Looking at Extremal Black Holes from Very Far Away

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Near-Extremal black holes

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Based on work to appear with

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MOTIVATION

- The conventional picture of (non-supersymmetric) extremal black holes, viz., one where there is non-trivial degeneracy at zero temperature, is somewhat curious.

- Not only does this naively violate the third law of thermodynamics, it also poses challenges for how near-extremal black holes Hawking radiate.

$$E = E_0 + \frac{T^2}{T_{\text{gap}}}\,, \qquad S = S_0 + \frac{T}{T_{\text{gap}}}\, S_1$$

- The fact that energy departures from extremality are quadratic, has important implications: at temperatures below the gap, the black hole is unable to decharge by emitting even a single Hawking quantum. Preskill, Schwarz, Shapere, Trivedi, Wilczek '91

- Various attempts have been made to address this puzzle, eg., black hole pair production, attractor mechanism, etc.

Hawking, Horowitz, Ross '94 Dabholkar, Sen, Trivedi '06

- The modern understanding of this situation is somewhat prosaic: the non-trivial degeneracy is illusory, and in fact near-extremal black holes have a vanishingly small degeneracy. They behave like a conventional quantum mechanical system with few low-lying excitations. Ghosh, Maxfield, Turiaci, '19 Iliesiu, Turiaci '20

MOTIVATION

* The essential point is that the semiclassical analysis needs to carried out with care. While the black hole is a dominant saddle, fluctuations around it are important.

$$\mathcal{Z} \sim \det\nolimits_0 e^{I_0} + \det\nolimits_1 e^{I_1} + \cdots$$

* The modification in the low temperature thermodynamics arises from the presence of zero modes of the extremal geometry localized in the near-horizon region.

$$\mathcal{Z} \sim T^{\frac{3}{2}} \exp\left(\underbrace{S_0 + S_1 \frac{T}{T_{\rm gap}}}_{\text{classical result}} + c \log S_0\right)$$

$T^{\frac{3}{2}}$: 1-loop det of gapless modes

$c \log S_0$: 1-loop corrections from gapped and gapless modes

Sen ’11-’12

Iliesiu, Murthy, Turiaci ‘22

* The zero mode contribution can be nicely isolated by examining how they get gapped in the near-extremal solution.

MOTIVATION

- ✴ The fastest way to derive this picture is to appeal to the enhanced SL(2,R) symmetry of the near-horizon region

  Kunduri, Lucietti, Reall ’07

- ✴ One can either use a dimensional reduction down to this AdS_2 spacetime leading to an effective JT gravity description, where quantum effects can be understood.

  Moitra, Trivedi, Vishal ’18

- ✴ Alternately, one can work in the full near-extremal geometry, and deduce that the spectrum of the quadratic fluctuation operator has an apposite set of zero modes.
- ✴ The latter perspective is efficacious in the case of rotating black holes, which typically have the AdS_2 factor fibered and warped over compact base manifold, e.g., the 4d Kerr black hole.

  Rakic, MR, Turiaci ’23 Kapec, Sheta, Strominger, Toldo ’23

- ✴ Broadly speaking, all of these analysis zoom into the throat and exploit the homogeneity of the AdS_2 factor to identify the zero modes in question.

  Camporesi, Higuchi ‘94 Sen ’11-’12

- ✴ Once one identifies the zero modes, a nice way to compute their contribution to the one-loop determinant is to work with a small temperature regulator, however, still staying within the near-horizon region.

  Iliesiu, Murthy, Turiaci ‘22

MOTIVATION

- Can we do better, viz., provide a perspective on the quantum corrections without referring to the near-horizon geometry directly, but rather working in the full near-extremal black hole spacetime?

  - To be clear, the aim is to not make explicit use of the near AdS_2 factor at all. For instance, the original calculation was carried out in the full geometry, but used a semi-holographic approach, splitting the evaluation into a near-horizon part and a far zone part.

    Iliesiu, Turiaci '20

  - Likewise, the idea here is different from the gravitational computation of the index in the full geometry, where one works at finite temperature and identifies a complex Euclidean saddle associated with a particular value of chemical potential.

    Cabo-Bizet, Cassani, Martelli, Murthy '18

    Kologlu, Iliesiu, Turiaci '21

    Anupam, Athira, Chowdhury, Sen '23

RESULTS

* **Claim 1:** The near-horizon zero modes of the extremal black hole uplift to *light off-shell* modes of the quadratic fluctuation operator around the near-extremal black hole saddle. The eigenvalues of such modes scale linearly with the Matsubara frequency in the near-extremal regime. Such modes arise from:
  - Universal graviton fluctuations that might be identified with the Schwarzian modes in the AdS_2 throat.
  - Isometries of the background geometry and background gauge fields, both of which have generically lead to zero modes, though the details depend on the geometry in question.

* **Claim 2:** Zero modes associated with rotational isometries appear to be subtle, especially in Ricci flat spacetimes. In particular, the oft used harmonic gauge breaks down in the near-horizon region, leading to misleading conclusions about the existence of the said zero modes (case in point: the rotational zero mode of Kerr is not smooth in harmonic gauge). Rakic, MR, Turiaci '23

OUTLINE

- Analytic eigenmodes in near-extremal BTZ geometry
- Numerical eigenmodes for Reissner-Nordstrom-AdS_4.
- Rotational zero modes in general & issues in Ricci-flat geometries
- Ensemble choices: near-horizon vs full geometry
- The curious case of the rotational mode in BTZ and its lessons

Exhibit 1: The BTZ geometry

THE BTZ SPACETIME: A REFRESHER

* Consider 3d gravity with a negative cosmological constant (counterterms suppressed)

$$I = -\frac{1}{16\pi\, G_N}\int \mathrm{d}^3x\,\sqrt{g}\left(R + \frac{2}{\ell_{\mathrm{AdS}}^2}\right) - \frac{1}{8\pi\, G_N}\oint \mathrm{d}^2x\,\sqrt{\gamma}\,K$$

* Work in the grand canonical ensemble fixing asymptotic thermal period and introduce a chemical potential for rotation. The saddle point configuration is the rotating BTZ geometry with line element

$$\mathrm{d}s^2 = f(r)\,\mathrm{d}t_{\mathrm{E}}^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\left(\mathrm{d}\varphi + \frac{\mathrm{i}\,r_+\,r_-}{r^2}\,\mathrm{d}t_{\mathrm{E}}\right)^2 \qquad f(r) = \frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2}$$

* Physical parameters:

$$T = \frac{r_+^2 - r_-^2}{2\pi r_+}\,, \qquad \Omega = \frac{r_-}{r_+}$$

$$M = \frac{r_+^2 + r_-^2}{8\,G_N}\,, \qquad J = \frac{r_+ r_-}{4\,G_N}$$

* Near-extremal limit: $T_L = \dfrac{r_+ - r_-}{2\pi}\,, \qquad T_R = \dfrac{r_+ + r_-}{2\pi} \qquad T_L \ll T_R$

PERTURBATIONS AROUND BTZ

✳ To investigate the fluctuations we will look at perturbations of the spin-2 fields.

