Near-extremal Quantum Field Theories

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— based on w.i.p. with A. Bagchi, S. Detournay, D. Grumiller, M. Riegler, and J. Simon;
w.i.p. with J.Simon;
2304.10102 and 2211.03770 with A. Castro, S. Detournay, and B. Mühlmann.

Solvay workshop on near-extremal black holes

Extremal and near-extremal black holes

- Definition 1: Extremal black holes have vanishing surface gravity on the horizon, i.e., their temperature is zero.
- Definition 2: There is a maximum charge and angular momentum for a given mass. When this bound is saturated, we have an extremal black hole.
- Generically, both definitions are equivalent but there are counterexamples. [Dias, Horowitz, Santos; 2109.14633]
- We will only focus on cases where temperature goes to zero.

Universal features of extremal black holes

- Extremal black holes develop an infinitely long throat in the near-horizon region. The proper distance from horizon to any point outside the horizon is infinite.
- For a large class of black holes, near-horizon region contains an AdS_2 factor. [Kunduri, Lucietti,Reall, Figueras, Rangamani; 0705.4214,0803.2998]
- This universal behavior doesn't survive addition of any finite energy excitation. [Maldacena,Michelson, Strominger; 9812073]
- There are several ways to see this: 1. 2d gravity Lagrangian is topological. So, stress-energy tensor vanishes.
  2. Consider 2d dilaton gravity models. Any non-zero stress-energy tensor implies dilaton diverges near the boundary destroying the AdS_2 asymptotics.
  3. Even going slightly away from extremality, near-horizon region is no-longer decoupled from the remaining spacetime. Proper distance from horizon to any point outside becomes finite.

Universal features of near-extremal black holes

- If this were the whole story, it would be of limited interest since it like studying just the ground state of a quantum mechanical system have and no finite energy excitations.
- However, this is not the whole story. For black holes with small deviations away from extremality, a universal description also emerges by keeping leading order effect of backreaction. [Almheiri, Polchinski 1402.6334]
- It is obtainted by correcting Einstein-Hilbert action by Jackiw-Teitelboim (JT) gravity action

$$I_{JT} = C_{JT} \int d^2x \sqrt{-g}\Phi \left(R + \frac{2}{\ell_2^2} \right) \tag{1}$$

- The onshell JT action is given by Schwarzian action [Maldacena, Stanford, Yang; 1606.01857]

$$I_{\mathrm{Sch}} = C_{\mathrm{Sch}} \int \mathrm{d}\tau \{f(\tau), \tau\} , \quad \{f(u), u\} = \frac{f'''}{f''} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2 . \quad (2)$$

$f(\tau)$ represents the reparametrizations of boundary AdS_2 given by

$$\mathrm{d}s^2 = \mathrm{d}\rho^2 - \left(e^{\rho/\ell_2} + \frac{\ell_2}{2}\{f(\tau), \tau\} e^{-\rho/\ell_2}\right) \mathrm{d}\tau^2 \quad (3)$$

- It also famously captures the low-energy regime of SYK model.
- The Schwarzian action describes a quantum mechanical model that is exactly solvable. The partition function is one-loop exact [Stanford, Witten '17]

$$Z_{\mathrm{Schw}} = \left(\frac{\pi}{\tilde{\beta}}\right)^{3/2} e^{\pi^2/\tilde{\beta}} , \quad \tilde{\beta} = \frac{\beta}{2C_{\mathrm{Sch}}} . \quad (4)$$

Near-extremal QFTs

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- We will refer to this regime as near-extremal regime of the dual field theory. In this regime, the field theory computation of quantities like correlation function and partition function should be consistent with Schwarzian theory.
- This regime should involve studying thermal field theory close to zero temperature amongst other limits.

