Solvay Workshop on: "Quantum Simulation"

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Geometry of "flux attachment" in the fractional quantum Hall effect.

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- flux attachment and the Laughlin state
- non-commutative geometry
- emergent geometry and metric of flux-attachment

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- FQHE occurs in "flat" Landau levels in a clean enough system so the repulsive two-body interaction dominates the (inhomogenous) one-body (potential) energy, but is small compared to the energy gaps separating partially-filled Landau levels from filled and empty ones

"bandwidth" (Landau level broadening)	**<<**	**interaction-energy**	**<<**	**"band gaps" (between Landau levels)**

- The same idea was applied to make "toy model" Bloch band "fractional Chern Insulator" systems in which exact numerical diagonalization revealed FQHE-like states

Neupert et. al and many others (Regnault, Sheng,....)

- so there is a proof in principle that zero-field lattice systems can show Laughlin-like FQHE states.

- How does this fit in with the Laughlin picture of FQHE in a Landau level?

$$2\pi \ell_B^2 = \frac{B}{(h/e)}$$

$$\Psi \propto \prod_{i<j} (z_i - z_j)^m \prod_i e^{-\frac{1}{4}|z_i|^2/\ell_B^2}$$

$$\boxed{z = x + iy}$$

- according to conventional wisdom the holomorphic structure of the Laughlin state has something to do with "being in the lowest Landau level": how can this translate to the lattice of a Chern insulator?

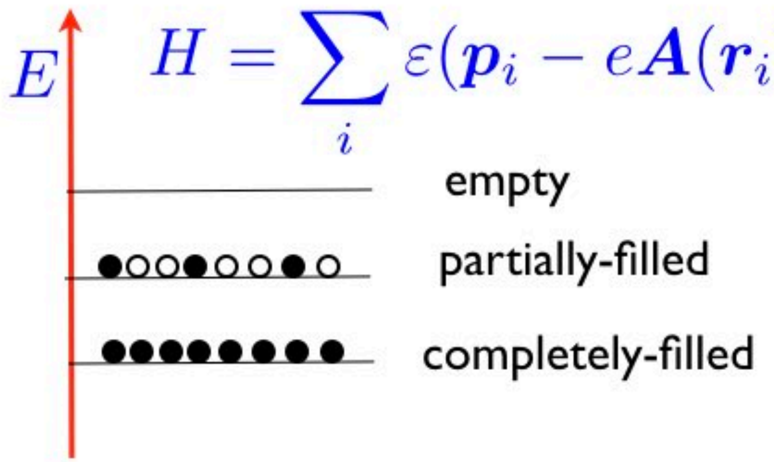
- In fact, the "holomorphic" structure of Laughlin and other "conformal block model wavefunctions" has **<u>nothing whatsoever to do with "being in the lowest Landau level"</u>**
- Instead, it derives from the non-commutative geometry of the "guiding centers" of Landau orbits, without any relation to the shape of those orbits around the center.

$$[R^x, R^y] = -i\ell_B^2$$

- thirty years after its experimental discovery and theoretical description in terms of the Laughlin state, the fractional quantum Hall effect remains a rich source of new ideas in condensed matter physics.
- The key concept is "**flux attachment**" that forms "**composite particles**" and leads to topological order.
- Recently, it has been realized that flux attachment also has interesting **geometric** properties

- the kinetic energy of electrons bound to a 2D surface through which a uniform magnetic flux passes undergoes Landau quantization into macroscopically-degenerate Landau levels

$$H = \sum_i \varepsilon(\boldsymbol{p}_i - e\boldsymbol{A}(\boldsymbol{r}_i))$$



one state in each level per London quantum of magnetic flux

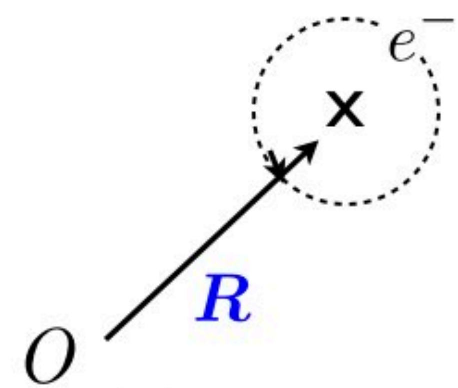
- In momentum space, the electrons move on closed contours of constant energy, similar to motion in phase space

$$[p_x, p_y] = i\hbar eB$$

$$\varepsilon(\boldsymbol{p}) = \frac{(p_x^2 + p_y^2)}{2m}$$

quantized Landau orbit
around guiding center

- Their residual real-space degree of freedom is the center of their circular orbit, called the "guiding center" $\boldsymbol{R}$



origin
(arbitrary)

$$\boldsymbol{R} = \boldsymbol{r} - (eB)^{-1}\hat{\boldsymbol{n}} \times (-i\hbar\boldsymbol{\nabla} - e\boldsymbol{A})$$

The origin ambiguity of $\boldsymbol{R}$ is also a gauge ambiguity

- Landau quantization of the orbital motion of the electrons leaves a residual problem of non-commutative geometry of the guiding centers of the orbits in a partially-filled Landau level.

$$H = \sum_{i<j} V(\boldsymbol{R}_i - \boldsymbol{R}_j)$$

no kinetic energy!

$$[R^x, R^y] = -i\ell_B^2$$

dynamics comes from non-commutative geometry!

$2\pi\ell_B^2 =$ **"quantum area"** through which one London quantum of magnetic flux passes

analogous to Planck area!

- Where can we find non-commutative geometry on a lattice?
- A topologically-non-trivial bandstucture must have at least <u>two</u> orbitals in the unit cell, but if we project into that band, there is <u>only one independent state per unit cell</u>
- The overlap matrix between orbitals is then rank-deficient, with a kernel of null eigenvalues

$$\{c_i, c_j^\dagger\} = S_{ij} \quad = \langle i|P|j\rangle$$

Projection into band

Orbitals are renormalized after projection so that

$$\langle i|P|i\rangle = 1$$

$$\{c_i, c_i^\dagger\} = 1$$

$$\{c_i, c_j^\dagger\} = S_{ij}$$

- Because of this, an “onsite” Hamiltonian

$$H = \sum_i E_i n_i + \sum_{i<j} V_{ij} n_i n_j$$

will have non-trivial dynamics

- band topology is encoded in the complex phase of S_{ij}, and geometry in the quantum distance measure

$$d_{ij} = 1 - |S_{ij}|$$

- Sij define the fuzzy “quantum lattice” that generalizes the classical lattice $S_{ij} = \delta_{ij}$

- A basis of orthogonal states of the projected band is obtained as the non-zero eigenstates of S

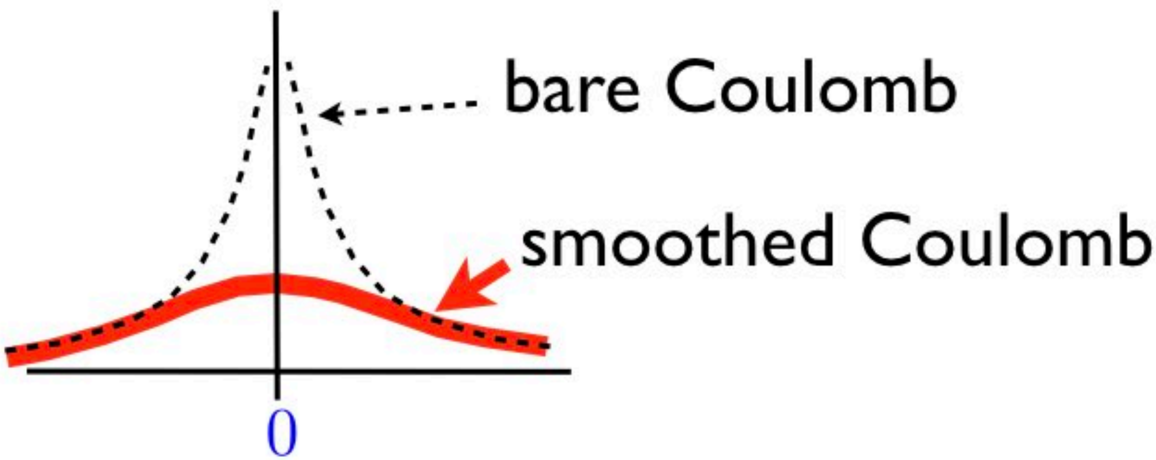
$$\sum_j S_{ij} u_{j\lambda} = s_\lambda u_{i\lambda} \qquad c_\lambda^\dagger = \frac{1}{\sqrt{s_\lambda}} \sum_i u_{i\lambda} c_i^\dagger$$

- For the basis of coherent (Gaussian) Landau level states, this leads to the holomorphic states, which are the non-zero eigenstates of

$$S(\boldsymbol{x}, \boldsymbol{x}') = e^{-\frac{1}{4}(z^* z - 2z^* z' + z'^* z')/\ell_B^2}$$

$$H = \sum_{i<j} V(\boldsymbol{R}_i - \boldsymbol{R}_j) \qquad [R^x, R^y] = -i\ell_B^2$$

- The interaction potential is very smooth because the short-distance singularity of the Coulomb interaction is smoothed out by the Landau orbit "form factor"

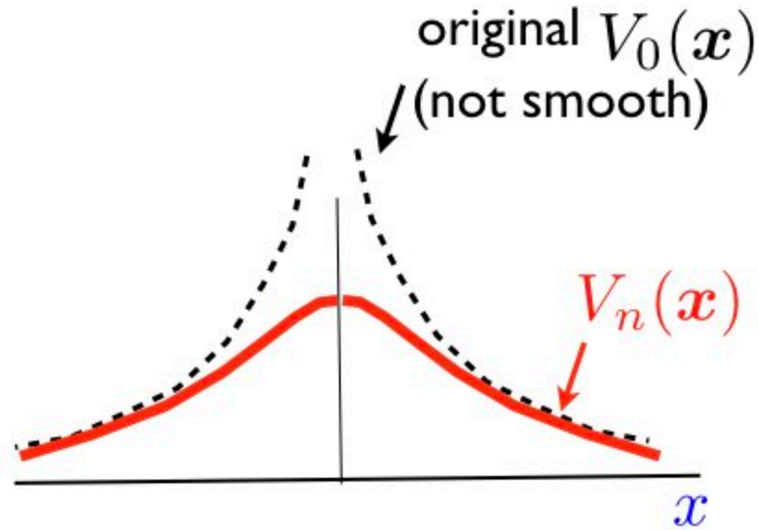


- The expansion of $V(r)$ about any point is absolutely convergent.
- This is needed for a function of non-commuting variables to "make sense"

$$[R^a, R^b] = -i\ell_B^2 \epsilon^{ab}$$

$$H = \sum_{i<j} V_n(\boldsymbol{R}_i - \boldsymbol{R}_j)$$

Identical quantum particles
(fermions or bosons)



We now have the final form of the problem:

- The potential $V_n(\boldsymbol{x})$ is a **very smooth** (in fact entire) function that depends on the form- factor of the partially-occupied Landau level

- The essential clean-limit symmetries are translation and inversion:

$$\boldsymbol{R}_i \mapsto \boldsymbol{a} \pm \boldsymbol{R}_i$$

$$H = \sum_{i<j} V(\boldsymbol{R}_i - \boldsymbol{R}_j) \qquad [R^x, R^y] = -i\ell_B^2$$

- This is a strongly interacting model with no free-particle limit! How can we study it?
- Numerical solution with a finite number N of particles has been the only quantitative source of information
- We can treat the rotationally-invariant case most easily $V(\boldsymbol{r}) = V(|\boldsymbol{r}|)$
- in 1983, Laughlin studied the N=3 case and was led to a remarkable model wavefunction

- The Laughlin state (Nobel 1998)

Landau level filling factor $\nu = 1/m$

$$\psi \propto \prod_{i<j} (z_i - z_j)^m \prod_i e^{-\frac{1}{4} z_i^* z_i / \ell_B^2}$$

- rotationally invariant, lowest Landau level, $z = x + iy$

- $m = 1$ is the uncorrelated Slater-determinant filled lowest Landau level, $m > 1$ is a highly correlated topologically-ordered state.

