INSTITUT INTERNATIONAL DE CHIMIE SOLVAY

HUITIÈME CONSEIL DE PHYSIQUE

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LES

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RAPPORTS ET DISCUSSIONS

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INTRODUCTION

LE HUITIÈME CONSEIL DE PHYSIQUE

Le huitième des Conseils de Physique, prévus par l'article 10 des statuts de l'Institut International de Physique fondé par Ernest Solvay, a tenu ses séances à Bruxelles, du 26 septembre au 2 octobre 1948, dans les locaux mis obligeamment à la disposition de l'Institut par l'Université Libre de Bruxelles, siège social de l'Institut.

Le Comité Scientifique de l'Institut était représenté par :

Sir Lawrence Bragg (Cambridge), *président*, M. N. Bohr (Copenhague), M. Th. De Donder (Bruxelles), Sir Owen W. Richardson (Alton Hants-G. B.), M. E. Verschaffelt (La Haye), M. H. A. Kramers (Leiden), *membres* et M. E. Henriot (Bruxelles), *secrétaire*.

MM. Debye (Ithaca, N.Y.), Joffé (Leningrad), Einstein (Princeton, U.S.A.), Joliot (Paris), membres du Comité Scientifique étaient absents.

Les rapporteurs étaient :

MM. C. F. Powell (Bristol), P. Auger (Paris), F. Bloch (California, U.S.A.), P. M. S. Blackett (Manchester), H. S. Bhabha (Bombay), L. de Broglie (Paris), R. E. Peierls (Birmingham), W. Heitler (Dublin), E. Teller (Chicago), R. Serber (Berkeley, U. S. A.), L. Rosenfeld (Manchester)

MM. B. Rossi (Cambridge, U.S.A.) et H. A. Bethe (Ithaca, N. Y.) absents, avaient transmis un rapport.

M. L. de Broglie, absent, s'était fait représenter par Mme M.-A. Tonnelat.

MM. N. Bohr, J. R. Oppenheimer et W. Pauli, ont fait des exposés complémentaires.

Les invités étaient :

MM. F. Bloch (California, U. S. A.), H. B. G. Casimir (Eindhoven), J. D. Cockcroft (Berks, G. B.), P. I. Dee (Glasgow), Dirac (Cambridge), Ferretti (Rome), O. R. Frisch (Cambridge), O. Klein (Stockholm), Leprince-Ringuet (Paris), Mlle L. Meitner (Stockholm), MM. C. Moeller (Copenhague), F. Perrin (Paris), J. R. Oppenheimer (Princeton, U. S. A.), W. Pauli (Zürich), L. Rosenfeld (Manchester), R. Serber (Berkeley, U. S. A.), P. Scherrer (Zürich), E. Schroedinger (Dublin).

M. le D^r L. Marton, du National Bureau of Standards, à Washington, de passage par Bruxelles, a été également invité.

Le Centre interuniversitaire de Physique Nucléaire était représenté par M. Marc de Hemptinne (Louvain), Président de son Comité Scientifique.

Mme Joliot-Curie (Paris), Sir M. L. E. Oliphant (Birmingham), et M. J. Schwinger (Cambridge, U. S. A.) n'avaient pu accepter l'invitation qui leur avait été envoyée.

MM. J. Timmermans, G. Balasse, J. Errera, O. Goche, P. Kipfer, L. Flamache, professeurs à l'Université Libre de Bruxelles étaient invités à assister comme auditeurs du Conseil.

M. M. Cosyns, directeur du Centre de Physique Nucléaire de l'Université de Bruxelles, absent, était remplacé par M. Occhialini, attaché à ce centre.

Le secrétariat était assuré par M. E. Stahel, Professeur honoraire à l'Université, *Secrétaire honoraire*, M. J. Géhéniau, Professeur ordinaire à l'Université, *Secrétaire*, Melle Dilworth, Attachée au Centre de Physique Nucléaire, MM. I. Prigogine, Chargé de cours, L. Groven, Chef de Travaux, L. Van Hove, Assistant, Yves Goldschmidt, Assistant au Centre de Physique Nucléaire, M. Van Styvendael, Aspirant du F. N. R. S., M. Demeur, Aspirant du F. N. R. S., Van Isacker, Assistant à l'Institut Royal Météorologique, *Secrétaires-Adjoints.*

La Commission administrative de l'Institut se composait de M. Jules Bordet, *président*, MM. E. J. Solvay, D^r F. Heger-Gilbert, E. Henriot, *membres*, M. F. H. van den Dungen, *secrétaire*.

On the Notions of Causality and Complementarity [1]

N. Bohr

After some introductory remarks recalling the great stimulation which the meetings of the Solvay Institute of Physics through the years had been for the clarification of fundamental physical problems the speaker read at the invitation of the chairman the following brief account of the situation as regards analysis and synthesis in atomic physics originally prepared for a symposium arranged by the periodical *Dialectica* (Vol II, p. 312, 1948).

The causal mode of description has deep roots in the conscious endeavours to utilize experience for the practical adjustment to our environments, and is in this way inherently incorporated in common language. By the guidance which analysis in terms of cause and effect has offered in many fields of human knowledge, the principle of causality has even come to stand as the ideal scientific explanation.

In physics, causal description, originally adapted to the problems of mechanics, rests on the assumption that the knowledge of the state of a material system at a given time permits the prediction of its state at any subsequent time. However, already here the definition of state requires special consideration and it need hardly be recalled that an adequate analysis of mechanical phenomena was only possible after the recognition that, in the account of a state of a system of bodies, not merely their location at a given moment but also their velocities have to be included.

In classical mechanics, the forces between bodies were assumed to depend simply on the instantaneous positions and velocities; but the discovery of the retardation of electromagnetic effects made it necessary to consider force fields as an essential part of a physical

[1] The purpose of this article is to give a very brief survey of some epistemological problems raised in atomic physics. A fuller account of the historical development, illustrated by typical examples which have served to clarify the general principles, will appear soon as a contribution by the writer to the Einstein volume in the series « Living Philosophers ».

system, and to include in the description of the state of the system at a given time the specification of these fields in every point of space. Yet, as is well known, the establishment of the differential equations connecting the rate of variation of electromagnetic intensities in space and time has made possible a description of electromagnetic phenomena in complete analogy to causal analysis in mechanics.

It is true that, from the point of view of relativistic argumentation, such attributes of physical objects as position and velocity of material bodies, and even electric or magnetic field intensities, can no longer be given an absolute content. Still, relativity theory, which has endued classical physics with unprecedented unity and scope, has just through its elucidation of the conditions for the unambiguous use of elementary physical concepts allowed a concise formulation of the principle of causality along most general lines.

However, a wholly new situation in physical science was created through the discovery of the universal quantum of action, which revealed an elementary feature of « individuality » of atomic processes far beyond the old doctrine of the limited divisibility of matter originally introduced as a foundation for a causal explanation of the specific properties of material substances. This novel feature is not only entirely foreign to the classical theories of mechanics and electromagnetism, but is even irreconcilable with the very idea of causality.

In fact, the specification of the state of a physical system evidently cannot determine the choice between different individual processes of transition to other states, and an account of quantum effects must thus basically operate with the notion of the probabilities of occurrence of the different possible transition processes. We have here to do with a situation which is essentially different in character from the recourse to statistical methods in the practical dealing with complicated systems that are assumed to obey laws of classical mechanics.

The extent to which ordinary physical pictures fail in accounting for atomic phenomena is strikingly illustrated by the well-known dilemma concerning the corpuscular and wave properties of material particles as well as electromagnetic radiation. It is further important to realize that any determination of Planck's constant rests upon the comparison between aspects of the phenomena which can be described only by means of pictures not combinable on the basis of classical physical theories. These theories indeed represent merely idealizations of asymptotic validity in the limit where the actions involved in any stage of the analysis of the phenomena are large compared with the elementary quantum.

In this situation, we are faced with the necessity of a radical revision of the foundation for description and explanation of physical phenomena. Here, it must above all be recognized that, however far quantum effects transcend the scope of classical physical analysis, the account of the experimental arrangement and the record of the observations must always be expressed in common language supplemented with the terminology of classical physics. This is a simple logical demand, since the word « experiment » can in essence only be used in referring to a situation where we can tell others what we have done and what we have learned.

The very fact that quantum phenomena cannot be analysed on classical lines thus implies the impossibility of separating a behaviour of atomic objects from the interaction of these objects with the measuring instruments which serve to specify the conditions under which the phenomena appear. In particular, the individuality of the typical quantum effects finds proper expression in the circumstance that any attempt at subdividing the phenomena will demand a change in the experimental arrangement, introducing new sources of uncontrollable interaction between objects and measuring instruments.

In this situation, an inherent element of ambiguity is involved in assigning conventional physical attributes to atomic objects. A clear example of such an ambiguity is offered by the mentioned dilemna as to the properties of electrons or photons, where we are faced with the contrast revealed by the comparison between observations regarding an atomic object, obtained by means of different experimental arrangements. Such empirical evidence exhibits a novel type of relationship, which has no analogue in classical physics and which may conveniently be termed « complementarity » in order to stress that in the contrasting phenomena we have to do with equally essential aspects of all well-defined knowledge about the objects.

An adequate tool for the complementary mode of description is offered by the quantum-mechanical formalism, in which the canonical equations of classical mechanics are retained while the physical variables are replaced by symbolic operators subjected to a non-commutative algebra. In this formalism Planck's constant enters only in the commutation relations

$$qp - pq = \sqrt{-1}\,\frac{h}{2\pi} \tag{1}$$

between the symbols q and p standing for a pair of conjugate variables, or in the equivalent representation by means of the substitutions of the type

$$p = -\sqrt{-1}\,\frac{h}{2\pi}\,\frac{\partial}{\partial q} \tag{2}$$

by which one of each set of conjugate variables is replaced by a differential operator. According to the two alternative procedures, quantum-mechanical calculations may be performed either by representing the variables by matrices with elements referring to the individual transitions between two states of the system or by making use of the so-called wave equation, the solutions of which refer to these states and allow us to derive probabilities for the transitions between them.

The entire formalism is to be considered as a tool for deriving predictions, of definite or statistical character, as regards information obtainable under experimental conditions described in classical terms and specified by means of parameters entering into the algebraic or differential equations of which the matrices or the wave-functions, respectively, are solutions. These symbols themselves, as is indicated already by the use of imaginary numbers, are not susceptible to pictorial interpretation; and even derived real functions like densities and currents are only to be regarded as expressing the probabilities for the occurrence of individual events observable under well-defined experimental conditions.

A characteristic feature of the quantum-mechanical description is that the representation of a state of a system can never imply the accurate determination of both members of a pair of conjugate variables q and p. In fact, due to the non-commutability of such variables, as expressed by (1) and (2), there will always be a reciprocal relation

$$\Delta\, q \,.\, \Delta\, p = \frac{h}{4\pi} \tag{3}$$

between the latitudes $\Delta\, q$ and $\Delta\, p$ with which these variables can be fixed. These so-called indeterminacy relations explicitly bear out the limitation of causal analysis, but it is important to recognize that no unambiguous interpretation of such relations can be given in words suited to describe a situation in which physical attributes are objectified in a classical way.

Thus, a sentence like « we cannot know both the momentum and the position of an electron » raises at once questions as to the physical reality of such two attributes, which can be answered only by referring to the mutually exclusive conditions for the unambiguous use of space-time coordination, on the one hand, and dynamical conservation laws, on the other. In fact, any attempt at locating atomic objects in space and time demands an experimental arrangement involving an exchange of momentum and energy, uncontrollable in principle, between the objects and the scales and clocks defining the reference frame. Conversely, no arrangement suitable for the control of momentum and energy balance will admit precise description of the phenomena as a chain of events in space and time.

Strictly speaking, every reference to dynamical concepts implies a classical mechanical analysis of physical evidence which ultimately rests on the recording of space-time coincidences. Thus, also in the description of atomic phenomena, use of momentum and energy variables for the specification of initial conditions and final observations refers implicitly to such analysis and therefore demands that the experimental arrangements used for the purpose have spatial dimensions and operate with time intervals sufficiently large to permit the neglect of the reciprocal indeterminacy expressed by (3). Under these circumstances it is, of course, to a certain degree a matter of convenience to what extent the classical aspects of the phenomena are included in the proper quantum-mechanical treatment where a distinction in principle is made between measuring instruments, the description of which must always be based on space-time pictures, and objects under investigation, about which observable predictions can in general only be derived by the non-visualizable formalism.

Incidentally, it may be remarked that the construction and the functioning of all apparatus like diaphragms and shutters, serving to define geometry and timing of the experimental arrangements, or photographic plates used for recording the localization of atomic objects, will depend on properties of materials which are themselves essentially determined by the quantum of action. Still, this circumstance is irrelevant for the study of simple atomic phenomena where, in the specification of the experimental conditions, we may to a very high degree of approximation disregard the molecular constitution of the measuring instruments. If only the instruments are sufficiently heavy compared with the atomic objects under investigation, we can in particular neglect the requirements of relation (3) as regards the control of the localization in space and time of the single pieces of apparatus relative to each other.

In representing a generalization of classical mechanics suited to allow for the existence of the quantum of action, quantum mechanics offers a frame sufficiently wide to account for empirical regularities which cannot be comprised in the classical way of description. Besides the characteristic features of atomic stability, which gave the first impetus to the development of quantum mechanics, we may here refer to the peculiar regularities exhibited by systems composed of identical entities, such as photons or electrons, and determining for radiative equilibrium or essential properties of material substances. As is well known, these regularities are adequately described by the symmetry properties of the wave-functions representing the state of the whole systems. Of course, such problems cannot be explored by any experimental arrangement suited for the tracing in space and time of each of the identical entities separately.

It is furthermore instructive to consider the conditions for the determination of positional and dynamical variables in a state of a system with several atomic constituents. In fact, although any pair, q and p, of conjugate space and momentum variables obeys the rule of non-commutative multiplication expressed by (1), and thus can only be fixed with reciprocal latitudes given by (3), the difference $q_1 - q_2$ between the space coordinates referring to two constituents of a system will commute with the sum $p_1 + p_2$ of the corresponding momentum components, as follows directly from the commutability of q_1 with p_2 and of q_2 with p_1. Both $q_1 - q_2$ and $p_1 + p_2$ can, therefore, be accurately fixed in a state of the complex system and we can consequently predict the values of either q_1 or p_1 if either q_2 or p_2, respectively, are determined by direct measurements. Since at the moment of measurement the direct interaction between the objects may have ceased, it might thus appear that both q_1 and p_1 were to be regarded as well-defined physical attributes of the isolated object and that, therefore, as has been argued, the quantum-mechanical representation of a state should not offer an adequate means of a complete description of physical reality. With regard to such an argumentation, however, it must be stressed that any two arrangements which admit accurate measurements of q_2 and p_2 will be mutually exclusive and that therefore predictions as regards q_1 or p_1, respectively, will pertain to phenomena which basically are of complementary character.

As regards the question of the completeness of the quantum-mechanical mode of description, it must be recognized that we are dealing with a mathematically consistent scheme which is adapted within its scope to every process of measurement and the adequacy of which can only be judged from a comparison of the predicted results with actual observations. In this connection, it is essential to note that, in any well-defined application of quantum mechanics, it is necessary to specify the whole experimental arrangement and that, in particular, the possibility of disposing of the parameters defining the quantum-mechanical problem just corresponds to our freedom of constructing and handling the measuring apparatus, which in turn means the freedom to choose between the different complementary types of phenomena we wish to study.

