Scale and conformal invariance for cold atomic gases

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Solvay chair for Physics 2022

Lecture 3

Scale invariance

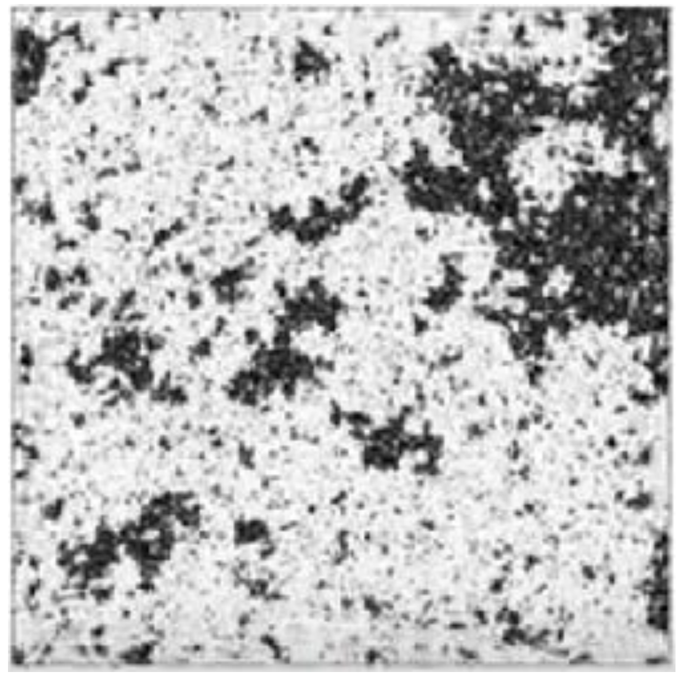
A concept that was introduced in the 70's in high energy physics

Can there be physical systems with no intrinsic energy/length scale?

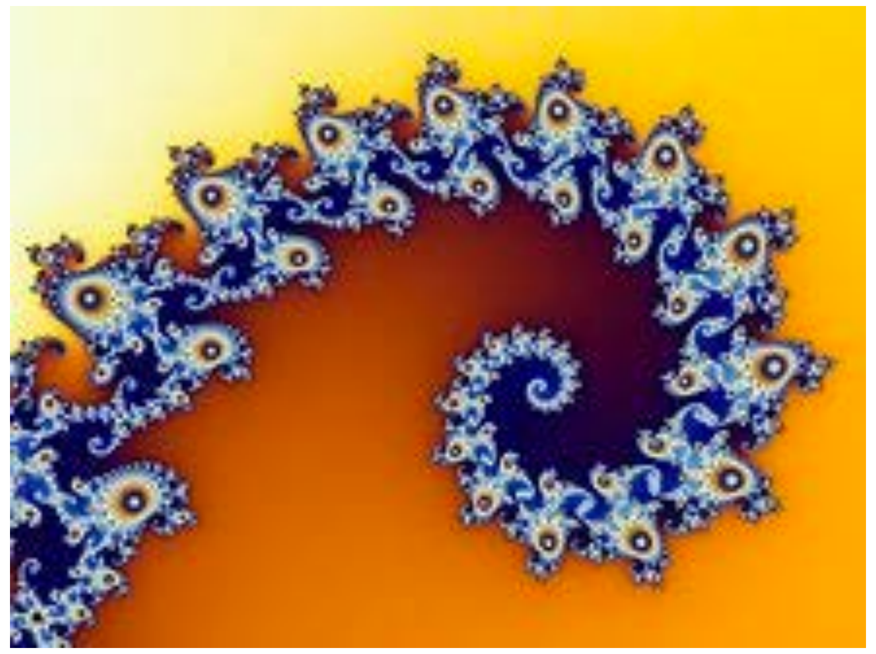
Need to explain the behavior of e^- - nucleon scattering cross-sections

This concept later found many applications in physics, maths, biology, etc.

Phase transitions and renormalization group



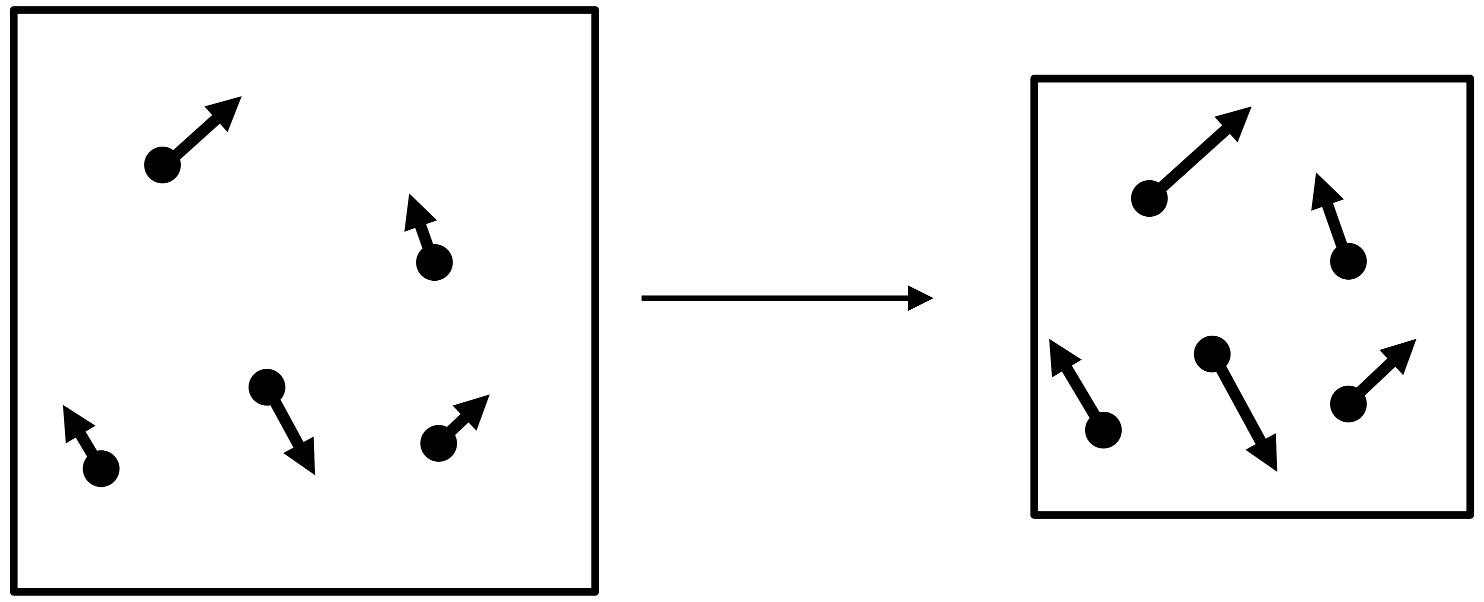
Fractals



Scale invariance in a gas of particles

Consider a fluid whose equations of motion, *i.e.* its action $\int E \, \mathrm{d}t$, are invariant in the following rescaling:

Positions: $\boldsymbol{r} \;\rightarrow\; \boldsymbol{r}/\lambda$ Time: $t \;\rightarrow\; t/\lambda^2$



Velocity: $\boldsymbol{v} \;\rightarrow\; \lambda \boldsymbol{v}$

Considerable simplification of the study of equilibrium properties and dynamics

Clearly $E_{\mathrm{kin}} \rightarrow \lambda^2 E_{\mathrm{kin}}$, implying that $\int E_{\mathrm{kin}} \, \mathrm{d}t$ is invariant

What about interactions? Can we achieve $E_{\mathrm{int}} \rightarrow \lambda^2 E_{\mathrm{int}}$ when $\boldsymbol{r} \rightarrow \boldsymbol{r}/\lambda$?

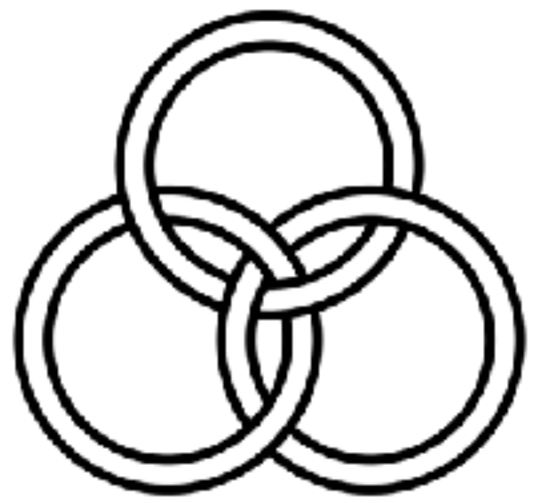
Gases with scale invariant interactions (1): the $1/r^2$ potential

$\boldsymbol{r} \to \boldsymbol{r}/\lambda \qquad E_{\rm int} \to \lambda^2 E_{\rm int}$

The simplest case: the $1/r^2$ potential

$$V = \sum_{i<j} \frac{g}{r_{ij}^2}$$

Calogero-Moser-Sutherland model in 1D



Efimov problem in 3D

For such a potential, there is no length scale associated to interactions

Reminder: for a power-law potential g/r^n, the relevant (quantum!) length scale ℓ is obtained by equating kinetic and potential energy

$$\frac{\hbar^2}{m\ell^2} = \frac{g}{\ell^n}$$

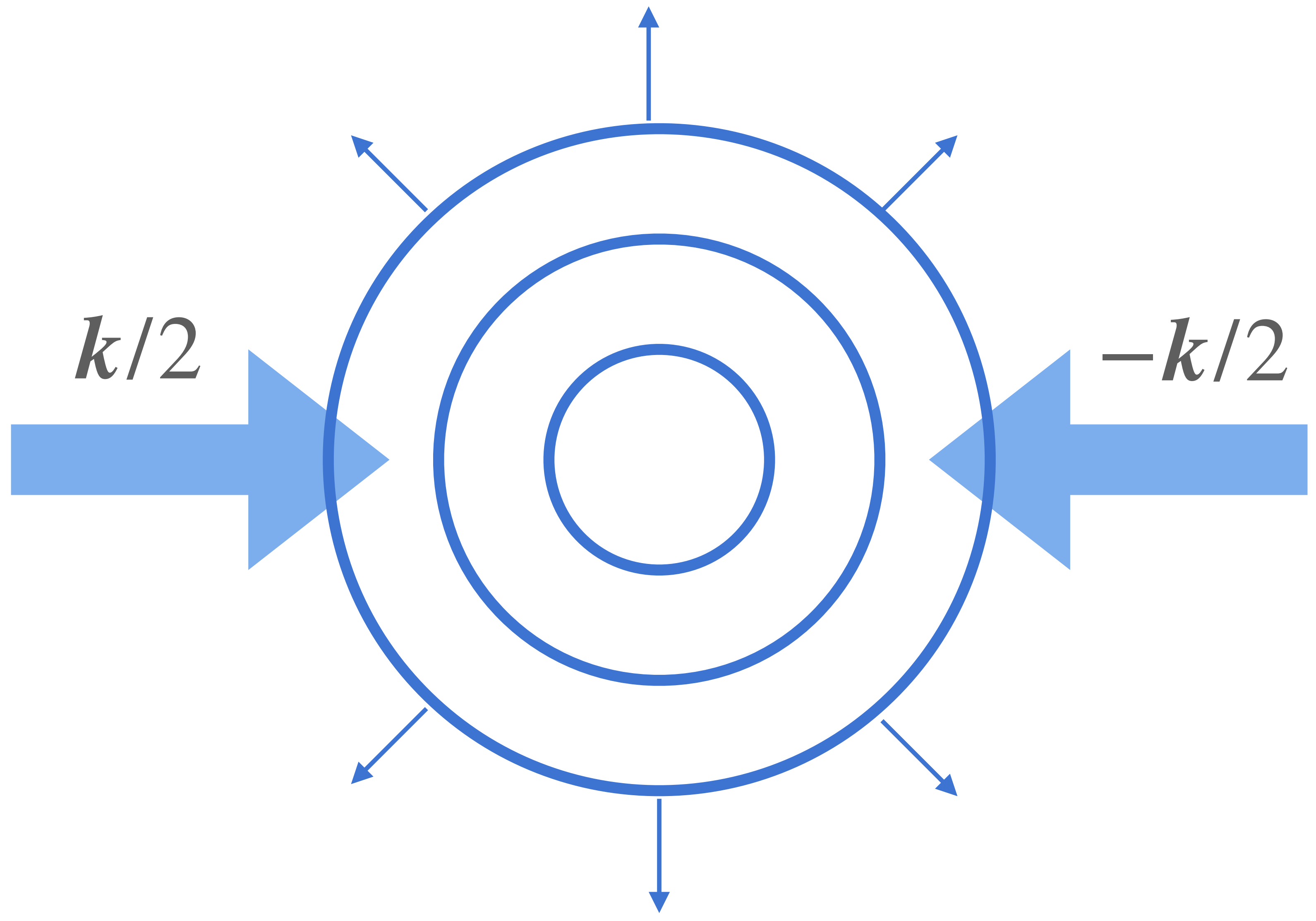
- Coulomb interaction ($n = 1,\ g = e^2$): $\ell =$ Bohr radius $\hbar^2/me^2$
- Van der Waals interaction ($n = 6,\ g = C_6$): $\ell =$ van der Waals radius $\propto \left(mC_6/\hbar^2\right)^{1/4}$

No characteristic length ℓ for $n = 2$!

Gases with scale invariant interactions (2): the unitary case

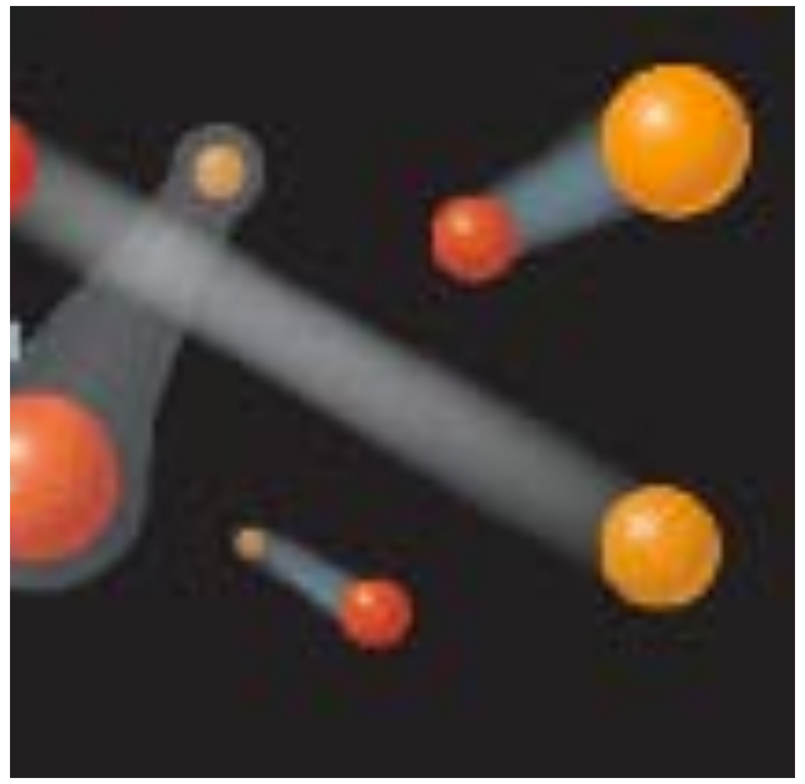
Collision between two atoms

s wave regime (low energy): $\psi(\boldsymbol{r}) \sim e^{i\boldsymbol{k}\cdot\boldsymbol{r}} - \frac{a}{1+ika}\frac{e^{ikr}}{r}$



a : scattering length

A Feshbach resonance allows one to reach the limit $a \to \infty$: $\psi(\boldsymbol{r}) \sim e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + i\,\frac{e^{ikr}}{kr}$

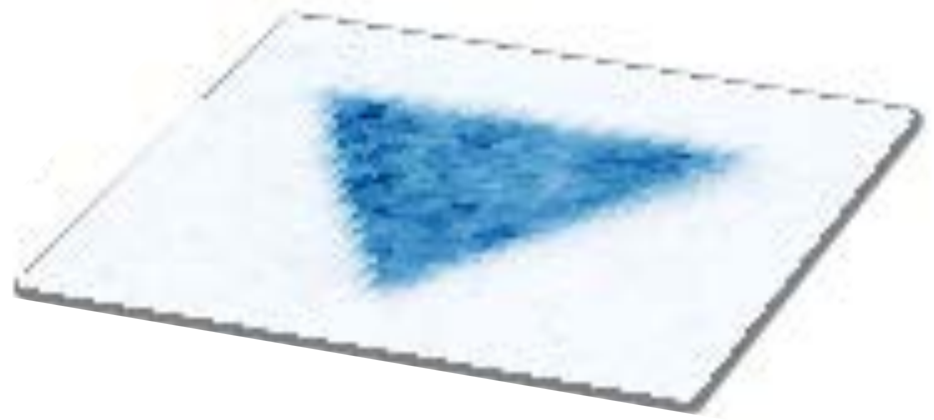


Unitary limit: the strongest interaction allowed by Quantum Mechanics

For bosons, this unitary 2-body physics comes with subtle 3-body effects (Efimov)

For spin 1/2 fermions, genuine scale invariant system: no length scale associated to interactions

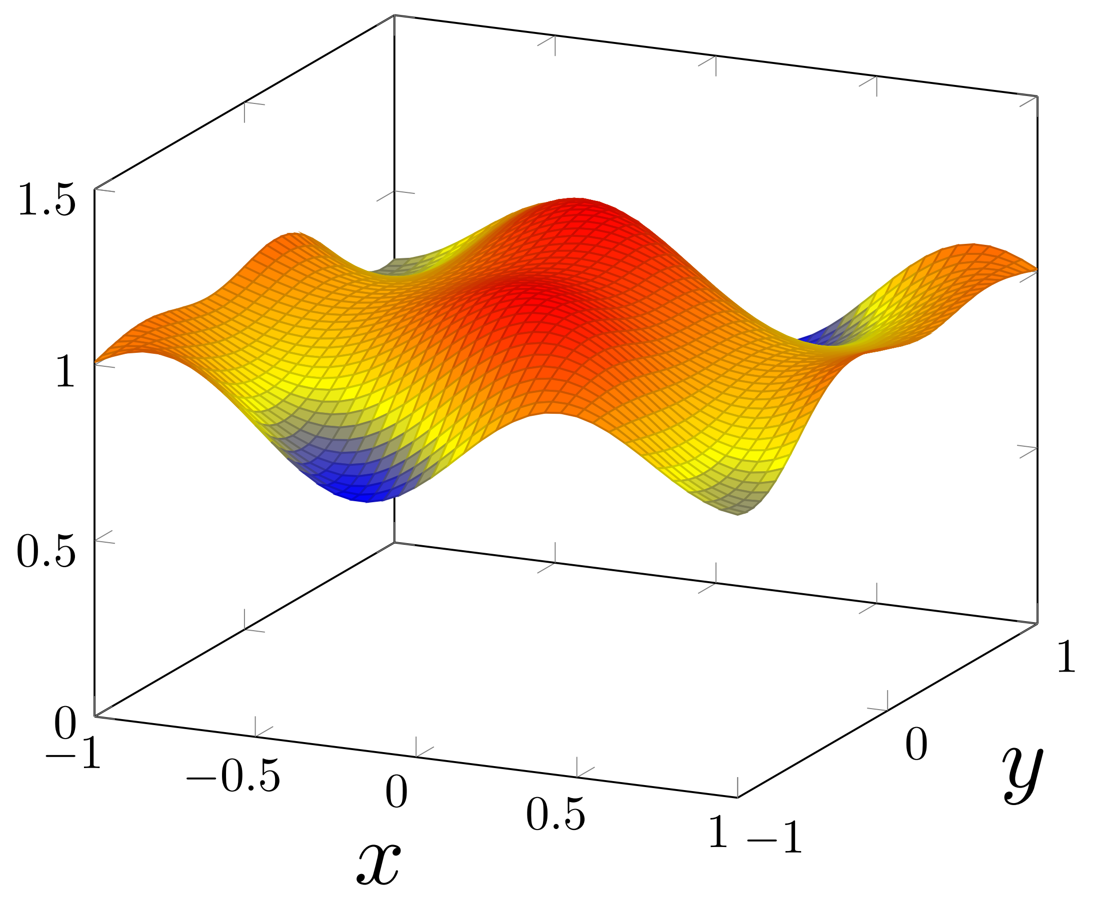
Gases with scale invariant interactions (3)