- We will answer the question of existence of near-extremal limit of QFTs by looking at QFTs in two dimensions.
- We will consider CFTs, warped CFTs and Carrollian CFTs.
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- A common charatersitic of these theories is that there have a Virasoro factor in their symmetry algebra.
- **Near-extremal CFTs** For a large class of 2d CFTs with large central charge, there exists a regime of parameters, namely, low temperature and large angular momentum where partition function and correlation functions are determined by Schwarzian theory. [Ghosh, Maxfield, Turiaci; 1912.07654]
- These results are in line with the bulk computations of near-extremal BTZ.

- **Near-extremal Warped CFTs (WCFTs)** For a large class of non-unitary WCFTs with large central charge, there exists a regime of parameters, where partition function is determined by warped-Schwarzian theory. It matches the low energy behavior of complex SYK mode. [AA, Castro, Detournay, Mühlmann; 2211.03770]
- These results are also in line with the bulk near-extremal limit of warped black holes. Based on this we conjectured that only non-unitary WCFTs have interesting holographic duals. [AA, Castro, Detournay, Mühlmann; 2304.10102]
- We also present the exact modular S-matrices of WCFTs that can be used to obtain density of states of near-extremal WCFTs. [AA, Simon; to appear]

- **Near-extremal Carrrollian CFTs (CCFTs):** CCFTs also contain a universal "near-extremal" sector. Partition function is dominated by vacuum character and looks similar to Schwarzian partition function.

  [AA, Bagchi, Detournay, Grumiller, Riegler, Simon; to appear]
- However, we do not yet know the bulk interpretation of this "near-extremal" regime of CCFTs.
- The putative bulk is 3d asymptotically flat spacetime. There are no black holes in 3d in absence of cosmological constant.
- There are flat space cosmologies but they only have one horizon.
- Nevertheless, this sector does exists from the field theory side. We also present modular S-matrices of Carrollian CFTs that can be used to derive the density of states in the near-extremal regime. [AA, Simon; AA, Bagchi, Detournay, Grumiller, Riegler, Simon; to appear]

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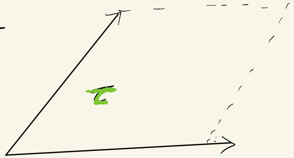
- Nevertheless, this sector does exists from the field theory side. We also present modular S-matrices of Carrollian CFTs that can be used to derive the density of states in the near-extremal regime. [AA, Simon; AA, Bagchi, Detournay, Grumiller, Riegler, Simon; to appear] In all three cases, there is another universal regime, i.e., Cardy regime.

Near-extremal CFT_2

Consider a CFT_2 on a torus with modular parameter τ



Modular invariance => partition function is invariant.

Dual channel

$$Z(\tau, \bar{\tau}) = Z\left(-\frac{1}{\tau}, -\frac{1}{\bar{\tau}}\right)$$

direct channel

Z can be decomposed as sum over characters

$$Z(\tau, \bar{\tau}) = \mathrm{Tr}\left(q^{(L_0 - c/24)} \, \bar{q}^{(\bar{L}_0 - \frac{c}{24})} \right)$$

$$= \chi_{\mathbb{1}}(\tau)\, \chi_{\mathbb{1}}(\bar{\tau}) + \sum_{\text{primaries}} \chi_h(\tau)\, \chi_{\bar{h}}(\bar{\tau})$$

$$q = e^{2\pi i \tau} \quad , \quad \bar{q} = e^{-2\pi i \bar{\tau}}$$

$\chi_h(\tau)$ are the characters

$$\chi_h(\tau) = \mathrm{Tr}_{\mathcal{V}_h}\left(q^{L_0 - \frac{c}{24}}\right)$$

→ Highest wt. rep. labelled by h

Introduce left and right temp.

$$\beta_L = -2\pi i \tau \quad , \quad \beta_R = 2\pi i \bar{\tau}$$

$$\Rightarrow \quad \chi_h(\beta_L) = \mathrm{Tr}_{\mathcal{H}_h}\left[e^{-\beta_L (L_0 - c/24)} \right]$$

$$\chi_{\bar{h}}(\beta_R) = \mathrm{Tr}_{\mathcal{H}_{\bar{h}}}\left[e^{-\beta_R (\bar{L}_0 - c/24)} \right]$$

Descendants are suppressed when $\beta_{L/R} \to \infty$ by Boltzmann factors. We need their contribution to obtain the prefactor of Z_{Schw}.

