The Kosterlitz-Thouless transition explored with atomic gases

Jean Dalibard

Solvay chair for Physics 2022

Lecture 2

This Solvay lecture series

Lecture 1 (inaugural), September 6: ***A brief history of cold atoms***

Lecture 2, September 20 (2:00 pm): ***From Bose-Einstein condensation to Kosterlitz-Thouless physics***

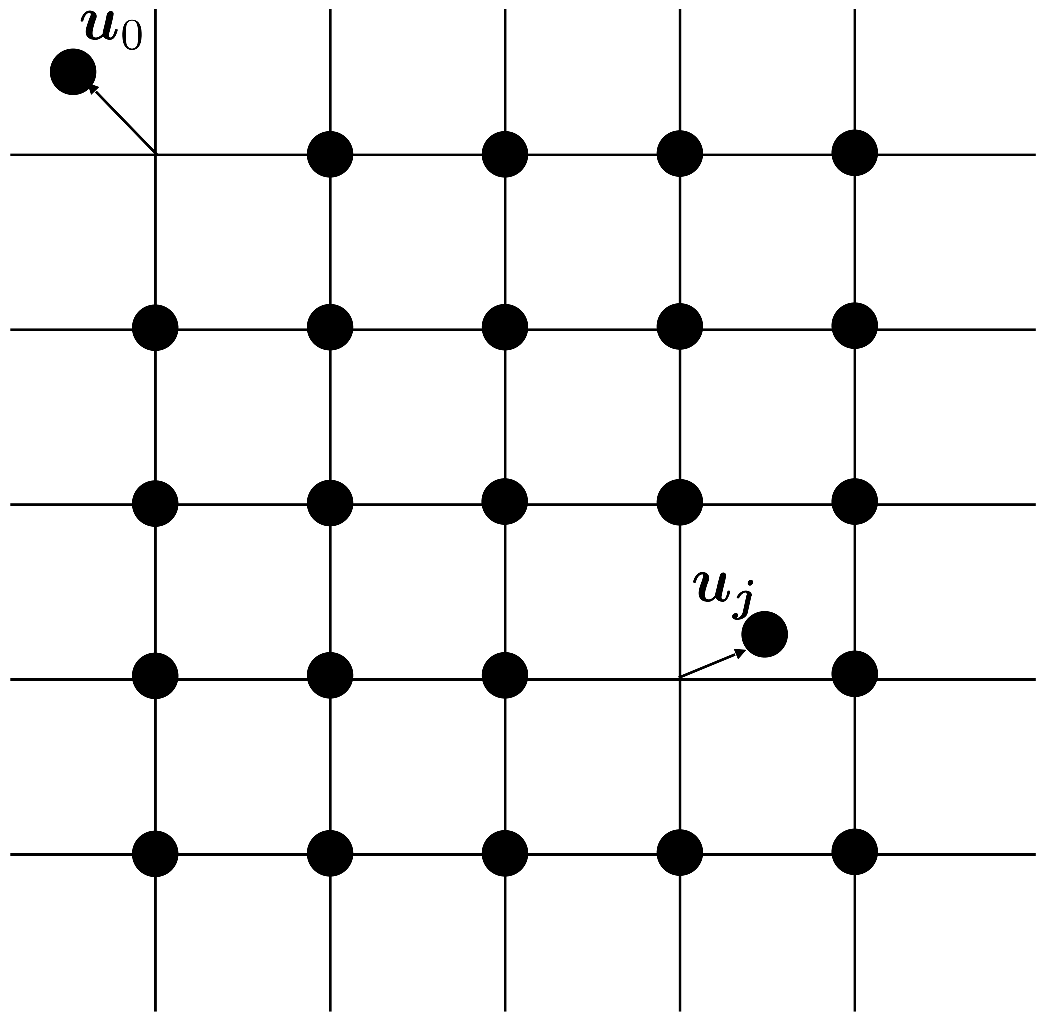
Lecture 3, September 23 (1:00 pm): ***Controlling atomic interactions: Scale invariance tested in the laboratory***

Lecture 4, September 28 (2:00 pm): ***A droplet of spin-1 atoms: the simplest many-body system***

Peierls, 1933: Physics in a low dimensional world

Would the usual physical objects like crystals or magnets exist in a 2D world?

At non-zero temperature there is no true crystalline order in dimension 1 or 2



$$\langle(\boldsymbol{u_j}-\boldsymbol{u}_0)^2\rangle \propto T\ \ln(R_j)$$

diverges at long distance

Generalization by Mermin-Wagner and Hohenberg (1966) for any system with short-range interactions:
no breaking of a continuous symmetry leading to a long-range order in the system ($T \neq 0$)

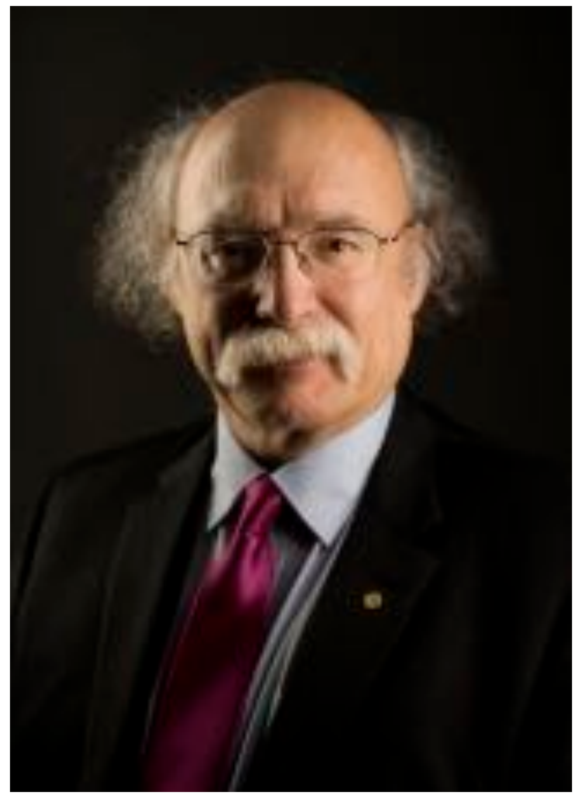
This result also applies to the case of Bose-Einstein condensation (U(1) symmetry)

Topological order

1973, Kosterlitz & Thouless (prelim: Berezinskii): Ordering, metastability and phase transitions in two-dimensional systems

J.M. Kosterlitz

D.J. Thouless



F.D. Haldane

In spite of Mermin-Wagner-Hohenberg theorem, “unconventional” phase transitions can still take place in 2D systems

Transition between two different kinds of disordered phases, that are topologically distinct

Outline of this lecture

1. The Peierls argument

2. The ideal 2D Bose gas

3. The Gross-Pitaevskii approach for the interacting 2D gas

4. The Kosterlitz- Thouless argument

5. Investigations with atomic, molecular and optical (AMO) systems

Hadzibabic & Dalibard, Riv. Nuo. Cim. 34, 389 + College de France lectures 2016-17

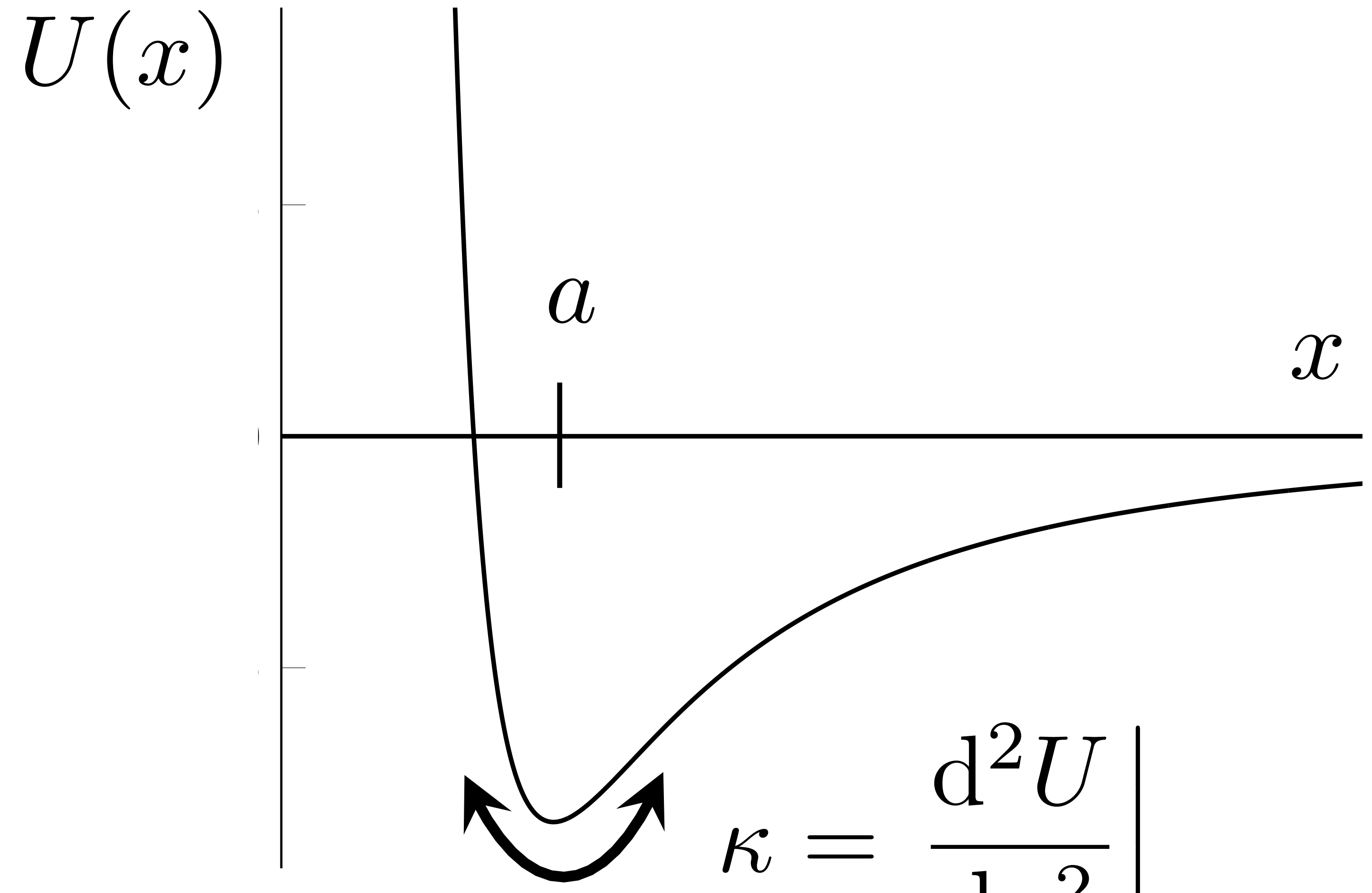
For superconductors: Benfatto, Castellani & Giamarchi, arXiv 1201.2307

1.

The Peierls argument in 1D, 2D, 3D

Simple argument in 1D: Piling up defects

Zero temperature + no quantum fluct.: ordered chain



$$\kappa = \left.\frac{\mathrm{d}^2 U}{\mathrm{d}x^2}\right|_{x=a}$$

Non-zero temperature:

We fix the position x_0 of the atom $j = 0$. The position of atom $j = 1$ can fluctuate:

$$x_1 = x_0 + a + \delta_1 \qquad \langle \delta_1 \rangle = 0 \qquad \langle \delta_1^2 \rangle \sim \frac{k_\mathrm{B} T}{\kappa}$$

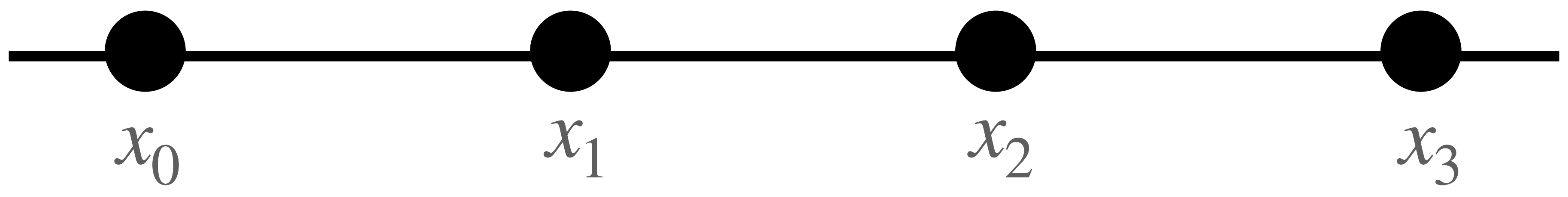
Then: $$x_2 = x_1 + a + \delta_2$$

$$\vdots$$

$$x_j = x_{j-1} + a + \delta_j \qquad \langle \delta_j^2 \rangle \sim \frac{k_\mathrm{B} T}{\kappa} \qquad \longrightarrow \qquad x_j = x_0 + ja + \Delta_j \qquad \Delta_j = \delta_1 + \delta_2 + \ldots + \delta_j$$

Sum of independent variables: $\langle \Delta_j^2 \rangle \sim \dfrac{k_\mathrm{B} T}{\kappa}\, j.$

Piling up defect (2)

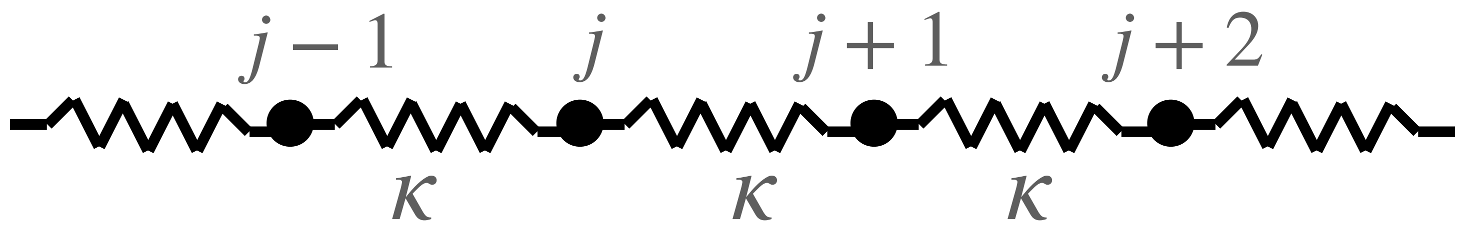


$$x_j = x_0 + ja + \Delta_j \qquad \langle \Delta_j^2 \rangle \sim \frac{k_\mathrm{B}T}{\kappa}\, j.$$

If $\langle \Delta_j^2 \rangle \gtrsim a^2$, i.e. $j \gtrsim \dfrac{\kappa a^2}{k_\mathrm{B}T}$, all information is lost regarding the position of atom j with respect to the crystal period

no long-range order

For a more rigorous argument, look at the collective modes of the chains (phonons):

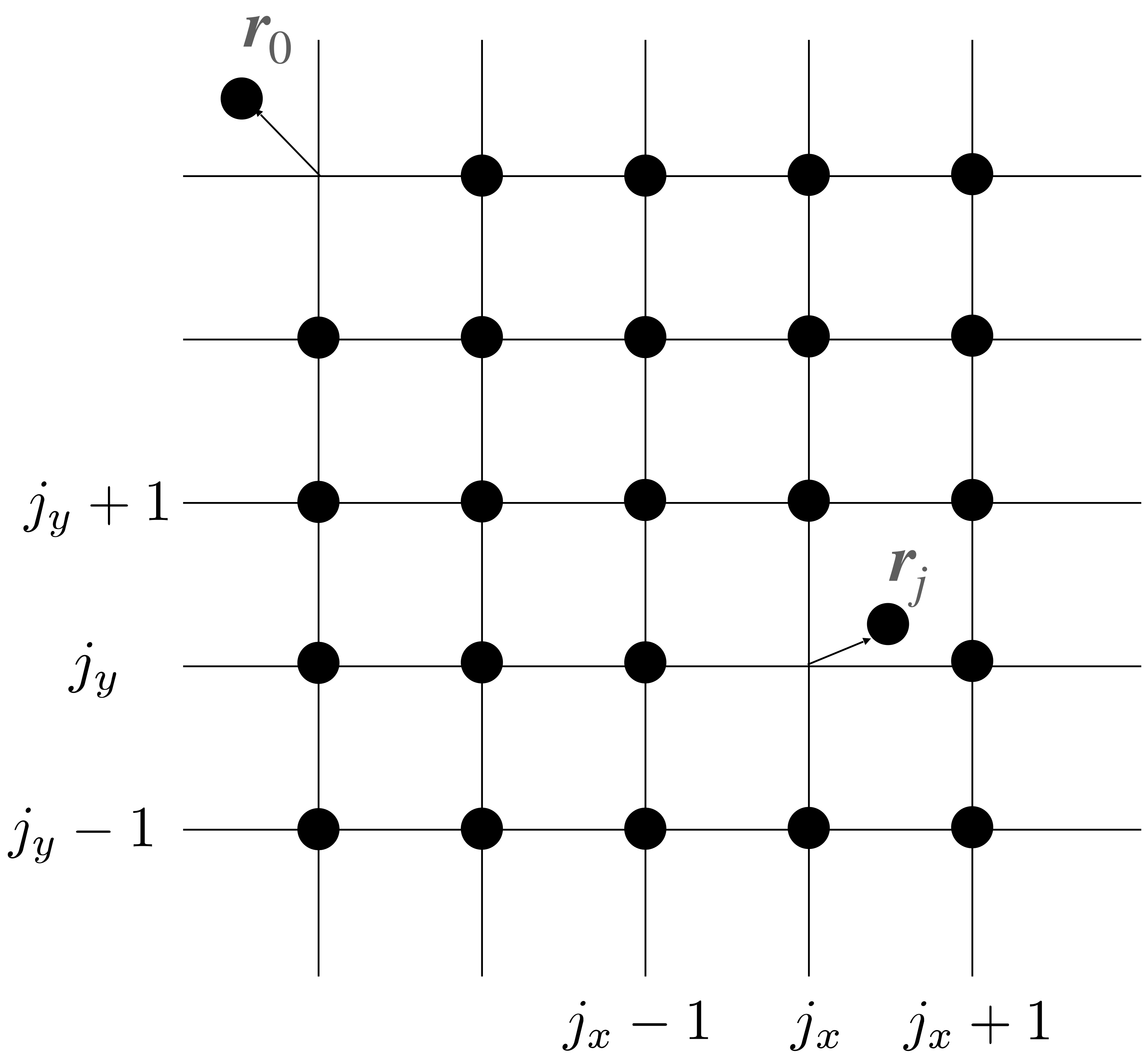


$$u_j = \frac{1}{\sqrt{N}} \sum_q \mathrm{e}^{\mathrm{i}qX_j}\, \tilde{u}_q \qquad X_j = ja \qquad E = \sum_q \frac{1}{2} m \dot{\tilde{u}}_q \dot{\tilde{u}}_{-q} + \frac{1}{2} m \omega_q^2 \tilde{u}_q \tilde{u}_{-q}$$

Collection of independent harmonic oscillators of wave vector q and frequency $\omega_q = 2\sqrt{\dfrac{\kappa}{m}}\sin(qa/2)$

Average at thermal equilibrium: $\langle \tilde{u}_q \tilde{u}_{q'}^* \rangle = 0$ if $q \neq q'$ and $\dfrac{1}{2} m\omega_q \langle |\tilde{u}_q|^2 \rangle = \dfrac{1}{2} k_\mathrm{B}T$

The 2D case



The detailed analysis is more complicated because of the two possible polarizations of the modes: parallel or perpendicular to $\boldsymbol{q}$

Scaling analysis: phonons $\boldsymbol{q} = (q_x, q_y)$ with $\omega_q = cq$ for low q

$$\frac{1}{2} m\omega_q \langle |\tilde{u}_q|^2 \rangle = \frac{1}{2} k_{\mathrm{B}} T$$

$$\boldsymbol{r}_j - \boldsymbol{r}_0 - \boldsymbol{j}a = \frac{1}{\sqrt{N}} \sum_{\boldsymbol{q}} \left(\mathrm{e}^{\mathrm{i}qja} - 1 \right) \tilde{\boldsymbol{u}}_q$$

Logarithmic divergence of $\langle (\boldsymbol{r}_j - \boldsymbol{r}_0 - \boldsymbol{j}a)^2 \rangle$ with the distance ja with a dominant contribution of $q \sim \pi / ja$

