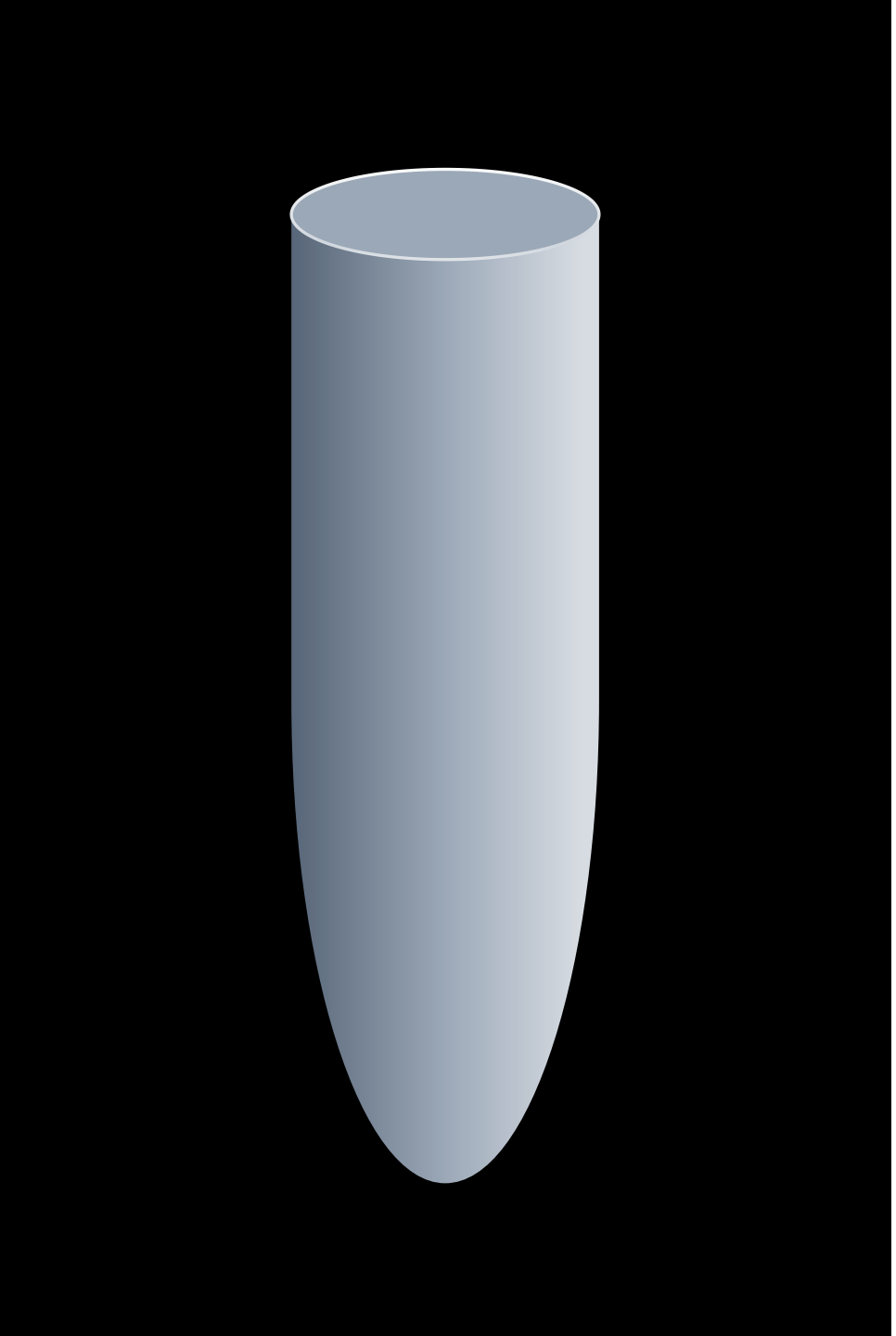


Lifting the Index

ROBERTO EMPARAN ICREA+ICCUB

NEAR-EXTREMAL BLACK HOLES @ SOLVAY/ULB

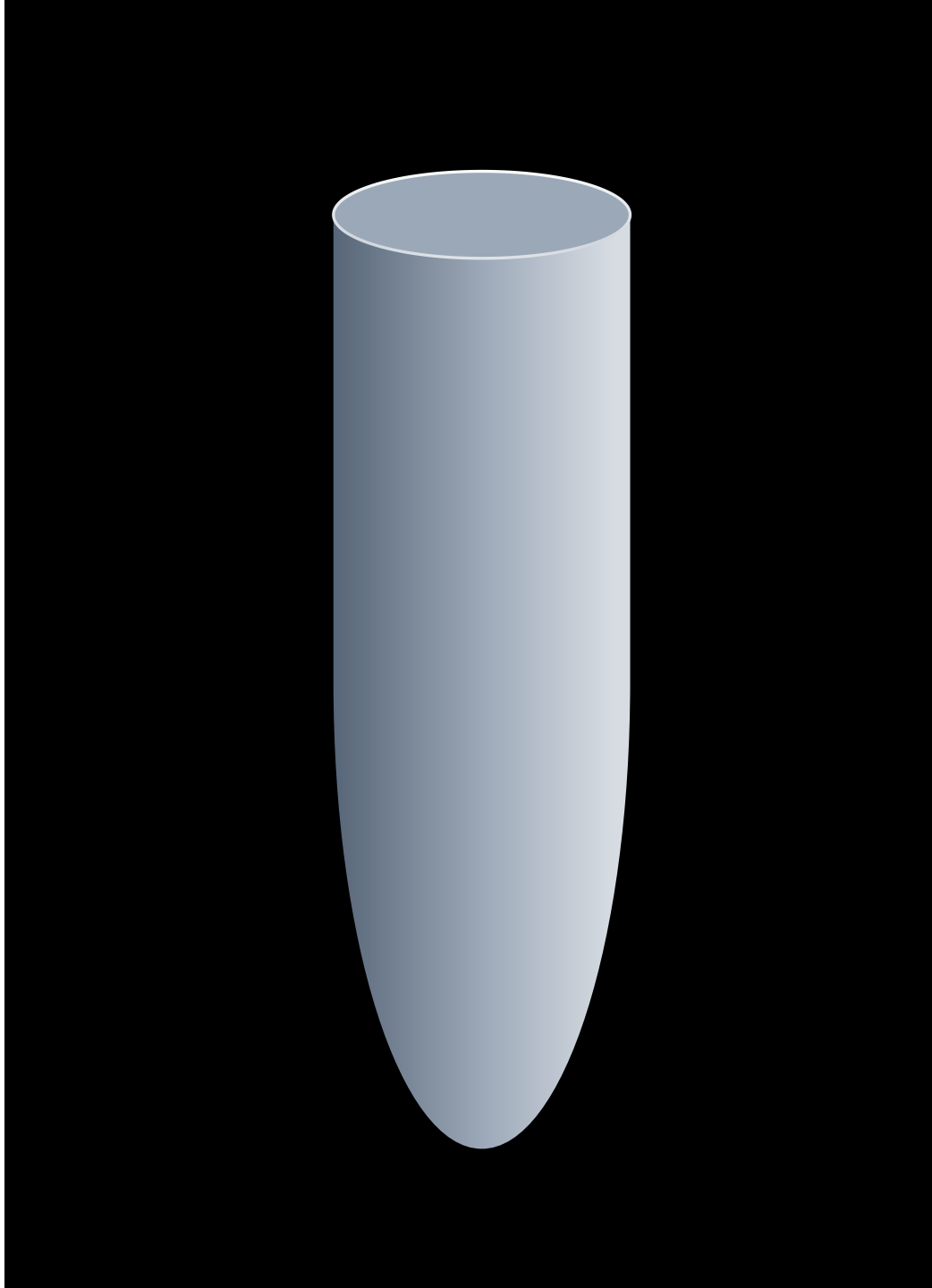
4 SEP 2024

WITH JAN BORUCH

LUCA ILIESIU

SAMEER MURTHY

TO APPEAR



Counting states

A basic problem in any quantum system

Counting with different weights:

$$\mathrm{Tr}\, e^{-\beta H}\ ,\ \mathrm{Tr}\, e^{-\beta(H-\mu Q)},\ \mathrm{Tr}\,(-1)^F e^{-\beta H}\ , \ldots$$

Counting states

$Z(\beta) = \mathrm{Tr}\, e^{-\beta H}$: governs thermodynamics

$N_B - N_F = \mathrm{Tr}\, (-1)^F e^{-\beta H}$: in susy systems,

independent of β & protected

Witten

Counting states

Euclidean path integral w/ appropriate periodicity conds

One-loop trace counts states

Black holes are quantum systems

Counting states?

Black holes are quantum systems

Counting states with Euclidean path integral

Gibbons+Hawking

$$Z(\beta) = \int_{g(0)=g(\beta)} \mathcal{D}g\, e^{-I_E[g]}$$

Trace without trace

$$Z(\beta) = e^{-\beta F} \approx e^{-I_E[g_{cl}]}$$

$$\rightarrow S = \frac{A_H}{4G}$$

From classical saddle point: no one-loop trace

More traces without trace?

$$\text{Index} = \text{Tr}\,(-1)^F e^{-\beta H}$$

Of interest for susy (BPS) black holes

Protected: must match weak-coupling counting

More traces without trace?

$$\text{Index} = \text{Tr}\,(-1)^F e^{-\beta H}$$

Can it be computed with the GPI?

