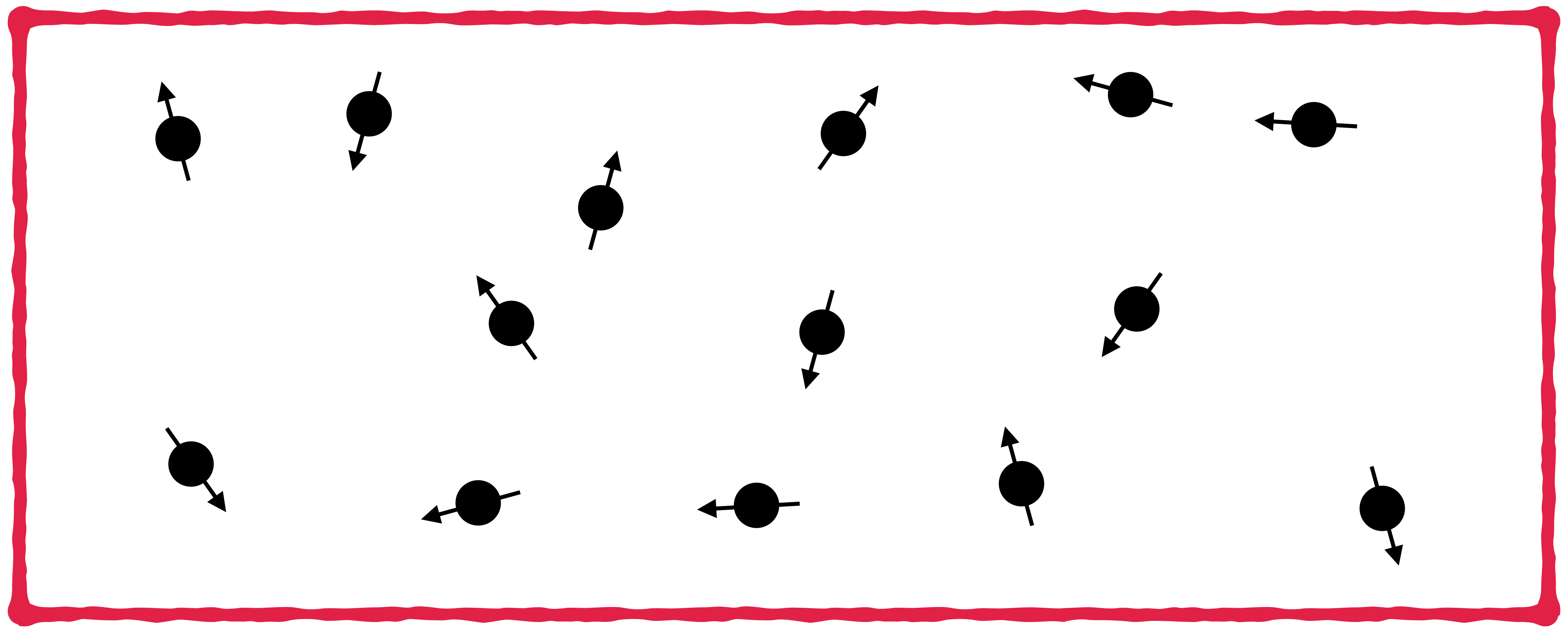
An assembly of spin 1 atoms:
The simplest many-body system

Jean Dalibard

Solvay chair for Physics 2022

Lecture 4

Spinor gases and fluids



Continuous degrees of freedom

$\vec{r}_i, \; \vec{p}_i$

infinite Hilbert space

Discrete degrees of freedom

$\vec{s}_i$

finite Hilbert space

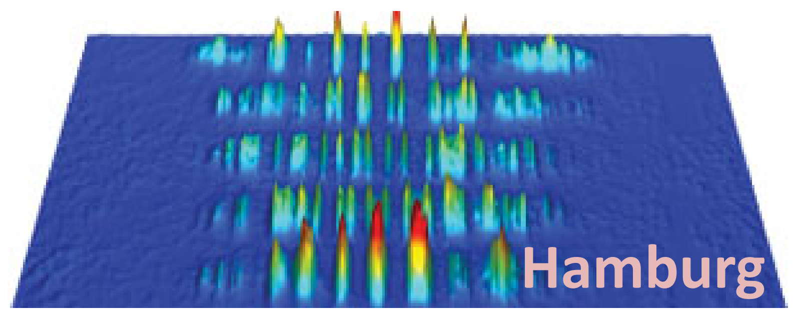
The existence of the spin provides a new degree of freedom,
as well as a source for new types of interactions

The richness of spinor physics

Stamper-Kurn & Ueda
Rev. Mod. Phys. (2013)

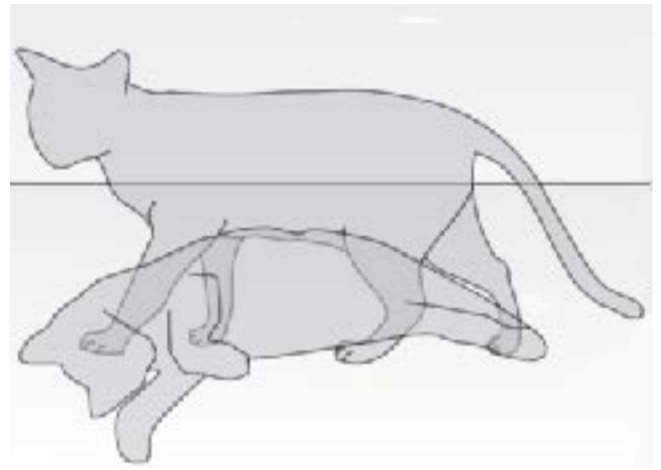
Coherent spin oscillation, spin mixing, dynamical instabilities

Georgia Tech, Hamburg, Hannover, Mainz-Munich, NIST, Paris,…



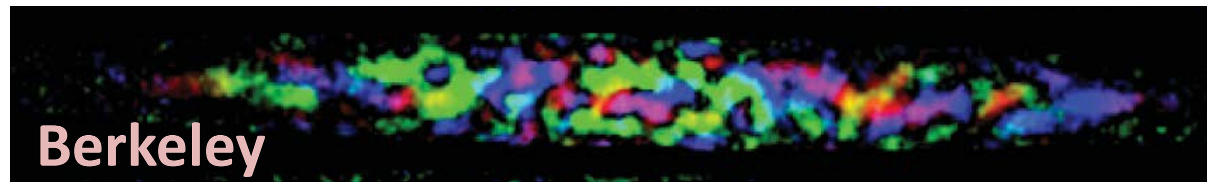
Spin squeezing & entanglement

Georgia Tech, Hannover, Heidelberg, Tsinghua, Paris…



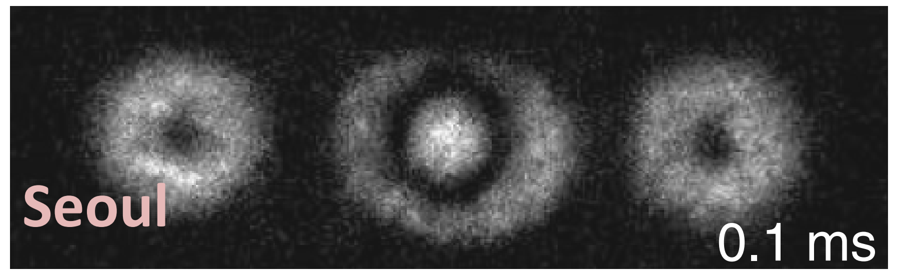
Quenched dynamics and pre-thermalization phenomena

Berkeley, Georgia Tech, Hamburg, Heidelberg,…



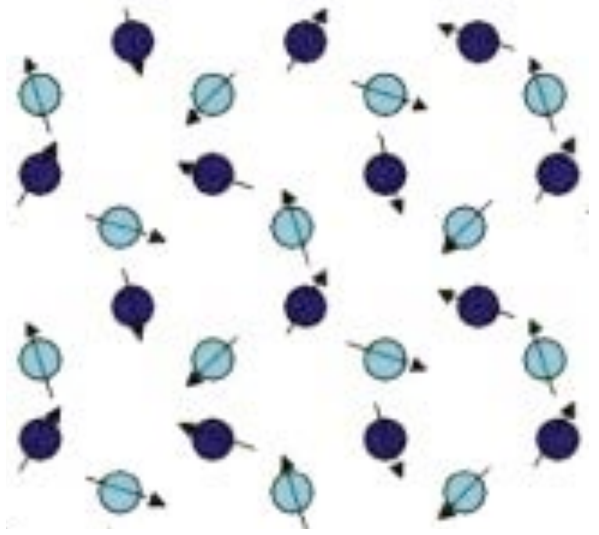
Topological defects

Boulder, MIT, Rochester, Seoul, Amherst,…

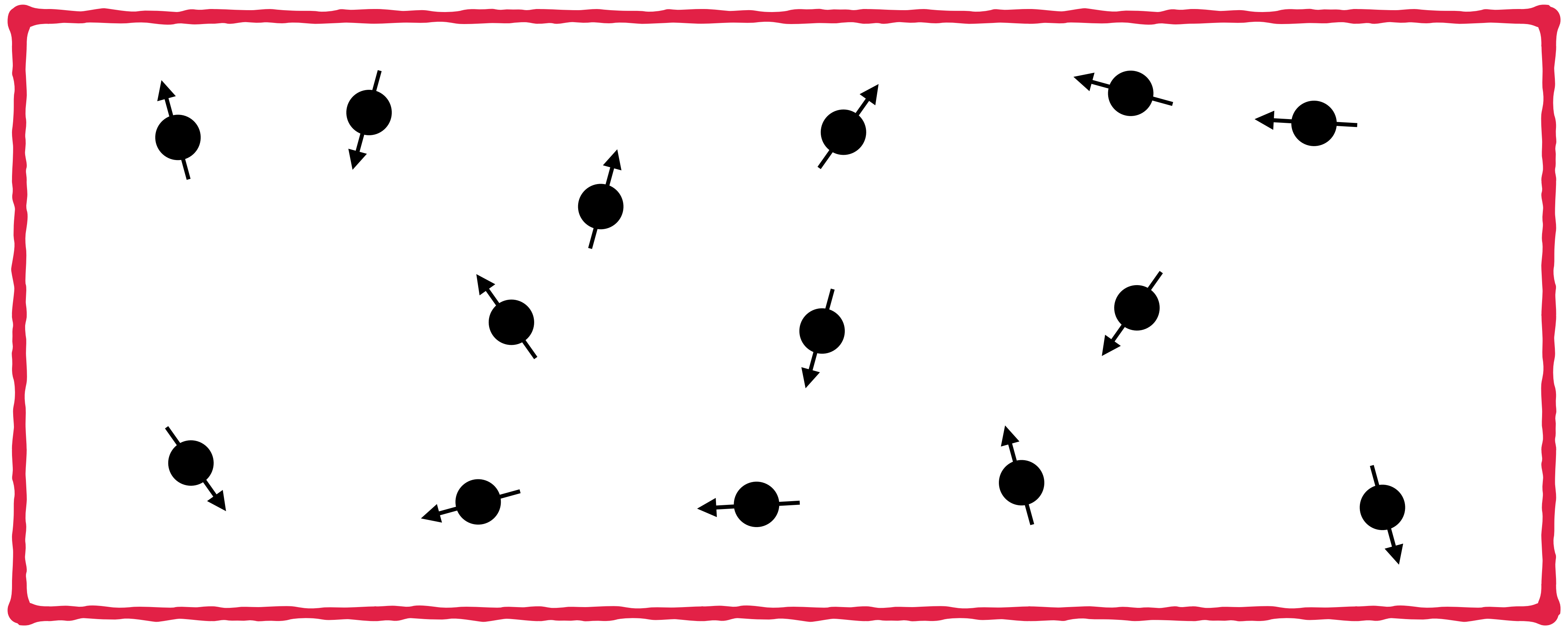


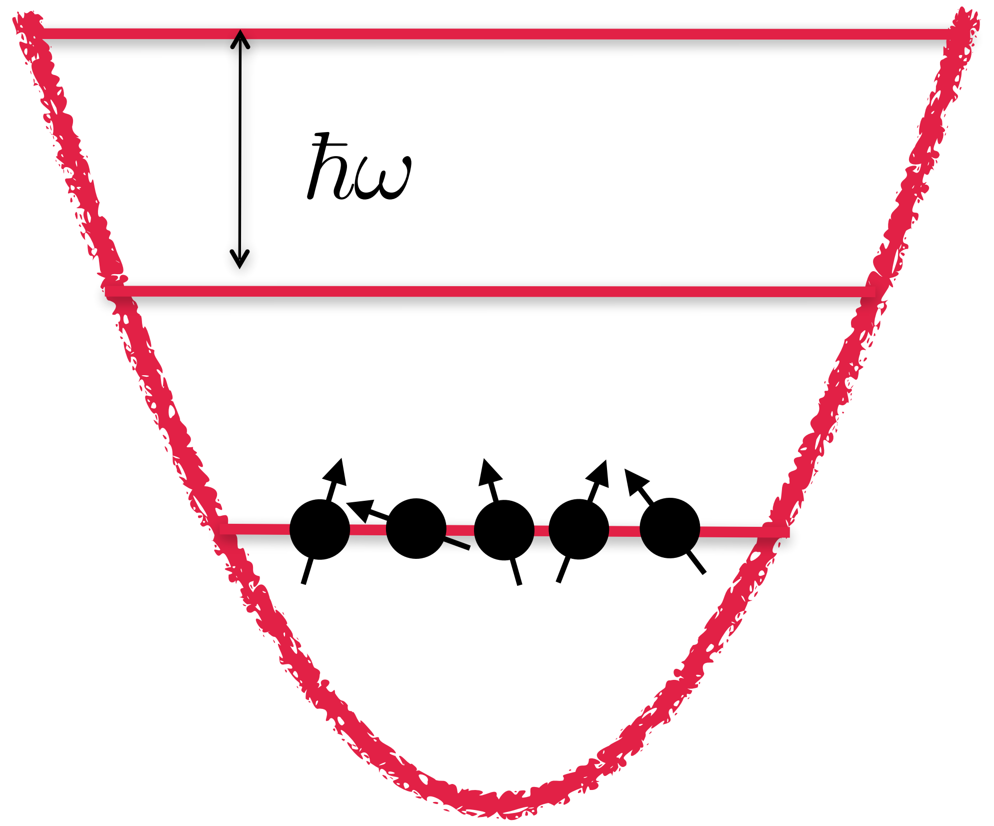
Model for Heisenberg spin lattice systems, dipolar gases

Stuttgart, Hamburg, Innsbruck, Paris-Nord, Boulder, Stanford, …



The single mode approximation





In this lecture, we will assume that all external degrees of freedom are frozen: $k_B T,\, E_{\text{int}} \ll \hbar\omega$

In good approximation, all atoms occupy the ground state of a tight laser trap

Only spin degrees of freedom are relevant (Single Mode Approximation = SMA)

Corresponding interactions: $\hat{H}_{\text{interaction}} = \dfrac{\alpha}{N} \sum_{i<j} \hat{\vec{s}}_i \cdot \hat{\vec{s}}_j$ $\qquad \alpha > 0$: antiferromagnetic

All-to-all coupling

Outline of this lecture

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2. Experimental implementation

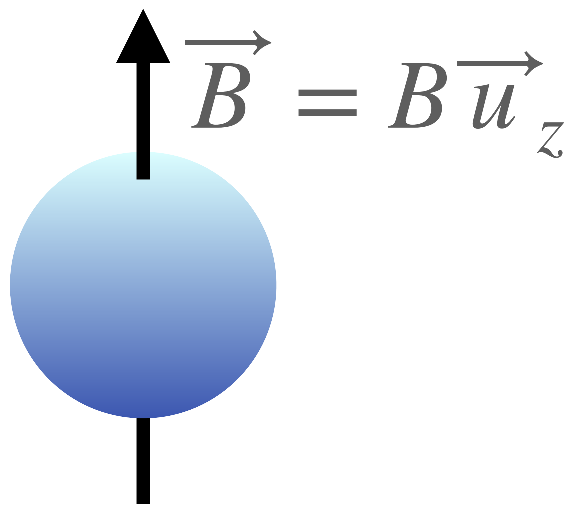
3. A mean-field analysis

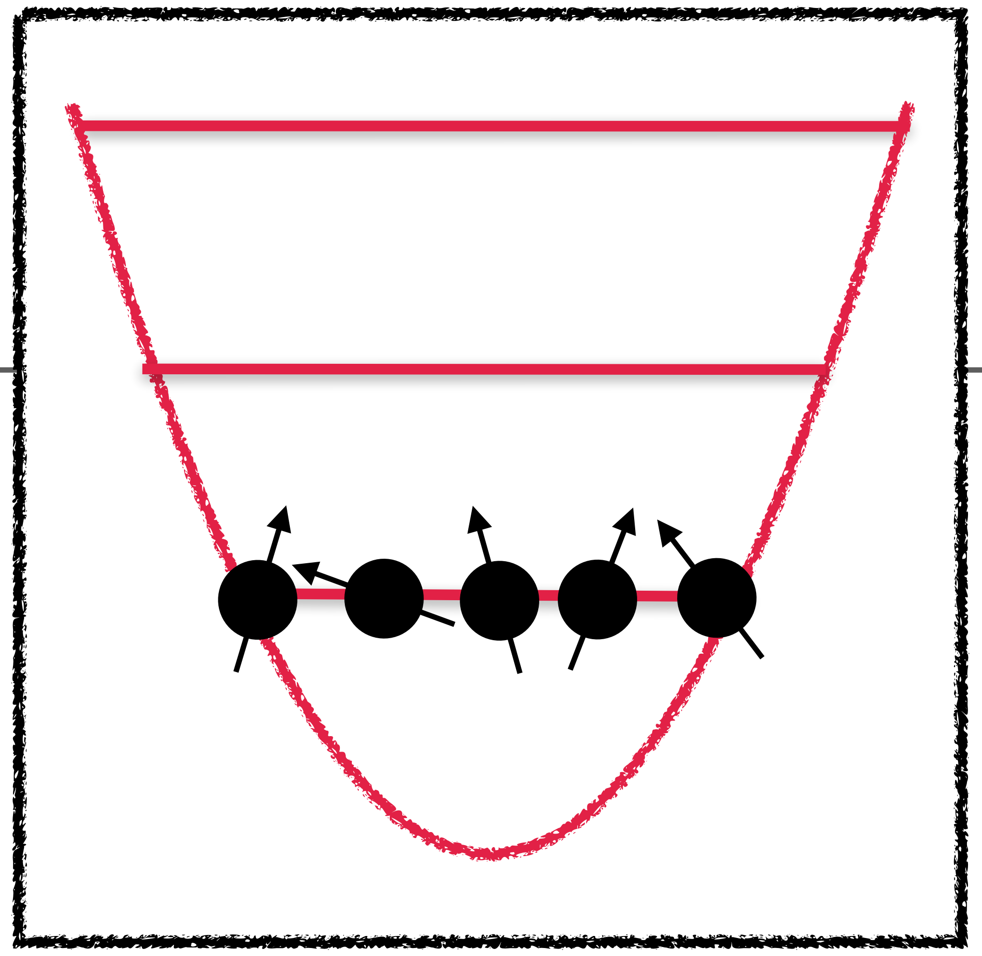
4. Bogoliubov approach and reversible many-body dynamics

5. Beyond Bogoliubov: Relaxation dynamics

6. A fragmented condensate

Symmetry and conserved quantity





Total spin: $\overrightarrow{S} = \sum_{i=1}^{N} \vec{s}_i$

Single atom Hamiltonian: Zeeman effect with $\overrightarrow{B}$ along the z axis

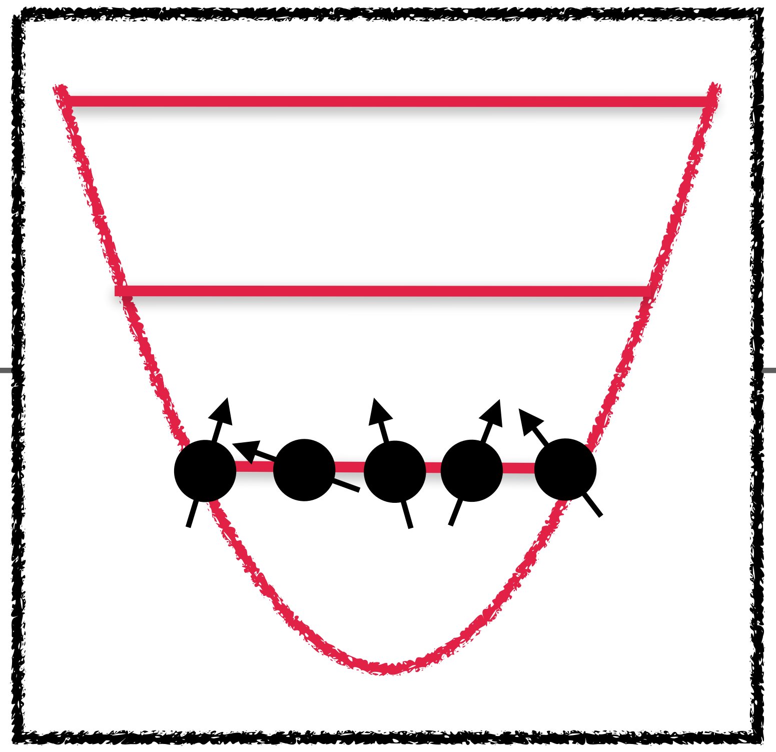
Conservation of $S_z = \sum_{i=1}^{N} s_{i,z}$

Binary interaction:

$$\hat{H}_{\text{interaction}} = \frac{\alpha}{N} \sum_{i<j} \hat{\vec{s}}_i \cdot \hat{\vec{s}}_j = \frac{\alpha}{N} \sum_{i<j} \left[\frac{1}{2} \left(\hat{s}_i^+ \hat{s}_j^- + \hat{s}_i^- \hat{s}_j^+ \right) + \hat{s}_{i,z} \hat{s}_{j,z} \right]$$

S_z is also conserved by the interaction Hamiltonian

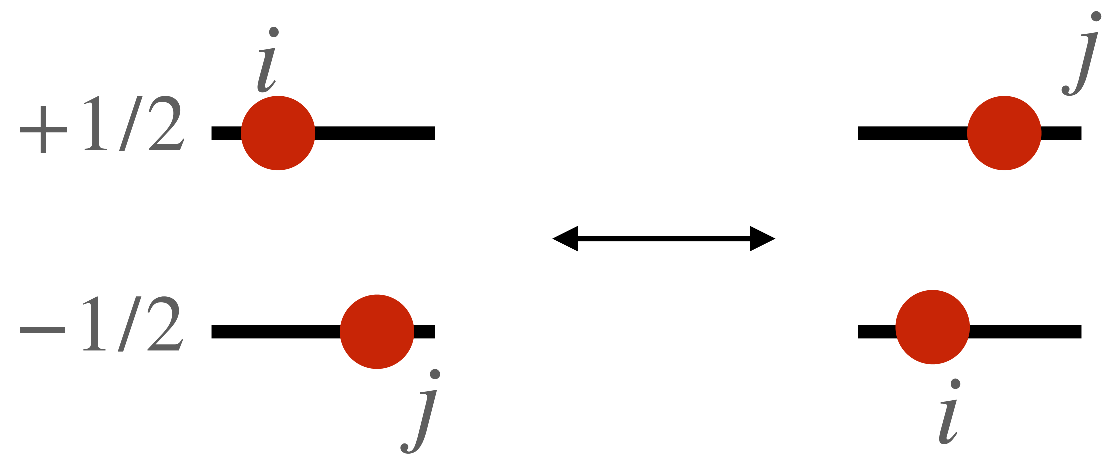
Which spin to get a non-trivial many-body dynamics?



$$\hat{H}_{\text{interaction}} = \frac{\alpha}{N} \sum_{i<j} \hat{\vec{s}}_i \cdot \hat{\vec{s}}_j = \frac{\alpha}{N} \sum_{i<j} \left[\frac{1}{2} \left(\hat{s}_i^+ \hat{s}_j^- + \hat{s}_i^- \hat{s}_j^+ \right) + \hat{s}_{i,z} \hat{s}_{j,z} \right]$$

Assembly of spins ½

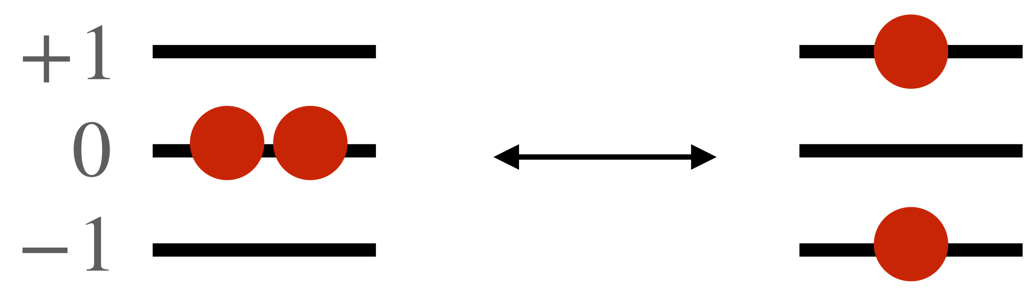
$$S_z = \frac{\hbar}{2} \left(N_{+1/2} - N_{-1/2} \right)$$



No relevant dynamics can happen

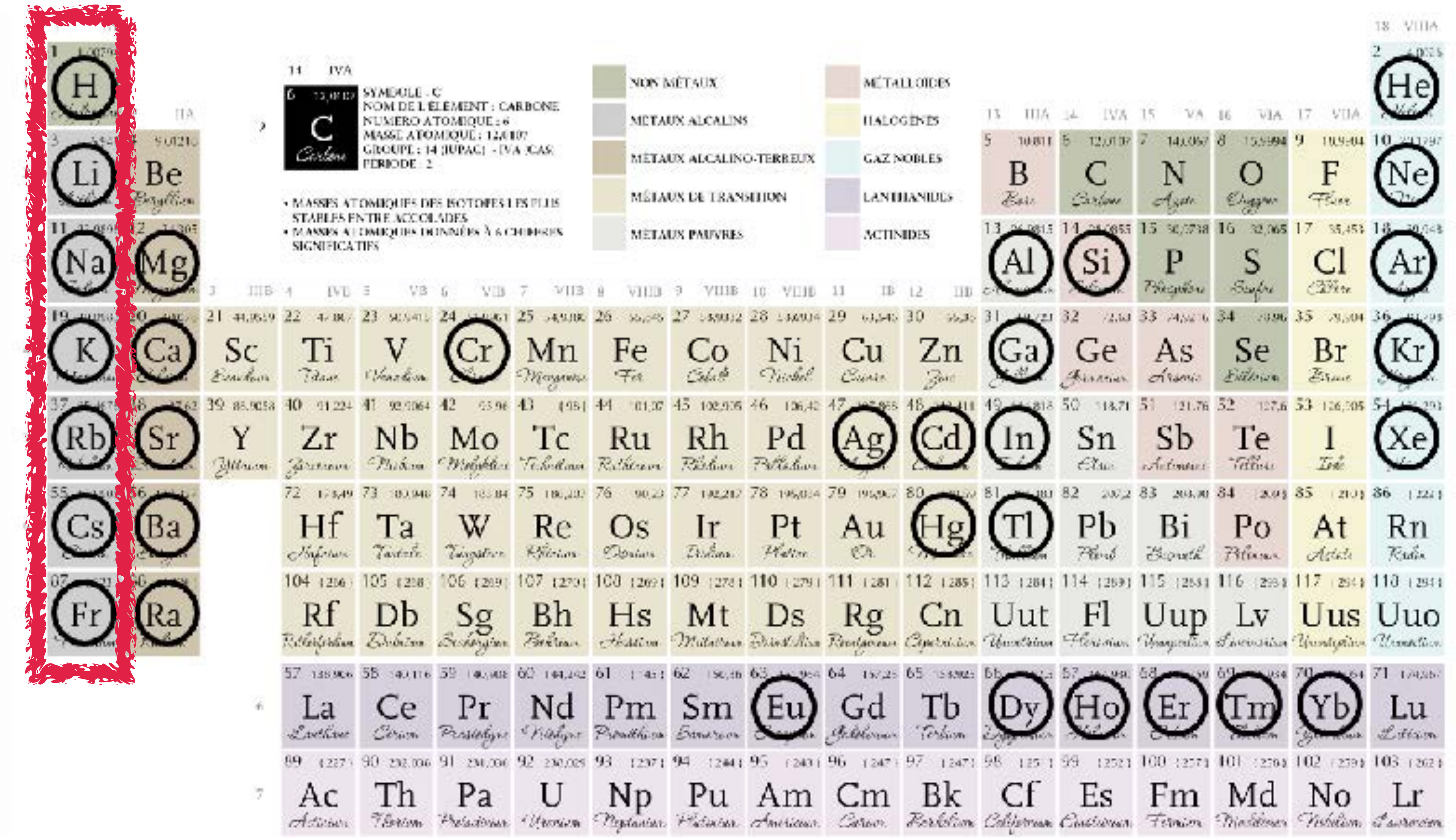
Assembly of spins 1

$$S_z = \hbar \left(N_{+1} - N_{-1} \right)$$



A sufficiently complex system to illustrate several aspects of many-body physics

Where do we get our spin 1 atoms?