Contact interaction in a 2D Bose gas: $\boldsymbol{r} \to \boldsymbol{r}/\lambda \qquad g\,\delta(\boldsymbol{r}) \to g\,\delta(\boldsymbol{r}/\lambda) = \lambda^2\, g\,\delta(\boldsymbol{r})$

Valid only for relatively weak interactions, so that a classical field description (Gross-Pitaevskii equation) is valid (otherwise, quantum anomaly from the regularisation of $\delta(\boldsymbol{r})$ *)*

Energy of the gas: $E(\psi) = E_{\text{kin}}(\psi) + E_{\text{int}}(\psi)$



$$E_{\text{kin}}(\psi) = \frac{\hbar^2}{2m}\int |\boldsymbol{\nabla}\psi|^2 \qquad\qquad E_{\text{int}}(\psi) = \frac{\hbar^2}{2m}\tilde{g}\int |\psi|^4$$

$\tilde{g}$: interaction strength

No singularity for the contact interaction at the classical field level

In 3D, $\tilde{g} = 4\pi a$ where a is the scattering length

In 2D, the interaction strength $\tilde{g}$ is dimensionless: no length scale associated with interactions

Outline of the lecture

Time-independent problems

Universality of the equation of state

Solitons in 2D

The Efimov effect

Time-dependent problems

Conformal invariance and the SO(2,1) dynamical symmetry

The breathing mode

Breathers

Scale-invariant equation of state

For a “standard” cold 3D gas, the scattering length a brings the energy scale $\epsilon \equiv \hbar^2/ma^2$

Exemple of an equation of state: $$n\lambda^3 = \mathscr{F}\left(\frac{k_B T}{\epsilon}, \frac{\mu}{\epsilon}\right)$$ *i.e., a 2-variable function*

For a scale-invariant Fermi gas ($a = 0$ or $a = \infty$), it must read $$n\lambda^3 = \mathscr{G}\left(\frac{\mu}{k_B T}\right)$$

Considerable simplification (1-variable function) which leads to $PV = \frac{2}{3}E$ T.L. Ho, 2004

Similarly for a 2D Bose gas: $$n\lambda^2 = \mathscr{G}\left(\frac{\mu}{k_B T}, \tilde{g}\right)$$ $\tilde{g}$ dimensionless coupling

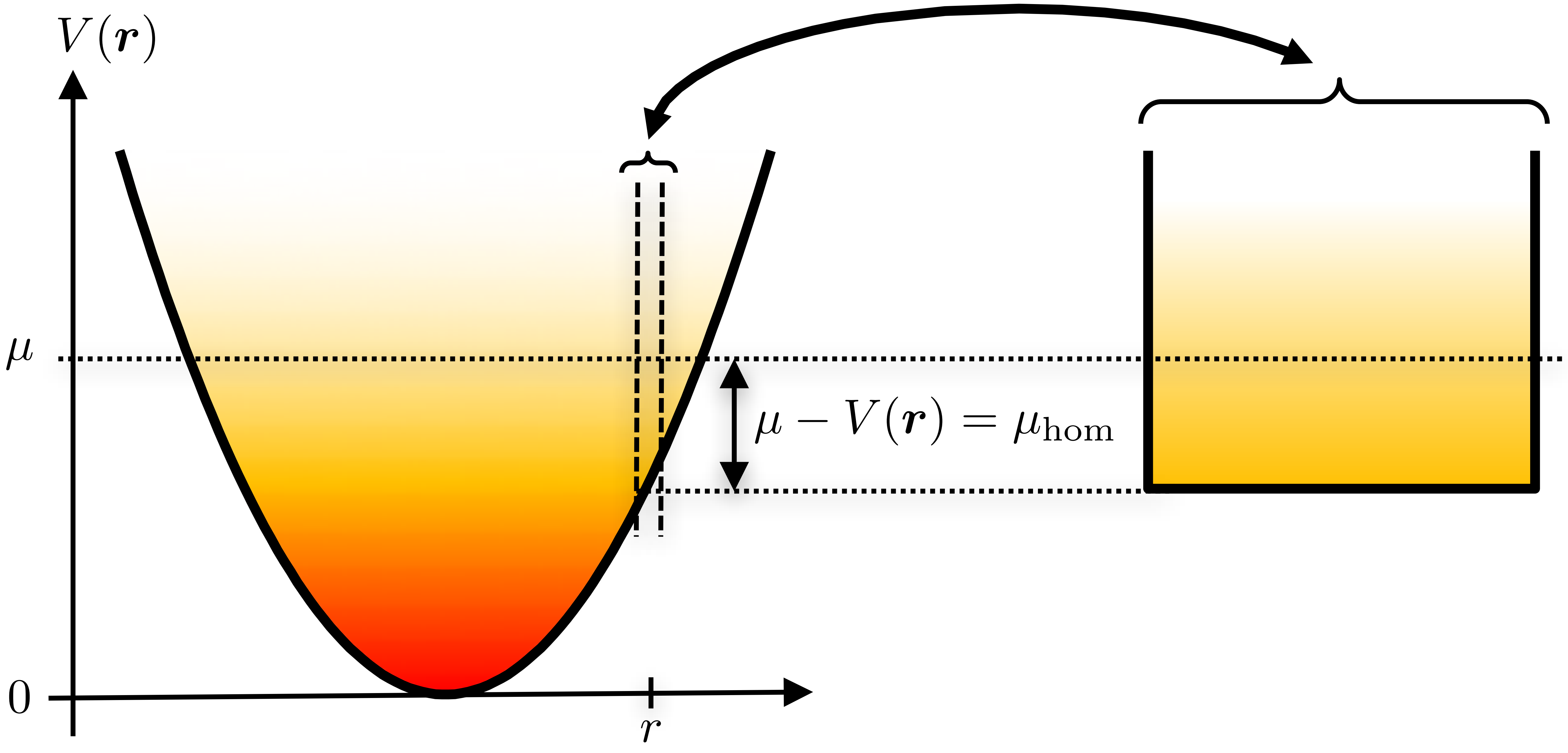
$$\longrightarrow \quad PV = E$$

Trapped atomic gases and local density approximation

Gas at equilibrium in a trap with temperature T and chemical potential μ

Link between the density at one point in the trap and that of a homogeneous system

$$T_{\text{hom.}} = T \qquad \mu_{\text{hom.}} = \mu - V(\boldsymbol{r})$$



Validity : mean free path, healing length << size of the gas

The equation of state of the 2D Bose gas

$$n\lambda^2 = \mathcal{G}\left(\frac{\mu}{k_B T}, \tilde{g}\right)$$

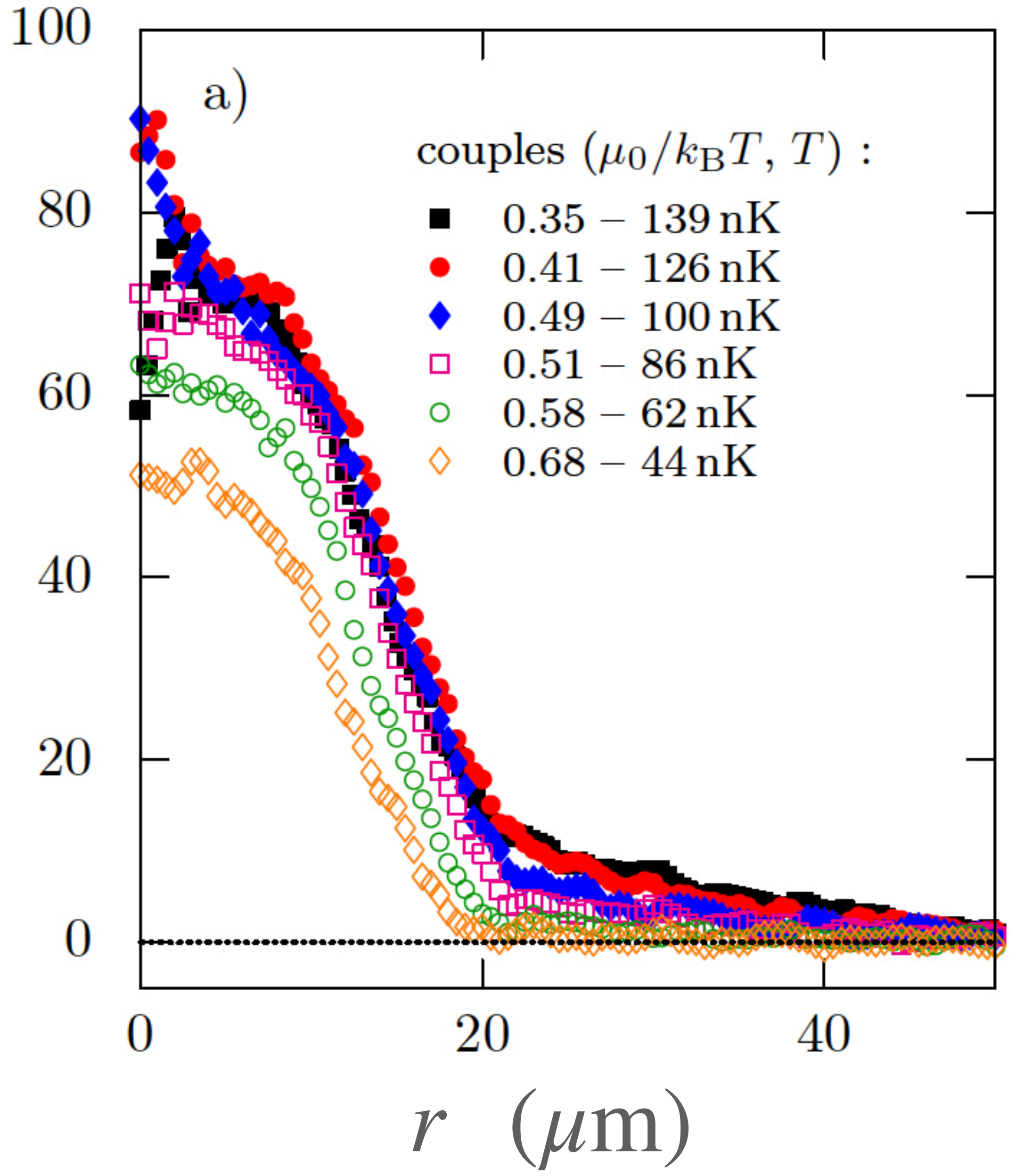
Theory using a classical-field analysis: Prokof'ev & Svistunov

Measurements : Chicago, Paris, Cambridge

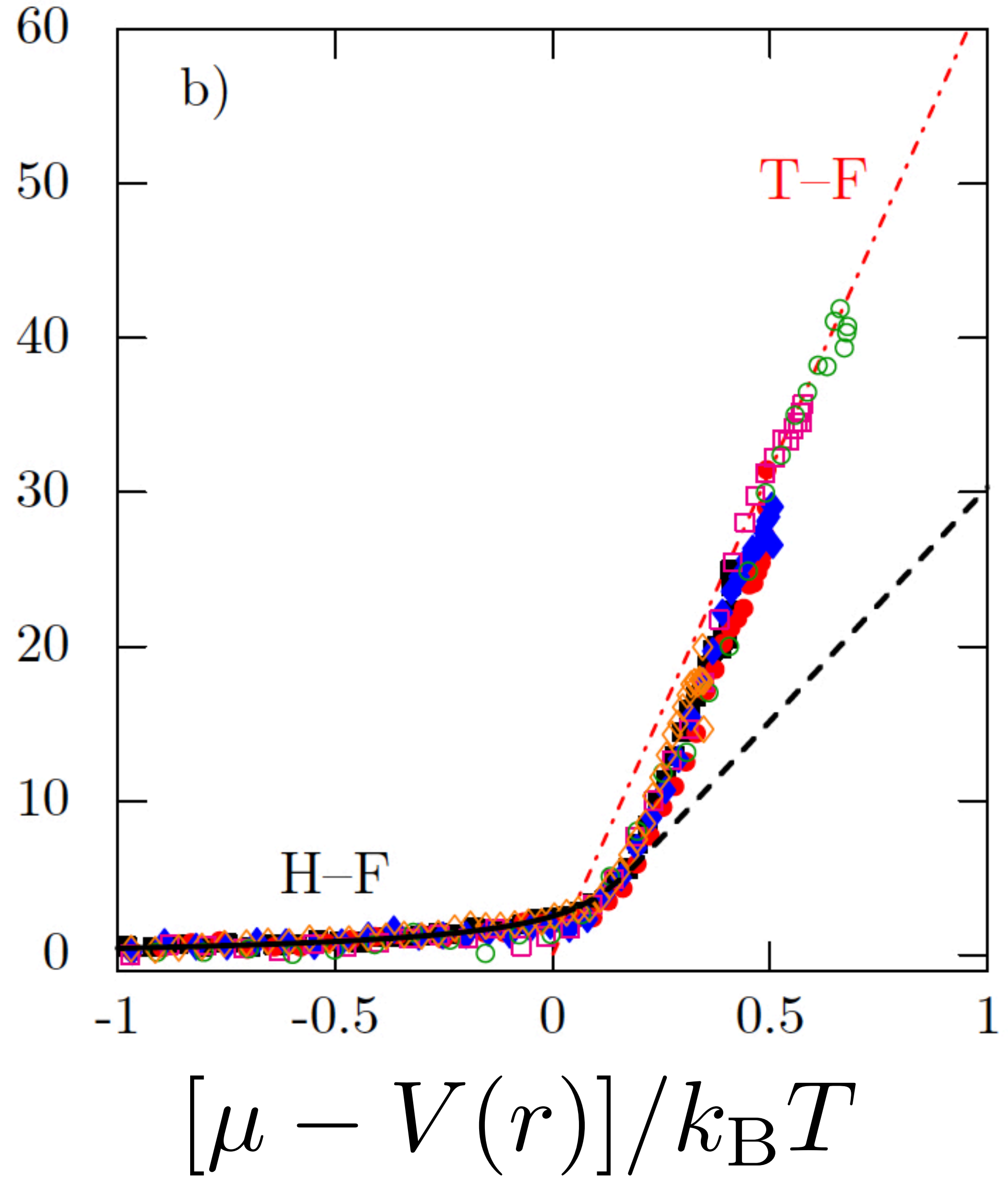
Smooth external trapping potential $V_{\text{trap}}(r)$ + local-density approximation: $\mu(r) = \mu(0) - V_{\text{trap}}(r)$

A single image gives access to the desired function $\mathcal{G}$: $n(r)\,\lambda^2 = \mathcal{G}\left(\frac{\mu(r)}{k_B T}\right)$

Density (at/µm²)



$\mathcal{D}_{\text{total}}$



Note the absence of any discontinuity or cusp: KT transition is of infinite order

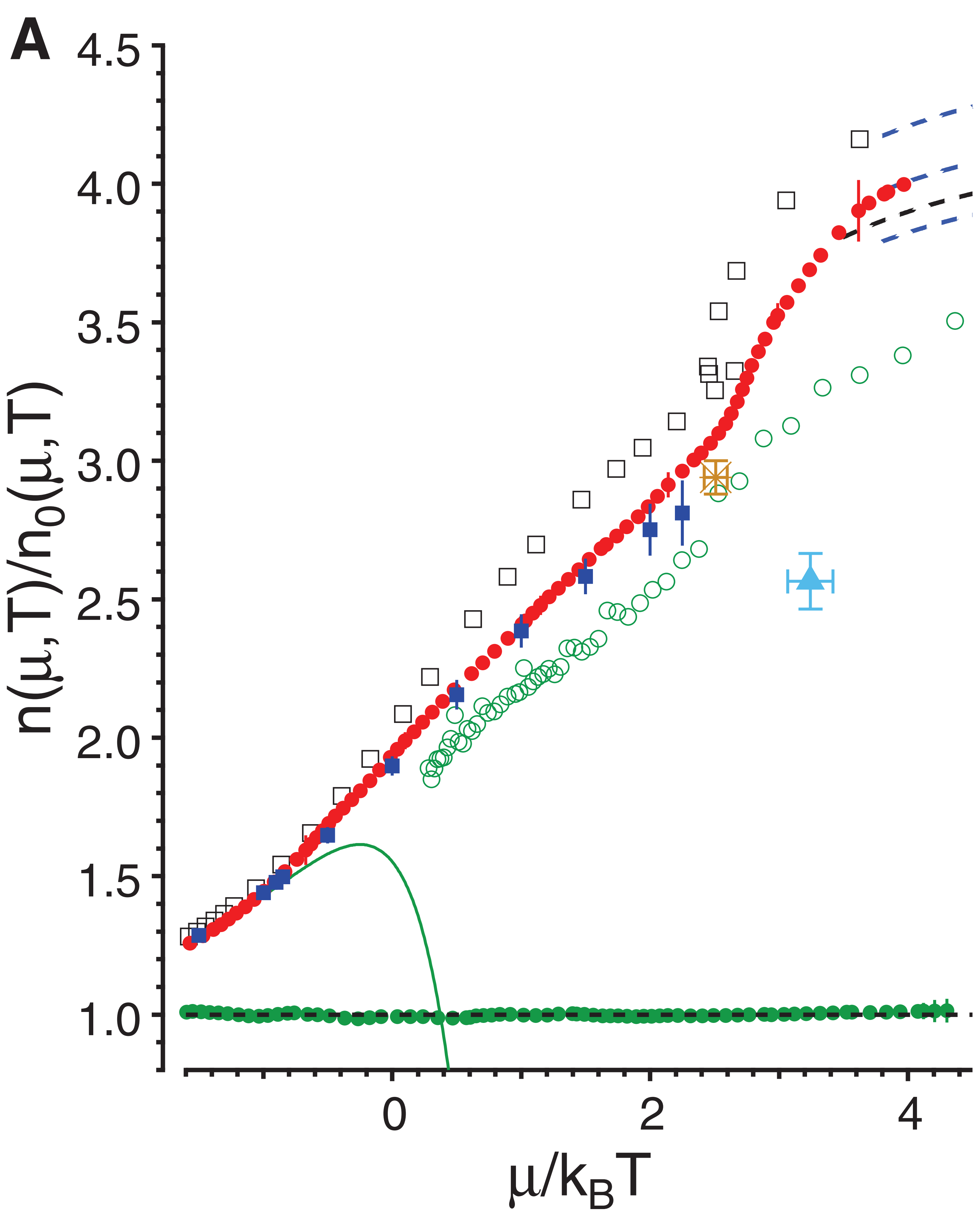
Yefsah et al., PRL **107**, 130401 (2011)

Desbuquois et al., PRL **113**, 020404 (2014)

The equation of state of the 3D unitary Fermi gas

Paris (Salomon group), MIT (Zwierlein group)

Ku et al., (2012)
Science **335**, 563



Red solid circles: experimental EoS.

