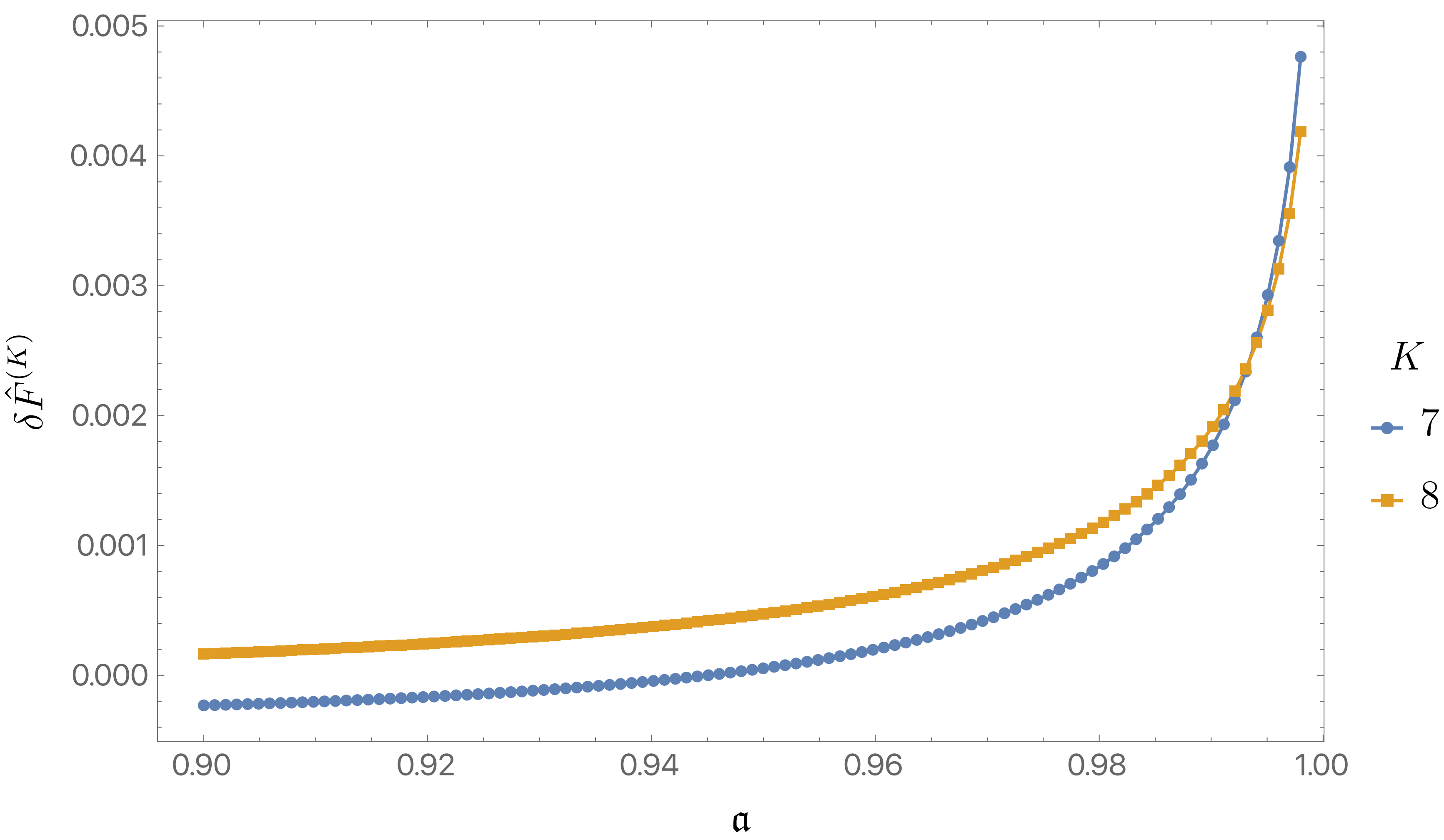
- Strongest magnetars have fields up to 10^{16} G Raynaud et al. [2003.06662] $\implies \mathfrak{q} \sim 2 \times 10^{-4}$

- Observable with future precision GW measurements?

Numerical results for astrophysics

- What about electrodynamic observables? Numerical analysis gives

$$\delta\hat{\mathcal{F}}^{(K)} = \frac{\kappa^2 J}{8\pi d_K}\frac{\mathcal{F}-\bar{\mathcal{F}}}{\bar{\mathcal{F}}} \sim (2 \ \text{to} \ 50) \times \mathfrak{q}^2 \ \text{ for } \mathcal{F} = \dot{x}^a F_{a\varphi}$$



$\mathfrak{q} = 0.01$

- Best-case astrophysical scenario:

$$\frac{\mathcal{F}-\bar{\mathcal{F}}}{\bar{\mathcal{F}}} \gtrsim \text{few percent}$$

Naive EFT estimate:

$$\delta\mathcal{F}^{(K)} \sim 16\mathfrak{q}^2$$

Discussion

EFT breaking?

- The derivative expansion of the bulk Lagrangian remains under control: yet-higher derivative Lorentz scalars do not diverge, and with smaller Wilson coefficients contribute negligibly to the action.
  $\Longrightarrow$ Calculations of the EFT-corrected background geometry remain robust.

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  $\Longrightarrow$ finite-size effects diverge, e.g., Love numbers associated with gravitoelectric tensor $E_{ab}^2 = (C_{acbd}\dot{x}^c \dot{x}^d)^2$ Cheung, Solon [2006.06665]

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- Infalling string: horizon looks like singular plane wave geometry, and a string can get highly excited if $\gamma \leq 1$

Future directions

- Adding large external magnetic field: Kerr-Newman black hole in a constant ambient magnetic field has exact solution. EFT-correct this spacetime to more precisely simulate astrophysical scenarios?

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- EFT corrections to black holes in AdS, in supersymmetric theories, or string theory (EFT-corrected GHS?)

Questions