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu} \qquad\qquad \tilde{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} g_{\mu\nu} h$$

✳ To investigate the fluctuations we will look at perturbations of the spin-2 fields. We pick a suitable gauge condition (harmonic gauge)

$$I_{\text{gf}} = \underbrace{\frac{1}{32\pi\, G_N} \int \mathrm{d}^3x\, \sqrt{g}\, \nabla^\mu \tilde{h}_{\mu\sigma}\, \nabla^\nu \tilde{h}_\nu{}^\sigma}_{\text{gauge fixing term}} + \underbrace{\frac{1}{32\pi\, G_N} \int \mathrm{d}^3x\, \sqrt{g}\, \bar{\eta}_\mu \left(-g^{\mu\nu}\nabla^2 - R^{\mu\nu}\right) \eta_\nu}_{\text{ghost action}}$$

✳ The quadratic action is governed by the spin-2 Lichnerowicz operator, which maps symmetric two-tensors to symmetric two-tensors

$$I \supset \frac{1}{16\pi G_N} \int \mathrm{d}^3x \sqrt{g}\, \tilde{h}^{\mu\nu} (\Delta_L h)_{\mu\nu}$$

$$(\Delta_L h)_{\mu\nu} = -\frac{1}{4} \nabla_\rho \nabla^\rho h_{\mu\nu} + \frac{1}{2} R_{\rho(\mu}\, h_{\nu)}{}^\rho - \frac{1}{2} R_{\mu\rho\nu\sigma}\, h^{\rho\sigma} - \left(R_{\sigma(\nu} - \frac{1}{4} g_{\sigma(\nu}\, R\right) h_{\mu)}{}^\sigma - \frac{\Lambda}{2} h_{\mu\nu}$$

SPIN-2 EIGENMODES

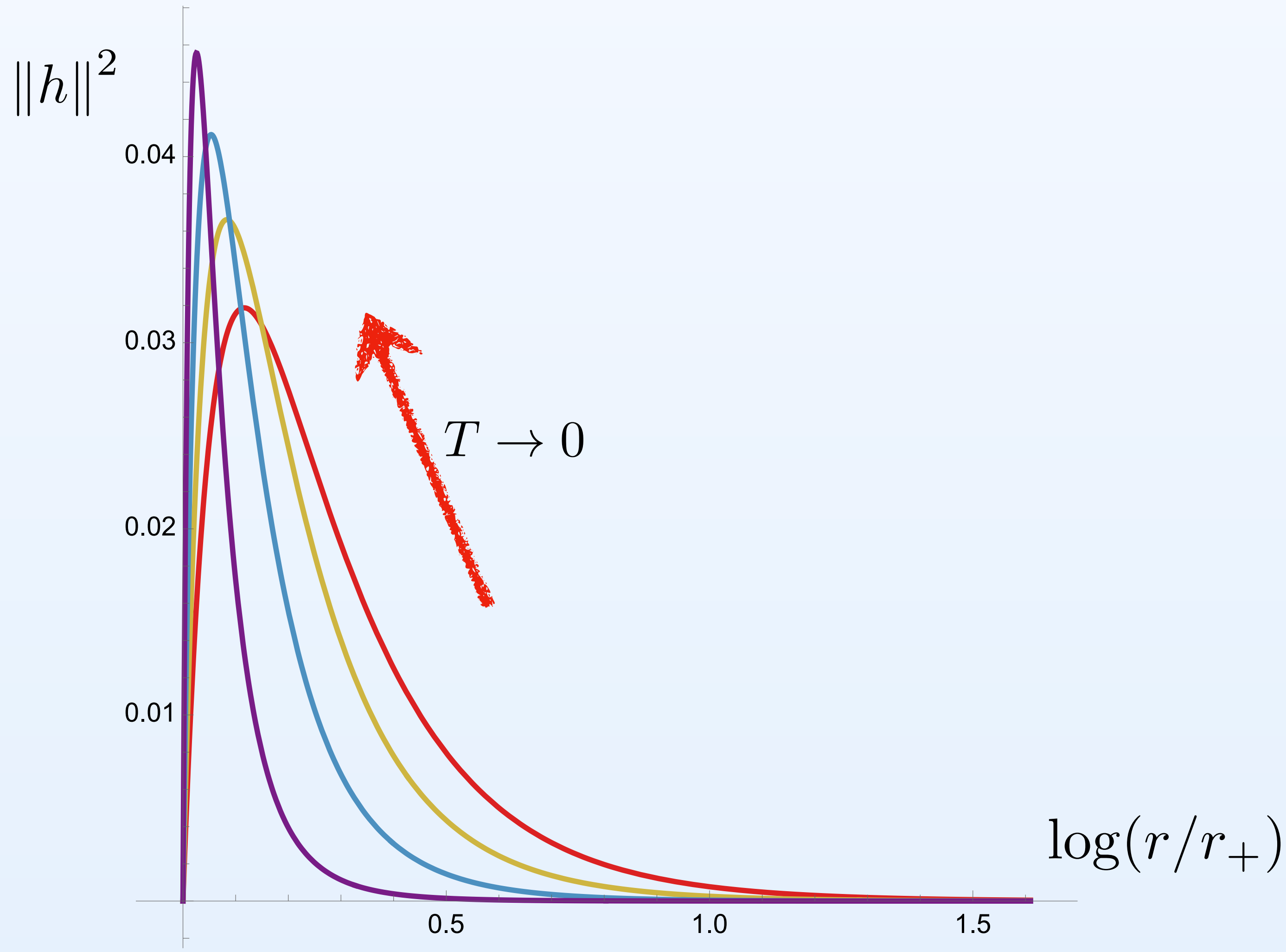
- In the near-horizon of the extremal BTZ geometry, which is AdS_2 fibered over a circle, one has set of Schwarzian zero modes, associated with the AdS_2 factor.
- These modes are in the kernel of the spin-2 Lichnerowicz operator. We wish to know what they correspond to in the full spacetime.
- The eigenvalue problem turns out to be easy to solve analytically, on the space of transverse traceless spin-2 fluctuations $\nabla^\mu h_{\mu\nu} = h^\mu{}_\mu = 0$

$$(\Delta_L h)_{\mu\nu} = \underbrace{\frac{|n|\,T_L}{T_L+T_R}\Big(1-|n|\frac{T_L}{T_L+T_R}\Big)}_{=\lambda_n} h_{\mu\nu} \qquad\qquad n \in \mathbb{Z}\backslash\{0,\pm 1\}$$

- The temporal dependence of the eigenmodes is fixed in terms of the Matsubara frequencies (parameterized by *n*). $\omega_n = \frac{2\pi\, n}{\beta}$
- They have some radial profile in terms of elementary functions and the rotational Killing field of the background remains an isometry of the perturbed (off-shell) geometry.

Datta, David '11 Castro, Keeler, Szepietowski '17

FEATURES OF THE EIGENMODES



* The norm of these modes gets concentrates inside the throat as we approach extremality.

* The actual wavefunction profiles beautifully match with those of the Schwarzian modes down the throat.

* The constant mode in time, and modes with one unit of Matsubara frequency are not normalizable, and hence excluded (thus, constrains allowed eigenmodes).

IMPLICATIONS

- ✳ We verify the existence of a single discretuum of modes at low temperature

$$(\Delta_L h)_{\mu\nu} = \lambda_n \, h_{\mu\nu} \, , \qquad \lambda_n \sim \alpha \, |n| \, T \, , \qquad n \in \mathbb{Z} \backslash \{0, \pm 1\}$$

- ✳ These modes give a power-law contribution to the one-loop determinant which become important at the gap scale

$$\log Z^{1-\text{loop}}_{\text{graviton}} \supset -\frac{1}{2} \sum_{n \neq 0, \pm 1} \log \left(\frac{\lambda_n}{16\pi \, G_N} \right) \quad = \frac{3}{2} \log \left(\frac{T}{T_q} \right) + \cdots \, . \qquad\qquad T_q = \frac{24}{c}$$

- ✳ This is the only family of eigenfunctions of the spin-2 Lichnerowicz operator with low eigenvalues near-extremality. In particular, the geometry does not appear to entertain modes associated with the rotational Killing field.