Can it reveal properties of holography, or dual quantum theory?

Gravitational Index

FINITE-T SUPERSYMMETRY & MULTI-CENTER SOLUTIONS

Charged BPS Black Holes

$$\mathrm{Index}(\beta, Q) = \mathrm{Tr}_Q (-1)^F e^{-\beta H}$$

$$= (N_B - N_F) e^{-\beta M_{\mathrm{BPS}}}$$

Index: finite T & periodic fermions

Euclidean Reissner-Nordström?

- Finite T & smooth horizon → contractible τ-circle,

  periodic fermions are singular

- Extremal RN: finite T → horizon cusp & wrong index

Hawking+Horowitz+Ross

$$\mathrm{Tr}_Q(-1)^F e^{-\beta H}$$

$$(-1)^F = e^{2\pi i J} = e^{\beta\Omega J}$$

$$\text{w/ } \beta\Omega = 2\pi i$$

$$\mathrm{Tr}_Q(-1)^F e^{-\beta H}$$

$$(-1)^F = e^{2\pi i J} = e^{\beta\Omega J}$$

w/ $\beta\Omega = 2\pi i$

$$= \mathrm{Tr}_Q e^{-\beta(H-\Omega J)}\Big|_{\Omega=2\pi i/\beta}$$

$$= Z_{grav}\left(\beta, \Omega = \frac{2\pi i}{\beta}, Q\right) \approx e^{-I_E\left(\beta, \frac{2\pi i}{\beta}, Q\right)}$$

Cabo-Bizet+Cassani+Martelli+Murthy, Choi+Kim[2]+Nahmgoong, Cassani+Papini, Bobev+Crichigno, Larsen+Lee, Iliesiu+Kologlu+Turiaci, Heydemann et al, Boruch et al …

Add rotation: Kerr-Newman

BPS bound: $M = Q$

Lorentzian is singular

Euclidean is OK if $\Omega_E = -i\Omega = \frac{2\pi}{\beta}$

Hartle+Hawking, Whitt, Yuille

Euclidean Kerr-Newman $\Omega = 2\pi i/\beta$

Cycles

$(\tau, \phi) \sim (\tau + \beta, \phi + i\beta\Omega_E)$: contractible at horizon

$$\sim (\tau + \beta, \phi + 2\pi)$$

$(\tau, \phi) \sim (\tau + \beta, \phi)$: non-contractible

Correct conditions for fermions in index:

$$\Psi(\tau, \phi) = -\Psi(\tau + \beta, \phi + 2\pi) = +\Psi(\tau + \beta, \phi)$$

Euclidean Kerr-Newman $\Omega = 2\pi i/\beta$

$$I_E = \beta Q - \pi Q^2$$

$$\text{Index}(\beta, Q) = (N_B - N_F)e^{-\beta M_{\text{BPS}}} \approx e^{-I_E}$$

$$N_B - N_F \approx e^{\pi Q^2} = e^{S_{BH}(\Omega=0)}$$

Justifies comparing $S_{BH}(\text{ext})$ to weak-coupling index

Index geometry as split center

General BPS solution* in N=2 Sugra: Israel-Wilson-Perjés (IWP)

Tod

$$ds^2 = \frac{1}{V\tilde{V}}(dt + \omega)^2 + V\tilde{V}d\mathbf{x}^2$$

$V, \tilde{V}$ harmonic in $\mathbb{R}^3$ $\qquad$ $\nabla \times \omega = \tilde{V}\nabla V - V\nabla\tilde{V}$

Euclidean: imaginary electric field + magnetic dipole

imaginary rotation

* w/ timelike Killing spinor

Index geometry as split center

Single-center IWP

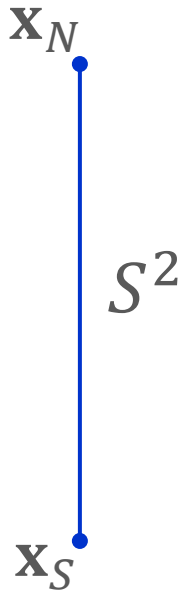
$$V = 1 + \frac{Q}{|\mathbf{x}-\mathbf{x}_0|} \qquad \tilde{V} = 1 + \frac{Q}{|\mathbf{x}-\mathbf{x}_0|} \qquad \omega = 0$$