Green solid circles: Ideal Fermi gas.

Blue solid squares: diagrammatic Monte Carlo calculation for density

Solid green line: third-order Virial expansion.

Open black squares: self-consistent T-matrix calculation.

Open green circles: lattice calculation

Orange star and blue triangle: critical point from the Monte Carlo calculations.

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→ Solitons in 2D

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Solitons for the Gross-Pitaevskii equation

Look for a stationary wave function ψ solution of the variational problem $\delta\left[E(\psi)\right] = 0$ for an attractive non-linearity $g < 0$

$$E[\psi] = \frac{1}{2}\int\left(\left|\nabla\psi\right|^2 + g\left|\psi\right|^4\right)\,\mathrm{d}^D r$$

$$\hbar = m = 1$$

$$\int|\psi|^2 = N$$

Relevant in optics, atomic physics, condensed matter…

Dimensional analysis for a wave packet of size ℓ:

$$\frac{E(\ell)}{N} \sim \frac{1}{\ell^2} - \frac{N|g|}{\ell^D}$$

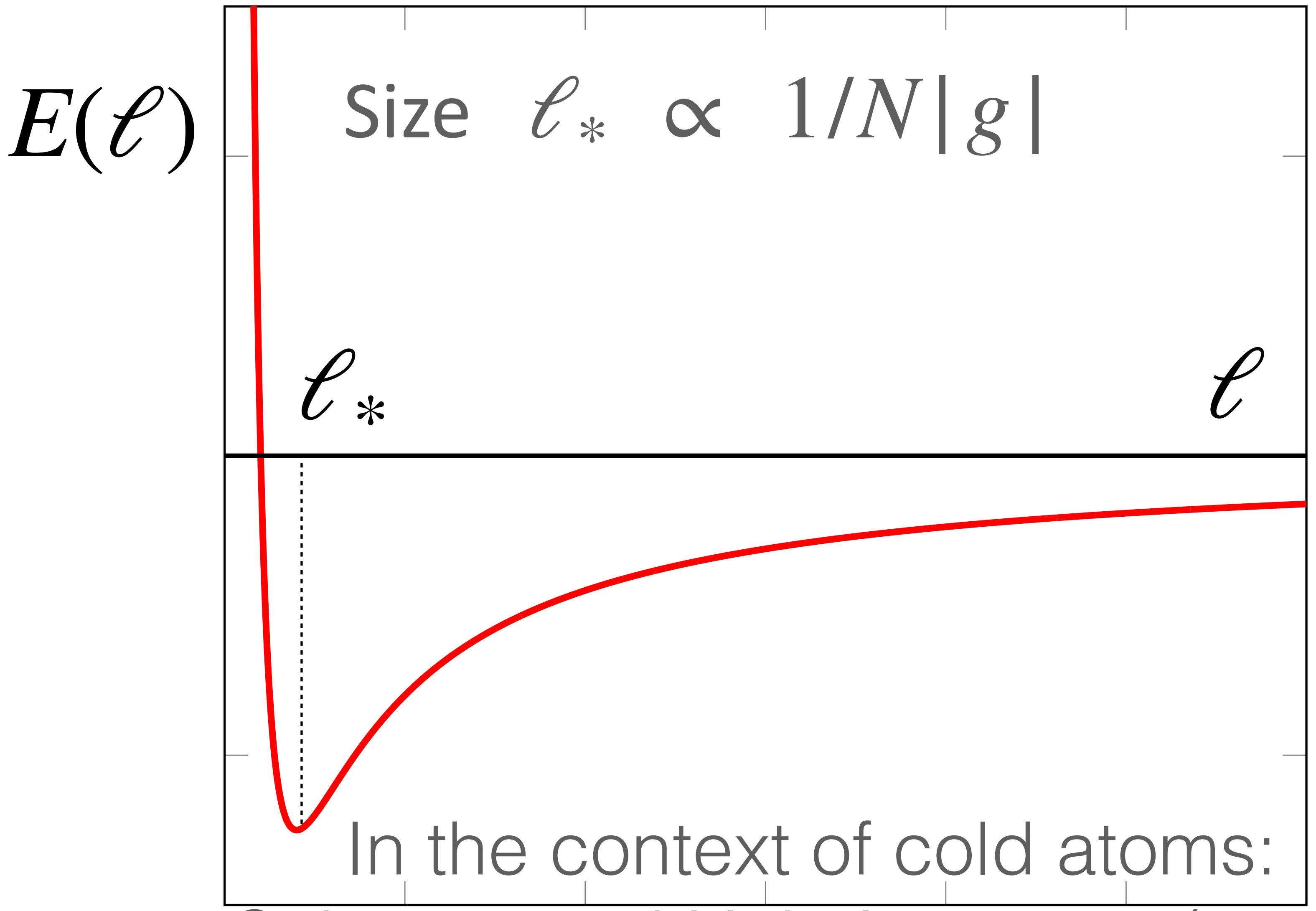
Crucial role of dimensionality D

Solitons in 1D, 2D, 3D

Wave packet of size ℓ in dimension D :

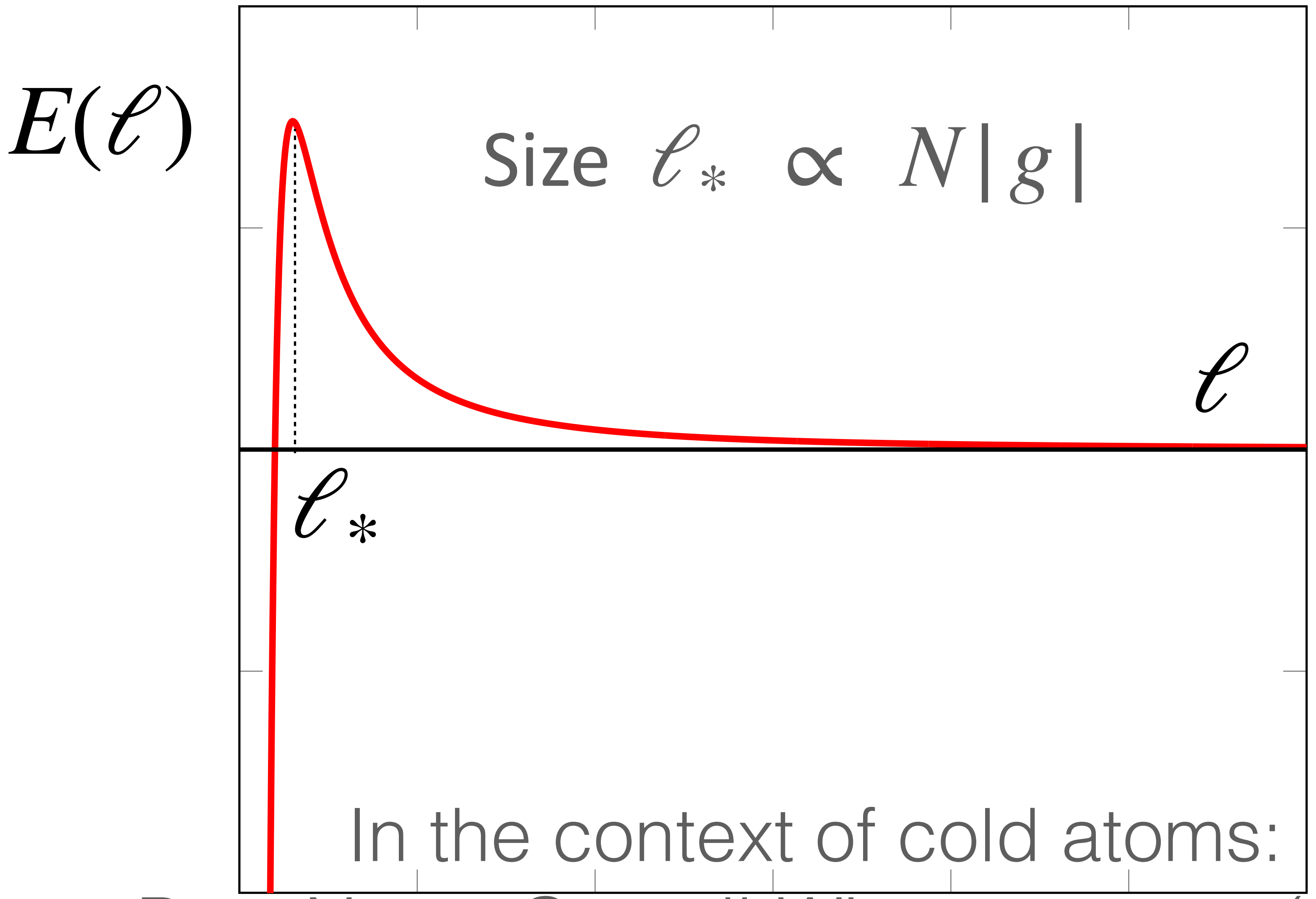
$$\frac{E(\ell)}{N} \sim \frac{1}{\ell^2} - \frac{N|g|}{\ell^D}$$

In 1D: Stable solution for any N and any g



In the context of cold atoms:
Salomon and Hulet's groups (2002)

In 3D: Dynamically unstable extremum



In the context of cold atoms:
BoseNova: Cornell-Wieman group (2001)

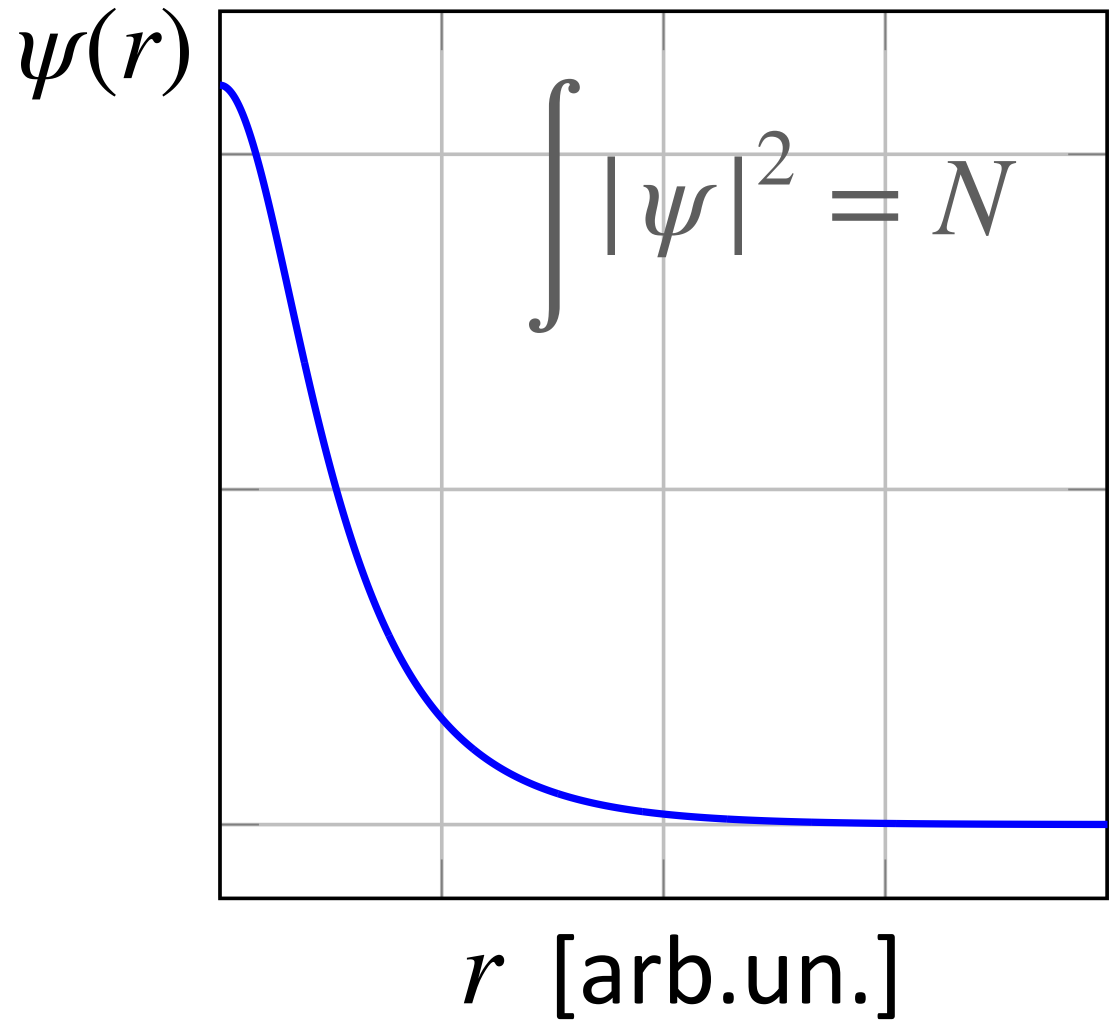
2D is a critical dimension: Stationary solutions can be obtained only for discrete values of $N|g|$

2D: the Townes soliton

$$E[\psi] = \frac{1}{2}\int\left(\left|\nabla\psi\right|^2 + g\left|\psi\right|^4\right)\mathrm{d}^2r$$

Chiao, Garmire & Townes, 1964

Radially symmetric, node-less solution of $-\frac{1}{2}\nabla^2\psi + g\,\psi^3 = \mu\,\psi$



Such a solution exists only if $(Ng)_{\mathrm{Townes}} = -5.85...$

It has $E = 0$ and $\mu < 0$

Once a particular solution is known, scale invariance provides a continuous family of solutions

$\phi(\boldsymbol{r}) = \lambda\,\psi(\lambda\boldsymbol{r})$ $\quad\mu_\phi = \lambda^2\mu\quad$ λ real

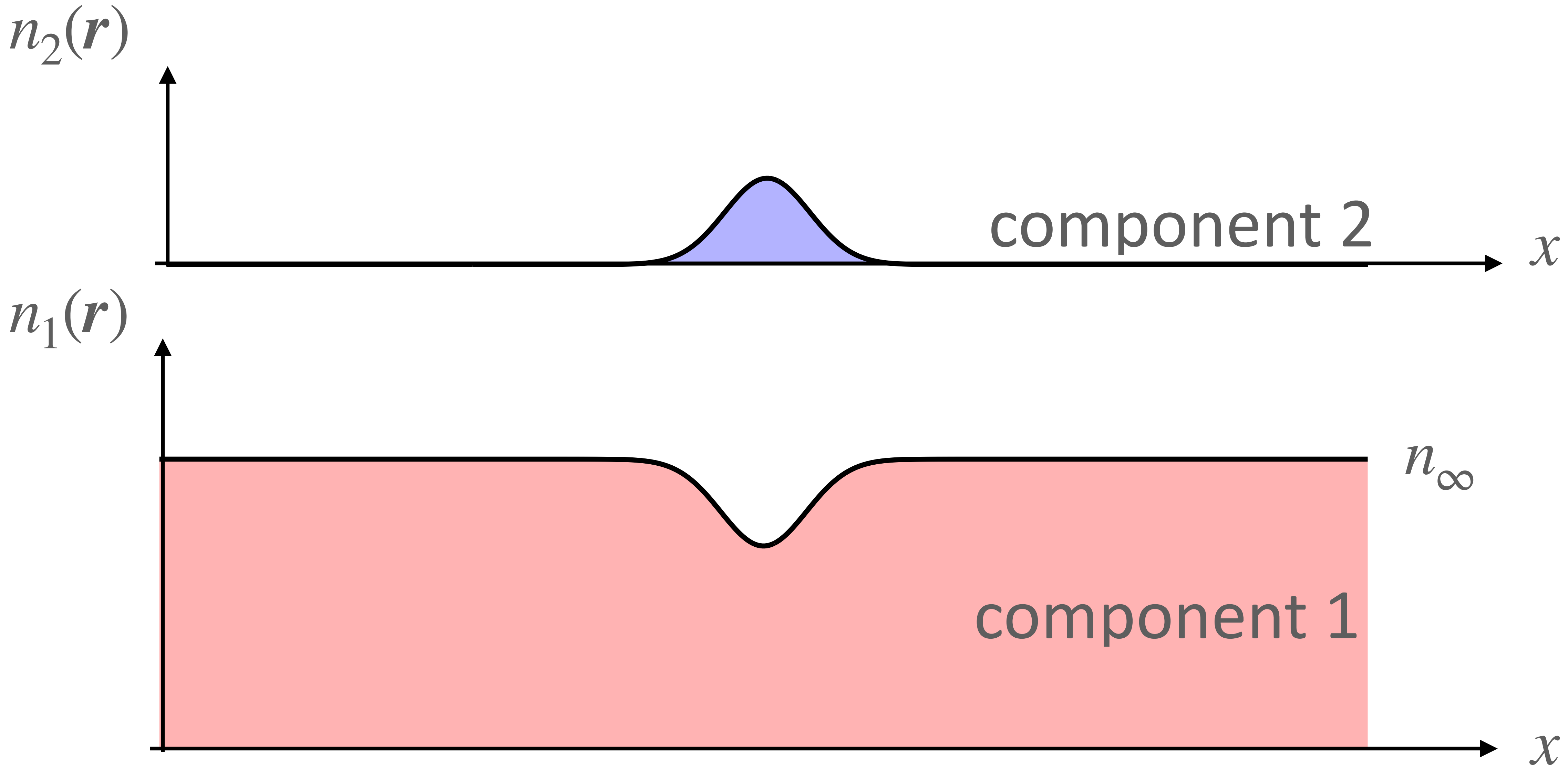
No particular length scale for the Townes soliton when it exists

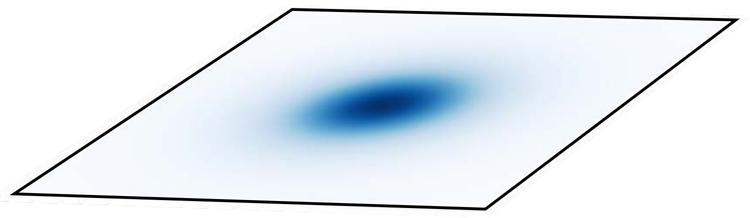
However: Instable with respect to a change in shape or in Ng

Observation of Townes soliton with cold atomic gases

Purdue group: Phys. Rev. Lett. 127, 023604 (2021), use of a Feshbach resonance with ^{133}Cs

Paris group: Phys. Rev. Lett. 127, 023603 (2021), use of a two-component gas with ^{87}Rb