In order to avoid logical inconsistencies in the account of this unfamiliar situation, great care in all questions of terminology and dialectics is obviously imperative. Thus, phrases often found in the physical literature, as « disturbance of phenomena by observation » or « creation of physical attributes of objects by measurements » represent a use of words like « phenomena » and « observation » as well as « attribute » and « measurement » which is hardly compatible with common usage and practical definition and, therefore, is apt to cause confusion. As a more appropriate way of expression, one may strongly advocate limitation of the use of the word *phenomenon* to refer exclusively to observations obtained under specified circumstances, including an account of the whole experiment.

With this terminology, the observational problem in atomic physics is free of any special intricacy, since in actual experiments all evidence pertains to observations obtained under reproducible conditions and is expressed by unambiguous statements referring to the registration of the point at which an atomic particle arrives on a photographic plate or to a corresponding record of some other amplification device. Moreover, the circumstance that all such observations involve processes of essentially irreversible character lends to each phenomenon just that inherent feature of completion which is demanded for its well-defined interpretation within the framework of quantum mechanics.

Recapitulating, the impossibility of subdividing the individual quantum effects and of separating a behaviour of the objects from their interaction with the measuring instruments serving to define the conditions under which the phenomena appear implies an ambiguity in assigning conventional attributes to atomic objects which calls for a reconsideration of our attitude towards the problem of physical explanation. In this novel situation, even the old question of an ultimate determinacy of natural phenomena has lost its conceptional basis, and it is against this background that the viewpoint of complementary presents itself as a rational generalization of the very ideal of causality.

The complementary mode of description does indeed not involve any arbitrary renunciation on customary demands of explanation but, on the contrary, aims at an appropriate dialectic expression for the actual conditions of analysis and synthesis in atomic physics. Incidentally, it would seem that the recourse to three-valued logic, sometimes proposed as means for dealing with the paradoxical features of quantum theory, is not suited to give a clearer account of the situation, since all well-defined experimental evidence, even if it cannot be analysed in terms of classical physics, must be expressed in ordinary language making use of common logic.

The epistemological lesson we have received from the new development in physical science, where the problems enable a comparatively concise formulation of principles, may also suggest lines of approach in other domains of knowledge where the situation is of essentially less accessible character. An example is offered in biology where mechanistic and vitalistic arguments are used in a typically complementary manner, In sociology too such dialectics may often be useful, particularly in problems confronting us in the study and comparison of human cultures, where we have to cope with the element of complacency inherent in every national culture and manifesting itself in prejudices which obviously cannot be appreciated from the standpoint of other nations.

Recognition of complementary relationship is not least required in psychology, where the conditions for analysis and synthesis of experience exhibit striking analogy with the situation in atomic physics. In fact, the use of words like « thoughts » and « sentiments », equally indispensable to illustrate the diversity of psychical experience, pertain to mutually exclusive situations characterized by a different drawing of the line of separation between subject and object. In particular, the place left for the feeling of volition is afforded by the very circumstance that situations where we experience freedom of will are incompatible with psychological situations where causal analysis is reasonably attempted. In other words, when we use the phrase « I will » we renounce explanatory argumentation.

Altogether, the approach towards the problem of explanation that is embodied in the notion of complementarity suggests itself in our position as conscious beings and recalls forcefully the teaching of ancient thinkers that, in the search for a harmonious attitude towards life, it must never be forgotten that we ourselves are both actors and spectators in the drama of existence. To such an utterance applies, of course, as well as to most of the sentences in this article from the beginning to the end, the recognition that our task can only be to aim at communicating experiences and views to others by means of language, in which the practical use of every word stands in a complementary relation to attemps of its strict definition.

RAPPORTS
ET
DISCUSSIONS

The magnetic field of massive rotating bodies

P.M.S. Blackett

I.

INTRODUCTION

In a recent paper (1) the author has drawn attention to the approximate validity for the earth, the sun and 78 Virginis, of the relation

$$P = -\beta_1 \frac{G^{1/2}}{2c} U \tag{1}$$

between the magnetic moment P and the angular momentum U, where G and c are the gravitational constant and velocity of light respectively and β_1 is a constant of the order of unity.

H. W. Babcock, whose measurements of the magnetic field of 78 Virginis were the first made on any star (2), independently drew attention to the proportionality of P and U for these bodies (3-4). It was pointed out by the author that already in 1923 the validity of equation (1) had been implicitly recognised by H. A. Wilson, but that this had fallen into oblivion. A detailed discussion of the nature of the experimental evidence was also given by the writer, together with a survey of the main theories which had been put forward at various times to explain the origin of the magnetic field of the earth and other large rotating bodies. The conclusion was reached that it was improbable that the validity of the empirical relation (1) was accidental in origin and that therefore one must consider the possibility that it represented a general property of massive rotating bodies of roughly spherical shape. If this were

(1) Blackett, *Nature*, **159**, p. 658 (1947).
(2) H.W. Babcok, *Ap. J.*, **105**, p. 105 (1947).
(9) H.W. Babcok, *P. A. S. P.*, **59**, p. 112 (1947).
(3) H.W. Babcock, *Phys. Rev.*, **72**, p. 83 (1947).

indeed the case, then it followed that an explanation of it must be sought in a new fundamental property of matter not contained within the structure of present day physical theory. Moreover it seemed likely that a full understanding of the effect was only likely to be achieved within the framework of a general theory embracing both gravitational and electro-magnetic phenomena.

However, it does not seem likely that any of the unified field theories which have been put forward hitherto would be able to explain the effect. For the observed phenomena demand some essential asymetry in nature e. g. between positive and negative electric charges, whereas most field theories, at anyrate all such theories as deal with macroscopic quantities only, contain no such essential asymetry.

Since in (1) the dipole moment is proportional to the angular momentum of the body, and since the latter quantity is the sum of contributions from the whole bulk of the body, it is hard to resist the conclusion that every part of the rotating body must contribute to the total magnetic field.

Now the only simple hypotheses in the form of a differential law which yields the integral relation (1) is the hypotheses of H. A. Wilson [1] that a mass element moving with velocity v produces a magnetic field at a distance r given by

$$H = -\beta_1 \frac{G^{1/2}}{c} m \frac{\bar{v} \cdot \bar{r}}{r^3} \tag{2}$$

in analogy with the magnetic field of a moving charge.

For a spherical body in which the mass density is any function of the distance R to the centre, it has been verified by Chapman [2] that the external magnetic field is that of a dipole with the magnitude given by the equation (1). It must be emphasised that equation (2) is certainly untrue if applied to free translating bodies, as it both gives magnetic fields which certainly do not exist and because it is inconsistent with the restricted principle of relativity. The writer pointed out however that in spite of these difficulties, it seemed useful to postulate the validity of (2) when applied to a mass element of a rotating rigid body, that is when the velocity of a mass element is given by

$$v = \overline{\omega} \cdot \overline{R}$$

[1] H.A. Wilson, *Proc. Roy. Soc.*, **104**, p. 451 (1923).
[2] Chapman, *Proc. Phys. Soc.*, **61**, p. 95 (1948).

when ω is the angular velocity of the body and R is the distance of the mass element from the centre of gravity.

Putting aside for the moment the obvious arbitarainess of this procedure, we can use (2) to calculate the external and internal field of given bodies and compare the results with experiment. No other simple expression consistent with the emperical relation (1) appears to exist. One must however bear in mind the possibility that ρ in (3) may possibly not be precisely equal to the local density, but night depend also to some extent on the nuclear constitution of the body, or on the local gravitational field.

An equivalent formulation of (2) is to state that a mass flux ρv, associated with a rotation, has the same magnetic field as that of a current density i, given by

$$i = -\beta_1 \frac{G^{1/2}}{c} \rho v \tag{3}$$

This is the relation used by Chapman in calculating the field of the sun and other bodies [1].

In this report, the argument of the previous work will be extended along the following main lines. In section II, an account will be given of recent experimental measurement of the magnetic field inside the earth's crust. These experiments appear to offer the possibility of unambiguous proff that the outer layer of the crust does actually contribute to the external magnetic field of the earth in spite of the fact that it is neither the seat of electric currents nor electric charges, nor magnetic materials, of such magnitude and properties as to allow the effect to be explained within the framework of known physical laws. It follows that the experiments alone, quite independently of the questionable validity of the astronomical evidence, can, in principle, provide definite proof of a new property of matter.

In section III the recent experimental measurements by Babcock of the magnetic field of certain stars will be described, and in section IV a short review will be given of the evidence as to the nature of the sun's magnetic field and its supposed radial limitation.

Section V describes some recent work of Bullard on the origin of the secular variation of the earth's field. This work allows certain deductions to be made about the origin of the main field.

(1) Chapman, « Solar Magnetisation and the suggested fundamental by Rotation », *M. N. R. A. S.* in the Press.

In section VI, two specific hypotheses are considered, wich are both consistent with the present observations, but which predict differend and, in principle, verifiable phenomena.

II.

THE MAGNETIC FIELD INSIDE THE EARTH

Dr. E. C. Bullard pointed out in a discussion that measurements in deep mines of the magnetic field of the earth, using standard survey instruments, should serve to distinguish between theories in which the crust of the earth above the place of observation does contribute to the magnetic field, and those theories in which it does not. The former will be called bulk theories and the latter core theories.

We will consider here only the main component of the earth's field, that is that part which corresponds to the field of a dipole situated at the centre of the earth and directed along the axis of rotation.

For a core theory, the vertical component V and the horizontal component H of the magnetic field will increase with depth, varying inversely as the cube of the distance from the centre. If d is the depth of observation and a the radius of the earth, then for $d << a$, the vertical and horizontal components will be given in terms of their values at the surface by the expression

$$V_d = V_o \left(1 + \frac{3\,d}{a}\right) \tag{4}$$

$$H_d = H_o \left(1 + \frac{3\,d}{a}\right) \tag{5}$$

For a bulk theory the variation of one or both components will be different, according to the particular assumption made as to the origin of the field.

Runcorn (1) was the first to calculate the variation of V and H downwards assuming that each mass element contributes a magnetic field given by (2), and found that V should *increase* approximately according to (4), as for a core theory, but that H should *decrease*

(1) Runcorn, « Result quoted by Hales and Gough », *Nature*, **160**, p. 746 (1947).

at about twice the rate. Chapman [1] corrected an unnecessary approximation in Runcorn's derivation and gave the following slightly different expression for H

$$H_d = H_o\,[1 - 3\,(5\rho_1/k\rho - 1)\,d/a] \tag{6}$$

where ρ_1 and ρ are the surface and mean densities of the earth and where k denotes the ratio I/I_o of the moment of inertia I of the earth to the moment of inertia I_o of a uniformly denser sphere of the same size and mass.

It is convenient to express the variation of V and H in the general forms

$$V_d = V_o\,(1 + \frac{Ad}{a}) \tag{7}$$

$$H_d = H_o\,(1 + \frac{Bd}{a}) \tag{8}$$

where A and B are constants, which are characteristic for a particular theory, but which are independent of V_o, H_o and the magnetic latitude.

For all core theories we have from (4) and (5) that $A = B = 3$.

For the bulk theory based on Wilson's hypothesis, we have $A = 3$ from Runcorn's result, and from (6)

$$B = -3\,(5\rho_1/k\rho - 1) \tag{9}$$

For the earth we have $\rho_1 = 2.8$, $\rho = 5.5$ and $k = 0.88$, giving $B = -5.7$. We see that this bulk theory gives a decrease of H downwards of nearly twice the magnitude of the increase for a core theory.

Other bulk theories will give different values for their constants. Suppose for instance that the earth is considered as a uniformly magnetised sphere. Then it is easily shown that V and H are the same at a small depth d as at the surface; in other words for such a theory, $A = B = O$.

2.1 Experimental measurements of V and H in Mines.

The first experiments to determine the horizontal component of the earth's field below the surface of the earth were made at the suggestion of Dr. E. C. Bullard in a mine in South Africa by Hales

(1) Chapman, *Nature*, **161**, p. 52 (1948).

and Cough [1]. From fifteen determinations made on three successive days, the difference Δ H between the horizontal field at the depth of 1463 metres and that at the surface was $(-25 \pm 4)\ \gamma$, where $\gamma = 10^{-5}$ gauss.

Corrections had to be applied to this for the magnetic effect of several shale bands and two dykes in the area of observation. These were estimated to produce a decrease of H between 6 and 14 γ, leaving a net *decrease* of H of between 11 and 19 γ, giving $\Delta H = -(15 \pm 5)\ \gamma$.

Hales and Cough compared this result with the expected value of $+ 11\ \gamma$ for a core theory and $- 26\ \gamma$ for a bulk theory based on (3), using Runcorn's expression.

Further measurements were made by Runcorn [2] and collaborators in a coal mine in Lancashire at a depth of 1240 metres. This mine is more suitable than the one in South Africa for such measurements as the area is less disturbed magnetically, as shown by a ground magnetic survey supplemented by geological evidence. A magnetic survey along the mine gallery used was made to find a position free from the effects of local magnetic materials, steel rails, pit shafts, etc.

Two variometers were set up and compared at the surface. One was left at the surface and read at regular intervals to dertemine the diurnal variation. The other was taken down the mine where readings were made over a period of a few hours, after which it was brought back to the surface and its readings compared with the surface instrument to determine any change of zero. By comparing the run of the readings at the surface and below ground, the effect of the diurnal variation could be largely eliminated. Corrections for the temperature of the mine (43^0 C) were then applied using the instrumental temperature coefficient as determined in a separate experiment.

Two separate determinations of the change of V and H were made with the following results

$$\Delta V = V_d - V_o = + (25 \pm 5)\ \gamma$$
$$\Delta H = H_d - H_o = - (50 \pm 10)\ \gamma$$

Putting in the values d = 1240 metres and a = 6380 KM in equation

(1) Hales and Gough, *loc. cit.*

(2) Runcorn, « Discussion at R.A.S. 27 Feb. 1948 » reported in *Nature*, **462,** (1948); *Phys. Soc.* in the Press.

(4), which we have shown should hold both for a core theory and for the particular bulk theory under consideration, we find $\Delta V = + 26\ \gamma$, in close agreement with the observations. This agreement between the observed and expected change of V provides a valuable check on the method of measurement and on the freedom of the place of observation from magnetic disturbances, and so gives one confidence in the measurement of the change in H.

In table 1 are set out the experimental data for the measurement of H in South Africa and Lancashire. In the last two columns are given the values to be expected on a core theory [equation(5)] and on the bulk theory [equation (6)].

TABLE 1

Change of Horizontal Field Under-ground measured in units of 10^{-5} gauss (1 gamma)

Place	Depth metres	V gauss	H gauss	ΔH obs. gamma	H(core)	H(bulk)
S. Africa. (26°0'S.28°0'E)	1463	.29	.15	—(15±5)	+ 14	— 27
Lancashire. (53°30'N.2°30'W)	1240	.44	.17	—(50±10)	+ 10	— 20
Mean	1350	.37	.16	—(33±12)	+ 12	— 23

It is seen that in both places the observed value of Δ H is negative and much nearer the value calculated for the bulk theory than for a core theory. In South Africa the observed value is about half the calculated value for the bulk theory and in Lancashire over twice as great. Till the origin of this discrepancy is found, the reliability of all the measurements must remain in some doubt. Possible causes are (a) the existence of undiscovered local magnetic anomalies, (b) change of zero of instruments during transit to and from mine (c) incorrectly determined or irregular temperature coefficient, (d) difference in density of surface rocks.

Note. — In a recent set of measurements in a coal mine in Kent, V was found to *decrease* by 22 γ and H to *increase* by 30 γ. The only reasonable explanation of the observed decrease of V is that it must be due to some magnetic rocks in the neighbourhood. A search for these rocks is in progress.