$\rightarrow$ Extremal Reissner-Nordström

Index geometry as split center

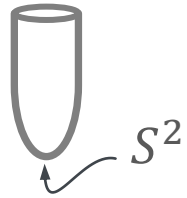
Two-center IWP

$$V = 1 + \frac{Q}{|\mathbf{x}-\mathbf{x}_N|} \qquad \tilde{V} = 1 + \frac{Q}{|\mathbf{x}-\mathbf{x}_S|}$$



North-South line in $\mathbb{R}^3$ is minimal S^2 in full space

Fixed point-set of $\partial_t + \Omega_E \partial_\phi$: bubbling



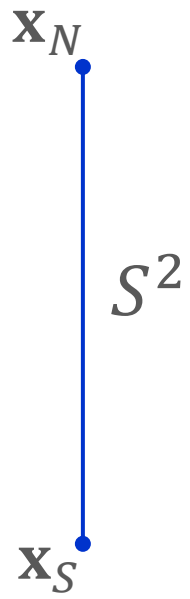
Hartle+Hawking, Whitt, Yuille, Dunajski+Hartnoll

Index geometry as split center

Boruch+Iliesiu+Murthy+Turiaci

Two-center IWP

$$V = 1 + \frac{Q}{|\mathbf{x}-\mathbf{x}_N|} \qquad \tilde{V} = 1 + \frac{Q}{|\mathbf{x}-\mathbf{x}_S|}$$



Regularity: no-conical + no Dirac strings

$\Rightarrow$ $|\mathbf{x}_N - \mathbf{x}_S|$ and Ω_E fixed in terms of β, Q

$\Rightarrow$ 2-center IWP = Euclidean Kerr-Newman index geometry

From Black hole to Index geometry

Boruch+Iliesiu+Murthy+Turiaci

Black hole: single center IWP

Index: split into two centers

- Impose regularity (no-conical + no Dirac strings)

  $\rightarrow$ solution fixed in terms of β, Q

- Dipoles generated, not independent

BPS 4D Black Holes

AND THEIR UPLIFTS TO 5D

In N=2 pure Sugra: extremal RN Black hole

In N=2 Sugra + vector supermultiplets

- Eg from type II_A in CY_3
- General solution: Shmakova

General solution (Shmakova)

$$ds^2 = -\frac{1}{\Sigma(x)}dt^2 + \Sigma(x)d\mathbf{x}^2$$

$A, B, C = 1, \ldots, n_V$

$$\Sigma(x) = \sqrt{Q^3 H^0 - (LH^0)^2}$$: entropy function

$$L = -H_0 + \frac{D_{ABC}H^A H^B H^C}{3(H^0)^2} - \frac{H^A H_A}{H^0}\,, \quad Q^{\frac{3}{2}} = \frac{1}{3}D_{ABC}y^A y^B y^C, \quad D_{ABC}y^A y^B = \frac{D_{ABC}H^A H^B}{H^0} - 2H_C$$

Simpler for $n_v = 1$

modulus charge

$(H^0, H^A, H_A, H_0) \leftrightarrow (D6, D4, D2, D0)$ Harmonic $H^0 = h^0 + \frac{p^0}{r}$ etc

$n_V = 3$: 4-charge black holes in 4D

D0-D4^3 : wrap D4 on 4-cycles intersecting over point

D2^3-D6 : wrap D2 on 2-cycles intersecting over point; D6 on CY

4 equal charges: RN bh 3+1 charges: $n_V = 1$ black holes

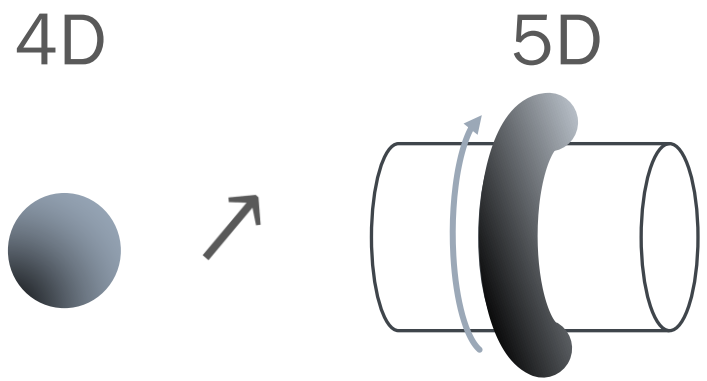
→ **Non-zero entropy BPS black holes**

Known microscopic state counting of index. Agrees with BH entropy

Uplift to 5D (11D M-theory on CY_3)

D0-$D4^3$ ↗ W-$M5^3$

intersect over line→string



5D Black string

3-charge + P

Uplift to 5D (11D M-theory on CY_3)