“Only logarithmic” in 2D: quasi-long range order

The 3D case

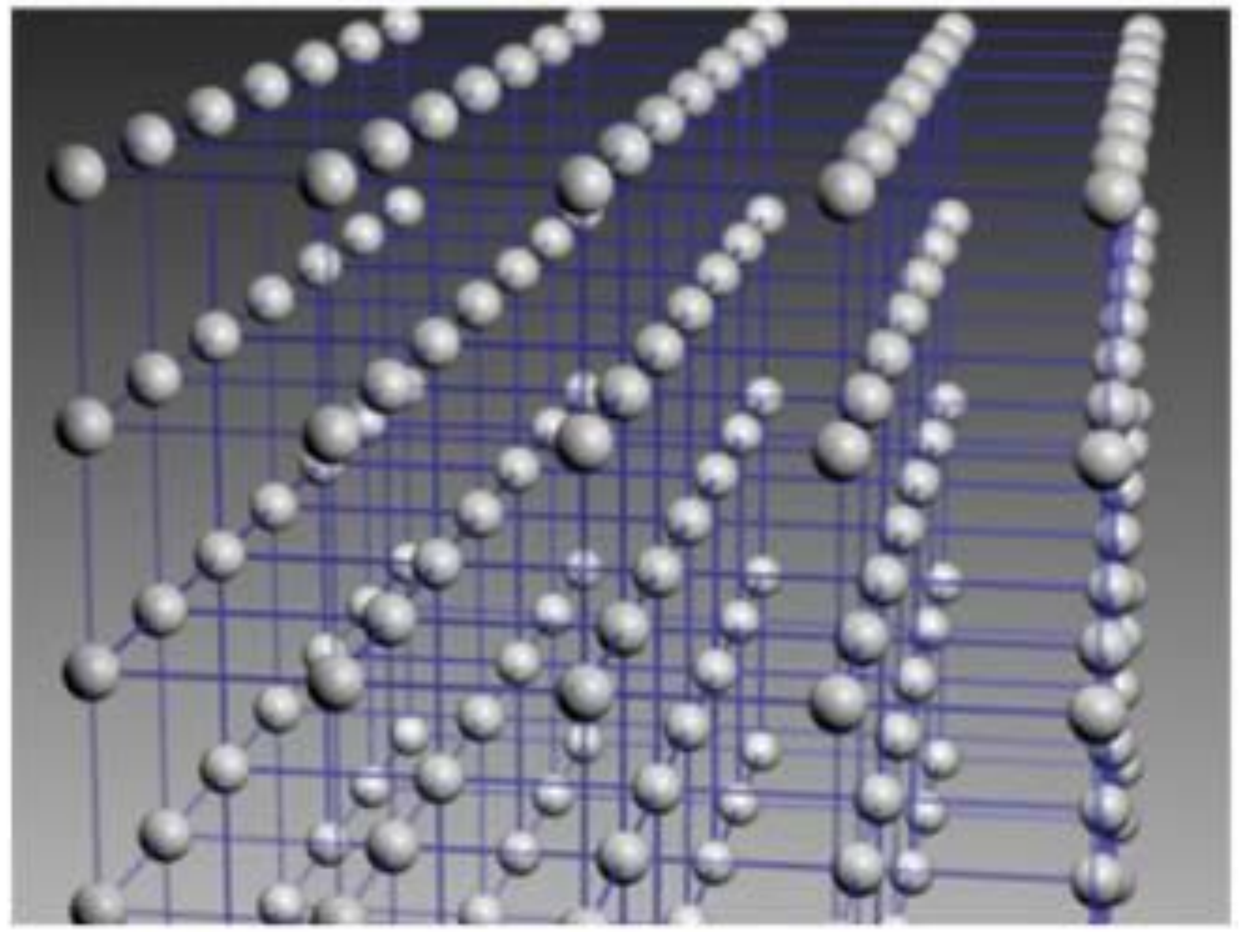


image I. Bloch

Same type of analysis:

$$\langle (\boldsymbol{u_j} - \boldsymbol{u}_0)^2 \rangle \sim \frac{a k_{\mathrm{B}} T}{\kappa} \int_{\mathrm{ZB}} \frac{\sin^2(\boldsymbol{q} \cdot \boldsymbol{R_j}/2)}{q^2} \, \mathrm{d}^3 q$$

$\mathrm{d}^3 q = q^2 \, \mathrm{d}q \, \mathrm{d}^2\Omega \longrightarrow$ no divergence anymore in $\boldsymbol{q} = 0$

One then finds that $\langle (\boldsymbol{r}_j - \boldsymbol{r}_0 - ja)^2 \rangle$ is independent of j

Cristalline order can exist over an infinite range

wikipedia

Mermin - Wagner - Hohenberg theorem

For a system with a dimension lower or equal to 2 and short-range interactions, there is no spontaneous breaking of a continuous symmetry at a non-zero temperature

- If the range is infinite, mean-field theory is valid and standard phase transitions predicted in this case can occur.

- Continuous symmetry: translation, Heisenberg magnetism, Bose-Einstein condensation. The theorem does not apply as such to discrete symmetries (Ising).

- At zero temperature the interacting Bose gas is condensed.

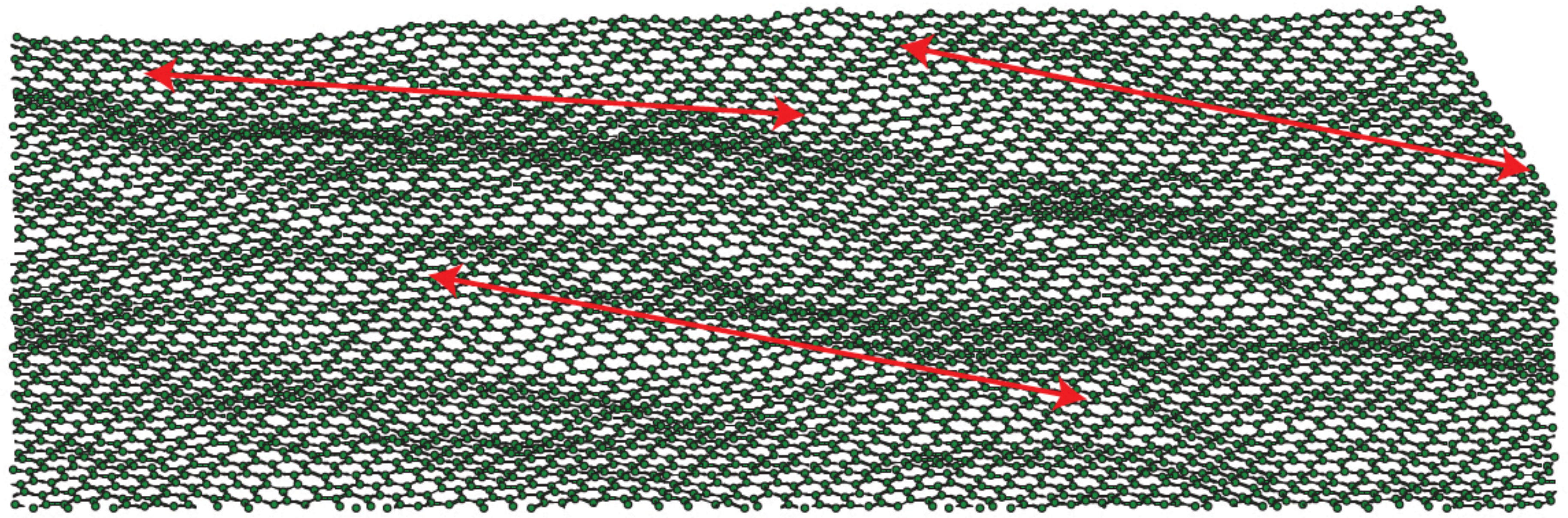
What about graphene?

Compatibility with Mermin - Wagner - Hohenberg theorem?

- Because of the slow increase of $\log(j)$ with j, the loss of crystal order appears only on very long length scales

  A finite size sample may exhibit a crystalline order

- The surface is rippled and the non-linear coupling between the fluctuations of the height and the displacements parallel to the surface induce an effective long-range component



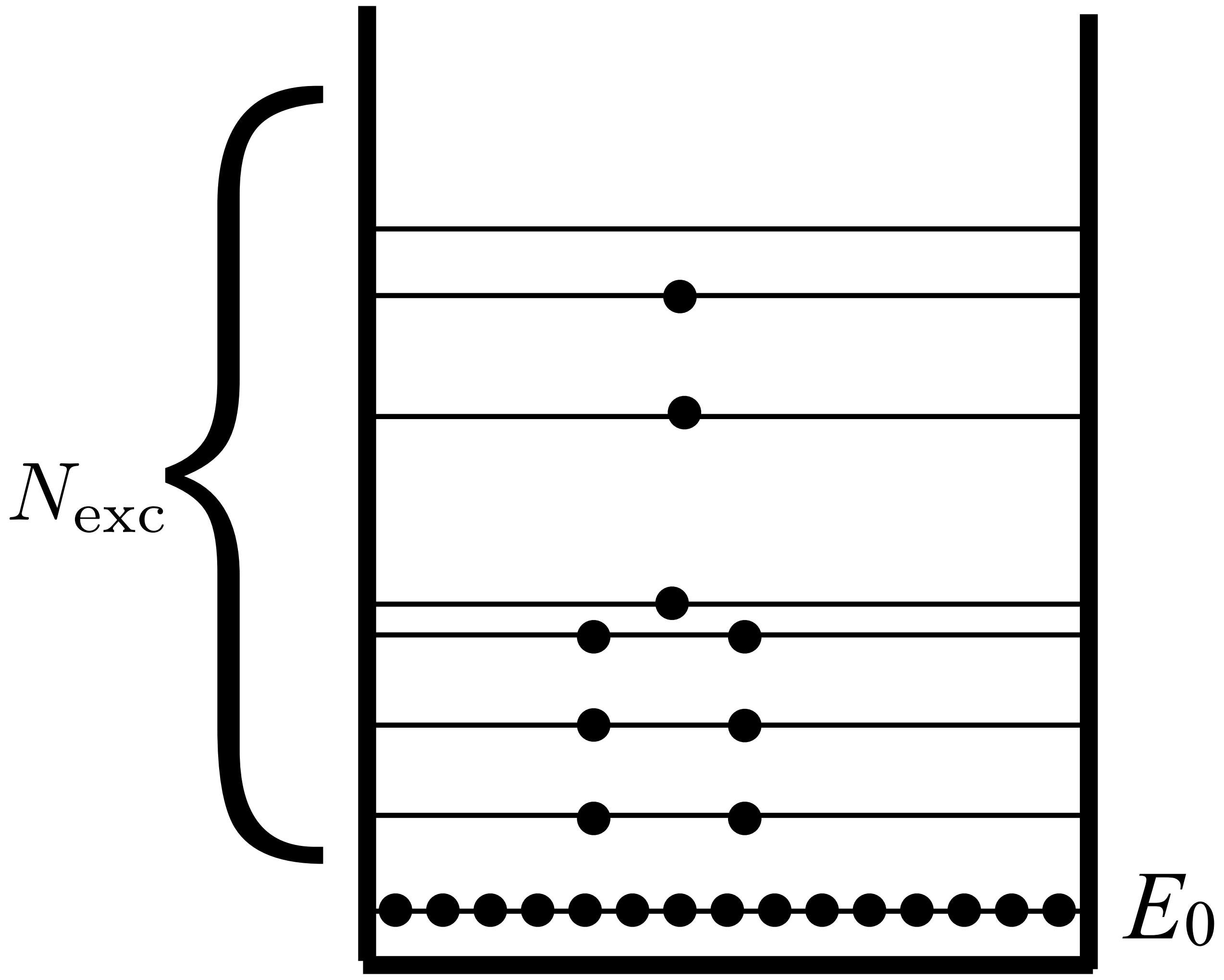
Fasolino et al, 2007
Nature materials 6.11, p. 858–861.

2.

The 2D ideal Bose gas

Einstein's saturated ideal gas

Bose particles confined in a box at fixed temperature



The number of particles that can be placed in the excited sates is bounded. Indeed the Bose law

$$N_{\boldsymbol{p}} = \frac{1}{\mathrm{e}^{(E_{\boldsymbol{p}}-\mu)/k_{\mathrm{B}}T} - 1}$$

is meaningful only if $\mu < E_0 = 0$

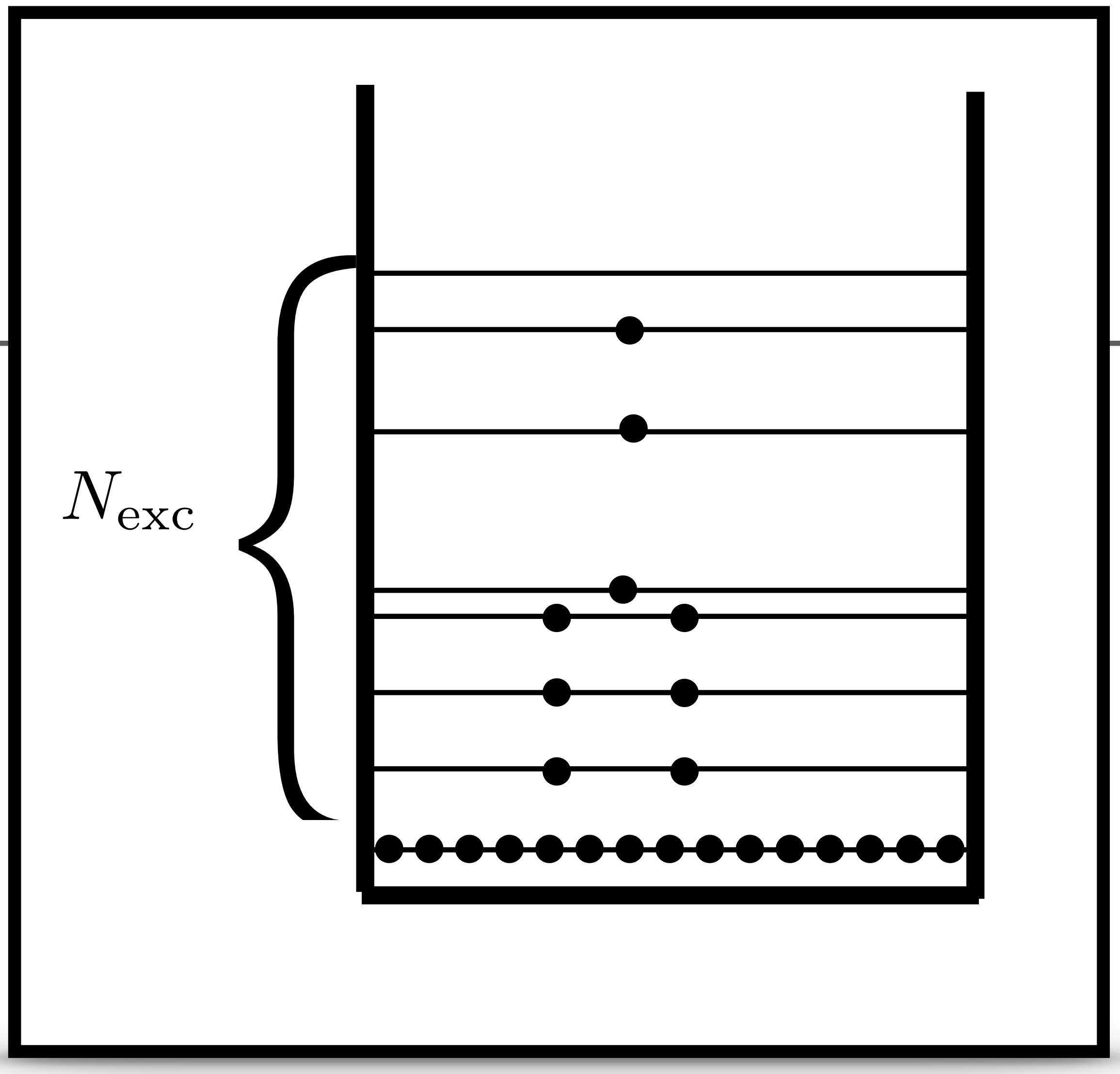
$$N_{\mathrm{exc}}(T,\mu) = \sum_{\boldsymbol{p}\neq 0} \frac{1}{\mathrm{e}^{(E_{\boldsymbol{p}}-\mu)/k_{\mathrm{B}}T} - 1} < \sum_{\boldsymbol{p}\neq 0} \frac{1}{\mathrm{e}^{E_{\boldsymbol{p}}/k_{\mathrm{B}}T} - 1}$$

obtained for $\mu \to 0$

Continuum limit using the density of states: $N_{\mathrm{exc}}(T,\mu) < \int_0^{+\infty} \frac{D(E)}{\mathrm{e}^{E/k_{\mathrm{B}}T} - 1}\,\mathrm{d}E$

Does this integral converge in $E=0$?

Einstein's saturated ideal gas (2)



$$N_{\mathrm{exc}}(T,\mu) < \int_0^{+\infty} \frac{D(E)}{\mathrm{e}^{E/k_{\mathrm{B}}T}-1}\,\mathrm{d}E$$

The convergence in $E=0$ depends on $D(E)$

- In 3D: $D(E) \propto \sqrt{E}$ and $\dfrac{1}{\mathrm{e}^{E/k_{\mathrm{B}}T}-1} \approx \dfrac{1}{\left(1+\frac{E}{k_{\mathrm{B}}T}\right)-1} = \dfrac{k_{\mathrm{B}}T}{E}$

  $\longrightarrow$ $k_{\mathrm{B}}T\displaystyle\int_0 \frac{1}{\sqrt{E}}\,\mathrm{d}E$ converges in $E=0$: BEC !

- In 2D: $D(E)$ is constant and the integral $k_{\mathrm{B}}T\displaystyle\int_0 \frac{1}{E}\,\mathrm{d}E$ diverges

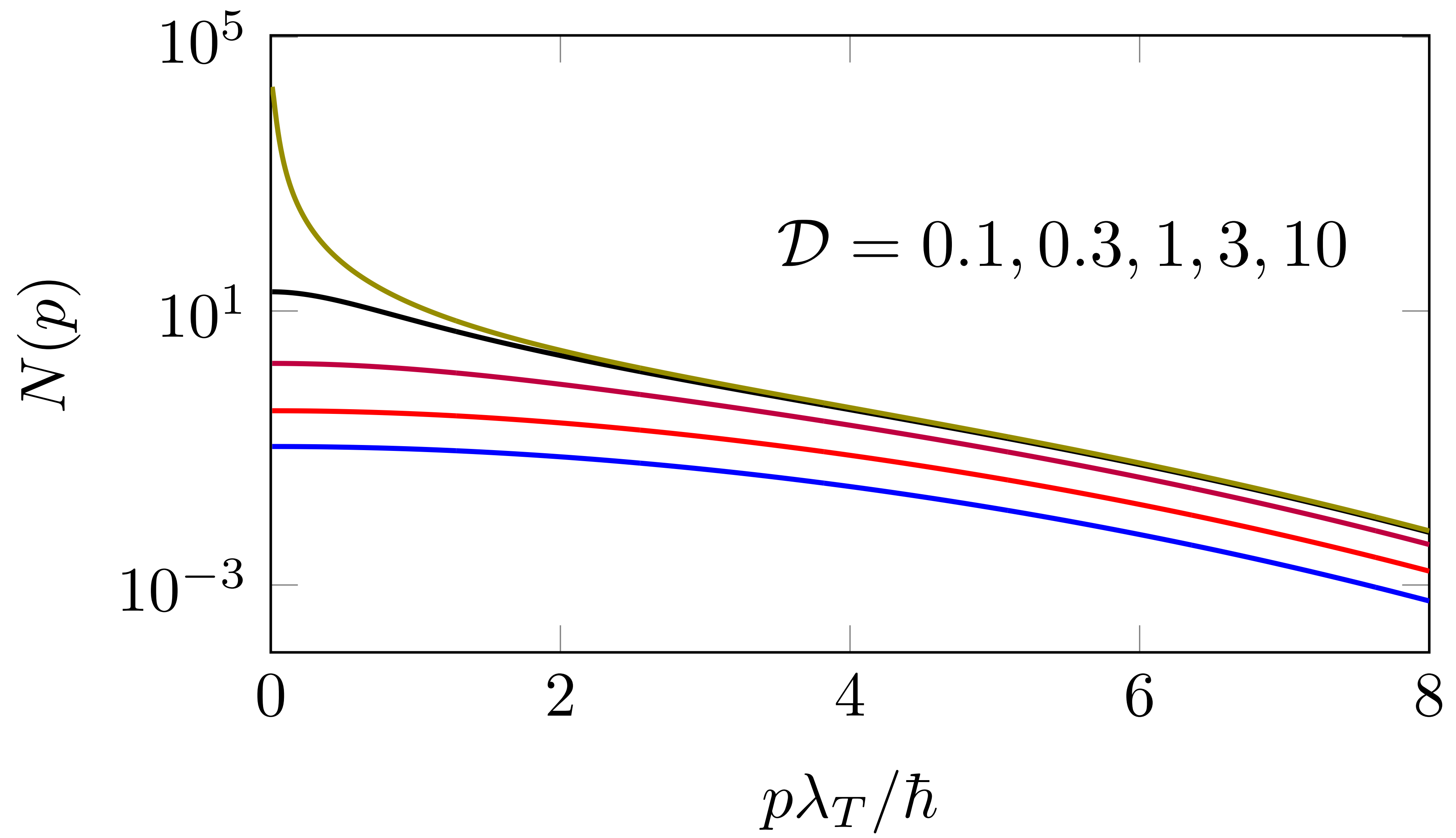
For a given T, one can put an arbitrarily large number of particles in the excited states by letting $\mu \to 0$: no BEC

Momentum distribution of the ideal 2D Bose gas

Quantum statistics still play a role, even in the absence of condensation:

Particles accumulate in the region of small momenta

Varying phase space density from $\ll 1$ to $\gg 1$



$$\lambda_T = \frac{\hbar\sqrt{2\pi}}{\sqrt{mk_\mathrm{B}T}}$$: thermal wavelength

$$\mathcal{D} = \rho\lambda_T^2$$: phase space density

$$\frac{p\lambda_T}{\hbar} = 1 \;\Leftrightarrow\; \frac{p^2}{2m} = \frac{1}{4\pi}k_\mathrm{B}T$$