This dictates the choice of modular transformed channel.

Dual channel

High right moving temperature $\beta_R \to 0$ projects the partition function on the vacuum character in the modular transformed channel

$$Z(\beta_L, \beta_R) = \chi_{\mathbb{1}}\left(\frac{2\pi i}{\beta_L}\right) \chi_{\mathbb{1}}\left(\frac{2\pi i}{\beta_R}\right) \left\{ 1 + \cdots\cdots \right\}$$

Corrections of order $\exp\left(-\frac{2\pi^2}{\beta_R} \bar{h}_{gap}\right)$

$\bar{h}_{gap}$: lowest $\bar{h}$ other than vacuum

$$\chi_{\mathbb{1}}\left(\frac{2\pi i}{\beta_R}\right) \approx \exp\left[\frac{c}{24}\frac{4\pi^2}{\beta_R}\right]$$

Assumption involved : no other state with $\bar{h}=0$ except vacuum

(Existence of twist gap)

$\bar{h}=0 \Rightarrow h=0$

Cardy Formula can be derived by taking $\beta_L \to 0$ as well

$$S_{Cardy} = \frac{\pi^2}{3}c\left(\frac{1}{\beta_L}+\frac{1}{\beta_R}\right)$$

Near-extremal limit:

$$c \to \infty$$

$\beta_R \sim \frac{1}{c} \to 0$ - Projects onto vacuum

$\beta_L \sim c \to \infty$ - Contains Schwarzian

Schwarzian appears in left moving sector.
Using explicit form of vacuum character

$$\chi_{\mathbb{1}}(\tau) = \frac{q^{-\left(\frac{c-1}{24}\right)}}{\eta(\tau)}(1-q)$$

$(1-q)$ → null state subtraction
$SL(2,\mathbb{R})$ invariant vacuum

$\eta(\tau)$: Dedekind eta fn.

$$\chi_{\mathbb{1}}\left(\frac{2\pi i}{\beta_L}\right) \sim \underbrace{\left(\frac{2\pi}{\beta_L}\right)^{3/2} \exp\left[\frac{\beta_L}{24} + \frac{c}{24}\frac{4\pi^2}{\beta_L}\right]}_{\sim Z_{schw}}$$

$$Z_{schw}(\tilde{\beta}) = \left(\frac{\pi}{\tilde{\beta}}\right)^{3/2} \exp\left[\frac{\pi^2}{\tilde{\beta}}\right]$$

$$\tilde{\beta} \sim \frac{\beta_L}{c}$$

$$\Rightarrow \boxed{Z^{CFT_2}_{\text{Near Ext.}} \propto Z_{\text{Schw.}} \left(1 + \cdots\right)}$$

One gets Cardy entropy even in the near-extremal regime upto log corrections.

Assumptions about CFT

1. Unitary : $h, \bar{h} \geq 0$

2. Compact : Unique vacuum $h = \bar{h} = 0$
   $sl(2)$ invariant

3. Twist gap :- Only vacuum has $\bar{h} = 0$
   $\Downarrow$
   No additional symmetries

Bulk Interpretation

- Boundary of a Euclidean BTZ has following identifications

$$(t_E, \phi) \sim (t_E, \phi + 2\pi) \sim (t_E + \beta, \phi + \theta)$$

β: Inverse temp.