- ✳ This result is consistent with the boundary CFT calculation in the grand canonical ensemble. Assuming a twist gap, the vacuum Virasoro block precisely predicts only the Schwarzian contribution.

Ghosh, Maxfield, Turiaci, '19

Pal, Qiao '23

$$\log Z^{\text{1-loop}}_{\text{graviton}} = \log \left[\prod_{n=2}^{\infty} \frac{1}{(1 - e^{-4\pi^2 T_L n})(1 - e^{-4\pi^2 T_R n})} \right]$$

$$\sim \underbrace{\frac{1}{24 \, T_L}}_{\text{Shift of extremal energy}} + \underbrace{\frac{3}{2} \log T_L}_{\text{Schwarzian mode}}$$

Exhibit 2: The Reissner-Nordstrom AdS geometry

REISSNER-NORDSTROM ADS BLACK HOLES

- Consider 4d Einstein-Maxwell theory with a negative cosmological constant

$$I = -\frac{1}{16\pi\, G_N} \int \mathrm{d}^4x\, \sqrt{g} \left(R + \frac{6}{\ell_{\mathrm{AdS}}^2} - F_{\mu\nu} F^{\mu\nu} \right) - \frac{1}{8\pi\, G_N} \oint \mathrm{d}^3x\, \sqrt{\gamma}\, K + I_{\mathrm{bdy}}$$

- The electrically charged RN-AdS_4 black holes are spherically symmetric solutions with charge chemical potential. For simplicity we assume the absence of charged matter (to suppress superradiance).

$$\mathrm{d}s^2 = f(r)\, \mathrm{d}t_{\mathrm{E}}^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\, \mathrm{d}\Omega_2^2\,, \qquad f(r) = \frac{r^2}{\ell_{\mathrm{AdS}}^2} + 1 - \frac{2m}{r} + \frac{Q^2}{r^2} \qquad A = \mathrm{i} \left(\frac{Q}{r} - \frac{Q}{r_+} \right) \mathrm{d}t_{\mathrm{E}}$$

- Near-horizon analysis reveals three families of zero modes in the extremal solution:
  - The Schwarzian modes from gravitons (associated to large diffeos in AdS_2)
  - The U(1) gauge modes (associated to large gauge transformations in AdS_2)
  - An SO(3) family of modes associated with rotations on the transverse sphere.

THE QUADRATIC FLUCTATION ANALYSIS

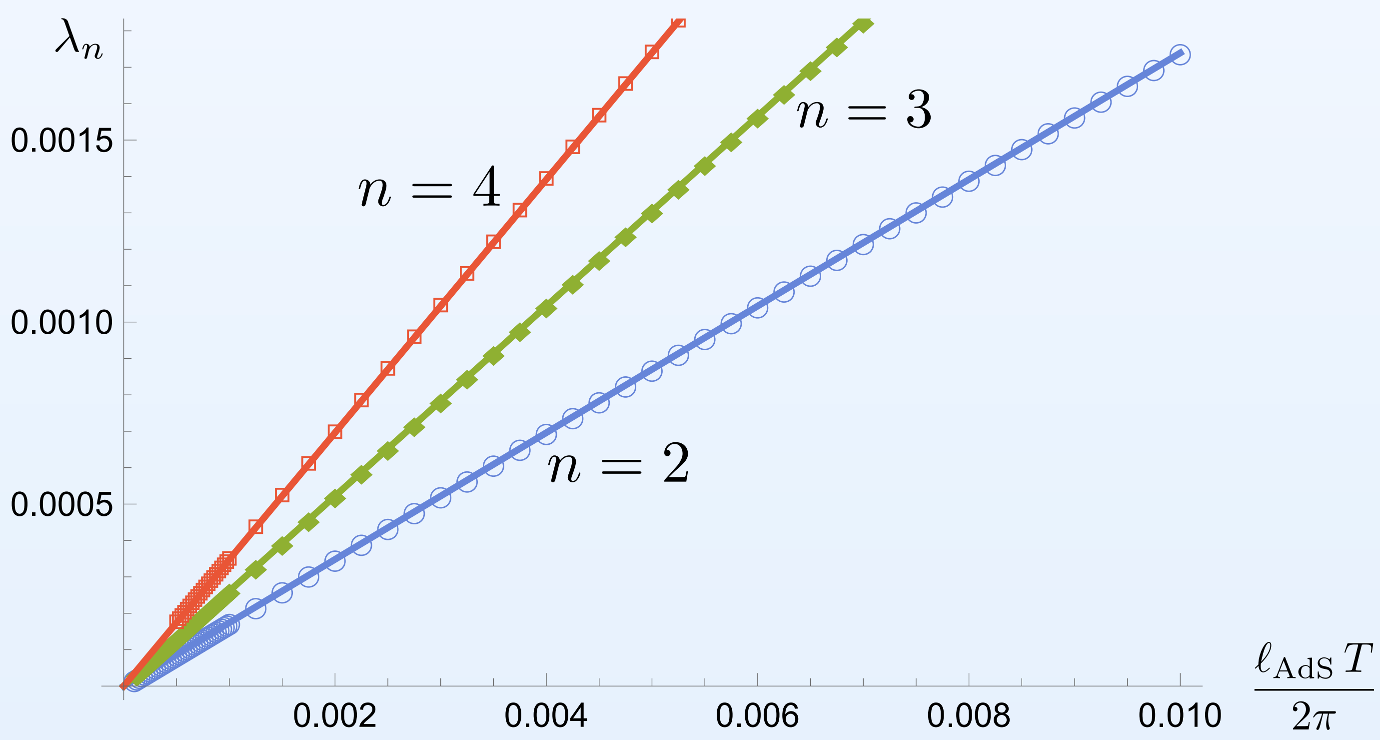
- The analysis of the quadratic fluctuations around the RN-AdS$_4$ black holes requires some groundwork, because
  - gravitons and photons are coupled
  - the Euclidean action is not sign definite.
- Latter is dealt with in Einstein-Hilbert theory by integrating the conformal mode along an imaginary direction. In Einstein-Maxwell this is no longer effective and has not been addressed hitherto.
  - Our proposal is to integrate the Maxwell field along the imaginary direction.
- We need to also ensure fluctuations are physical. Else one has to independently compute the eigenspectrum of the ghost operators.
- To address these issues, we work with a physical gauge choice Marolf, Santos '22 (+ Liu '23)