- Laughlin thought of his state as a Lowest Landau Level (LLL) wavefunction, using the fact that in the symmetric gauge, and with rotational symmetry, a LLL one-particle wavefunction has the form

$$\psi(x,y) = f(z) e^{-\frac{1}{4} z^* z / \ell_B^2}$$

a holomorphic function

- It also has no obvious continuously-variable parameter (m must be an integer)

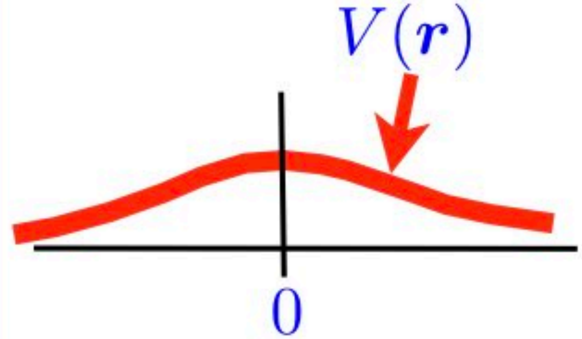
- **<u>In fact, as we will see, both these commonly-held beliefs are incorrect!</u>**

$$\psi \propto \prod_{i<j}(z_i - z_j)^m \prod_i e^{-\frac{1}{4} z_i^* z_i / \ell_B^2}$$

- The Laughlin state was soon shown to be the exact ground state of a "toy model" short-range interaction, and a very good approximation to the ground state of the Coulomb interaction
- It has been interpreted in terms of "flux attachment", (composite bosons and composite fermions) and its excitations identified as obeying (Abelian) "fractional statistics" and exhibiting "topological order"
- Its holomorphic part was recognized as a "conformal block" of an (Abelian) conformal field theory correlation function, leading even more interesting non-Abelian model states, which could be used for "topological quantum computing"

$$H = \sum_{i<j} V(\boldsymbol{R}_i - \boldsymbol{R}_j)$$

$$[R^x, R^y] = -i\ell_B^2$$



- The only parameter in this model is the interaction, **no explicit mention of the Landau level (lowest or otherwise)**
- Non-commutative geometry is intrinsically "fuzzy", so has no valid "Schrödinger" real-space wavefunction formalism, just a Heisenberg Hilbert-space picture

$$\psi(\boldsymbol{x}) = \langle \boldsymbol{x} | \Psi \rangle$$

Schrödinger Heisenberg

$$\langle \boldsymbol{x} | \boldsymbol{x}' \rangle = 0$$
$$\boldsymbol{x} \neq \boldsymbol{x}'$$

needed for Schrödinger, but fails in non-commutative quantum geometry

$$H = \sum_{i<j} V(\boldsymbol{R}_i - \boldsymbol{R}_j) \qquad [R^x, R^y] = -i\ell_B^2$$

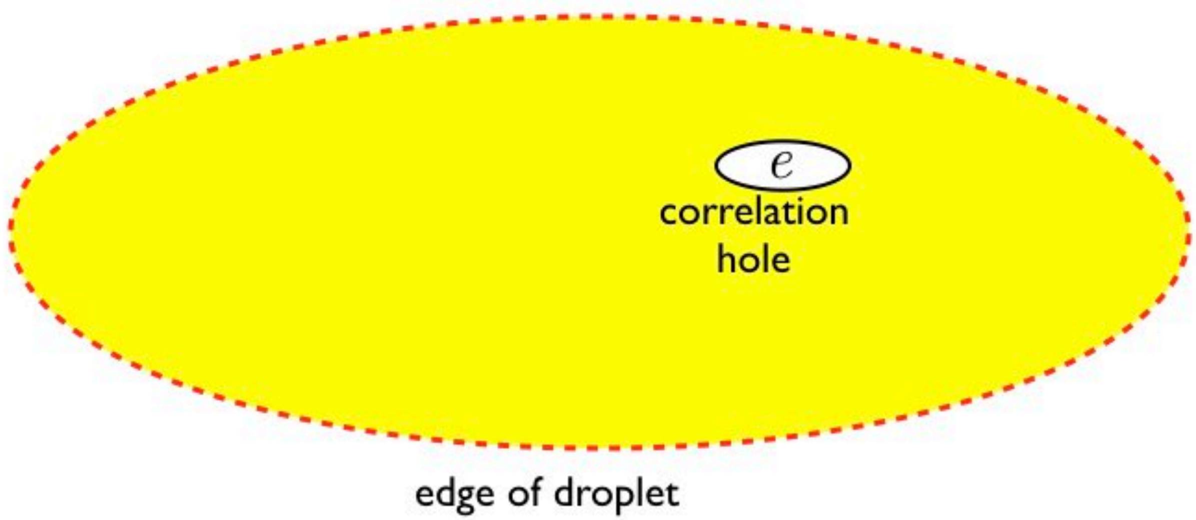
- so what is the actual meaning of the "Laughlin wavefunction"?
- choose **any** unit-determinant ("unimodular") metric g_{ab}
- diagonalize $L = \frac{g_{ab}}{2\ell_B^2} R^a R^b = \frac{1}{2}(a^\dagger a + a a^\dagger)$

$$|\Psi_L(g)\rangle = \prod_{i<j} (a_i^\dagger - a_j^\dagger)^m |0\rangle \qquad \begin{array}{c} a_i|0\rangle = 0 \\ [a_i, a_j^\dagger] = \delta_{ij} \end{array}$$

Heisenberg form of the (unnormalized) Laughlin state

- for $m > 1$ (but not $m = 1$), the metric is a **hidden parameter** of the state

- The original form of the Laughlin state is a finite-size droplet of N particles on the infinite plane.
- Somewhat confusingly, in this droplet state the metric parameter fixes both the shape of the droplet state **<u>and</u>** the shape of the correlation hole around each particle formed by "flux attachment":



- to remove the edge, compactify on the torus with N_Φ flux quanta:

- An unnormalized holomorphic single-particle state has the form

$$|\psi\rangle = \prod_{i=1}^{N_\Phi} \sigma(a^\dagger - w_i)|0\rangle, \qquad \sum_{i=1}^{N_\Phi} w_i = 0$$

generalized Weierstrass sigma function

$$\boxed{\sigma(z) = e^{\frac{1}{2}C_2 z^2} z \prod_{L\neq 0} \left(1 - \frac{z}{L}\right) e^{\frac{z}{L} + \frac{1}{2}\left(\frac{z}{L}\right)^2}}$$

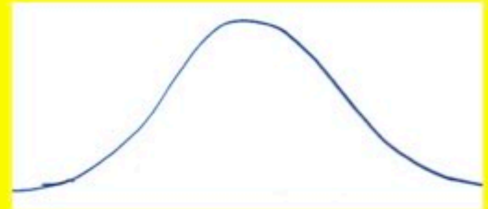
C_2 is an "almost holomorphic modular invariant"

Filled Landau level $N = N_\Phi$

$$|\Psi_{\text{filledLL}}\rangle = \sigma(\textstyle\sum_i a_i^\dagger) \displaystyle\prod_{i<j} \sigma(a_i^\dagger - a_j^\dagger)|0\rangle$$

independent of choice of metric, after normalization

This is the **entire** problem: nothing other than this matters!



$$H = \sum_{i<j} U(\boldsymbol{R}_i - \boldsymbol{R}_j)$$

$$[R^x, R^y] = -i\ell_B^2$$

like phase-space, has Heisenberg uncertainty principle

- H has translation and inversion symmetry

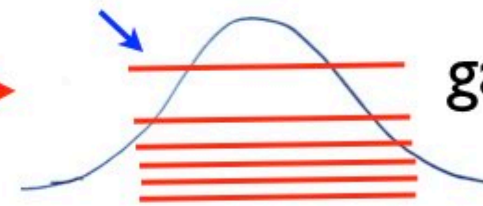
$$[(R_1^x + R_2^x), (R_1^y - R_2^y)] = 0$$

$$[H, \textstyle\sum_i \boldsymbol{R}_i] = 0$$

- generator of translations and electric dipole moment!

$$[(R_1^x - R_2^x), (R_1^y - R_2^y)] = -2i\ell_B^2$$

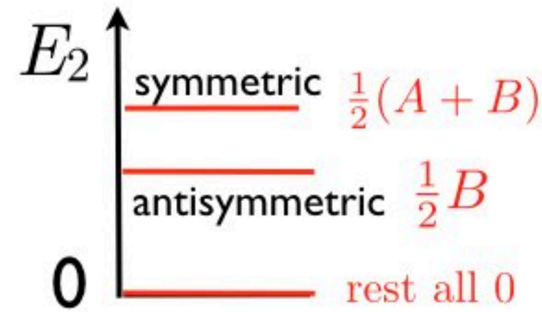
- relative coordinate of a pair of particles behaves like a single particle



two-particle energy levels

- Solvable model! ("short-range pseudopotential")

$$U(r_{12}) = \left(A + B\left(\frac{(r_{12})^2}{\ell_B^2}\right)\right) e^{-\frac{(r_{12})^2}{2\ell_B^2}}$$