θ: Angular potential

- Define: $\beta_L = \beta - i\theta \quad , \quad \beta_R = \beta + i\theta$

- Grand Canonical partition function is

$$Z(\beta, \theta) = \mathrm{Tr}\left[e^{-\beta H - i\theta J} \right]$$

$$= \mathrm{Tr}\left[e^{-\beta_L (L_0 - c/24) - \beta_R (\bar{L}_0 - \frac{c}{24})} \right]$$

Mass ↖

$$H = L_0 + \bar{L}_0 - \frac{c}{12}$$

$$J = \bar{L}_0 - L_0$$

← Angular Momentum

- Near extremality $\Rightarrow$ $\beta \gg 1$

For near-extremal BH to dominate $\Rightarrow$ $\beta + i\theta \ll 1$

large $M \sim J$

(since $M \sim J$ near extremality)

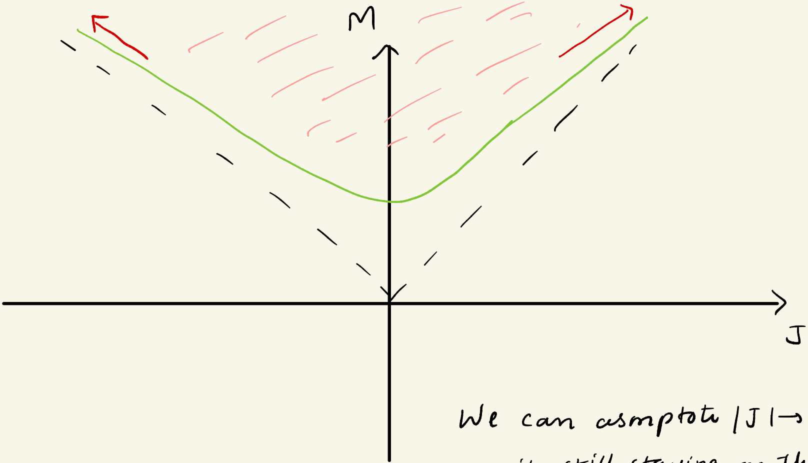
$\beta_R \ll 1$

$\beta_L \gg 1$

we get Same conditions used in near ext. CFT

$\beta_L \beta_R < 2\pi^2$ for BTZ to dominate

or $(M^2 - J^2) > 2\pi$



We can asmptote $|J| \to M$ while still staying on the desirable side of Hawking page phase transition.

Summary till now

Partition Function CFT_2 written in term of characters

Modular S Transformation $\Big\downarrow$ $\beta_R \sim \frac{1}{c} \to 0$

Vacuum Character Dominates

$\Big\downarrow$ $\beta_L \sim c \to \infty$

Schwarzian appears in left vacuum character

Generalising this procedure to other QFTs

partition function of a modular invariant/covariant QFT, $Z(\tau_p, \tau_q) = \sum \chi_{p,q}(\tau_p, \tau_q) = \chi_p(\tau_p)\chi_q(\tau_q)$

↓ Modular S Transformation — $\tau_q \to 0$ to project onto vacuum

Vacuum Character Dominates (Already universal)

↓ Take apt. limit on τ_p

"Schwarzian-like"/"Near-extremal" partition function.

Near- extremal Carroll CFT_2

- Carroll symmetries arise as the $c \to 0$ limit of Poincare symmetries, making space absolute and time relative; opposite of Gallilean symmetries. [Lévy-Leblond '65, Sen Gupta '66]

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- Carroll symmetries arise on a Carroll manifold defined by the pair $(\tau^\mu, h_{\mu\nu})$ a degenerate symmetric tensor $h_{\mu\nu}$ and a vector τ^μ generating the kernel of $h_{\mu\nu}$,

$$h_{\mu\nu}\tau^\mu = 0. \tag{5}$$

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- For $d-$dimensional flat Carroll spacetimes, $\tau^\mu = \partial_t$ and $ds^2 = \sum_i^{d-1} (dx^i)^2$, Conformal Carroll algebra, $\mathfrak{ccar}_d$, is generated by the isometries

$$\mathcal{L}_\xi \tau^\mu = -\lambda \tau^\mu, \quad \mathcal{L}_\xi h_{\mu\nu} = 2\lambda h_{\mu\nu}. \tag{6}$$

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- $\mathfrak{ccar}_d$, is isomorphic to the $(d+1)$-dimensional Bondi–van der Burgh–Metzner–Sachs (BMS) algebra, $\mathfrak{bms}_{d+1}$, which is the algebra of $d+1$ dimensional asymptotically flat spacetimes. [Duval, Gibbons, Horvathy '14].