While awaiting further measurements and experimental check on all these points, it seems useful to take a crude mean of the two results, giving the figures in the last row of the table. Taken in this way, it is reasonable to claim that the measurements so far made give some evidence, even if not yet very strong evidence, against a core theory and in favour of the particular bulk theory under discussion.

The main objective of these and future experiments of this type is the precise determination of the coefficients A and B of equations (7) and (8) at different localities and depths, and under materials of different density and physical properties. On the bulk theory of the Wilson type, these coefficients should be independent of locality and depht but vary with the surface density according to (9) ([1]).

It is convenient to collect together in Table 2 the observed values of A and B and the values expected on the different theories.

TABLE 2

Predicted and Observed Values of Coefficients A and B.

Coefficient	Core	Bulk (Wilson)	Bulk (uniform mag:)	Observations	
				PLACE	OBS. VALUE
A	+ 3.00	+ 3.00	0	Lancashire	+ 2.8
B	+ 3.00	— 5.7	0	S. Africa Lancashire	— 4.3 } mean — 14.6 } — 9.9

This presentation of the data shows clearly how much better the observations agree with the fundamental bulk theory than with either a core theory or a bulk theory depending on the assumption of uniform magnetisation.

2.2 Can the observation be explained by the accepted laws of physics?

Though it has been shown that the observations are roughly consistent with the postulate that each part of a rotating body contributes a magnetic field given by (2) or (3), it is necessary to enquire

([1]) See also section V for a somewhat different theory.

whether any alternative explanation for the observations can be found, using only the accepted laws of physics.

The only accepted ways in which the aerth's crust could contribute to the magnetic field of the earth are :

(*a*) by being electrically charged, so as to give a magnetic field by the rotation of the earth,

(*b*) by being the seat of a system of real electric conduction currents,

(*c*) by possessing a relatively high magnetic susceptibility.

The first possibility (*a*) can be ruled out immediately, as already at the time of Sutherland's work (1904-1908), it was recognised that the charge density σ required ($\sigma \doteqdot 0.3G^{1/2} \rho$) would give rise to an electric field inside the earth of the order of 10^8 ev/cm. Such a field certainly does not exist.

On hypothesis (*b*) a system of circulating conduction currents would have to exist with a current density $i \doteqdot 0.3\ G^{1/2}\ \rho\ v$, where v is the velocity of the part of the earth considered due to the rotation of the earth. This implies a current density of about 10^{-8} amps. $cm.^{-2}$ near the equator. Though earth currents of a local and ephemeral nature do exist, it is quite certain that no such systematic circulation of current of this order of magnitude round the earth's axes can possibly exist. Since the resistivity of the earth's crust may vary from 10^5 ohm. cm^{-1} for limestone to 10^{11} $ohm.cm^{-1}$ for granite or sandstone, one would observe potential differences along the E — W direction ranging from 10^0 tot 10^8 volts per kilometre. It is quite certain that such potential differences do not exist.

Even if such currents did exist in the crust, their origin and maintenance could certainly not be found within the accepted laws of physics. It is only in the liquid core of the earth that the physical conditions for the maintenance of a net current circulating round the axes could possibly exist. However, attempts by Elsasser, Frenkel and others to derive such a current system from the convective motion and temperature differences in the liquid core have not proved markedly successful [1]. It is quite certain that there is no possible mechanism for such a system of real conduction currents in the rigid crust.

It is worth noting that it is easy to derive in quite a simple and

[1] See also section IV.

direct way, the value of the current density i that must be postulated to exist in the crust to explain any gieven observed change of H downwards, as expressed by the corresponding value of the constant B. The expression found is

$$B = 3 - 4 \pi ia/H_o \tag{10}$$

where H_o is the surface field and a is the radius of the earth. Taking B (obs) as about — 6, and giving H its value of 0.3/gauss for the equator, we find that $i = 3.6 + 10^{-9}$ amps. cm.$^{-2}$, in rough agreement with the previous estimate.

To explain the observed decrease of H downward by the third possibility (c), that is by the hypothesis that the crust itself is magnetic, requires that the intensity of magnetisation of the earth is far higher than the actual measured value. Let 1_s denote the intensity of magnetisation of the aerth's crust and 1_o the average intensity of magnetisation of the whole earth. Then it can easily be shown that the constant B in (8) is given by (1)

$$B = 3 (1 - 1_s/1_o) \tag{11}$$

Since the dipole moment of the earth, $P = \frac{3}{4} \pi a^3.1°$ has the value $8 \,.\, 10^{25}$ gauss. cm^3, we find that 1_o has the value 0.08 gauss. Giving B its observed value of about — 6, we find that $1_s = 0.24$. (1)

In the observations in the Lancashire mine, the rocks above were mainly sandstone and mudstones, with a measured susceptibility of the order of 10^{-5}. Hence there is not the slightest possibility of explaining the observed decrease of H by means of the magnetic properties of the rocks of the crust.

We conclude therefore that the accepted laws of physics will not be adequate to explain the observed decrease of H below the surface, if this decrease is finally established as a world wide phenomenon and not due only to local irregularities.

(1) For a uniformly magnetised sphere, $I_s = I_o$, so $B = O$.

III.

BABCOCK'S OBSERVATIONS OF THE MAGNETIC FIELDS OF STARS

Babcock has followed up his pioneer work on 78 Virginis by measurements of the field of several other stars (1). In Table 3 (column 1 to 4) are given the results for five stars of spectral classes Ao to Fo inclusive. Fields of the order or greater than 1000 gauss were found for four of them, while one gave no detectable field. Babcock does not state the minimum detectable field, but we will assume that it is 500 gauss. Babcock also found no detectable field for three stars of later spectral type, α Canis Majoris (F5), ε Pegasi (E O) and α Tauri (K 5). One star BD $18^0 3789$ was found to have a magnetic field which varied periodically between + 7800 and — 6500 gauss. This star, like all but one of the others showing magnetic fields, belong to a small class of A and early F stars called spectrum variables, in which the relative intensity of certain weak lines varies periodically with the time. The period of magnetic variation of B D $18^0 3789$ appears to be the same as that of its spectrum variation, which is 9.25 days. One star β COR.B. which shows a magnetic field but is not in the list of 20 spectrum variables given by Deutsch (2), is nevertheless stated by Babcock to possibly be one.

No mechanism to explain the property of spectrum variability has been put forward, though it is clear that such stars must undergo some type of mechanical and thermal oscillation. In one spectrum variable (α^2 Canum Venaticorum) periodic variations in velocity up to $\pm$ 10 K. M. per sec. have been observed, presumably associated with such an oscillation.

Babcock considers it likely that the spectrum variability and the possession of a magnetic field may be closely related properties of a star, and further that the mechanism of underlying both may be related to the slow solar cycle of sunspot numbers and polarity and of the form of the Solar corona.

These considerations led him to the view that the sun may be a magnetic variable, and that this might account for certain discrepancies between the measured field of the sun at different times. (See section IV).

(1) Babcock, *P. A. S. P.*, **59**, p. 112 (1947); *Phys. Rev.*, **74**, p. 489 (1948).
(2) Deutsch, *A. J.*, **105**, p. 283 (1947).

TABLE 3

Data for Stars of Classes A_0 to F_0 for which measurements of magnetic field have been made by Babcock.

1 star	2 class	3 Magnetic measured	4 Field assumed	5 R_1	6 M_1	7 v KMsec-1	8 β_1
B.D.18 3789 . .	Aö	+7800 —6500	1000	2.3	2.7	107	0.49
α Can. Maj. . .	Aö	0	< 500	2.3	2.7	107	<0.41
78 VIR.. . . .	A2	1500	1500	2.0	2.3	107	1.08
γ EQU	Fö	>1900	>1900	1.4	1.5	102	<1.03
β COR. BOR. .	Fö	>1000	>1000	1.4	1.5	102	>0.54
						mean =	$\overline{\beta}$ 0.71

Babcock argues that the striking variation of magnetic field observed for B D 18 3789 makes it unlikely that the magnetic field of this star can have a fundamental origin, such as had been postulated by both him and by the writer from the observed proportionality of the magnetic moment and the angular momentum for the earth, the sun and 78 Virginis. For clearly the angular momentum of a star cannot change in the way required to explain the change in magnetic field. It could be argued, however, that a star, which did possess a magnetic dipole determined by its angular momentum, and which was in such a state of mechanical and thermal oscillation as to give rise to the phenomenon of spectral variability would be also expected to undergo electromagnetic oscillations which might explain the observed effect. Moreover, all « non-fundamental » or « specific » theories of the origin of the field of astronomical bodies, such as those of Elsasser or Frenkel, ultimately relate the origin of the field to the rotation of a substantial portion of the body — in the case of the earth to the liquid core. Rapid changes in the magnitude and direction of the angular momentum of that *part* of a star to which is attributed its magnetic moment are as little likely to occur as are rapid charges in the angular momentum of the whole star. It appears therefore that the phenomenon of magnetic variability requires the hypothesis of some kind of electro-magnetic oscillation about a mean field, whichever origin, a fundamental or a specific one, is assumed for the mean field.

It will be notificed from Table 3 that the mean fields of all the four stars for which a field has been established, are of the order of 1000 gauss, though one of them shows a large oscillation about this value.

It seems difficult to supposes that the origin of the field of a star with a constant magnetic field can be essentially different from the origin of the field of a magnetic variable; for if the two mechanisms were quite different one would not expect roughly the same mean field. We will therefore assume the origin of the *mean* fields of stars with constant and variable magnetic fields are essentially the same, and we will use data for both types of stars to estimate the values of the constant β_1 in equation (1) and (3).

While awaiting further details of Bancock's measurements of the variable field of B D $18^0 3789$, we will assume that its mean field is $(7800 - 6500) / 2 = 650$ gauss, as shown in Table 3.

We will only consider stars of type Ao to Fo inclusive, since (*a*) no magnetic measurements have been made on earlier types and (*b*) it is known that high rotational velocities disappear rather suddenly betwwen the types F2 and F5.

To calculate β_1, it is convenient to re-write the expression (18a) of the previous paper, by introducing the peripheral velocity $v = \overline{\omega}.\overline{R}$, instead of ω, and replacing R and M by their values R_1 and M_1 expressed in terms of the sun's radius Rs and mass Ms. In this way we get

$$\beta = \frac{5}{2} \cdot \frac{c}{G^{1/2}} \cdot \frac{Rs}{Ms \,.\, k\eta} \; \frac{R_1^2 \, H}{M^1 \; v} \qquad (12$$

The quantity k is the ratio of the moment of inertia of the star to that of a uniformly dense body of the same mass and radius, while η is a quantity introduced to take into account the non-uniform rotation of differents parts of the star. It is defined as the ratio of the actual angular momentum of the star to that of a body of the same size, mass and equatorial peripheral velocity. Inserting the values $G^{1/2}/c = 8.62 \times 10^{-15}$ $cm^{1/2}$ $gm^{1/2}$, $R_s = 6.97 \times 10^{10}$ cm, $M_s = 2.0 \times 10^{33}$ gm we get

$$\beta = \frac{706}{k\,\eta} \cdot \frac{R_1^2 \, H}{M_1 \; v} \qquad (13)$$

Since it is not possible at present to measure H and v on the same star, it is necessary to use the mean value of v as determined for each spectral type. Westgate has given the frequency of occurence of equatorial velocities for O and B and for A type stars. This data was reproduced in the former paper from a table compiled by Becker [1]. The mean velocity for A type stars is found to be 102 KM/sec. for O and B types, and 107 KM/sec. for A types. Since Westgate states that early F stars have similar rotational velocities to O and B types, we will assume the former value for the F O stars.

In Column 5 and 6 of Table 3 are also given the values of radius and mass in terms of the sun for the spectral class as given by Becker.

For k we will assume as previously the value 0.16 derived for the point-convective model. For η, again as previously, we will provisionally assume the value of unity, though we will discuss the possible effect of non-uniform motion later.

The resulting values of β_1 as calculated from (13) are given in the last column. Since they are calculated assuming that each star has the mean velocity for its class, whereas we know that the actual velocities range at least from 25 to 250 KM/sec., the calculated values of β_1 should show a similar 10 to 1 dispersion. It will be of great interest to see whether further experimental observations do show a frequency distribution of magnetic fields similar to that of rational velocities. With the data available, all that can be done is to take the crude mean of all the calculated values of β_1, without regard to the inequality signs, as representing the best determinable value of β_1 for these stars. This is found to be 0.71. Assuming this mean value of $\overline{\beta_1}$ as determined in this way to be an approximation to the true value of β_1 in equations (1) and (3), we can use the known frequency of given rotational velocities to calculate the expected frequency of occurence of stars with given magnetic fields. Westgate's [2] data shows that roughly one quarter of A type stars have peripheral velocities lying in each of the velocity ranges, 0 to 50, 50 to 100, 100 to 160, and 160 to 250 KM/sec. Using (13) to calculate H_ρ, we find that one would expect about one quarter of A stars to have magnetic flelds lying in each of the ranges, 0 to 500, 500 to 1000, 1000 to 1600, and 1600 to 2500 gauss. With the present assumed minimum detectable field of about 500 gauss, one would therefore expect about a quarter of these stars to give no detectable field. Babcock observed 1 out of 5. We can conclude that Babcock's measurements are not inconsistent with the validity of relation (1), with $\beta_1 = 0.71$. Assuming this to be true we can calculate from (2) the mean fields of the spectral classes from B O to F O given in Table 4.

(1) Becker, *Sterne and Sternsysteme* (1942).
(2) Westgate, *A. J.*, **78**, p. 41 (1933); **79**, p. 98 (1945).

TABLE 4

Mean expected magnetic field of early spectral types

Spectral Type	M_1 Mass	R_1 Radius	v Velocity KM/sec	$\omega = v/R$ Ang. vel. rel. to sun	$H\rho$ magnetic field gauss.
B.0	15	6.9	102	7.4	500
B.5	6	4.3	102	12	510
A.0	2.7	2.3	107	23	850
A.5	1.8	1.6	107	33	1170
F.0	1.5	1.4	102	36	1210
sun G.0	1.0	1.0	2.0	1.0	30 (1)

It is interesting to note that both angular rotation and the mean magnetic field increases as one goes from A to F O stars and has its maximum value for the latter type. The apparent sudden drop in rotation and so presumably in magnetic field sets in between types F2 and F5.

The reason that the value of 0.71 for β_1 obtained here is markedly lower than the value 1.15 derived in the former paper from the data for 78 VIR alone is that it now appears from Babcock's further measurements of the fields of other stars that 78 VIR has probably a larger magnetic field than the average for its class and so is presumably rotating also faster than the average. It is worth emphasising that even if the relation (1) turns out essentially to have no general theoretical validity, it is very probable that within a given spectral class the magnetic field of a star is likely to be closely correlated with its angular momentum, So most of the analysis of this section may prove to be valid even if the fundamental explanation of the field is abandoned in favour of a specific theory; though of course in this case the constant β_1 would have to be given another interpretation.

If the magnetic field has the fundamental origin which we have postulated, then the absolute value of $\overline{\beta 1}$ is a great interest. Probably the greatest uncertainty in the calculated mean value of 0.71 lies in uncertainty in the assumed values of k and η in (12) and (13). In a private communication Professor Freundlich has expressed the view that the most probable value of k is that calculated for a poly-

(1) See section IV.

trope with $\eta = 3$; this value is 0.20. Using this value instead of the value 0.16 assume above, we find $\overline{\beta 1} = 0.57$. This new value is considerably closer to the value of 0.30 for the earth than the earlier value given in the previous paper.