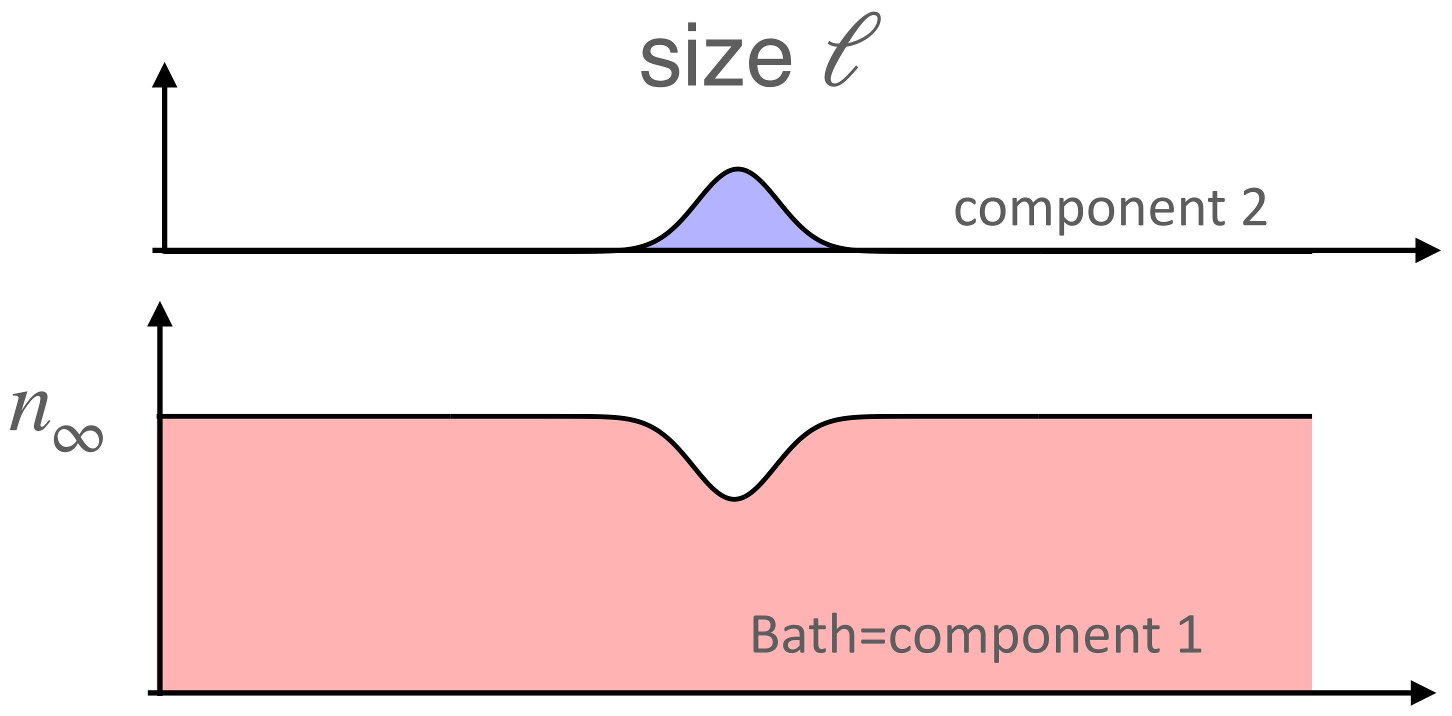


Each fluid is described by a 2D Gross-Pitaevski equation and is stable: $g_{ii} > 0$ for $i = 1{,}2$

- Component 1 extends to infinity with the asymptotic density n_∞
- Component 2 contains N_2 atoms

The two fluids are (slightly) non-miscible: $g_{12} > \sqrt{g_{11} g_{22}}$

The weakly-depleted bath



Bath healing length $\xi_1 = \dfrac{1}{\sqrt{2 g_{11} n_\infty}}$

$$\mu_2 \psi_2 = \left(-\frac{\hbar^2}{2m} \nabla^2 + g_{12} n_1 + g_{22} n_2 \right) \psi_2$$

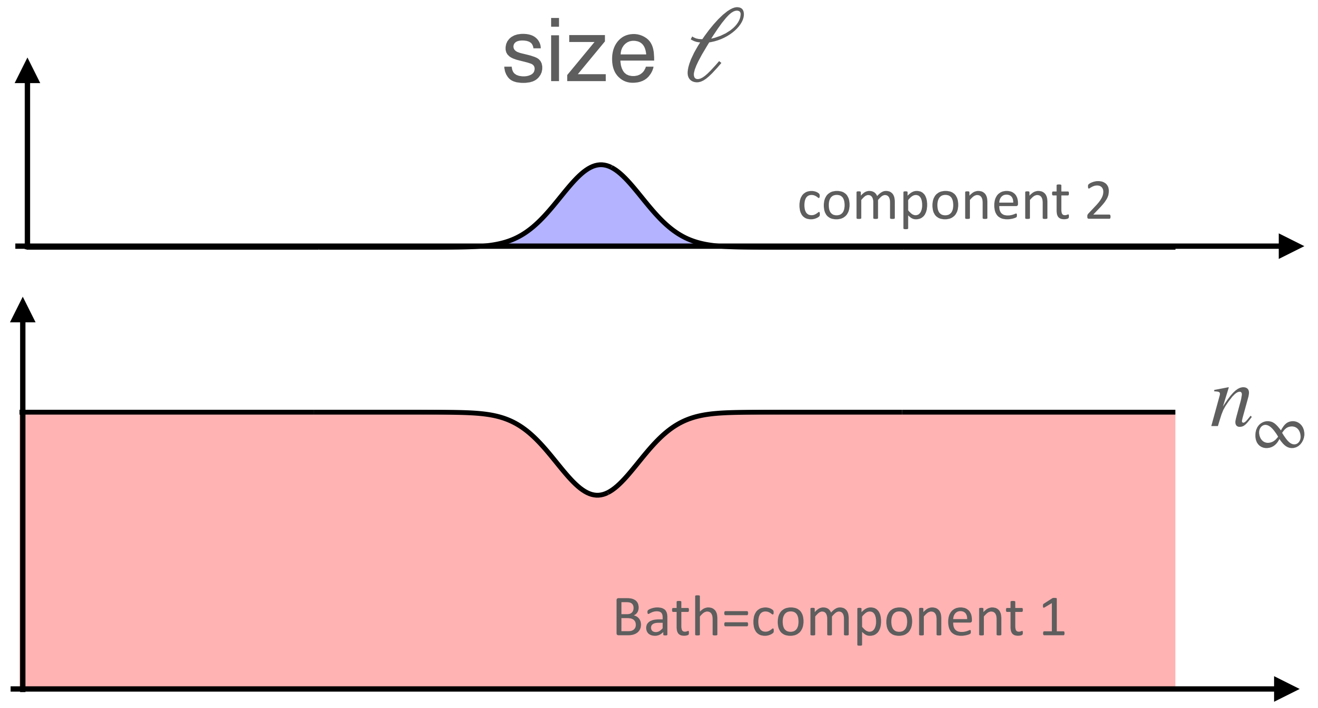
$$\mu_1 \psi_1 = \left(-\frac{\hbar^2}{2m} \nabla^2 + g_{11} n_1 + g_{12} n_2 \right) \psi_1 \qquad \mu_1 = g_{11} n_\infty$$

Assume that $n_2 \ll n_1 \approx n_\infty$ everywhere (weak depletion of comp. 1) and that $\ell \gg \xi$ (large extension of comp. 2)

Thomas-Fermi approximation for the bath (component 1):

$$\mu_1 = g_{11} n_1 + g_{12} n_2 \longrightarrow n_1 = n_\infty - \frac{g_{12}}{g_{11}} n_2$$

The minority component



$$\mu_2 \psi_2 = \left(-\frac{\hbar^2}{2m}\nabla^2 + g_{12} n_1 + g_{22} n_2 \right) \psi_2$$

$$n_1 = n_\infty - \frac{g_{12}}{g_{11}} n_2$$

Simple equation for the component 2: $$\mu \psi_2 = \left(-\frac{\hbar^2}{2m}\nabla^2 + g_{\text{eff}}\, n_2 \right) \psi_2$$

$$\mu = \mu_2 - g_{12} n_\infty$$

$$g_{\text{eff}} = g_{22} - \frac{g_{12}^2}{g_{11}}$$

Bare interaction (repulsive)

Interaction mediated by the bath:

- always attractive
- independent of the bath density

Non-miscibility criterion:

$$g_{12}^2 > g_{11} g_{22} \quad \Leftrightarrow \quad g_{\text{eff}} < 0$$

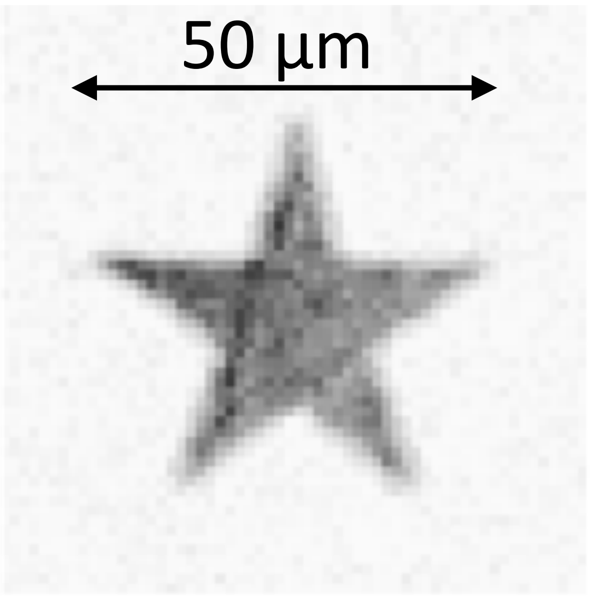
Our experimental setup (rubidium)

Frozen motion along the vertical direction z

$$\omega_z/2\pi = 4\,\mathrm{kHz}$$

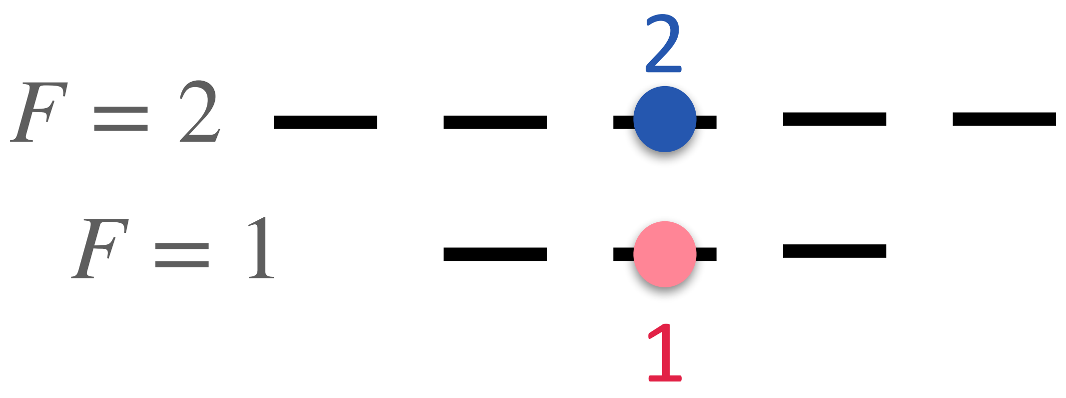
Initial confinement in the xy plane:

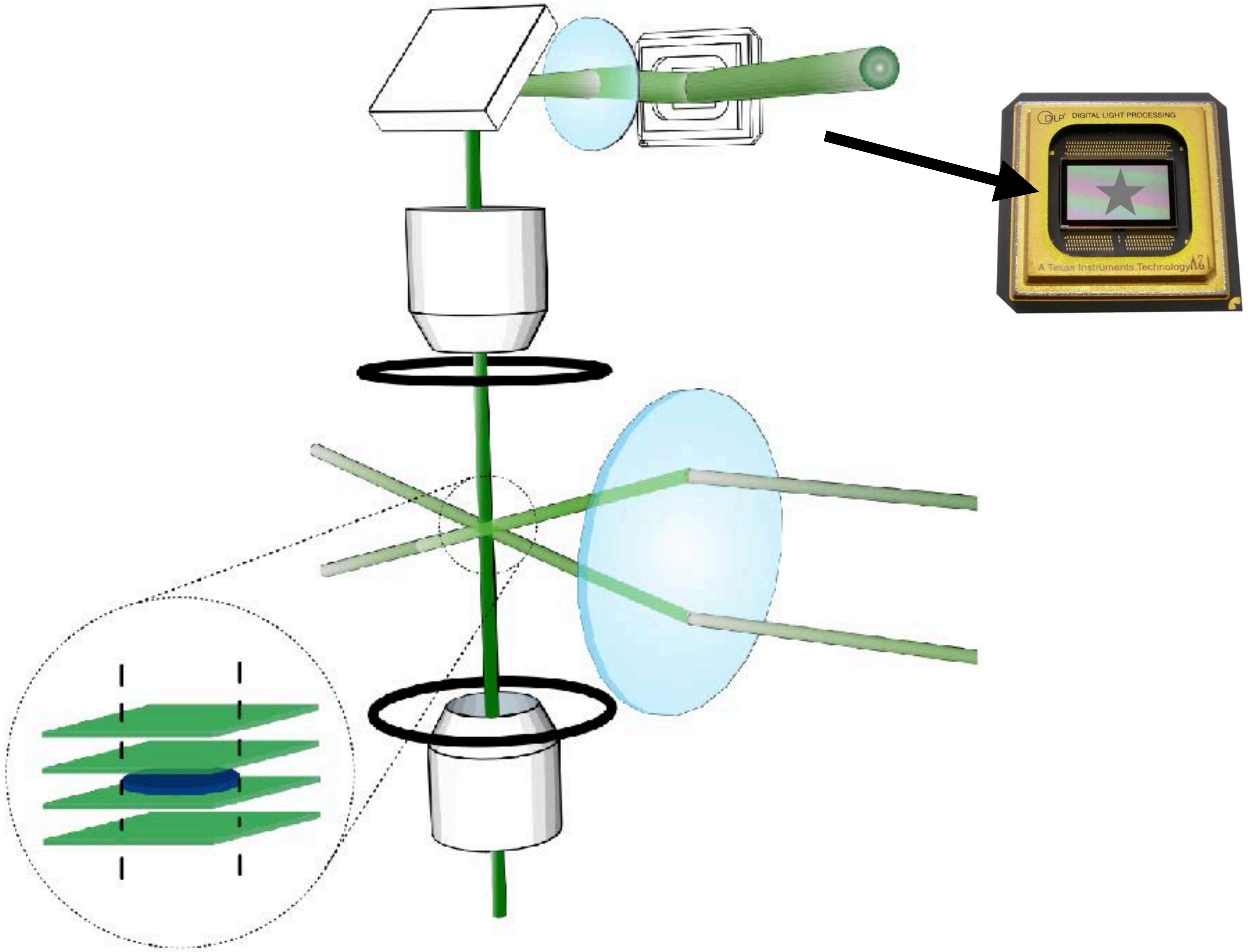
Box-like potential with arbitrary shape



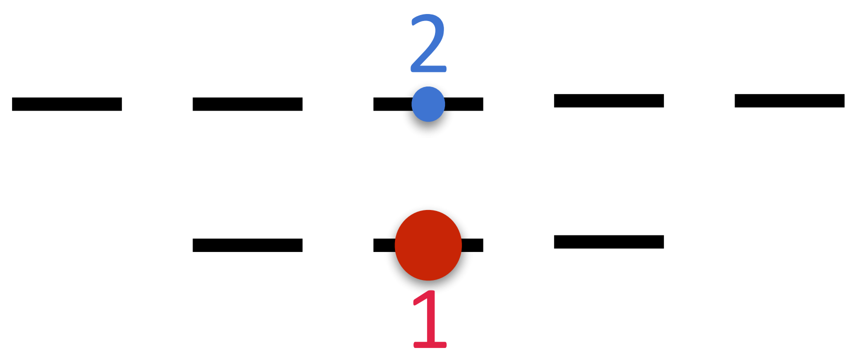
Uniform gas with up to 10^5 atoms

Density up to 100 atoms/μm^2

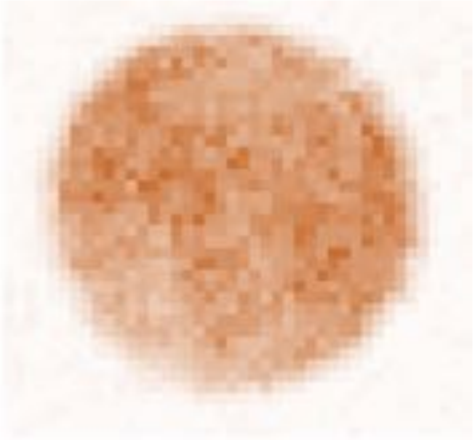




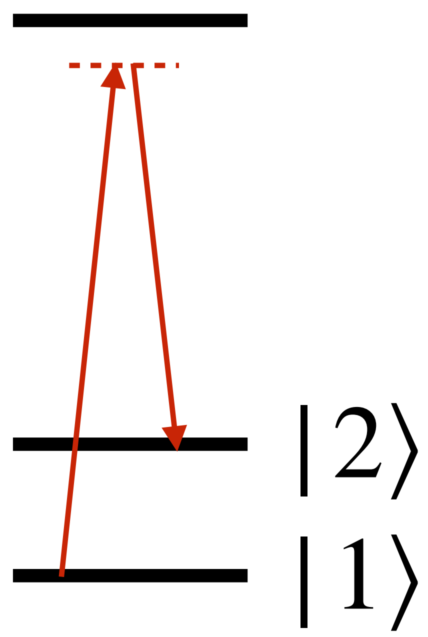
Our approach to Townes soliton creation



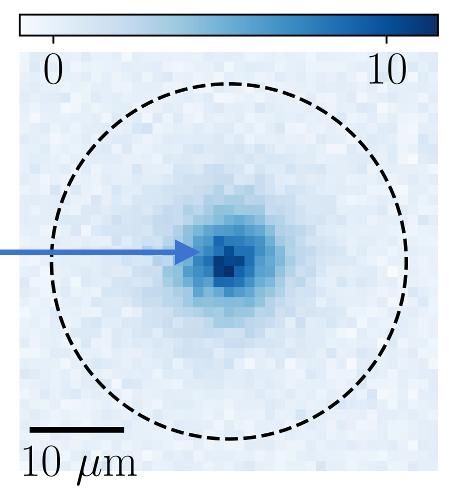
- Prepare a uniform 87Rb gas in the internal state $|1\rangle$