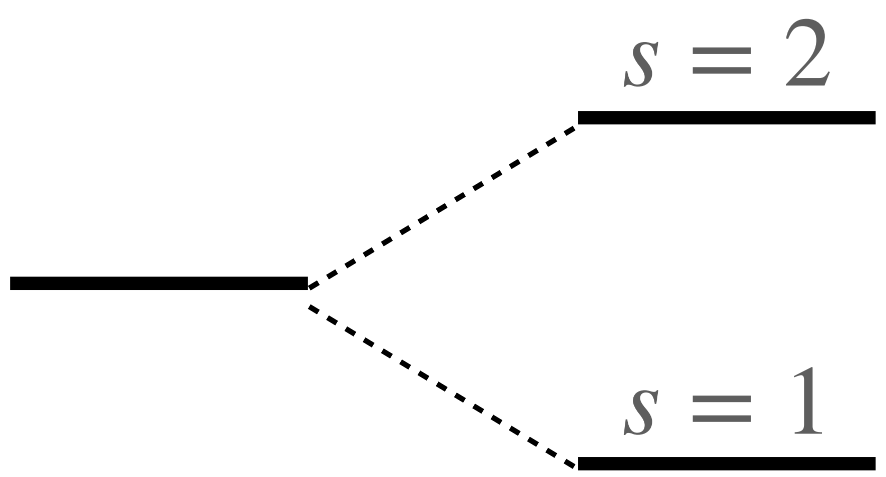


Alkali-metal atoms:

Coupling of the outer electron spin $s_e = \dfrac{1}{2}$ and the nuclear spin s_n

with for ^{7}Li, ^{23}Na, ^{39}K, ^{87}Rb : $s_n = \dfrac{3}{2}$

Hyperfine structure of the ground state of a single atom:



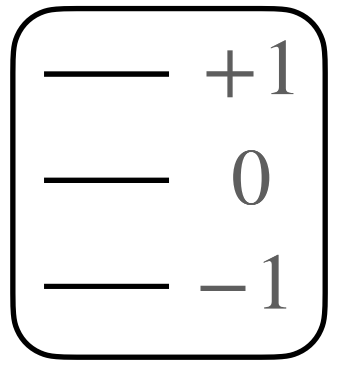
Plays no role, except for inducing a non-linear Zeeman effect for $s = 1$ (next slides)

The state of interest in this lecture

The Hilbert space of the problem

N spin 1 atoms in the single mode regime (N even)

Drawing for $N = 4$



Occupation number basis

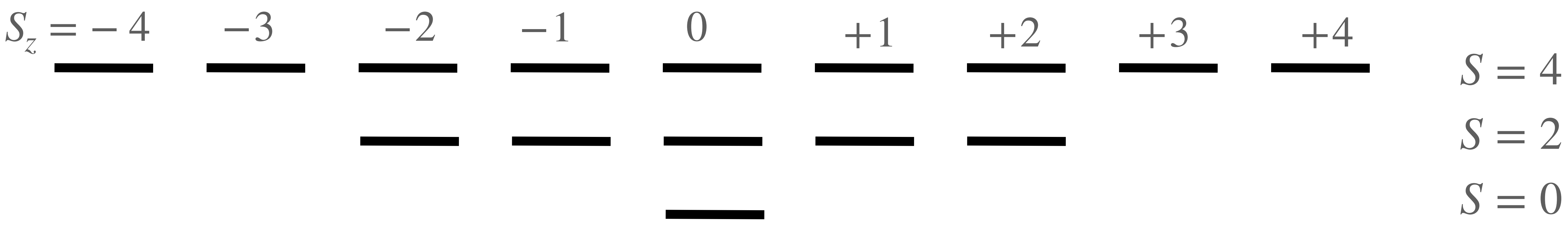
$$|N_-, N_0, N_+\rangle$$

with $N_- + N_0 + N_- = N$

Hilbert space with dimension $d = \frac{1}{2}(N+1)(N+2)$ (i.e. $d = 15$ for $N = 4$)

Total spin basis

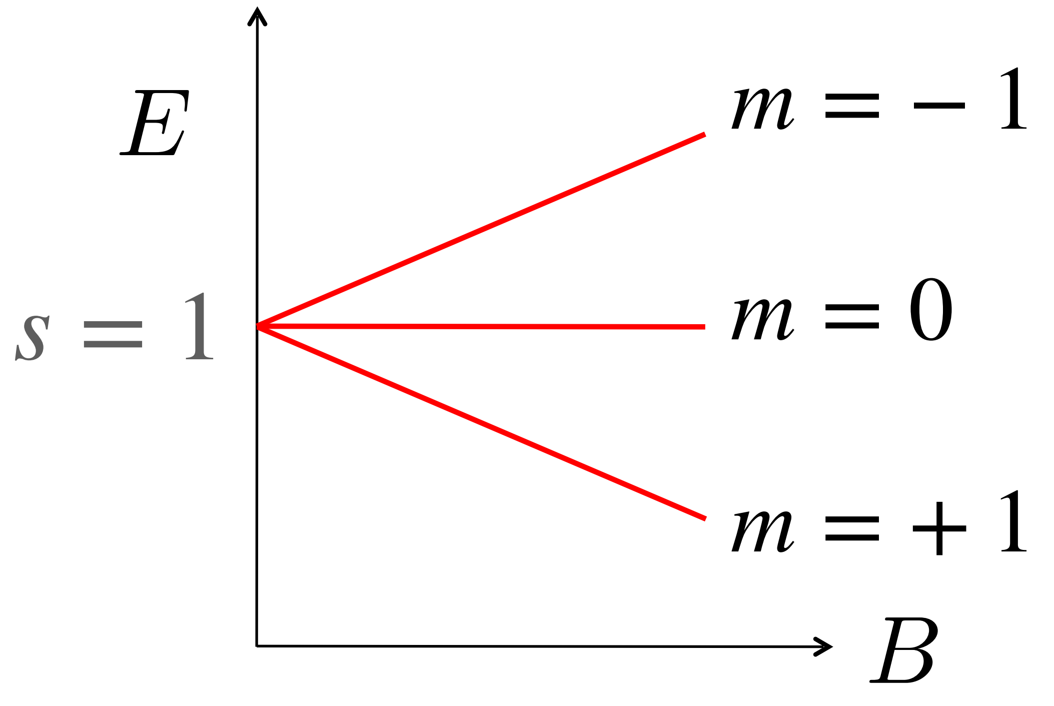
$$|N, S, S_z\rangle$$



If S_z is a conserved quantity, we work along a given column of these two diagrams

The relevant Hamiltonian (1): Zeeman energy

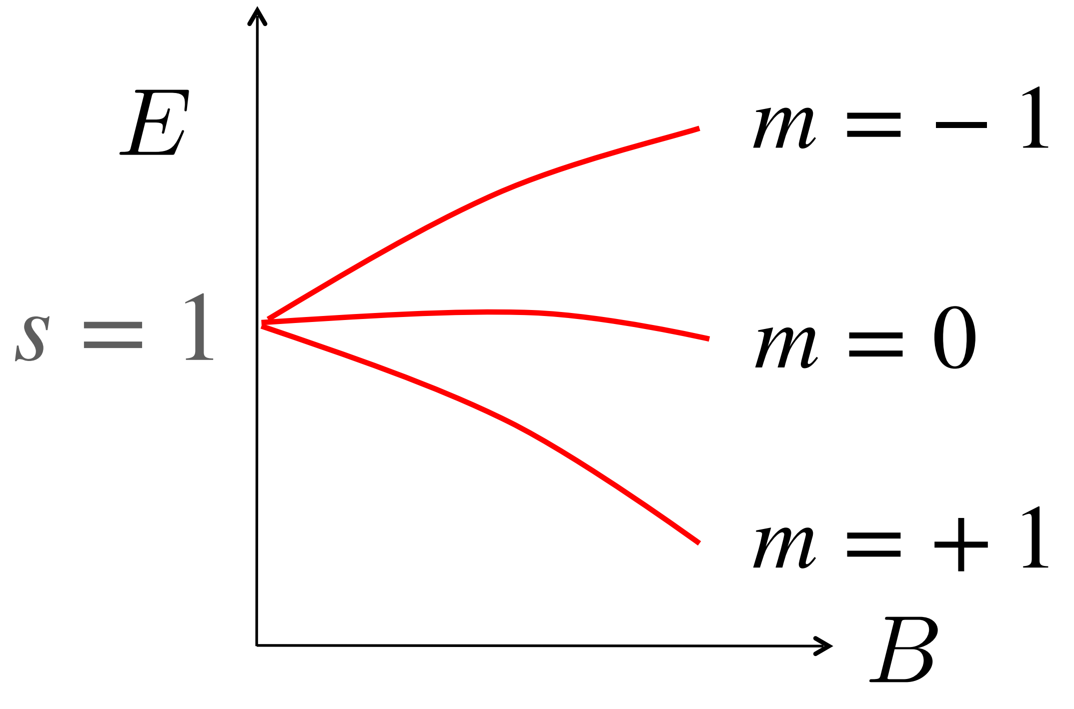
Linear Zeeman effect:



$$\hat{H}^{(1)}_{\text{Zeeman}} = -\mu B \hat{S}_z = -\mu B \left(\hat{N}_+ - \hat{N}_- \right)$$

Since S_z is a conserved quantity, $H^{(1)}_{\text{Zeeman}}$ does not contribute to the dynamics

Quadratic Zeeman effect:



$$\hat{H}^{(2)}_{\text{Zeeman}} = qB^2 \left(\hat{N}_{+1} + \hat{N}_{-1} \right) \ + \ \text{constant}$$
$$= -qB^2 \hat{N}_0 \ + \ \text{constant}$$

$$N = N_{-1} + N_0 + N_{+1}$$
$$\beta \equiv qB^2$$

$q = 277$ Hz/G$^2 > 0$: favours the accumulation of atoms in $m = 0$

The value and the sign of q could be changed by rf dressing or a time-modulation of B

The relevant Hamiltonian (2): Interaction energy

Ho (1998)
Ohmi & Machida (1998)

Real magnetic interactions (dipole-dipole) are negligible at our temperature scale

Only van der Waals interactions (described by a contact potential) are significant

For a collision between two spin 1 atoms, the total spin can be:

$$\begin{cases} S = 0 \text{ (symmetric spin state)} \\ S = 1 \text{ (anti-symmetric spin state)} \\ S = 2 \text{ (symmetric spin state)} \end{cases}$$

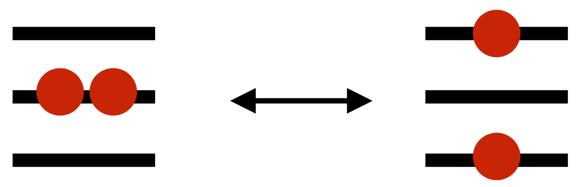
Here, symmetric orbital state (same spatial mode) $\Rightarrow$ Only $S = 0$ and $S = 2$ channels are relevant

$$\hat{H}^{(1,2)}_{\text{interaction}} = \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \left[g_0 \hat{\mathscr{P}}_{S=0} + g_2 \hat{\mathscr{P}}_{S=2} \right]$$

$$= \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2) \left[\bar{g} + g_s \hat{\boldsymbol{s}}_1 \cdot \hat{\boldsymbol{s}}_2 \right]$$

$$\bar{g} = \frac{g_0 + 2g_2}{3} \longrightarrow$$ plays no role in the SMA

$$g_s = \frac{g_2 - g_0}{3} \longrightarrow$$ responsible for



Na: $g_s > 0$ (antiferro) Rb: $g_s < 0$ (ferro)

For Na: $a_0 = 2.51$ nm, $a_2 = 2.80$ nm

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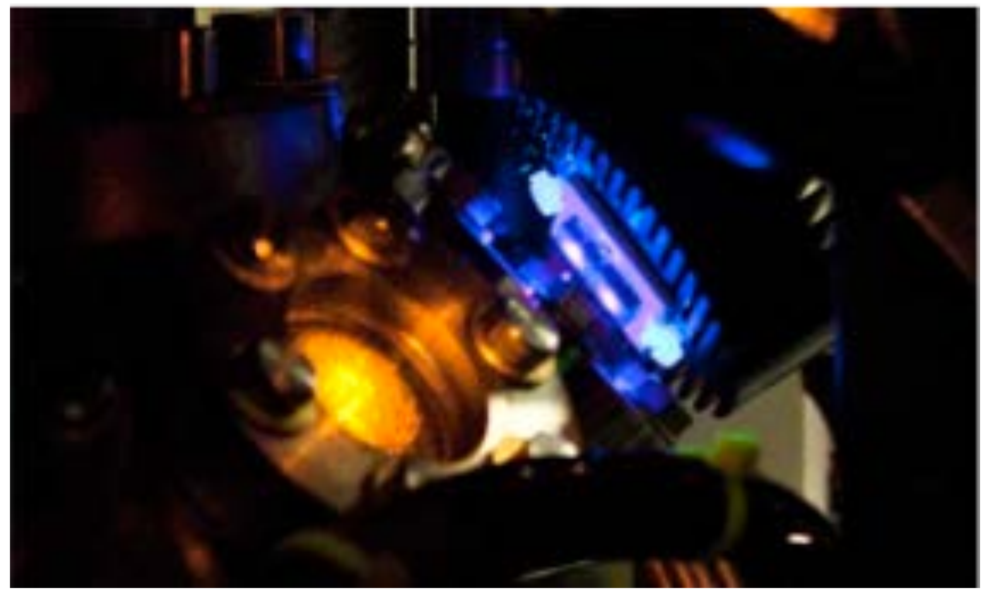
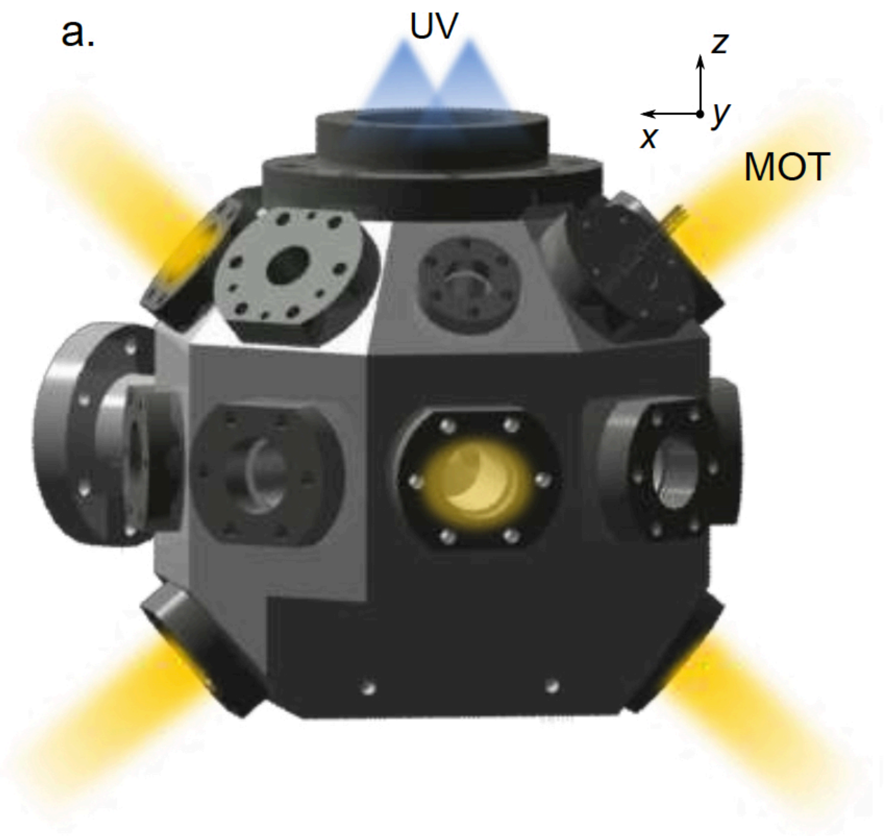
The experimental system

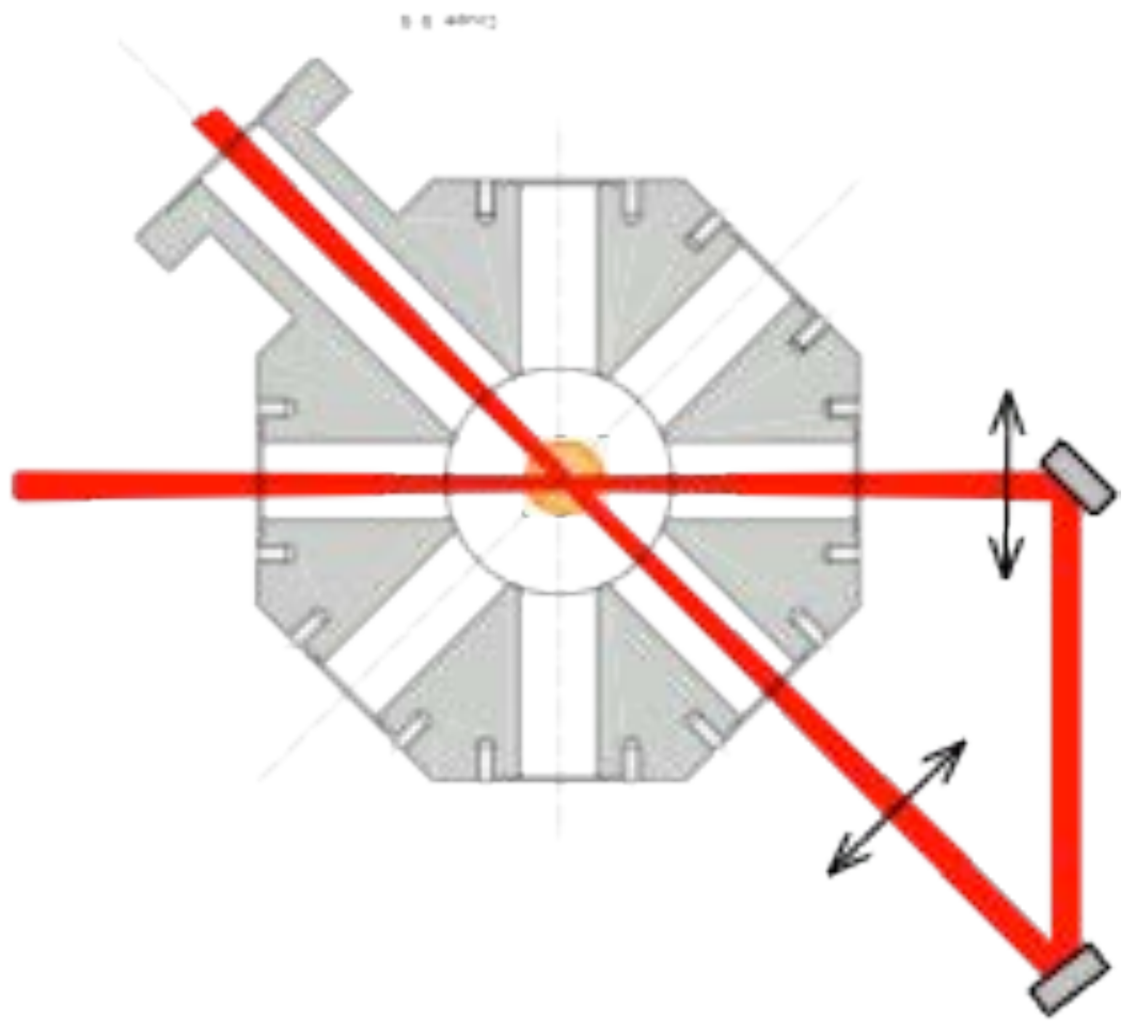
David Jacob, Lingxuan Shao, Vincent Corre, Tilman Zibold, Luigi De Sarlo, Emmanuel Mimoun, Jean Dalibard and Fabrice Gerbier, Phys. Rev. A 86, 061601(R) (2012)



Room temperature vapour cell of Sodium (using UV light-induced desorption)

Magneto-optical trap in the vapour cell





Evaporation in the $s = 1$ ground state in a crossed dipole trap + dimple

$\omega/2\pi = 0.5$ to 3 kHz $\qquad \hbar\omega \gtrsim k_B T,\ E_{\mathrm{int}}$

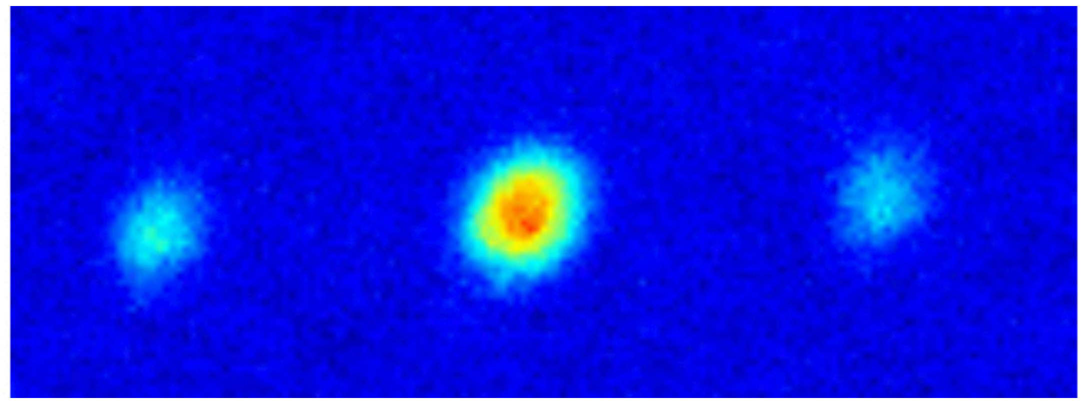
Quasi-pure BEC in SMA with an adjustable atom number between 100 and 5000

Magnetisation: Detection and control

Diagnostic of the sample by Stern-Gerlach analysis

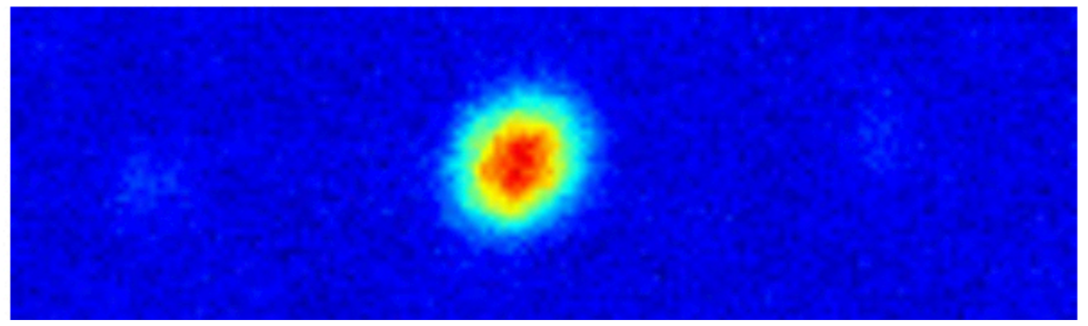
Absorption imaging

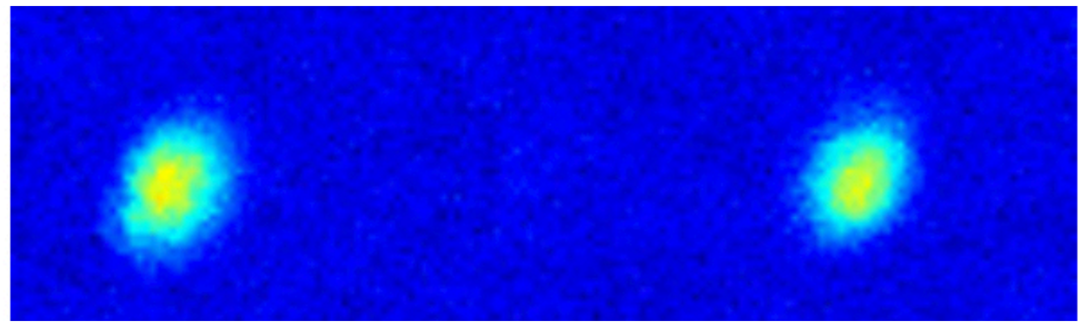
Check of the single-mode approximation:
same spatial profile for $m = -1, 0, +1$