Some recent theoretical work by Schwarzchild [1] suggests that the interior parts of a star may be rotating much slower than the outside, leading to a value of η considerably less than unity, and so to correspondingly increased values of β_1. Chapman [2] has shown that Schwarzchild's model for the sun gives $\eta . \simeq 1/2$ leading to values of β_1 of about 2.0. However Schwarzchild's theroy appears to lead to a larger decrease of rotational period at the poles of the sun compared with the value at the equator, than is actually observed. So it is probable that η is not as low as Chapman calculates and therefore β_1 not as high.

An other source of possible error is in the values assumed for the mass and radius of stars of given spectral type. Babcock has pointed out to me that the stars with measured fields are peculair stars, e. g. 78 VIR is A2p not A2, and that such stars may not have quite the same size and masses the normal types. Deutsch estimates that these peculiar stars are probably about 1 magnitude brighter than the corresponding normal stars, and are bluer. He considers for instance than an A0 star may have the same size and mass as a B.8 star. Since from (13), β is proportional to R_1^2/M_1, we see from Table 4 that the value of β_1 would be increased. However Babcock also suggest that the greater absolute brightness of these stars may be due to the fact that they are probably appreciably flattened by rotation and so have a larger value of gravity at the poles than at the equator. Such a larger value would lead to increased brightness. One must remember that the only stars of a given type which the magnetic field can at present be measured are that small fraction for which the axis of rotation is nearly in the line of sight. For only in such cases is the rational broadening of the lines sufficiently small to allow the Zeeman effect to be measured. If, therefore the polar region of rotationally flattened stars are brighter than the equatorial region, one will expect that stars for which the magnetic field can be measured will be brighter than normal, but will not necessarily be larger or heavier.

(1) Schwarzchild, *A. J.*, **106**, p. 437 (1947).
(2) Chapman, in the press.

Similar considerations may possibly apply to the spectrum variables, though I have not found this mentioned in the literature. The spectral lines of spectrum variables are quite narrow, of the order of 1 or 2 A.U. broad, as can be seen in the spectra reproduced in the paper by Deutsch. This indicates a component of rotational broadening along the line of sight less than about 50 KM/sec. So these stars are either (*a*) oriented at random but with much small rotation than is normal for A type stars, or (*b*) have normal rotations but are oriented along the line of sight. According to the first hypothesis, the correlation of large measured magnetic fields with spectrum variability noted by Babcock, would imply an anti-correlation of large magnetic field with large rotation — a conclusion so improbable as to be rejected. Hence one concludes that the second hypothesis is correct, that is that the spectrum variables have roughly the normal rotation, but have their axes along the lines of sight. It is not, however, necessary to assume that the property of spectrum variability in a special property only of the polar region of such stars. For the lines which show this property are generally not only sharp but rather weak, and so might well tend to escape detection in a star oriented at a large angle to the line of sight, due to the rotational broadening. If this is the correct interpretation, the observed correlation of large magnetic fields with spectrum variability should be reformulated as a correlation between the *observability*, rather than the *existence* of magnetic fields and spectrum variability, both being dependent on absence of large rotational broadening due to approximate parallelism of the axis of rotation with the line of sight.

IV.

THE SUN'S MAGNETIC FIELD

The original measurements by Hale and his collaborators between 1912 and 1918 of the general magnetic field of the sun seemed to give convincing evidence that the field has a dipole character with a value at the pole of about 53 gauss. The method used was that of detection of slight traces of circular polarisation in the wings of the lines. The latitude variation was found to be that expected for a dipole and in addition a time variation with a period of 31 1/2 days, revealed by a variation of field at a given latitude, was detected which showed that the magnetic axis made an angle of about 6^0 with the mechanical axis.

In spite of the convincing nature of these results, considerable doubt has been expressed at various times as to the reality of the field. This doubt arose because a few of Hale's original observers failed to detect the effect. The measurements of most of the observers agreed, however, among themselves reasonable well.

In a recent discussion of the subject Cowling (1) has written : « Then average observed separation of the Zeeman components is of the same order as the probable error of a single measurement. Because of its smallness, its reality is sometimes doubted. The present author sees no reason to share these doubts. To ensure that personal bias should not colour the results, the original observer measuring the Zeeman displacements was kept in ignorance of the hemisphere and latitude of the observations. Thus if he were prone to imagine a non-existent phenomenon, he would give it sometimes one sign, sometimes the other, and in the aggregate his estimates would cancel out. Since, in spite of this, regular results were obtained, the phenomenon being observed was clearly not imaginary. In fact, though all observers have not bean able to identify real Zeeman displacements, all those who have been able to identify them have found effects with the same sign. »

« Moreover, no convincing explanation of the observations has been given, other than the existence of a magnetic field. The phenomenon requiring explanation is a difference in polarization between the two wings of a spectral line. Effects producing a pure displacement or pure broadening of the line cannot explain the polarization; no one has yet suggested a satisfactory alternative to the explanation in terms of a magnetic field. »

Hale found that different spectral lines gave different values for the magnetic field. In general, weak lines gave the greatest field (about 50 gauss) and strong lines a weaker field (of the order or less than 10 gauss). Since it is generally assumed that strong lines originate at greater heights than weak ones, this was interpreted as indicating that the field fell off rapidly with height. Hale concluded that the field fell from 50 gauss to less than 10 gauss within a radial distance of some 300 K.M. Cowling, however, doubts if this rapid decrease with height need be considered as real.

(1) Cowling, *M. N., R. A. S.*, **105**, p. 166 (1945).

This supposed rapid falling off of field with height has been held by Chapman and Rosseland to show that the lines of magnetic force must be nearly horizontal. But this would give a variation of polarisation across the disc quite different from that found by Hale. On the whole it seems more likely that the failure to detect the Zeeman effect of strong lines must be in some way due to their greater width.

The only other measurements are those of Thiessen (1) who used an interferometric method with a circular analyser. In a series of measurements in 1945 he found a magnetic field for the Fe 6173 line corresponding to a polar field of 53 ± 12 gauss. The variation with latitude and longitude were not measured, but the states that he verified many times that the circular polarization diminished to zero at the equator and the poles, as it should. However, quite recently (2), 1947-1948, Thiessen has repeated his measurements on the same line and has failed to find any field as large as 5 gauss! He states that the new measurements were more extensive than the old ones and probably more reliable. He points out, however, that the later measurements were made at sun spot maximum, whereas the earlier ones were some two years before this. He plans further measurements at the next sun spot minimum.

Very recently Babcock (3) has reported a series of measurements of the sun's field using a Lummer Plate crossing a grating, together with an analyser for circulary polarised light. Among 42 sets of readings made between 1940 and 1947, magnetic fields between 6 and 60 gauss were found in 18, while the remaining 24 gave no measurable field or slight negative values. He concludes that « Hale's surmise that the field may be variable appears to be supported ». These new results justify the assumption that the sun's mean polar field can be taken as approximately 30 gauss.

As has already been mentioned, Babcock suggested that a possible explanation of these discrepancies may be that the general magnetic field of the sun varies with the phase of the sun spot cycle, just as the field of B D 18.3789 varies with the phase of its spectrum variability; moreover, he speculates that the fundamental mechanisms may be the same for these two stars, though of course, the magnitude of the field and its period of variation are very different. On this

(1) Thiessen, *Annal. d'Astrophysique*, **9**, p. 101 (1948).

(2) Private communication.

(3) Babcock, *P. S. A. P.*, **60**, p. 244 (1948).

view, presumably the sun would be considered a spectrum variable too. And in a sense it must be, for since the spectrum of a sunspot differs from the normal solar spectrum owing to its lower temperature, the total average solar spectrum at sun spot maximum must differ, however slightly, from that at sun spot minimum.

Cowling surveys the various theories that have been proposed to account for the sun's field and finds that none of them give the right order of magnitude. The least unplausible is that the magnetic field arises from electric currents resulting from the thermal motion of a rotating mass. This theory is analogous to that proposed by Elsasser to account for the earth's field. Cowling supposes that convective motion occurs in an unstable central region. The rising and falling masses are deflected to the east and west due to the Coriolis force. These Coriolis forces set up pressure differences in the gas, which give rise to a relative diffusion of electrons and ions and so produces an electric current. Cowling shows that the direction of the magnetic field produced is that observed. However, by making reasonable assumptions as to the temperature difference, velocity of rise and fall, and electric conductivity, Cowling finds that the surface magnetic field of the order only of 10^{-6} gauss would be produced. So this theory must be abandoned.

Cowling can find no other theory which promises any better success, except to remark on the very unlikely possibility that the core of the sun might be capable of permanent magnetisation. He writes : « This demands that hot ionised material is capable of acquiring a regular crystal-like structure, a possibility normally disregarded. In view of the difficulties of other hypotheses, the possibility may however be worthy of further study ».

We can sum up the situation as follows : — (*a*) The sun probably possesses a dipole field, whose polar strenght may possibly fluctuate from about 50 gauss to less than 10 gauss, giving a mean field of about 30 gauss. (*b*) No explanation of the origin of the field has been found using only known properties of matter and none seem likely to be found. (*c*) Though no explanation of the field of sunspots has been found, it seems not excluded that the field may arise from a compression of existing lines of force by fluid motions.

It is of some interest to note that if the field of the sun is 53 gauss, then the values of β_1, deduced from relation (1) is 1.14, whereas the mean value for the stars with measured fields is 0.68. If, however, the mean field of the sun over a sun spot cycle is in fact only about

30 gauss, one obtains a value of 0.65 that is closely the same as that obtained for the stars.

Two methods exist by which information about the magnitude of the sun's field outside the chromosphere could be obtained, in principle; these are the form of the solar corona and the effect of the sun's field on the intensity of cosmic rays at the earth (1). But no reliable information appears to be obtainable as yet from these phenomena.

Even if the sun's general field is supposed to have a fundamental origin, a quite different explanation must be found for the supposed obliquity of the magnetic axis, and for sunspot fields.

Presumably the former would be explained by postulating subsidiary induced dipoles as in the theory of the earth's secular variation developed by Elsasser and Bullard (Section VI).

It was recognised already long ago by Sutherland, and has recently been re-emphasised by Chapman, that the field of a sunspot is vastly lager than would be expected if it had the same fundamental origin as the main field. No fully satisfactory theory seems to have been found to explain the magnetism os sunspots, and it could be argued that when one is found, the same mechanism might serve to explain the main field. It is possible that this may prove to be the case, but on the other hand it is also possible that the explanation of the field of a sunspot may lie in a drastic distortion of the main field, through some complicated hydrodynamic motions, such as have been discussed by Alfven and by Elsasser. Elsasser, in fact, appears to conclude from his analysis of such motions that their main effect is to compress or expand existing lines of force the freezing in effect- rather than to create new ones, and so do not provide a theory of the main field.

V.

COLLECTED RESULTS

From the data of Tables 3 and 4 of the previous paper it can be deduced that the ratio of the mean angular momentum of the 5 stars to that of the sun, is 223 ± 50. Taking the polar field of the sun as 30 gauss, the ratio of the mean magnetic moment of the five stars

(1) See recent note by Alfven, *P. R.*, **72**, p. 88 (1947).

to that of the sun is found to be 230 $\pm$ 40. So we see that for the mean of these five stars compared with the sun, the magnetic moments proportional to the Angular Momenta within the experimental error.

In Figure I, are shown the values of $\log_{10}U$ and $\log{}_{10}P$ for the earth, the sun and the mean of the 5 stars. The value of k for the earth is taken as 0.88, and for the sun and stars as 0.20 (see page 36).

The slope of the straight line drawn through the three points is 1.025, that is unity within the experimental error. In these calculations η has been taken as unity, that is all the bodies have been assumed to be in uniform rotation.

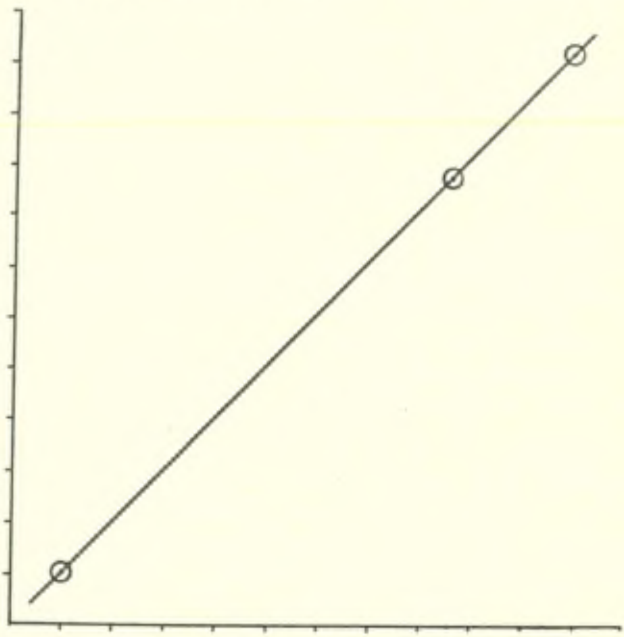


Figure 1.

An alternative method of displaying the results is shown in Figure 2, in which the observed field Hobs relative to that of the earth is plotted on a log-log scale against the calculated field.

This latter is calculated from the expression H and Mwk/R which is derived directly from equation (1). This presentation of the results has the advantage over that shown in Figure I of comparing directly the measured field with the calculated field. The range

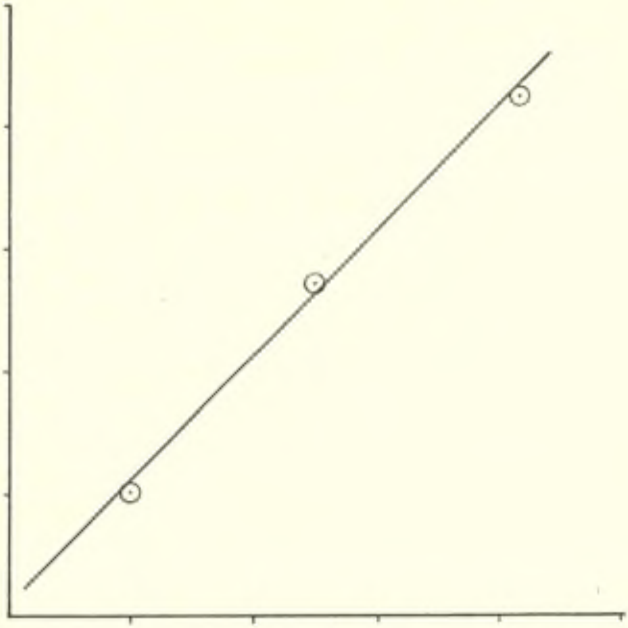


Figure 2.

of measured and calculated fields is about 1800 to 1, whereas the range of P and U is about 10^{10} to 1. The must greater range of the latter quantity arises because U and $M w R^3$ and P and HR^3, and R for the sun and stars is of the eorder of one hundred times greater than for the earth.

VI.

THE SECULAR CHANGE IN THE EARTH'S FIELD

The situation outlined in the former paper, that no plausible theory of the earth's main field relying only on the known laws of physics, still persists. However, some progress has been recently made by Elsasser and by Bullard (1) (2) in explaining the origin of its large secular variation, which may amount to 25 % of its average value.

Elsasser showed by a statistical treatment of the 42 harmonic

(1) Elsasser, *Phys. Rev.*, **60**, p. 159 (1941); **69**, p. 106 (1946); **70**, p. 202 (1946).
(2) Bullard, *M. N., R. A. S.*, Geophys : Suprl., **25**, p. 246 (1948).

components of the earth's field, that the non-dipole part of the field can be represented by some 10 dipoles distributed in random positions within a distance of half the radius from the earth's centre.