4D 5D

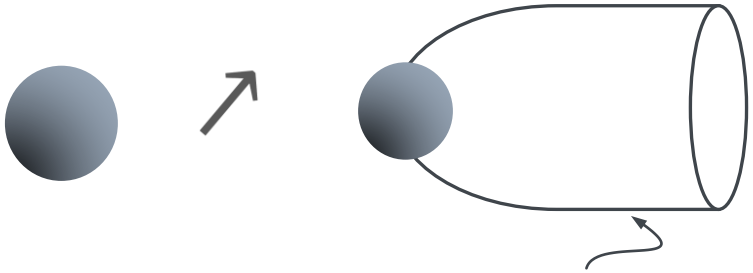
D0-D4^3 ↗ W-M5^3

intersect over line→string

5D Black string

3-charge + P

D2^3-D6 ↗ M2^3-KKm



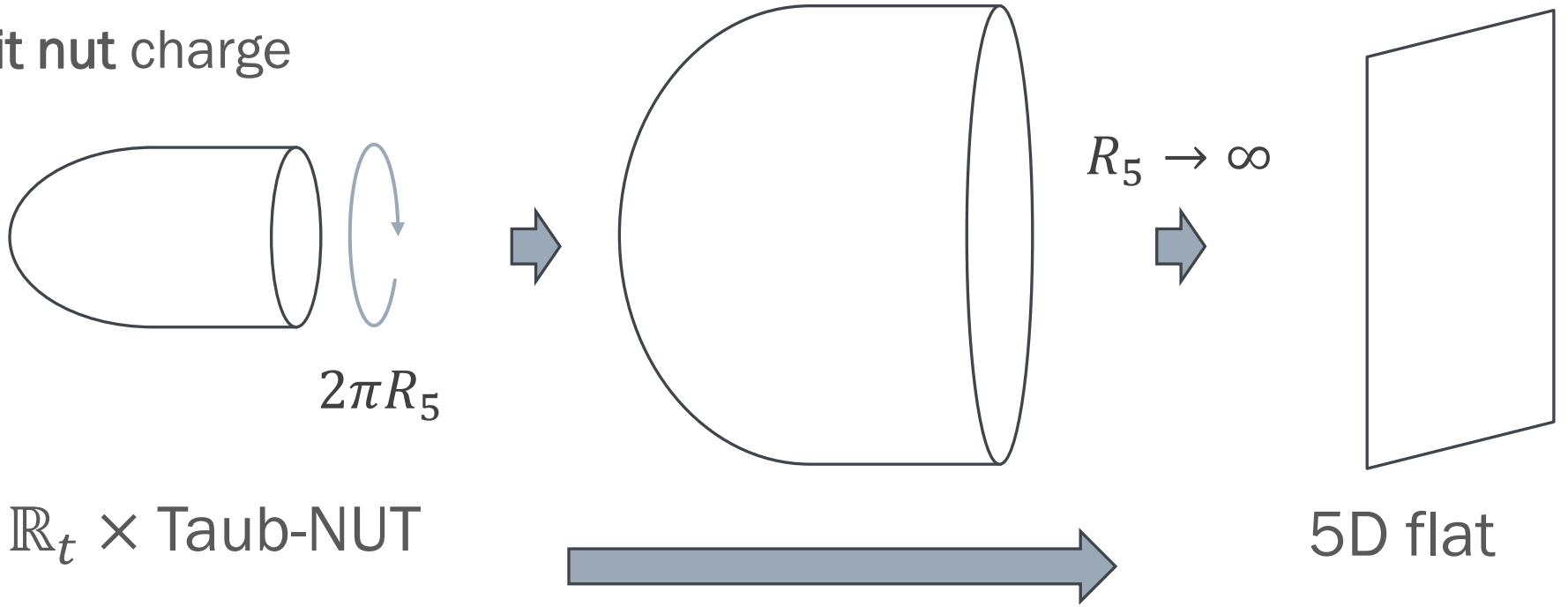
5D Black hole

3-charge + nut

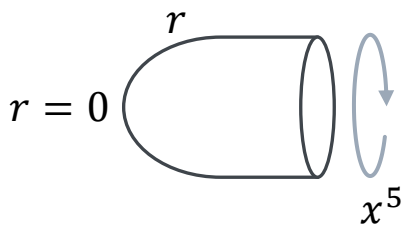
Decompactify $R_5 \to \infty$

Taub-NUT

Unit nut charge



Decompactify $R_5 \to \infty$



$R_5 \to \infty$

Euclidean Taub-NUT

$$ds^2 = -dt^2 + R_5 H^0 dr^2 + \frac{R_5}{H^0}\left(d\psi + p^0 \cos\theta \, d\phi\right)^2 + r^2 d\Omega_2$$