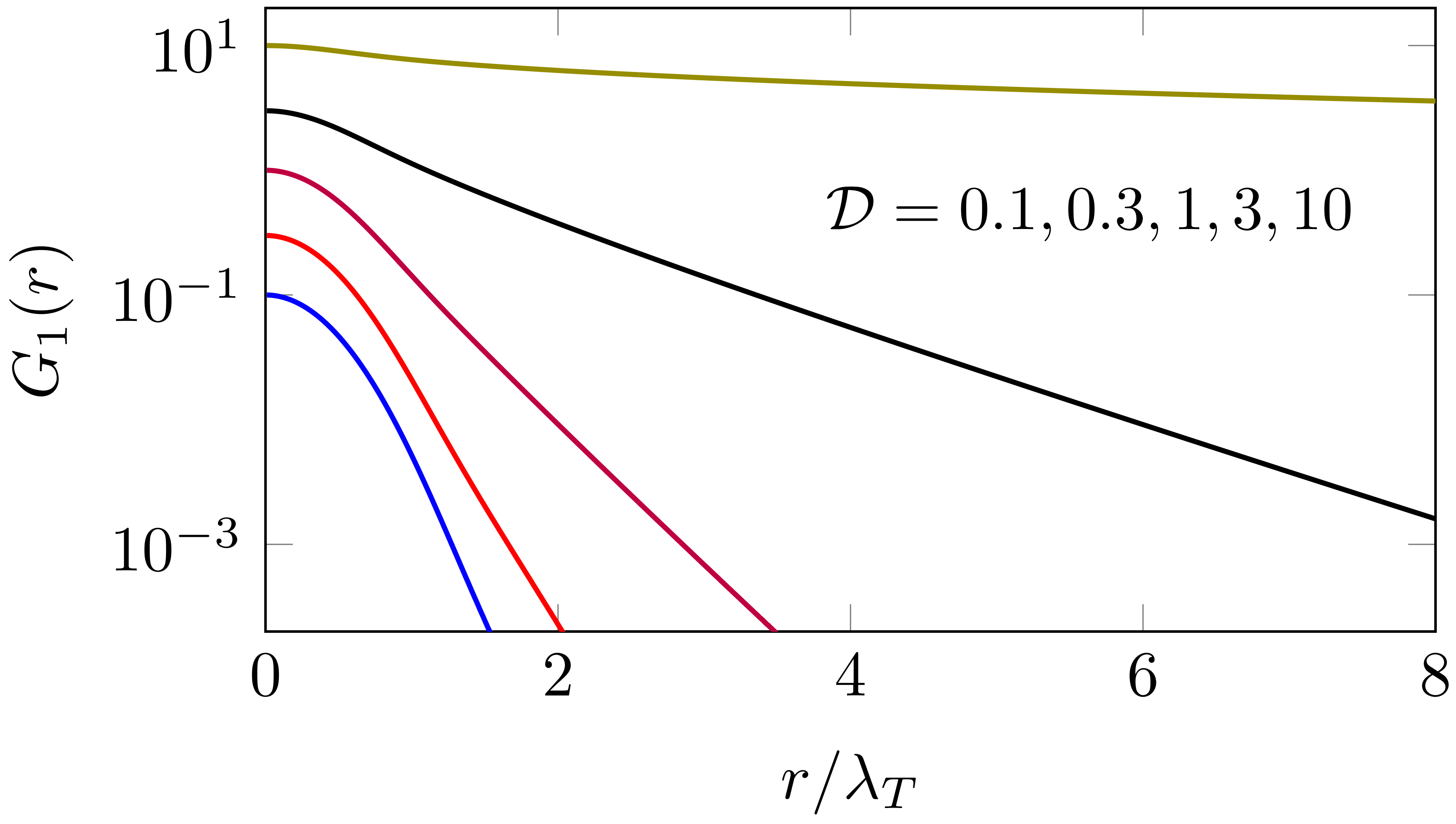
$$N(\boldsymbol{p}) = \frac{1}{\mathrm{e}^{(p^2/2m-\mu)/k_\mathrm{B}T} - 1} \approx \frac{k_\mathrm{B}T}{\frac{p^2}{2m} + |\mu|}$$

Lorentz distribution

Spatial coherence of the ideal 2D Bose gas

Characterized by the one-body correlation function

$$G_1(\boldsymbol{r},\boldsymbol{r}') = \langle \boldsymbol{r}|\hat{\rho}_1|\boldsymbol{r}'\rangle$$ *: Fourier transform of the momentum distribution*



Lorentz momentum distribution

$\longrightarrow$ Exponential G_1

$$G_1(r,0) \propto \mathrm{e}^{-r/\ell}$$

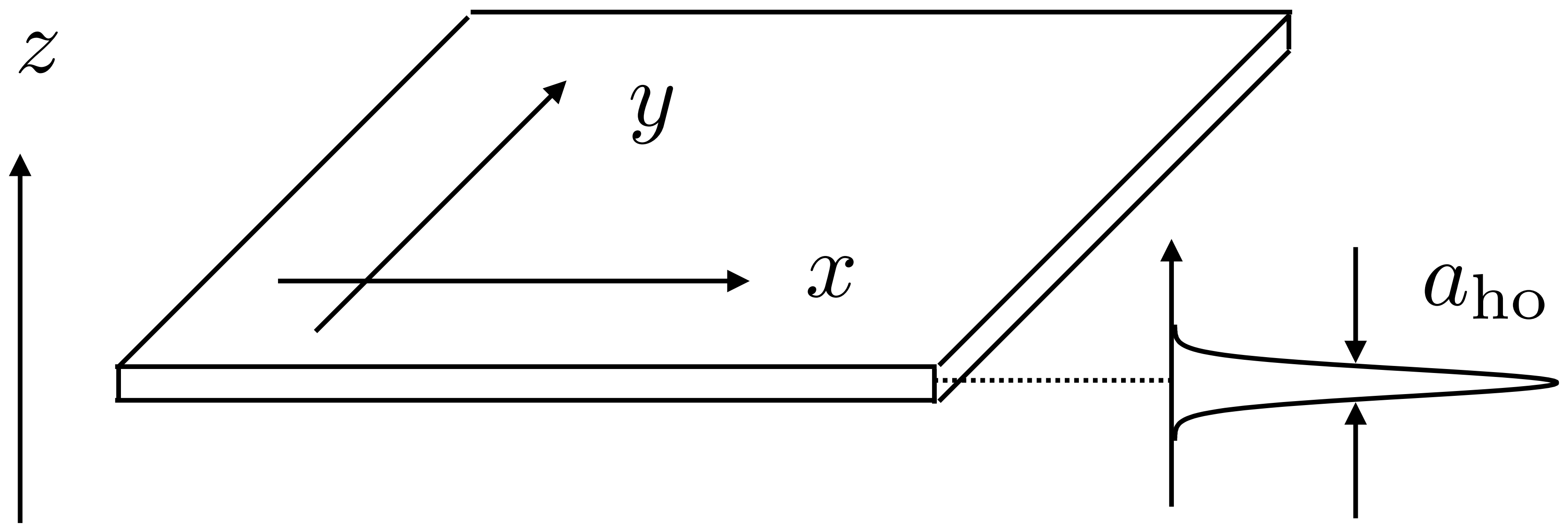
The coherence length ℓ increases with $\mathscr{D}$

An exponential decay is by essence "fast" (even if ℓ can be large): no emergence of quasi-long range order at this stage…

3.

The Gross-Pitaevskii approach for the interacting 2D gas

strong harmonic
confinement along z



Contact interactions described
by the 3D scattering length a

The 2D Gross-Pitaevskii energy

Description of the state of the gas by the classical field $\psi(x, y)$: $E = E_{\text{kin}} + E_{\text{int}}$

$$E_{\text{kin}} = \frac{\hbar^2}{2m} \int |\boldsymbol{\nabla} \psi|^2 \, \mathrm{d}^2 r$$

$$E_{\text{int}} = \frac{\hbar^2}{2m} \, \tilde{g} \int |\psi(\boldsymbol{r})|^4 \, \mathrm{d}^2 r$$

$$\tilde{g} = \sqrt{8\pi} \frac{a}{a_{\text{ho}}}$$

Dimensionless parameter describing the strength of contact interactions

Ground state in a box $L \times L$: $\psi(\boldsymbol{r}) = \sqrt{\rho_0}$ $\qquad \rho_0 = \frac{N}{L^2}$

At non-zero temperature, phase and density fluctuations:

$$\psi(\boldsymbol{r}) = \sqrt{\rho(\boldsymbol{r})} \, \mathrm{e}^{\mathrm{i}\theta(\boldsymbol{r})} \xrightarrow[\text{valid for large phase space densities}]{\text{Freezing of density fluctuations due to repulsive atomic interactions,}} \psi(\boldsymbol{r}) \approx \sqrt{\rho_0} \, \mathrm{e}^{\mathrm{i}\theta(\boldsymbol{r})}$$

$$\mathscr{D} \gg 1 \qquad \mathscr{D} = \rho_0 \lambda_T^2 \qquad \lambda_T^2 = \frac{2\pi\hbar^2}{m k_B T}$$

The interaction energy is a constant: only the kinetic energy is relevant for the dynamics

Phase coherence in an interacting 2D Bose gas

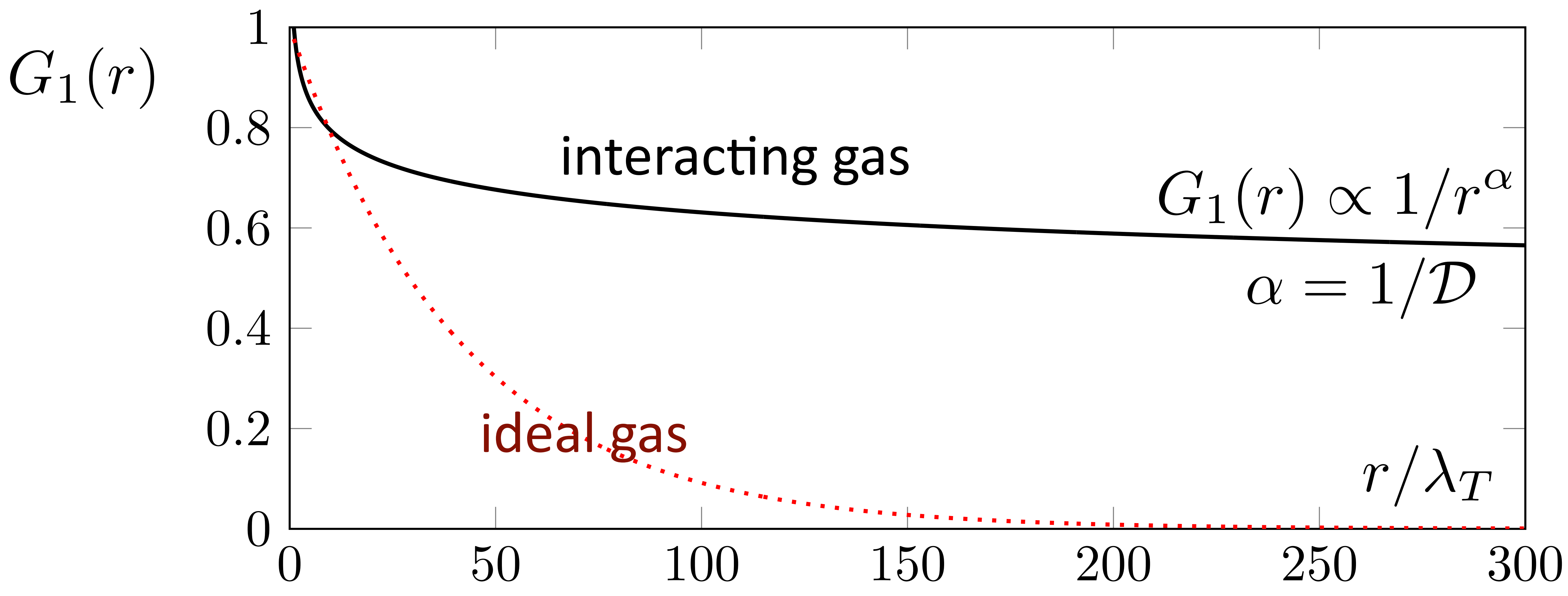
$$\psi(\boldsymbol{r}) \approx \sqrt{\rho_0}\ \mathrm{e}^{\mathrm{i}\theta(\boldsymbol{r})}$$

$$E_{\text{kin}} = \frac{\hbar^2}{2m} \int |\boldsymbol{\nabla}\psi|^2 \ \mathrm{d}^2 r \longrightarrow E_{\text{kin}} \approx \frac{\hbar^2}{2m}\ \rho_0 \int (\boldsymbol{\nabla}\theta)^2 \ \mathrm{d}^2 r$$

Fourier expansion of the phase $\theta(\boldsymbol{r}) = \sum_{\boldsymbol{q}} c_{\boldsymbol{q}}\, e^{i\boldsymbol{q}\cdot\boldsymbol{r}}$ *phonon excitations (no vortex!)* cf. Peierls

At thermal equilibrium: $\langle |c_{\boldsymbol{q}}|^2 \rangle \propto k_{\mathrm{B}} T \longrightarrow \left\langle \left[\theta(\vec{r}) - \theta(0)\right]^2 \right\rangle \approx \frac{2}{\mathscr{D}} \ln(r/\lambda_T)$ $\quad \mathscr{D} = \rho_0 \lambda_T^2$

One-body correlation function $G_1(r) = \langle \psi(\boldsymbol{r})\ \psi^*(0) \rangle \ \approx\ \rho_0\ \langle \mathrm{e}^{\mathrm{i}[\theta(\boldsymbol{r}) - \theta(0)]} \rangle \ = \rho_0 \mathrm{e}^{-\frac{1}{2}\langle [\theta(\boldsymbol{r}) - \theta(\boldsymbol{0})]^2 \rangle} = \left(\frac{\lambda_T}{r}\right)^{\frac{1}{\mathscr{D}}}$



$\mathscr{D} = 10$

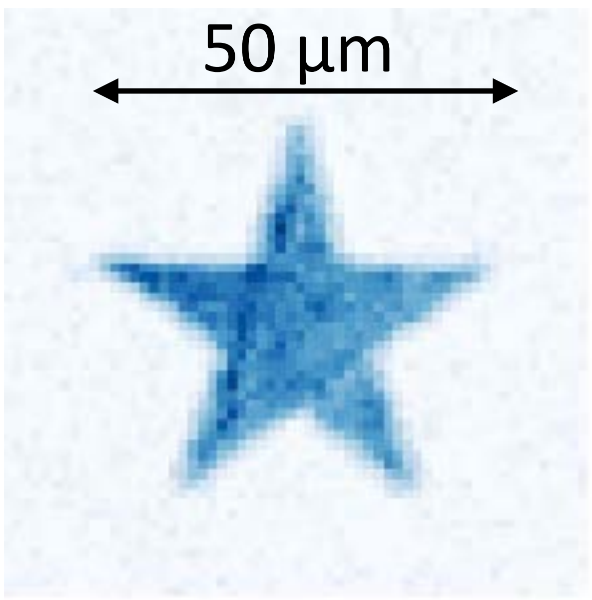
A uniform 2D gas in the lab

Frozen motion along the vertical direction z

$$\omega_z/2\pi = 4\,\mathrm{kHz}$$

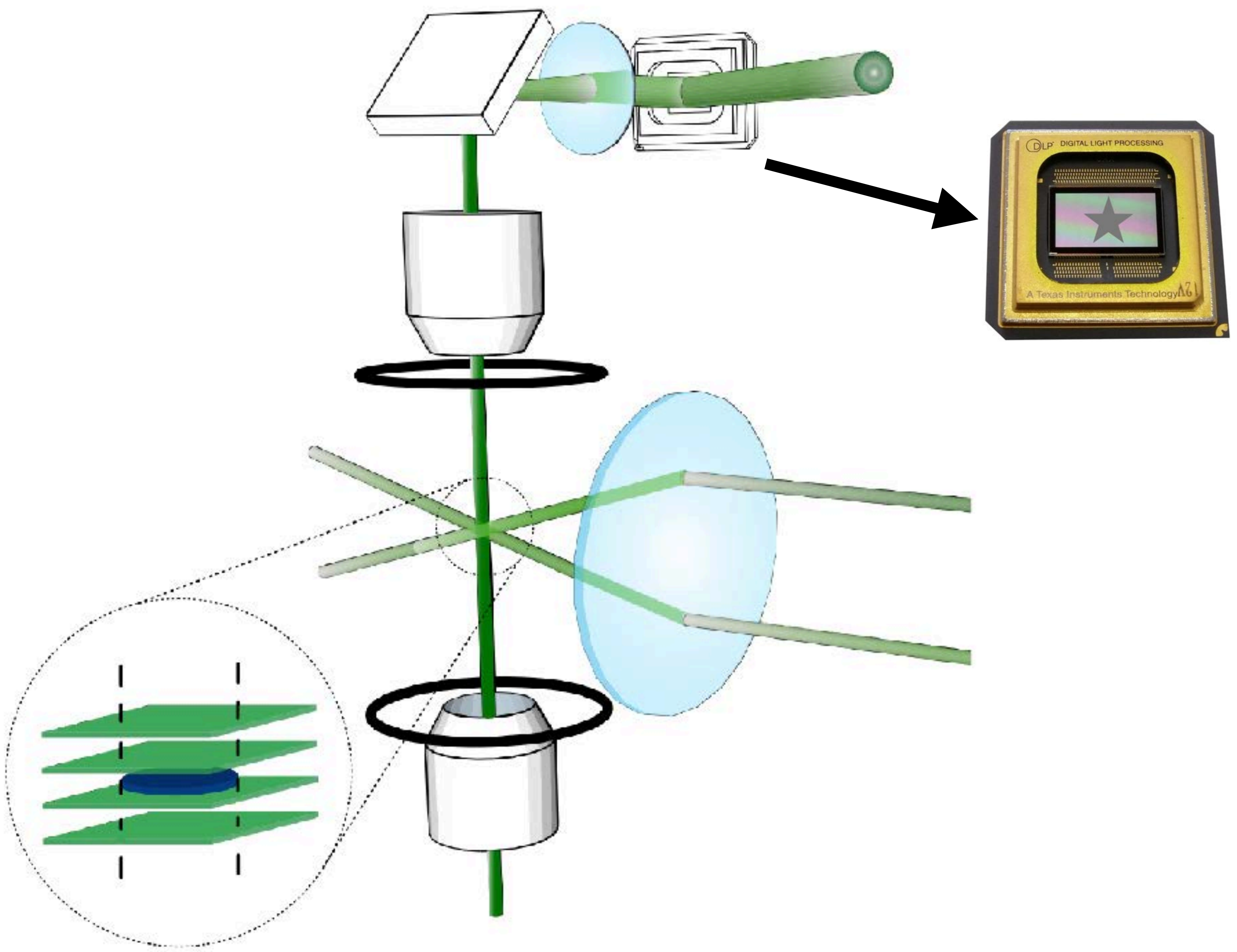
Initial confinement in the xy plane:

Box-like potential with arbitrary shape



density up to 100 atoms/µm^2

Uniform gas with $\sim 10^5$ atoms



PhDs: R. Saint-Jalm, E. Le Cerf, B. Bakkali-Hassani, J.-L. Ville, C. Maury, G. Chauveau

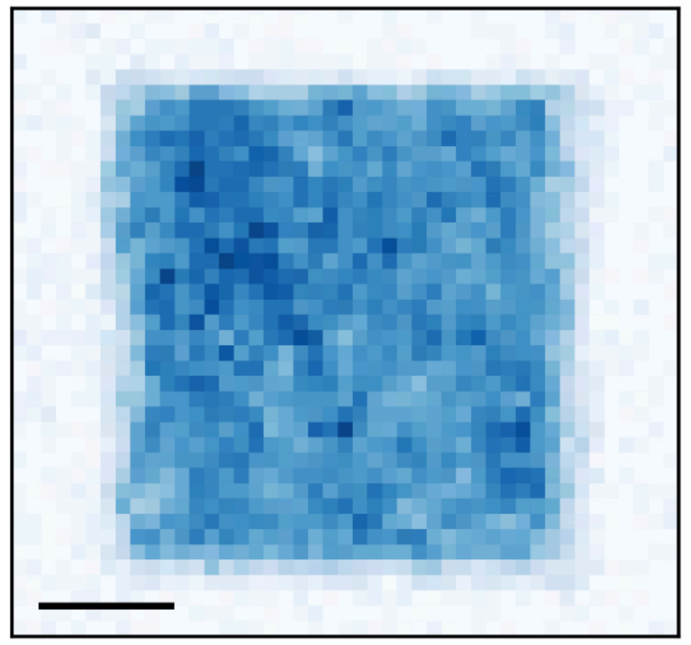
Postdocs: M. Aidelsburger, P.C.M. Castilho, Y.-Q. Zhou

PIs: S. Nascimbene, J. Beugnon, J. Dalibard

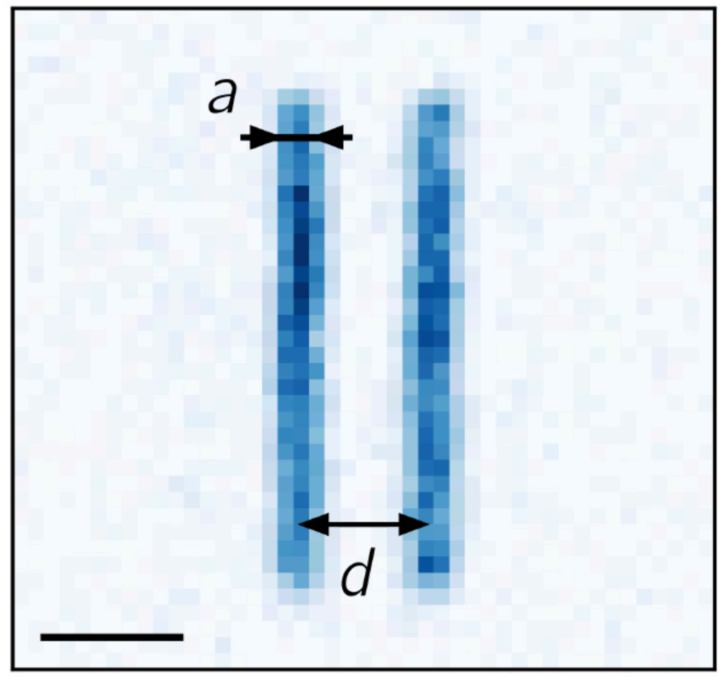
Accessing the correlation function $G_1(\boldsymbol{r}, \boldsymbol{r}')$