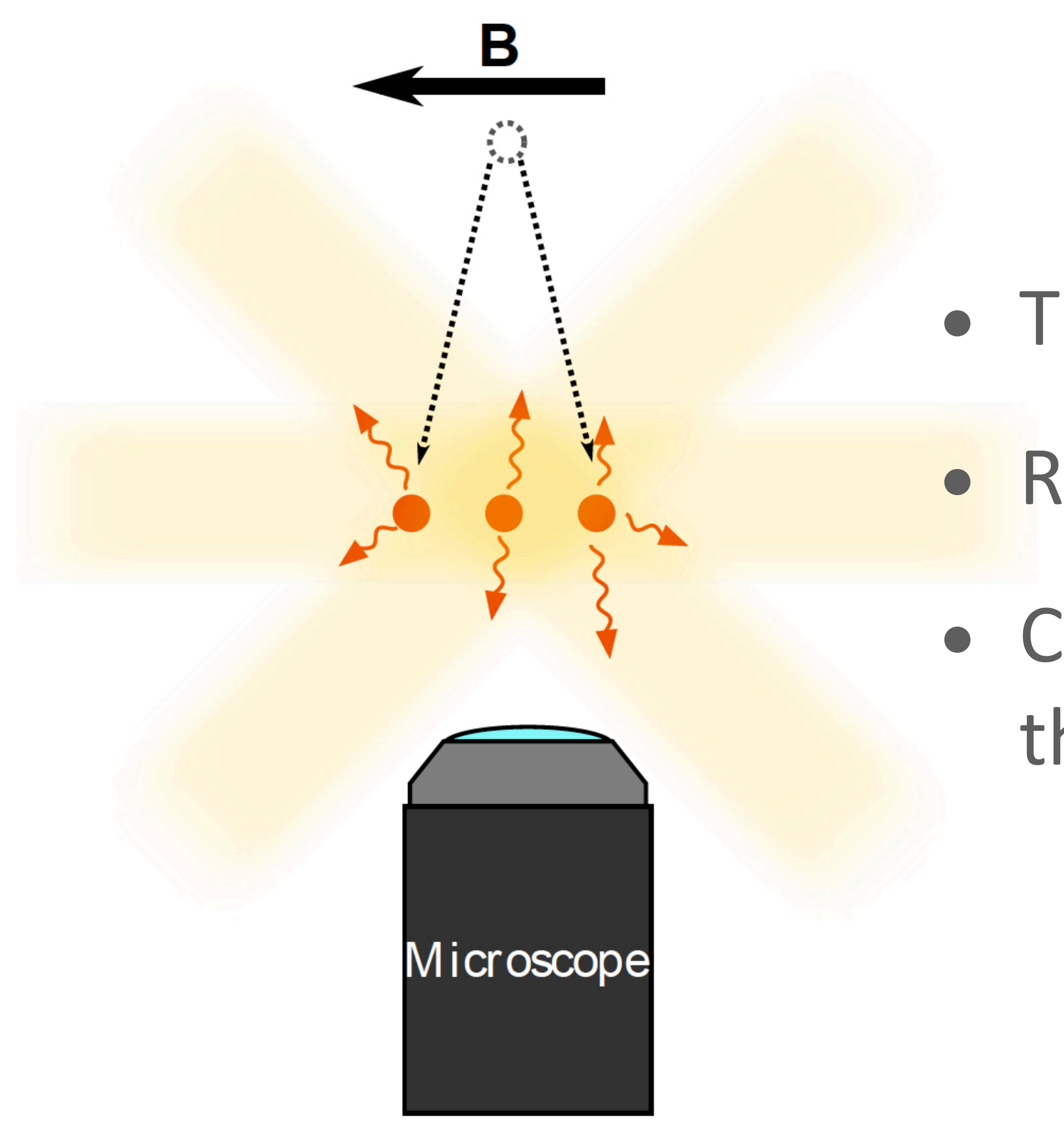
Using a combination of magnetic field gradient and radio-frequency pulses, we can prepare the atoms:

- all in $m = 0$,
- or in a superposition of $m = \pm 1$
- or in a superposition of $m = 0$ and $m = +1$
- or whatever…





Spin resolved fluorescence imaging at the single atom level



An Qu, Bertrand Evrard,
Jean Dalibard, Fabrice Gerbier,
Phys. Rev. Lett. **125**, 033401 (2020)

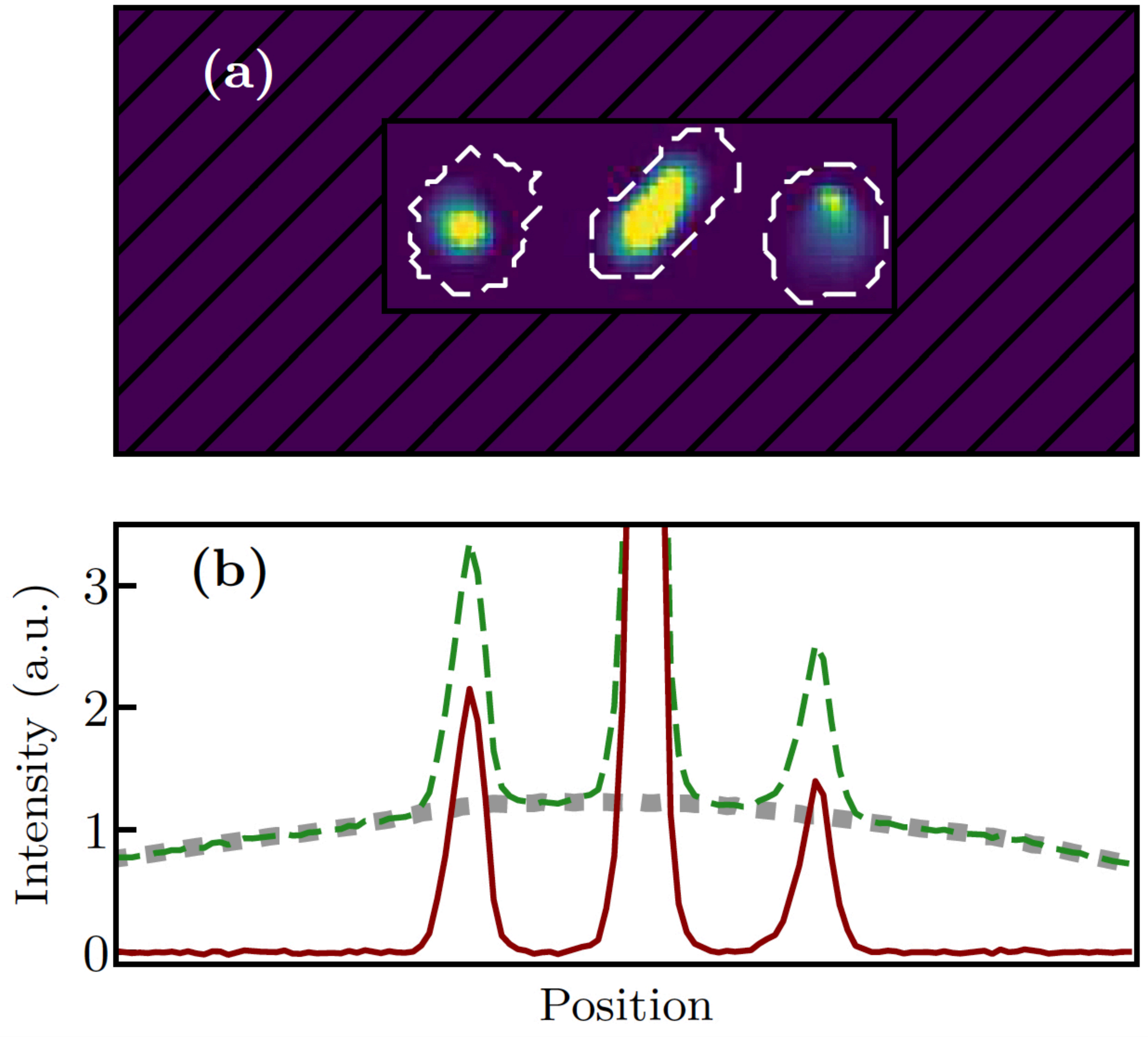
- Time-of-flight in the presence of $\vec{\nabla} B$
- Recapture in an optical molasses
- Collect the emitted fluorescence light during the molasses phase

Optimal molasses duration: ~ 5 ms

Determine optimized regions of interest

After background removal, the residual shot noise corresponds to a sensitivity of

$N \approx 1.6$ atom



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A mean-field approach to the ground state of the N spin system

The two non-commuting contributions to the Hamiltonian $(\alpha, \beta > 0)$

Antiferromagnetic interactions: $\hat{H}_{\text{interaction}} = \dfrac{\alpha}{N} \sum_{i<j} \hat{\vec{s}}_i \cdot \hat{\vec{s}}_j = \dfrac{\alpha}{2N} \hat{\mathbf{S}}^2$ + const.

$\hat{\mathbf{S}} = \sum \hat{\mathbf{s}}_i$: total spin

Quadratic Zeeman effect: $\hat{H}_{\text{Zeeman}} = \beta \left(\hat{N}_{+1} + \hat{N}_{-1} \right)$ + constant

Mean-field approach

Trial wave-function where all atoms occupy the same spin state $|\Psi\rangle = |\psi\rangle^{\otimes N}$

$$|\psi\rangle = \begin{pmatrix} \sqrt{n_{+1}}\, \mathrm{e}^{\mathrm{i}\phi_{+1}} \\ \sqrt{n_0}\, \mathrm{e}^{\mathrm{i}\phi_0} \\ \sqrt{n_{-1}}\, \mathrm{e}^{\mathrm{i}\phi_{-1}} \end{pmatrix}$$

6 real parameters but
- Irrelevant global phase
- Fixed norm: $n_{+1} + n_0 + n_{-1} = 1$

$\longrightarrow$ 4 independent real parameters

The ground state in the mean-field approach

Zhang, Yi, You (2003)

Minimize $\langle\psi|\hat{H}|\psi\rangle$ with

$$\hat{H} = \frac{\alpha}{2N}\hat{\mathbf{S}}^2 + \beta\left(\hat{N}_{+1} + \hat{N}_{-1}\right) \quad \text{and} \quad |\psi\rangle = \begin{pmatrix} \sqrt{n_{+1}}\,\mathrm{e}^{\mathrm{i}\phi_{+1}} \\ \sqrt{n_0}\,\mathrm{e}^{\mathrm{i}\phi_0} \\ \sqrt{n_{-1}}\,\mathrm{e}^{\mathrm{i}\phi_{-1}} \end{pmatrix}$$

magnetization $M_z = n_{+1} - n_{-1}$
$= S_z/N$

Second-order phase transition with the population n_0 as the order parameter

Critical magnetic field value $\beta_c = \alpha\left(1 - \sqrt{1 - M_z^2}\right)$

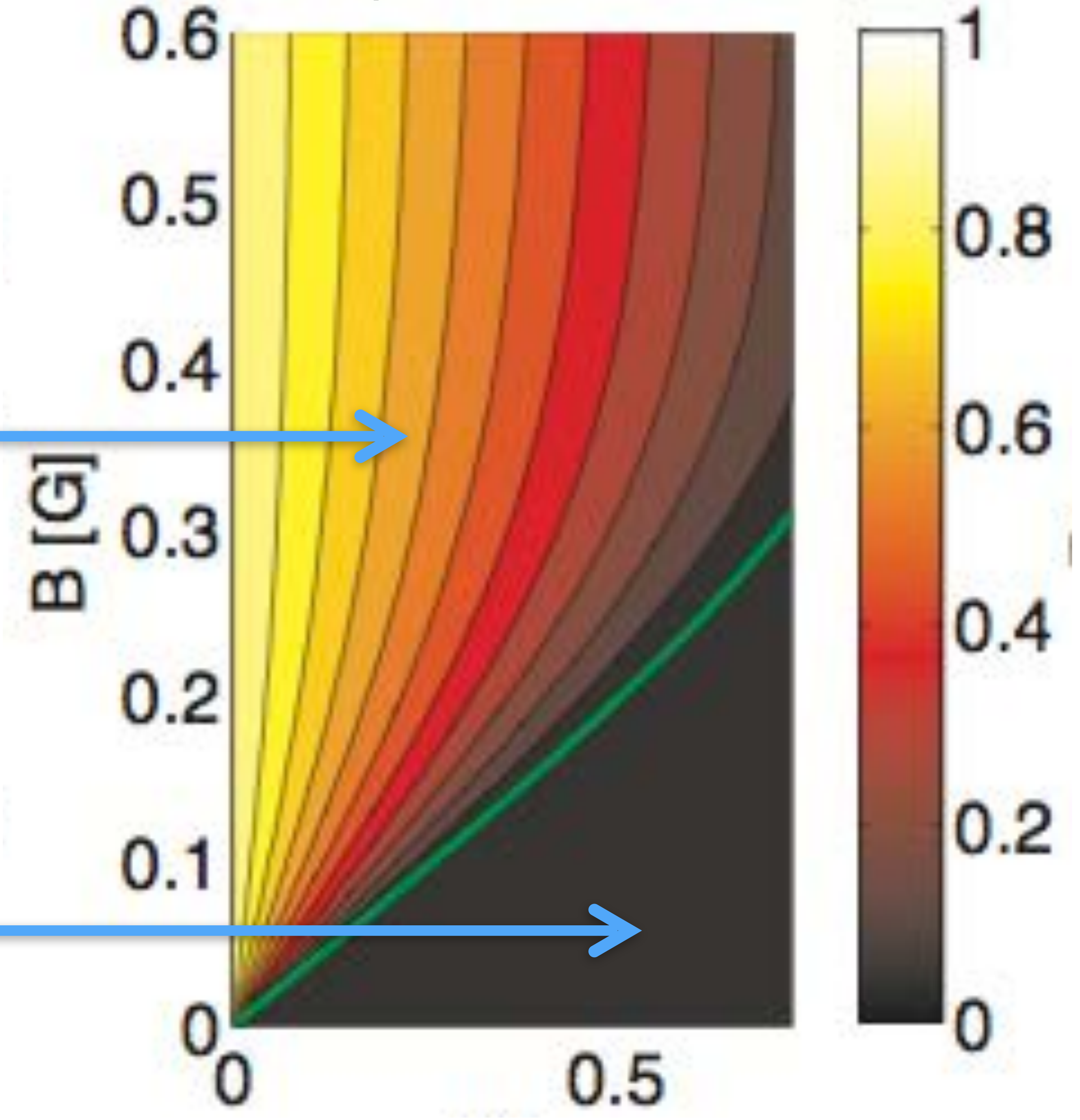
For $\beta > \beta_c$, the three populations are non-zero

$$n_0 \neq 0$$

For $\beta < \beta_c$, only $n_{\pm 1}$ are non-zero

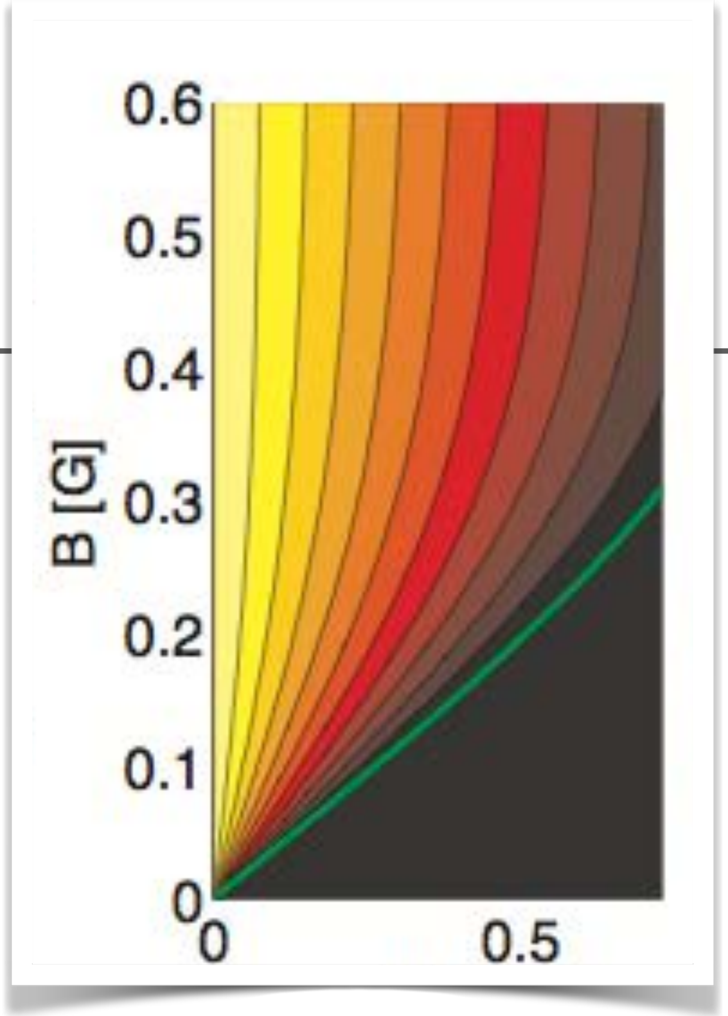
$$n_0 = 0$$

Population n_0



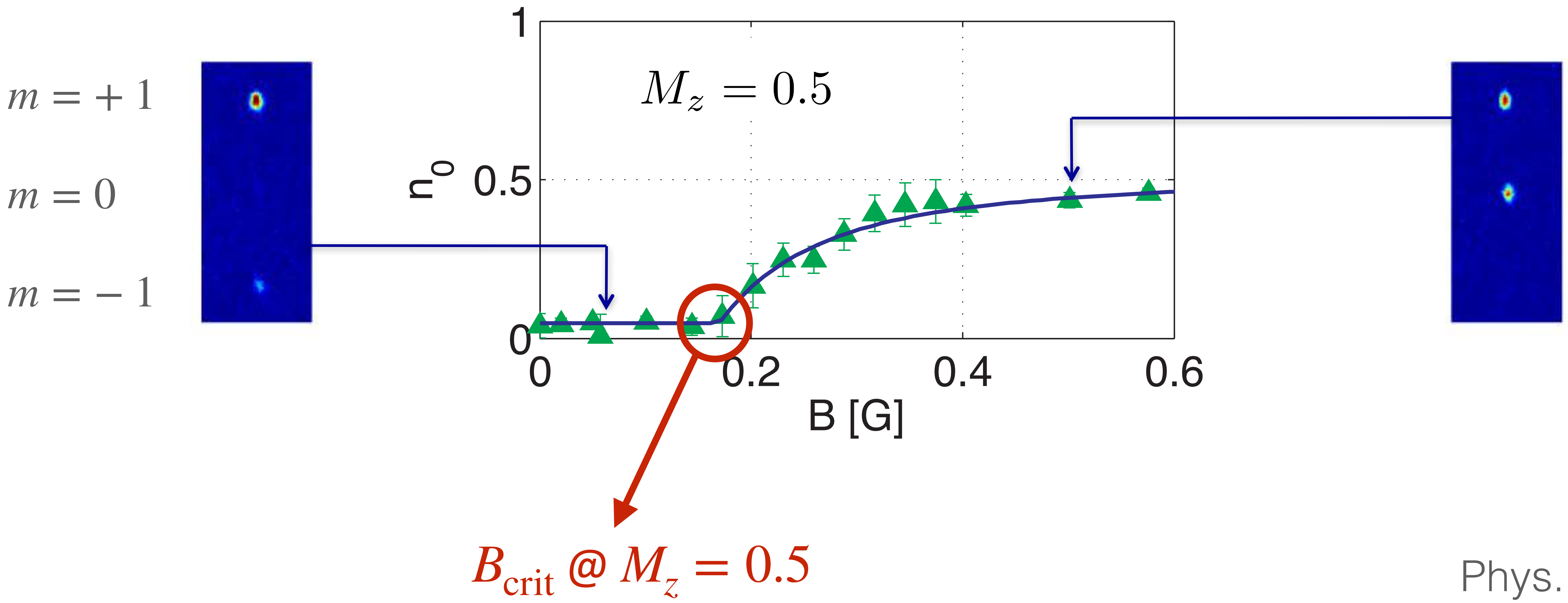
magnetization M_z

Experimental determination of the phase diagram



- Prepare the system ($N = 5000$) with a given magnetization (here $M_z = 0.5$)
- Vary the magnetic field
- Measure the fraction of atoms in $m = 0$

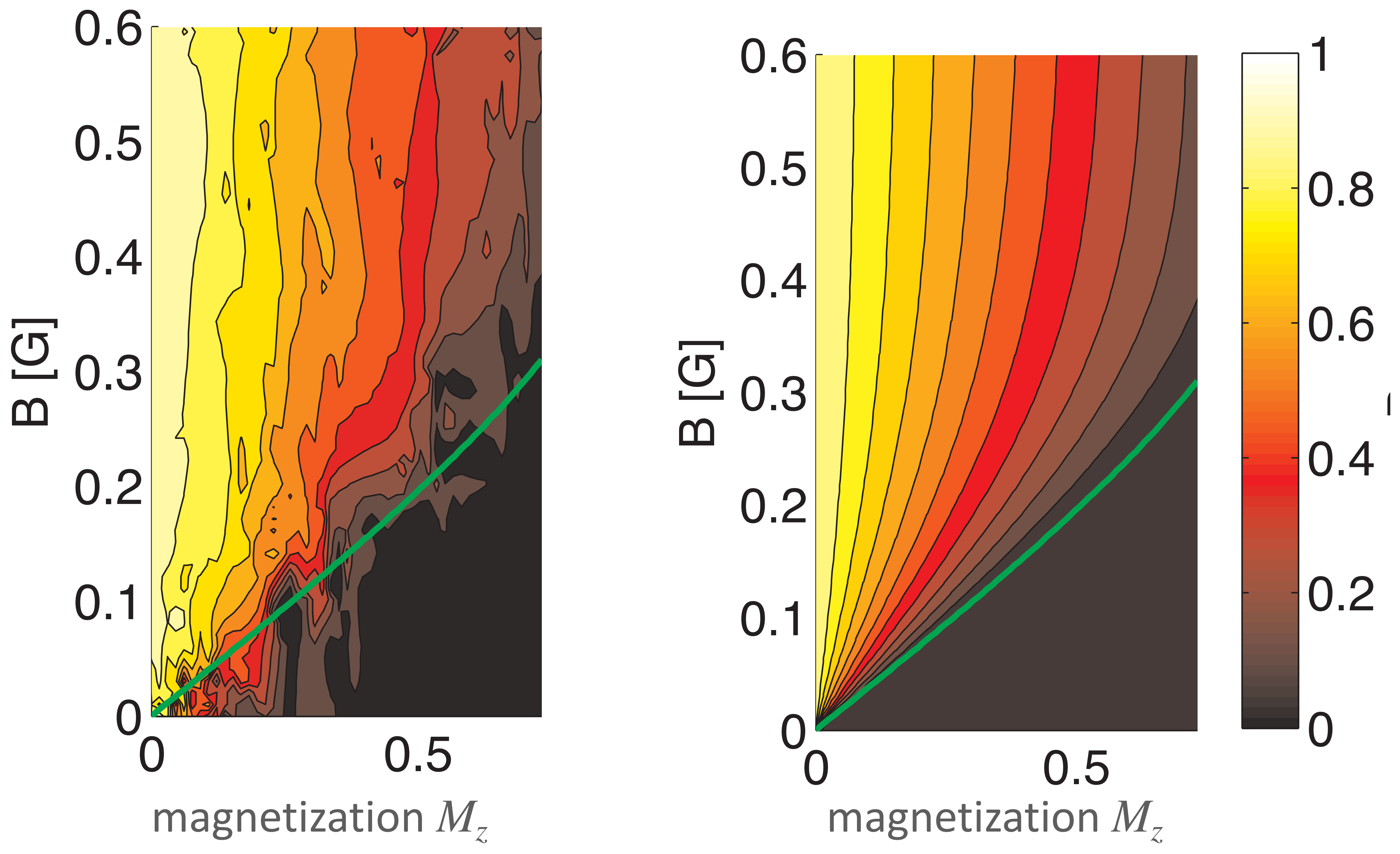
$$|m=0\rangle + |m=0\rangle \leftrightarrows |m=+1\rangle + |m=-1\rangle$$



Phys. Rev. A 86, 061601(R)

Experimental determination of the phase diagram (2)