- We are interested in $d = 2$, i.e., $\mathfrak{bms}_3$ or $\mathfrak{ccar}_2$. It consists of semidirect sum of Virasoro and an abelian algebra. Expanding the generators in Fourier modes

$$[L_n,\, L_m] = (n-m)\, L_{n+m} + c_L(n^3 - n)\, \delta_{n+m,\,0} \tag{7}$$

$$[L_n,\, M_m] = (n-m)\, M_{n+m} + c_M(n^3 - n)\, \delta_{n+m,\,0} \tag{8}$$

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- L_ns are superrotations and M_ns are supertranslations. $L_0, L\pm1, M_0, M_{\pm1}$ generate global subalgebra $\mathfrak{isl}(2,\mathbb{R})$ coresponding to the isometries of 3d Minkowski. $c_L = 0$ and $c_M \neq 0$ for Einstein gravity.

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- The 2d QFTs with these symmetries are Carroll CFT_2 (CCFT_2)—natural holographic duals to 3d asymptotically flat gravity.

Carroll Partition Function, modular transformations

- We define the partition function of a Carroll CFT_2 as

$$Z_{\mathfrak{ccar}}(\beta_{\mathfrak{car}}, \theta_{\mathfrak{car}}) = \mathrm{Tr}\, e^{-\beta_{\mathfrak{car}} H + i\theta_{\mathfrak{car}} J}\,, \tag{10}$$

where $\beta_{\mathfrak{car}}$ is the inverse Carroll temperature, $\theta_{\mathfrak{car}}$ is the angular potential and

$$H = M_0, \quad J = L_0. \tag{11}$$

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- One can obtain Carroll CFT_2 from a Lorentzian CFT_2 in the limit of vanishing speed of light

$$t \to \epsilon t, \quad \phi \to \phi, \qquad \epsilon \to 0. \tag{12}$$

$$L_n = \mathcal{L}_n - \bar{\mathcal{L}}_{-n}, \qquad M_n = \epsilon(\mathcal{L}_n + \bar{\mathcal{L}}_{-n}) \tag{13}$$

$$c_L = c - \bar{c}, \qquad c_M = \epsilon(c + \bar{c}), \tag{14}$$

$$\beta_{\text{CFT}} = \beta_{\mathfrak{car}}, \qquad \theta_{\text{CFT}} = \epsilon\theta_{\mathfrak{car}} \tag{15}$$

- This limiting procedure also provides a way to obtain Carroll modular transformations. We start with CFT_2 modular transformations $\text{PSL}(2,\mathbb{Z})$,

$$\tau \to \frac{a\tau + b}{c\tau + d} \qquad ad - bc = 1 \quad \text{with} \quad a, b, c, d \in \mathbb{Z}. \tag{16}$$

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- The relation between CFT_2 and Carrollian modular parameters, $\sigma \equiv i\beta_{\mathfrak{car}}/2\pi$, $\rho \equiv \theta_{\mathfrak{car}}/2\pi$, yields the expansion

$$\tau = \sigma + \epsilon\,\rho \to \frac{a\sigma + b}{c\sigma + d} + \epsilon\,\rho\,\frac{ad - bc}{(c\sigma + d)^2} + \mathcal{O}(\epsilon^2)\,, \tag{17}$$

leading to Carroll modular transformations

$$\sigma \to \frac{a\sigma + b}{c\sigma + d} \qquad \rho \to \frac{\rho}{(c\sigma + d)^2}\;. \tag{18}$$

σ transforms like τ and ρ transforms like imaginary part of τ.

- If σ is thought of as coordinate on the base manifold, $\mathcal{H}$, on which Carroll modular transformations act, ρ transforms like a vector in $\mathcal{T}_\sigma\mathcal{H}$.