The main conception of Bullard and Elsasser is based on the observed fact that the field of the earth varies in a related way over certain large areas of the earth, of the order of a thousand miles or so in diameter. The period of variation, of the order of 100 years, is far too rapid to suppose that the cause lies in any thermal or mechanical changes in the solid outer 3000 KM of the earth. It is, therefore, necessary to seek the origin of the changing field in the liquid core of the earth, for only here can mechanical and thermal motions take place quickly enough.

Bullard shows that the observed secular change of field in S. Africa over the last 100 years is consistent with the slow growth of a magnetic dipole near the interface between core and crust. He shows quantitatively how such a dipole can arise from a slowly growing eddy in the outer region of the liquid core, provided one assumes the pre-existence of the main field. Bullard calculates that a mass of electrically conducting fluid rotating in the main field will give rise to electrical eddy currents and so to an induced magnetic dipole, of such a character as to give an additional field at the surface of the type observed in a secular change. Making plausible numerical estimates of the size of the eddies, Bullard can explain at least the order of magnitude of the observed changes. Though there are still difficulties to be overcome before such explanation can be made quantitatively satisfactory, it does for the first time offer a plausible and already semi-quantitative origin of the secular variation, provided, and only provided, one assumes the pre-existence of a main field.

Bullard points out that his theory of the secular variation may provide, when more fully worked out, a test between a core theory and a bulk theory of the main field. For the magnitude and direction of the magnetic dipole induced in a rotating eddy in the core will depend on the magnitude and direction of the main field near the surface of the core, and this will be markedly different according to wether the main field itself arises in the core, or whether it originates in the whole bulk of the earth. So far, it is not possible to judge which origin is indicated. In fact neither theory at present came adequate.

From this work of Bullard we can conclude that a plausible theory of the secular variations can be found by supposing that they are due to the interaction of motion of the liquid core with the main field, but that the main field itself is not which to be so explained, but must rather be assumed to arise in some other way.

The partial success of Bullard's theory of the secular variation gives, therefore, some indirect support for the view that the main field itself does not arise in the core. Now it is only in the liquid core that the main field can conceivably arise, provided the normal laws of physics are assumed. For there is almost certainly no possible physical mechanism in the solid outer part which could give rise to the main field by known physical principles, e. g. currents, charges or parmanent magnetism. So if the main field does not arise in the liquid core, then we conclude that it cannot be explained at all by the accepted laws of physics, and so must arise from some new property of matter.

It has long be recognised that the secular variations appear to be a regional rather than an earth wide phenomena, (see for instance Chapman and Bartels, Vol. I, p. 130). Elsasser showed by a statistical analysis of the spherical harmonic components of the earth's fields that the non-dipole part of the field can be represented by some 10 dipoles distributed near the outside of the liquid core. On this interpretation the obliquity of the magnetic axis would seem to be due to a number of essentially local disturbances, rather than to a systematic earth wide phenomenon.

VII.

DISCUSSION

The experimental evidence discussed in the former sections is certainly not as yet adequate to prove conclusively that the earth's magnetic field arises from a new property of matter. Further measurements of the field of the sun and the stars and of the variation of the earth's field below the surface, are clearly urgently needed. However, the existing evidence is sufficient to justify a careful search for new possibilities of testing the hypothesis of a fundamental origin of the field. One such possible test arises out of the following considerations.

We will start by assuming the approximate validity of equation

(3), that is that a mass flux $\rho\ v$, associated with a rotation ω, has the same magnetic field as that of a current density i given by

$$i = -\beta^1 \frac{G^{1/2}}{c} \rho\ v \qquad (3)$$

where $v = \overline{\omega}.\overline{R}$. It will be convenient to call this current i, the virtual rotational electric current, or more simply, the virtual current associated with a rotational mass flux.

In seeking to find an explanation of this virtual current, two different theoretical possibilities present themselves in the first instance for consideration and lead to significantly different predictions.

The first is that equation (3) is to be taken as exact, that is that ρ is to be taken as simply the mass density, that is, the sum of the masses of all protons, neutrons and electrons in unit volume. On this hypothesis, the constant β_2, as determined experimentally either from the external or internal field of a rotating body, should always be found to have the same value. In fact, β_1 which from (3) is proportional to $i/\rho\ v$, and so can be considered as the virtual current per unit mass flux, would be the same for all bodies of whatever nuclear constitution.

The second hypothesis is that the virtual electric current may depend not on the total mass of all the particles, but on their total electric charge, that is, may, on the total number of electrons (or protons) in unit volume. Since the ratio of number of electrons to the number of nucleons varies only by a factor of a little over two between heavy or light elements, this hypothesis is consistent with the *approixmate* validity of (3). However, this variation is, in principle, detectable by the differente in the magnetic field produced by the rotation of matter of different nuclear constitutions. The difference arises, of course, simply from the fact that the neutrons, on this hypothesis, contribute to the mass but not to the virtual ciurrents, so that for instance, a rotating mass of hydrogen would have a larger magnetic effect per unit mass than a mass of heavier elements.

The second hypothesis would be in conformity the view that the magnetic effect of rotating matter is in some way bound up with some minute inequality in the behaviour of positive and negative charges.

We see then that our two hypotheses satisfy one of the main requirements of good hypotheses, that they suggest further experiments : Since the second one has the more interesting consequences it is worth while considering it in more detail, and formulating it explicitly. To do this we replace the mass density ρ in (3) by (M/e) σ_o, where σ_o is the charge density of electrons (or protons), and e/M is the ratio of the charge to the mass of the proton. We will define σ_o as essentiallt a positive quantity, just as is ρ in (3). Then we obtain.

$$i = -\beta_2 \frac{M\ G^{1/2}}{e\ \ c}\ \sigma_o\ v \qquad (14)$$

where β_2 is a new constant, again of the order of unity.

Now this second hypothesis can be tested in principle by determining the magnetic field of astronomical bodies with different nuclear constitutions (e. g. the earth and the sun, or two stars of markedly different hydrogen content) or (*b*) by measurements of the vertical variation of magnetic field of the earth in mines and under the ocean.

We can, in principle, distinguish experimentally between our two hypotheses by finding which of the two equations (3) or (4)1 gives the same value of the constants β_1 and β_2 for bodies of different nuclear constitution.

From these equations we have

$$\frac{\beta_1}{\beta_2} = \frac{\sigma_o}{\rho} = \frac{\Sigma\ Z}{\Sigma\ W}$$

where Z and W are the atomic numbers and atomic weights of all the component nuclei. In table 5 are given the values of *f* for certain elements and mixtures of elements. If (14) is correct, but if we use (3) for convenience to calculate β_1, then the relative values determined experimentally for any two materials will be given by the corresponding values of *f*. If we compare the value of *f* for limestone ($f = 0.50$) and water ($f = 0.56$), we see that it is not completely excluded that measurements of the field underground and underwater might be made of sufficient accuracy to test between the two hypotheses. If the same value of β_1 is obtained, the first hypotheses is indicated; if the difference is about 11 %, then the second.

TABLE 5

Total Charge to Mass Ratio $f = \beta_1/\beta_2 = \Sigma Z/\Sigma W$

Elements

Element	Z	W	f
H	1	1	1.00
O	8	16	0.50
Fe	26	55.8	0.47
Pb	82	207	0.39

Compounds and Mixtures

Substance	Composition	f
Water	H_2O	0.56
Limestone	$CaCO_3$	0.50
Olivine	$Mg_2Fe_2SiO_4$	0.49
light Stellar Matter . . .	(90% H. by weight 10% other light elements)	0.92
heavy Stellar matter	(40% H. by weight 60% other light elements)	0.62

Of greater immediate interest, however is the calculated difference between the value of f for the earth, assumed mainly of composition similar to olivine ($f = 0.48$), and that for the probable constituents of the sun i. e. 40% of hydrogen by weight ($f = 0.65$). The ratio of these two values of f is 1.29. Now it will be remembered that the value of β_1, for the stars is found to be about 0.68, and for the earth 0.30. So it is not excluded that some part of the observed difference of β_1 for these bodies may be due to the fact that the earth has lost most of its hydrogen while the sun and normal stars have not. If stars exist consisting almost entirely of hydrogen, one would expect exceptionally large fields, i. e. 60% above those of normal stars. If, as has been suggested, the very dense matter in some very dense stars consists mainly of neutrons, formed by the combination of electrons and protons, then on our hypothesis, they should possess a much smaller magnetic field than if of normal matter.

Since Jupiter has not lost its hydrogen, the value of β_1, appropriate to it, will be appreciably higher than that of the earth [1].

(1) See former paper.

The considerations show that future experiments may be able to decide between the hypothesis that the virtual currents depend on the mass of a body or on the electric charges in it.

Using (14) we can calculate β_2 for the earth and the stars. This is most conveniently done by using the already calculated values of β_1 and using the values of $f = \beta_1/\beta_2$ from Table 5. The results are given in Table 6. In all cases η is taken as unity.

TABLE 6

Body	k	β_1	material	β_1/β_2	β_2
Earth	0 88	0.3	olivine	0.48	0 62
Sun (H=30G).	0.16	0 65	40%H	0 65	1 00
5 Stars. . . .	0.16	0.71	»	0.65	1.09

One sees immediately that the difference between the value of β_2 for the earth, on the one hand and the sun and the stars on the other, is smaller than the corresponding difference of the two values of β_1. The values are also considerably nearer unity.

If, however, Chapman's value of $\eta \simeq 1/2$, based on Schwarzchild's distribution of ω inside a star, is correct, then the value of β_2 for the sun and the stars will become about 2.0 instead of 1.0.

It is just possible that the low values of β_1 for the earth might be due to a reduction of the external field by the induced dipoles in the core which, in Elsasser Bullard's theory are the cause of the secular variation.

We conclude therefore that the limited and highly inaccurate data at present available, on the whole supports our second hypothesis, as expressed in equation (14).

Now the quantity $G^{1/2}\,M/e = 0.92 \times 10^{-18}$, appearing in equation (14) represents the ratio between the gravitational mass of a proton and its electrostatic charge, and is a quantity that must clearly play a fundamental part in any future unified field theory and in cosmology, just as the fine structure constant $e^2/\hbar c = 1/137$ plays an essential role in quantum theory. Let us write

$$\frac{G^{1/2}M}{e} = \varepsilon \tag{15}$$

Then, expressing i in electrostatic units, (14) becomes

$$i = -\beta_2\,\varepsilon\,\sigma_o\,v \tag{16}$$

where β_2 is experimentally determined as about 1.0 ± 0.4.

Corresponding to this modification of (3) and (14), we must now modify (1). Our new equation, corresponding to our second hypothesis, is clearly

$$P = 1/2\ \beta_2\ \varepsilon\ E \tag{17}$$

where E can be called the electrical angular momentum of the body defined by

$$E = \int \omega\ \sigma_o\ p^2\ dV \tag{18}$$

when p is the perpendicular distance of a volume element dV from the axis.

This general form would allow one, for instance, to calculate P for a star with any given distribution of elements of varying nuclear constitution in its interior.

The second hypothesis has also the theoretical advantage of expressing the dipole moment in terms of electric as well as mechanical properties of matter (1).

A final highly speculative theoretical argument may be permitted. If the magnetic field of massive rotating bodies has a fundamental origin, then it seems plausible to suppose that the field is in some way bound up with the problem of the relativity of rotational motion. Now it is already proved by Schiff's (2) analysis of Oppenheimer's paradox, that an observer near a fixed spherical condenser, who is rotating relative to the galaxies (i. e. in « absolute » rotation), has to introduce fictitious electric currents, which are everywhere equal and anti-parallel to the real convective currents due to the motion of the charges of the condenser relative to himself (3). These fictitious currents have the magnetic field of ordinary currents, but are not associated with the movement of real charges. Schiff's fictitious currents are in form, but not of course in magnitude, not unlike the virtual currents which we have introduced as the simplest way of explaining the observed facts of the magnetism of rotating bodies. So we see that the conception of virtual or fictitious currents is by no means foreign to the conceptions underlying the treatment of absolute rotation.

(1) See Tzu, *Nature*, **160**, p. 746 (1947) and Arley, *Nature*, **161**, p. 596 (1948)

(2) Schiff, *Proc. Nat. Acad. Sci.*, **25**, p. 391 (1939).

(3) These fictitious currents come into existence by the distortion of the observer's metric by the rotation of the galaxies relative to him. In a closely similar way, a rotating observer has to introduce the fictitious Coriolis and centrifugal forces (Thirring).

Of course, Schiff's virtual currents introduce nothing essentially new into physics and of course vanish for an uncharged condenser (i. e. for a natural body); they are merely one aspect of conventional electromagnetic theory and so cannot provide an explanation of our supposed new phenomena expressed by (16) wich must, of course, essentially be connected with grantational phenomena. To explain this new phenomena, some new feature must be introduced. This new feature must clearly introduce some asymmetry between positive and negative electricity — to explain the actual direction of the earth's field, i. e. the negative sign in (1).

Note. — In almost all « specific » theories, this essential asymmetry enters through the differences of mass of the negatively charged electron and positively charged proton.

It will be remembered that in the early fundamental theories of Schuster, Sutherland, Wilson and Swann, the essential asymmetry is introduced in the form of an arbitrarily assumed difference of the order of 10^{-22} between the forces between positive and negative charges. Though these theories are quite untenable and are long since abandoned, they serve to emphasise the necessity of introducing some asymmetry between positive and negative charges.

Guided by the analogy of Schiff's theorem, we can perhaps usefully introduce this desired asymmetry by supposing that the virtual currents that have to be introduced by a rotating observer are slightly less (i e. by the order of ε) for positive than for negative charges. So for a neutral body, i. e. one with equal numbers of real positive and negative charges, the postulated virtual currents due to the rotation will not cancel exactly the real convective currents, but leave a small excess negative current.

Or to express this suggestion more simply and vaguely, it seems possible that the origin of the field of a rotating body, as observed by an observer rotating with it, lies in a slight difference, of magnitude ε in the behaviour of positive and negative charges when in absolute rotation, but which does not appear at all in pure translational motion.

The fact that the magnitude of this difference $\varepsilon = G^{\frac{1}{2}} M/c$ is proportional to the square root of the gravitational constant, shows that the phenomenon is essentially connected with gravitation, and so would vanished in the limit of $G = O$, just as quantum phenomena vanish in the limit of $h = o$.

H. Dirac [1] is correct in supposing that the large non-dimensional number $1/\varepsilon^2 = C^2/GM^2$ is not a constant but has increased linearly with the age of the Universe, it follows that ε was much larger when the world was young and consequently that rotative bodies had a much larger magnetic field than today.

APPENDIX

THE MAGNETIC FIELD OF WHITE DWARFS

The writer pointed out that if White Dwarfs were formed by the collapse of main sequence stars, one would expect them to possess magnetic fields of the order of a million gauss, and that possibly the great width of the spectral lines of most of these stars might in fact be due to this cause. This arises because one would expect the angular momentum of the star and so, according to relation (1), its magnetic dipole momentum to be conserved during the collapse. Owing to the small final radius of the collapse star, a very large magnetic field of the surface would be expected.

In a private communication Babcock has reported observations of the Balmer lines of 40 Eridani B, using an analyser for circularly polarised light. He found no sign of any Zeeman effect. A. D. Thakeray [2] has obtained spectra of Wolf 1346, and has also failed to find any evidence for any Zeeman effect.