- Laughlin state

$$|\Psi_L^m\rangle = \prod_{i<j}\left(a_i^\dagger - a_j^\dagger\right)^m |0\rangle$$

$$a_i|0\rangle = 0 \qquad a_i^\dagger = \frac{R^x + iR^y}{\sqrt{2}\ell_B}$$

$$E_L = 0 \qquad [a_i, a_j^\dagger] = \delta_{ij}$$

maximum density null state

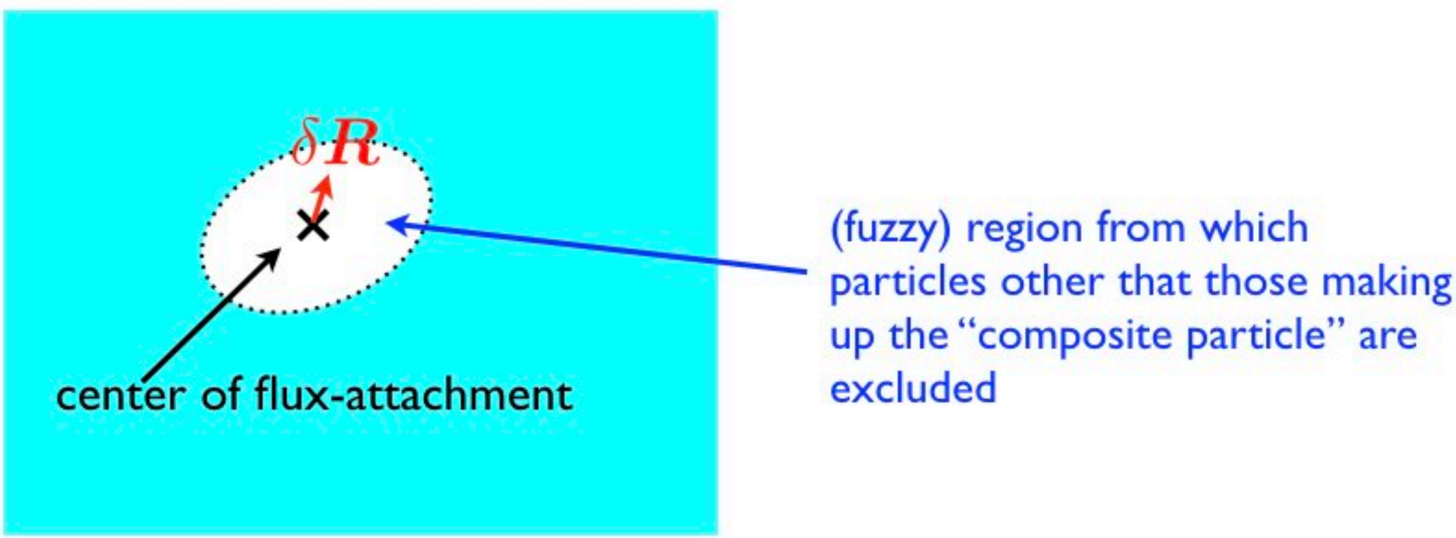
- m=2: (bosons): all pairs avoid the symmetric state $E_2 = \frac{1}{2}(A+B)$

- m=3: (fermions): all pairs avoid the antisymmetric state $E_2 = \frac{1}{2}B$

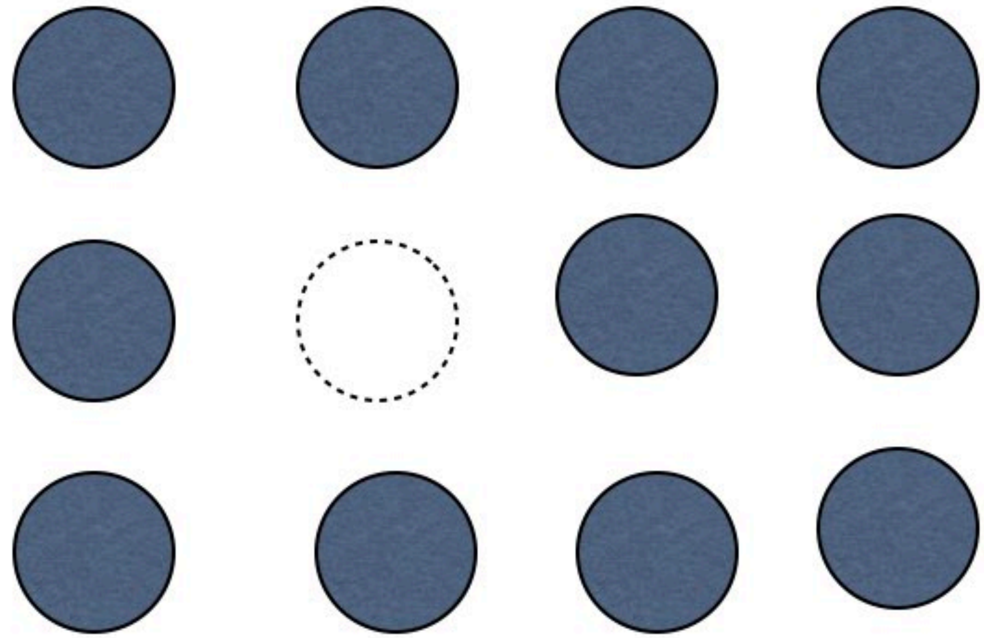
- the essential unit of the 1/3 Laughlin state is the electron bound to a correlation hole corresponding to three "units of flux", or three of the available single-particle states which are exclusively occupied by the particle to which they are "attached"
- In general, the elementary unit of the FQHE fluid is a "composite boson" of p particles with q "attached flux quanta"
- This is the analog of a unit cell in a solid....

- Flux attachment is a gauge condensation that removes the gauge ambiguity of the guiding centers, giving each one a "natural" origin, so they define a physical electric dipole moment of the "composite particle" in which they are bound by the "attached flux".

- This is analogous to how the "the vector potential becomes an observable" (in a hand-waving way) in the London equations for a superconductor.



- quantum solid
- unit cell is correlation hole
- defines geometry



- repulsion of other particles make an attractive potential well strong enough to bind particle

solid melts if well is not strong enough to contain zero-point motion (Helium liquids)

- In Maxwell's equations, the momentum density is

$$\pi_i = \epsilon_{ijk} D^j B_k \qquad D^i = \epsilon_0 \delta^{ij} E_j + P^i$$

- The momentum of the condensed matter is

$$\boldsymbol{p} = \boldsymbol{d} \times \boldsymbol{B}$$

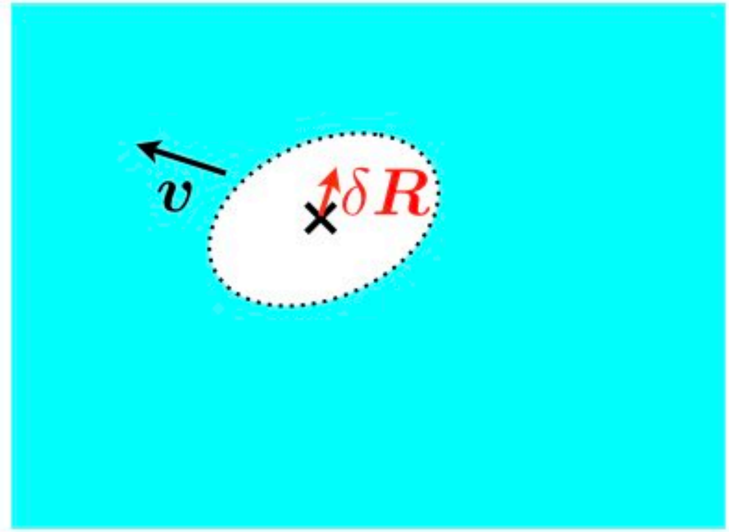
electric dipole moment

- in 2D the guiding-center momentum then is

$$p_a = eB\epsilon_{ab}\delta R^b$$

- The electrical polarization energy of the dielectric composite particle then gives its energy-momentum dispersion relation, with no involvement of any "Newtonian inertia" involving an effective mass

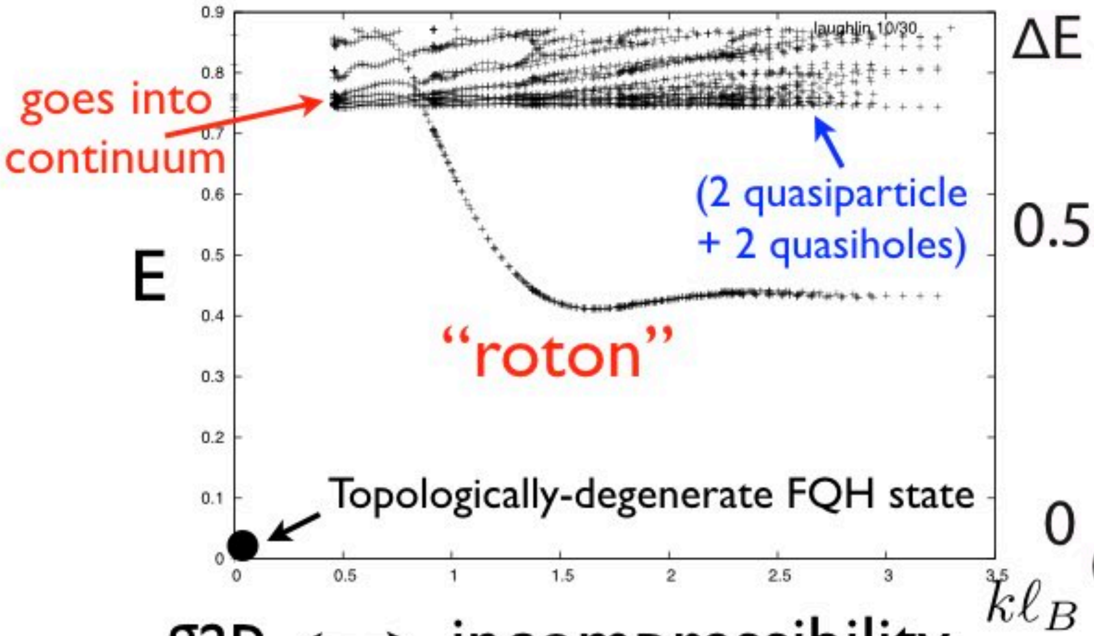
- The Berry phase generated by motion of the "other particles" that "get out of the way" as the vortex-like "flux-attachment" moves with the particle(s) it encloses can be formally-described as a Chern-Simons gauge field that cancels the Bohm-Aharonov phase, so that the composite object propagates like a neutral particle.

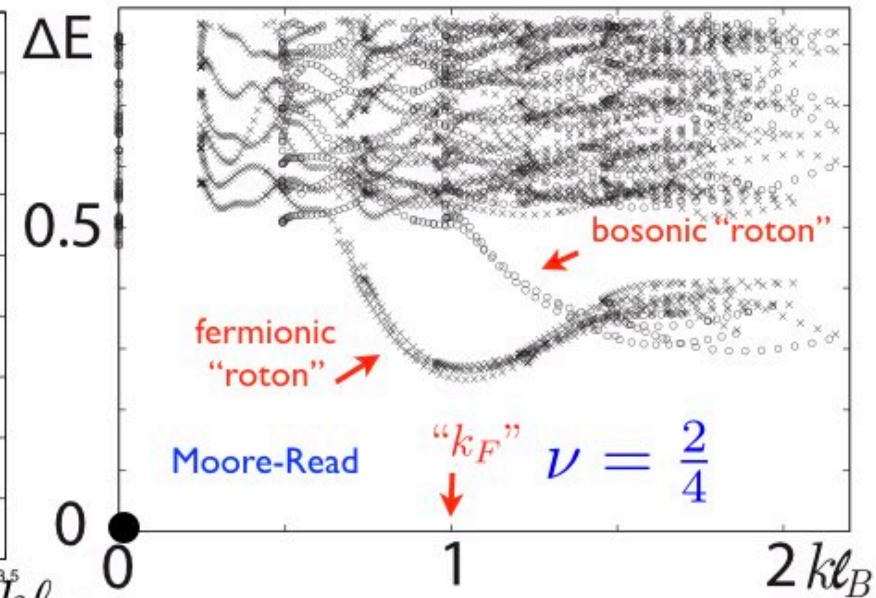


- If the composite particle is a boson, it condenses into the zero-momentum **(zero electric dipole-moment)** inversion-symmetric state, giving an incompressible-fluid Fractional Quantum Hall state, with an energy gap for excitations that carry momentum or electric dipole moment ("quantum incompressibility", no transmission of pressure through the bulk).