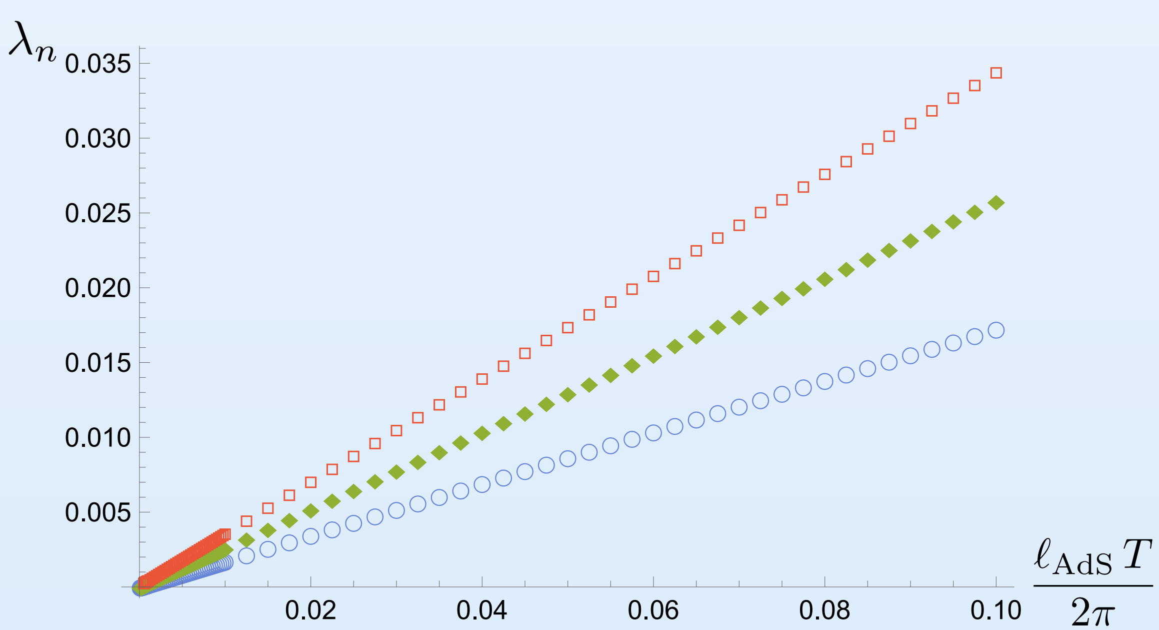
$$\nabla^\mu \tilde{h}_{\mu\nu} - \frac{1}{2} F_{\nu\mu}\, a^\mu + A_\nu\, \frac{1}{2} \nabla^\mu a_\mu = 0 \qquad \nabla^\mu a_\mu = 0\,.$$

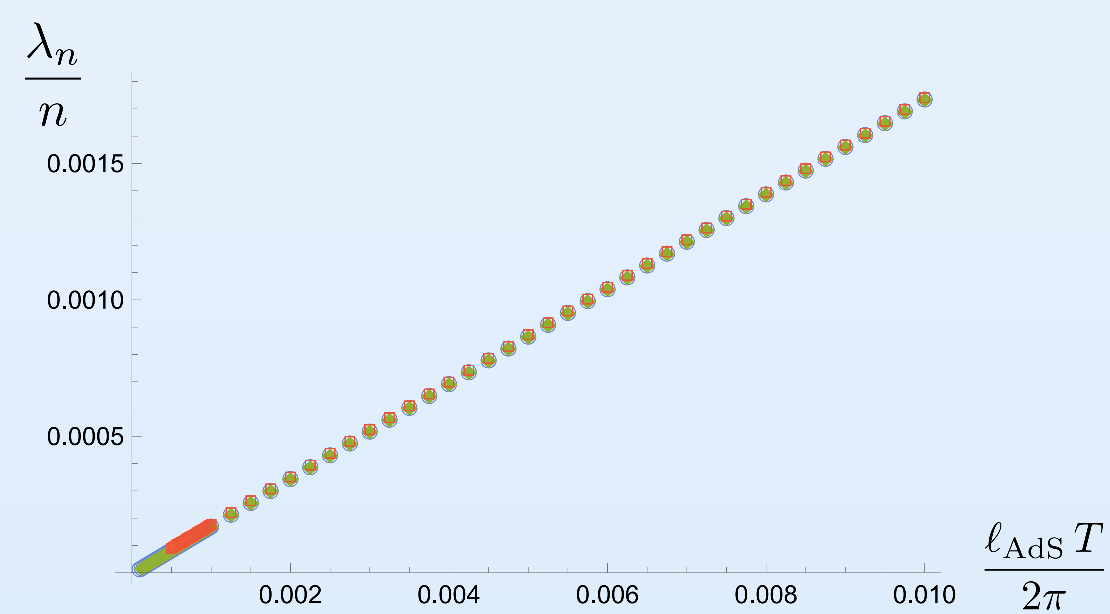
- Working in symmetry sectors, we obtain self-adjoint quadratic fluctuations operators which we diagonalize.

RESULTS 1: SCHWARZIAN MODES

$$\mathfrak{D}X = \lambda_n X\,, \qquad \lambda_n \sim \alpha\,|n|\,T\,, \qquad n \in \mathbb{Z}\backslash\{0, \pm 1\}$$