$$H^0 = \frac{1}{R_5} + \frac{p^0}{r} \qquad \psi \sim \psi + 4\pi p^0$$

$$p^0 = 1 \qquad r = \frac{\hat{r}^2}{4R_5} \qquad R_5 \to \infty$$

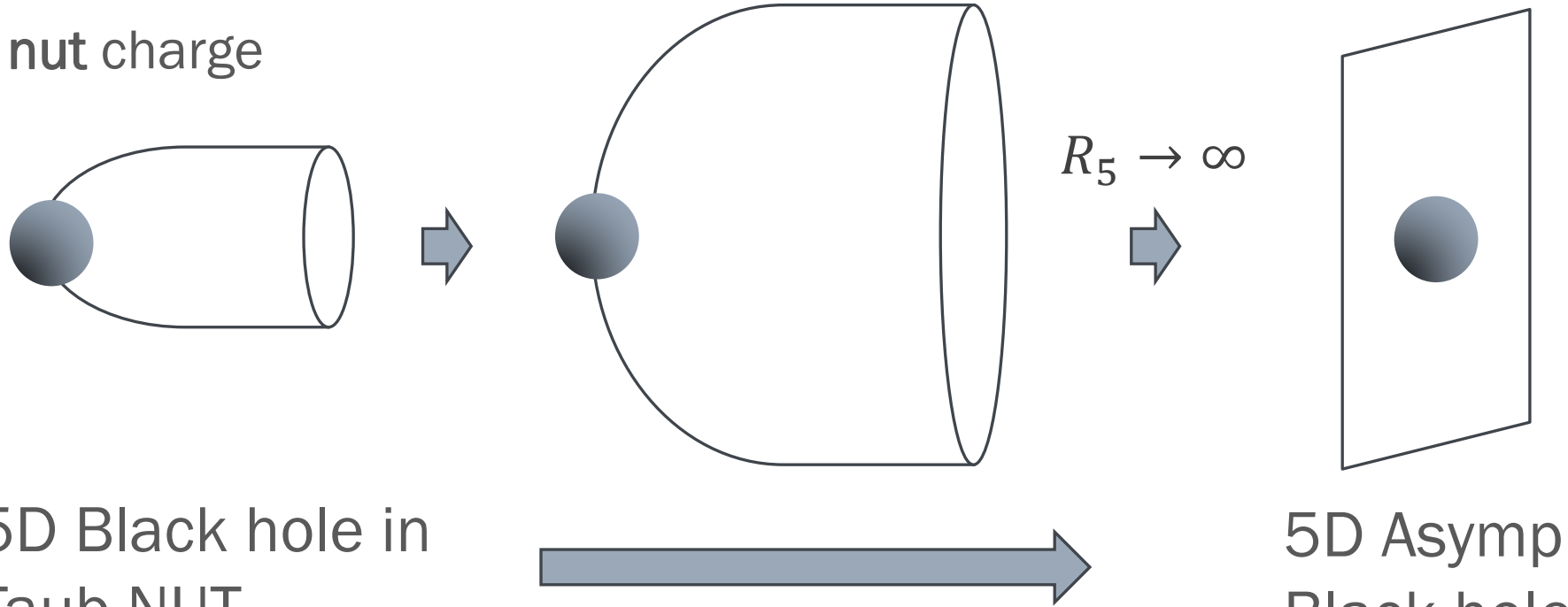
$$ds^2 \to -dt^2 + \underbrace{d\hat{r}^2 + \frac{\hat{r}^2}{4}\left((d\psi + \cos\theta \, d\phi)^2 + d\Omega_2\right)}_{\mathbb{R}^4}$$

4D/5D lift

Gaiotto+Strominger+Yin

$M2^3$-KKm

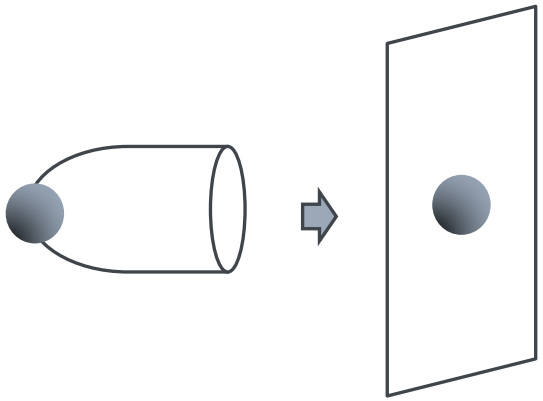
Unit nut charge



5D Black hole in Taub-NUT

5D Asymp Flat Black hole

4D/5D lift



4D/5D lift

Metric($D2^3$-D6) $\rightarrow [\nearrow + (R_5 \rightarrow \infty)] \rightarrow$ Metric($M2^3$)