- All FQH states have an elementary unit (analogous to the unit cell of a crystal) that is a composite boson under exchange.

- It may be sometimes be useful to describe this boson as a a bound state of composite fermions (with their own preexisting flux attachment) bound by extra flux (Jain's picture)





gap $\longleftrightarrow$ incompressibility

Collective mode with short-range V_1 pseudopotential, 1/3 filling (Laughlin state is exact ground state in that case)

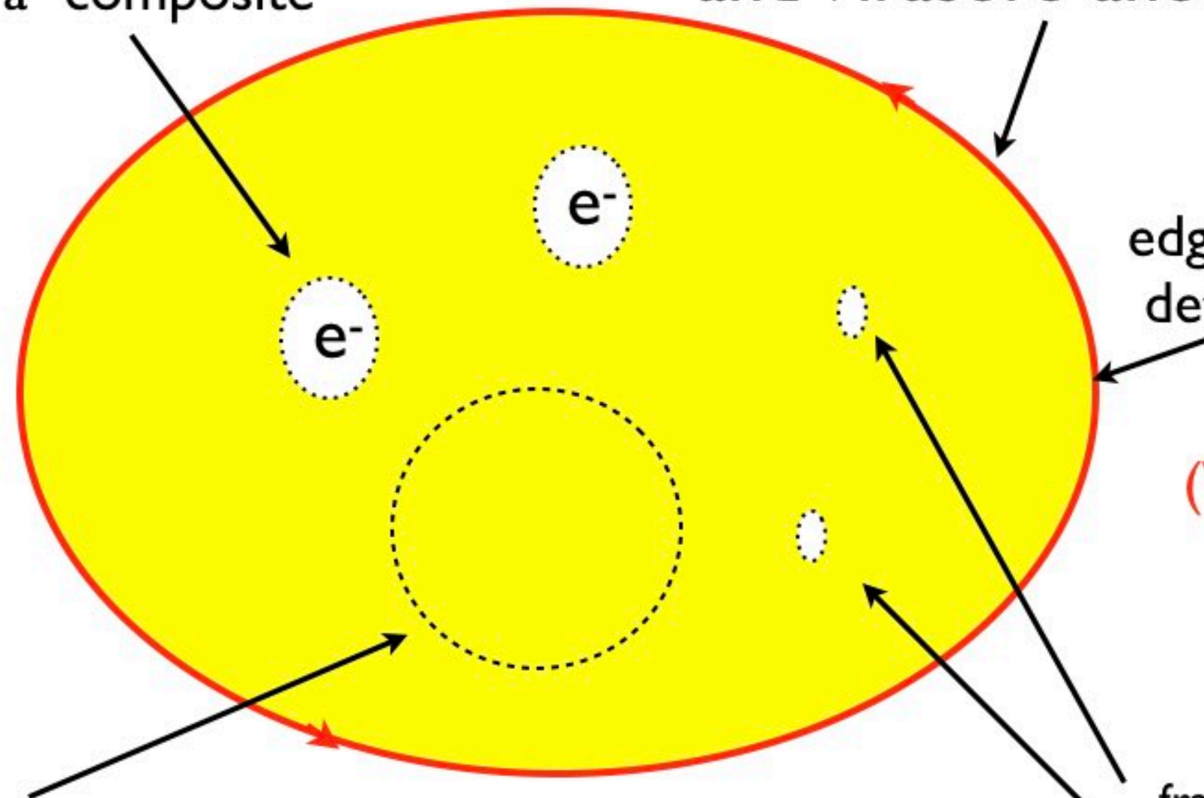
Collective mode with short-range three-body pseudopotential, 1/2 filling (Moore-Read state is exact ground state in that case)

- momentum $\hbar$k of a quasiparticle-quasihole pair is proportional to its **electric dipole moment $\mathbf{p}_e$**

$$\hbar k_a = \epsilon_{ab} B p_e^b$$

gap for electric dipole excitations is a MUCH stronger condition than charge gap: fluid **does not transmit pressure through bulk!**

• Anatomy of Laughlin state

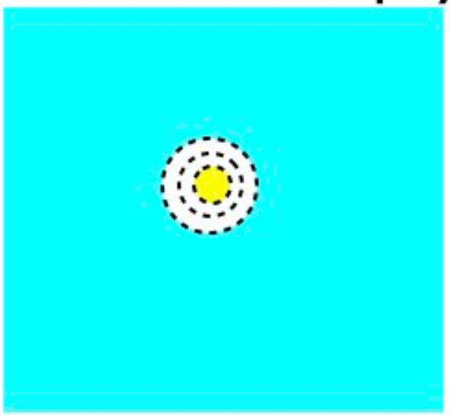


- the essential unit of the 1/3 Laughlin state is the electron bound to a correlation hole corresponding to "units of flux", or three of the available single-particle states which are exclusively occupied by the particle to which they are "attached"

- In general, the elementary unit of the FQHE fluid is a "composite boson" of p particles with q "attached flux quanta"

- This is the analog of a unit cell in a solid....

- The Laughlin state is parametrized by a unimodular metric: what is its physical meaning?



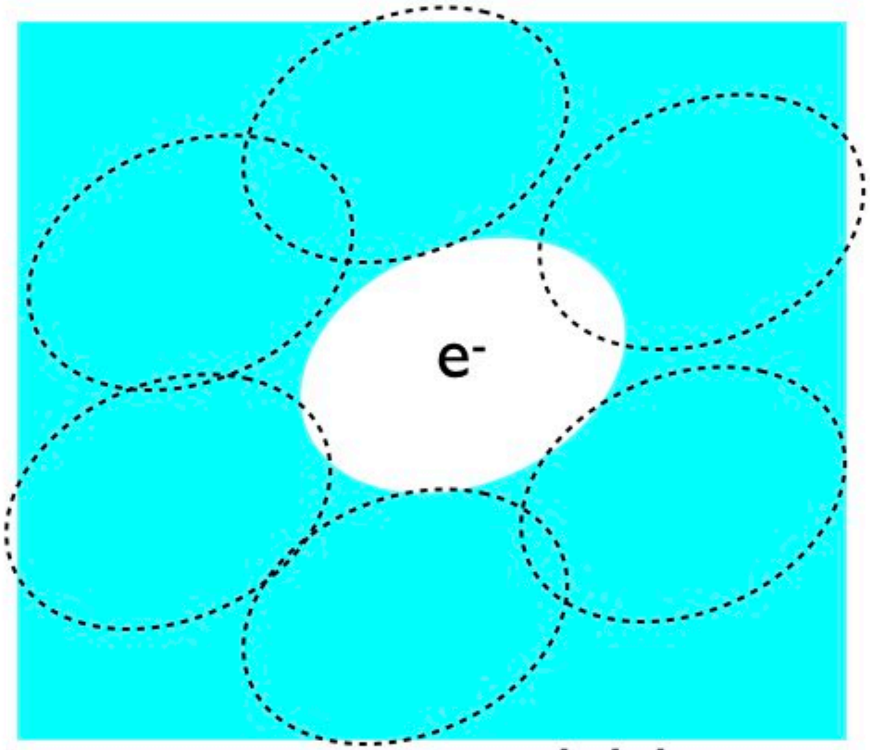
correlation holes in two states with different metrics

- In the $\nu = 1/3$ Laughlin state, each electron sits in a correlation hole with an area containing 3 flux quanta. The metric controls the *shape* of the correlation hole.

- In the $\nu = 1$ filled LL Slater-determinant state, there is <u>no</u> correlation hole (just an exchange hole), and this state does <u>**not**</u> depend on a metric

but no broken symmetry

- similar story in FQHE:



- "flux attachment" creates correlation hole
- defines an emergent geometry
- potential well must be strong enough to bind electron
- continuum model, but similar physics to Hubbard model
- new physics: Hall viscosity, geometry............

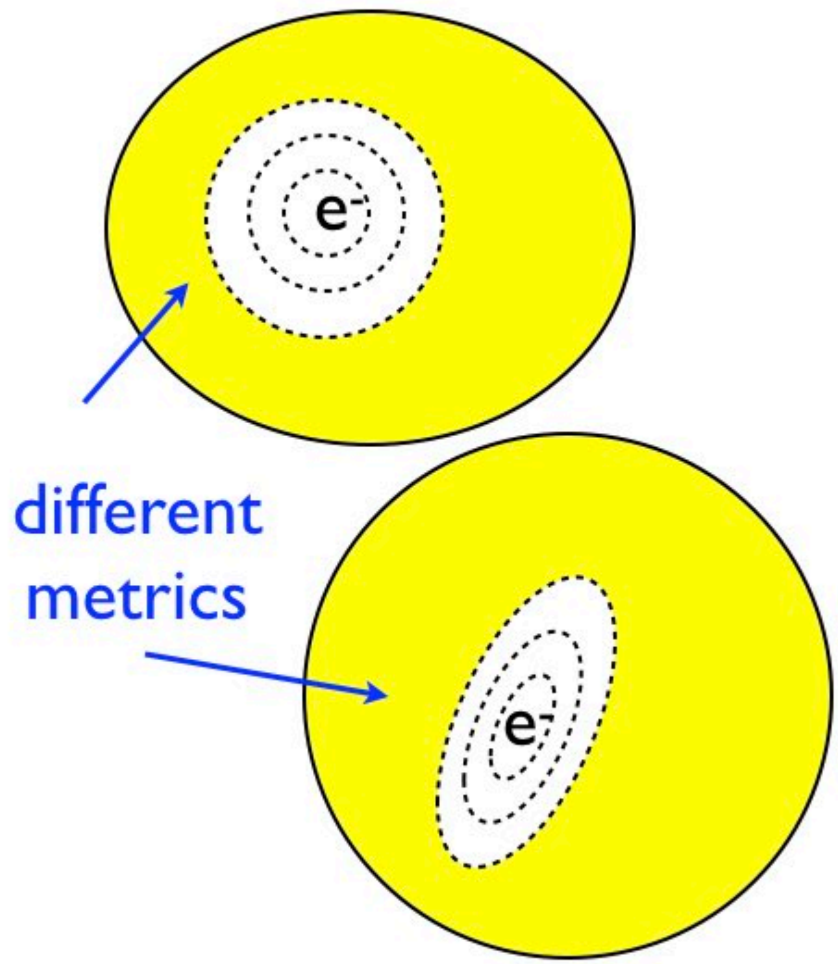
- composite boson: if the central orbital of a basis of eigenstates of L(g) is filled, the next two are empty