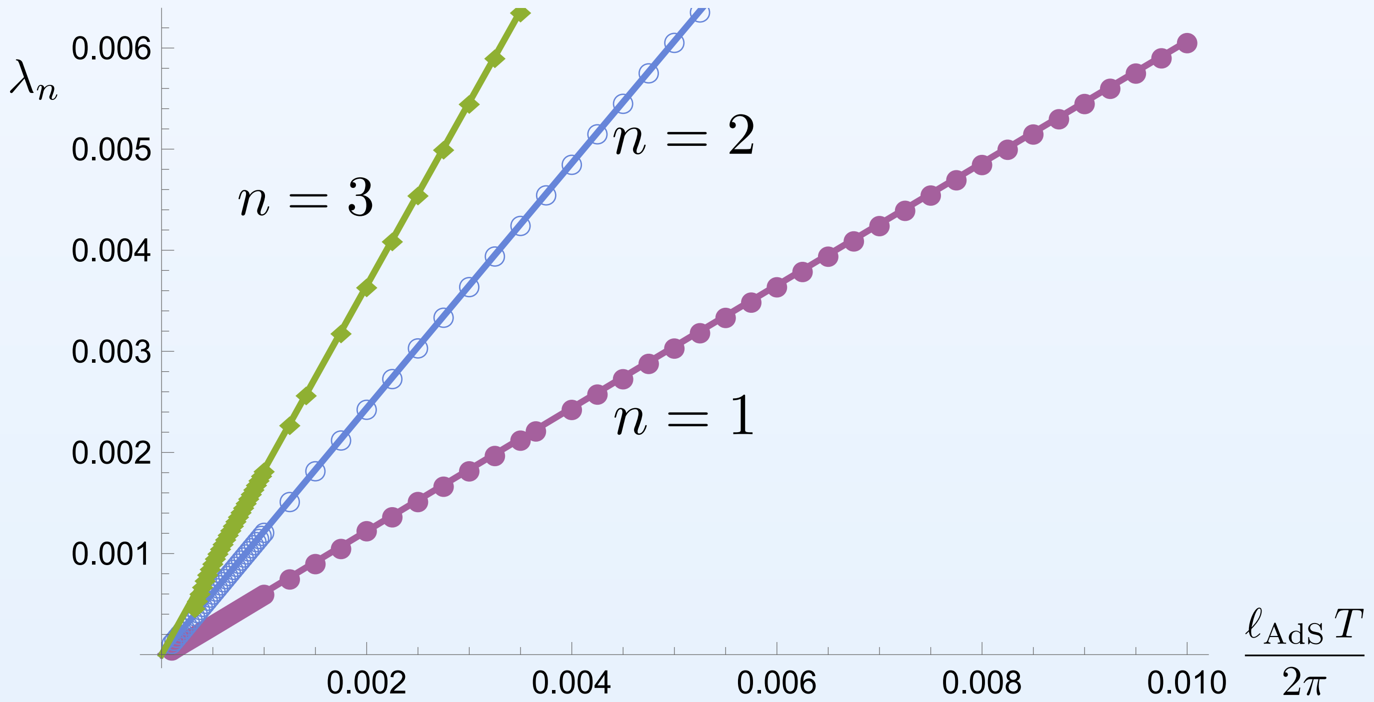


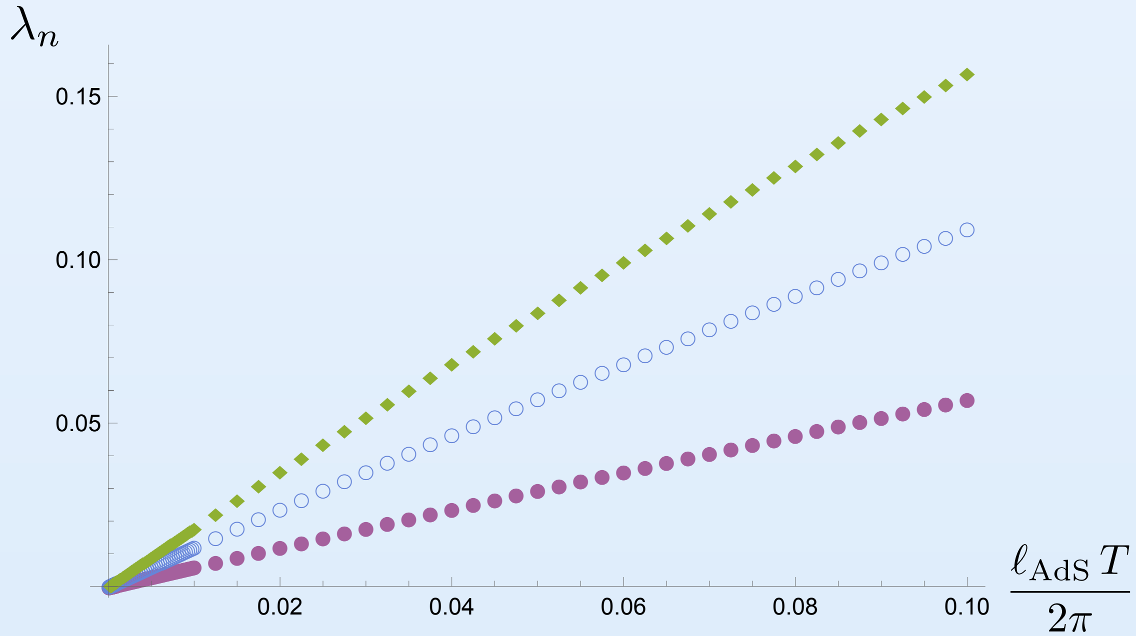


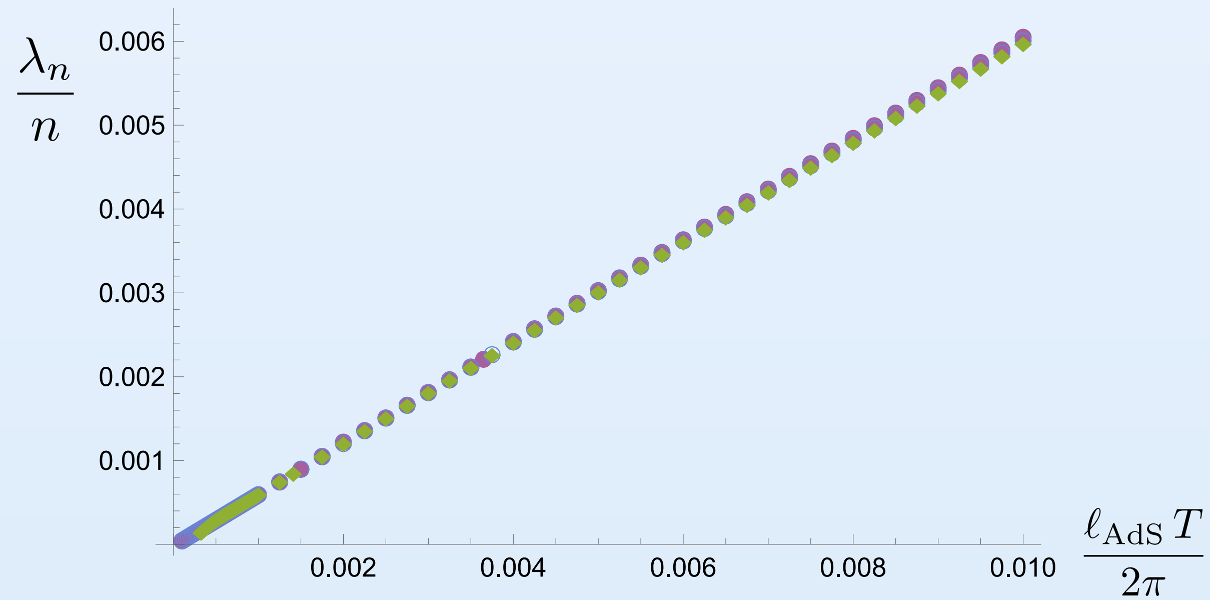


RESULTS 2: U(1) GAUGE MODES

$$\mathfrak{D}X = \lambda_n X\,, \qquad \lambda_n \sim \alpha\,|n|\,T\,, \qquad n \in \mathbb{Z}\backslash\{0\}$$