Investigation via a "Young slit" experiment

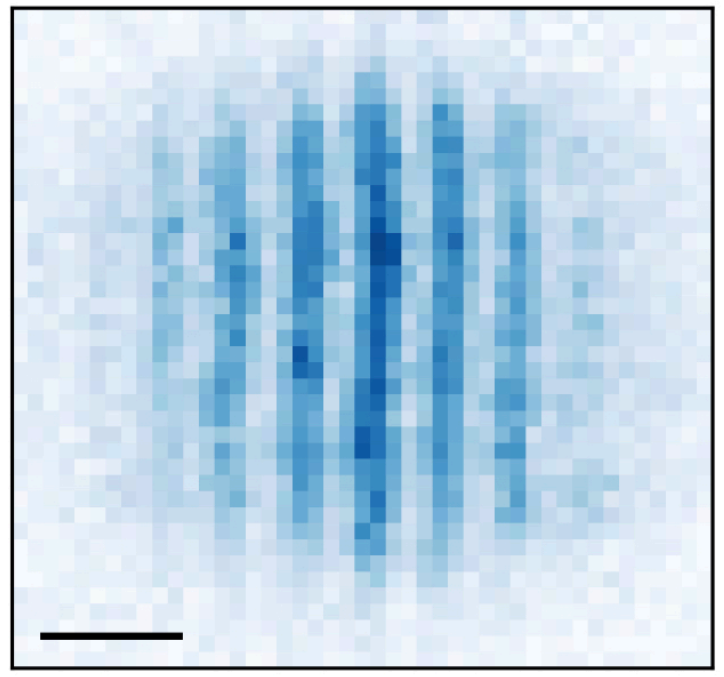
Initial state



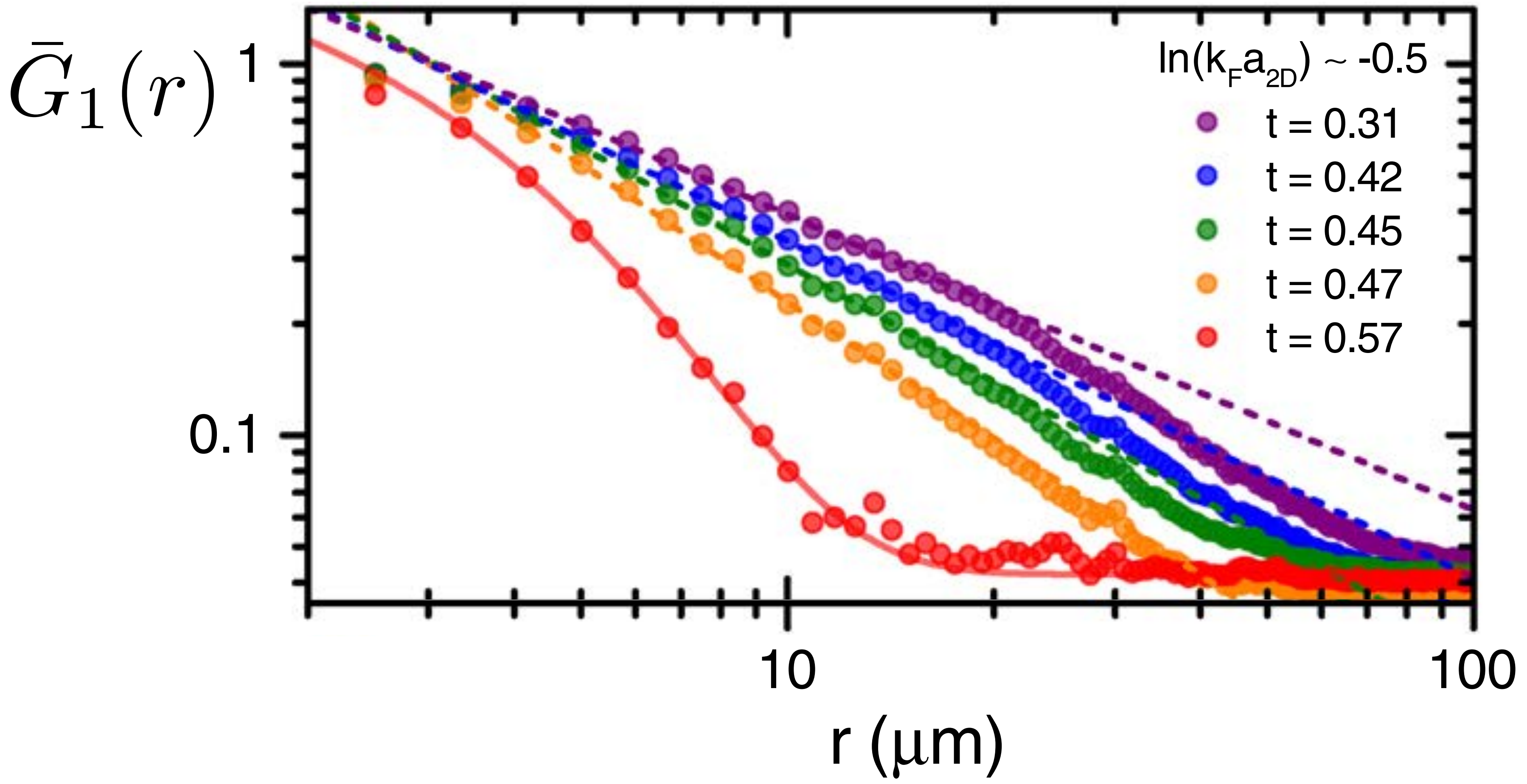
Isolate two slits using a shaped laser beam



Interference after time-of-flight reveals the coherence between the slits



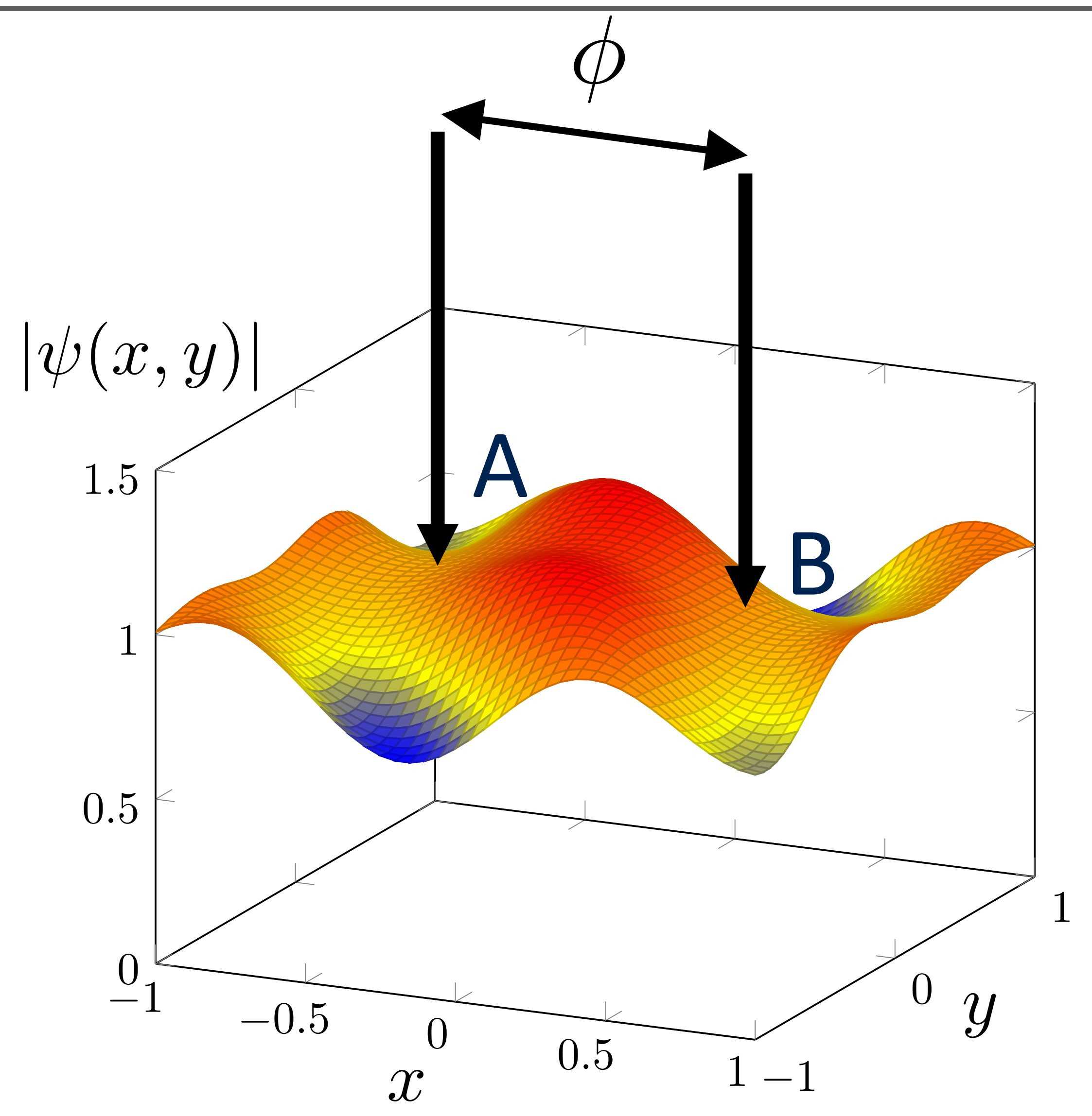
Experiments along this line: Heidelberg (Jochim, 2015), Oxford (Foot, 2022), Paris (Glorieux, 2022)



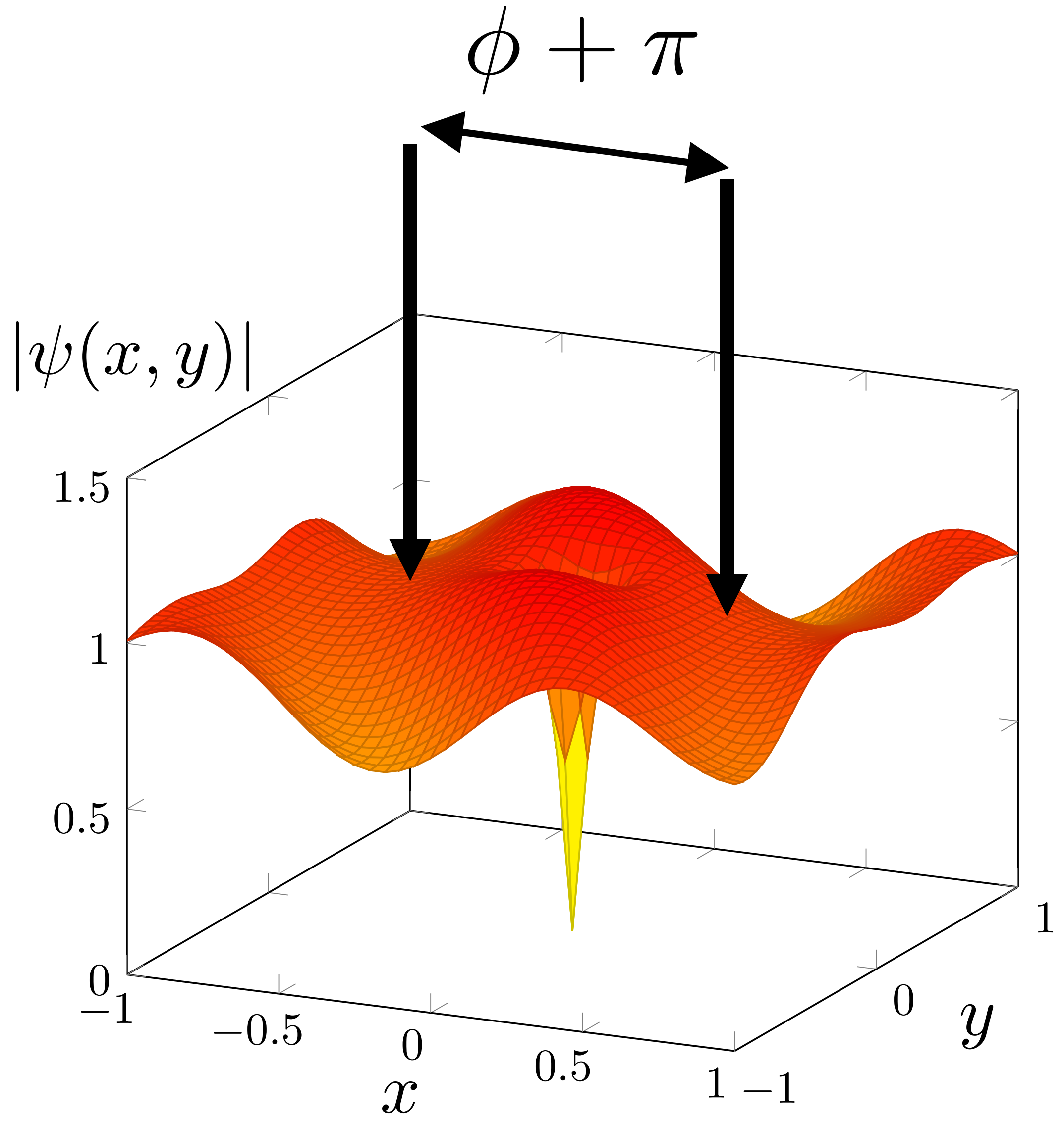
The algebraic decay of G_1 holds at low temperature, but is replaced by a faster, exponential decay at higher T.

Murthy et al., PRL **115** 010401 (2015)

A game changer: vortices



Consider two points A and B between which there exists a significant phase coherence if one restricts to phonon excitations



If an isolated vortex has a significant probability to appear in the vicinity of the AB segment, the relative phase will strongly fluctuate:

$$\phi \quad \rightarrow \quad \phi + \pi$$

If isolated vortices have a spatial density ρ_v, one can expect that any phase ordering will be lost over a distance $\sim \rho_v^{-1/2}$

The Kosterlitz-Thouless transition explored with atomic gases

Jean Dalibard

Solvay chair for Physics 2022

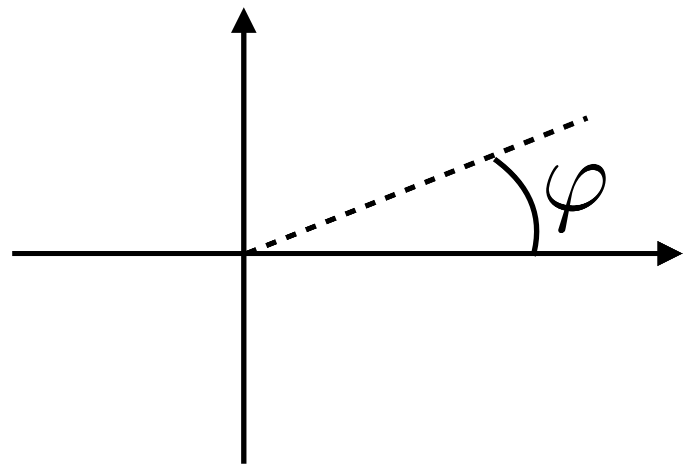
Lecture 2, part 2

4.

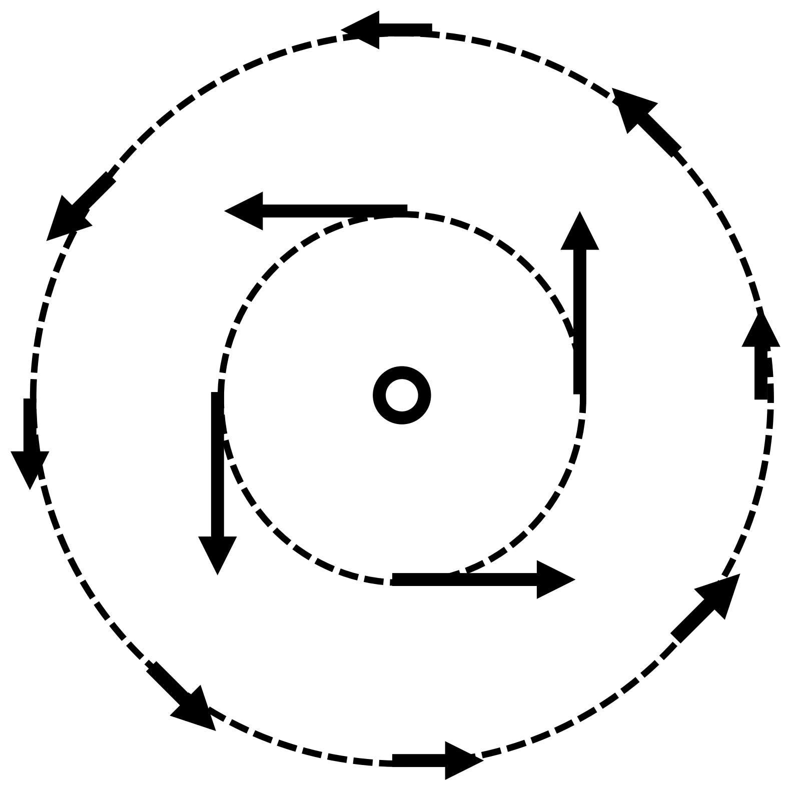
The Kosterlitz- Thouless argument

Velocity field of a vortex

Example of a vortex in $\boldsymbol{r} = 0$: $\psi(\boldsymbol{r}) = \sqrt{\rho(r)}\,\mathrm{e}^{\mathrm{i}\varphi}$ $\theta(\boldsymbol{r}) = \varphi$

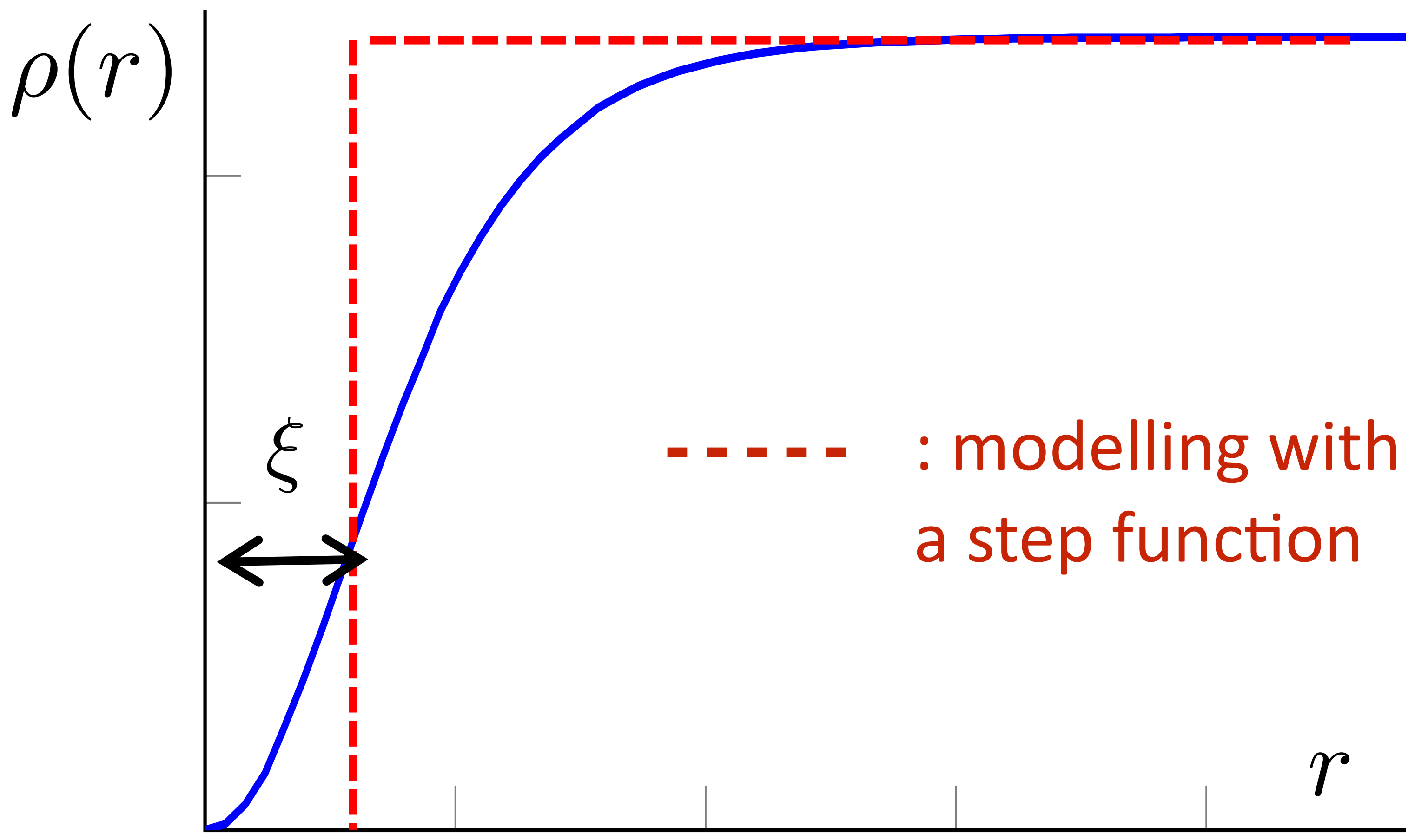


$$\boldsymbol{v}(\boldsymbol{r}) = \frac{\hbar}{m}\boldsymbol{\nabla}\theta = \frac{\hbar}{mr}\boldsymbol{u}_\varphi$$



$$\oint \boldsymbol{v}(\boldsymbol{r}) \cdot \mathrm{d}\boldsymbol{r} = 2\pi\frac{\hbar}{m}$$