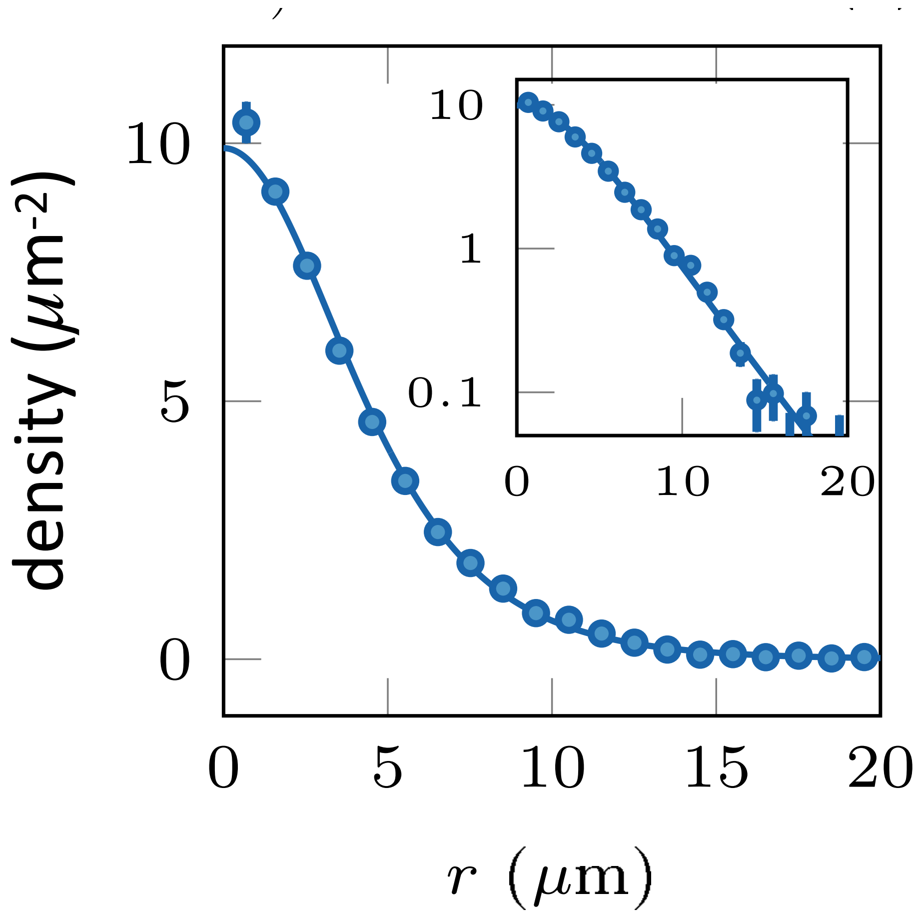
- Transfer in a spatially resolved way a small fraction of atoms in state $|2\rangle$



Atoms in $|2\rangle$



Atoms in $|1\rangle$ are still here, but not imaged

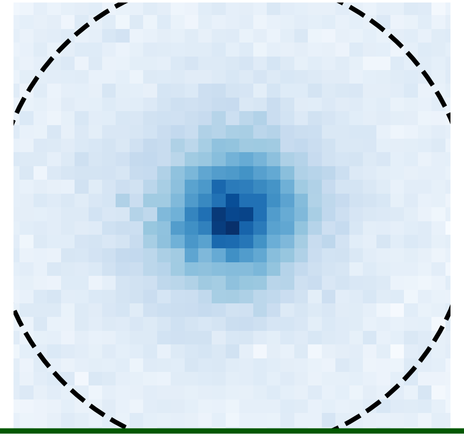


Townes profile with very good precision

- Look at the evolution of this “bubble” of atoms $|2\rangle$ immersed in a bath of $|1\rangle$

3D scattering lengths: a_{11}=100.9 a_0, a_{12}=100.4 a_0, a_{22}= 94.9 a_0 → 2D coupling strengths: g_{11}=0.160, g_{12}=0.159, g_{22}=0.151 →

$$g_{\text{eff}} = g_{22} - \frac{g_{12}^2}{g_{11}} \approx -0.0076$$

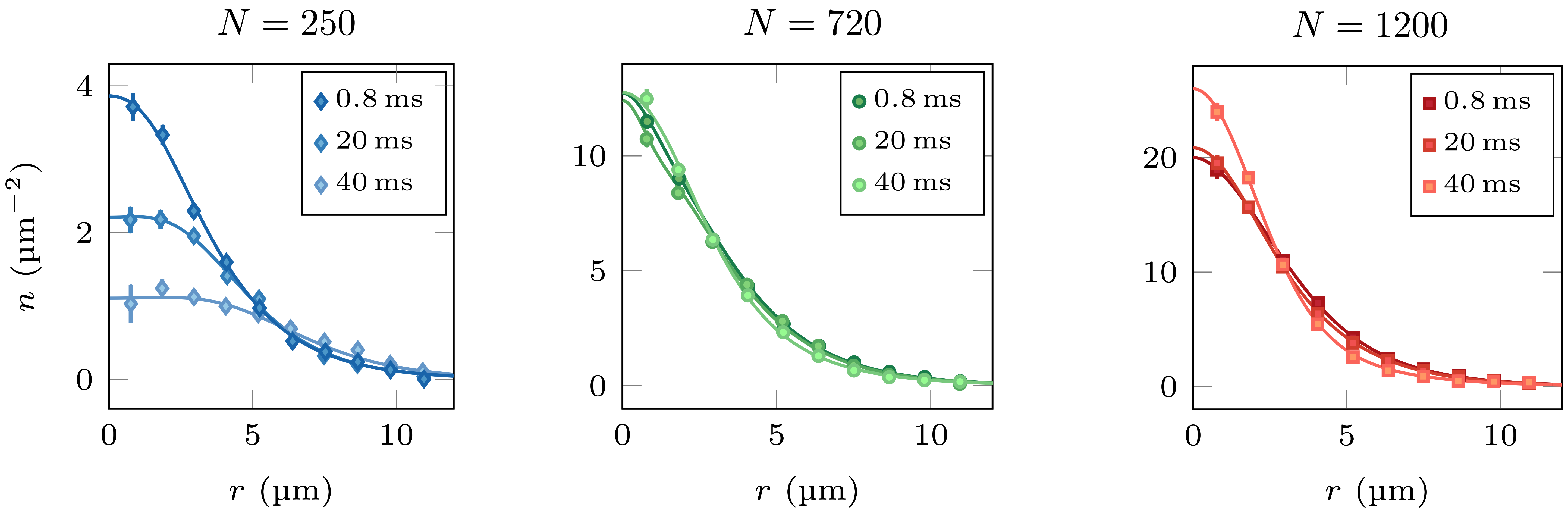


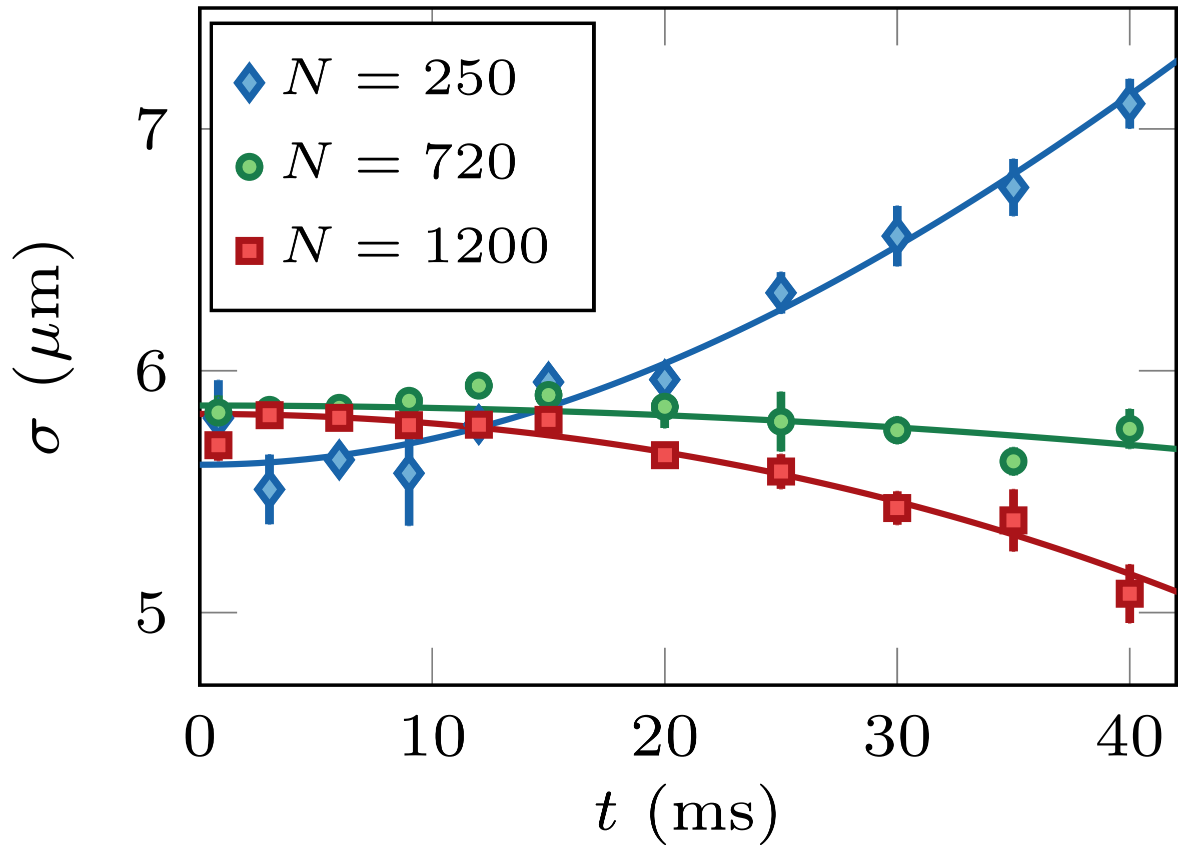
Observation of a Townes soliton

$$g_{\text{eff}} = g_{22} - \frac{g_{12}^2}{g_{11}} \approx -0.0076$$

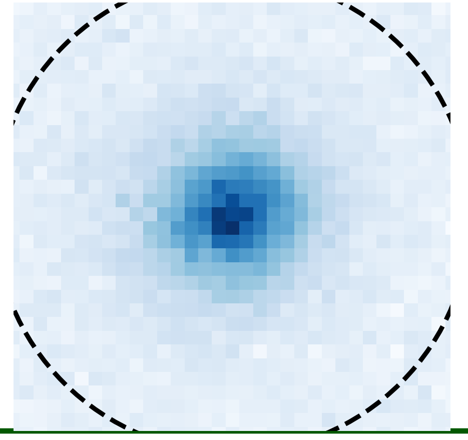
For our parameters, the threshold $N_{\text{Townes}}\,|\,g\,| = 5.85$ corresponds to $N_{\text{Townes}} \approx 770$

Here we print the Townes pattern with a given size $\sigma_0 = 5.7\ \mu\text{m}$, but with different atom numbers



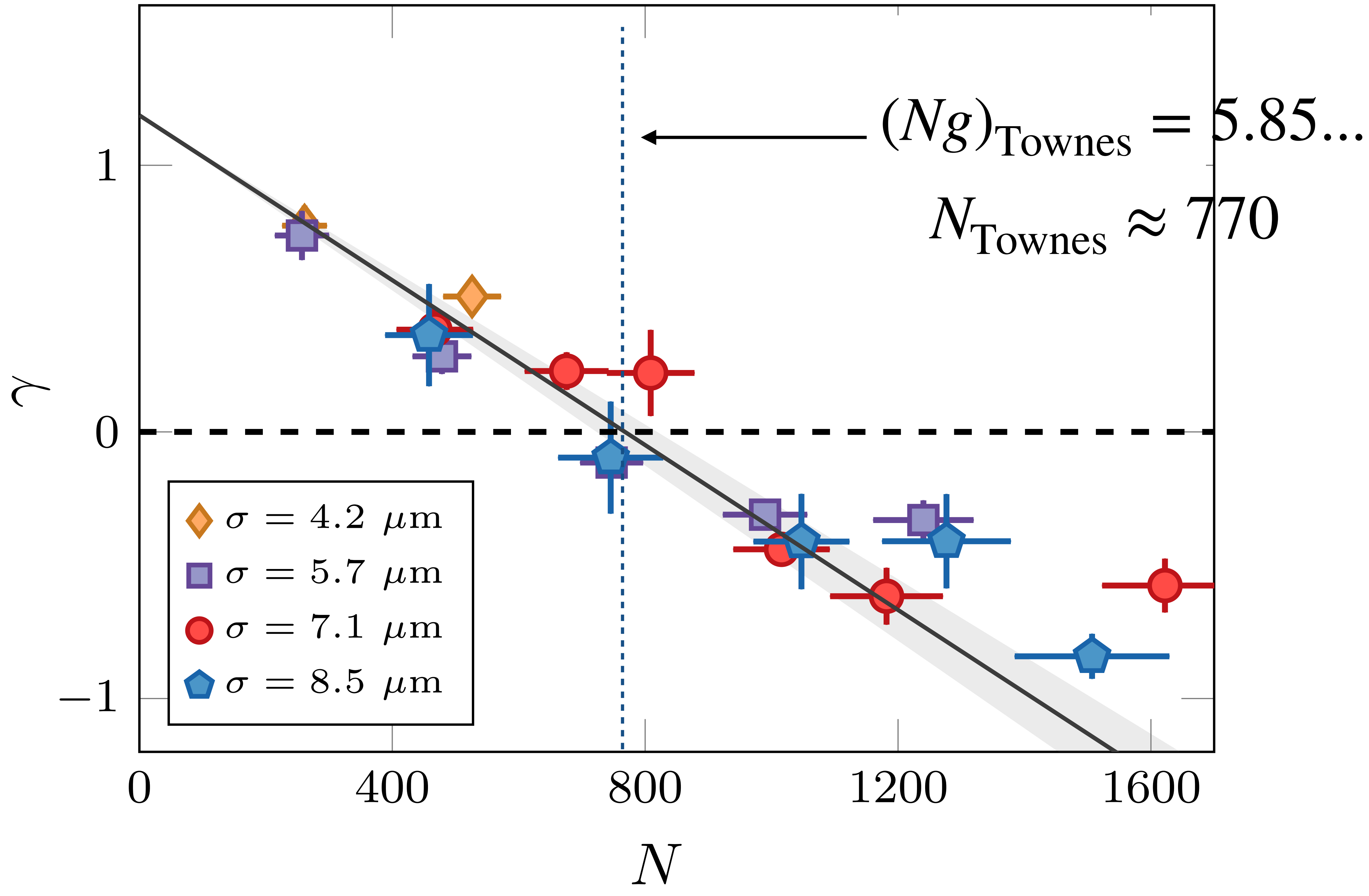


Scale invariance of Townes soliton



Expansion factor

$$\propto \frac{\mathrm{d}}{\mathrm{d}t}\langle r^2 \rangle$$



The stable shape is always obtained for ≈ the same atom number, irrespective of the size

PRL 127, 023603 (2021) see also PRL 127, 023603 (2021) by Chen & Hung

Scale and conformal invariance for cold atomic gases

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Solvay chair for Physics 2022

Lecture 3, part 2

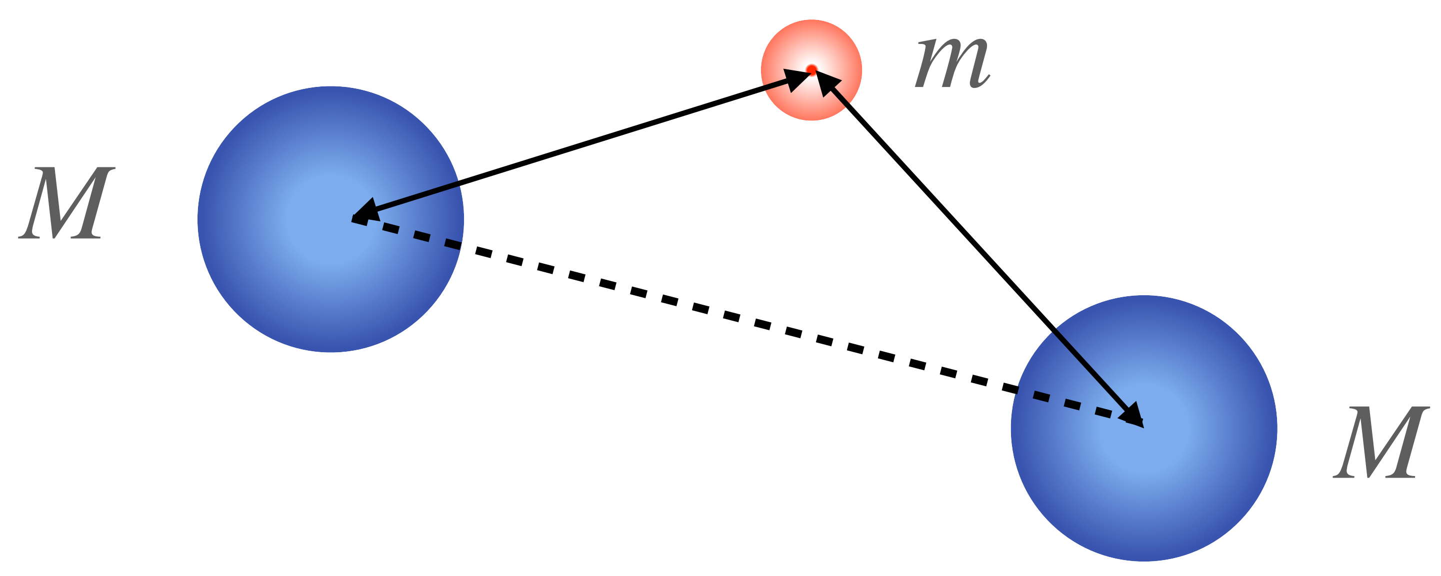
Outline of the lecture

Time-independent problems

Universality of the equation of state

Solitons in 2D

➡ The Efimov effect — Efimov, 1970



Fonseca et al, 1979

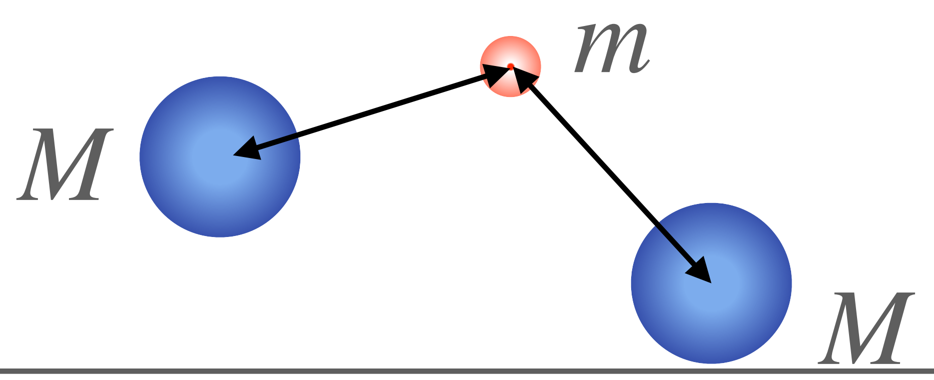
We look here at the 3-body problem "Heavy + Heavy + Light"

→ No direct interaction "Heavy-Heavy"

→ Heavy-Light contact interaction with scattering length a

Limit $a \to \infty$: no two-body bound state "Heavy + Light"

The relative motion of the heavy particles



$a = \infty$

Motion in the $1/R^2$ potential created by the heavy-light resonant interaction:

$$-\frac{\hbar^2}{M}\nabla^2\Psi(\boldsymbol{R}) - \frac{g}{R^2}\Psi(\boldsymbol{R}) \;=\; E\,\Psi(\boldsymbol{R})$$

$$g = \Omega^2\,\frac{\hbar^2}{2m}$$

$$\Omega = 0.57\cdots$$

We look for $E < 0$

Radial wave equation ($\ell = 0$) for $u(R) \equiv R\,\Psi(R)$:

$$-\frac{\mathrm{d}^2 u}{\mathrm{d}R^2} - \frac{\beta}{R^2}u(R) \;=\; \epsilon\, u(R)$$

$\beta \propto M/m$

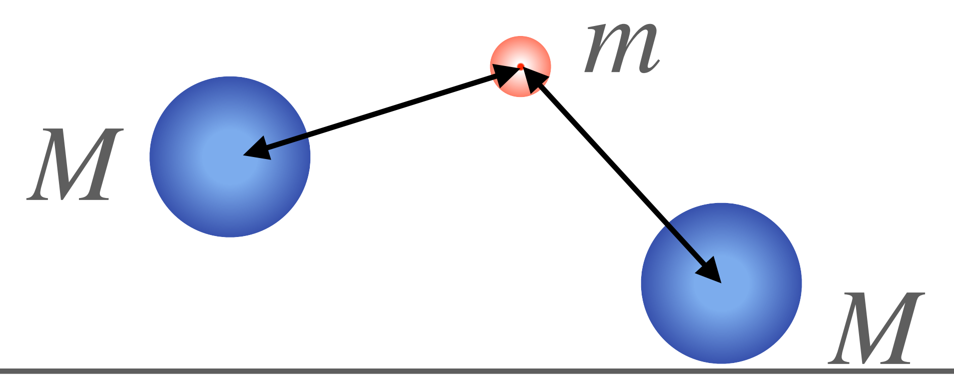
For $\beta > 1/4$, there exists solutions with negative energies (i.e. bound states of the trimer system)

Scale invariance of g/R^2: if $\Psi(\boldsymbol{R})$ is a solution for energy E, then $\Phi(R) = \Psi(\lambda R)$ is solution for $\lambda^2\,E$.