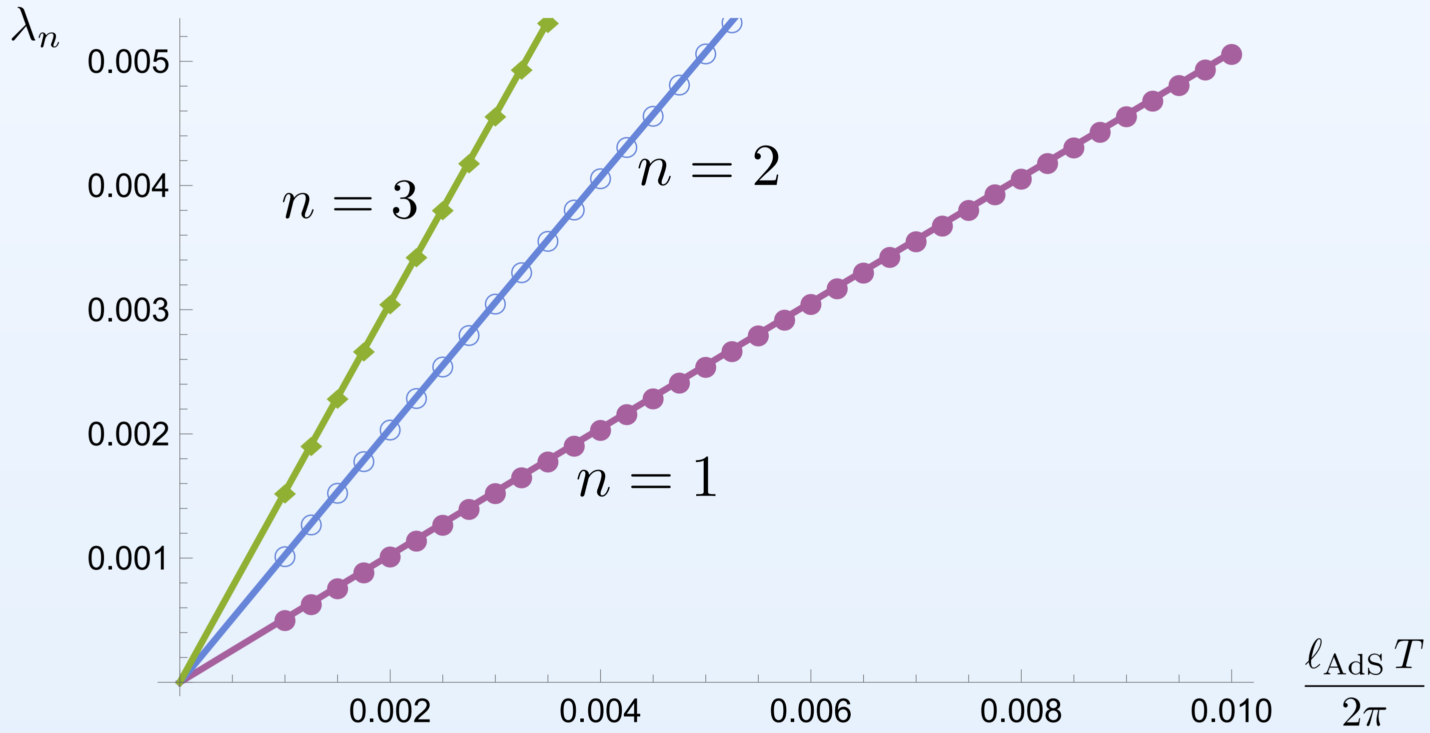


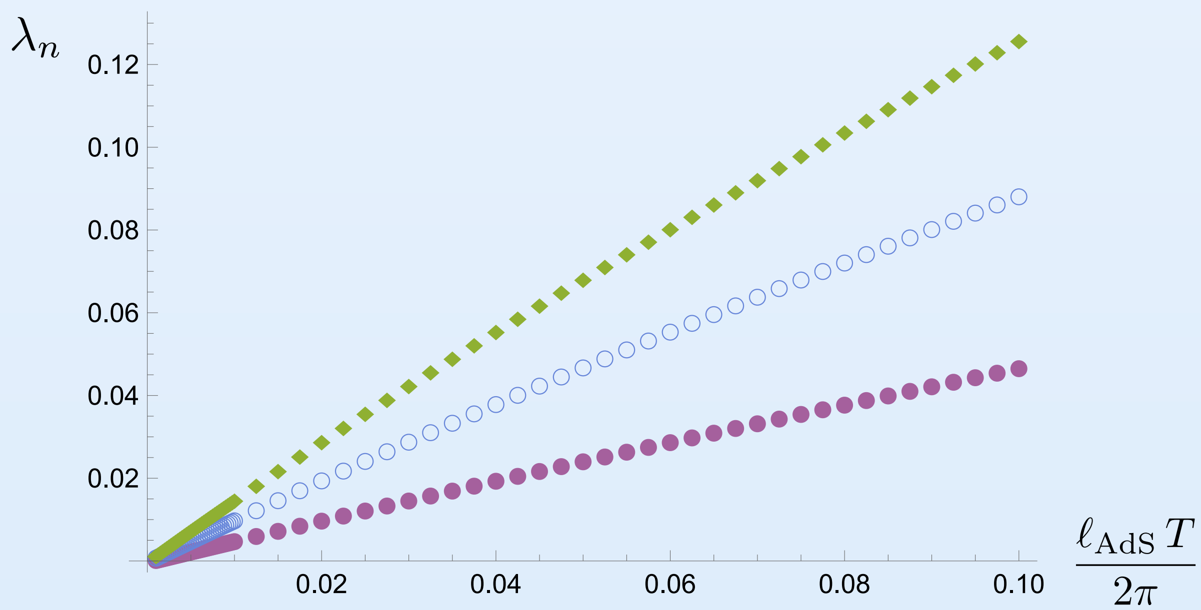


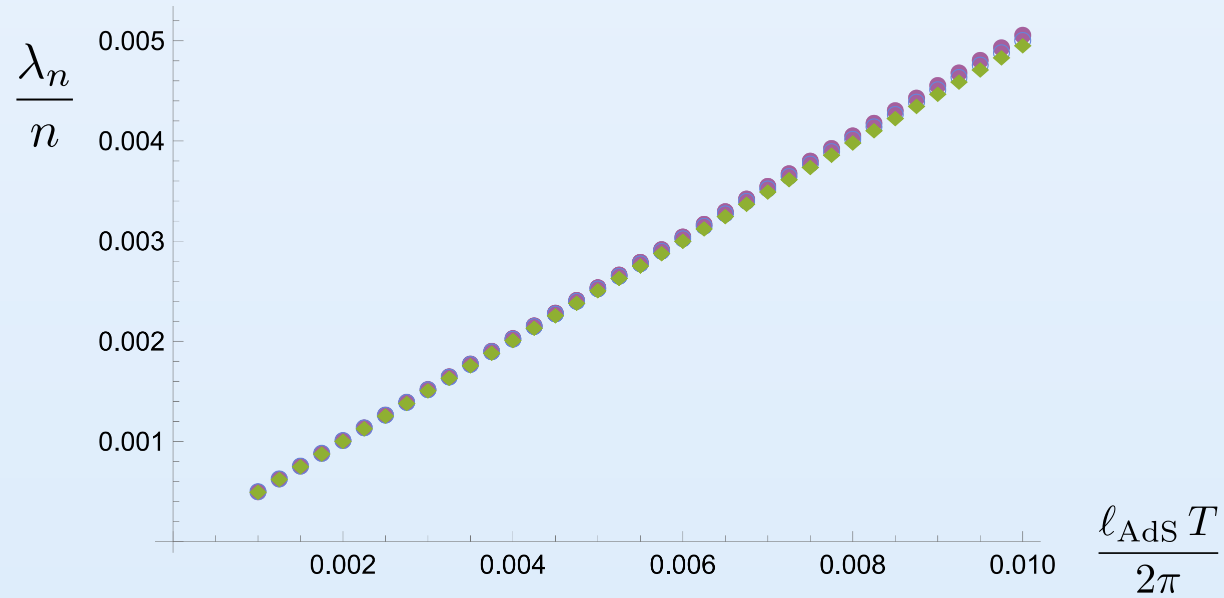


ROTATIONAL ZERO MODES (L=1, M=0)

$$\mathfrak{D}X = \lambda_n X\,, \qquad \lambda_n \sim \alpha\,|n|\,T\,, \qquad n \in \mathbb{Z}\backslash\{0\}$$