Density profile close to the vortex location



$$\xi = \frac{1}{\sqrt{2\tilde{g}\rho}}$$ *Healing length*

$\tilde{g}$: Dimensionless interaction parameter

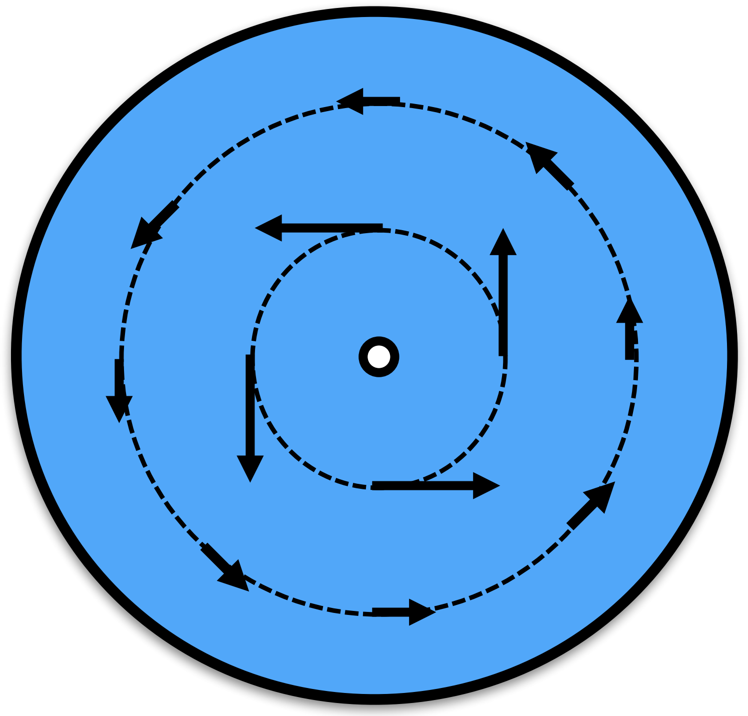
$\propto$ 3D scatt. length / thickness

Energy of a vortex

Kinetic energy (vortex at the center of a disk of radius R):

$$E_{\text{kin}} = \frac{1}{2} m \int \rho(r)\, v^2(r)\, \mathrm{d}^2 r$$

$$v(r) = \frac{\hbar}{mr}$$



$$\approx \frac{1}{2} m\rho \frac{\hbar^2}{m^2} \int_\xi^R \frac{1}{r^2}\, r\, \mathrm{d}r$$

$$= \pi \frac{\hbar^2 \rho}{m} \ln(R/\xi)$$

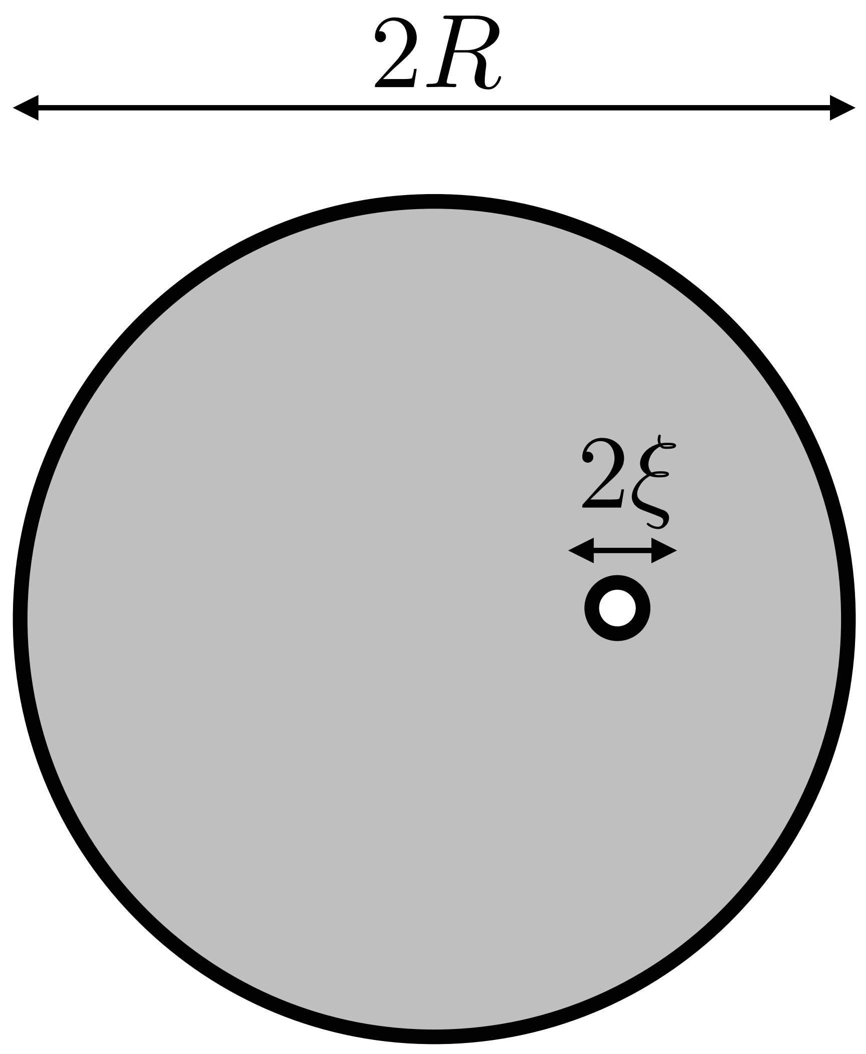
Prefactor : robust
Inside the log : depends on the model for the core

Diverges with system size

Interaction energy: one must create a hole of size ξ in the fluid $\epsilon_0 \sim \frac{\hbar^2 \rho}{m} \ll E_{\text{kin}}$

Is the existence of an isolated vortex likely?

$\mathcal{D} = \rho \lambda_T^2 \qquad \lambda_T^2 = \dfrac{2\pi\hbar^2}{m k_B T}$



Number of independent « boxes » to place the vortex: $W = R^2/\xi^2$

Probability for a vortex to exist in such a box: $p \approx \mathrm{e}^{-E_{\mathrm{kin}}/k_B T}$

$$\frac{E_{\mathrm{kin}}}{k_B T} = \frac{1}{k_B T} \frac{\pi \hbar^2 \rho}{m} \ln(R/\xi) = \frac{\mathcal{D}}{2} \ln(R/\xi)$$

Probability for a given box: $p \approx \exp\left[-\dfrac{\mathcal{D}}{2} \log\left(\dfrac{R}{\xi}\right)\right] = \left(\dfrac{\xi}{R}\right)^{\mathcal{D}/2}$

Total probability: $\mathcal{P} = W p \approx \left(\dfrac{\xi}{R}\right)^{-2+\mathcal{D}/2}$

entropic term (-2), **energetic term** ($\mathcal{D}/2$)

Renormalization:

$\rho, \ \mathcal{D} \longrightarrow \rho_s, \ \mathcal{D}_s$

superfluid component

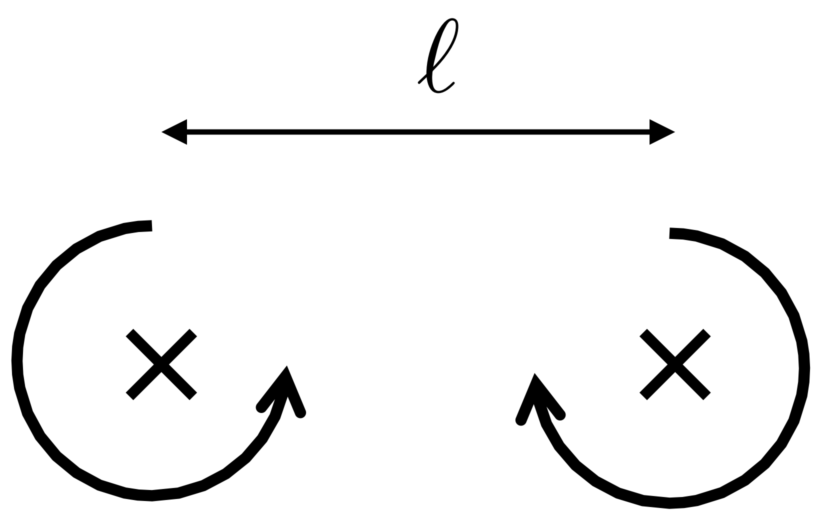
Large $\mathcal{D}_s$ — $\mathcal{D}_s = 4$ — small $\mathcal{D}_s$ → Temperature

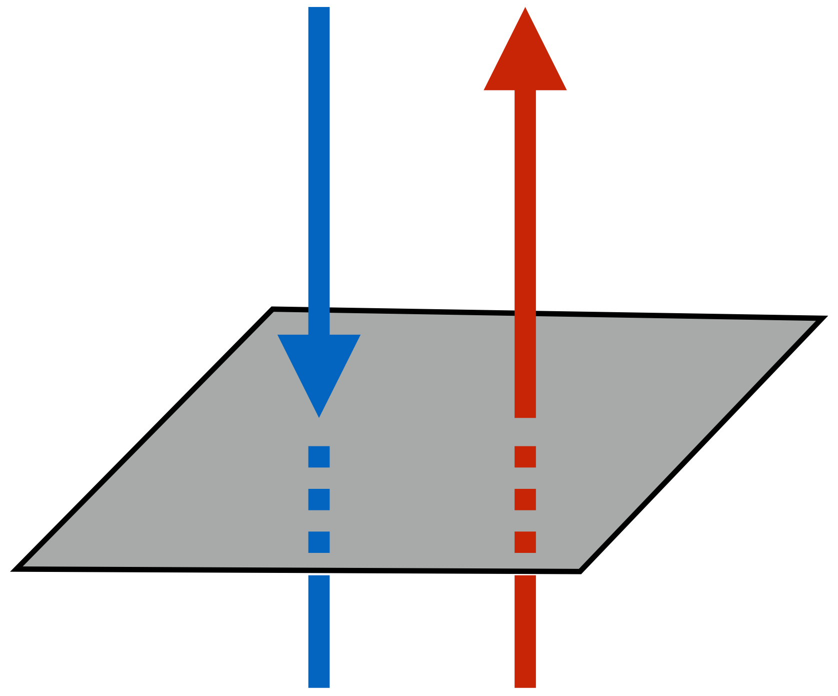
No isolated vortices, quasi-long range order

Proliferation of isolated vortices, loss of quasi-long range order

What about vortex pairs?

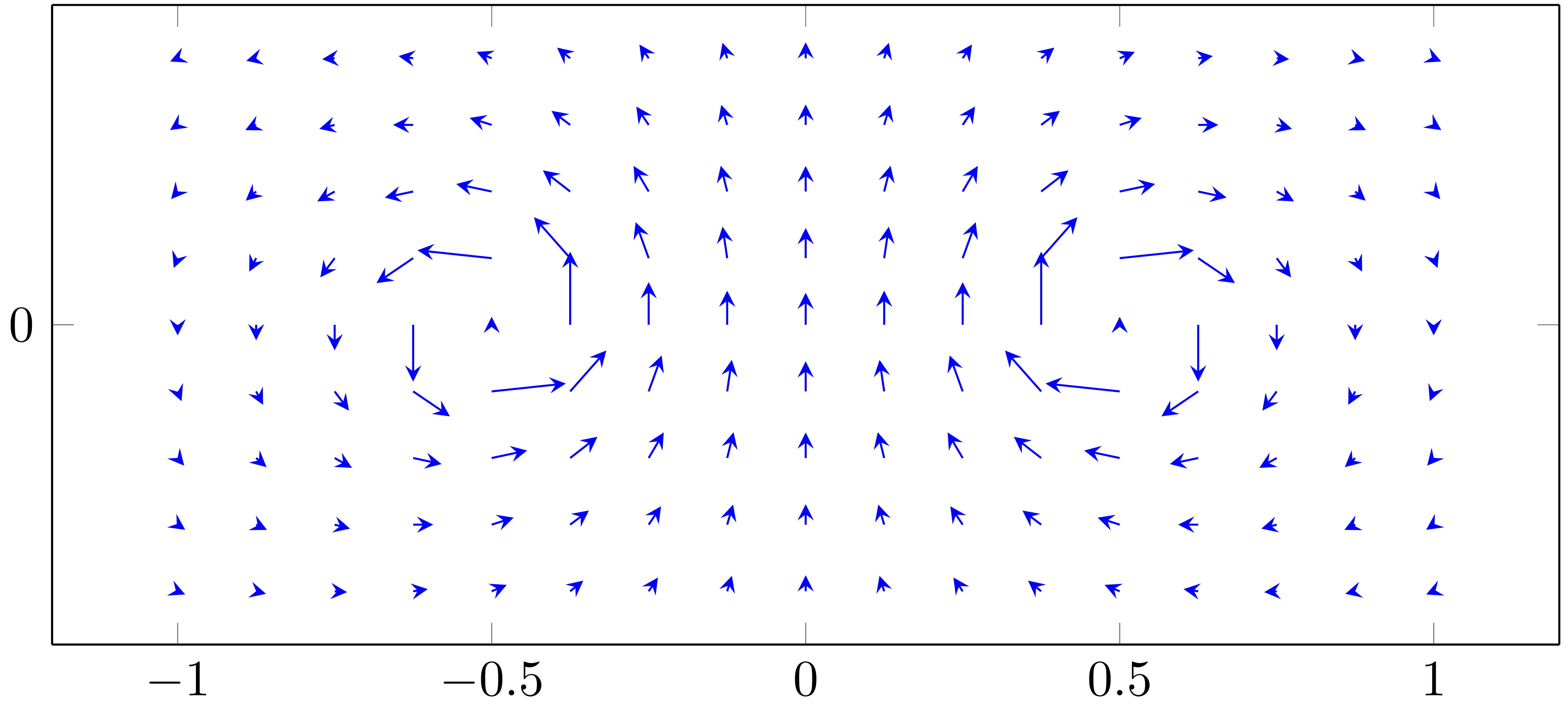
Superposition of the velocity fields created by each vortex





Magnetic analogy: Field created by parallel wires with opposite currents

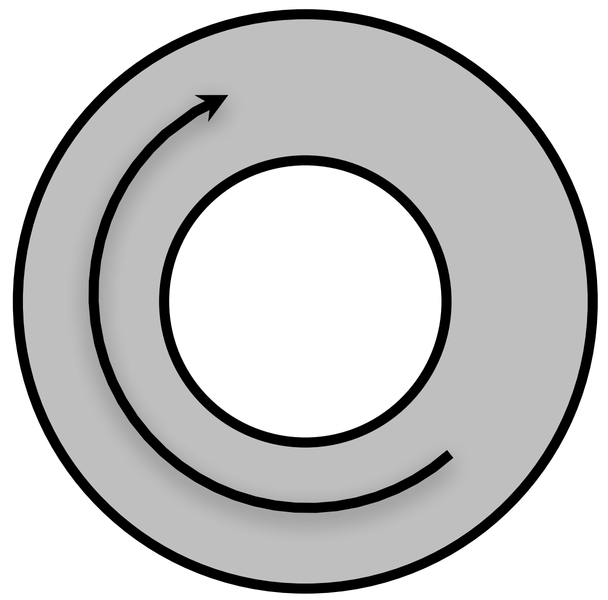
Dipolar field: Decreases as $1/r^2$ at infinity instead of $1/r$ for an isolated vortex



Finite energy even for an infinite sample: always present at non-zero temperature

Vortices and superfluidity

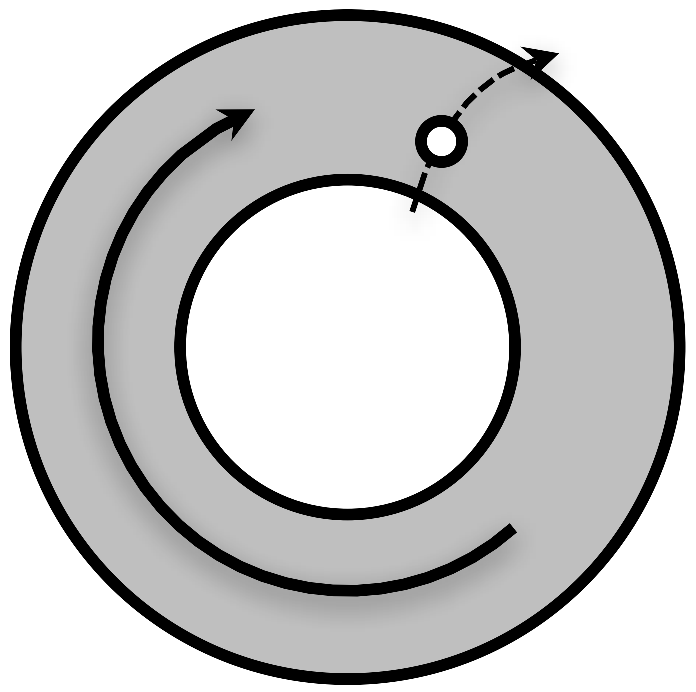
Current in a ring, corresponding to a phase winding $2\pi N$ of the field $\psi(\boldsymbol{r})$



Is this current metastable ?

If isolated vortices exist in the ring, they may cross it:

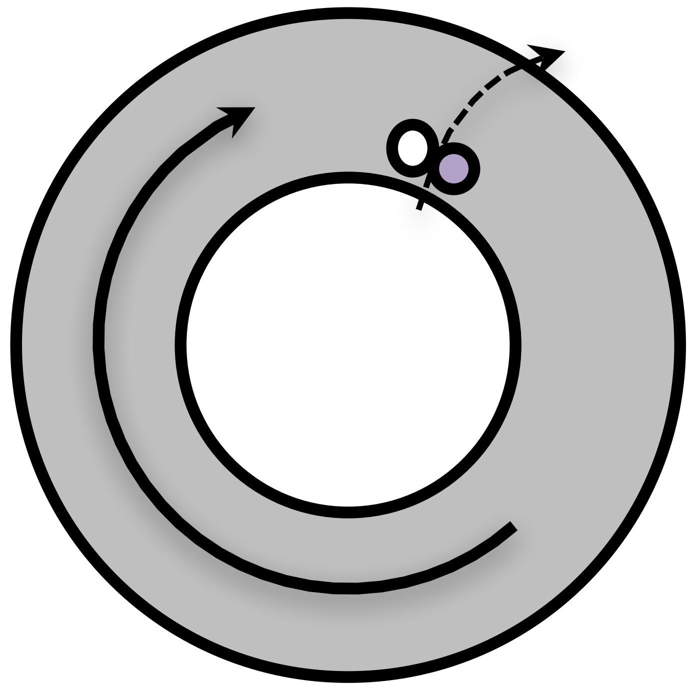
$$N \to N \pm 1$$



Fluctuations of the current, which will thus be damped and will tend to zero

Isolated vortices destroy the superfluidity

A pair of vortices of opposite charges has no effect on the current



Vortex pairs do not destroy the superfluidity

To summarize

The BKT transition occurs between two different types of states

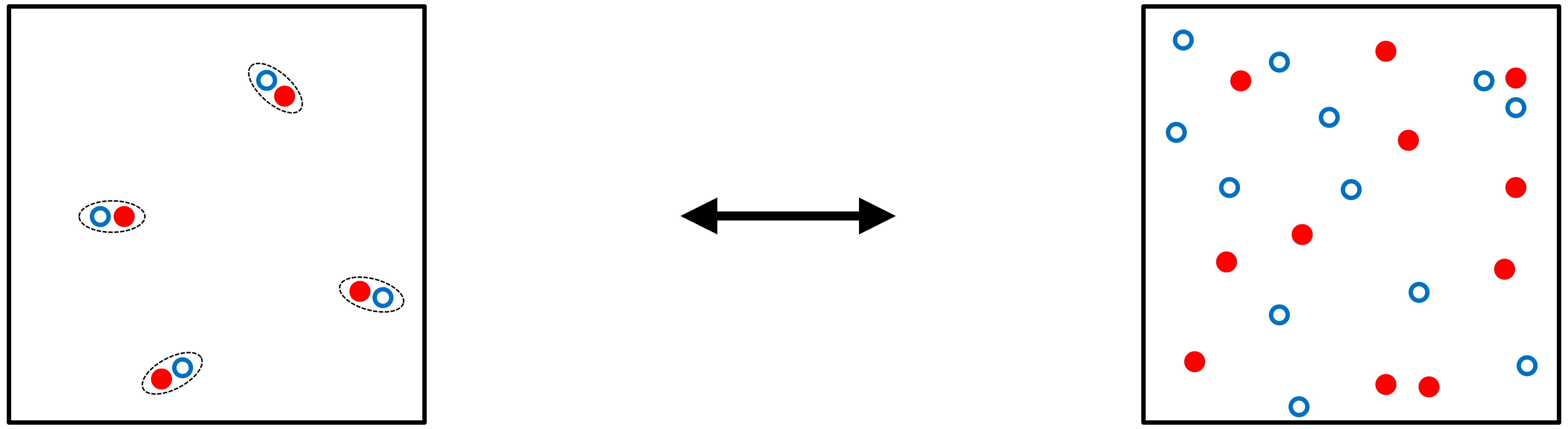
Superfluid state | **Normal state**

$\mathcal{D}_s > 4$ | $\mathcal{D}_s = 0$

0 — T_c — Temperature

$G_1(r) \propto 1/r^{\alpha}$ | $G_1(r) \propto \mathrm{e}^{-r/\ell}$

$\alpha = 1/\mathcal{D}_s$



Universal law for the critical value of the superfluid density: $\mathcal{D}_{\mathrm{s,crit}} = 4$ $\alpha_{\mathrm{crit}} = 1/4$