Fraction n_0 for various M_z and various magnetic fields (note the symmetry $M_z \leftrightarrow -M_z$)



no adjustable parameters

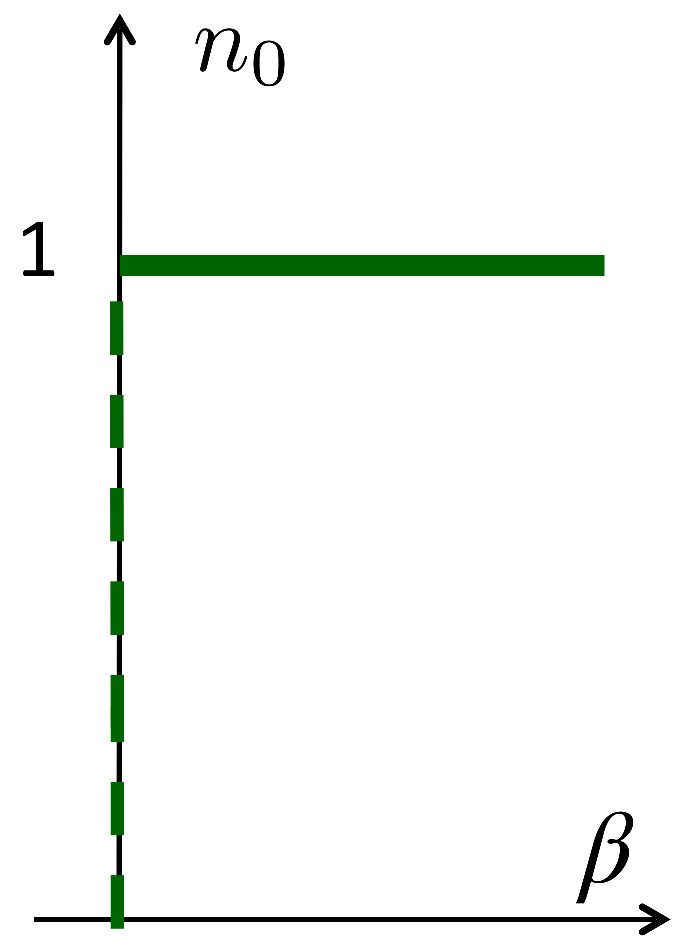
D. Jacob et al., Phys. Rev. A **86**, 061601(R)

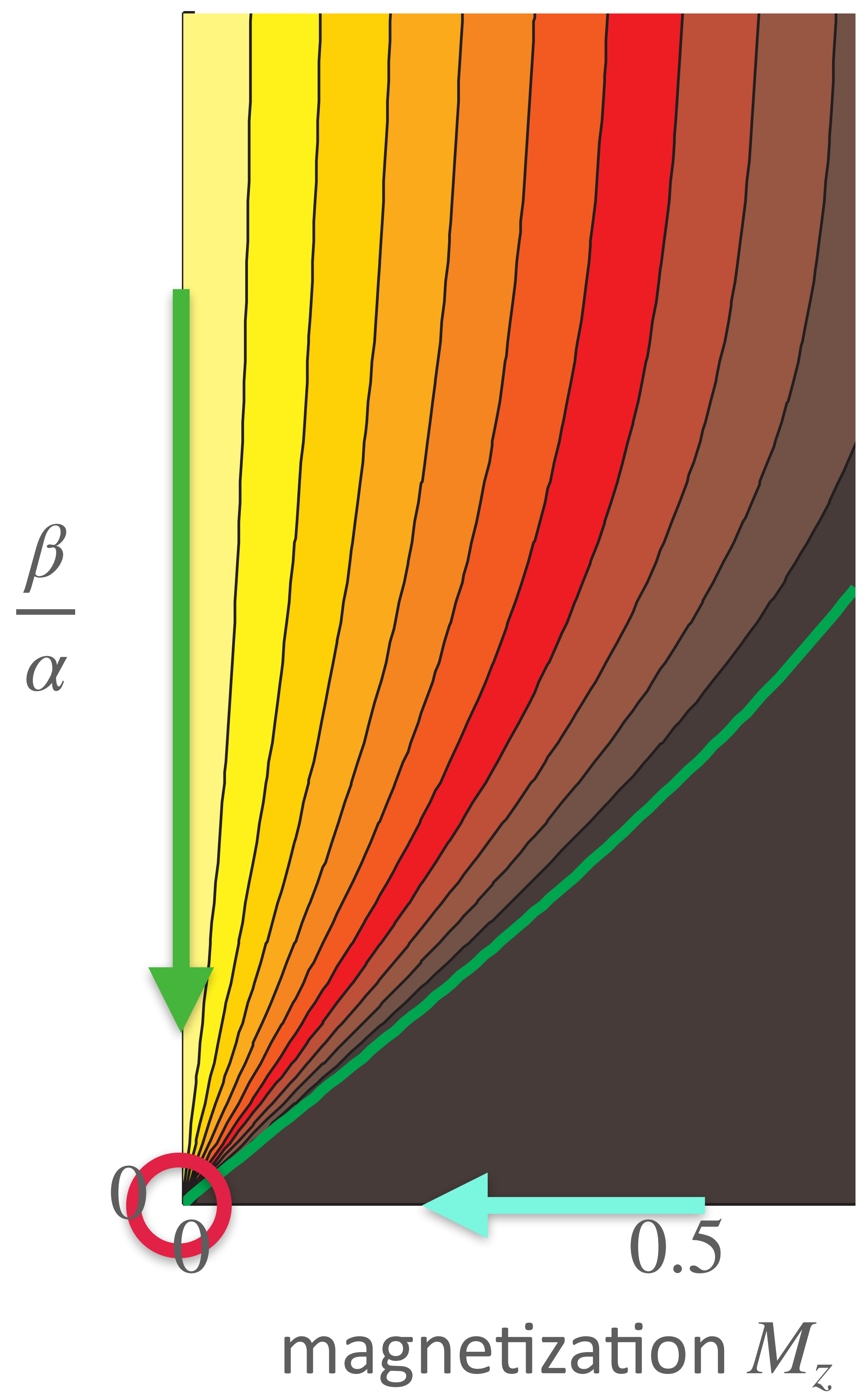
Previous measurements: NIST for $M_z > 0.5$, Georgia Tech (but not SMA)

A singular point in the mean-field analysis: $M_z = \beta = 0$

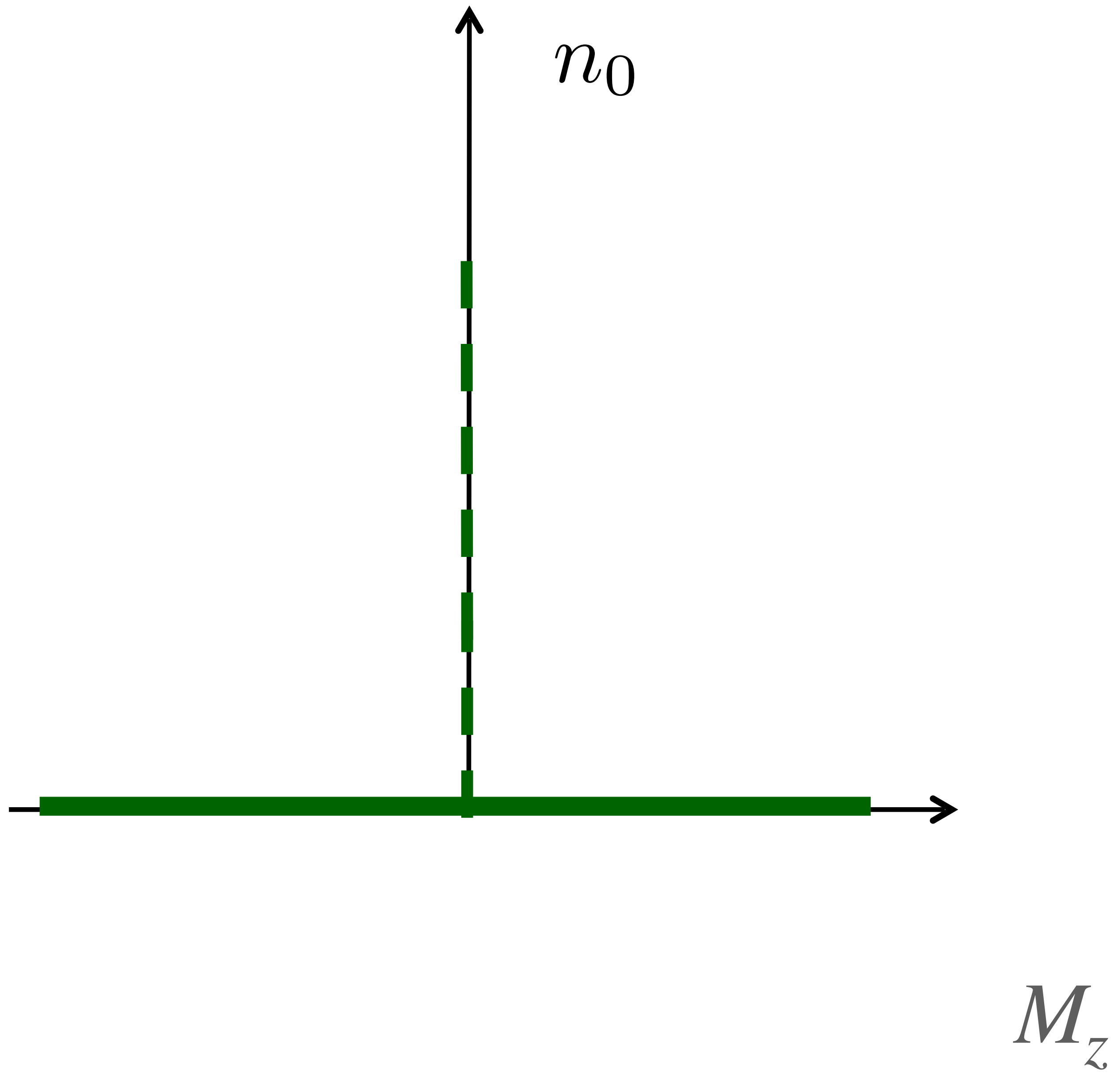
Population n_0 of the $m = 0$ state

Ground state along the line $M_z = 0$





Ground state along the line $\beta = 0$



Important role of Quantum Fluctuations around this point!

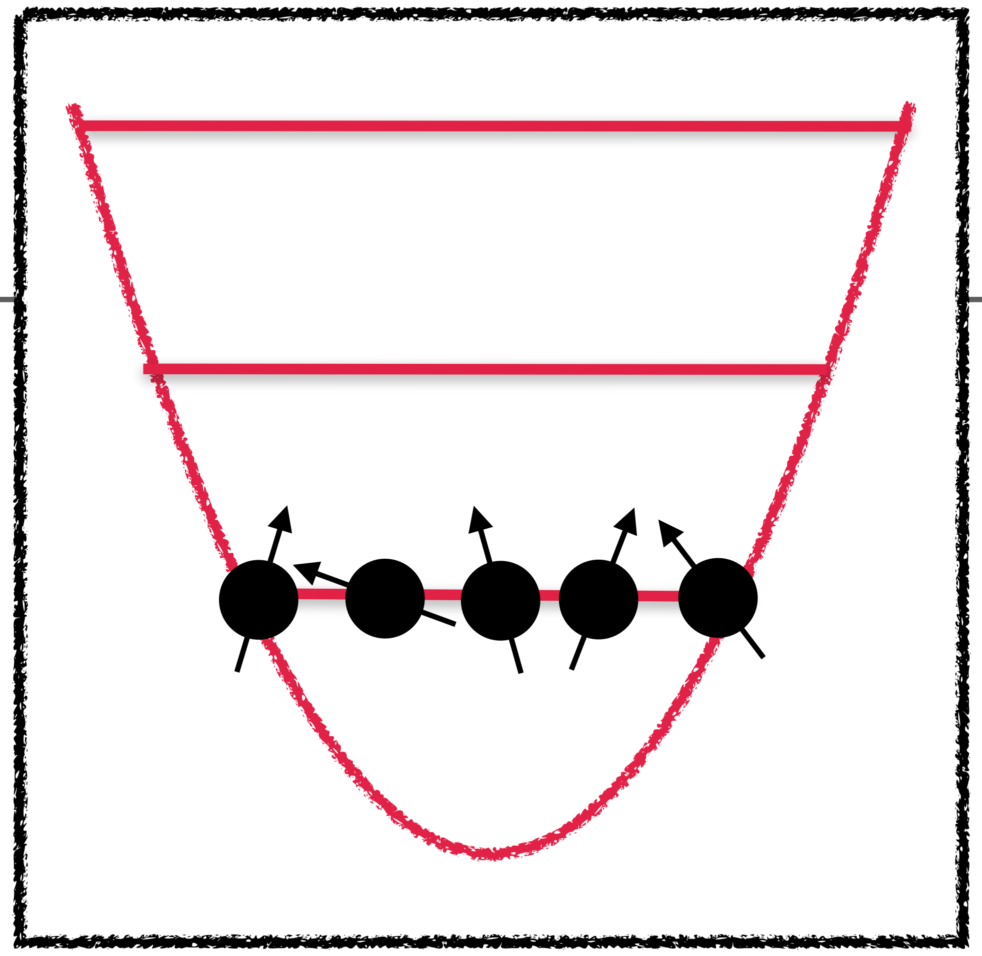
An assembly of spin 1 atoms:
The simplest many-body system

Jean Dalibard

Solvay chair for Physics 2022

Lecture 4, part 2

Outline of this lecture



$$|m=0\rangle + |m=0\rangle \quad \longleftrightarrow \quad |m=+1\rangle + |m=-1\rangle$$

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4. Bogoliubov approach and reversible many-body dynamics

   Bertrand Evrard, An Qu, Jean Dalibard and Fabrice Gerbier,
   Phys. Rev. Lett. **126**, 063401 (2021)

5. Beyond Bogoliubov: Relaxation dynamics

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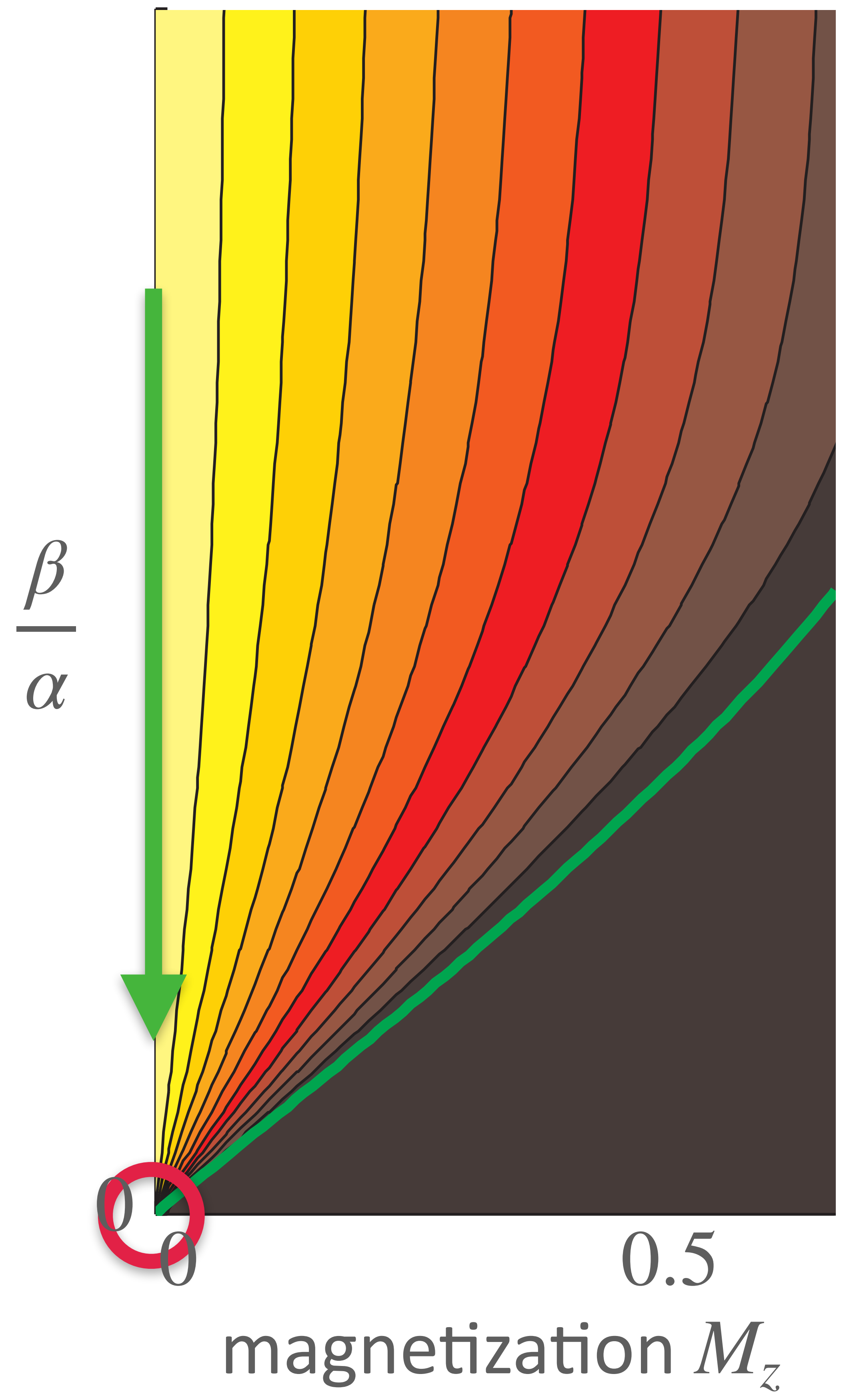
The various regimes to be studied

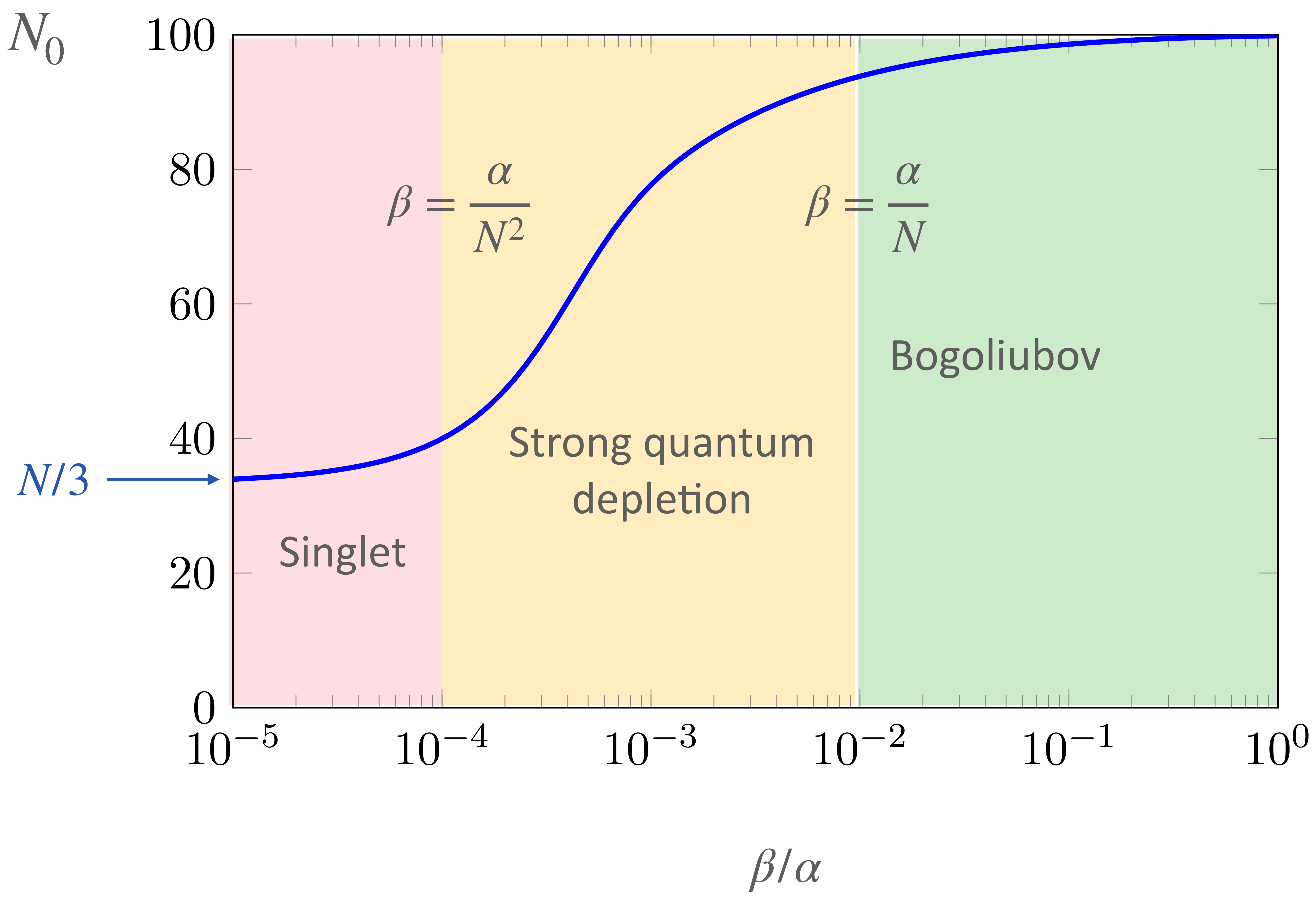
$$\hat{H} = \frac{\alpha}{2N}\hat{S}^2 + \beta\left(\hat{N}_{+1} + \hat{N}_{-1}\right)$$

Mean-field analysis

Population n_0 of the $m = 0$ state



Ground state obtained by exact diagonalization for $N = 100$ atoms and $S_z = 0$

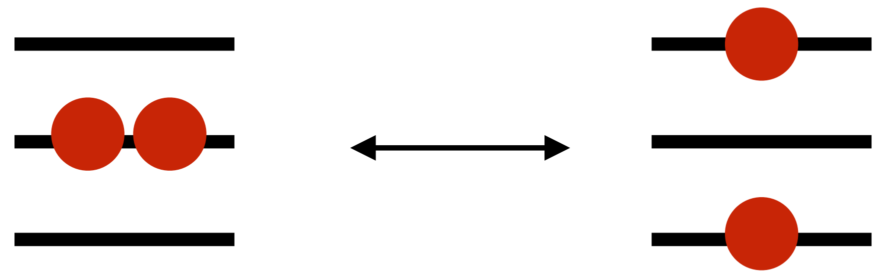


Bogoliubov approach (1)

G. I. Mias, N. R. Cooper, and S. M. Girvin (2008)
Y. Kawaguchi and M. Ueda (2012)

$\hat{H} = \dfrac{\alpha}{2N}\hat{S}^2 + \beta\left(\hat{N}_{+1} + \hat{N}_{-1}\right)$: Let's write the Hamiltonian in the "number" basis

→ Interaction term: $\dfrac{\alpha}{2N}\hat{S}^2 = \dfrac{\alpha}{N}\left[\hat{a}^\dagger_{+1}\hat{a}^\dagger_{-1}\hat{a}_0^2 + \left(\hat{a}_0^\dagger\right)^2\hat{a}_{+1}\hat{a}_{-1}\right] + \ldots$



where $\ldots = \dfrac{\alpha}{2N}\left[\hat{S}_z^2 + 2N + (2\hat{N}_0 - 1)\left(\hat{N}_{+1} + \hat{N}_{-1}\right)\right]$ is already diagonal

→ Zeeman term: $\beta\left(\hat{N}_{+1} + \hat{N}_{-1}\right) = \beta\left(\hat{a}^\dagger_{+1}\hat{a}_{+1} + \hat{a}^\dagger_{-1}\hat{a}_{-1}\right)$

Take advantage of $N_0 \gg N_{\pm 1}$ by setting $\hat{a}_0 \approx \hat{a}_0^\dagger \approx \sqrt{N}$

$$\hat{H} \approx \alpha\left(\hat{a}^\dagger_{+1}\hat{a}^\dagger_{-1} + \hat{a}_{+1}\hat{a}_{-1}\right) + (\beta + \alpha)\left(\hat{a}^\dagger_{+1}\hat{a}_{+1} + \hat{a}^\dagger_{-1}\hat{a}_{-1}\right) + \text{constant}$$

Quadratic Hamiltonian: Elementary brick at the basis of the Bogoliubov method

Bogoliubov approach (2)

G. I. Mias, N. R. Cooper, and S. M. Girvin (2008)
Y. Kawaguchi and M. Ueda (2012)

$$\hat{H} = \frac{\alpha}{2N}\hat{S}^2 + \beta\left(\hat{N}_{+1} + \hat{N}_{-1}\right) \longrightarrow \hat{H} \approx \alpha\left(\hat{a}^\dagger_{+1}\hat{a}^\dagger_{-1} + \hat{a}_{+1}\hat{a}_{-1}\right) + (\beta+\alpha)\left(\hat{a}^\dagger_{+1}\hat{a}_{+1} + \hat{a}^\dagger_{-1}\hat{a}_{-1}\right) + \text{constant}$$