$$|\Psi_L^3\rangle = \prod_{i<j}\left(a_i^\dagger - a_j^\dagger\right)^3 |0\rangle$$

$$L(g)|\psi_m\rangle = (m + \tfrac{1}{2})|\psi_m\rangle$$

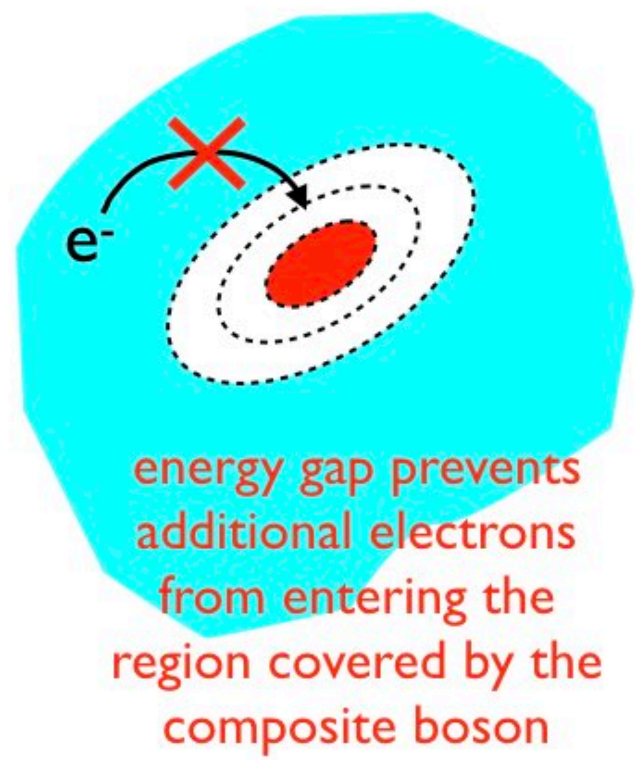
- this correlation hole is equivalent to "attachment of three flux quanta" or vortices that travel with the particle, generating a Berry phase that cancels the Bohm-Aharonov phase and transmutes Fermi to Bose exchange statistics.

- this shape of the corelation hole - and hence its correlation energy - varies with the metric g_{ab}

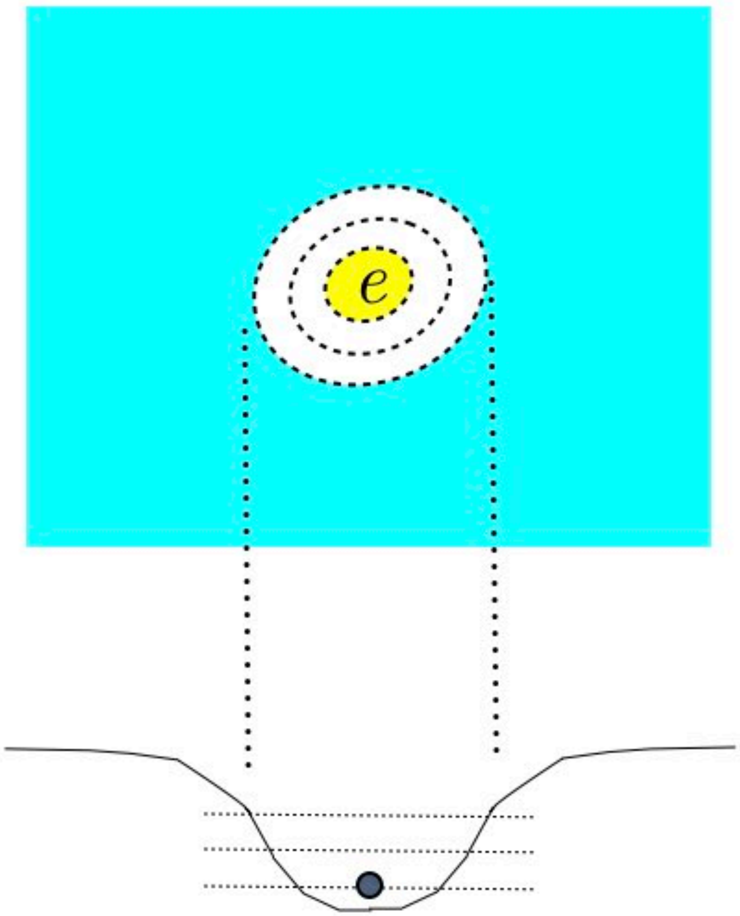


- Origin of FQHE incompressibility is analogous to origin of **Mott-Hubbard gap** in lattice systems.
- There is an energy gap for putting an **extra particle** in a quantized region that is **already occupied**

- **On the lattice** the "quantized region" is an atomic orbital with a fixed shape
- **In the FQHE** only the **area** of the "quantized region" is fixed. The **shape** must adjust to minimize the correlation energy.



$1/3$ Laughlin state



If the central orbital is filled, the next two are empty

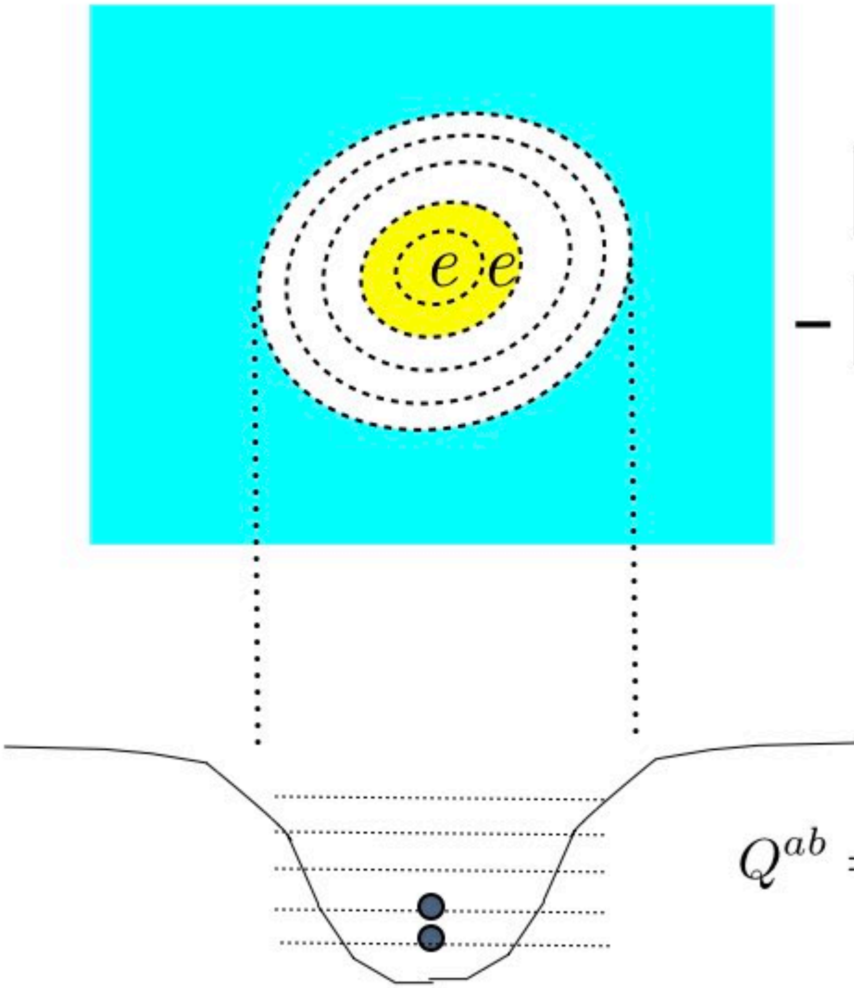
The composite boson has inversion symmetry about its center

It has a "spin"

$$
\begin{array}{rccccr}
 & \frac{1}{2} & \frac{3}{2} & \frac{5}{2} & & \\
 & \boxed{1} & \boxed{0} & \boxed{0} & \cdots\cdots & L = \frac{1}{2} \\
- & \boxed{\frac{1}{3}} & \boxed{\frac{1}{3}} & \boxed{\frac{1}{3}} & \cdots\cdots & - L = \frac{3}{2} \\
\hline
 & & & & & s = -1
\end{array}
$$

the electron excludes other particles from a region containing 3 flux quanta, creating a potential well in which it is bound

$2/5$ state



$$
\begin{array}{ccccc}
\frac{1}{2} & \frac{3}{2} & \frac{5}{2} & & \\
\end{array}
$$

$$
\begin{array}{|c|c|c|c|c|}
\hline
1 & 1 & 0 & 0 & 0 \\
\hline
\end{array}\ \cdots\cdots \qquad L = 2
$$

$$
-\ \begin{array}{|c|c|c|c|c|}
\hline
\frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} & \frac{2}{5} \\
\hline
\end{array}\ \cdots\cdots \qquad -L = 5
$$