Total (superfluid+normal) phase-density at the critical point: $\mathcal{D}_{\mathrm{total}} \approx \ln\left(\frac{380}{\tilde{g}}\right) > 4$

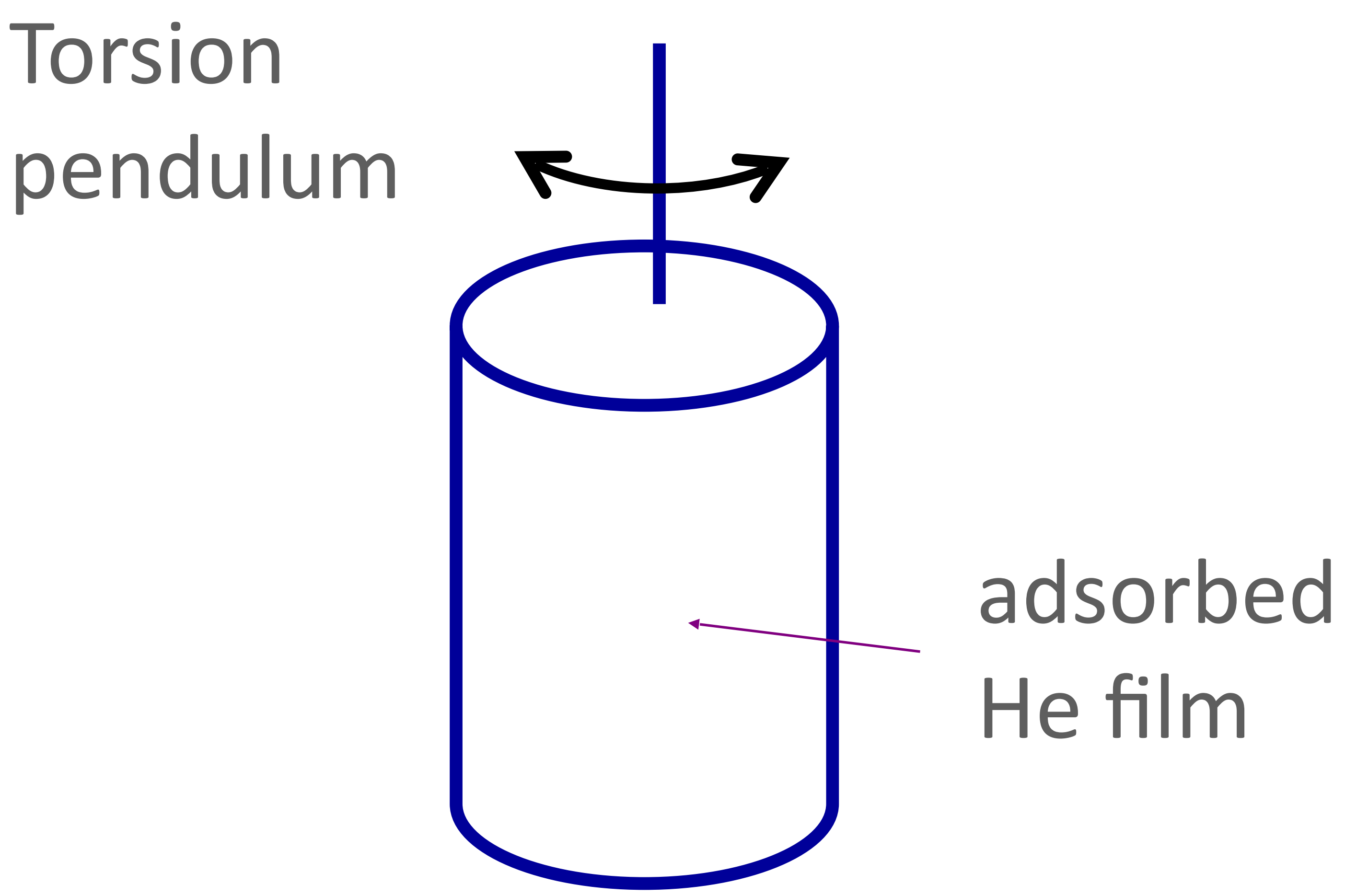
$\tilde{g}$: Dimensionless interaction parameter

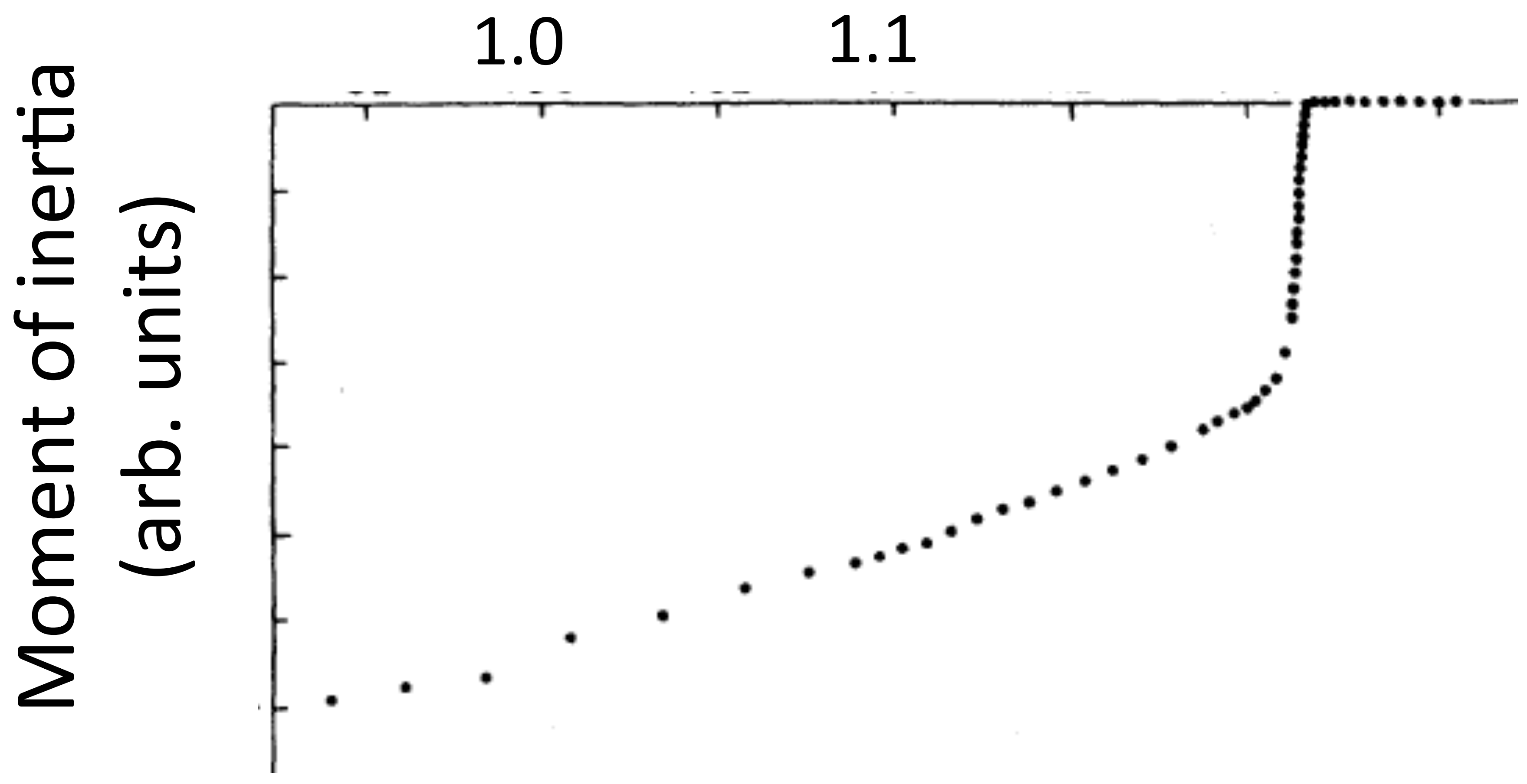
Prokofev and Svistunov

First experimental evidences

Superfluidity of liquid helium films

Bishop and Reppy, 1978





Also 2D superconducting films, colloidal particles, arrays of tunnel junctions,…

What about atomic, molecular and optical (AMO) systems?

4.

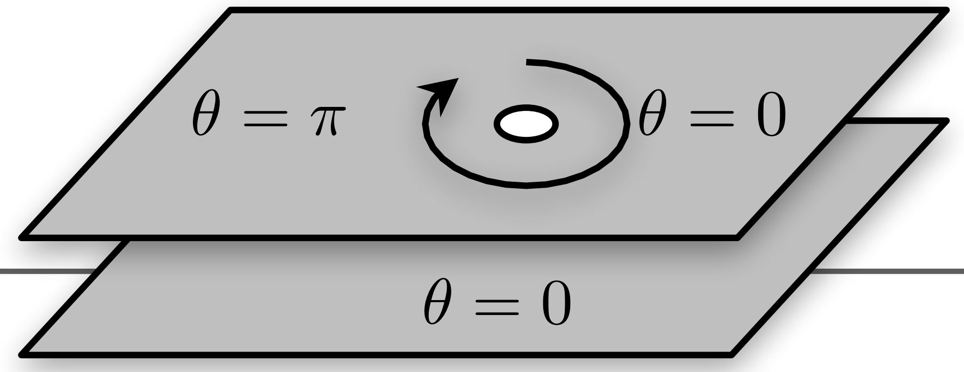
Investigation with AMO systems

Observation of vortices

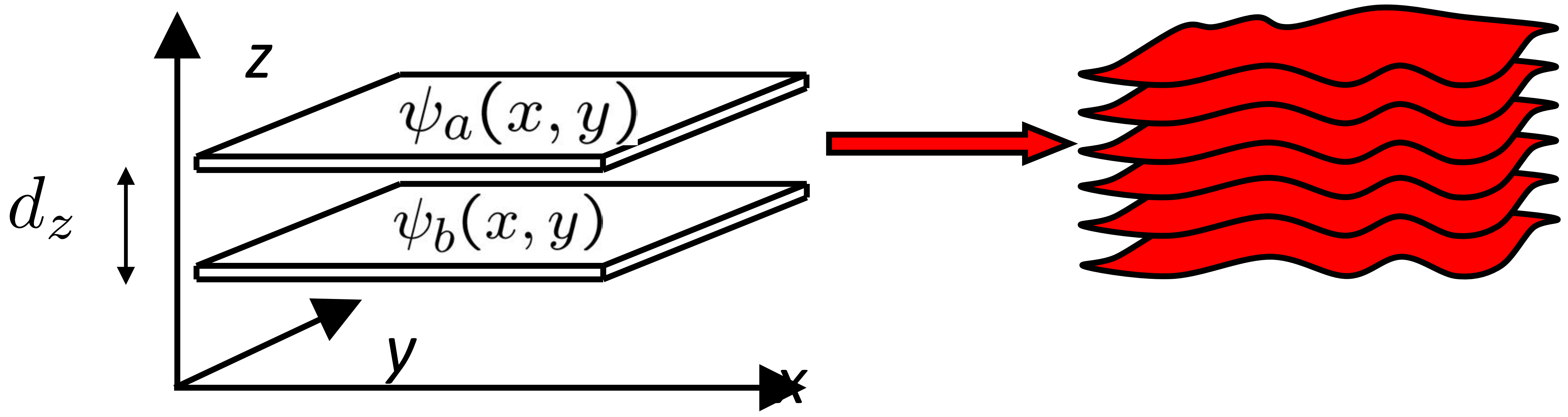
Superfluidity

Sound and superfluid jump

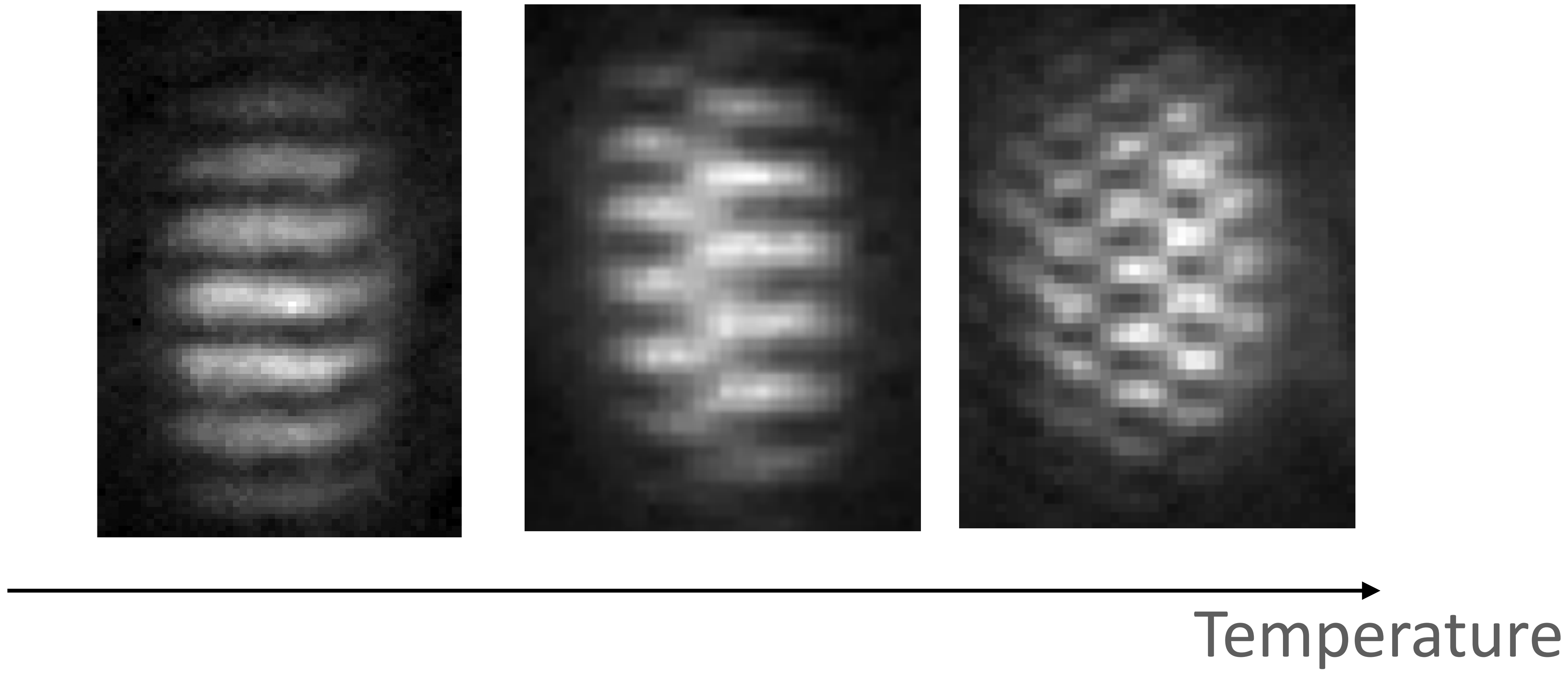
Direct observation of vortices



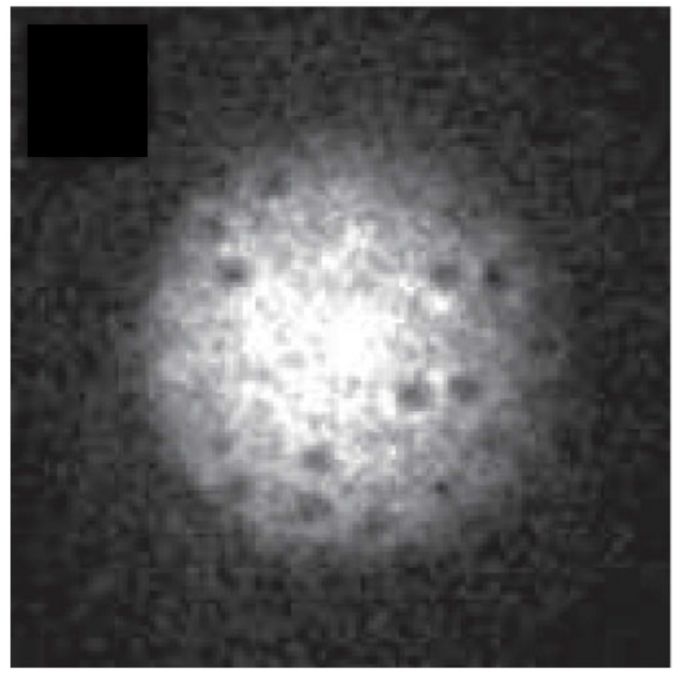
Observation of phase defects



Paris group, Nature 441 1118 (2006)



Observation of density holes in the cloud



Group of J.-I. Shin (Seoul), PRL 110, 175302 (2013)

Boulder, NIST-Gaithersburg, MIT, Cambridge, Chicago, Palaiseau, Amherst, Hamburg,Villetaneuse, Heidelberg, Tokyo-Stanford, Heidelberg, Melbourne,…

4.

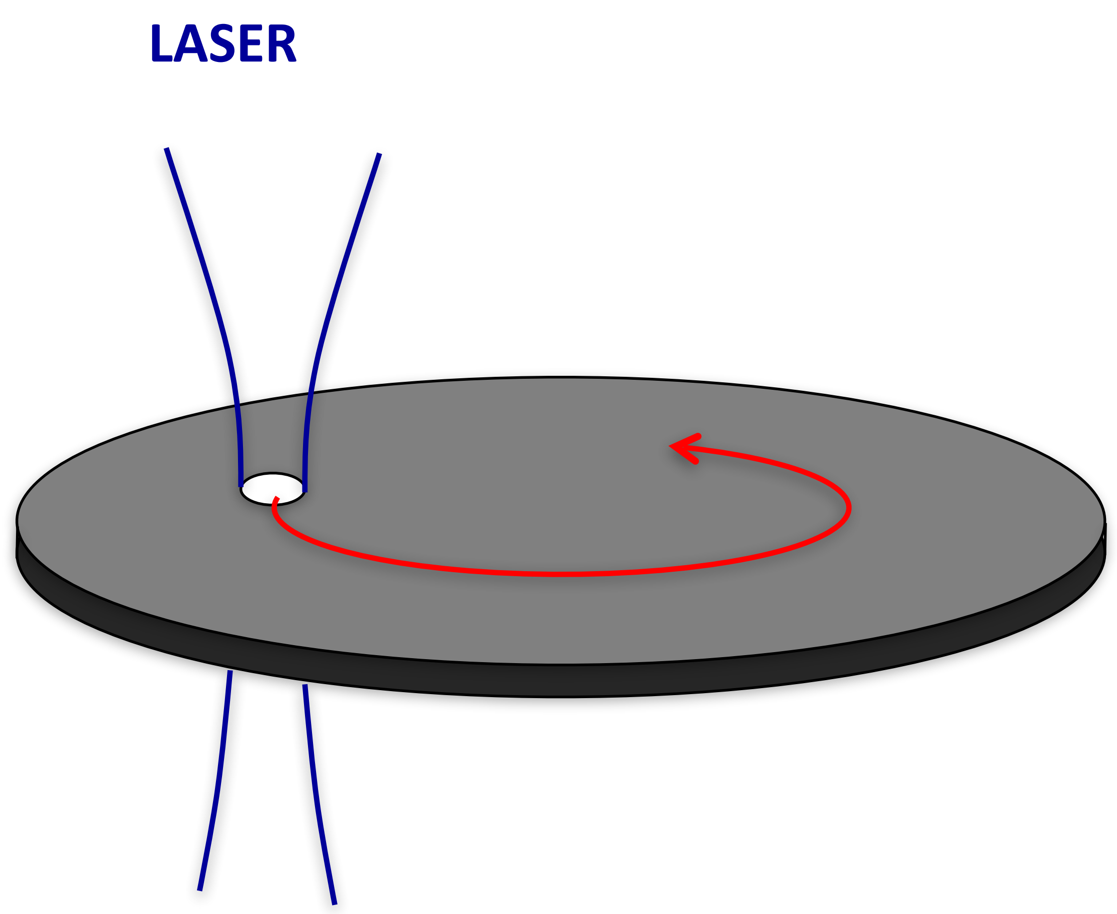
Investigation with AMO systems

Observation of vortices

→ Superfluidity

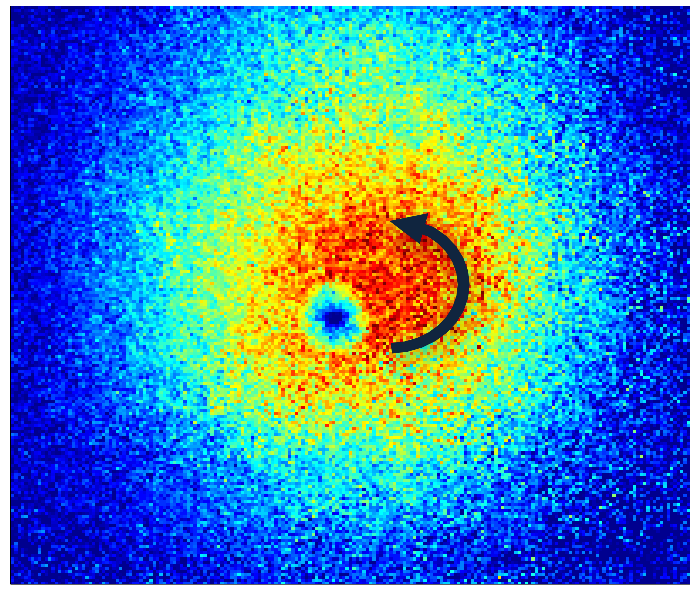
Sound and superfluid jump

Testing superfluidity with a Rb atomic gas



Does a moving impurity “heat” the sample?