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- The Carroll modular group is generated by composing S and T transformations

$$S:\ \sigma \to -\frac{1}{\sigma} \qquad \rho \to \frac{\rho}{\sigma^2} \qquad\qquad T:\ \sigma \to \sigma + 1 \qquad \rho \to \rho\,. \tag{19}$$

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- They satisfy the usual identities

$$S^2 = \mathbb{1} \qquad (ST)^3 = \mathbb{1} \tag{20}$$

Carroll Characters

- The states in a 2d Carrollian CFT are labelled with the eigenvalues of L_0 and M_0:

$$L_0|\Delta,\xi\rangle = \Delta|\Delta,\xi\rangle \qquad\qquad M_0|\Delta,\xi\rangle = \xi|\Delta,\xi\rangle\,. \tag{21}$$

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- A generic descendant takes the form

$$|\Psi\rangle = L_{-n_1}L_{-n_2}\dots L_{-n_q}M_{-m_1}M_{-m_2}\dots M_{-m_r}|\Delta,\xi\rangle_p \qquad n_i, m_j > 0$$

- There is another type of representation — induced representation, which is built out of states anihilated by all supertranslations (except for M_0):

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- Generic states in induced representation are obtained by acting with arbitrary combinations of L_n generators (not necessarily $n > 0$) on $|\Delta, \xi\rangle_I$:

$$|\Phi\rangle = L_{n_1} L_{n_2} \dots L_{n_m} |\Delta, \xi\rangle_I\,. \tag{24}$$

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- The highest weight and induced representations turn out to have identical characters.

- For non-vacuum states, the Carroll characters are given by

$$\chi_{(c_L,c_M,\Delta,\xi)}(\sigma,\rho) = \frac{e^{\frac{2\pi i\sigma}{12}}\, e^{-2\pi i(\sigma\frac{c_L}{2}+\rho\frac{c_M}{2})}\, e^{2\pi i(\sigma\Delta+\xi\rho)}}{\eta(\sigma)^2} \tag{25}$$

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  where $\eta(\sigma)$ is the Dedekind eta-function.

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- The Carroll partition function is then the sum of Carroll characters

$$Z_{\mathfrak{ccar}}(\sigma,\rho) = \sum_{\text{primaries}} D(\Delta,\xi)\, \chi_{(c_L,c_M,\Delta,\xi)}(\sigma,\rho)\,. \tag{27}$$

where $D(\Delta,\xi)$ is multiplicity of the primaries with weight (Δ,ξ).

Summary until now

- Carroll CFT_2 is a two-dimensional QFT invariant under $\mathfrak{ccar}_2$ or $\mathfrak{bms}_3$ symmetries.
- It can be obtained as a limit of a CFT_2.
- Thermal CCFT_2 is invariant under Carroll modular transformations that act on the upper half plane as well as its tangent space.
- CCFT_2 partition function can be expressed as the sum of Carroll characters. These characters are same for both induced and highest weight representations.

Vacuum Dominance and Universal Carroll Sectors

Are there any universal sectors present in a generic class of 2d Carroll CFTs?

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- Modular invariance of the 2d Carrollian partition function under the S- transformation implies

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- We can thus write the partition function in terms of characters in the S-dual channel.

$$Z_{\mathfrak{ccar}}(\sigma, \rho) = \sum_{\text{primaries}} \chi_{(c_L, c_M, \Delta, \xi)}\left(-\frac{1}{\sigma}, \frac{\rho}{\sigma^2}\right) . \tag{29}$$

- We look for regimes where the vacuum character is the dominant contribution to the partition function in the S-dual channel

$$\frac{\chi_{(c_L,c_M,\Delta,\xi)}}{\chi_{(c_L,c_M,0,0)}}\left(-\frac{1}{\sigma},\frac{\rho}{\sigma^2}\right)\to 0, \quad \forall\Delta,\xi\neq 0\ . \tag{30}$$

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- It turns out that there are two regimes/ sectors where this happens

Sector	Physical parameters
1.Cardy	$\beta_{\mathfrak{car}}\Omega_{\mathfrak{car}}\to 0^+,\ \Omega_{\mathfrak{car}},\beta_{\mathfrak{car}}<0$
2.Cardy – Near extremal	$\beta_{\mathfrak{car}}\Omega^2_{\mathfrak{car}}\to 0^-\ ,\ \Omega_{\mathfrak{car}},\ \beta_{\mathfrak{car}}<0$

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- Carroll temperature is negative for both of the regimes which is in line with the negative temperature for Flat space cosmologies in the dual theory.