So far then the evidence is against the hypotheses that White Dwarfs have magnetic fields of the order of a million gauss. However, the experiments are not quite conclusive since the analysers used were for circularly polarised light, which is only suitable to reveal a Zeeman pattern if the axis of the star's dipole is nearly along the line of sight. On the average, however, the angle between these two directions will be of the order of 60°. For such large angles the Zeeman pattern will consist mainly of plane or elliptical polarised rather than circularly polarised components, and so would be detected with an analyser for partial plane or elliptical polarisation rather than one for circular polarisation. If the former were used one would expect the *shape* of the line contour to change with the

(1) Dirac, *Nature*, **139**, p. 373 (1934).

(2) Thakeray. M.N. R.A.S. **107**, p. 463 (1947).

direction of the analyser, as contrasted with a *shift* of the line as observed with a circular analyser.

Observation of this kind would be difficult but till they are carried out the question as to whether White Dwarfs do have large magnetic fields must be left open.

It is of course possible that White Dwarfs are not formed by the *collapse* of normal stars. This is the view of Schatzman (1), who considers that they have probably a quite different cosmological origin. Alternatively perhaps some mechanism such as planet or ring formation may have operated to remove a large part of the angular momentum. It seems, however, rather unlikely that such a mechanism can always come into play when a star collapses. It seems widely accepted that novae and supernovae are due to such a process of collapse, brought about perhaps to exhaustion of hydrogen, or possibly by the setting in of energy loss by neutrino emission, as in the theory of Gamow and Schoenberg. If such processes do exist the resulting small and dense stars should have a large magnetic field if they were originally rotating as fast, say, as the sun. It is of course possible that stars of this kind do exist, but then they are not White Dwarfs. An alternative possibility is that the matter in the interior of these stars consists mainly of neutrons and that consequently (according to [14]) the magnetic field is small.

(1) Schatzman. *Annales d'Astrophysique*, **10**, p 93 (1947).

Discussion du rapport de M. Blackett

M. Teller. — The men in the Department of Terrestrial magnetism of the Carnegie Institution in Washington have been representing for some years the field of the earth by a big constant dipole and about a dozen smaller dipoles. These smaller dipoles are located at about 1/2 the radius of the earth, at the interface of the stony and metallic phases.

These smaller dipoles show secular variations.

2. If the magnetic moment of the sun is varying one may espect a variation in the cut-off energy for cosmic rays.

Considering, however, the presence of ions and electrons in interplanetary space one is lead to expect a high conductivity and self-induction. As a result changes in the magnetic field of the sun will be confined to a region near the sun.

The solar field at and outside the earth orbit will stay fixed.

Thus actually no change, in the cut-off energy will occur.

M. Blackett. — The fact that the magnetic field at a given level of the sun's photosphere varies with the time, does not necessarily imply that the external dipole field also does so.

M. Oppenheimer. — One can try to explain the existence of a magnetic field of rotating bodies in three ways:

1. Some complex hydrodynamic and electrical phenomena;

2. A new property of matter;

3. A consequence of the mere fact of rotation, and of the assymetry between electrons and protons. Some effects of this kind were investigated but turn out to be much too small. Thus the Thomas precession is neither of the correct form not correct order. Similarly Follin has investigated the effects of acceleration in aligning the moments of the Dirac electron, using essentially Pauli's general relativistic formulation of the Dirac equation. The calculated effects are wholly insignificant.

M. Peierls. — The equation proposed by Blackett is not symmetric as between positive and negative electricity. This can hardly be a fundamental assymmetry, since it would then be hard to understand why in every other respect the laws of nature are perfectly symmetrical.

Rather one would to ascribe it to the difference in the masses of the positively and negatively charged particles, in a world built of negative protons and positive electrons the effect would be in the opposite direction.

An example of this type of asymmetry would be a slight difference of charge as between proton and electron.

In this case the neutrino which is emitted in decay must have a small charge; it is an interesting speculation how, what limit could be put on such a charge from the failure to detect neutrinos.

This assumption would in any way, however, give no direct explanation of Blackett's term, since the residual charge of « neutral » matter would be neutralized by conduction.

M. Teller. — If the neutrino carries a charge 10^{-18} e the range of neutrinos is expected to be 10^{36} times longer than that of electrons.

Thus a 1 MeV neutrino will have a range of about 10^{34} cm. in condensed matter. By an appropriate inversion of the β process neutrinos should be absorbed in nuclei; at the same time the nucleus in question emits an electron. The cross section for this process is about 10^{-45} cm^2 and the corresponding mean *free* path of neutrinos in condensed matter is about 10^{23} cm.

The stopping effect of a charge 10^{-18} e is therefore negligible even when compared to the extremely small interaction between neutrinos and matter which is predicted by present theory.

It would be practically impossible to detect a charge as small as 10^{-18} e on an isolated particle.

M. Casimir. — I should like to ask how accurately it is known that a non-ionized atom is really neutral. I remember that many years ago De Haas at Leyden showed me some experiments by means of which he had studied this question. A beam of atoms passed through electric fields by which all ions were removed and was then collected in a Faraday cage. The current was found to be zero with a fairly high degree of precision. However, with more modern means the accuracy could probably be much increased.

M. Oppenheimer. — Some experiments on neutrality of mater are in progress in Rabi's laboratory.

M. Blackett. — Recalls the attempt by Sutherland in 1904 and 1908 to explain the earther field by postulating small difference between the forces between positive and negative particles.

M. F. Perrin. — Le signe de l'effet est-il le même dans les étoiles que pour le soleil et la terre?

M. Blackett. — Answers that the sign of the rotation is not known for these stars.

M. Oppenheimer. — The double beta decay experiment of Feiermann on Cd^{113} can give some evidence in favour of neutrality of matter, since it observes the transition $Z \rightarrow Z + 2, + 2$ electrons without neutrino emission.

M. P. Auger. — Vous avez signalé que le moment de rotation orbital des planètes autour du soleil est beaucoup plus grand que celui du soleil sur lui même.

Le champ magnétique ainsi créé d'après votre formule ne devrait-il pas avoir une considérable influence sur le spectre d'énergie des rayons cosmiques qui peuvent atteindre la terre?

M. Blackett. — It is not known wether the rotation of the galaxy (or of the planets of the solar system) give rise to a magnetic field, proportional to the total angular momentum. Babcock assumes that it does. I have a feeling that it is only the rotation of each body about its axis which contributes to the field, not the rotation of the bodies on their orbits. But this is only a guess.

To simplify the essential point at issue, consider a double star, each component of which has zero absolute rotation. Has such a system a magnetic field? My guess would be no.

M. P. Auger. — Il me semble bien difficile de comprendre comment cette formule qui paraît représenter une propriété fondamentale de la matière en rotation s'applique à des corps sphériques en rotation sur eux-mêmes et pas à des rotations orbitales.

Il faudrait que la nature des forces qui maintiennent la matière en rotation sur sa trajectoire soit responsable de la différence, force de gravitation pour les planètes, force de pression (chocs atomiques) pour la matière solaire ou terrestre. Il y aurait alors là un effet de la polarisation électrique due à l'action de la gravitation sur les noyaux des atomes soutenus par les chocs sur leur cortège electronique?

M. Rosenfeld. — Reminds of a note in *Nature* (160, 746, 1947) by Tzu, in which it is shown by simple considerations of invariance and dimensions that a macroscopic theory of gravitation and electromagnetism, involving no other fundamental constants that the gravitation constant G and the velocity of light *c*, can only yield Blackett's formula by introducing a relation equivalent to the proportionality of charge and mass densities, which is unacceptable.

In view of the mushroom-like growth of the more or less fanciful « unified theories » claiming to yield Blackett's formula, this result, although negative, is very useful in providing a simple criterion which allows us to dismiss all such claims.

On the other hand, an example of a theory based on atomic laws and actually yielding a relation between magnetic moment and *spin* angular momentum very similar to Blackett's is provided by Pauli's five-dimensional projective formulation of the fundamental equations of gravitation, electromagnetism and matter.

In fact the generalized Dirac equation for the electron, on this theory, contains an additional term proportional to the combination of constants $G^{1/2}/c$ and giving rise to an extra magnetic moment.

However the net effect of such a term for a system of electrons with random orientation of spins would be much too small to be brought in relation with the earth's magnetic field.

On the Abundance and Origin of Elements

by M. G. Mayer and E. Teller

I.

THE ABUNDANCE OF ELEMENTS

Discussions about the origin of the universe are certainly much older than science itself. In the last few decades the subject has gotten a new formulation under the influence of Einstein's general theory of relativily and a new impetus by the astronomical discovery that the universe is expanding. These astronomical observations, together with others, seem to indicate an age of the universe of approximately 2×10^9 years. The subject of the present discussion will be the abundance of the chemical elements and their isotopes. We shall see that these abundances show some striking regularities and we shall attempt to find out what conditions may have given rise to the observed abundances. In this way we hope to find additional clues as to the state of the universe in its early stages.

The extensive investigations on the abundances of elements have been summarized by V. M. Goldschmidt in 1937 (1). This summary, which includes some of V. M. Goldschmidt's most excellent work, also brings out very clearly the great uncertainty that still continues to exist in our knowledge of the relative abundances of chemical substances. Of course a uniform abundance ratio is not to be expected. The relative abundance of elements found in certain locations will have been influenced by chemical separation processes. Furthermore, according to our present beliefs, the lightest elements

(1) V. M. Goldschmidt, *Det Norske Videnskaps,* Akademi i Oslo, I. *Mat.-Naturv.,* Klasse 1937, No. 4.

continue to be formed and destroyed at present near the centers of the stars.

There are three main sources of evidence for the abundance of elements. First, the abundance ratio in the earth's crust; second the abundance ratio in meteorites; and third, the spectra of the sun and of the stars.

The first of these three lines of evidence is the least reliable. When the earth was formed the overwhelming part of gaseous matter seems to have escaped or else was never any part of the condensing material which formed our planet. Thus, of the elements, H, He, O, N, C (which escaped in the form of CO_2), and of the rare gases, we cannot find a fair sample on the earth. Apparently the oxygen retained was not sufficient to oxidize all of the iron in the earth and the superabundant iron forms at present the liquid core of our planet. It is generally assumed that the metals which are less easy to oxidize than iron are still predominately in the metallic form, and if these metals are soluble in iron then their greater portion will be now found dissolved in the core. Thus, gold is probably rare and valuable not because its abundance in the universe is low, but because it does not easily oxidize and most of it has been dissolved in the liquid iron which fills the space inside the inner half of the earth's radius. Other elements are rare in the earth's crust because they participate in the formation of mixed crystals in the solid mantle of our earth which is composed of the oxides and silicates of the most abundant elements. The numerous elements which then remain are enriched in the outer few miles of our earth's surface. One of these elements for instance is uranium, whose cosmic abundance is not great, but which has been concentrated in the outer layers of our planet for us to use or misuse. The study of abundance ratios of the elements in the earth's crust is helpful for our present discussion only when careful corrections are made as to the chemical separation processes which may have served to enrich or deplete these elements on the earth's surface. One example for fruitful study of relative abundances in the crust of our planet is the investigation of the rare earths. These elements are so similar in their chemical behavior that one may expect to find their relative abundances little changed by the processes in which our earth was formed.

The study of meteorites is for most elements the best source in the assessments of relative cosmic abundances of elements. Indeed, the meteorites are fragments of an old planet which seems to have

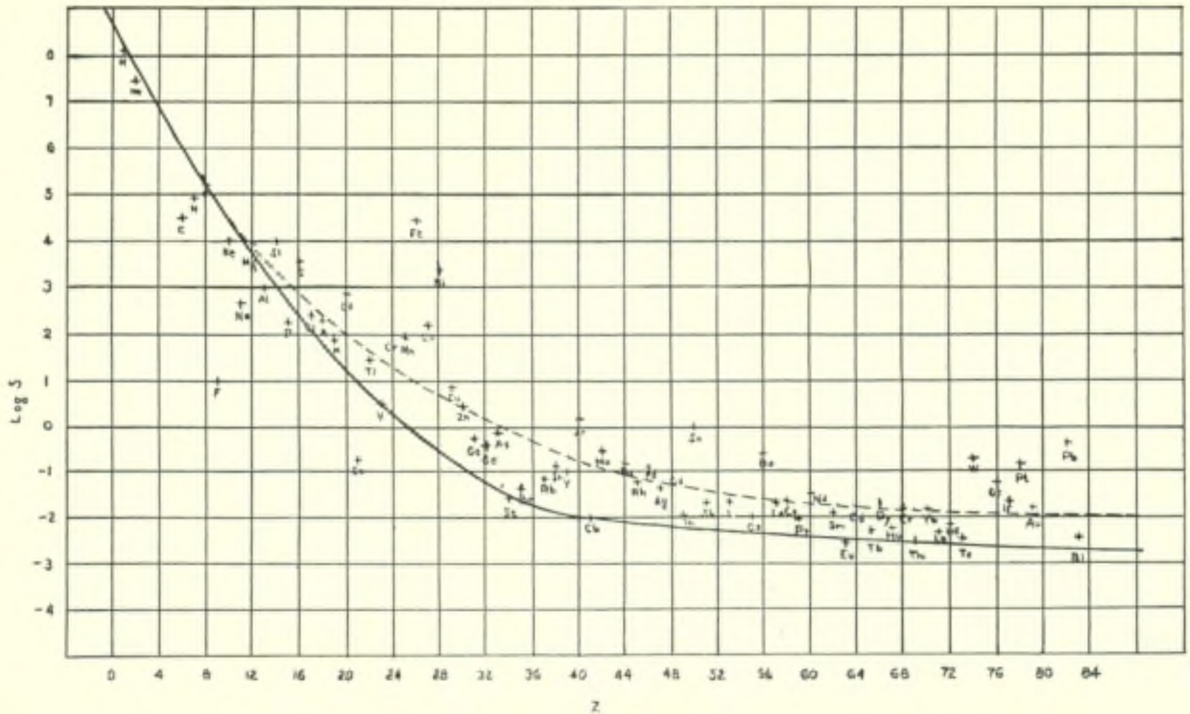


Figure 1.

been composed like our own earth of an iron core and an oxide-silicate mantle. In evaluating abundances the right proportion of the iron and silicate masses has to be established. In order to do that the relative frequencies of the corresponding metallic and stone meteorites have been studied and geophysical data have been utilized. The planet whose breakup has given rise to the meteorites seems not to have been able to retain the volatile elements enumerated in the beginning of the previous paragraph. For the abundances of the non-volatile elements, however, the studies of the meteorites have been used for assessing probable cosmic abundances.

The volatile elements appear to be retained only in the major planets, the sun, and the stars. Indeed, the small density of Jupiter and Saturn as well as the detailed study of equilibrium conditions in the sun and stars indicate that H and He together are likely to form more than 90% of all matter. More accurate data are difficult to obtain for these two elements. The spectra of the sun and the stars are our only source of information on the other volatile elements, and should help us to establish the abundance of H and He. These spectroscopic studies, however, are difficult because the result is strongly influenced by the excitation conditions and reabsorption of the spectral lines. Further progress in the knowledge of the detailed structure of the photosphere of the sun and the stars will undoubtedly lead to more firmly established results.

In Figure 1 the ten-base logarithm of the abundance of the various chemical substances is plotted against their atomic number Z ([1]). As indicated above, most abundances were obtained from meteoritic studies. The abundances of volatile elements are evaluated with the help of solar and steller spectra and the ratio of the rare-earth abundances has been established by analysis of material obtained from the earth's crust. The abundances have been plotted only up to Bi. Elements heavier than Bi are radioactive and their abundances are clearly connected with their lifetimes rather than with the peculiar way in which these elements originated.