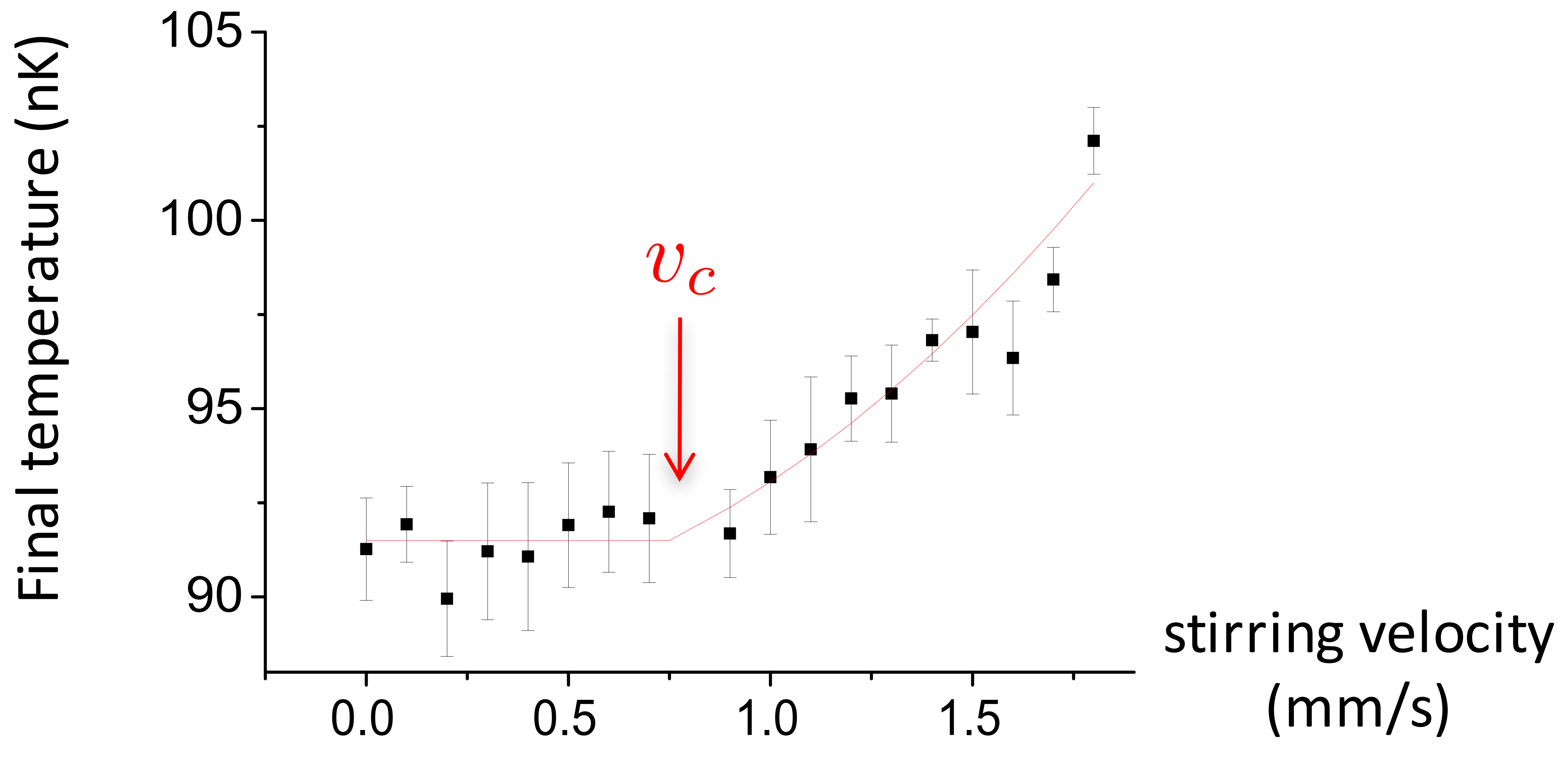
Impurity: focused laser beam that repels the atoms



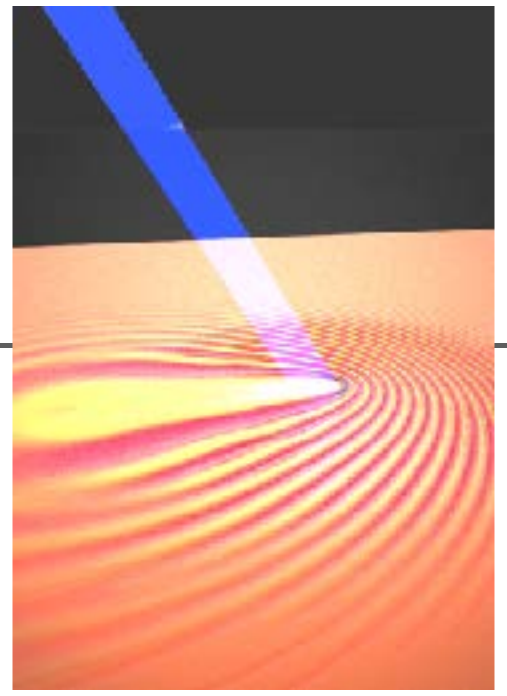
For given μ,T, we stir for 200 ms and
measure the slight increase of temperature

Desbuquois et al.,
Nature Physics 8 645 (2012)

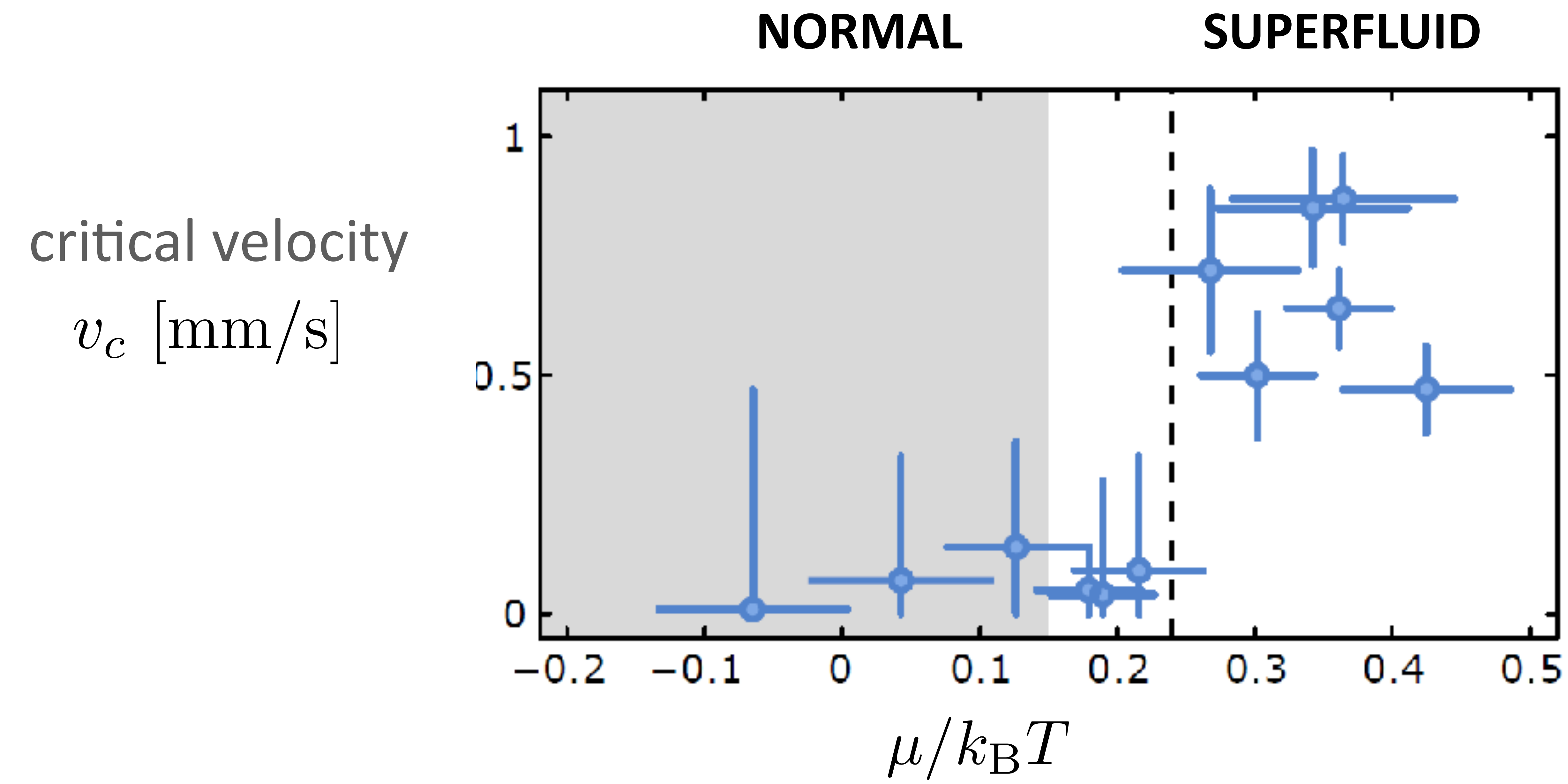
Related experiment in Seoul (2015, Yong-Il Shin’s group)



The critical velocity in 2D



Critical velocity measured for various μ, T



$v_c \approx 0.3$ to $0.6\ c_s$

c_s : sound velocity

$\tilde{g} = 0.1$

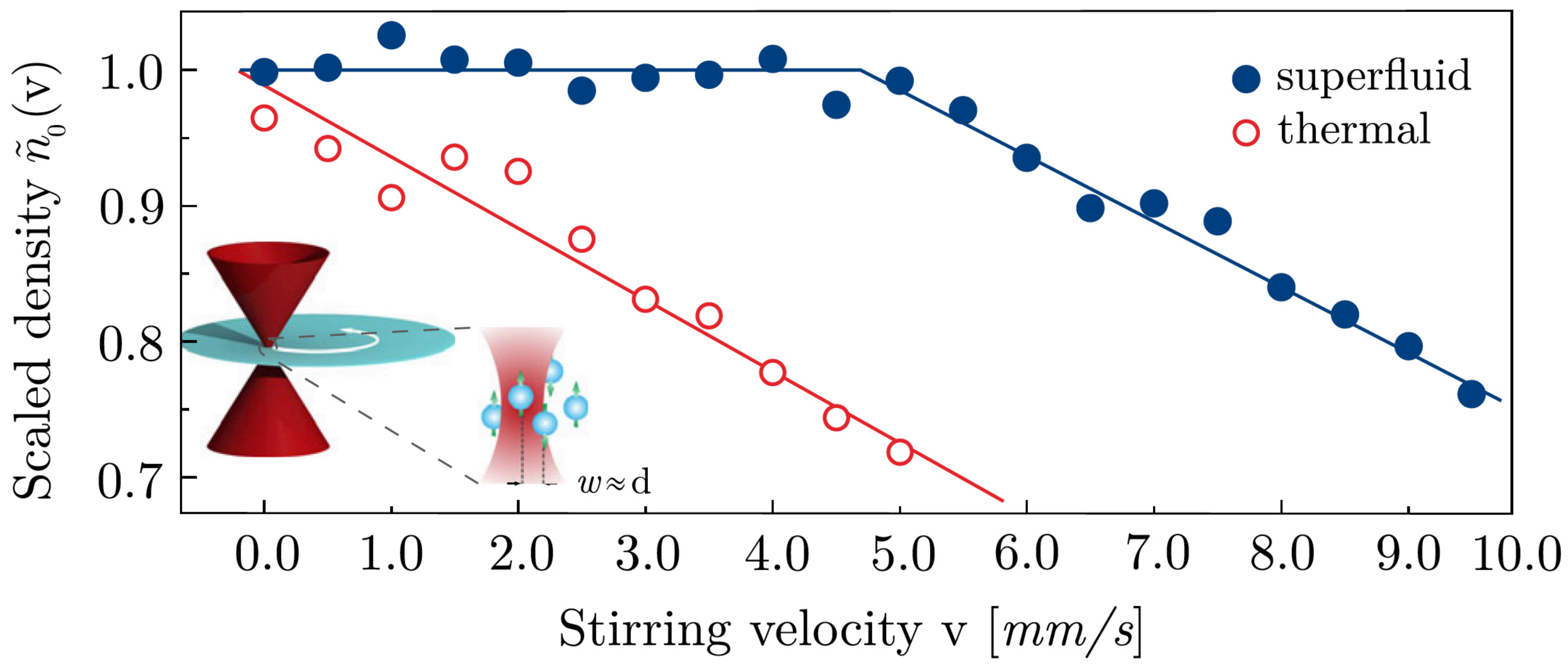
Critical $\mu/k_B T$ in excellent agreement with classical field simulations (Mathey's team)

Desbuquois et al.,
Nature Physics 8 645 (2012)

Vijay Pal Singh et al.,
Phys. Rev. A 95, 043631 (2017)

Testing superfluidity with boson molecules

Hamburg 2015 (Moritz's group): strongly interacting 6Li2



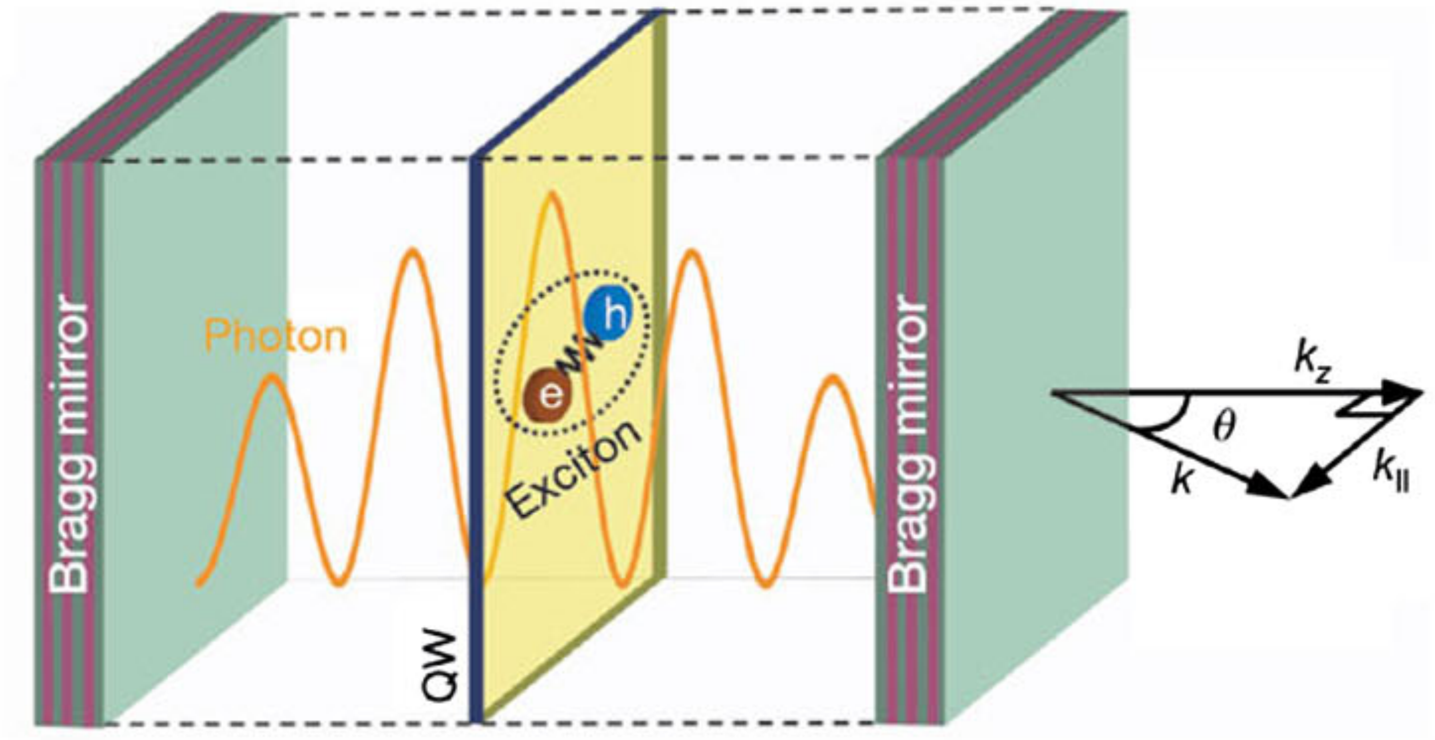
$\tilde{g} \approx 1$

Weimer et al.
Phys. Rev. Lett.
114 095301 (2015)

Here also excellent agreement with classical field simulations (Mathey's team)

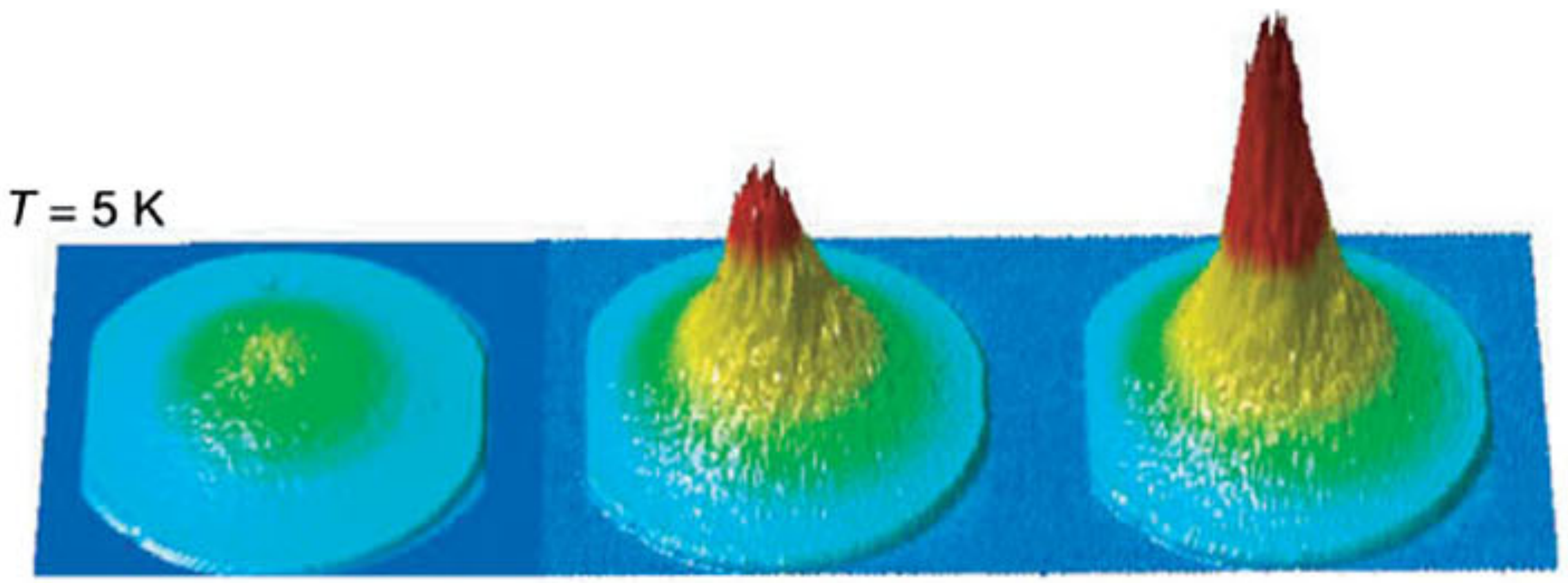
Superfluidity of polariton fluids

Hybrid objects in 2D, partly photon, partly exciton (electron-hole pair in a quantum well)



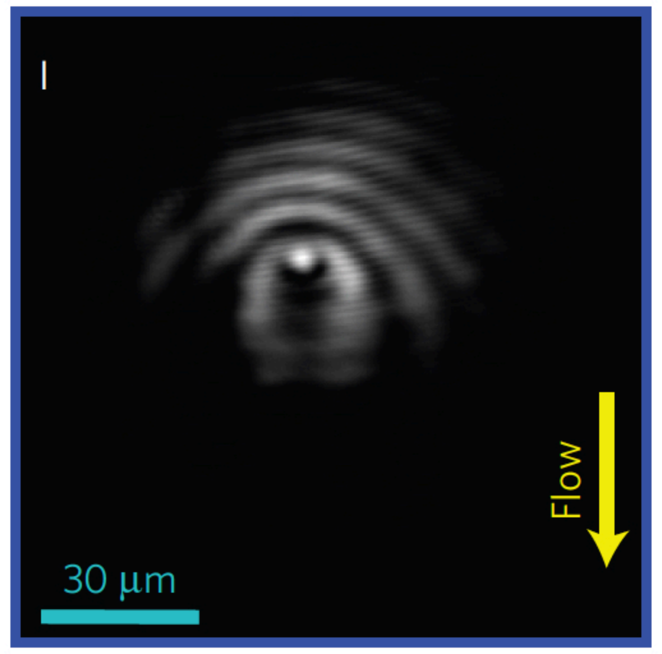
- Very low effective mass
- Interactions due to the exciton part

Quasi-condensation

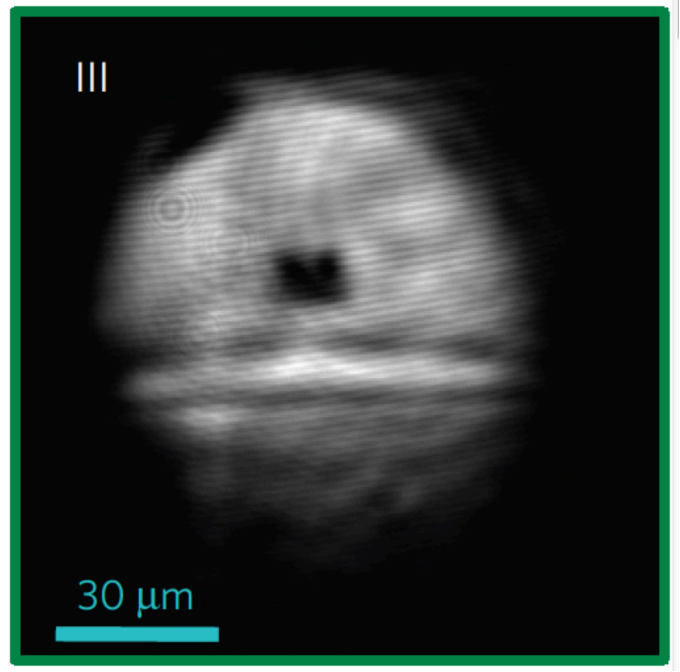


Kasprzak et al.,Nature 443 409 (2006)

Flow around a static defect



low dens: 1 µm^{-2}



large dens: 40 µm^{-2}

Amo et al., Nat. Phys. 5 805 (2009)

4.