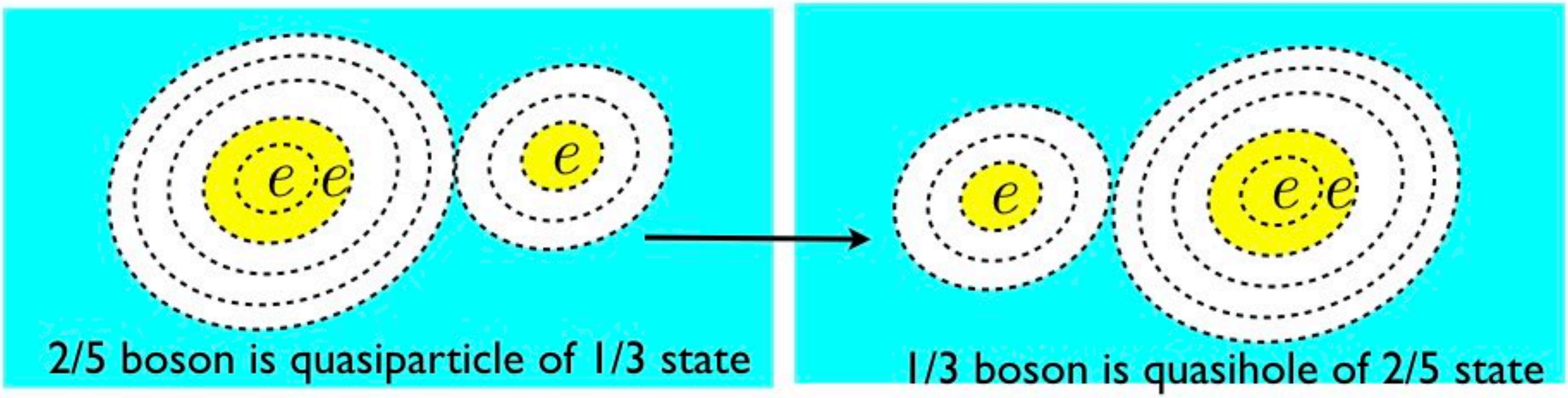
$$
s = -3
$$

$$
L = \frac{g_{ab}}{2\ell_B^2} \sum_i R_i^a R_i^b
$$

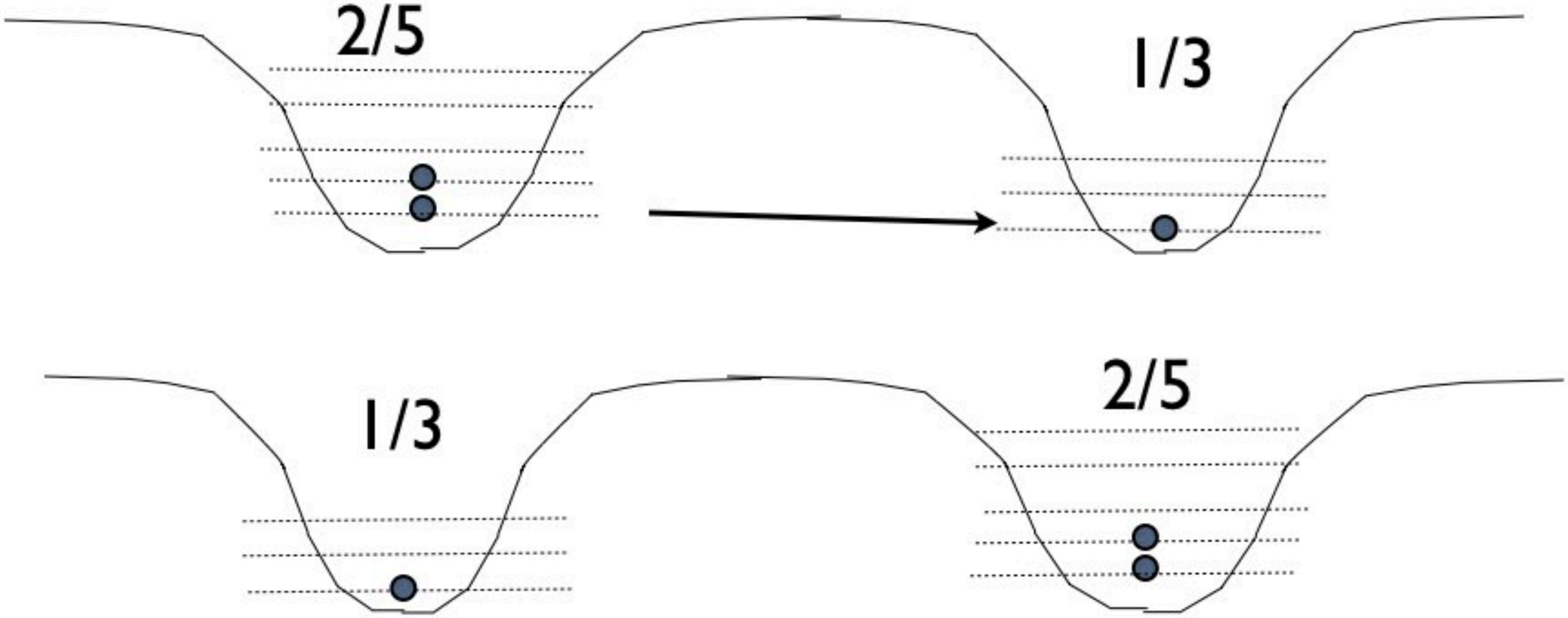
$$
Q^{ab} = \int d^2 r\, r^a r^b \delta\rho(r) = s\ell_B^2 g^{ab}
$$

second moment of neutral
composite boson
charge distribution

hopping of a "composite fermion" (electron + 2 flux quanta)



Jain's "pseudo Landau levels"

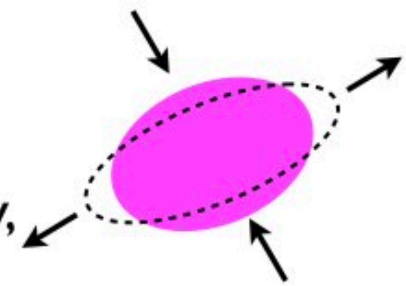


- The composite boson behaves as a neutral particle because the Berry phase (from the disturbance of the the other particles as its "exclusion zone" moves with it) cancels the Bohm-Aharonov phase

- It behaves as a boson provided its statistical spin cancels the particle exchange factor when two composite bosons are exchanged

p particles	$(-1)^{pq} = (-1)^p$	fermions
q orbitals	$(-1)^{pq} = 1$	bosons

- The metric (shape of the composite boson) has a preferred shape that minimizes the correlation energy, but fluctuates around that shape



$\delta E \propto (\mathrm{distortion})^2$

- The zero-point fluctuations of the metric are seen as the O(q^4) behavior of the "guiding-center structure factor" (Girvin et al, (GMP), 1985)
- long-wavelength limit of GMP collective mode is fluctuations of (spatial) metric (analog of "graviton")

FDMH, Phys. Rev. Lett. **107**, 116801 (2011)

- Furthermore, the local electric charge density of the fluid with $\nu = p/q$ is determined by a combination of the magnetic flux density and the Gaussian curvature of the intrinsic metric

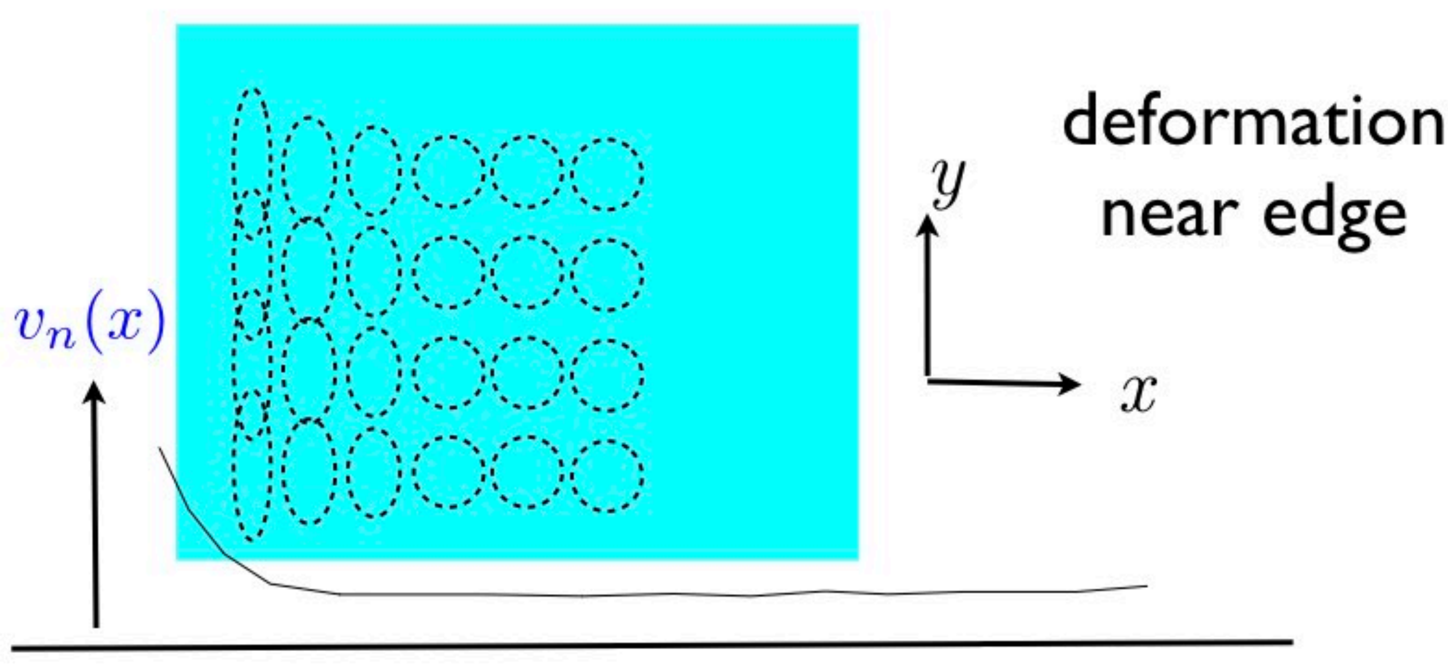
$$J^0_e(\boldsymbol{x}) = \frac{e}{2\pi q}\left(\frac{peB}{\hbar} - sK_g(\boldsymbol{x})\right)$$

Topologically quantized "guiding center spin"

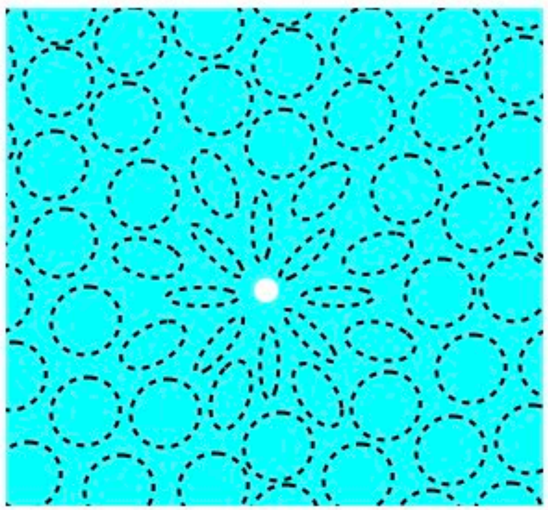
Gaussian curvature of the metric

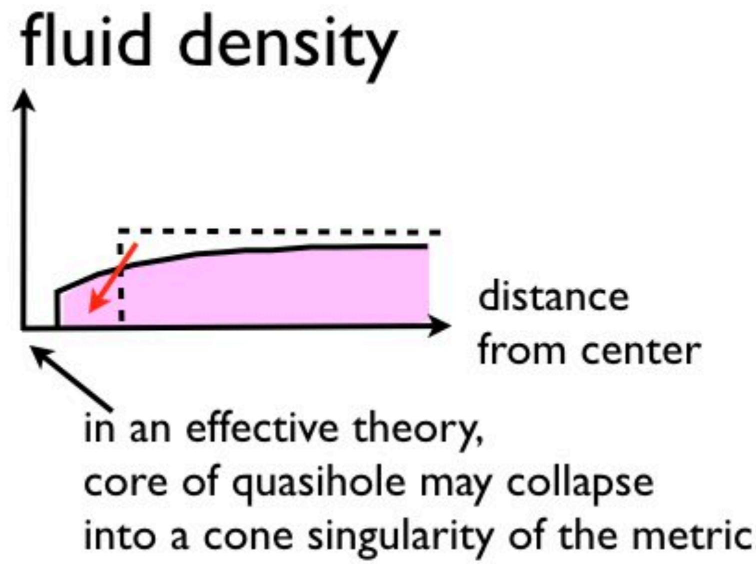
- In fact, it is locally determined, if there is an inhomogeneous slowly-varying substrate potential

$$H = \sum_i v_n(\boldsymbol{R}_i) + \sum_{i<j} V_n(\boldsymbol{R}_i - \boldsymbol{R}_j)$$



- “skyrmion”-like “cone”-like structure moves charge away from quasihole by introducing negative Gaussian curvature





- effective bulk action: $\sigma_H = \dfrac{(pe)^2}{2\pi\hbar K}$

$U(1)$ Chern-Simons field

$U(1)$ condensate field

"spin connection" of the metric

$$S = \int d^2x dt\, \mathcal{L}_0 - \mathcal{H}$$

$$\mathcal{L}_0 = \frac{\hbar}{4\pi}\epsilon^{\mu\nu\lambda}\left(K^{-1} b_\mu \partial_\nu b_\lambda + \beta\omega_\mu\partial_\nu\omega_\lambda\right) + J_b^\mu\left(\hbar(\partial_\mu\varphi - b_\mu - S\omega_\mu) + peA_\mu\right)$$

$$\mathcal{H} = \sqrt{g}\left(\varepsilon(\boldsymbol{v}, B) - U(g, B, P) - (E_a + \epsilon_{ab}v^b B)P^a\right)$$

kinetic energy of flow

metric-dependent correlation energy

- In the standard incompressible FQH states, the bulk interior of the fluid is described by a gapped topological field theory (TQFT).
- The gapless edge degrees of freedom are a direct sum of unitary representations of the Virasoro algebra.
- Can there be continuous second order transitions between FQH states at which the bulk gap collapses?