Continuous spectrum from $E = -\infty$ to $E = 0$?

Need for some type of boundary condition at short distance (e.g. hard core)

The relative motion of the heavy particles (2)



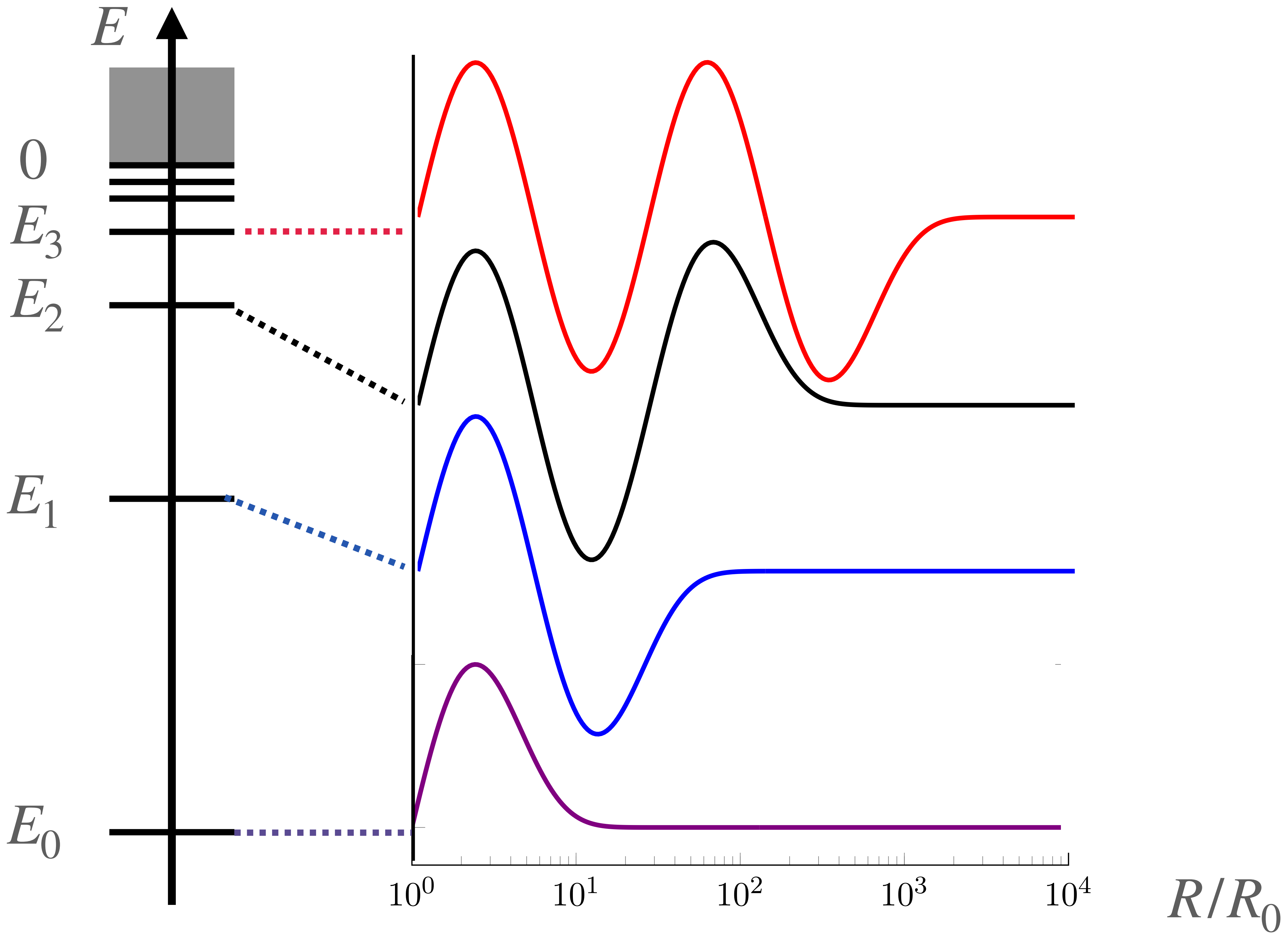
$a = \infty$

Impose a hard core in $R = R_0$

→ Breaks the continuous scale invariance

→ Keeps a discrete scale invariance: infinite sequence of bound states $E_n = E_0/\lambda^{2n}$ where λ depends on M/m

plot for
$s_0 \approx 1.9$



(logarithmic scale)

$$\Psi(R) \leftrightarrow E$$
$$\Phi(R) = \Psi(\lambda R) \leftrightarrow \lambda^2 E$$

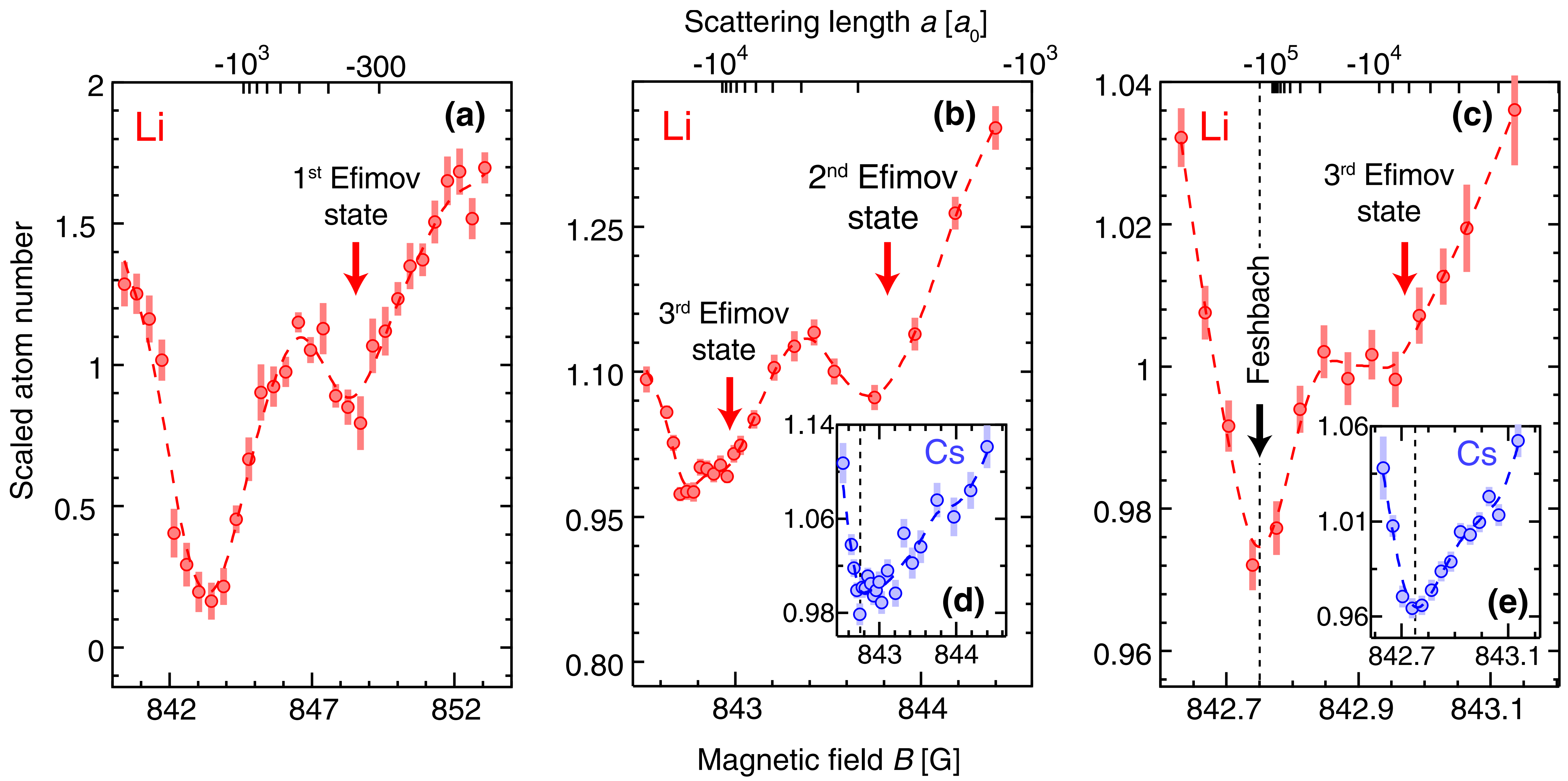
$$\lambda = e^{\pi/s_0} \quad \text{with} \quad s_0 = \left(\frac{M}{2m}\Omega^2 - \frac{1}{4}\right)^{1/2}$$

$$\Omega = 0.57\cdots$$

Efimov physics in a M-m-M system

Many beautiful experiments with cold gases, starting with pioneering work at Innsbruck on Cs (2006)

For $M - m - M$ systems, Chicago and Heidelberg (2014) with Cs - ^{6}Li -Cs ($\lambda \approx 5$ instead of ≈ 23 for equal masses)



$$\frac{a_2}{a_1} = 5.1\,(2)$$

$$\frac{a_3}{a_2} = 4.8\,(7)$$

Predicted ratio for Li-Cs-Cs: 4.88

Tung et al, PRL 113, 240402 (2014), Chin's group

see also Pires et al, PRL 112, 250404 (2014), Weidemuller's group

Outline of the lecture

Time-independent problems

Universality of the equation of state

Solitons in 2D

The Efimov effect

Time-dependent problems

Conformal invariance and the SO(2,1) dynamical symmetry

The breathing mode

Breathers

From scale to conformal invariance

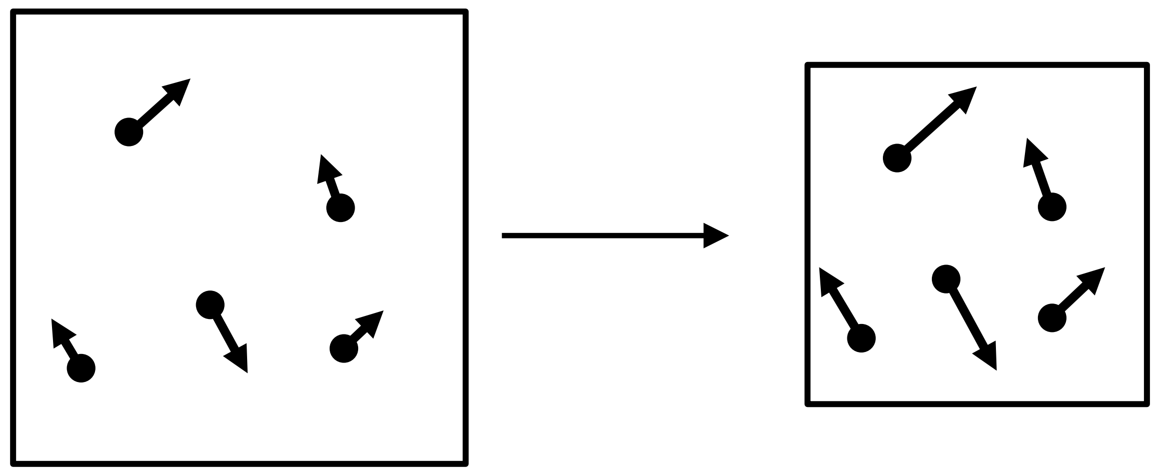
Niederer, 1972-73
Pitaevskii & Rosch, 1997

In addition to the standard Galilean transformations (translations, rotations), there exist 3 types of transformations that leave the unitary 3D Fermi gas or the 2D Bose gas invariant:

Dilatations: $\boldsymbol{r} \to \boldsymbol{r}/\lambda$ $\quad t \to t/\lambda^2$

Time translations: $\boldsymbol{r} \to \boldsymbol{r}$ $\quad t \to t + t_0$

"Expansions": $\boldsymbol{r} \to \dfrac{\boldsymbol{r}}{\gamma t + 1}$ $\quad t \to \dfrac{t}{\gamma t + 1}$



3-parameter group: dynamical symmetry associated with the SO(2,1) two-dimensional Lorentz group

Can be extended to a harmonic trap, with a slight modification of the transformations

The SO(2,1) symmetry in a nutshell

$$\hat{H}_{\text{kin}} = \sum_j \frac{\hat{\boldsymbol{p}}_j^2}{2m} \qquad \hat{H}_{\text{int}} = \frac{1}{2}\sum_{i \neq j} V(\hat{\boldsymbol{r}}_i - \hat{\boldsymbol{r}}_j) \qquad \hat{H}_{\text{pot}} = \sum_j \frac{1}{2} m\omega^2 \hat{\boldsymbol{r}_j}^2$$

Define the three operators:

$$\begin{cases} \hat{L}_1 = \dfrac{1}{2\hbar\omega}\left(\hat{H}_{\text{kin}} + \hat{H}_{\text{int}} - \hat{H}_{\text{pot}}\right) \\ \hat{L}_2 = \dfrac{1}{4}\displaystyle\sum_j \left(\hat{\boldsymbol{r}_j} \cdot \hat{\boldsymbol{p}_j} + \hat{\boldsymbol{p}_j} \cdot \hat{\boldsymbol{r}_j}\right) \\ \hat{L}_3 = \dfrac{1}{2\hbar\omega}\left(\hat{H}_{\text{kin}} + \hat{H}_{\text{int}} + \hat{H}_{\text{pot}}\right) \end{cases}$$

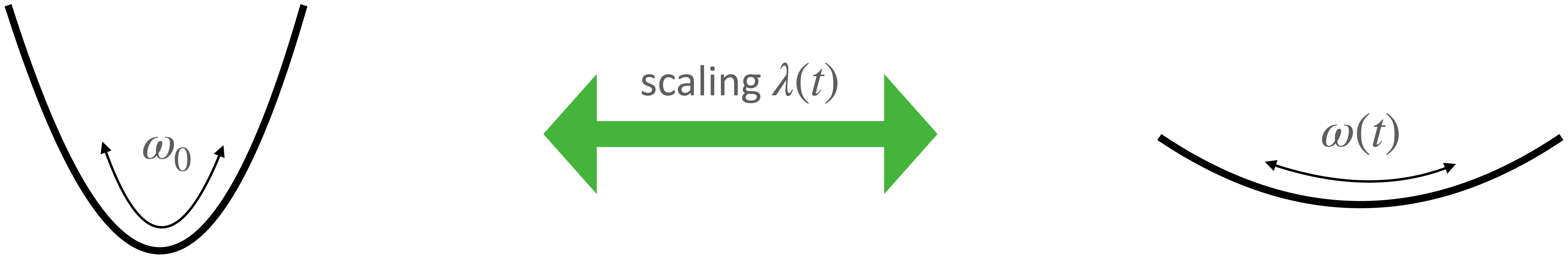
(total Hamiltonian)

Commutation relations: $[\hat{L}_1, \hat{L}_2] = -\mathrm{i}\hbar\hat{L}_3 \qquad [\hat{L}_2, \hat{L}_3] = \mathrm{i}\hbar\hat{L}_1 \qquad [\hat{L}_3, \hat{L}_1] = \mathrm{i}\hbar\hat{L}_2$

Close to an angular momentum (SO(3)), but not quite

The invariant is here: $\hat{L}_1^2 + \hat{L}_2^2 - \hat{L}_3^2$

Linking various time-dependent solutions



Conformal invariance allows one to link the solution of the N-body Schrödinger equation in a trap of frequency ω_0 to the solution in a trap with frequency ω, for the same initial state.

ω may possibly depend on time, and even be zero (untrapped case)

The scaling parameter $\lambda(t)$ is the solution of the Ermakov equation:

$$\frac{\mathrm{d}^2\lambda}{\mathrm{d}t^2} + \omega^2(t)\,\lambda(t) = \frac{\omega_0^2}{\lambda^3(t)}$$

Pitaevskii & Rosch, 1997; Kagan et al 1997; Castin & Dum 1997 ; Castin & Werner, 2004-06 ; Son et al, 2006-07; Nishida & Tan, 2008

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➡ The breathing mode — Pitaevskii & Rosch, 1997

Breathers

A smoking gun of SO(2,1) symmetry: The breathing mode

Pitaevskii & Rosch, 1997

- Prepare an arbitrary shape for the gas at $t = 0$
- Let the atoms evolve in a 2D harmonic potential of frequency ω in the presence of interactions
- Measure $\langle r^2 \rangle \propto \langle \hat{H}_{\text{pot}} \rangle$ after an evolution time t : Perfectly periodic evolution with frequency 2ω

Direct consequence of the commutation relations, using Heisenberg picture:

$$\hat{L}_1 = \frac{1}{2\hbar\omega}\left(\hat{H}_{\text{kin}} + \hat{H}_{\text{int}} - \hat{H}_{\text{pot}}\right) \qquad \hat{L}_2 = \frac{1}{4}\sum_j \left(\hat{\boldsymbol{r}}_j \cdot \hat{\boldsymbol{p}}_j + \hat{\boldsymbol{p}}_j \cdot \hat{\boldsymbol{r}}_j\right) \qquad \hat{L}_3 = \frac{\hat{H}}{2\hbar\omega}$$

$$\begin{cases} \dfrac{d\hat{L}_1}{dt} = \dfrac{i}{\hbar}[\hat{H}, \hat{L}_1] = -\,2\omega\,\hat{L}_2 \\ \dfrac{d\hat{L}_2}{dt} = \dfrac{i}{\hbar}[\hat{H}, \hat{L}_2] = +\,2\omega\,\hat{L}_1 \end{cases} \longrightarrow \quad \frac{d^2\hat{L}_1}{dt^2} + (2\omega)^2\hat{L}_1 = 0$$

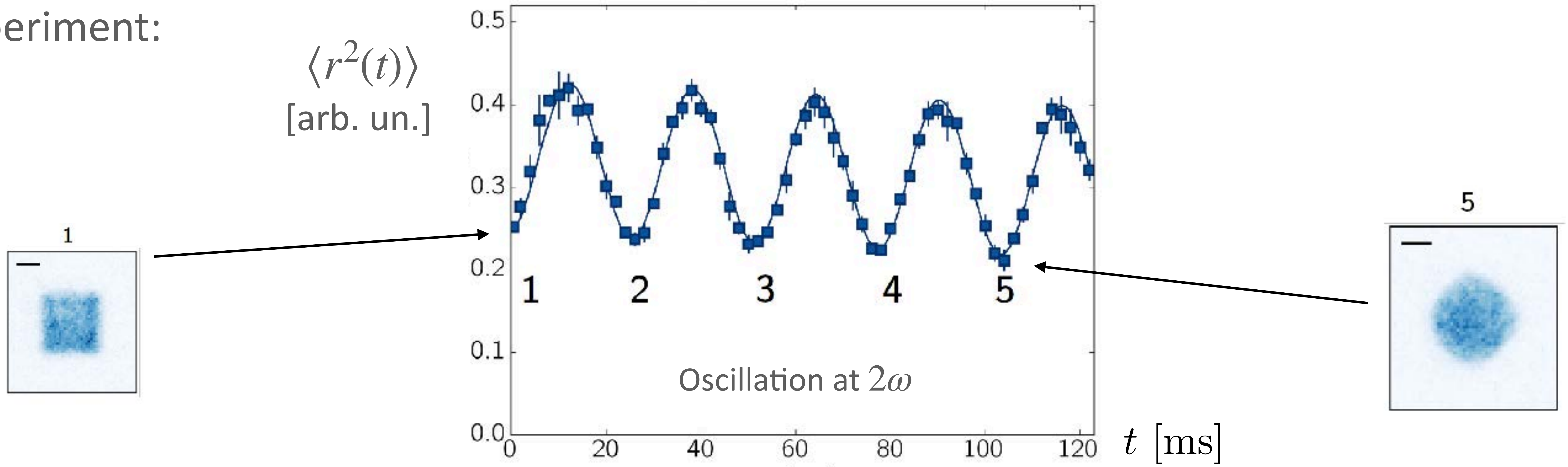
out-of-phase oscillation of $E_{\text{kin}} + E_{\text{int}}$ and E_{pot}

A smoking gun of SO(2,1) symmetry: The breathing mode

Pitaevskii & Rosch, 1997

- Prepare an arbitrary shape for the gas at $t = 0$
- Let the atoms evolve in a 2D harmonic potential of frequency ω in the presence of interactions
- Measure $\langle r^2 \rangle \propto \langle \hat{H}_{\text{pot}} \rangle$ after an evolution time t : Perfectly periodic evolution with frequency 2ω

A 2D experiment:



Saint-Jalm et al,
Phys. Rev. X 9, 021035 (2019)

→ In 2D, the scale invariance holds only at the classical field level. What about quantum corrections?