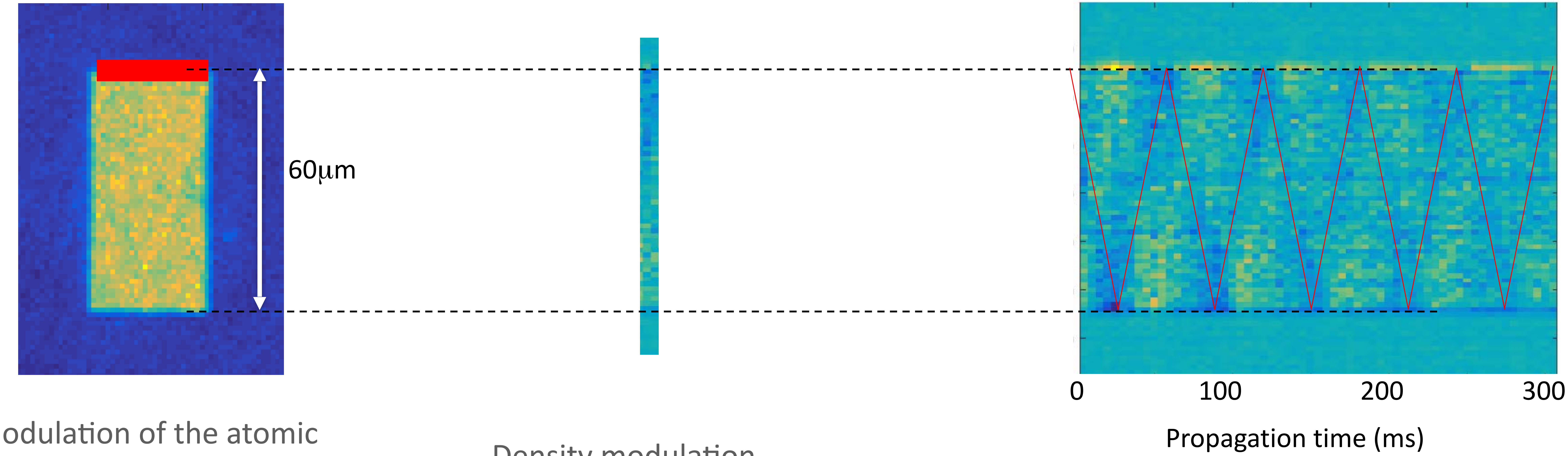
Investigation with AMO systems

Observation of vortices

Superfluidity

Sound and superfluid jump

Propagation of sound waves in a 2D gas



Modulation of the atomic density for a short period (20 ms) on one edge

Density modulation right after the excitation

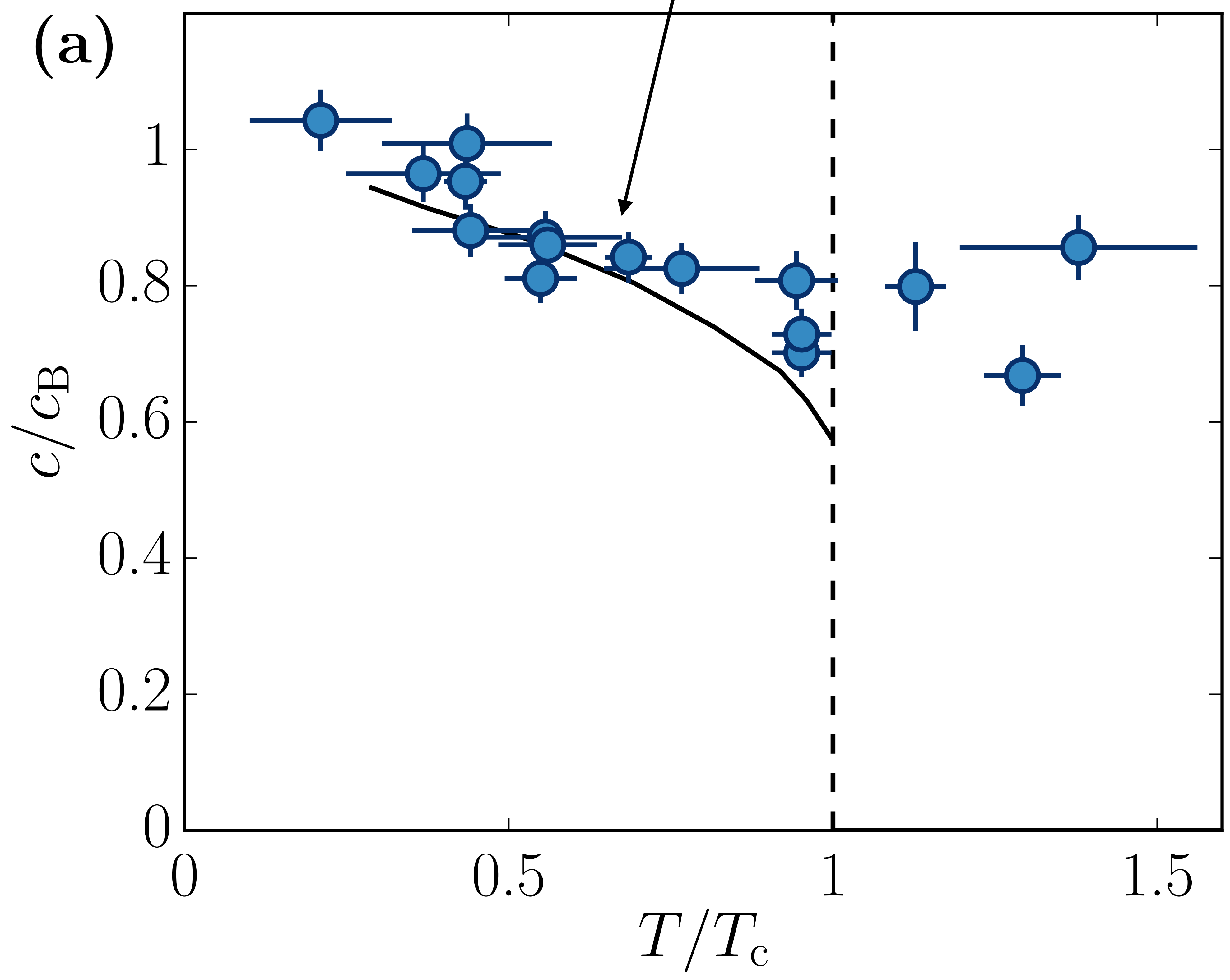
Multiple bounces of the wave packet
Sound velocity 2mm/s

J.-L. Ville et al., Phys. Rev. Lett. 121, 145301 (2018)

The sound velocity in our 2D gas

J.-L. Ville et al., Phys. Rev. Lett. 121, 145301 (2018)

Observation of “second sound”,
related to the superfluid component



^{87}Rb, $\tilde{g} = 0.15$

Line: prediction by Ozawa and Stringari, based on the equation of state calculated by Prokofev and Svistunov

The two-fluid model (both 3D and 2D)

London

Tisza

The essence of the model: superfluid + normal components

$$\rho = \rho_s + \rho_n$$
Total density

$$\boldsymbol{j} = \rho_s \boldsymbol{v}_s + \rho_n \boldsymbol{v}_n$$
Total current

- The entropy of the fluid is attributed entirely to the normal fluid
- The superfluid flow is irrotational, except for quantized vortices (which are not relevant for sound waves)

Propagation of a weak perturbation with a low frequency *ω*

- Superfluid hydrodynamics: $\omega \ll \mu/\hbar$ (i.e., wavelength >> healing length)
- Normal hydrodynamics: $\omega \ll \Gamma_{\text{coll}}$ (i.e., wavelength >> mean free path)

Leads to two wave equations: $$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 P \qquad \frac{\partial^2 \tilde{s}}{\partial t^2} = \frac{\rho_s}{\rho_n}\, \tilde{s}^2\, \nabla^2 T$$ $\longrightarrow$ Two types of sound waves

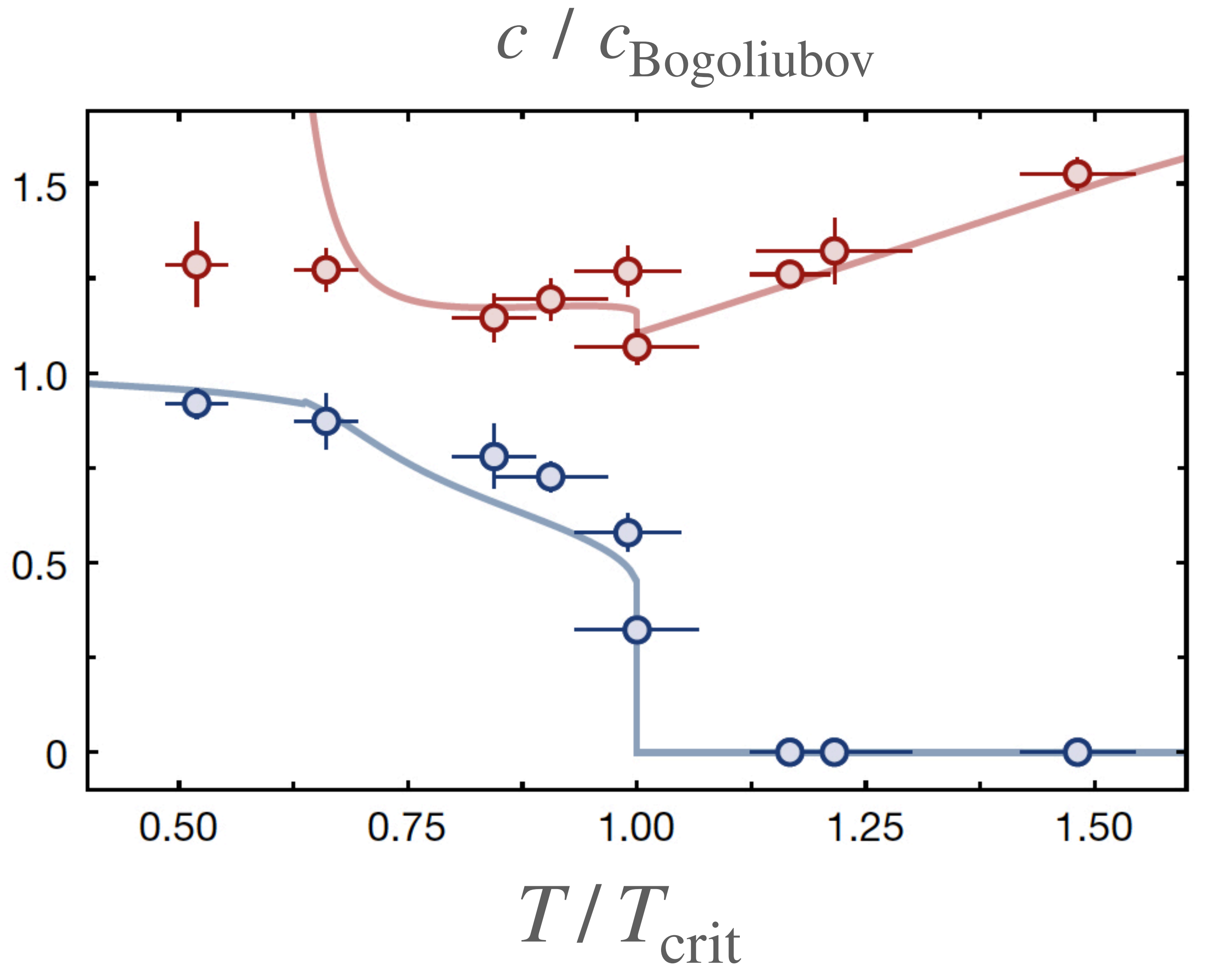
P : pressure $\qquad$ $\tilde{s}$: entropy/unit mass

Bi-square equation for the speed of sound: $c^4 - \alpha c^2 + \beta = 0$ with α, β functions of ρ_s/ρ_n

Observation of the two sounds

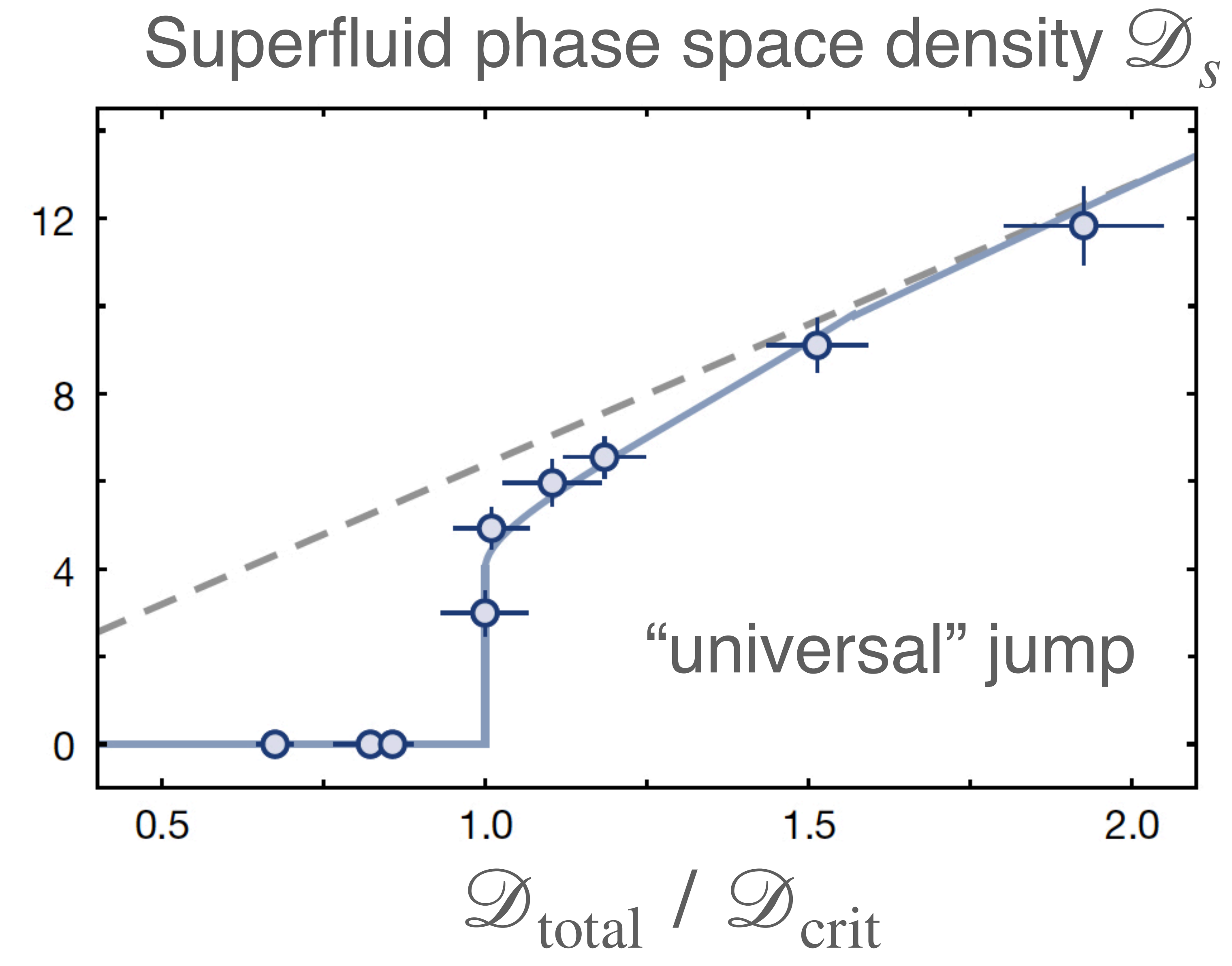
Cambridge : Hadzibabic group (Nature, 2021)

^{39}K, $\tilde{g} = 0.64$



$$\frac{\partial^2 \rho}{\partial t^2} = \nabla^2 P$$

$$\frac{\partial^2 \tilde{s}}{\partial t^2} = \frac{\rho_s}{\rho_n}\, \tilde{s}^2\, \nabla^2 T$$



Summary

From Peierls to Berezinskii - Kosterlitz - Thouless

R. Peierls 1907-95

VADIM L'VOVICH BEREZINSKIĬ (1935-1980)

J.M. Kosterlitz

D.J. Thouless

No breaking of a continuous symmetry in a 2D system at $T \neq 0$

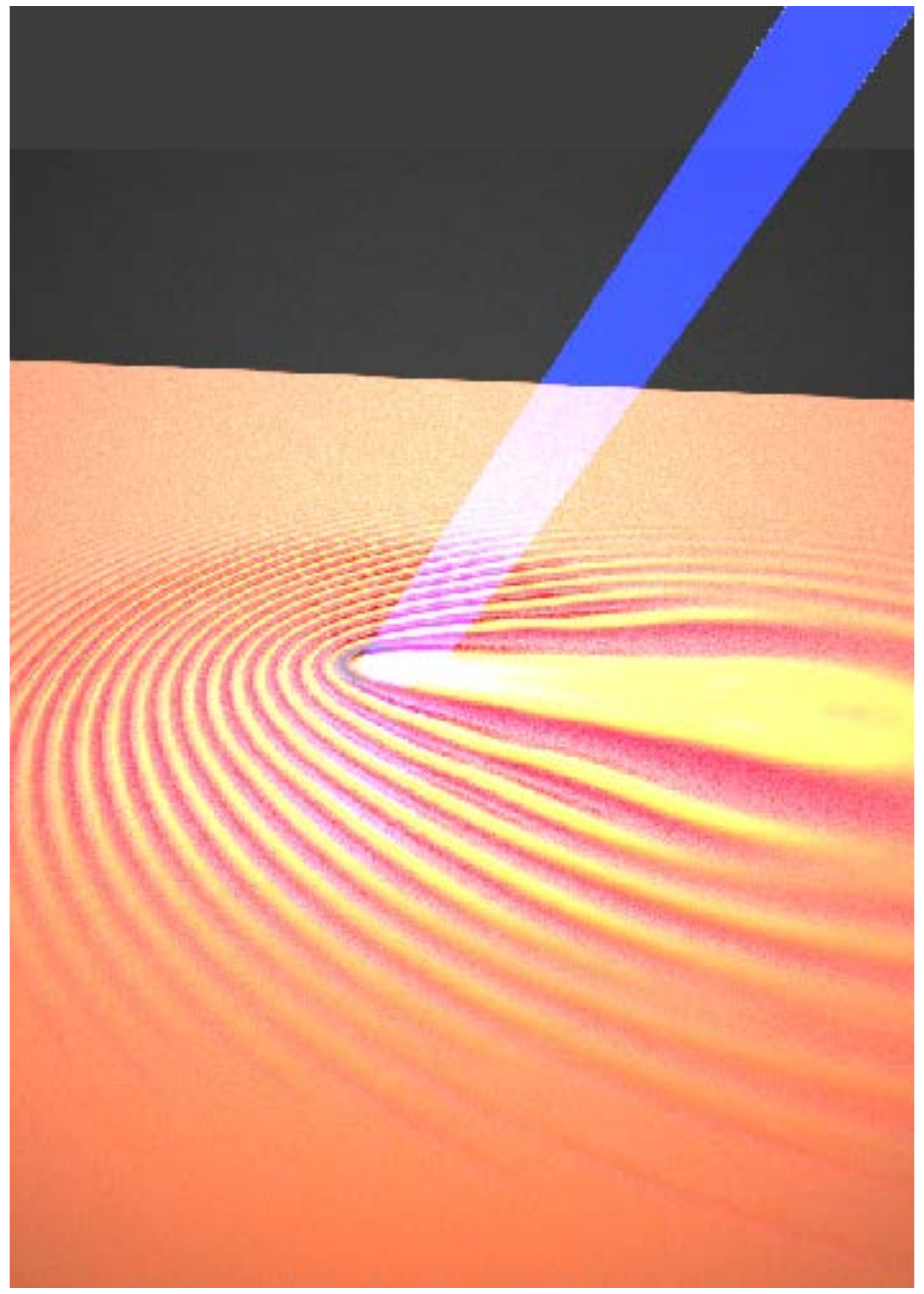
BKT : A non conventional phase transition is still possible

Superfluid transition

The role of AMO systems

Quantum fluids with atoms, molecules, photons, polaritons, have provided a unique insight in several aspects of BKT physics

- Superfluid behavior and critical point
- Visualisation of vortices
- Sound propagation
- Evidence for algebraic decay: $G_1(r) \propto r^{-\alpha}$
  $\alpha \approx 1/4$ at the critical point (Oxford 2022)



Current and future developments

- Influence of disorder
- Dynamics across the phase transition: revisiting the Kibble-Zurek mechanism