- The (fermion) "Gaffnian" model (Steve Simon et al)
- This is a model 2/5 state that (a) is an exact zero-energy state of a (three-body) interaction (b) has a non-unitary representation of the Virasoro algebra on its edge and (c) as a consequence is believed to have bulk gapless neutral excitations (Read).

- It is a Jack polynomial with a "root configuration exclusion statistics rule" of "not more than two particles in five consecutive orbitals"

- The "Gaffnian" interaction penalizes three-body states

$(z_1 - z_2)(z_2 - z_3)(z_3 - z_1)$ 11100

$(z_1 - z_2)(z_2 - z_3)(z_3 - z_1) \times \left((z_1 - z_2)^2 + (z_2 - z_3)^2 + (z_3 - z_1)^2\right)$ 11001

$$H = V_0 P_{111} + V_2 P_{11001}$$

- On the torus, the 2/5 Gaffnian zero-energy states has a 10-fold degeneracy corresponding to the two sets of 5 “motifs”

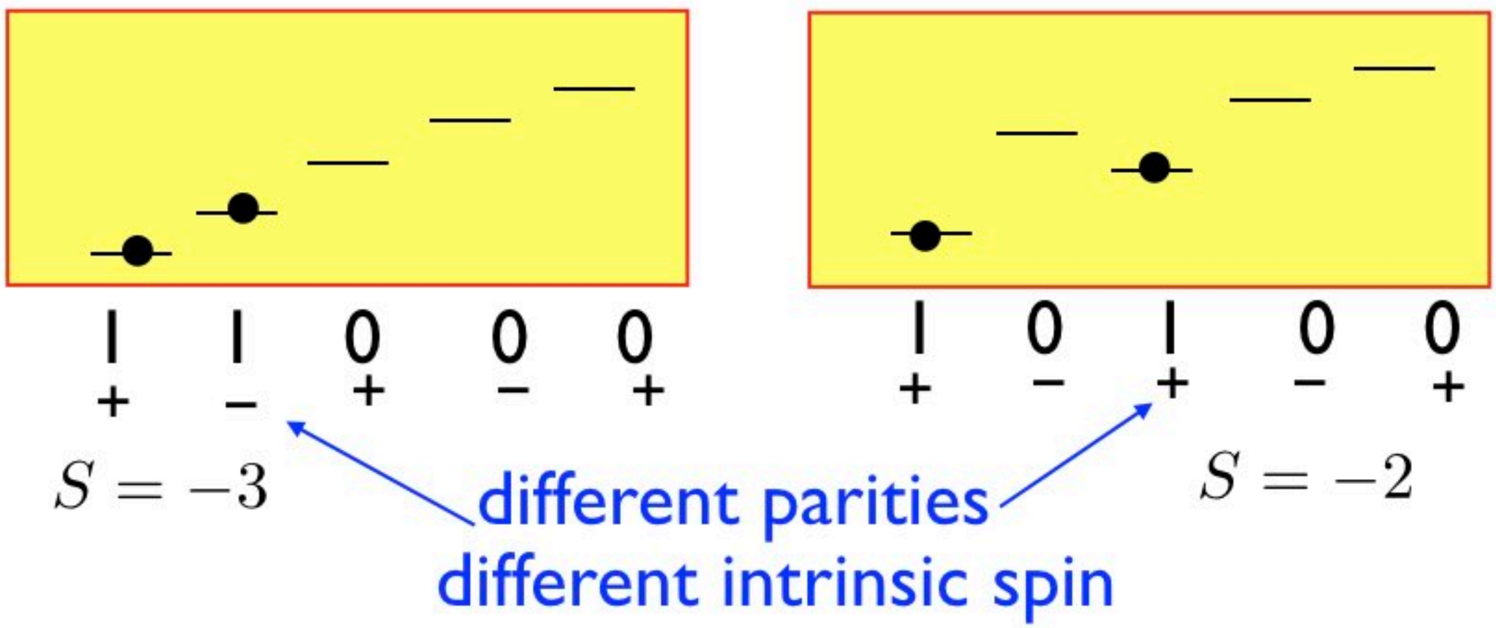
11000 01100 00110 00011 10001

10100 01010 00101 10010 01001

↑ lowest weight (most to left)

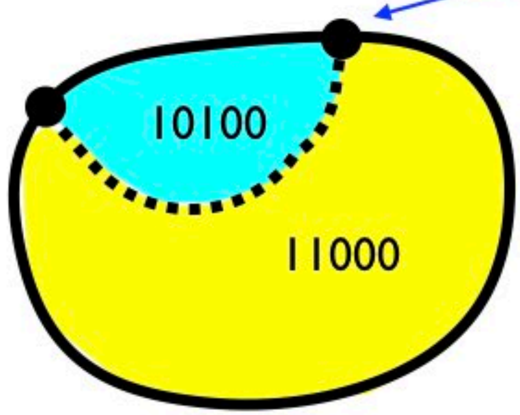
- A degeneracy beween two internal states of the 2/5 “composite boson” with different parity.

- In higher Landau levels the "10100" pattern may replace 11000 as the stable 2/5 pattern because of competition between the "vacancy potential" that favors putting the second particle in the second orbital, and repulsion from the first particle, which pushes it outwards



- Domain wall between states with different Wen-Zee term carries momentum density (electric dipole moment) but no chiral modes (no U(1) c Virasoro anomaly)

negative weight primary field of non-unitary CFT ???



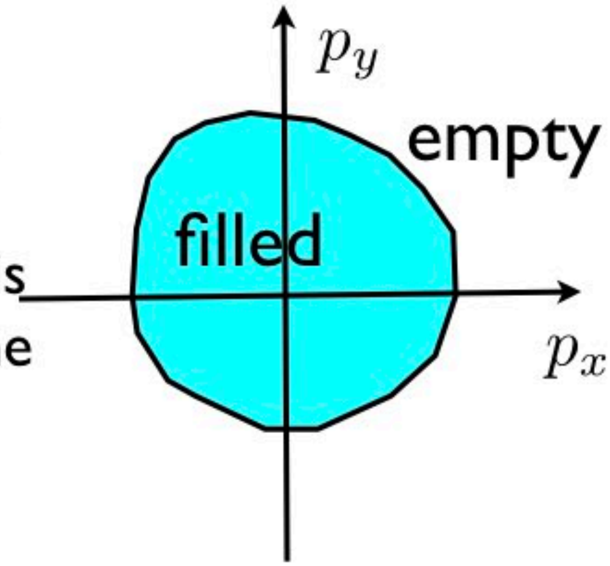
- gapless bulk if domain wall energy is zero or negative

<u>transmits pressure!</u>

- sliding of domain wall attachment point removes momentum from edge (non-unitary virasoro on edge)

- Many open questions about the gapless critical state (e.g. what is the dynamical critical exponent z (1or 2?))
- does charge gap exist for all ratios of the two parameters?
- develop a Full interpretation of the non-unitary Virasoro representation.

- The composite particle may also be a fermion. Then one gets a Fermi surface in momentum-space = electric dipole space, and a gapless anomalous Hall effect which is quantized when the Berry phase cancels the Bohm-aharonov phase. (HLR-type state)



- There must be a distribution of dipole moments (or momentum) of the composite fermions, centered at the inversion-symmetric zero-moment state which has lowest energy. These are quantized by a pbc, and no two composite fermions can have the same diople moment.

- Fermi surface quasiparticle formulas for anomalous Hall effect (FDMH 2006)
- in 2D:

$$\sigma_H = \frac{e^2}{2\pi\hbar}\left(n + \frac{\phi}{2\pi}\right)$$

Integer determined
at edge

$$e^{i\phi}$$

Berry phase for
moving a quasiparticle around
Fermi surface (arc)

- holomorphic representations of guiding-center states

$$\frac{R^a}{\sqrt{2}\ell_B} = w^a a^\dagger + w^a a \qquad [a, a^\dagger] = 1$$

$$(w_a)^* w_a = \tfrac{1}{2}(g_{ab} + i\epsilon_{ab}) \qquad w_a = g_{ab} w^b \quad \det g = 1$$

- This is the Girvin-Jach formalism, except they implicitly assumed the metric g_{ab} was the Euclidean metric of the plane. **In fact, it is a <u>free choice</u>, not fixed by the any physics of the problem.**

- Then, once a metric (i.e., a complex structure) has been chosen, a one-particle state can be described as

$$|\Psi\rangle = f(a^\dagger)|0\rangle \qquad a|0\rangle = 0$$

holomorphic

- Both the "vacuum" $|0\rangle$ and the function f(z) vary as the metric is changed (a Bogoliubov transformation)

- Normalization/overlap:

$$\langle\Psi_1|\Psi_2\rangle = \int \frac{dz \wedge dz^*}{2\pi i} f_1(z)^* f_2(z) e^{-z^* z}$$

- When compactified on the torus with flux N_Φ , the modular-invariant formulation is

$$f(z) \propto \prod_{i=1}^{N_\Phi} \tilde{\sigma}(z - w_i) \quad \sum_i w_i = 0$$

Bravais lattice in complex plane

$$\tilde{\sigma}(z|\{L\}) = e^{\frac{1}{2}C_2(\{L\})z^2} \sigma(z|\{L\})$$

"almost holomorphic modular invariant" (Eisenstein series)

Weierstrass sigma function

- In the Heisenberg-algebra reinterpretation

$$|\Psi\rangle = \prod_{i=1}^{N_\Phi} \tilde{\sigma}(a_i^\dagger - w_i)|0\rangle \qquad \sum_i w_i = 0$$

one particle
$N = 1$

- The filled Landau level is

$$|\Psi\rangle = \left(\prod_{i<j} \tilde{\sigma}(a_i^\dagger - a_j^\dagger)\tilde{\sigma}(\textstyle\sum_i a_i^\dagger) \right) |0\rangle$$

filled Level
$N = N_\Phi$

- The Laughlin states are

$\nu = \frac{1}{m}$ Laughlin state
$N_\Phi = mN$

$$|\Psi\rangle = \left(\prod_{i<j} \tilde{\sigma}(a_i^\dagger - a_j^\dagger)^m \right) \prod_{k=1}^{m} \tilde{\sigma}(\textstyle\sum_i a_i^\dagger - w_k)|0\rangle \quad \sum_{k=1}^{m} w_k = 0.$$

- A previously unknown (?) identity that I recently guessed and found was indeed true, and which dramatically transforms calculations on torus (e.g., orders of magnitude Monte-Carlo speedup)

$$\langle\Psi_1|\Psi_2\rangle = \int_{\Box} \frac{dz \wedge dz^*}{2\pi i} f_1(z)^* f_2(z) e^{-z^* z}$$

$$= \frac{1}{N_\Phi} {\sum_z}' \qquad z \in \{\tfrac{mL_1+nL_2}{N_\Phi}\}$$

$(N_\Phi)^2$ points

replace integral over fundamental region by a modular-invariant finite sum

- with Ed Rezayi, I found a remarkable clean composite Fermi liquid model state on the flat torus, inspired by a construction by Jain on the sphere.
- On the torus, the state is precisely equivalent to the usual treatments of the Fermi gas with a pbc.
- It is very accurate as compared to exact diagonalization of the Coulomb interaction, and amazingly "almost" (99.99%) particle-hole symmetric at $\nu = 1/2$.