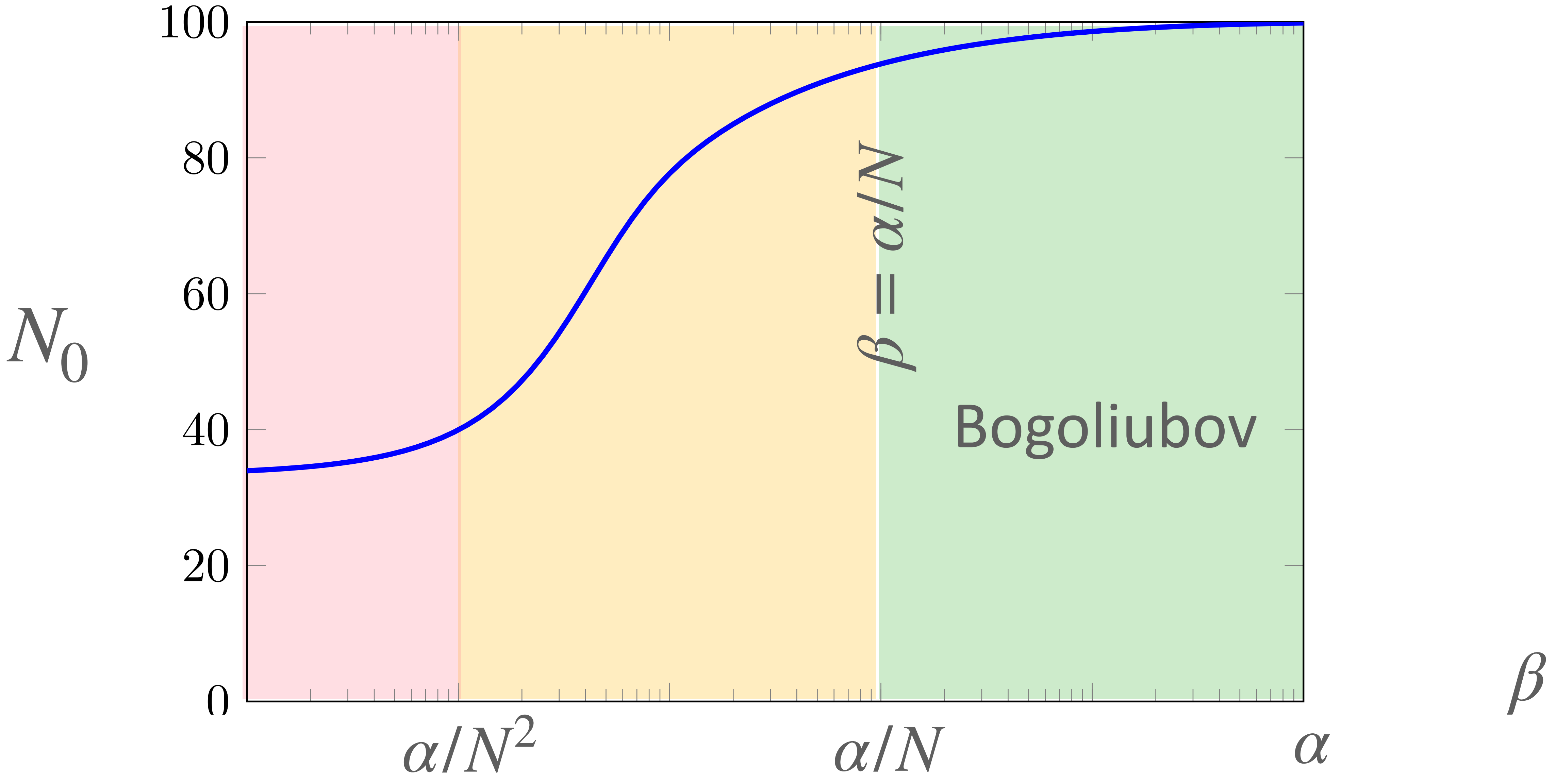
Canonical transformation: $\hat{b}_{+1} = u\hat{a}_{+1} + v\hat{a}^\dagger_{-1}$, $\hat{b}_{-1} = u\hat{a}_{-1} + v\hat{a}^\dagger_{+1}$, leading to :

$$\hat{H} = \hbar\omega\left(\hat{b}^\dagger_{+1}\hat{b}_{+1} + \hat{b}^\dagger_{-1}\hat{b}_{-1}\right) \qquad \hbar\omega = \sqrt{\beta(\beta+2\alpha)}$$

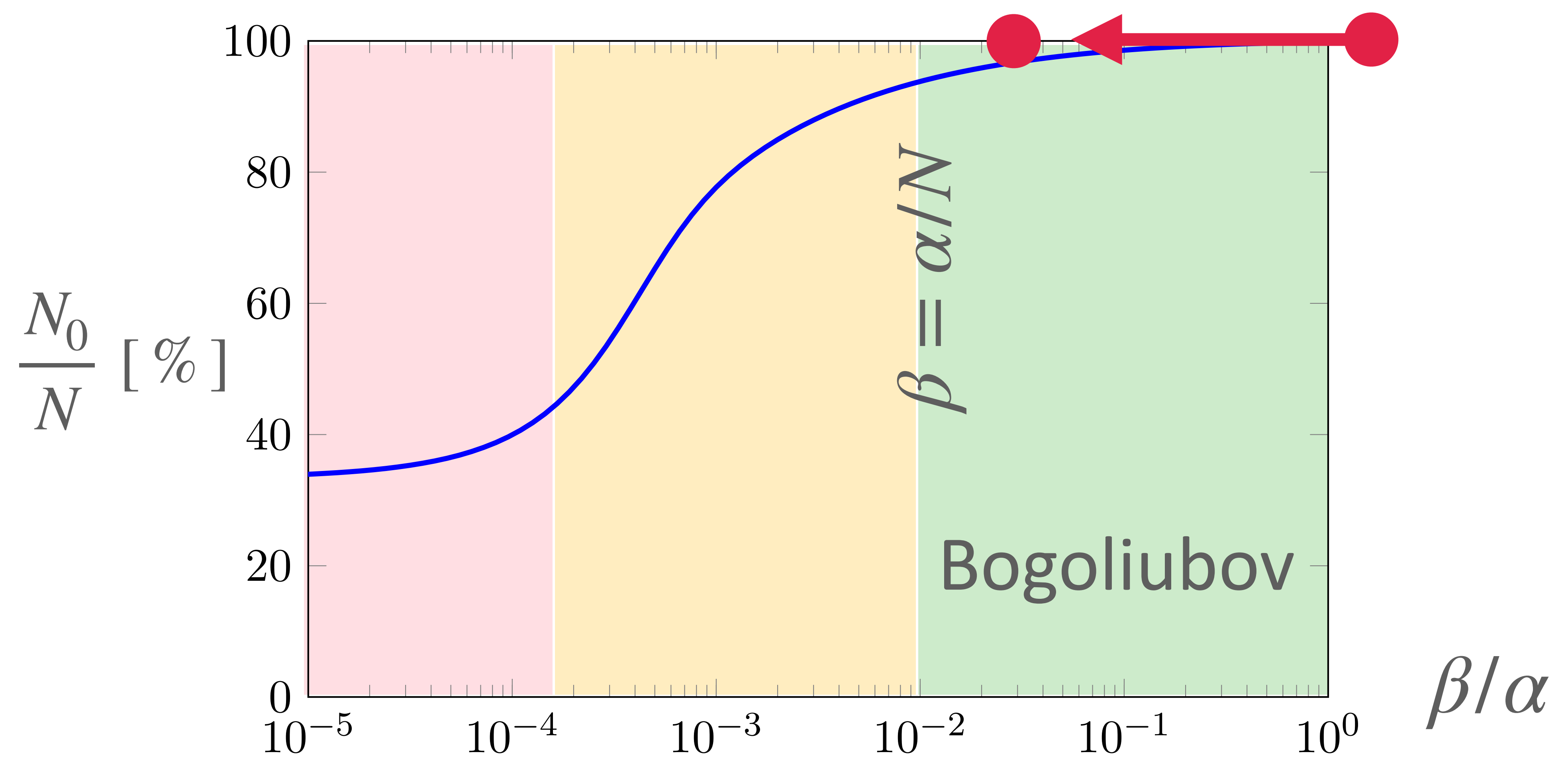
Linear energy spectrum

Approach valid as long as $N_{\pm 1} \ll N \approx N_0$, which requires $\beta \gg \alpha/N$

Calculation for $N = 100$



Experimental test starting from all atoms in $m = 0$ ($N_0 = N$)

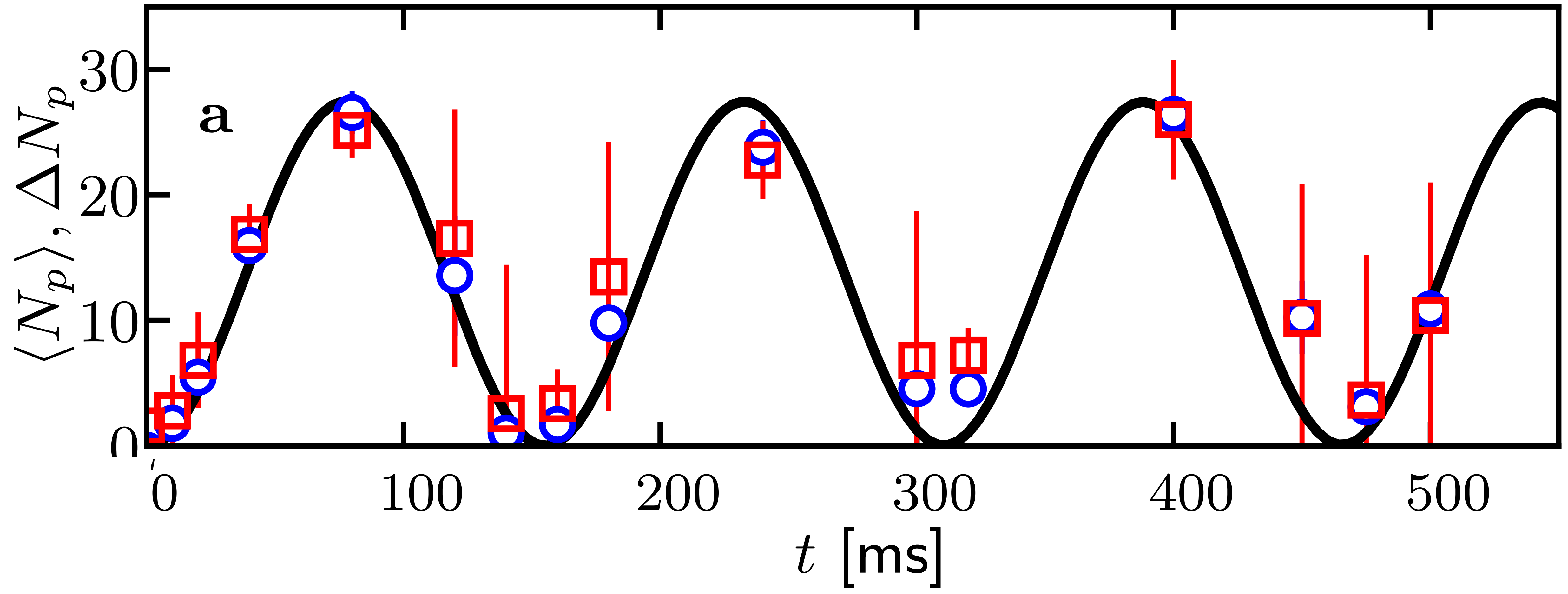


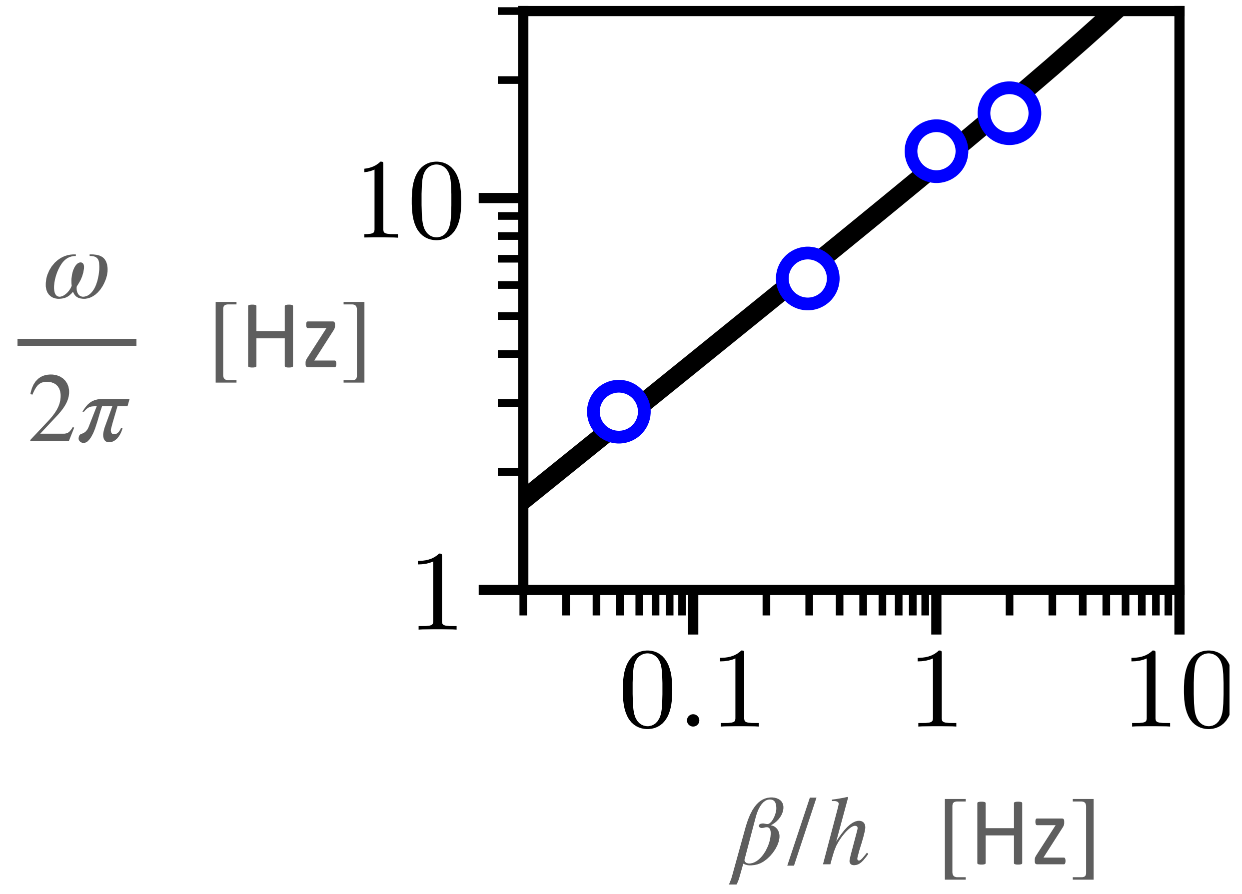
Rapid quench of the magnetic field (β) from a large to a low value

Expected: Oscillating dynamics of the number of pairs $m = \pm 1$

$$\bar{N}_{\text{pair}} = \left(\frac{\alpha}{\hbar\omega}\right)^2 \sin^2(\omega t) \qquad \hbar\omega = \sqrt{\beta(\beta + 2\alpha)}$$

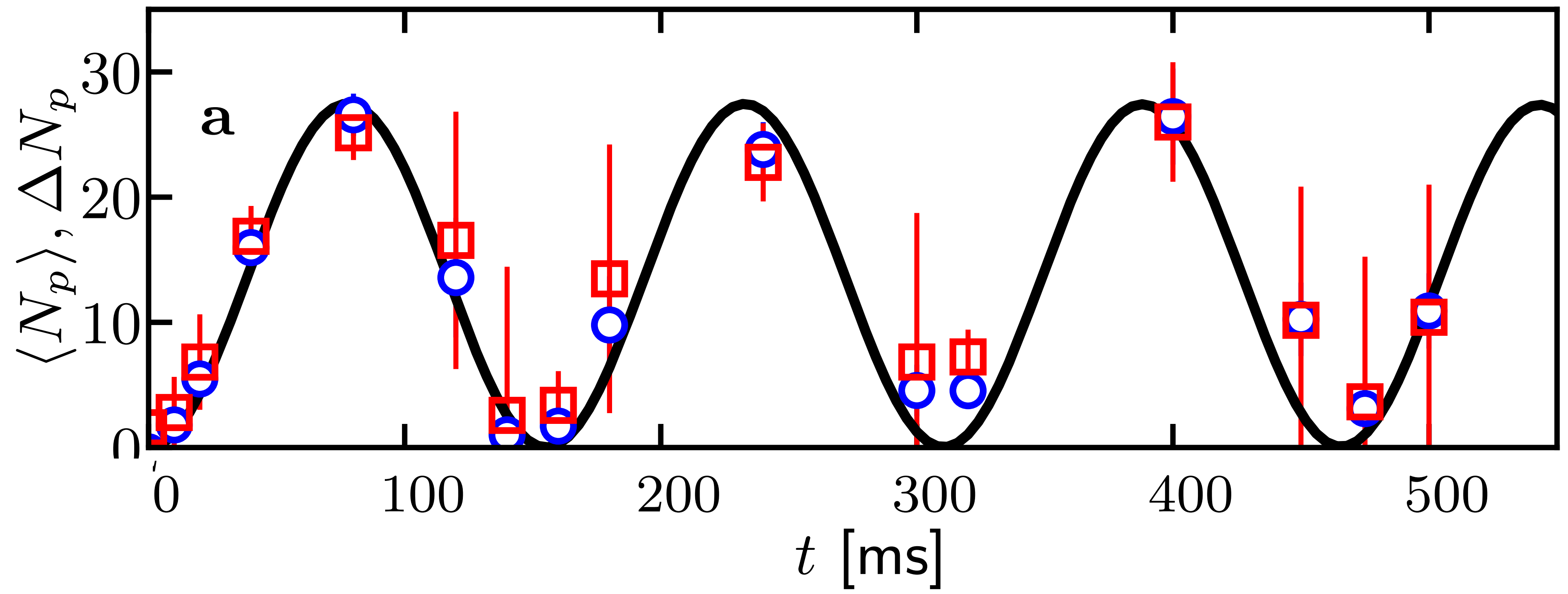
$N = 5000, \quad \alpha/h = 17\,\text{Hz}, \quad \beta/h = 0.31\,\text{Hz}$





The pair distribution function

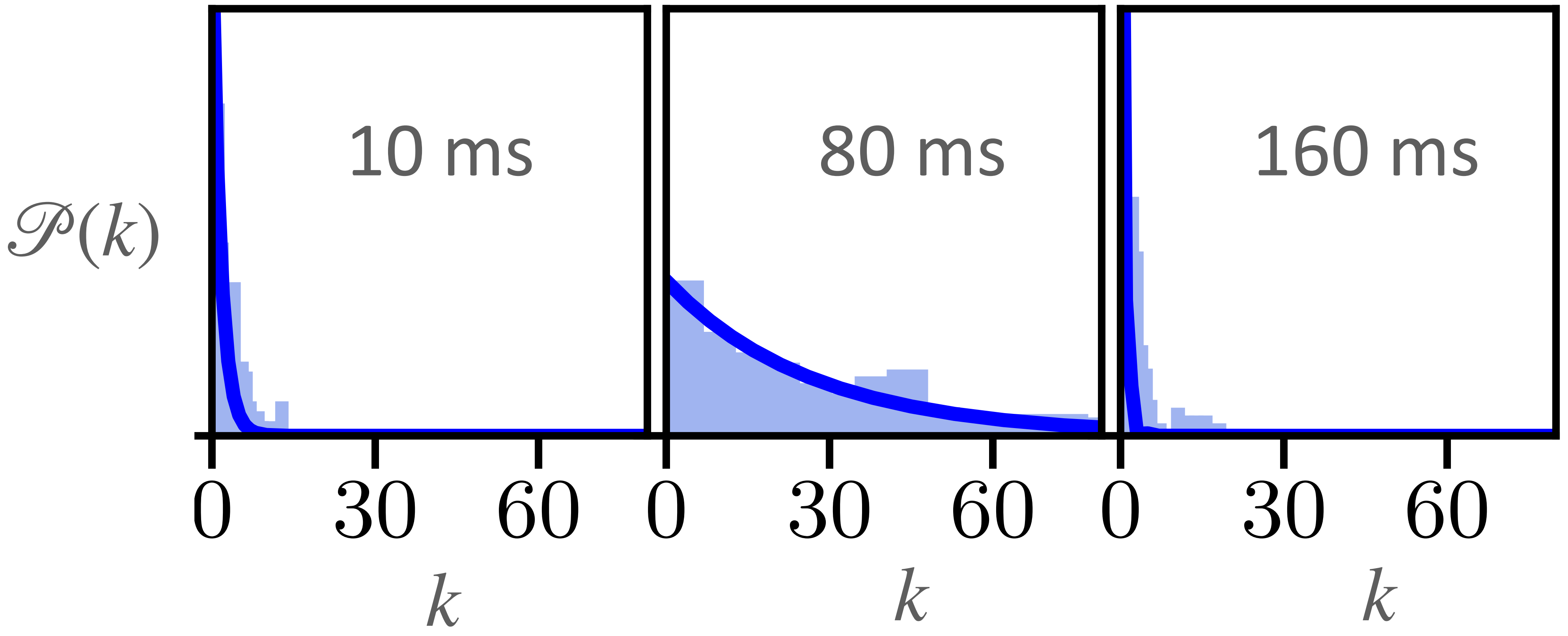
$$\bar{N}_{\rm pair} = \left(\frac{\alpha}{\hbar\omega}\right)^2 \sin^2(\omega t)$$



At any time, one expect a two-mode squeezed vacuum state $\propto \sum_{k=0}^{N/2} \left[\xi(t)\right]^k |k, N-2k, k\rangle$

where the pair distribution function corresponds to: $\mathscr{P}(k) = \frac{1}{\bar{N}_{\rm pair}} \exp\left(-\frac{k}{\bar{N}_{\rm pair}}\right)$

$$\bar{N}_{\rm pair}(t) = \frac{|\xi(t)|^2}{1-|\xi(t)|^2}$$



Outline of this lecture

1. An assembly of spin 1 particles

2. Experimental implementation

3. A mean-field analysis

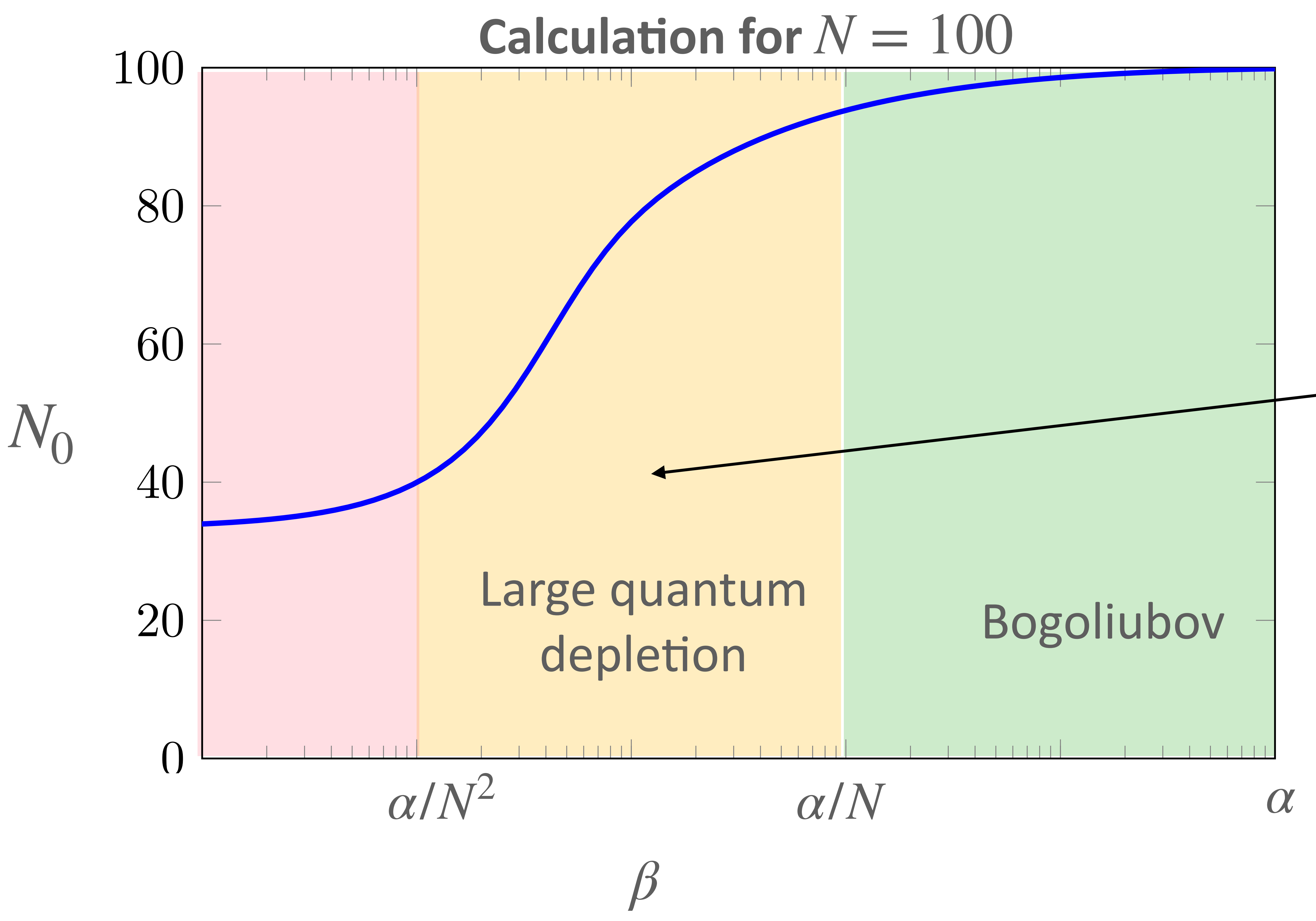
4. Bogoliubov approach and reversible many-body dynamics

5. Beyond Bogoliubov: Relaxation dynamics

Bertrand Evrard, An Qu, Jean Dalibard and Fabrice Gerbier,
Phys. Rev. Lett. **126**, 063401 (2021), Phys. Rev. A **103**, L031302 (2021)

6. A fragmented condensate

Beyond the Bogoliubov regime



$$\hat{H} = \frac{\alpha}{2N}\hat{\mathbf{S}}^2 + \beta\left(\hat{N}_{+1} + \hat{N}_{-1}\right)$$