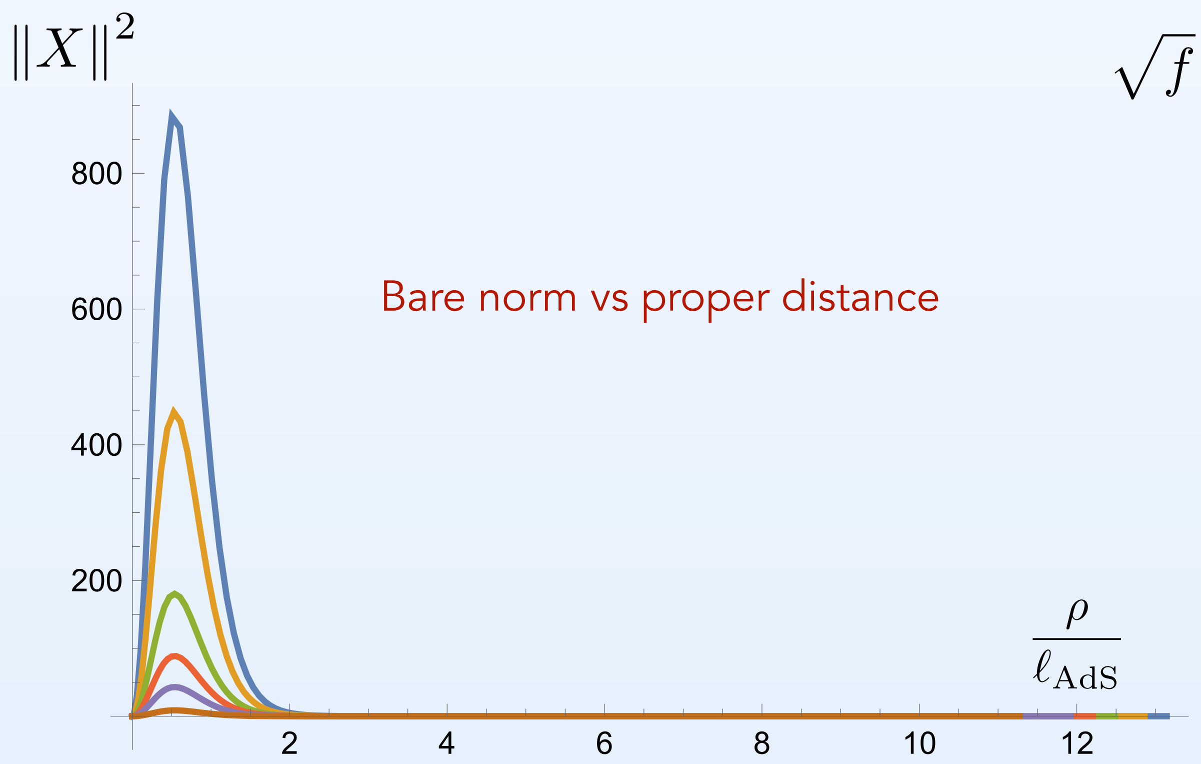


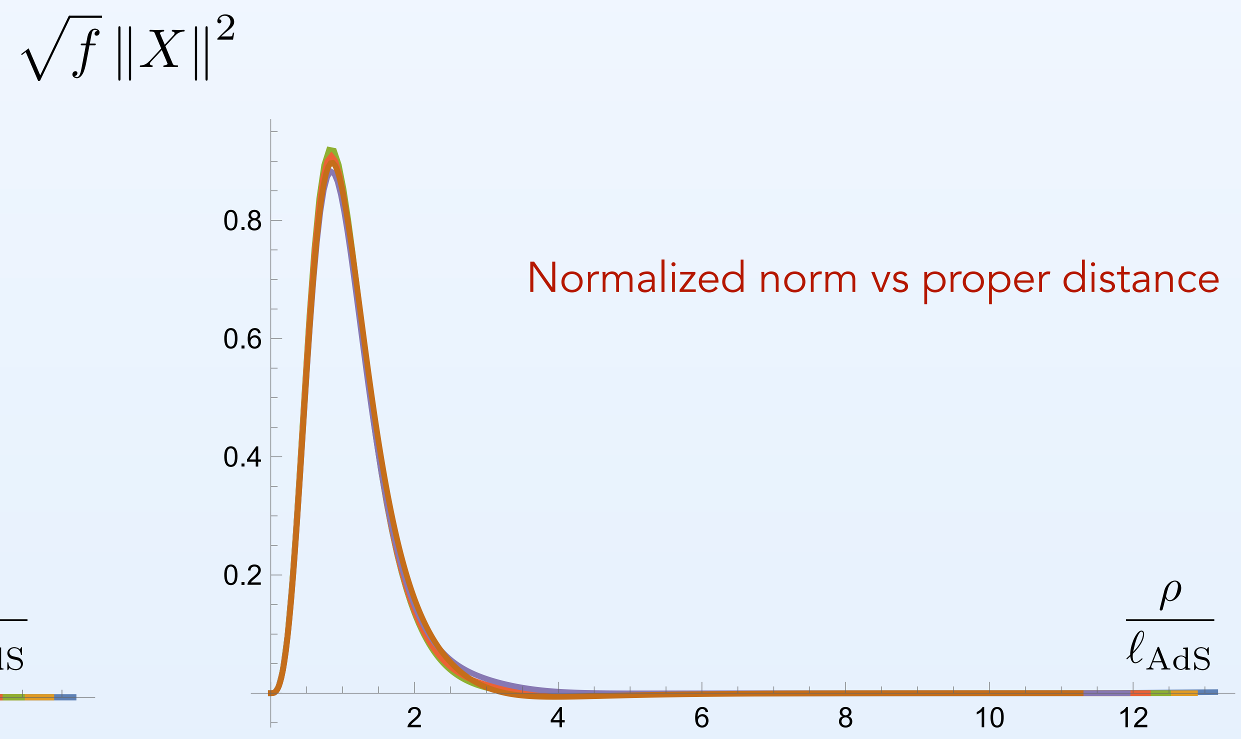




LOCALIZATION NEAR THE HORIZON

$$\mathfrak{D} X = \lambda_n X\,, \qquad \lambda_n \sim \alpha\,|n|\,T\,, \qquad n \in \mathbb{Z}\backslash\{0\}$$





- Examining the norm defined on the space of fluctuations, we can see that the fluctuations are nicely concentrated in the near-horizon region.
- The plots above are for the rotational modes, but similar features can be seen in the other sectors.

IMPLICATIONS OF THE LIGHT MODES

- The existence of these light off-shell modes implies that the 1-loop determinant around RN-AdS$_4$ has a low-temperature contribution which scales as

$$\log Z_{\text{1-loop}} \supset \underbrace{\frac{3}{2}\log\frac{T}{T_q}}_{\text{Schwarzian}} + \underbrace{\frac{3}{2}\log\frac{T}{T_q}}_{\text{Rotation}} + \underbrace{\frac{1}{2}\log\frac{T}{T_q}}_{\text{gauge U(1)}}$$

- This is the expected behaviour about a single saddle. To obtain the answer in the grand canonical ensemble, we should sum over suitable integral shifts of the gauge and rotational chemical potentials

$$\beta\mu \mapsto \beta\mu + 2\pi i\, n\,, \qquad \beta\Omega \mapsto \beta\Omega + 2\pi i\, m\,,$$

- The result in the grand canonical ensemble gets contribution from all the saddles, each of which will have a different saddle point answer, but a similar temperature scaling.

- After Legendre transforming to the canonical ensemble, the near-horizon analysis suggests only a contribution from the Schwarzian zero modes.

Rotation zero mode peculiarities

ROTATIONAL ZERO MODES OF ROTATING BLACK HOLES

* The analysis in the near-horizon geometry of extreme Kerr black hole, where one expects a single rotational zero mode, revealed a puzzle. The putative mode was was not smooth in harmonic gauge.

Rakic, MR, Turiaci '23

* A similar analysis in Kerr AdS_4 or the Kerr-Newman solution reveals the expected zero mode within harmonic gauge (and Lorenz gauge for the Maxwell field).

* The issue in the case of Kerr is breakdown of harmonic gauge which is defined in terms of the functional

$$\mathsf{GF}_\mu = \nabla^\nu \left(h_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, h \right)$$

* A zero mode exists if it is generated by a large diffeomorphism which satisfies the gauge condition.

$$h_{\mu\nu} = 2\,\nabla_{(\mu}\xi_{\nu)}$$

* This is the case for the Schwarzian mode, but not so for the rotational zero mode in the aforementioned geometries, which is to be generated using

$$\xi = H(\tau, y)\,\partial_\phi \qquad\qquad H(\tau, y) = e^{i\,n\,\tau} \left(\frac{y-1}{y+1} \right)^{\frac{|n|}{2}}$$

ROTATIONAL ZERO MODES OF ROTATING BLACK HOLES

- A perturbation that doesn't satisfy the gauge condition of choice can be fixed up by a compensating diffeo, which has to satisfy an inhomogenous elliptic equation

$$h_{\mu\nu} \mapsto h_{\mu\nu} + \nabla_\mu \zeta_\nu + \nabla_\nu \zeta_\mu \qquad \left(\delta^\mu{}_\nu \nabla^2 + R_\nu{}^\mu\right) \zeta_\mu = -\mathsf{GF}_\nu$$

- Failure of the gauge condition is equivalent to the elliptic operator appearing in the lhs having a non-zero kernel.
- For Ricci flat backgrounds like Kerr this is the case; one can construct a 1-form from the well-known eigenmodes of the scalar Laplacian on AdS_2.
- Equivalently, one can also see that that the ghost kinetic operator in harmonic gauge has a zero mode.
  - We believe (but have not yet checked) that there will be similar issues with harmonic gauge in the full near-extremal Kerr geometry.
- This issue persists for all Ricci flat rotating black holes, but does not infect solutions like BMPV, where one can indeed find the zero modes in question while remaining in harmonic gauge.