Metric changes under change in R_5. Index should be invariant

Index ($D2^3$-D6) = Index ($M2^3$ in TN) = **Index ($M2^3$)**

4D AF BH

5D AF BH

4D/5D lift

Lift: need one scalar for 5D circle radius = KK dilaton

N=2 pure Sugra: no scalar ⇒ cannot be lifted

Need at least N=2 Sugra + 1 vector supermultiplet

Lifting the index

TWO-CENTER 4D TO 5D

4D/5D lift for index

Rotating non-extremal bh in N=2 Sugra + vectors

∃ 4D “5-charge Kerr-Newman” w/3 vectors (Chow+Compère)

Take one to be KK-monopole: uplift+decompactify

⇒ 5D “3-charge Kerr-Newman”: index geometry

4D/5D lift for index

Multi-center approach (index as split black hole)

4D Israel-Wilson-Perjés $\in$ N=2 sugra

$\Rightarrow$ cannot be uplifted+decompactified

Need “IWP multicenter” for N=2 sugra + vector supermultiplets

General multi-center solution: Bates-Denef

$$ds^2 = -\frac{1}{\Sigma(x)}(dt + \omega)^2 + \Sigma(x)d\mathbf{x}^2$$

$$\Sigma(x) = \sqrt{Q^3 H^0 - (LH^0)^2} \qquad * d\omega = \langle dH, H \rangle$$

Harmonic $(H^0, H^A, H_A, H_0) \leftrightarrow (D6, D4, D2, D0)$

$$H^0 = h^0 + \sum_i \frac{p_i^0}{|\mathbf{x}-\mathbf{x_i}|} \quad \text{etc}$$

General multi-center solution: Bates-Denef

$$ds^2 = -\frac{1}{\Sigma(x)}(dt + \omega)^2 + \Sigma(x)d\mathbf{x}^2$$

$$\frac{\text{Majumdar–Papapetrou}}{\text{Shmakova}} = \frac{\text{IWP}}{\text{Bates–Denef}}$$

pure sugra

sugra+vectors

static

rotating

Reproduces IWP for pure Sugra

W/ one vector, uplifts to general (timelike) solution of 5D minimal sugra

Gauntlett+Gutowski+Hull+Pakis+Reall

5D index from lifted two-center

Single-center $D2^3$-D6 w/unit D6 charge

Split two-center: $\frac{D2^3\ D6}{2}|_N + \frac{D2^3\ D6}{2}|_S$

Generate imaginary $D0\overline{D0}$, $D4\overline{D4}$ dipoles, rotation

Uplift: $\frac{M2^3KKm}{2}|_N + \frac{M2^3KKm}{2}|_S$

Uplifted geometry

$$ds_{5D}^2 = -\frac{1}{Q^2}\left(dt + \omega + L\left(d\psi + A^0\right)\right)^2 + Q ds_{4D}^2(\text{TaubNUT})$$

$$ds_{4D}^2(\text{TaubNUT}) = \frac{1}{H^0}\left(d\psi + A^0\right)^2 + H^0 d\mathbf{x}^2$$

with $Q(H)$ and $L(H)$ as before.

If nut charge $p^0 = 1$ then can decompactify to 5D AF multicenter

Two-center lifted geometry

At asymptotic infinity it satisfies the correct periodicities for index

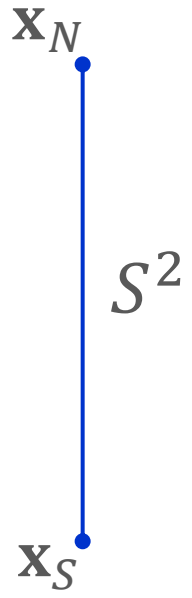
Near horizon (non-extremal) it is s*mooth*

Halved KK monopoles?

½ nut charge is usually problematic

Dirac strings would be observable

Not a problem since line between monopoles is S^2 "Dirac surface", which can't be shrunk to an observable string



5D index geometries

5D black hole index with two vectors first found as “3-charge rotating black hole” (Cvetic+Youm) in BPS limit with imaginary rotation

Anupam+Chowdhury+Sen

We’ve verified that the lifted two-center Bates-Denef solution with $n_v = 3$ is the same solution as in Anupam+Chowdhury+Sen