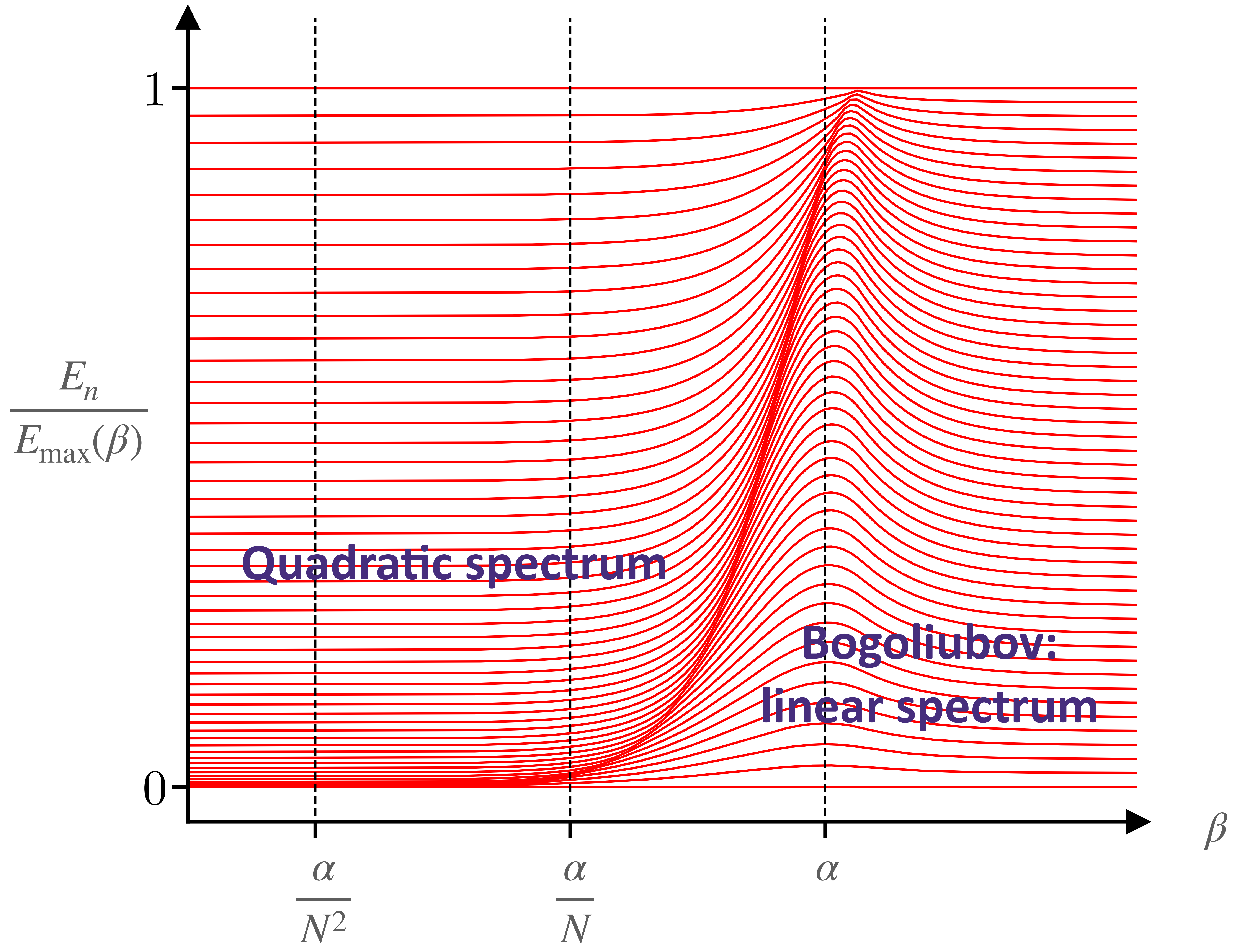
Region of interest here: $\beta \ll \alpha/N$

For states such that $\sqrt{\langle \hat{S}^2 \rangle} \sim \sqrt{N}$ and $\langle \hat{N}_m \rangle \sim N$,

the term $\dfrac{\alpha}{2N}\hat{S}^2$ dominates in the Hamiltonian

$\sim$ quadratic energy spectrum: $E_S = \dfrac{\alpha}{2N} S(S+1)$

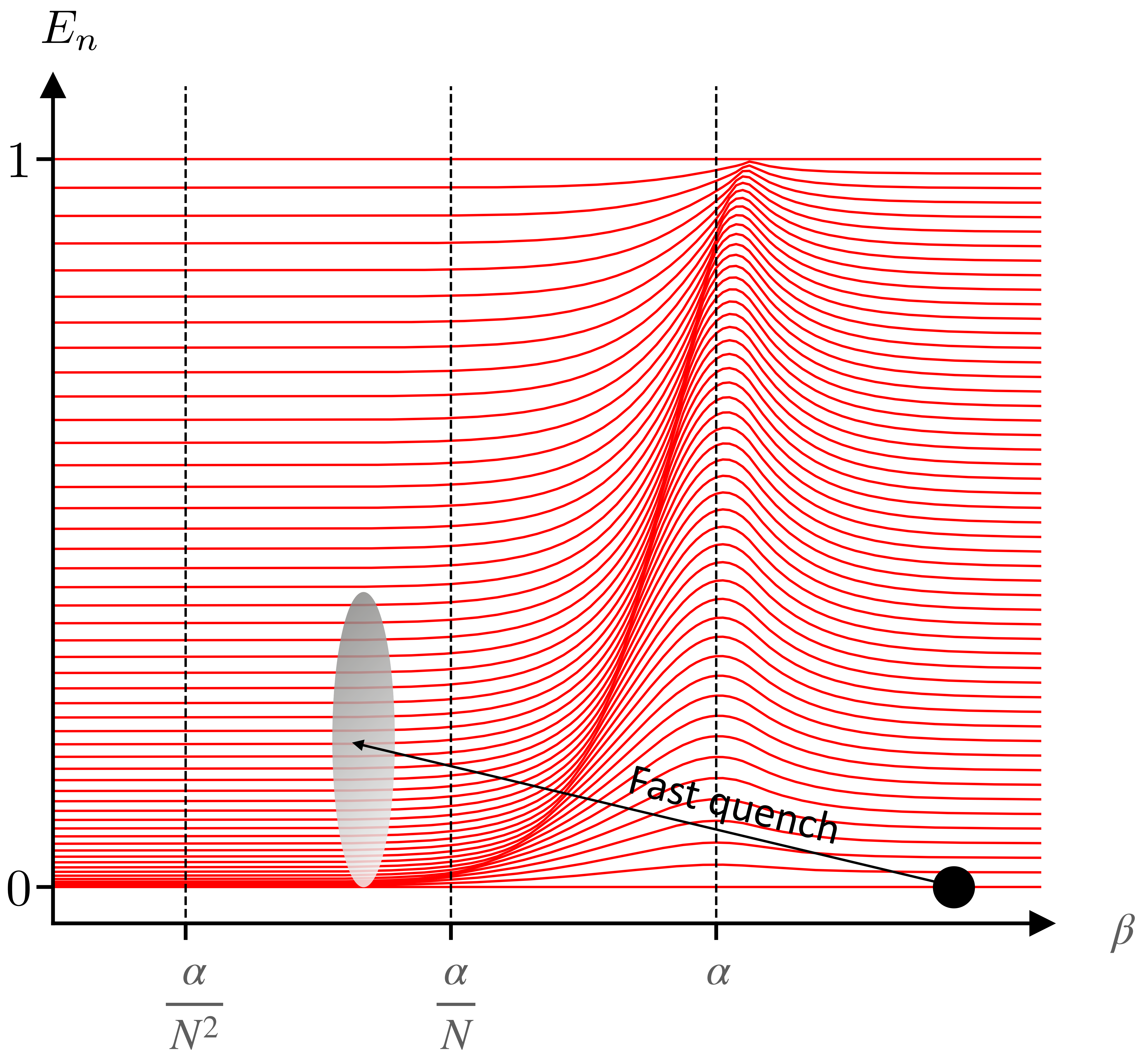
Evolution of the energy spectrum



Calculation for $N = 100$ and $S_z = 0$

$$\hat{H} = \frac{\alpha}{2N}\hat{\mathbf{S}}^2 + \beta\left(\hat{N}_{+1} + \hat{N}_{-1}\right)$$

Relaxation dynamics (1)



Initial state: $|\Psi_i\rangle = |0,N,0\rangle \equiv |m=0\rangle^{\otimes N}$

Decomposition on the spin basis:

$$|\Psi_i\rangle = \sum_S c_S |S, S_z = 0\rangle$$

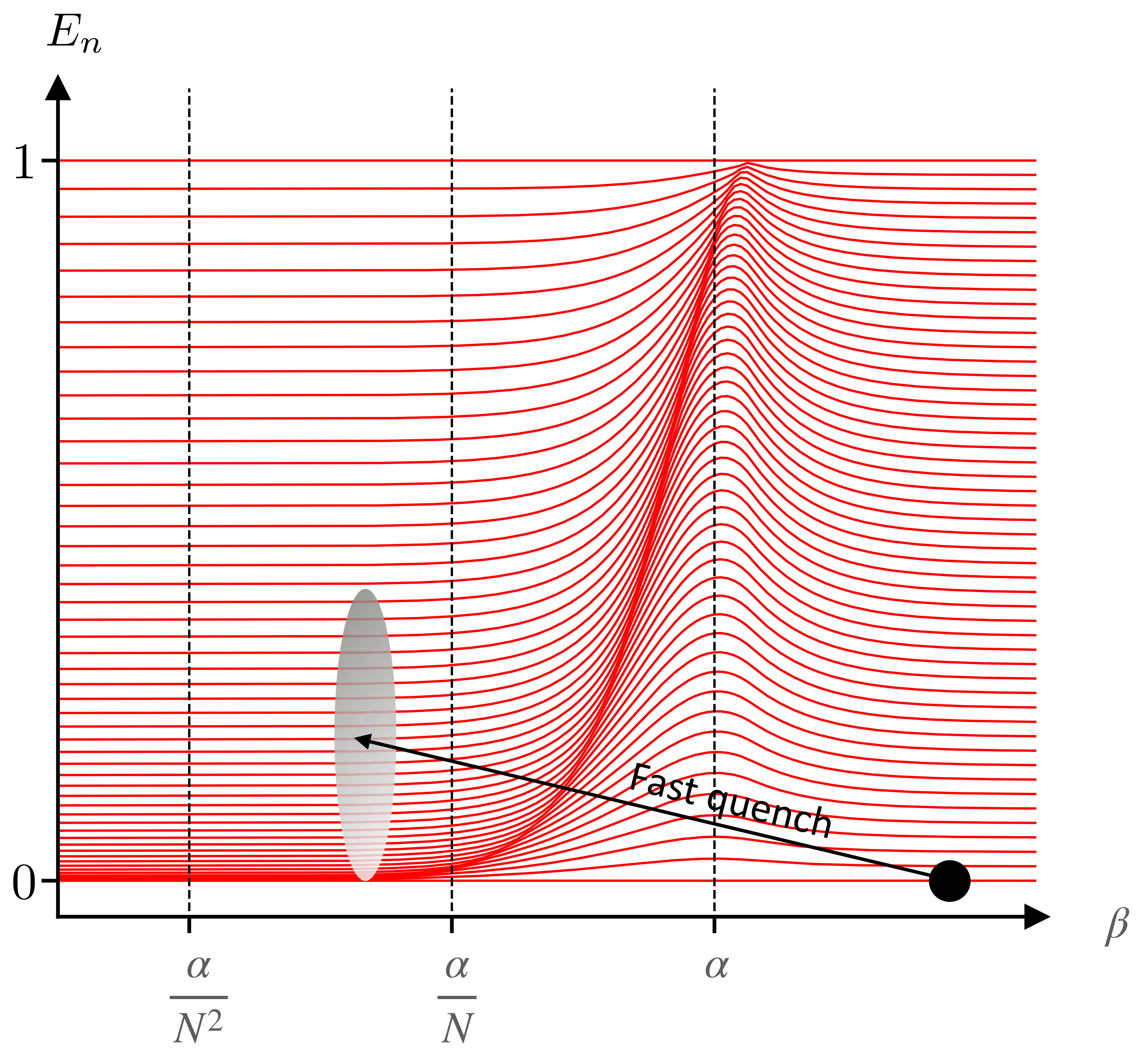
$$c_S \approx \sqrt{\frac{2S}{N}} \exp(-S^2/4N)$$

Subsequent evolution:

$$|\Psi(t)\rangle = \sum_S \mathrm{e}^{-\mathrm{i}\alpha S(S+1)t/2N\hbar} c_S |S, S_z = 0\rangle$$

What is the evolution of N_0 ?

Relaxation dynamics (2)



$$|\Psi_i\rangle = \sum_S c_S |S, S_z = 0\rangle \qquad c_S \approx \sqrt{\frac{2S}{N}}\ \exp(-S^2/4N)$$

$$|\Psi(t)\rangle = \sum_S \mathrm{e}^{-\mathrm{i}\alpha S(S+1)t/2N\hbar}\ c_S\ |S, S_z = 0\rangle$$

What is the evolution of $\bar{N}_0(t) = \langle \Psi(t) | \hat{N}_0 | \Psi(t) \rangle$?

A useful relation: $\langle S, 0 | \hat{N}_0 | S', 0 \rangle \approx \frac{N}{2} \delta_{S,S'} + \frac{N}{4} \left(\delta_{S,S'-2} + \delta_{S,S'+2} \right)$

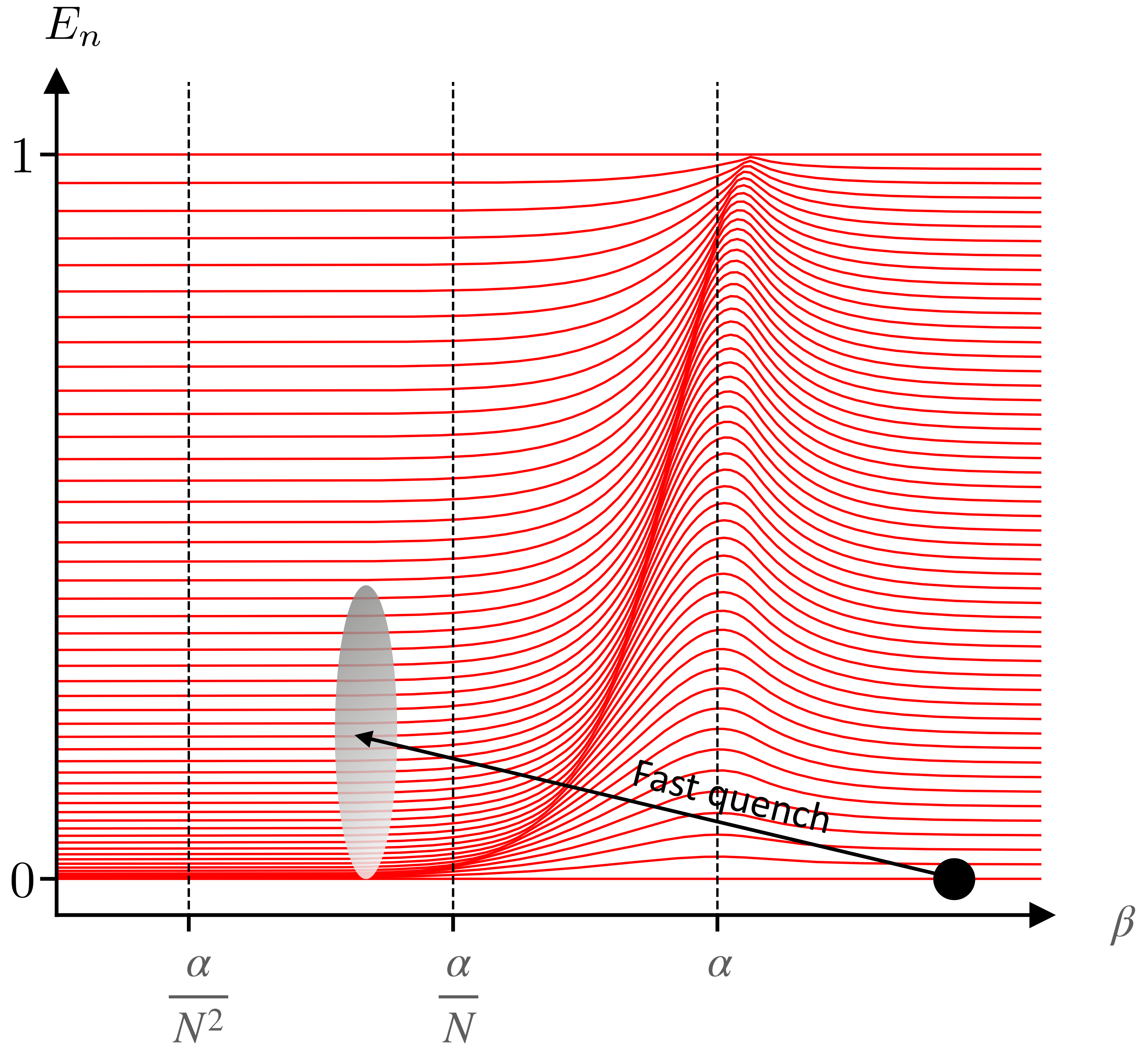
"Universal" dynamics due to the non-linearity of the spectrum: $\bar{N}_0(t) = N\left[1 - \tau D(\tau)\right]$ $\qquad \tau = \sqrt{\frac{2}{N}}\ \frac{\alpha t}{\hbar}$

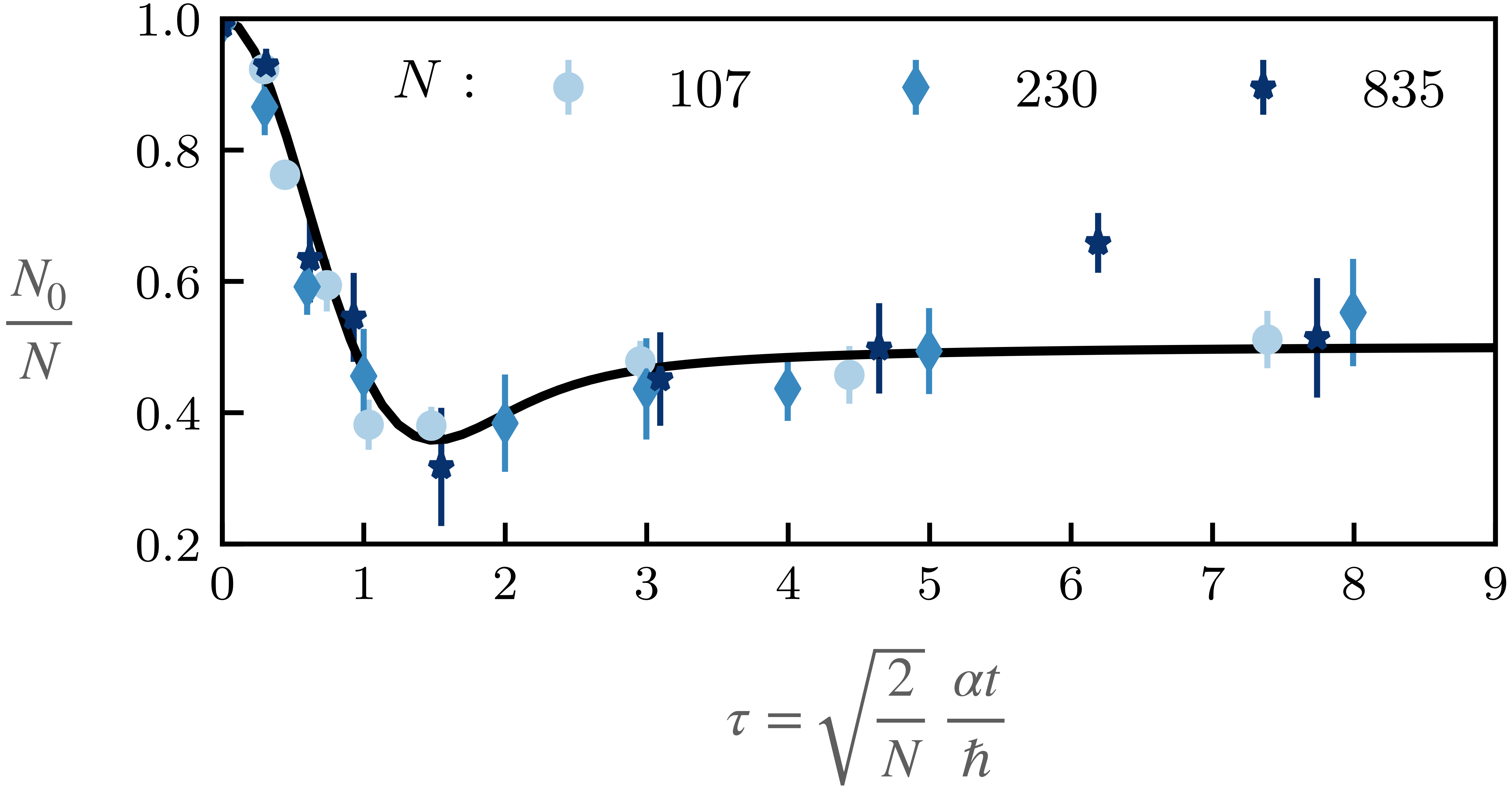
$$D(\tau) = \int_0^{+\infty} \sin(2x\tau)\, \mathrm{e}^{-x^2}\, \mathrm{d}x$$

Dawson function

Relaxation dynamics (3)

$D(\tau) = \int_0^{+\infty} \sin(2x\tau)\, e^{-x^2}\, dx$





Continuous line: $1 - \tau D(\tau)$ with $D(\tau)$: Dawson function

Alternative approach based on a quantum trajectory approach (dissipation into a fictitious environment)
L. Fernandes, M. Wouters, J. Tempere, Phys. Rev. A **105**, 013305 (2022)

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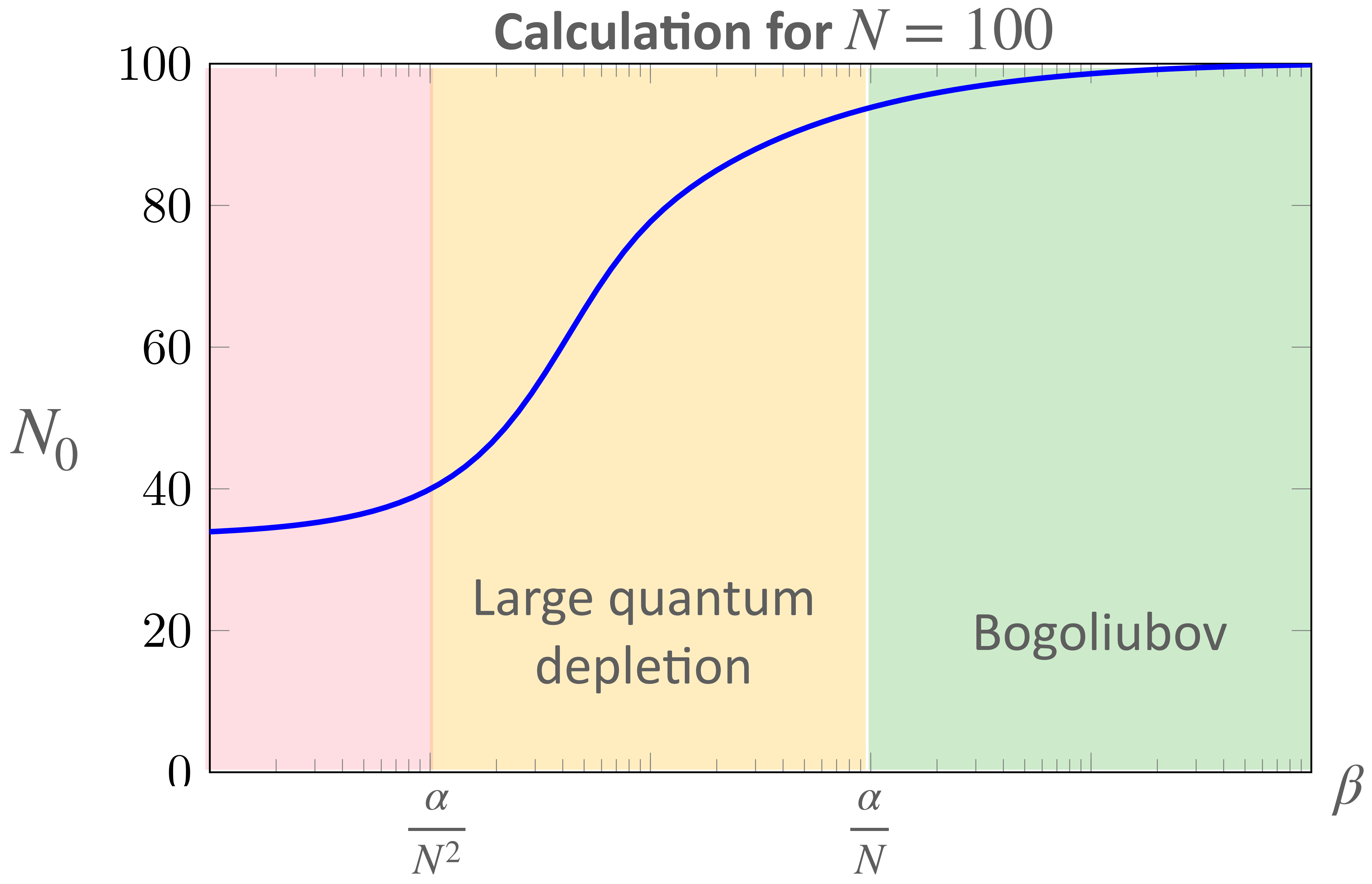
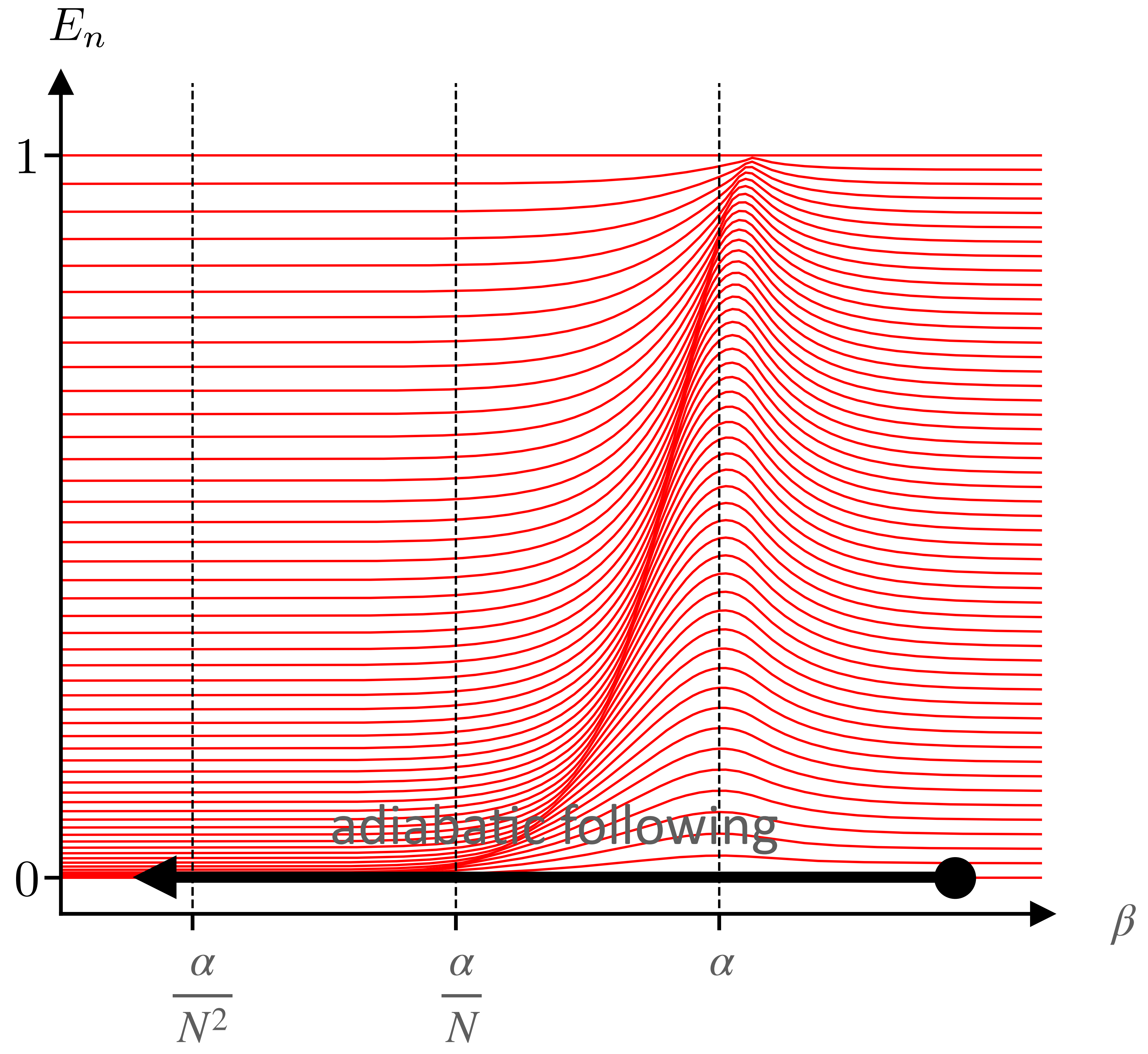
Bertrand Evrard, An Qu, Jean Dalibard, Fabrice Gerbier,
Science **373**, 1340 (2021)