- Composite Fermi liquid (HLR-like) at $\nu = \frac{1}{m}$

gives Chern-Simons

gives bf? / Z2

$$f(\{z_i\}) = \prod_{i<j} \tilde{\sigma}(z_i - z_j)^{(m-2)} \det_{ij} M_{ij} \prod_{k=1}^{m} \tilde{\sigma}(\textstyle\sum_i z_i - w_k)$$

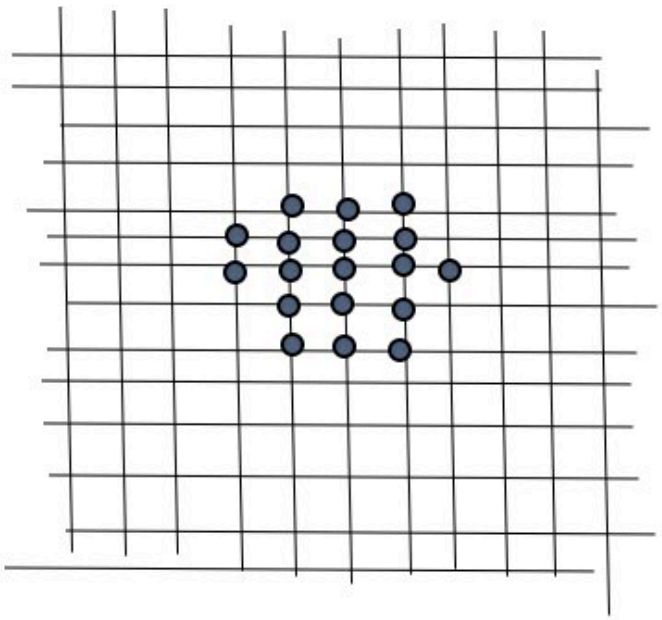
$$\sum_{\alpha=1}^{m} w_\alpha = \sum_{j=1}^{N} d_j = N\bar{d}$$

Fermi (Bose) for m even (odd)

$$M_{ij}(\{z_k\}; \{d_k\}) = e^{d_j^* z_i/m} {\prod_{k\neq i}}' \tilde{\sigma}(z_i - z_j - d_i + \bar{d})$$

a set of dipole moments $d_i \in \frac{L}{N}$ (particle number, not flux)

- There are vastly more possible choices of dipole "occupations" than independent states: The "good" ones are clusters that minimize $\sum_i |d_i - \bar{d}|^2$



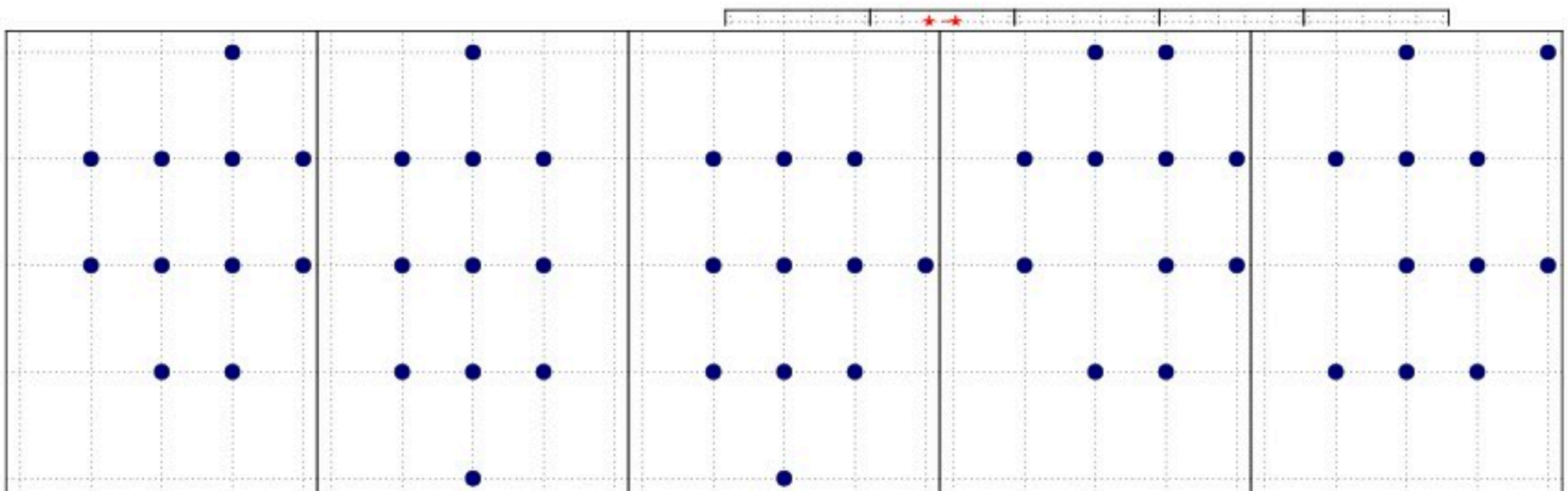
Computing ph symmetry

(with Scott Geraedts)

model state is numerically **very** close to p-h symmetry when k's are clustered

```
# Z_{COM}  overlap with PH-conjugate
overlap
0 0.999998870263 1.1297367517e-06
1 0.999999369175 6.3082507884e-07
2 0.99999860296 1.39704033186e-06
3 0.99999860296 1.3970403312e-06
4 0.999999369175 6.30825078063e-07
5 0.999998870263 1.12973675237e-06
6 0.999999369175 6.30825079173e-07
7 0.99999860296 1.39704032942e-06
8 0.99999860296 1.39704032909e-06
9 0.999999369175 6.30825078507e-07
```

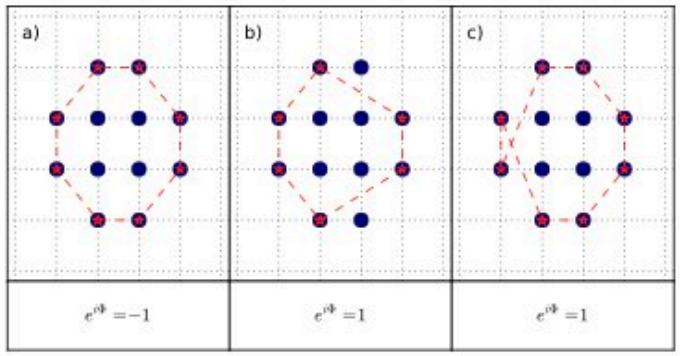
- particle-hole symmetry, and Kramers Z_2 structure (Scott Geraedts and Jie Wang)

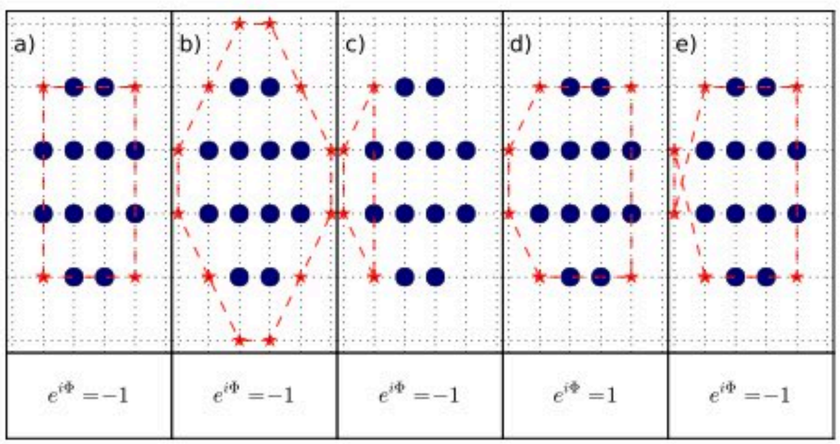


ED ΔE	0.0	ED ΔE	0.000404	ED ΔE	0.000268	ED ΔE	0.005555	ED ΔE	0.0
overlap	0.981955	overlap	0.736717	overlap	0.992142	overlap	0.226429	overlap	0.276141
ΔE	0.000446	ΔE	0.001419	ΔE	0.000531	ΔE	0.009271	ΔE	0.011863
$\langle H^2 \rangle - \langle H \rangle^2$	6.81819e-06	$\langle H^2 \rangle - \langle H \rangle^2$	6.5675e-06	$\langle H^2 \rangle - \langle H \rangle^2$	4.88251e-06	$\langle H^2 \rangle - \langle H \rangle^2$	1.58755e-05	$\langle H^2 \rangle - \langle H \rangle^2$	3.72072e-05
PH symmetry	0.993543	PH symmetry	0.987538	PH symmetry	0.995312	PH symmetry	0.856664	PH symmetry	0.964223

A many-body ansatz for Berry phase

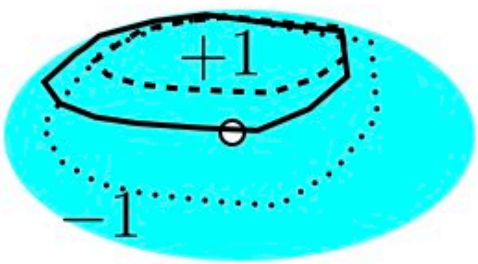
we confirmed all paths are real, by Kramers





These are ED results on exact Coulomb interaction states, with the exact particle hole symmetry, with occupation patterns obtained by finding the model states they have high overlap with

- is there an analog of Dirac cone point ?



State with the quantum numbers of an inversion symmetric Fermi sea with a single hole at the center (has an even number of particles)

A state on the Torus with these quantum numbers is a parity doublet

- as a hole is moved into the bulk, the ansatz must fail as if goes through the inversion-symmetric point!