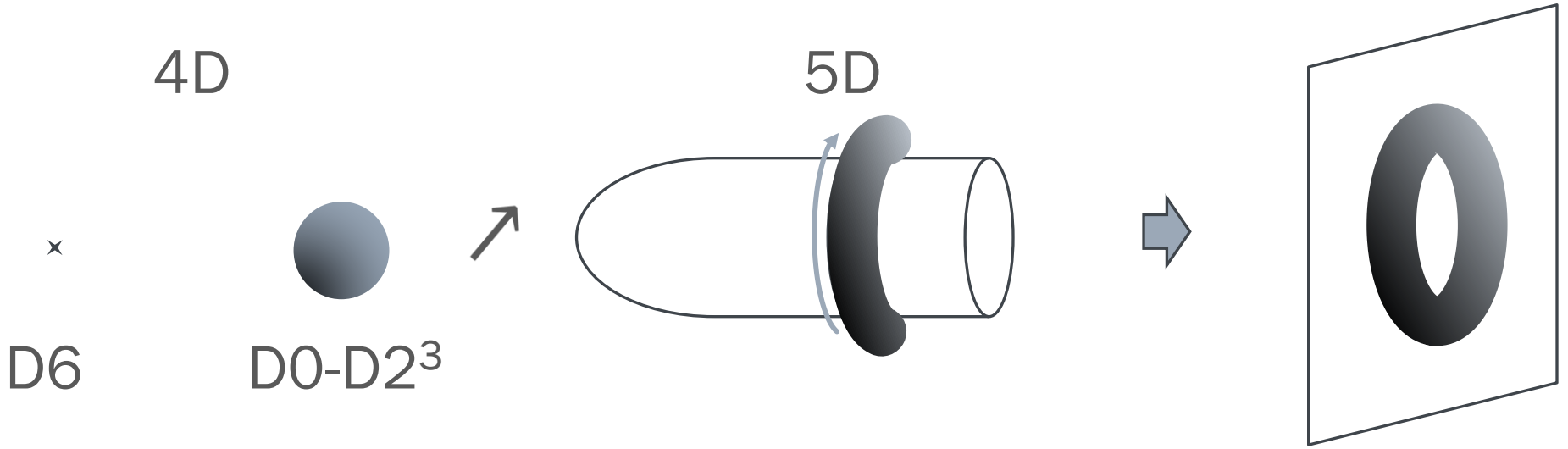
5D index geometries

Lifted multi-center allows to construct 5D index geometries that are not known otherwise, eg, with *arbitrary number of vectors*

5D Black Ring from 4D

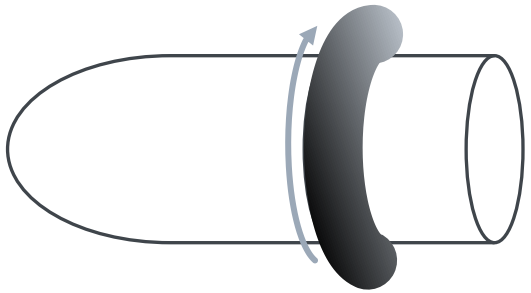
Elvang+RE+Mateos+Reall
Gaiotto+Strominger+Yin
Bena+Kraus+Warner

5D BPS black rings (and black Saturns etc) can be constructed as lifts of 4D BPS two-center solutions (∉ Shmakova)



5D Black Ring index

5D BPS black rings (and black Saturns etc) can be constructed as lifts of 4D BPS two-center solutions



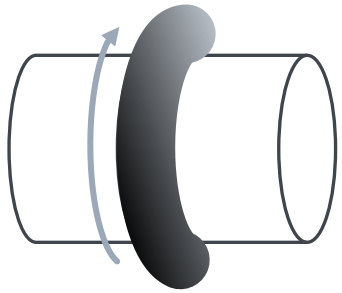
Splitting the black ring, the index is a 4-center solution

5D Black String index

With zero total D6 charge, we can obtain index of MSW string.

$M5^3$+W from index of $D4^3$+D0

Uplift w/out decompactification



5D Black String index

$M5^3$+W has an AdS_3 throat

Its index can be computed from a non-extremal AF geometry with arbitrary β that does not have an AdS_3 throat *anywhere*

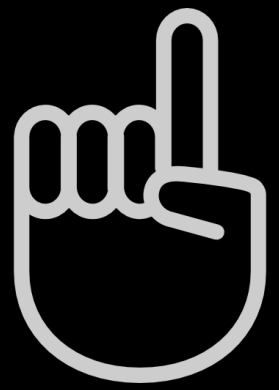
$\Rightarrow$ MSW CFT_2 index from a nowhere-AdS_3 AF geometry (!)

Outlook

LIFTING INDICES

- Susy indices consistently computed in gravity
- Non-trivial test that the Gravitational Path Integral is sensible, versatile and powerful
- Supersymmetric finite-temperature saddles are necessary
- Why is gravitational index β independent?
- 4D/5D lift of indices works and gives novel index geometries
- More precise tests of gravitational index vs micro index

Thank you



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