The zero-field limit

$$\hat{H} = \frac{\alpha}{2N}\hat{\mathbf{S}}^2 + \beta\left(\hat{N}_{+1} + \hat{N}_{-1}\right)$$





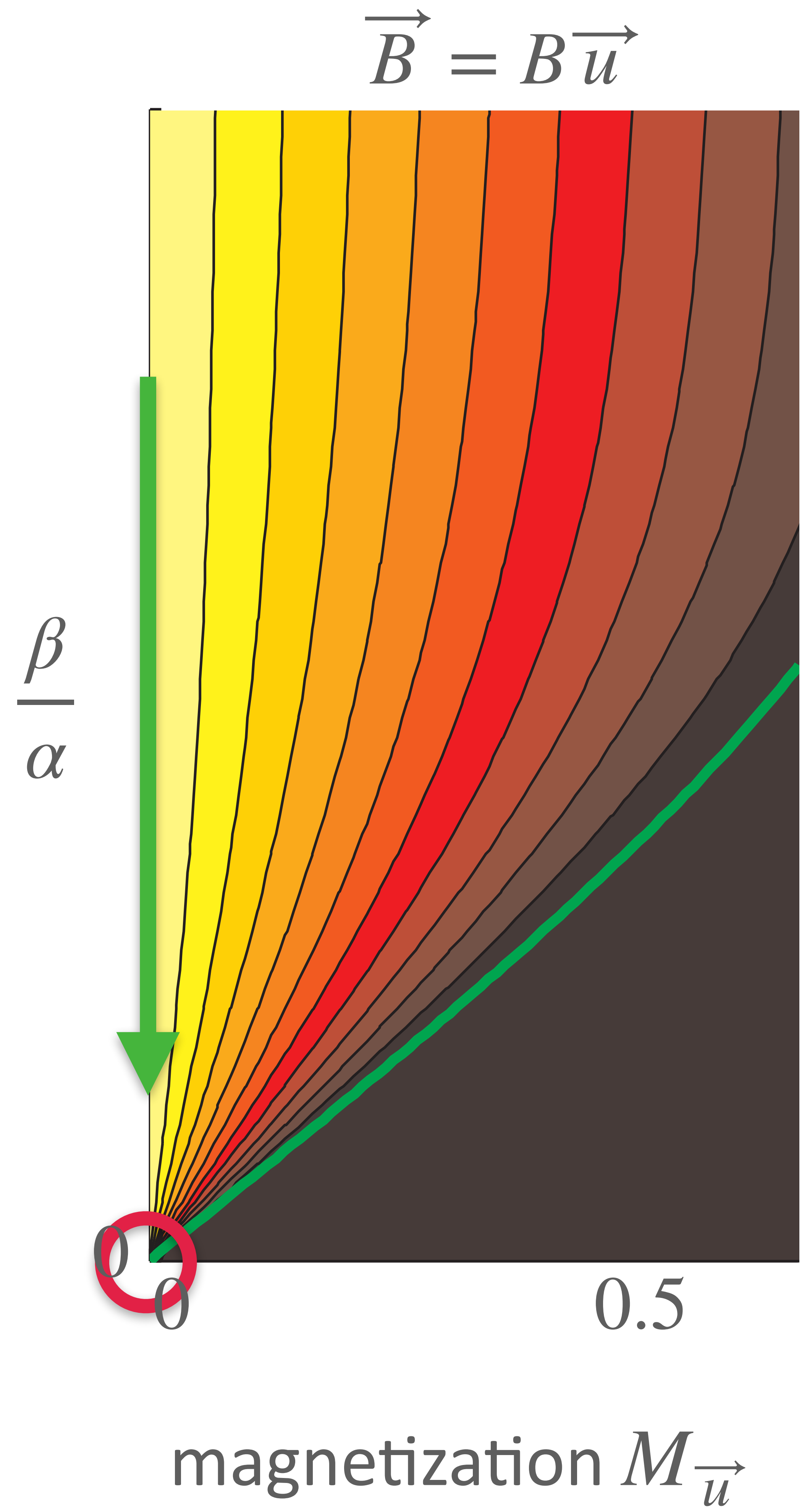
For $\beta \lesssim \alpha/N^2$, the Zeeman energy $\beta\bar{N}_0 \lesssim \beta N$ is always small compared to the interaction energy, $\alpha\hat{S}^2/N$, even for the low part of the spectrum $E(S=2) - E(S=0) = \frac{3\alpha}{N}$

What is the ground state of the system at zero magnetic field?

The ground-state in a mean-field point of view

$$\hat{H} = \frac{\alpha}{2N}\hat{S}^2 + \beta\left(\hat{N}_{+1} + \hat{N}_{-1}\right)$$

Population n_0 of the $m = 0$ state



For a given orientation of the magnetic field axis $\overrightarrow{u}$ and a zero magnetization along this axis, the limit $B \to 0$ gives

$$|\mathrm{GS} : \overrightarrow{u}\rangle = \left(|m = 0\rangle_{\overrightarrow{u}}\right)^{\otimes N}$$

spin-nematic (or polar)

In the mean-field point-of-view, the ground state of the system is the statistical mixture:

$$\rho_{\mathrm{GS}} \propto \int \left(|m = 0\rangle_{\overrightarrow{u}}\right)^{\otimes N} \left(\langle m = 0|_{\overrightarrow{u}}\right)^{\otimes N} \, \mathrm{d}^2 u$$

Spontaneous breaking of the rotational symmetry: for each realization of the experiment, an orientation of $\overrightarrow{u}$ is randomly chosen

A random magnetic field creating $\beta \sim \dfrac{\alpha}{N^2}$ *is sufficient to break the symmetry*

Infinitesimally small field in the thermodynamic limit

Fluctuations in the mean-field point of view

$$\hat{H} = \frac{\alpha}{2N}\hat{S}^2 + \beta\cancel{\left(\hat{N}_{+1} + \hat{N}_{-1}\right)}$$

Mean-field ground state

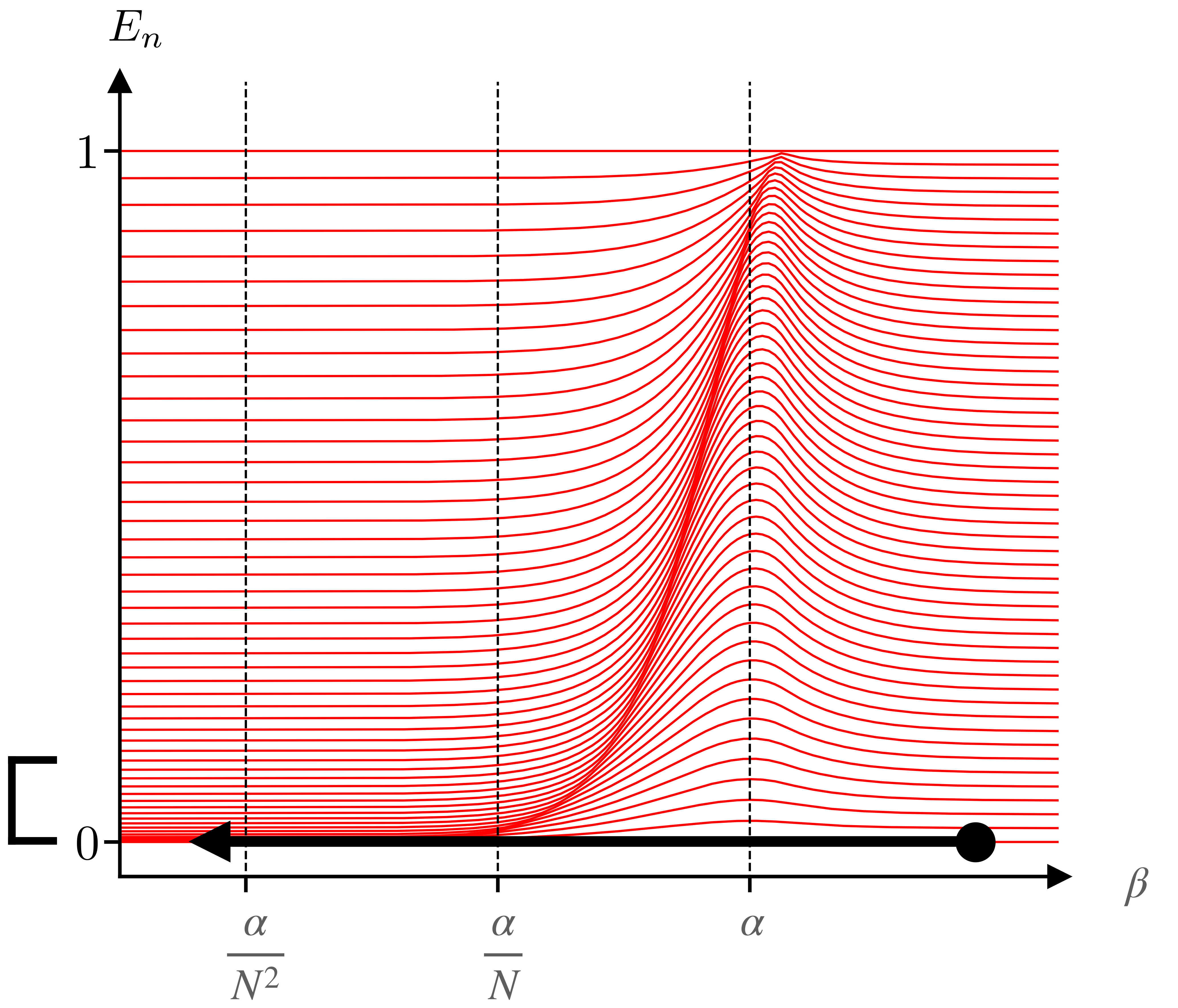
$$\rho_{\mathrm{GS}} \propto \int \left(|m=0\rangle_{\vec{u}}\right)^{\otimes N}\left(\langle m=0|_{\vec{u}}\right)^{\otimes N}\ \mathrm{d}^2 u$$

Average spin: $\langle \hat{\vec{S}} \rangle = \mathrm{Tr}\left(\hat{\vec{S}}\hat{\rho}_{GS}\right) = 0$

Spin fluctuations: $\langle \hat{\vec{S}}^2 \rangle = \mathrm{Tr}\left(\hat{\vec{S}}^2\hat{\rho}_{GS}\right) = 2N$

$$\Delta S \propto \sqrt{N}$$

In this mean-field approach, $\sim \sqrt{N}$ spin states are populated in the expected ground state



The difference with the true ground-state vanishes in the thermodynamic limit

P. W. Anderson, Phys. Rev. **86**, 694–701 (1952)

The true ground state of the $N-$spin system in zero field

Law, Pu, Bigelow, 1998
Ho & Yip, 2000

$$\hat{H} = \frac{\alpha}{2N}\hat{\mathbf{S}}^2 + \cancel{\beta\left(\hat{N}_{+1} + \hat{N}_{-1}\right)}$$

$\alpha > 0$ (antiferromagnetic interaction)

Minimize the total spin, while staying compatible with the exchange symmetry for bosons

Assume N even: the state $S = 0$ by forming a condensate of $N/2$ pairs in the singlet state of $(s = 1) \otimes (s = 1)$:

- For two spin-1 particles a and b, singlet state: $\frac{1}{\sqrt{3}}\left(|0{,}0\rangle - |+1, -1\rangle - |-1, +1\rangle\right)$ in the basis $|m_a, m_b\rangle$

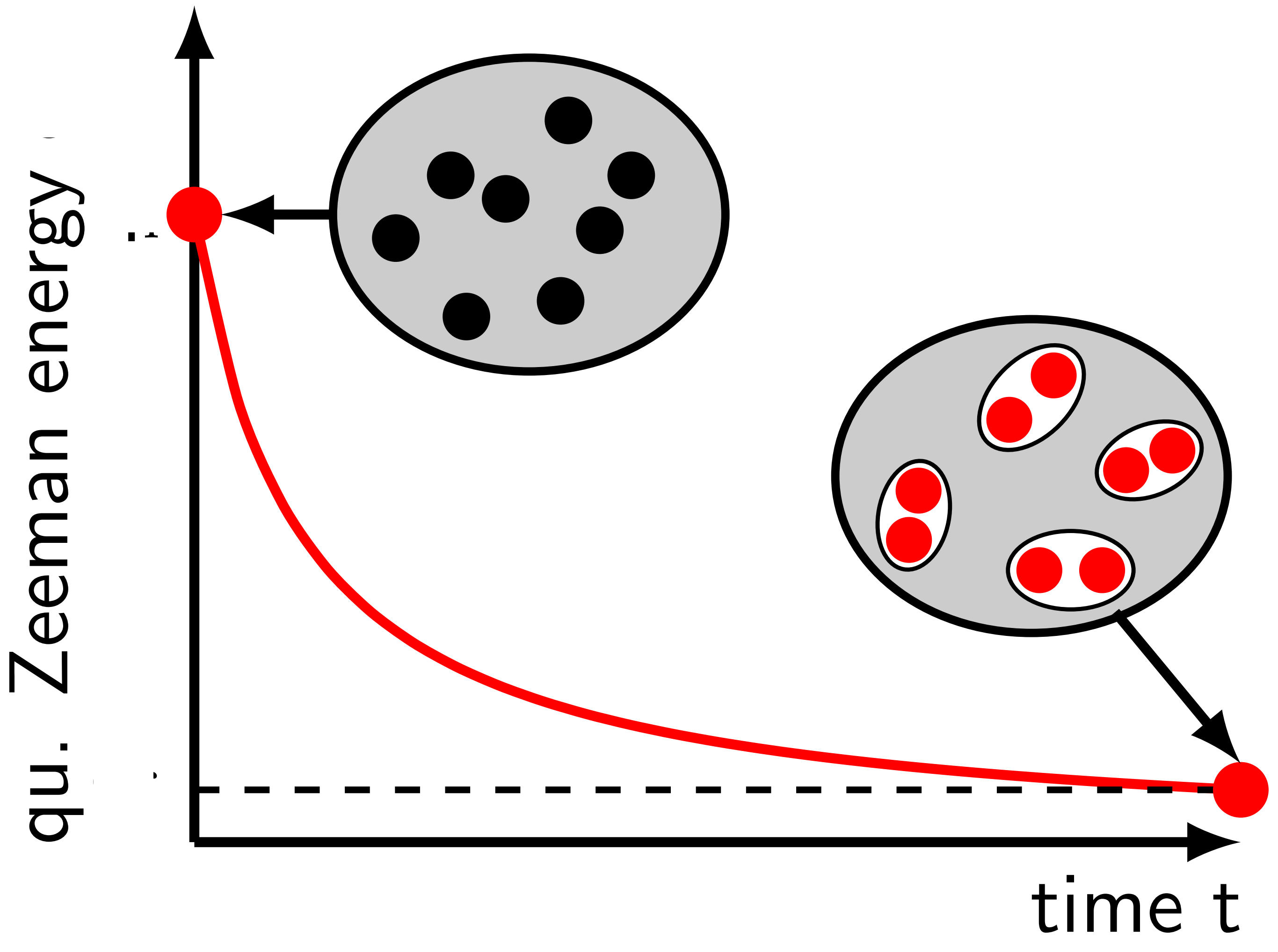
$$\frac{1}{\sqrt{6}}\left[\left(\hat{a}_0^\dagger\right)^2 - 2\hat{a}_{+1}^\dagger\hat{a}_{-1}^\dagger\right]|\text{vac}\rangle$$

- For N spin-1 particles, collective spin singlet state: $\frac{1}{\sqrt{(N+1)!}}\left[\left(\hat{a}_0^\dagger\right)^2 - 2\hat{a}_{+1}^\dagger\hat{a}_{-1}^\dagger\right]^{N/2}|\text{vac}\rangle$

Koashi & Ueda, 2000; Castin & Herzog, 2001; Ashab & Leggett, 2002;
Muellller et al, 2006; Barnett et al, 2010; De Sarlo et al, 2013

Producing the collective spin singlet in the lab

$$\hat{H} = \frac{\alpha}{2N}\hat{S}^2 + \beta\left(\hat{N}_{+1} + \hat{N}_{-1}\right)$$



How to reach it?

Start with a large ($\beta > \alpha$) field $\overrightarrow{B} = B\overrightarrow{u}_z$, with all atoms in $|m_z = 0\rangle$

Adiabatic following of the ground state down to a very low field (milliGauss) such that $\beta \sim \alpha/N^2$

Adiabatic criterion on the evolution of the gap ΔE to the first excited state: $$\frac{\mathrm{d}\Delta E}{\mathrm{d}t} \ll \frac{(\Delta E)^2}{\hbar}$$

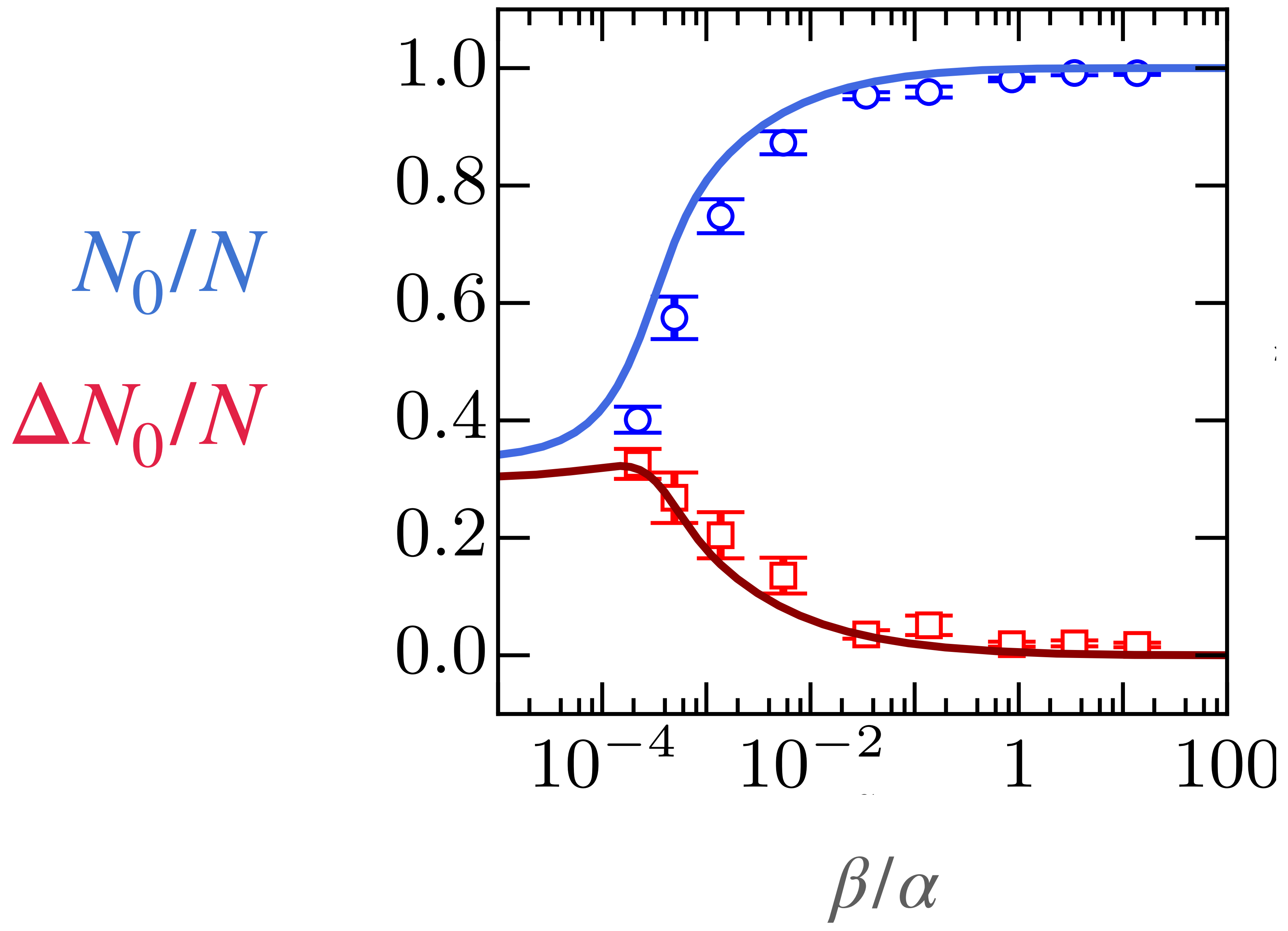
In practice, optimised ramp from 1 Gauss to 4 milliGauss in 1 second

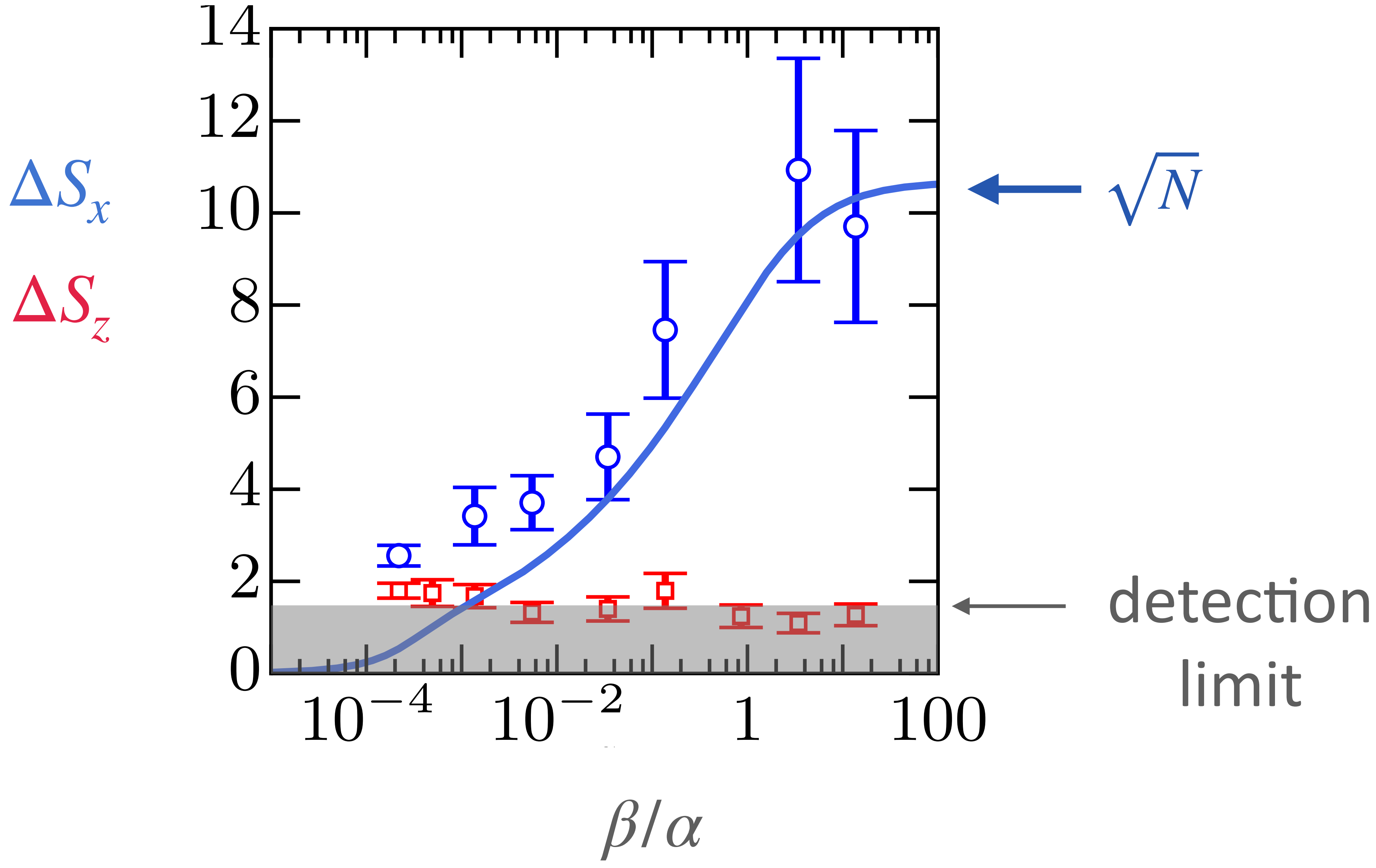
$N = 100$ atoms $\qquad \alpha = h \times 20\,\text{Hz} \qquad \beta_f = h \times 0.004\,\text{Hz}$