→ Are there shapes that lead to a fully periodic motion (i.e. all moments $\langle r^n \rangle$ are periodic) ?

Quantum anomaly for $\langle r^2 \rangle(t)$

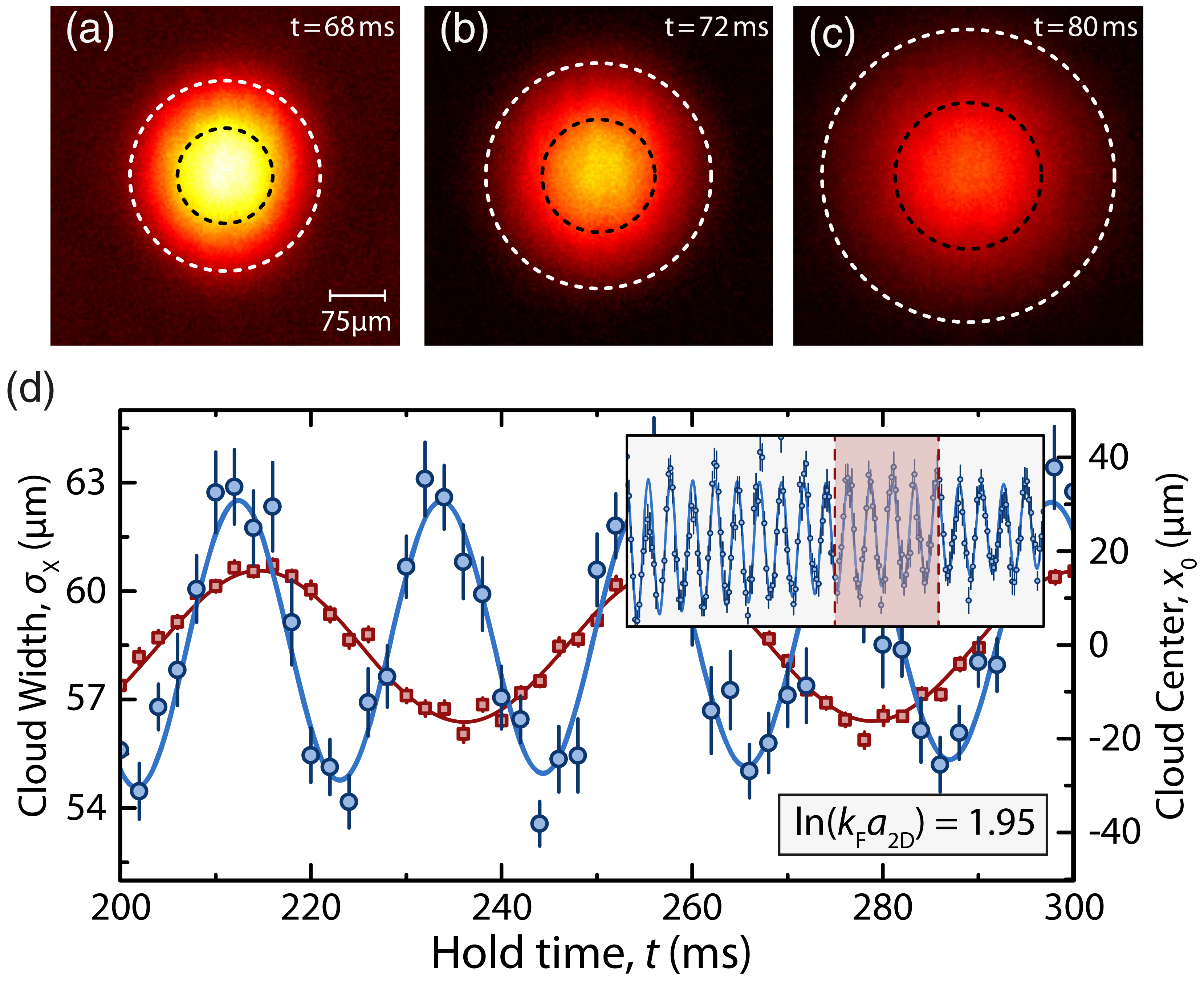
Olshanii, Perrin, Lorent PRL **105**, 095302 (2010)

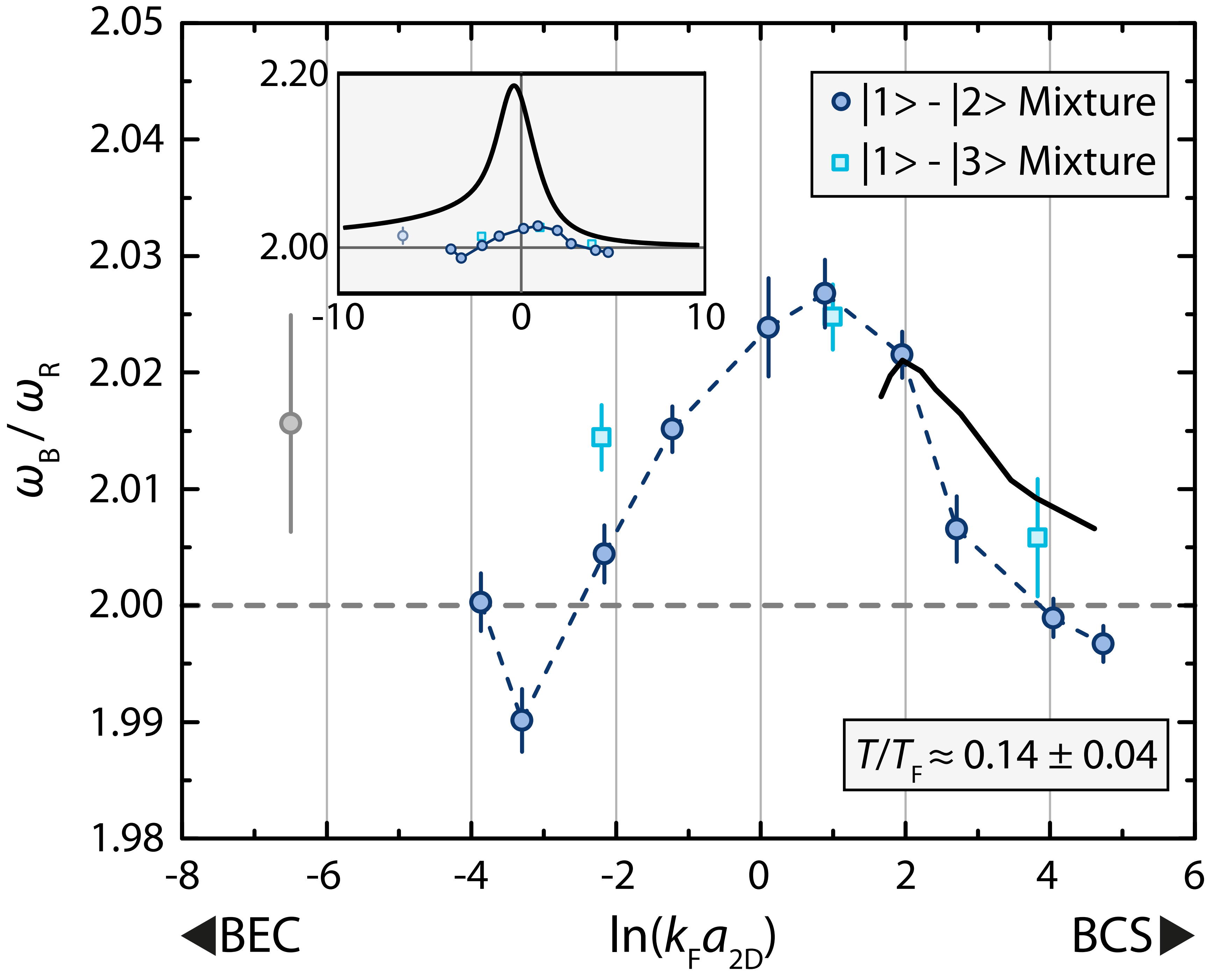
Hofmann, PRL **108**, 185303 (2012)

In 2D, the scale/conformal invariance holds only at the classical field level

The necessary regularization of the $\delta^{(2D)}(\boldsymbol{r}_i - \boldsymbol{r}_j)$ function for a quantum field treatment breaks this symmetry

Recent investigations wit a 2D Fermi gas close to the unitary point:





M. Holten et al,
PRL **121**, 120401 (2018)
[Jochim's group, Heidelberg]

see also T. Peppler et al, PRL **121**, 120402 (2018) [Vale's group, Swinburne]

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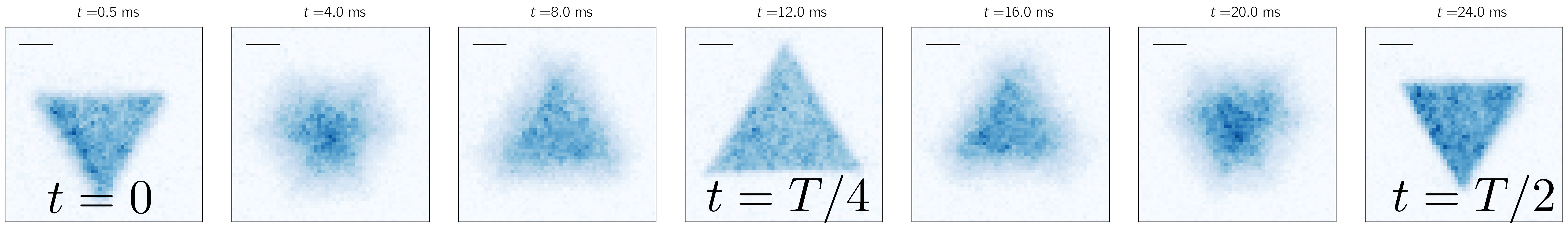
➡ Breathers

Are there shapes that lead to a fully periodic motion at 2ω (i.e. all moments $\langle r^n \rangle$ are periodic) ?

The equilateral triangle in the hydrodynamic ($Ng \gg 1$) regime

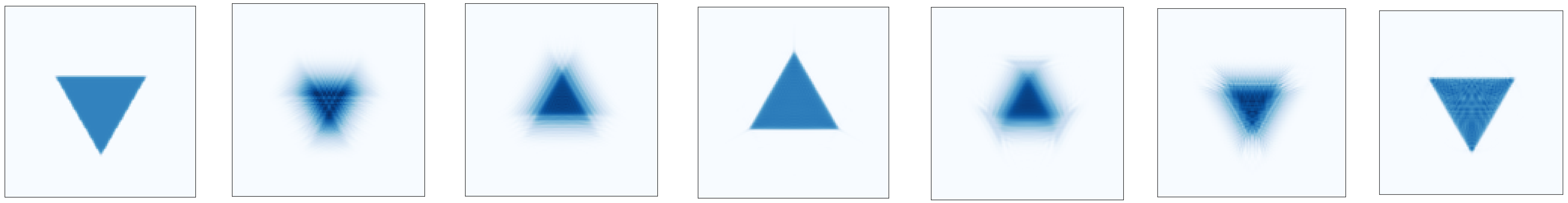
Saint-Jalm et al,
Phys. Rev. X 9, 021035 (2019)

Experimentally, in a harmonic trap of frequency ω:



period $T/2$ with $T = 2\pi/\omega$

Numerically, solution of the Gross-Pitaevskii equation on a 1024x1024 grid:



Initial state $|\psi_i\rangle$: uniform filling of the triangle

Overlap with wave function at $T/2$: $|\langle\psi_i|\psi_f\rangle| > 0.995$

Does not seem to occur for any other polygonal shape!

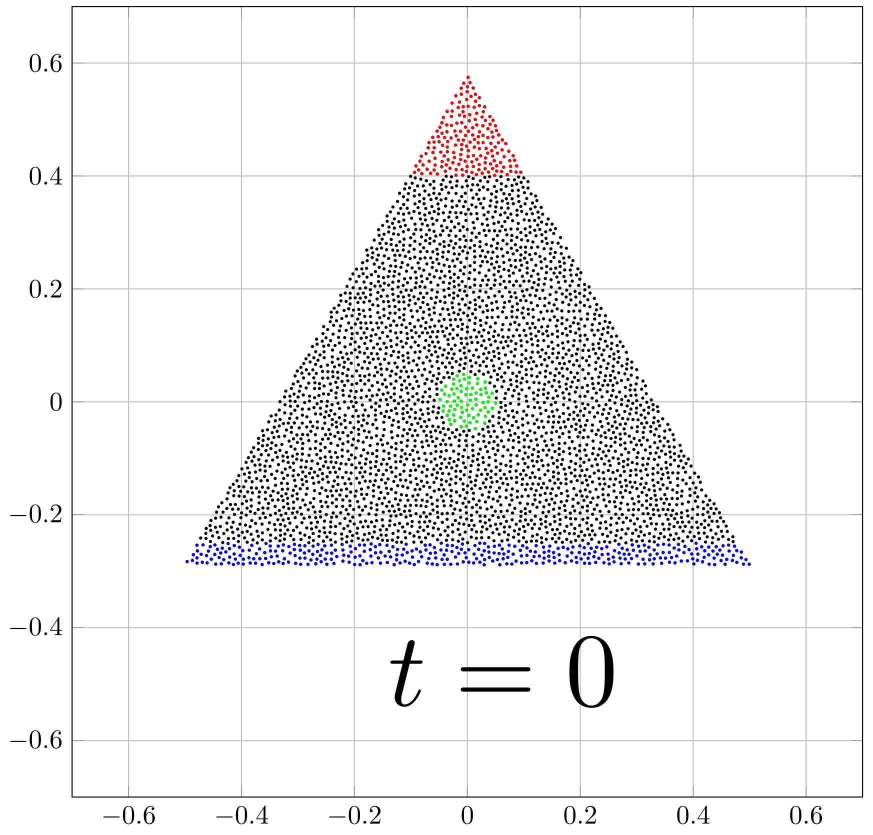
Do such breathers also show up for other 2D systems with SO(2,1) symmetry?

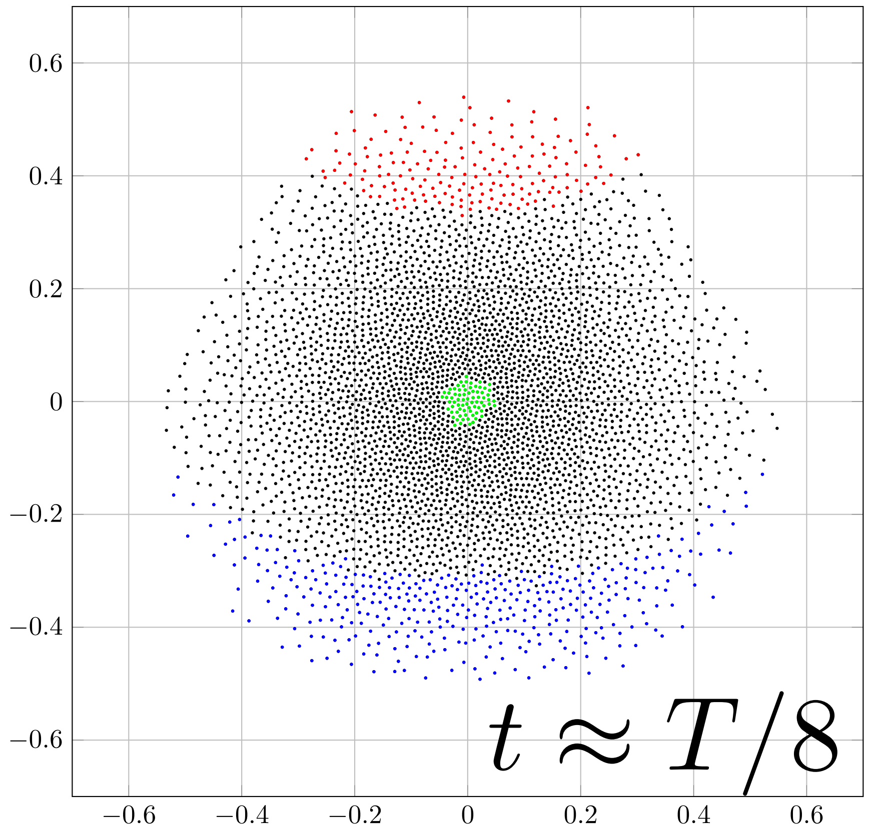
A simple test: Classical particles interacting with $V(r) \propto \frac{1}{r^2}$ potential $V(r/\lambda) = \lambda^2 V(r)$