- The partition function in the Cardy regime is well-approximated by

$$Z^{(1)}_{\mathfrak{ccar}}(\sigma,\rho) \approx \exp\left\{2\pi^2\left[\frac{c_L}{|\beta_{\mathfrak{car}}\Omega_{\mathfrak{car}}|}+\frac{c_M}{|\beta_{\mathfrak{car}}\Omega^2_{\mathfrak{car}}|}\right]\right\}. \tag{31}$$

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- In the Cardy-Schwarzian regime, the partition function is

$$Z^{(2)}_{\mathfrak{ccar}}(\sigma,\rho) \approx Z^{(1)}_{\mathfrak{ccar}}(\sigma,\rho)\left(\frac{1-e^{-\frac{4\pi^2}{|\beta_{\mathfrak{car}}\Omega_{\mathfrak{car}}|}}}{\eta\left(\frac{2\pi i}{\beta_{\mathfrak{car}}\Omega_{\mathfrak{car}}}\right)}\right)^2. \qquad (32)$$

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- In both of the regimes regimes, one finds BMS-Cardy formula for the entropy to the leading order excluding generically small corrections

$$S^{(1)}_{\mathfrak{ccar}} \approx S^{(2)}_{\mathfrak{ccar}} \approx 4\pi^2\left(\frac{c_L}{|\beta_{\mathfrak{car}}\Omega_{\mathfrak{car}}|}+\frac{c_M}{|\beta_{\mathfrak{car}}\Omega^2_{\mathfrak{car}}|}\right). \tag{33}$$

Near-extremal sector of $CCFT_2$

- There is a subsector of the Cardy-Near extremal regime that leads to a "Schwarzian-like" partition function. In this subsector $\beta_{\mathfrak{car}}\Omega_{\mathfrak{car}} \gg 1$ in addition to $\beta_{\mathfrak{car}}\Omega^2_{\mathfrak{car}} \to 0^-$.

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$$Z^{\mathrm{NE}}_{\mathfrak{ccar}} \approx \frac{(2\pi)^5}{(\beta_{\mathfrak{car}}\Omega_{\mathfrak{car}})^3}\exp\left\{\frac{\beta_{\mathfrak{car}}\Omega_{\mathfrak{car}}}{12} + 2\pi^2\left[\frac{c_L - 1/6}{|\beta_{\mathfrak{car}}\Omega_{\mathfrak{car}}|} + \frac{c_M}{|\beta_{\mathfrak{car}}\Omega^2_{\mathfrak{car}}|}\right]\right\}.$$

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- The prefactor indicates the contribution of six zero modes corresponding to six global generators $M_{0,\pm 1}, L_{0,\pm 1}$.
- This is in contrast to the 3 zero modes of Schwarzian theory.

Summary of near-extremal Carrollian CFTs

- A generic class of 2d Carroll CFTs has two universal sectors—Cardy and Cardy-Near extremal.

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- Both of these sectors reproduce BMS-Cardy entropy while Cardy-Near extremal regime has a subsector that leads to a “Schwarzian-like” partition function.
- They require negative Carroll temperatures which is in line with the thermodynamics of Flat space cosmologies.
- However, since there are no black holes in 3d Einstein gravity with $\Lambda = 0$, the bulk interpretation of the Schwarzian sector is unclear. Can’t be tied to near-extremality.

Near-extremal density of states from modular S-matrices

- Now, we will see the appearance of "Schwarzian" in the near-extremal desnity of states using modular S-matrices.

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- Or

$$\chi_i(\tau') = \int \mathrm{d}P\, S_{i,P}\, \chi_P(\tau) \tag{35}$$

for continuous spectrum.