Diagnosis of the singlet state: one- and two-body observables

Measurements performed either in the $z-$basis or after rotation of the state with adjustable angles and axes using Larmor precession and Rabi flopping

$N \approx 100$ atoms





$\Delta N_0 \sim N_0$: super-Poissonian fluctuations

Expected for the singlet state:

$$N_0 = \frac{N}{3},\ \Delta N_0 = \frac{2N}{3\sqrt{5}} + \mathcal{O}(\sqrt{N})$$

$\Delta S_{x,y,z} \ll \sqrt{N}$: squeezed spin state

Expected for the singlet state:

$\Delta S_{x,y,z} = 0$ for N even

Diagnosis of the singlet state: many-body state

$N \approx 100$ atoms

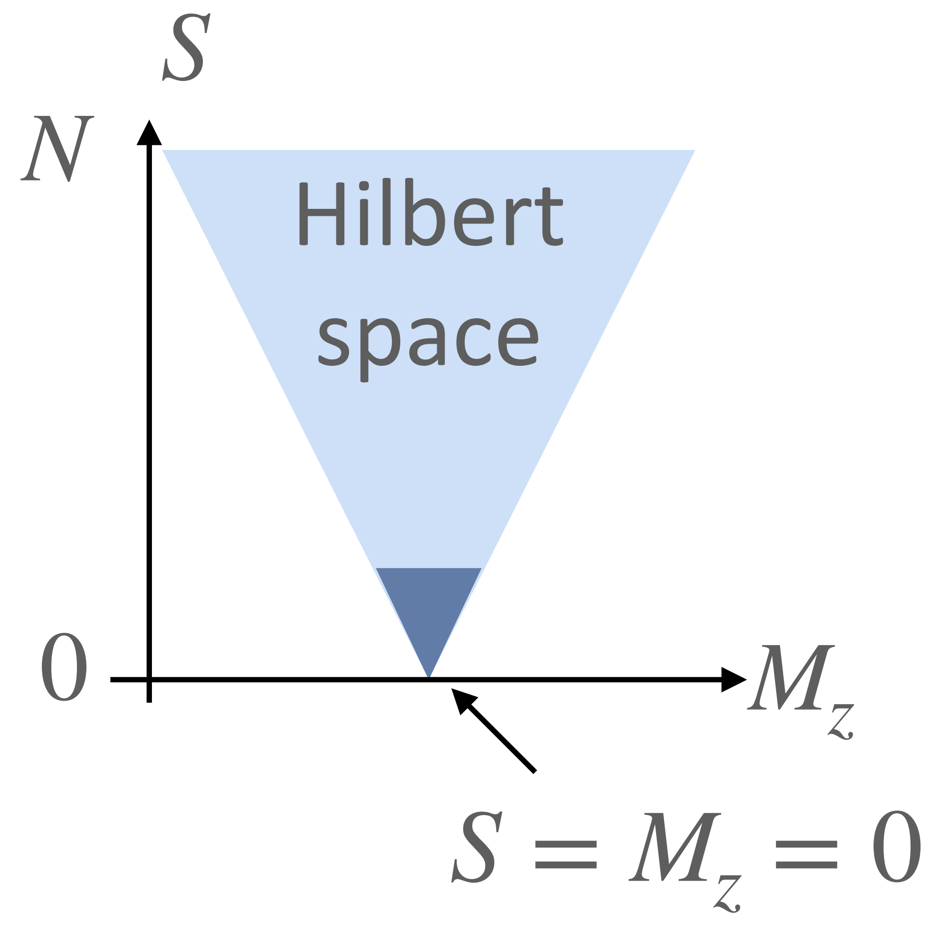
Set of 1100 shots giving $N_{+1}^{(i)}, N_0^{(i)}, N_{-1}^{(i)}$ for $i = 1,\ldots,1100$
with various angles and axes

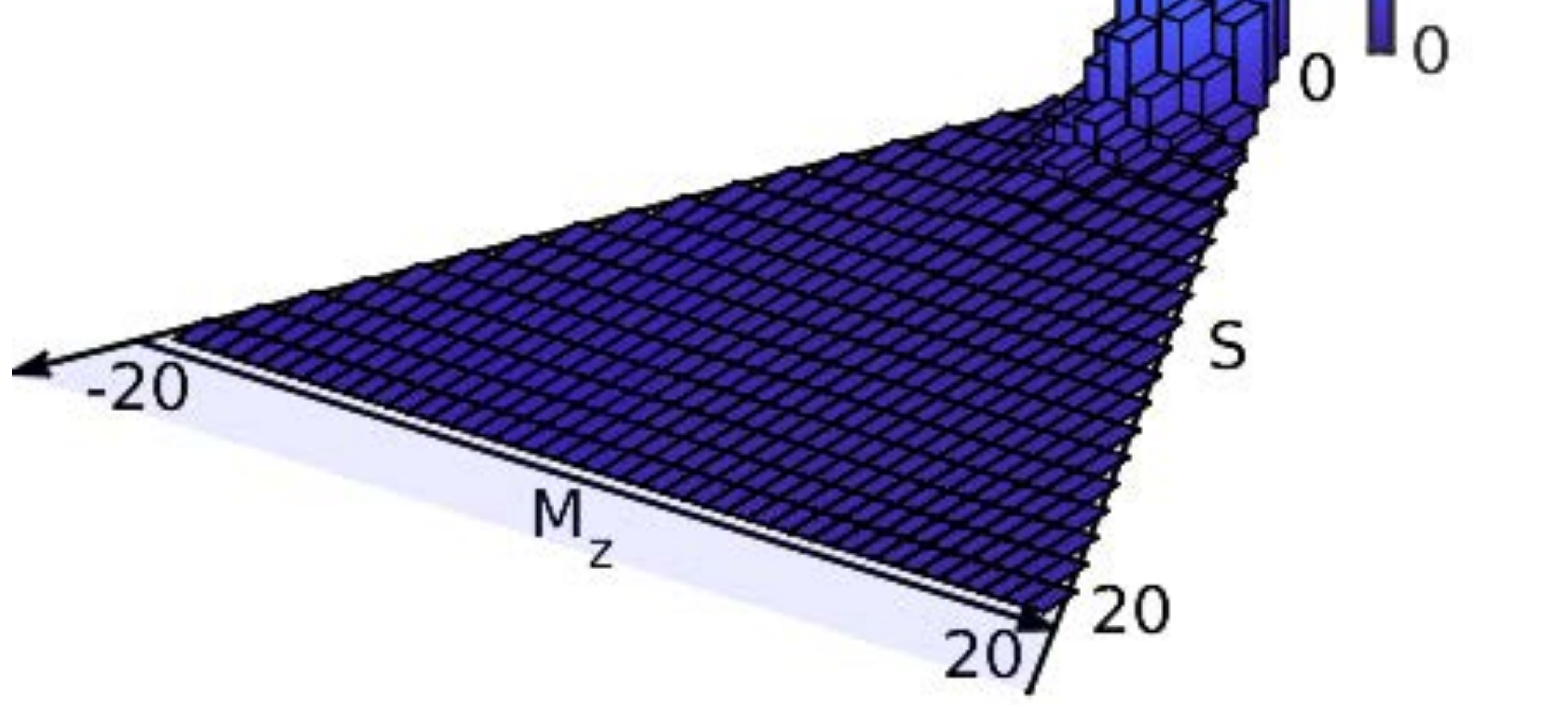
Reconstruction of the many-body density matrix ρ using a maximum likelihood algorithm

$$\max_{\rho} \mathscr{P}\left(\rho \,/\, \{N_m^{(i)}\}\right)$$ (Lvovsky, 2004)

- The density matrix ρ is essentially diagonal in the basis $|S, M_z\rangle$
- The first four spin manifolds contain 90% of the population, meaning a very low entropy: $S^{(100\text{ particles})} \approx 3k_B$

Spin temperature: 30 pK, comparable to $\dfrac{\alpha}{N} = 10$ pK



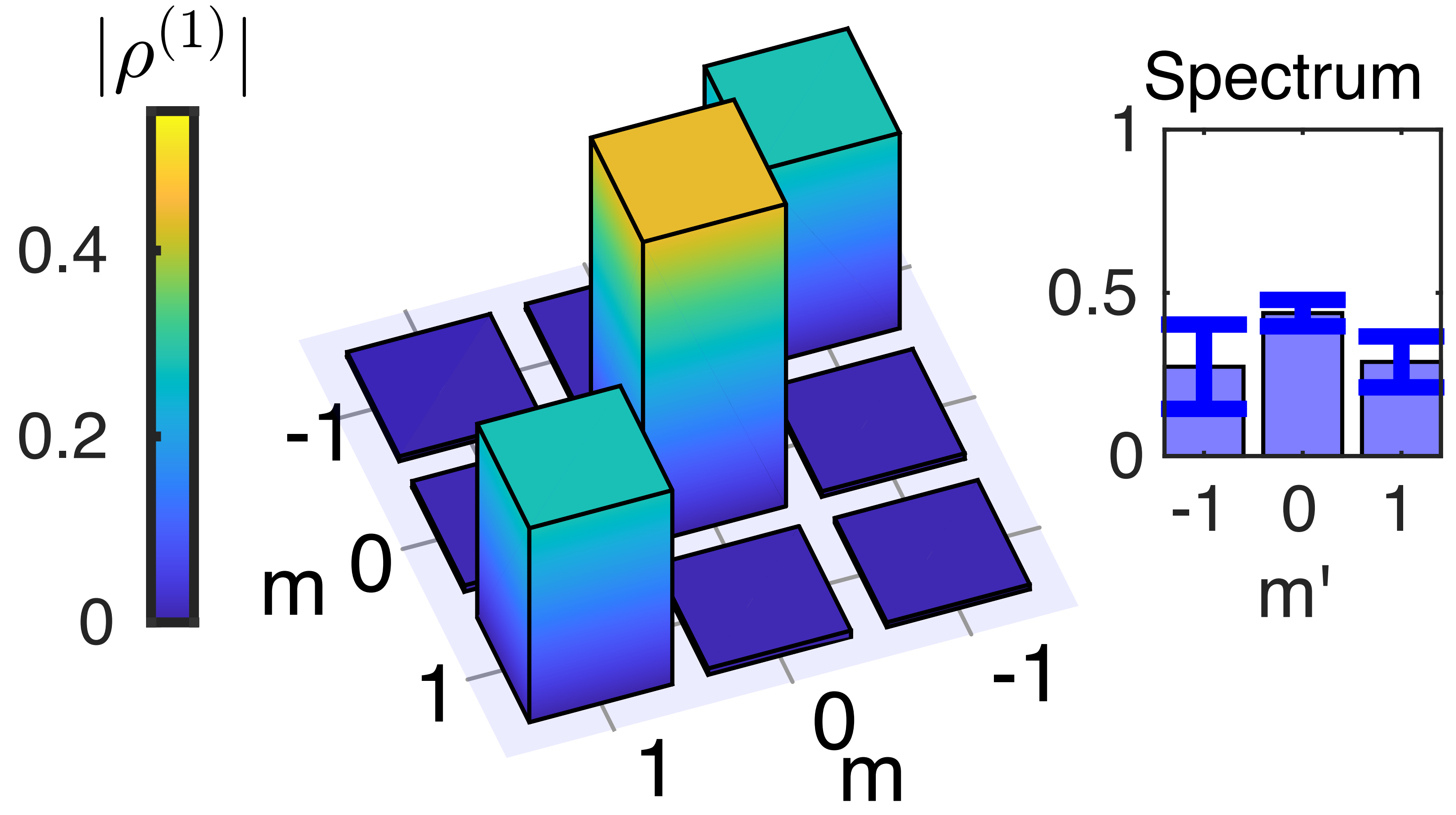


One-body density matrix: A fragmented BEC

$N \approx 100$ atoms

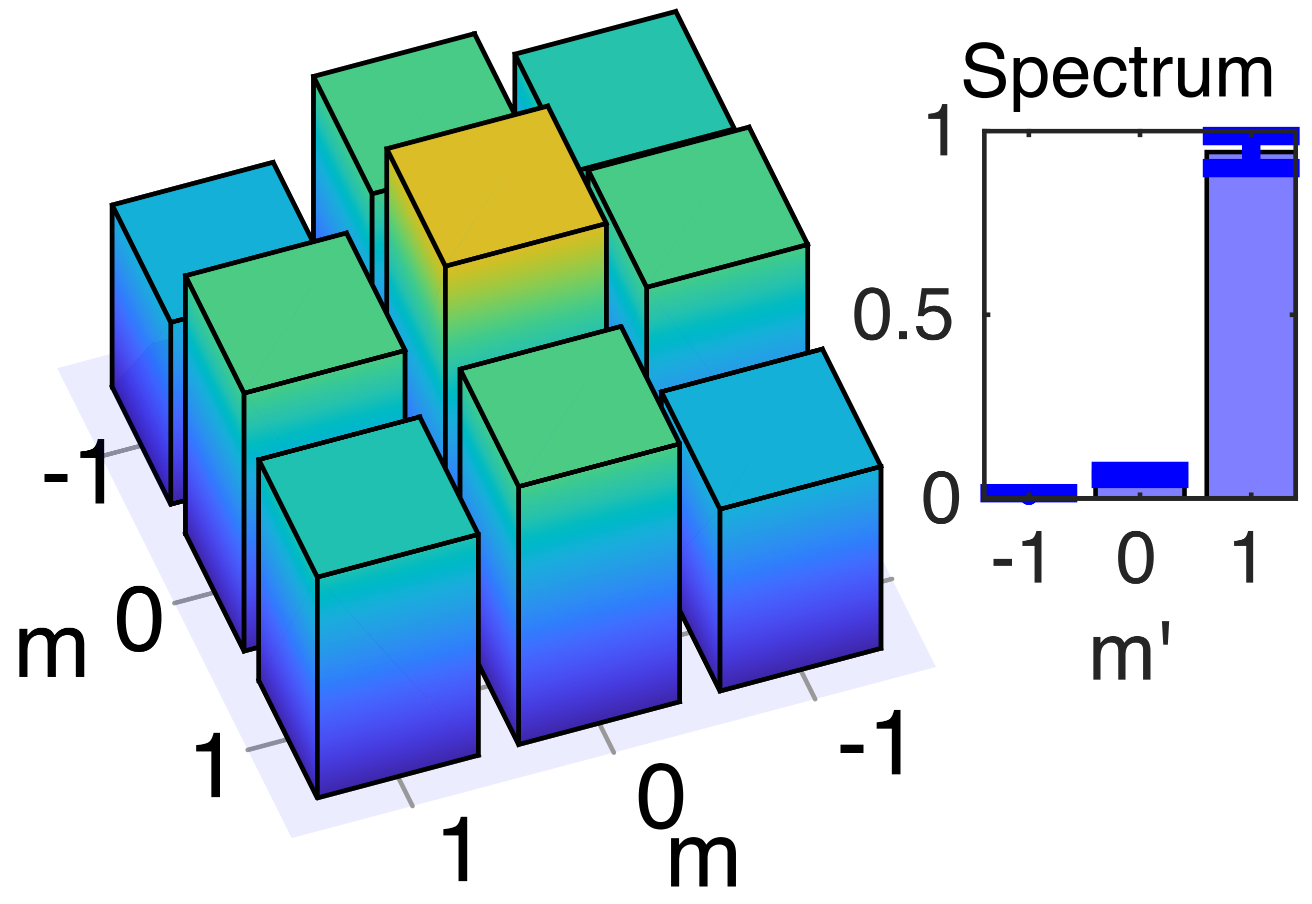
The set of measurements of $N^{(i)}_{+1}, N^{(i)}_{0}, N^{(i)}_{-1}$ allows us to reconstruct the nine coefficients of $\langle m \,|\, \hat{\rho}^{(1)} \,|\, m' \rangle$

(A) Fragmented BEC



The N-body measurement indicates that it is not a mere thermal state !

(B) Single BEC $\left(|\psi\rangle\right)^{\otimes N}$



Only one macroscopic eigenvalue

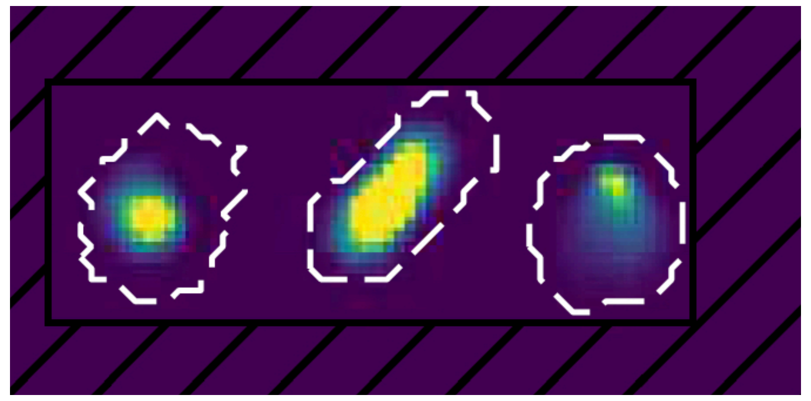
Predicted long ago (Nozières & Saint James, 1982) but little experimental evidence of such a full N-body state so far

E. J. Mueller, T.-L. Ho, M. Ueda, and G. Baym, Phys. Rev. A **74**, 033612 (2006)

See X.-Y. Luo et al., Science **355**, 620 (2017) for evidence of a two-fragment BEC

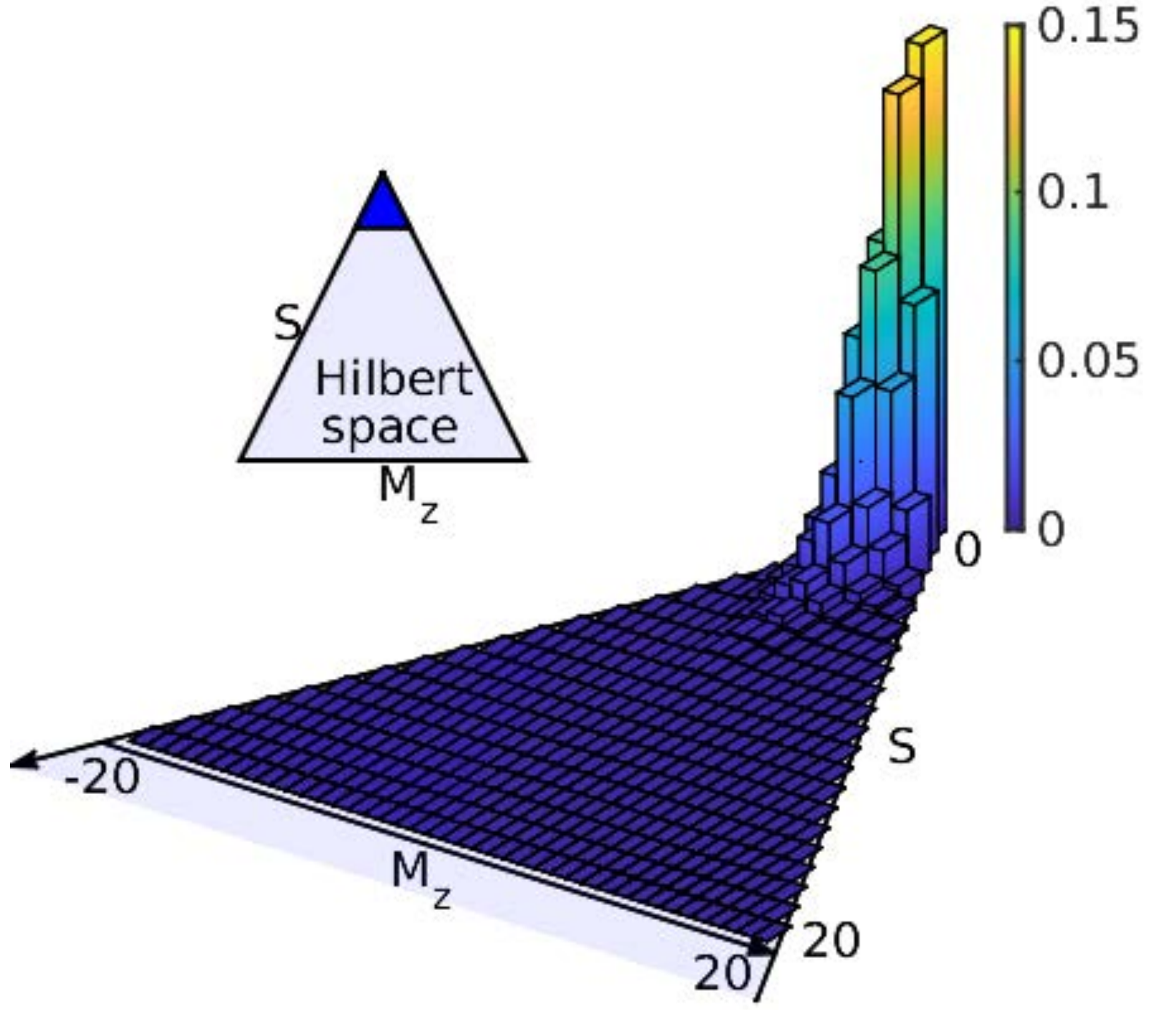
Conclusions

Assembly of spin 1 atoms in the same spatial mode, coupled with a detection at the single atom level



Unique system to illustrate many aspects of many-body physics, which also provides a very useful tool for quantum metrology

- Phase transition at the mean-field level
- Production of correlated pairs of atoms, with a record squeezing parameter
- Possibility to produce and characterized a massively entangled state: a singlet (spin 0) state made out of $N \gg 1$ spin 1 atoms

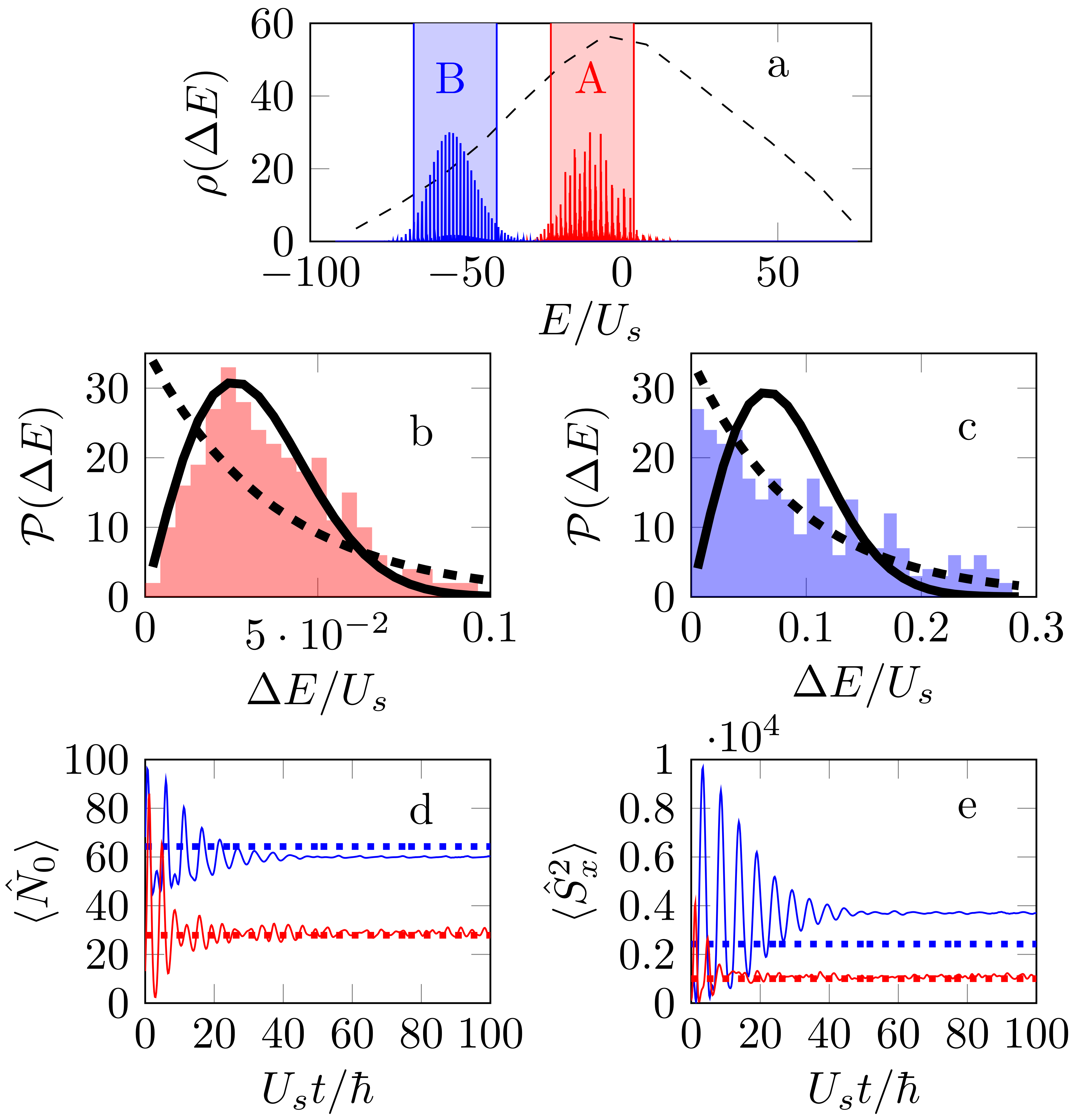


For well chosen couplings, this system can even exhibit a chaotic behavior and thus provides a tool to investigate the thermalization of a closed system

Evrard et al., PRL **126**, 063401 (2021)

$$\hat{H}' = \hat{H} + \Omega \hat{S}_x \, .$$

$$\hat{\mathcal{R}}_x(\theta)|m = 0\rangle^{\otimes N}$$



See also M. Garcia-March, S. van Frank, M. Bonneau, J. Schmiedmayer, M. Lewenstein, and L. F. Santos, New J. Phys. 20, 113039 (2018)