Ensemble choices

COMMENTS ON ENSEMBLES

Near-horizon with SL(2,R) symmetry	*Full spacetime geometry*
✴ AdS_2 asymptotics requires that we fix the physical charges and not the associated holonomies. E.g., $A \sim (\mu_{\rm nor} + Q_{\rm nn}\, r)\, dt$ Sen '08-'09	✴ In asymptotically flat or AdS_d with d>4 the physical boundary conditions involve fixing holonomies, and not the charges,.g., $A \sim \left(\mu_{\rm non} + \frac{Q_{\rm nor}}{r^{d-3}}\right) dt$ Marolf, Ross '06
✴ The zero modes only exist in the canonical ensemble: fixed thermal period and charges.	✴ The light off-shell modes described herein exist for fixed holonomies.

✴ The Schwarzian mode is universal, and gives a $T^{\frac{3}{2}}$ contribution to the 1-loop det.

✴ The 1-loop contribution from the gauge and rotation modes is computed in the regulated near-horizon geometry by passing to the fixed holonomy ensemble, summing over shifts, and then Legendre transforming back. A direct computation would be, of course, much more satisfactory.

✴ The upshot is that their contribution in the fixed charge ensemble is T independent.

The curious case of the rotational mode in BTZ

ROTATIONAL ZERO MODES IN BTZ

- Let us return to the extremal BTZ solution, whose near horizon, as mentioned is a fibration of AdS_2 over a circle

$$\mathrm{d}s^2 = \left(y^2-1\right)\mathrm{d}\tau^2 + \frac{\mathrm{d}y^2}{y^2-1} + \left(\mathrm{d}\phi + \mathrm{i}(y-1)\mathrm{d}\tau\right)^2$$

- In this geometry, there is in addition to the Schwarzian modes, a zero mode associated with the angular isometry (contrary to an earlier claim in Rakic, MR, Turiaci '23)

$$h_{\mu\nu} = 2\,\nabla_{(\mu}\xi_{\nu)}$$

$$\xi = H(\tau,y)\partial_\phi + \nabla^a H\partial_a\,, \qquad a\in\{\tau,r\}$$

$$H(\tau,y) = e^{i\,n\,\tau}\left(\frac{y-1}{y+1}\right)^{\frac{|n|}{2}}$$

- This is extremely curious for we now appear to have one too many zero modes.
  - First of all, these modes do not appear to extend into the full spacetime.
  - Second, and more importantly, in this case, we can use holography to predict the 1-loop determinant in both the canonical and grand canonical ensembles, and learn that it scales as $T^{\frac{3}{2}}$.

LOW TEMPERATURE, HIGH SPIN UNIVERSALITY

* Asymptotic high spin density of states in a 2d CFT has a nice universal limit that can be recognized as the Schwarzian contribution.

Ghosh, Maxfield, Turiaci, '19

* Consider a 2d CFT with Virasoro symmetry (no conserved currents), with a modular invariant partition function that has a character decomposition:

$$Z_{\rm CFT}(\tau,\bar\tau) = \chi_{\rm vac}(\tau)\,\overline{\chi}_{\rm vac}(\bar\tau) + \sum_{h,\bar h} \chi_h(\tau)\,\overline{\chi}_{\bar h}(\bar\tau)$$

$$\chi_{\rm vac}(\tau) = q^{-\frac{c-1}{24}}\,\frac{1-q}{\eta(\tau)}\,, \qquad \chi_h(\tau) = q^{h-\frac{c-1}{24}}\,\frac{1}{\eta(\tau)}\,, \qquad q = e^{2\pi i \tau}$$

* The limit of interest is low temperatures and fixed angular momentum:

$$T \sim \mathcal{O}\big(c^{-1}\big)\,, \qquad J \sim \mathcal{O}\big(c^{3}\big)\,, \qquad c \gg 1$$

$$Z(T,J) \sim \left(\frac{\pi\, c}{12}\, T\right)^{\frac{3}{2}} J^{-\frac{3}{4}}\, \exp\left[2\pi\sqrt{\frac{c}{6}}\, J - \beta\left(J - \frac{1}{12}\right) + \frac{\pi\, c}{12}\, T\right]$$

* The temperature dependence is the same in the grand canonical ensemble.

THE WEIGHT OF REGULATED ZERO MODES

- We have thus far assumed that turning on a small temperature regulator lifts the near-horizon zero modes, with action scaling as T/T_q.
- The coupling, or equivalently the thermal scale at which these modes are activated, is set by the susceptibility

Schwarzian scale set by specific heat
$$T_q = 4\pi^2 \frac{1}{\frac{\partial S}{\partial T}\Big|_{\mathrm{Q}}}$$

Gauge field: charge susceptibility
$$T_q^{\mathrm{U}(1)} = \frac{1}{K}, \qquad K = \frac{\partial Q}{\partial \mu}\Big|_{T=0}.$$

Rotation: angular momentum susceptibility
$$T_q^{\mathrm{rot}} = \frac{1}{K}, \quad K = \frac{\partial J}{\partial \Omega}\Big|_{T=0}$$

- A mode will contribute only if the scale T_q is non-vanishing, should it vanish then the mode remains strongly coupled.

A RESOLUTION AND ITS LESSONS

* An extremal BTZ black hole is rotating at the speed of light, and its angular velocity is fixed to be unity in AdS units (it approaches the superradiant limit from below).
* The angular momentum susceptibility blows up, and suggests that the rotational zero mode of the near-horizon geometry is non-normalizable.
* The end result is that there is only the Schwarzian zero mode, both in the near-horizon and in the full geometry. The former computes the grand canonical 1-loop determinant, while the latter computes the canonical 1-loop determinant. Both give a contribution of $T^{3/2}$, which is consistent with the holographic result.
* Discarding the rotational zero mode also gives the correct $\log S_0$ contribution.
* Implication: the asymptotically flat Reissner-Nordstrom geometry has a divergent charge susceptibility, suggesting that the gauge zero mode does not contribute to the 1-loop determinant.
* We are attempting to check this from our numerical analysis, by taking the flat space limit. It seems plausible that the gauge zero mode does not extend into the full geometry.

Summary & Open Questions

SUMMARY & OPEN QUESTIONS

- The near-horizon zero mode of extremal black holes uplift to light off-shell modes in a near-extremal geometry. Verified in several examples: BTZ, RN-AdS, hyperbolic AdS black holes.
- The computation in the full geometry clarifies some issues for rotational and gauge zero modes.
- The results obtained from the semiclassical gravity path integral can be shown told hold in semiclassical string theory, i.e., the result is robust to finite string length corrections.

  Ferko, Murthy, MR ’24
- Our analysis here involved off-shell modes of the (complex) Euclidean black hole saddle.
- The one-loop determinant around black holes has been argued to be given by a beautiful formula in terms of on-shell quasinormal modes.

  Denef, Hartnoll, Sachdev ‘09
- How does the near-extremal result arise from the quasinormal modes?

  wip w/ Hewei F Jia

  Jia, MR ’24

  M Kolanowski

Thank You!