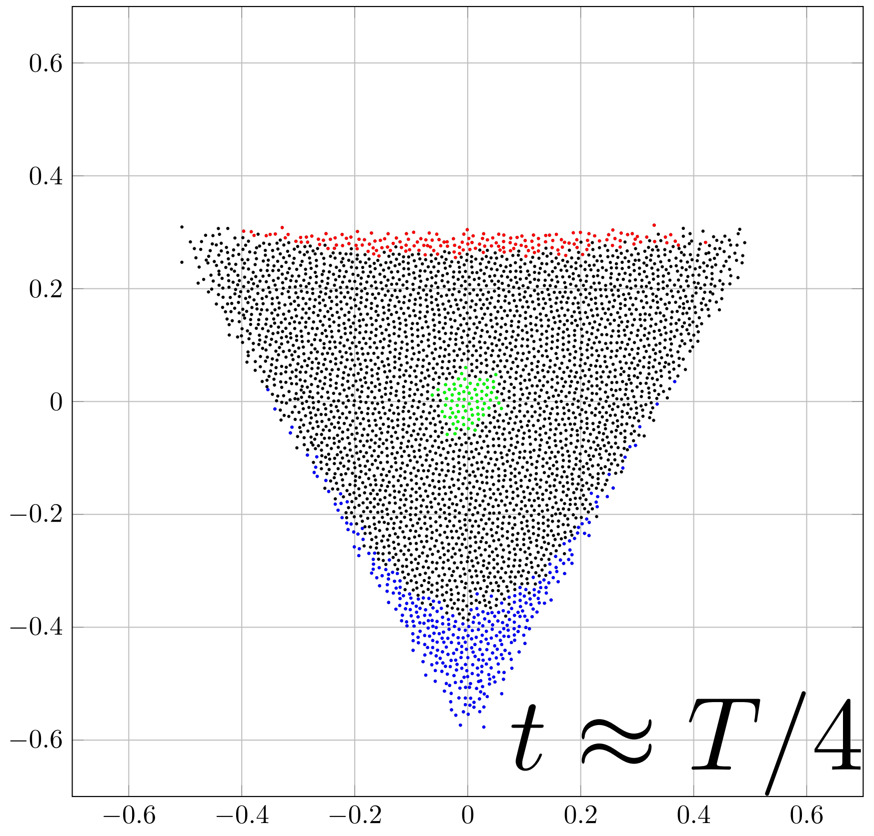
Simulation with

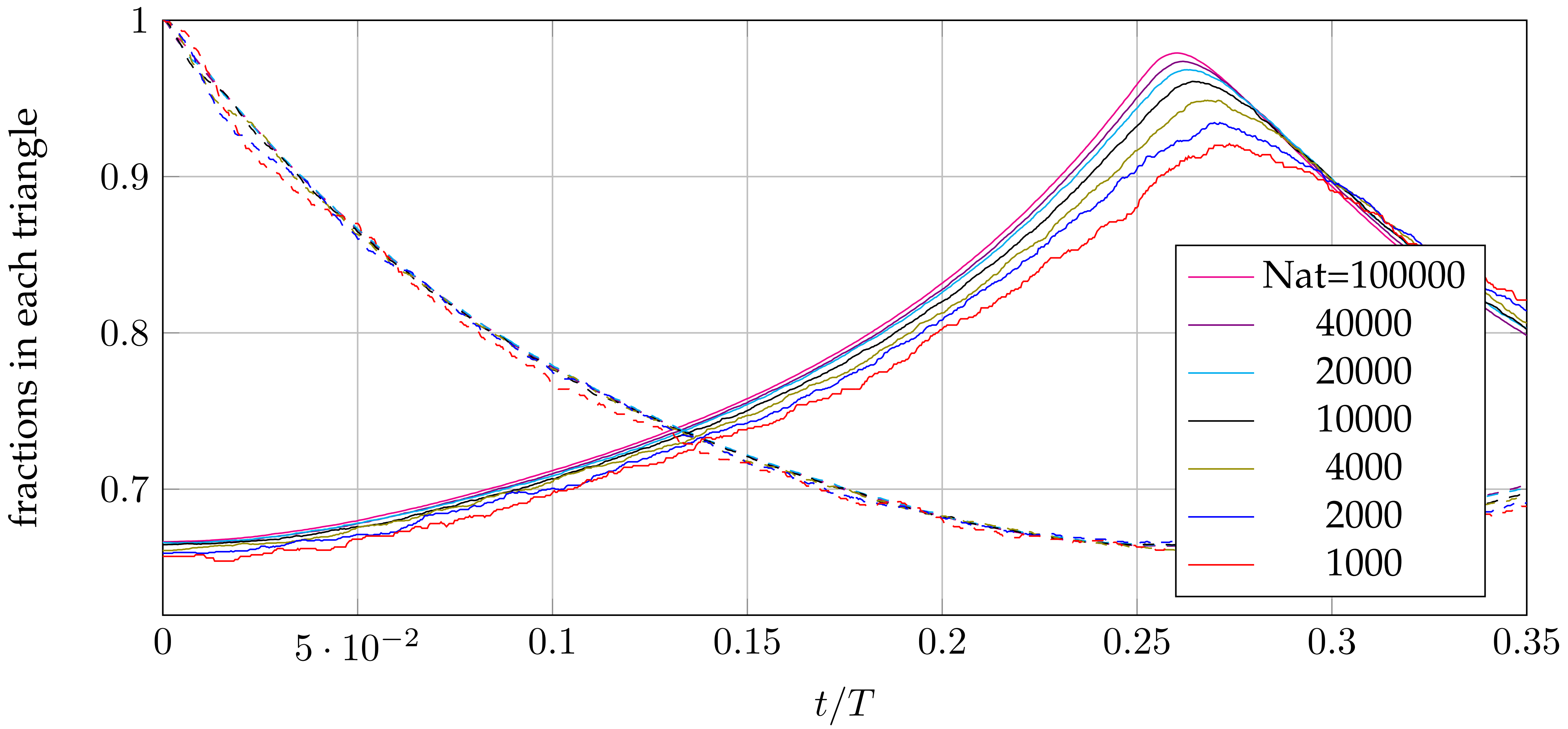
4000 particles

$\boldsymbol{v}_j(0) = 0$









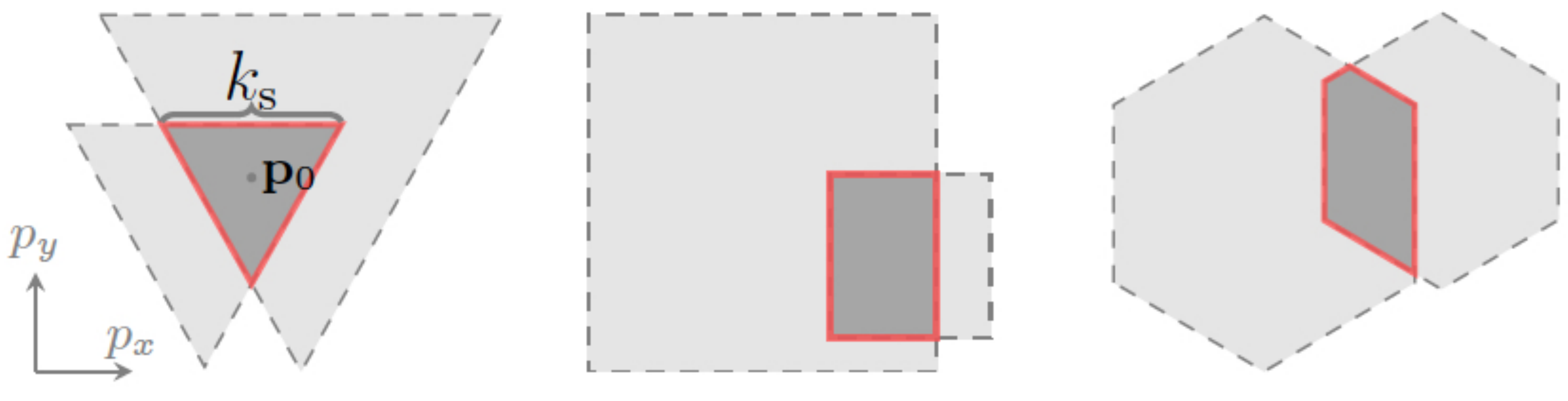
Two recent theoretical insights

Shi, Gao & Zhai, Phys. Rev. X 11 041031 (2021): "Ideal-Gas Approach to Hydrodynamics"

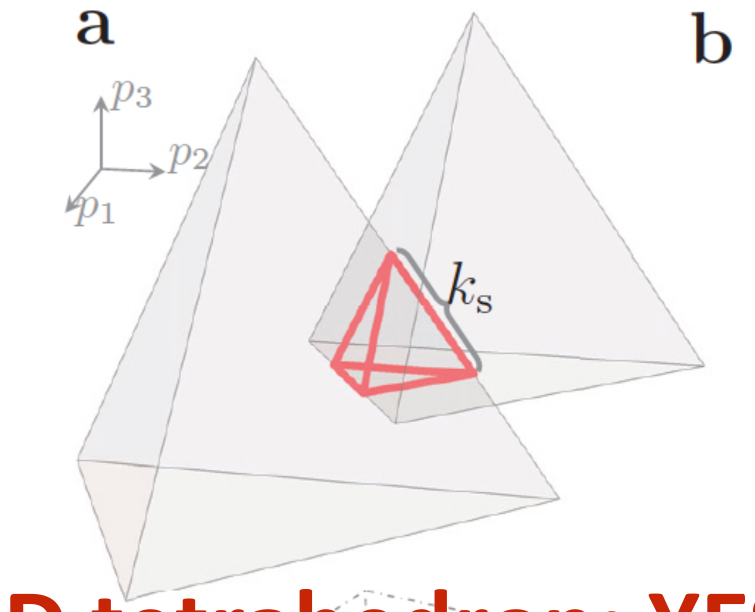
"There exist situations that the solution to a class of interacting hydrodynamic equations with certain initial conditions can be exactly constructed from the dynamics of noninteracting ideal gases"

In the proof, scale invariance appears as a necessary, but not sufficient, condition

Specific shapes : the overlap area of two homothetic equilateral triangles is always of the same shape



YES NO NO

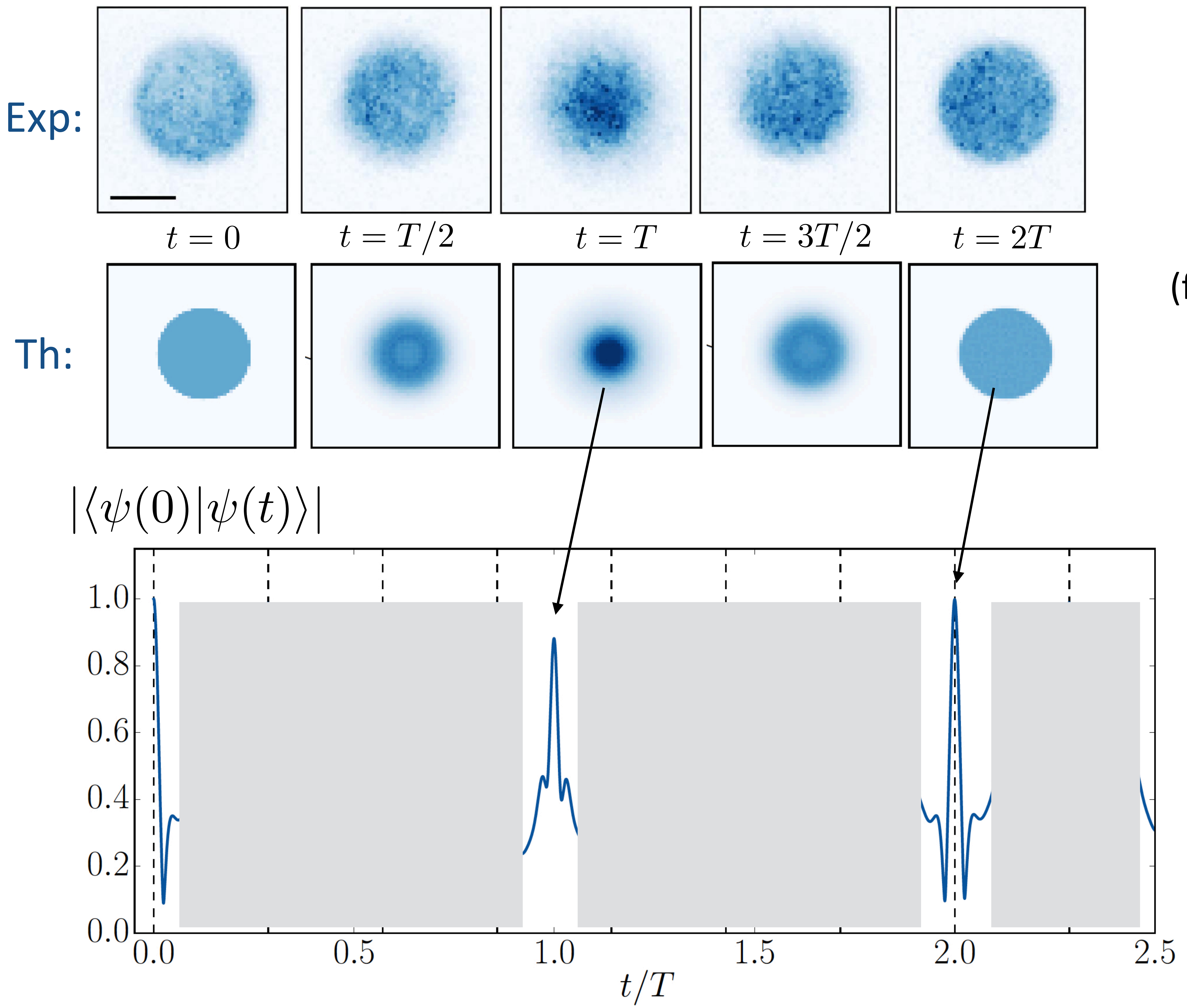


3D tetrahedron: YES

Olshanii et al, SciPost Phys. 10, 114 (2021): "Triangular Gross-Pitaevskii breathers and Damski-Chandrasekhar shock waves"

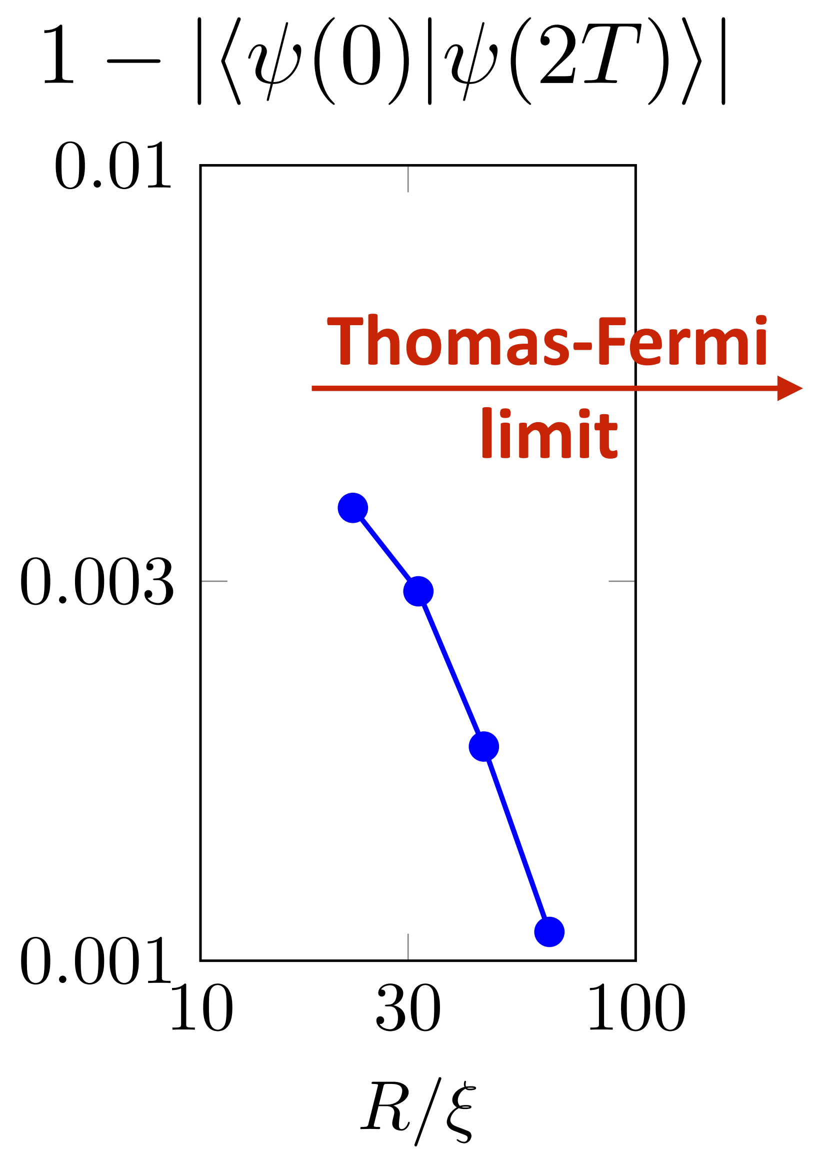
The shock wave created by the initial density jump does not induce further catastrophes in the hydrodynamic equations

Other examples of breathers? Only one so far: Disk



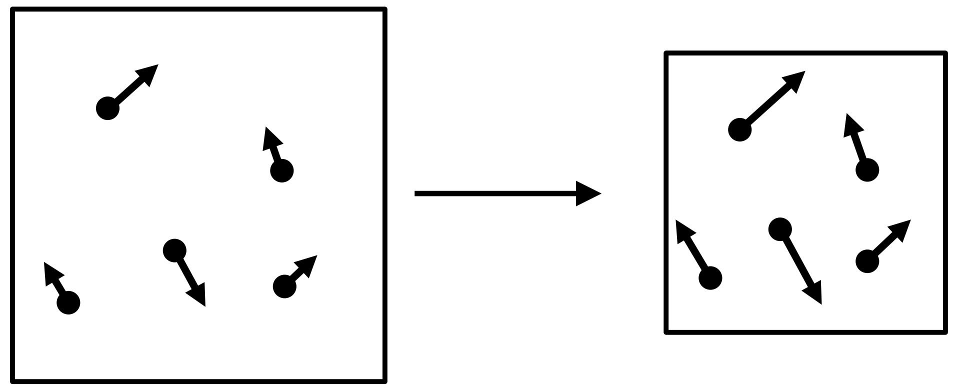
Period $2T$ with $T = 2\pi/\omega$

(for triangles, the period was $T/2$)



Summary

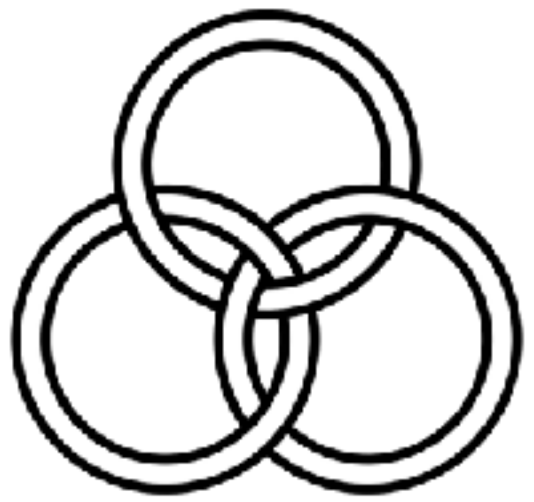
Conformal invariance: example of a dynamical (or hidden) symmetry



Transformations that leaves the equations of motion invariant

Valid either at the quantum-field level (3D unitary gas) or at the classical-field level (2D Bose gas)

A situation valid in any dimension: the $1/r^2$ interaction potential



Can this potential be simulated for a many-body quantum gas, besides the now well-understood Efimov effect?