CFTs/ Virsoro modular S-matrix

- Modular S-transformation:

$$S : \tau \to -\frac{1}{\tau} \tag{36}$$

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- Using Liouville parametrization, let

$$h - \frac{c-1}{24} =: P^2 \ , \quad \frac{c-1}{6} =: Q^2 \ , \quad Q =: b + b^{-1} \ . \tag{37}$$

Then, characters of Virsoro algebra are

$$\chi_P(\tau) \quad = \quad \frac{e^{2\pi i \tau P^2}}{\eta(\tau)}(1 - \delta_{vac} q) \ , \quad q := e^{2\pi i \tau} \tag{38}$$

$$\delta_{\text{vac}} = \begin{cases} 1, & \text{for vacuum} \\ 0, & \text{otherwise} \end{cases} . \tag{39}$$

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$$\chi_{P'}\left(-\frac{1}{\tau}\right) = \int_{-\infty}^{\infty} \frac{\mathrm{d}P}{2}\ S(P';P)\chi_P(\tau). \tag{44}$$

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  for vacuum character. This is the Plancherel measure for the continuous principal series representations of the quantum group $\mathcal{U}_q(sl_2)$. [Ponsot,Teschner; '99]

- One can use $S(P;\mathbb{1})$ to find the Schwarzian desity of states as follows.
- Since CFT partition function can be approximated by (left) vacuum characters in the S-transformed channel

$$Z_{\mathrm{CFT}} \sim \chi_{\mathbb{1}}\left(-\frac{1}{\tau}\right) = \int \mathrm{d}P\; S(\mathbb{1};P)\chi_P(\tau) \tag{47}$$

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- Thus, the above integral is dominated by $P \sim b$

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So, in the near extremal limit $D(P) \sim S(\mathbb{1}; P) \sim \sinh\big(2\pi b^{-1} P\big)$
$P \sim \sqrt{h} \sim \sqrt{Energy}$

Near-extremal Carrollian DoS from modular S-matrices

- Using a Liouville inspired parametrization, let

$$\xi - \frac{c_M}{2} =: P_M^2 \ , \quad \Delta - \frac{c_L - 1/6}{2} =: P_L \ , \quad c_L - \frac{1}{6} =: Q_L \ . \quad (51)$$

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- It is given by

$$S(P_L', P_M'; P_L, P_M) = 2 \frac{|P_M'|}{|P_M|^2} \frac{P_M}{P_M'} \sin\left[2\pi \left(\frac{P_M'}{P_M} P_L + \frac{P_M}{P_M'} P_L'\right)\right] \ . \tag{53}$$

for non-vacuum characters. [AA, Simon; to appear]

- For vacuum characters

$$S(\mathbb{1}; P_L, P_M) = 8\frac{P_M}{|P_M|^2}\ \sinh\left[2\pi\left(\sqrt{\frac{c_M}{2}}\frac{P_L}{P_M} + \frac{P_M}{\sqrt{c_M/2}}\frac{Q_L-2}{2}\right)\right] \sinh^2\frac{\pi P_M}{\sqrt{c_M/2}}\,. \quad (54)$$

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- $P_L \to 0$ and P_L/P_M doesn't diverge as $1/P_M$ in the near-extremal regime. So, DoS indeed goes to zero as $P_M \sim \sqrt{E} \to 0$ (E being "energy above extremality"). The behavior is similar to Schwarzian

Summary and future directions

- We observed that three different two-dimensional QFTs have a universal near-extremal sector different from the universal Cardy sector of these theories.
- Common characteristics of these theories that lead to the near-extremal sector: Modular symmetry, atleast one copy of Virsoro, and two dimensions.
- Can one find such sectors in other QFTs, particularly in higher dimensions?
- There should be a mechanism for this to happen since higher dimensional near-extremal blackholes also show the Schwarzian behavior and their putative duals are higher dimensional QFTs. Cardy in higher dimensions?

Thank you!