Inspection of Figure 1 shows that all elements can be naturally divided into two classes, the heavy and the light elements, with a somewhat arbitrary division line in the neighborhood of Ge or Se. The light elements include all the abundant elements. One might

([1]) This figure has been drawn from data recently compiled by Harrison Brown at the University of Chicago.

discern among the light elements a systematic change of abundance in that the heaviest of the abundant light elements, Fe, is less abundant than the most abundant elements in the lightest group, H, He, C and O. It is difficult to say whether this is really due to a general trend of decreasing abundance with increasing atomic weight, or whether the only real and significant fact is that H, and possibly He, are by order of magnitude more abundant than the other elements. The light elements furthermore show the marked peculiarity that the abundance varies from element to neighboring element quite strongly by factors which occasionally become as big as 10,000. The heavy elements, on the other hand, are invariably much less abundant than the abundance peaks in the light elements. Thus from Fe to Ga the abundance drops by a factor of nearly a million. For heavy elements no great fluctuations of abundance occur. The abundance not only seems to be reasonably similar for neighboring elements, but there also is hardly any trace of a systematic variation. The only element with an exceptional abundance seems to be Re which is less abundant than the average of the other heavy elements by a factor greater than 100. Recently the abundance of Re was re-examined by Dr. Harrison Brown in Chicago. His preliminary results indicate an abundance 200 times greater than that given in this figure ([1]). Thus the single exception from the rule of uniform abundances seems to have been due to experimental error.

Theories of the origin of elements use often as their starting point a smoothed curve of abundances, as indicated in the figure by the dotted line. It is clear that this line deviates from the observed abundances in some regions by much more than the experimental uncertainty, and it does not seem certain whether it is better to try to explain such a smooth curve or to find essentially different explanations for the origin of light elements whose abundance cannot be represented by a smooth function and the origin of heavy elements whose abundance can be represented in first approximation by a constant.

([1]) We are indebted to Dr. Harrison Brown for this communication.

II.

REGULARITIES IN THE ABUNDANCES OF ISOTOPES

The isotopic composition of substances is, on the whole, remarkably constant and one may expect that the observed isotopic compositions actually correspond to averages in the cosmos. There seem, however, to be some exceptions to this statement. Thus, according to spectroscopic evidence, the ratio of heavy and light hydrogen in the photosphere of the sun is less than one part in 100,000, while on the earth this ratio is one part in 5,000. It is not known whether this discrepancy is due to the fact that heavy hydrogen has been used up in the sun by thermonuclear reactions or whether the heavy hydrogen isotope is more abundant on the earth because in the formation of the earth most of the hydrogen escaped, and in the small amount that was retained the heavier isotope was enriched due to effects of the earth's gravitational field.

Another example of the same kind is the abundance of the carbon isotopes. In some cool stars, molecular bands of carbon have been observed. In the vibrational structure of these bands the lines due to C^{12}-C^{12} molecules are well separated from lines corresponding to C^{12}-C^{13} and C^{13}-C^{13} molecules. From the relative intensities of these lines the abundance ratios of the carbon isotopes can be estimated. In some stars these abundance ratios turn out to be within observational error the same as on the earth, that is, the abundance of the heavier isotope is about 100 times smaller than that of the lighter one. Some other stars, however, show isotopic ratios distinctly closer to unity. In view of the fact that carbon can participate in thermonuclear reactions now in progress, this discrepancy in isotopic composition need not have a significant bearing on the discussion of the origin of elements.

Some isotopic ratios have undoubtedly been changed by radioactive decay after the earth's crust has solidified. Differences in isotopic composition of lead minerals is well known. The content of He^3 in natural helium differs according to the origin of the helium. The great isotopic abundance of A^{40} is undoubtedly due to the fact that the bulk of argon escaped when the earth was formed and that most of the argon that is now in our atmosphere is due to the β-decay of K^{40}.

On the whole, however, we are probably justified in identifying observed isotopic compositions with the original ones established at the time when elements were formed.

Interesting regularities in isotopic abundances have been discussed recently by Suess (1). This author makes the assumption that the abundances of heavy nuclear species should lie on a smooth surface if the abundance is plotted against the number of neutrons and protons contained in the nucleus. This hypothesis seems to be in contradiction with the experimental evidence on the abundance of some elements, but Suess shows that a fair agreement may be obtained if the abundances of chemical elements are multiplied by factors well within experimental error. In this process the ratio of isotopic abundances is of course left unchanged, and thus the number of points which come to lie on a reasonably smooth surface is considerably greater than the number of parameters which Suess has adjusted. It should be pointed out, however, that even after the adjustment is performed, some nuclear species fail to lie on the smooth surface.

One of the isotopic regularities which Suess points out is particularly striking. This is : among the heavier elements, the heaviest stable isotope is as a rule much more abundant than the lightest stable isotope. One may say in greater detail : for elements with even Z, heavier than Se, the heaviest isotope is never rare. For the same elements the lightest isotope is almost always rare (that is, less abundant than 1.4%). Exceptions from this last rule are the following : Zr^{90}, with 48% abundance; Mo^{92}, with 14.9% abundance; Ru^{96}, with 4.7% abundance, Nd^{142}, with 26% abundance, and Sm^{144}, with 3% abundance. Of these five exceptions, two, namely Zr^{90} and Mo^{92}, have 50 neutrons, while two others, Nd^{142} and Sm^{144} have 82 neutrons. It has been shown (2) that nuclei containing 50 or 82 protons or neutrons have a remarkably high degree of stability. Nuclei with 50 or 82 particles are also the ones wich deviate most strongly from the smooth surface of abundances constructed by Suess. We do not understand as yet the reason for the peculiar stability of the 50th and 82nd particles but there is no doubt that this extra stability is closely related with the excessive abundances of these nuclear species.

(1) Hans E. Suess, *Z. Naturforschg*, **2a**, pp. 311-321 (1947) and pp. 604-608 (1947).

(2) M. G. Mayer, *Phys. Rev.*, **74**, p. 235 (1948).

III.

GENERAL CONCLUSIONS CONCERNING ORIGIN OF ELEMENTS

The experimental evidence given so far permits us to put forth a number of simple qualitative arguments.

A. The behavior of elements lighter than selenium and elements heavier than selenium is markedly different. For the former, the abundance of elements is violently fluctuating between low and high values; for the latter the abundances are uniformly low. For heavy elements of even Z values the heaviest isotopes are more abundant than the lightest ones; for the lighter elements the reserve statement seems to be more correct. It seems, therefore, probable that the heavy and light elements have been formed by different processes.

B. The light elements may have been formed by thermonuclear reactions. The apparently greater abundance of lighter isotopes may be explained by assuming that the build-up process has proceeded by adding protons to already existing nuclei. The great differences between abundances of light nuclei may be explained by the sensitive dependence at the effective cross sections on the temperature and by the great variability of cross sections for different kinds of processes.

C. There is conclusive evidence that at the time of production of the heavy nuclei, the proportion of neutrons considerably exceeded that which is now present in nuclei. Evidence for this neutron-excess comes from two sources : First, without such a neutron excess it is not possible to understand that the heavy isotopes of heavy elements are much more abundant than the lightest isotopes. Second, in absence of a neutron excess it is very hard to find any method by which the heavy nuclei could be built up at all. The only alternative would be to build up the heavy nuclei by reactions between charged particles. Such reactions would, however, require extremely high temperatures, and at these temperatures it is not possible to prevent disintegration of uranium and other fissionable nuclei. A neutron-excess within the heavy nuclei may stabilize them against fission.

Of the three statements proposed above, the last one seems inescapable. In fact, this last conclusion forms a part of every theory on the origin of heavy elements which so far has been proposed. The first two conclusions are more doubtful. In fact, previous theories on the origin of elements have made assumptions which to a greater or lesser extent disagree with these two conclusions. We shall now proceed to discuss the theories that have been proposed to explain the origin of elements.

IV.

THE NEUTRON-CAPTURE THEORY

It has been proposed by von Weizsacker many years ago that all nuclei may be built up by successive neutron capture followed by β-decay. This theory has been discussed repeatedly. Recently it has been elaborated in considerable detail by Gamow and collaborators (1). According to this theory the main factor determining the abundance of elements is a competition between neutron capture and β-decay of the neutrons. In order to calculate the former process it has been assumed that at the time at which the elements were formed the universe was in a dense and rapidly expanding state. By proper adjustment of the original temperatures and densities one obtains abundances shown in Figure 1 by the solid curve. This curve of course deviates from actual abundances in some regions quite strongly, but is nevertheless a good representation of the smoothed distribution. The curve drops for light elements with increasing nuclear charge, and then stays nearly constant in the region of heavy elements. This is due to the behavior of the capture cross section for fast neutrons. For elements in the first row of the periodic table, these cross sections are small, they increase rapidly with nuclear mass, and obtain a considerably higher and roughly constant value for heavier elements (2). Thus, under neutron

(1) R. A. Alpher, H. Bethe and G. Gamow, *Phys. Rev.*, **73**, p. 803 (1948); R. A. Alpher and R. Herman, « On the Relative Abundance of the Elements » (in press); G. Gamow, « The Evolution of the Universe », *Nature* (in press); R. A. Alpher and R. Herman, « Thermonuclear Reactions in the Expanding Universe » (in press); R. A. Alpher, « A Neutron-Capture Theory of the Formation and Relative Abundance of the Elements » (in press).

(2) D. J. Hughes, *Phys. Rev.*, **70**, p. 106*a* (1946). See also MDDC 27, Apr. 29, (1946). Used 1MeV pile neutrons.

bombardment, more light nuclei will be found in a steady state than heavier ones, while the abundance of the heavier elements is roughly constant. Eventually the light nuclei should be used up and all matter should be transformed into heavy nuclei. Before this happens, however, the bombarding neutrons transform into protons and element formation stops.

Gamow's theory also seems to give an explanation for the particulary great abundance of elements containing 50 or 82 neutrons or protons. Indeed it has been observed that such nuclei have low neutron capture cross sections and are thus again found with greater abundance in a steady state. This explanation, however, is open to the following criticism. In the steady state which we are considering, neutron capture does not lead directly to the observed nuclei, but rather to nuclei containing a greater number of neutrons from which the stable nuclear species are obtained by β-decay. One should therefore expect that small neutron cross sections for nuclei with 50 or 82 neutrons or protons should lead to great abundance of nuclei which are obtained from the subsequent β-decay. For this reason, according to Gamow's theory nuclei containing somewhat fewer neutrons than 50 or 82 and somewhat more protons than 50 or 82 would be particularly abundant. This, however, does not correspond to observations.

In order to obtain the correct ratio of elemental abundances, Gamow, Alpher, and Herman assume that the density ρ varies as $\frac{4.8\times10^{-4}}{t^{3/2}}$ gm/cm^3 where t is the age of the universe in seconds. The temperature T varies as $\frac{1.5\times10^{10}}{t^{1/2}}$ ^{0}K. Assuming these values one finds that in the initial stages of the universe the density of radiation is much greater than the density of matter. Under these conditions the rate of expansion and the variation of temperature with time is given by formulae in which only the velocity of light and the constant of gravitation enter. The only parameter which Gamow adjusts is the constant entering in the expression for the density. Using this formalism, Gamow attempts to explain not only the origin of elements but also the formation of galaxies. The galaxies are formed, according to Gamow, by gravitational instability at the time when expansion of the universe has caused sufficient cooling.

The scope of phenomena considered in Gamow's work is very great. Yet not all details of the abundance of elements are satisfactorily explained by the simple neutron-capture process. Gamow and his collaborators attempted to introduce corrections in the neighborhood of the lighter elements. They believe that subsequent thermonuclear reactions will change abundances of these lighter elements and thus lead to the marked deviations of observed abundances from the solid curve in Figure 1.

Another difficulty arises when one attempts to explain by Gamow's theory isotopic abundances. In fact, the simplest formulation of this theory will not explain why in nature two, and sometimes even three, stable isobaric nuclei are found. According to Gamow's theory one would indeed expect only the formation of the least charged isobar. Actually the more charged member of the isobaric pair is usually less abundant than the isobar carrying lesser charge, but there are some marked exceptions from this rule, and the great frequency with which isobaric pairs occur require in any case an explanation. One may assume that the heavier isobar has been formed in an isomeric state which could continue to increase its charge by β-disintegration. However, almost all isobaric pairs have even mass number and the stable isobars are nuclei containing an even number of protons and an even number of neutrons. Nuclei of this kind hardly ever seem to show the phenomenon of isomers.

Another explanation for the formation of the heavier isobars was proposed by Fermi and Turkevitch (1). Let us consider the formation of an isobar containing Z protons and N neutrons. The formation of this isobar is prevented by the stability of a nucleus with $Z - 2$ protons and $N+2$ neutrons, the latter nucleus forming the end-point of β-decay chains for nuclei with mass $Z+N$. Let us now consider the nuclei with mass $Z+N-1$. The β-decay chain of these nuclei may end in a nucleus of charge $Z-1$ and neutron number N. This nucleus might capture one of the late-comers among the neutrons, thus forming a nucleus of charge $Z-1$ and neutron number $N+1$. The nucleus of charge Z and neutron number N can be now formed by β-decay. This explanation breaks down, however (2), in case the β chain for nuclei with mass $Z+N-1$ ends in a stable nucleus of charge $Z-2$ and neutron number $N+1$. Thus

(1) This explanation actually preceded Gamow's work.

(2) The above objection has been raised by Fermi.

it seems extremely hard to explain on the neutron capture theory the formation of a nucleus like Mo^{92}. The β-decay chain for mass 92 ends in Zr^{92}, while for mass 91 the end point of the chain is Zr^{91}. The possibility still remains that Zr^{91} may have an isomeric state, and in this way, Mo^{91} may be formed by β-decay. Such an isomeric state, however, does not seem to be found up to the present time. The abundance of Mo^{92} is 14.9% and we are therefore faced with the necessity of explaining the presence of a fairly abundant isotope. It should also be pointed out that the same situation as discussed in the case of Mo^{92} is encountered quite often.

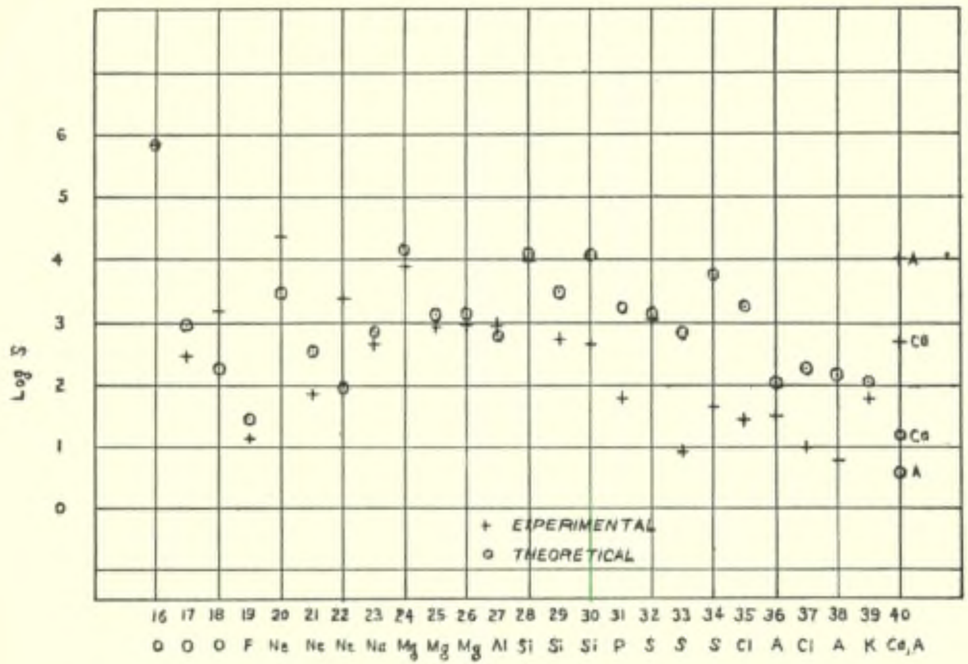
It is by no means excluded that the discussion of secondary processes such as thermonuclear reactions and photoneutron emission will give a satisfactory explanation of isotopic abundances, the great abundance of elements containing 50 or 82 particles, and the great fluctuation of abundances among the lighter elements. Gamow's theory of the origin of elements cannot, however, be considered final as long as these points remain to be cleared up.

V.

EQUILIBRIUM THEORIES

There is an unmistakable correlation between binding energy and abundance of nuclei. This fact has led to the early suggestion that chemical elements were formed in thermodynamic equilibrium ([1]). These theories showed some success in explaining relative abundances of lighter elements. A good example of this calculation is shown in Figure 2, which represents the results of calculations by Lattes and Wataghin for nuclear abundances between oxygen and calcium. The correspondence between the experimental and theoretical is evident, but differences of more than a factor of 10 occur occasionally. In many cases these discrepancies cannot be due to

([1]) J. Chandrasekhar and L. Heinrich, *Astrophys. J.*, **95**, p. 288 (1942); G. Wataghin, *Phys. Rev.*, **66**, p. 149 (1944); C. Lattes and G. Wataghin, *Phys. Rev.*, **69**, p. 237 (1946) and **70**, p. 430 (1946); C. F. von Weizsacker, *Phys. Zeitschr.*, **38**, p. 176 (1937); **39**, p. 633 (1938); C. B. van Albada, *Bull. of Astr. Inst. Netherlands*, **X**, p. 374 (1946) and *Astrophys. J.*, **105**, p. 393 (1947); O. Klein, G. Beskow and L. Treffenberg, *Arkiv. f. Mat., Astr. o Fysik, Bd.*, **33B**, No 1 (1946); G. Beskow and L. Treffenberg, *ibid. Bd.*, **34A**, No. 13 (1946) and *Bd.*, **34A**, No. 17 (1947); H. Jensen and H. Suess, *Naturwissen.*, **32**, p. 374 (1944) and **34**, p. 131, (1947).



Log S
0 1 2 3 4 5 6
+ EXPERIMENTAL
o THEORETICAL
A
Ca
16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40
O O O F Ne Ne Ne Na Mg Mg Mg Al Si Si Si P S S S Cl A Cl A K Ca,A

Figure 2.

experimental uncertainties, since ratios of isotopic abundances rather than abundances of elements are involved.

The simple equilibrium theory is, however, completely inadequate to explain the existence of heavy elements. In fact, 1 nucleus like uranium has both a high energy and a high free energy. The nucleus can disintegrate into fission fragments by liberating almost 200 MeV. Even if we should assume exceedingly high temperatures, so that the Boltzman-factor will not prevent the formation of uranium, we should still fail to form this element unless we also introduce densities comparable to those found in atomic nuclei.

It has been suggested that heavy and light elements have been formed under different conditions, that is, at different temperatures or at different densities. One of these possibilities has been worked out in great detail and with considerable success by Klein, Beskow, and Treffenberg (loc. cit.). They assume that elements were formed in a very hot and isothermic star, in which the density varied from a high value near the core to a low value near the surface. The heavy elements were formed near the core where the density actually was comparable to the densities of nuclei. The light elements, on the other hand, were formed near the surface of the star.

The great densities found near the core, together with the Pauli exclusion principle, will force electrons into states of exceedingly high kinetic energy. Thus it becomes energetically favorable that these electrons shall be captured by protons and form neutrons. In this way the thermodynamic equilibrium leads to a great neutron excess near the core. Thus, we see again that the formation of heavy elements is tied to an excess number of neutrons. This neutron excess also helps to explain the fact that uranium and trans-uranic elements have survived in spite of the possibility of fission. Indeed, the elements formed near the center of this star were elements much more rich in neutrons than are our present elements. Neutron rich isobars of uranium are likely to be much more stable with respect to fission. Uranium has been formed, according to this theory, at a later stage by β-decay. At this later stage, the temperature may have been low enough and nuclear collisions rare enough so that the uranium nucleus would have had a good chance to escape any excitation that would have led to fission.

Equilibrium theories are quite capable of explaining great fluctuations of abundances in the region of light elements. In building up heavy elements, however, these theories have to postulate great neutron excesses. One difficulty mentioned in connection with the neutron capture theory must be raised therefore against the equilibrium theory also. In order to obtain the nuclei as we know them today, we must assume that at some time the temperature must have dropped and the reaction rates could no longer maintain equilibrium. At the beginning of this stage, great densities and neutron excess must have been still present, otherwise, one could not explain the survival of heavy elements and in particular of uranium. If we assume that the density decreased after the temperature had dropped, then we are led back to the conclusion that the present nuclear species must have been formed from earlier heavy nuclei by a series of β-decays. In such a situation, however, one would expect that of isobaric pairs or triplets only the one carrying the least number of protons can be formed. One cannot consider the equilibrium theories as complete as long as a detailed and successful discussion of the later stages of element formation is missing.

VI.

PRODUCTION OF HEAVY ELEMENTS BY FISSION

If you find on a desert island a human footprint in the sand, two explanations seem possible. First, the particles of sand might have arranged themselves by chance in the pattern of a footprint. We object to this explanation as having a very low probability. The second, and more usual explanation, is to assume a human being who has made this footprint. This human being of course, considered as a statistical assembly of particles, has a very much lower probability even than the footprint which we wanted originally to explain. This method of explaining an improbable situation by another vastly more improbable one is, in fact, one of our normal procedures in drawing conclusions concerning the past (1).

In the previous section it was pointed out that it is difficult to explain the formation of the uranium nucleus since it has both a high energy and low a-priori probability. In order to explain its stability we were led to the picture of a substance containing a great excess of neutrons and having a density comparable to that of atomic

(1) The illustration given above is due to Weizsacker.

nuclei. This substance, considered as a statistical assembly, is in itself vastly more improbable than the uranium nucleus. We shall call this substance Ylem, a word introduced by Gamow to designate the primordial substance from which elements were made (1).

We shall now consider the breakup of Ylem into atomic nuclei. In doing so, we shall disregard the gravitational processes which have been utilized in the work of Klein and his collaborators. Actually, at the time that Klein's star breaks up, gravitational forces may be unimportant in determining the process of element-formation. On the other hand, it is not necessary for the present discussion that the Ylem should have formed at any time the core of a star. Its origin might have been different.

It is not necessary definitely to fix the ratio between neutrons and protons in the Ylem from which we want to form the heavy elements. In the next section we shall indeed discuss in some detail the possibility that this substance contained an extremely great excess of neutrons, and we shall see that the final result may be expected not to depend sensitively on the original ratio of neutrons and protons.

In the present section we shall make a few simple assumptions about the fission of this primordial substance and subsequent processes, and we shall find that these simple assumptions will enable us to calculate isotopic ratios. In particular, we shall assume that fission proceeds qualitatively in the same way as has been observed in the case of uranium, that is, breakup of the nucleus is followed by neutron evaporation and this in turn is followed by β-decay, leading finally to stable nuclei. The significant difference between our present picture and that of usual fission is that we are dealing here with a substance in which the neutron excess is much greater and in which also neutrons are presumably bound with much smaller energies than they are bound in atomic nuclei. Therefore, one must assume that evaporation of a great number of neutrons will follow the fission process. The great number of neutrons as well as fluctuations of energy due to the fission process itself makes it plausible that the energy available for the last evaporation processes should have a Gaussian distribution.

(1) According to Gamow, Ylem is an obsolete English noun. Professor Pauli pointed out that the concept of a primordial substance was discussed by the Greek philosophers under the name ὕλη.

More precisely we assume that after a number of evaporation processes the resultant fixed, but not yet stable, isotope will be left with a Gaussian energy distribution. Then the probability P(Z,N) of the process terminating at a definite isotope with a neutron number N is

$$P(Z,N) = K_Z(E_{N,Z} - E_{N-1,Z})e^{-1/\alpha^2(E_{N,Z}-E_o)^2} \quad (1)$$

$E_{N,Z}$ is the binding energy of an isotope containing N neutrons; E_O is the binding energy of the nucleus for which P(Z,N) is a maximum. Both E_O and α, the spread of isotopes, are functions of Z. K_Z is a normalization factor. The factor $(E_{N,Z}-E_{N-1,Z})$ is the binding energy of the last retained neutron. Formule (1) is valid only if the spread, α, corresponds at least to several units, in which case the probability of evaporating down to N, but not to N—1 neutrons, is proportional to the binding energy of the last neutron.

The nuclear energies were obtained from the semi-emperical formula for the mass of a nucleus ([1]).

$$M = A - 0.00081\ Z - 0.00611\ A + 0.014\ A^{2/3}$$
$$+0.083\left(\frac{A}{2}-Z\right)^2 A^{-1} + 0.000627\ Z^2\ A^{-1/3} + \delta \quad (2)$$

where $\delta = 0$ for A odd

$\delta = -0.036\ A^{-3/4}$ for A even Z even

$\delta = +0.036\ A^{-3/4}$ for A even Z odd

From this formula, the value of Z for which the energy is a minimum at constant A can be calculated. These points of Z and A will be referred to as the stability line. The observed asymmetrical distribution of elements can be explained only if the maximum of the Gauss distribution lies at higher neutron numbers than correspond to the stability line. In order to introduce the least possible number of parameters we assume that the total binding energy for the most probable end-point of evaporation exceeds by a constant amount the binding energy on the stability line at equal Z value. The best agreement was obtained by assuming for this energy difference .036 mass units. This roughly corresponds to an excess of 5 neutrons. We also assume a constant value for α equal to .024 mass units.

([1]) N. Bohr and J. A. Wheeler, *Phys. Rev.*, **56**, p. 426 (1939); von Albada, *Astrophys. J.*, **105**, p. 393 (1947).

During the evaporation process the nuclear charge does not change. We assume that within the range of applicability of our calculations all even Z values are produced as fission fragments with equal probability and no odd Z values are produced. A justification of favoring the even Z values lies in the fact that at great neutron excesses pairs of protons are likely to form configurations similar to α particles, while a single proton can be found in a configuration similar to that of a triton nucleus. These configurations are likely to be better approximations to the actual wave functions in the Ylem than they are in atomic nuclei. Indeed, in Ylem neutron binding energies are low and thus the substance will have a more loose structure, so that α— particles and even tritons might form a more self-contained sub-unit than is the case in actual nuclei. The great difference in binding energy between the α— particle and the triton then helps to explain why only even nuclei are formed.

The assumption that all even nuclei are produced with equal probability was made in order to explain the constant abundance of the heavy elements. There is no reason to assume that fission fragments less heavily charged than Germanium should not be formed. The great abundance of these light elements, however, seems to indicate that they may have a different origin. In this case, the isotopic ratios of light elements will not be influenced strongly by the relatively few light nuclei which have been formed by fission.

Having obtained from Formulae 1 and 2 the distribution of nuclei at which neutron evaporation stops, we can obtain the stable nuclei by considering the subsequent β-decay processes. If these β-decays have started from a nucleus with a great neutron excess then the β-decay process can of course not go beyond the stable isobar carrying the least positive charge. The more heavily charged isobars are explained in our theory as due directly to neutron evaporation. The fact that they are relatively rare is a consequence of our assumption of a Gaussian distribution for the energy available for the evaporation and of the values of the constants which appear in Equation (1). These constants have been so chosen that isobars with greater Z—values can be formed by processes which correspond to the high energy tail of the Gaussian distribution.

In Figure 3 relative abundances of isotopes have been plotted for

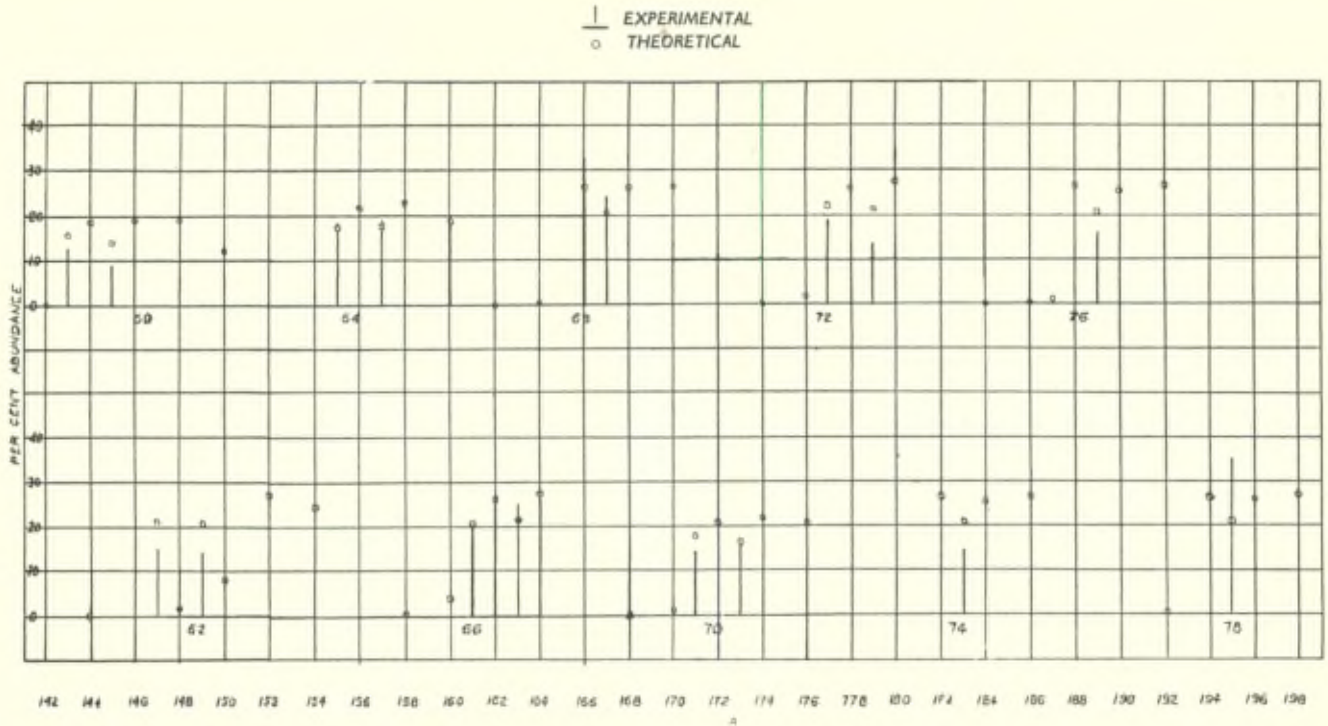


Figure 3.

a number of even elements [1]. The circles correspond to calculated abundances, the lines show the experimental values. Below each group of lines the nuclear charge is indicated. We see that in some cases an extremely good agreement is obtained. The best example of a good agreement is $Z = 64$. It would be, however, strange if such good agreement would be obtained in all instances. Our assumption about the Gaussian distribution need not correspond quite closely to reality. The values of energies as obtained from Formula (2) are certainly only approximate. We have also neglected in our discussion possibility of delayed neutron emission following a β-decay. Finally, we have neglected any secondary processes such as, for instance, recapture of an evaporated neutron by another nucleus or photoneutron emission. Indeed, discrepancies are found in almost all elements. For $Z = 62$ we find the remarkable phenomenon that the even mass number isotope of mass 148 is less abundant than the two neighboring odd mass number isotopes. This is found experimentally and also by theory. The theory however predicts much too low an abundance for this isotope. This is due to the fact that a more lightly charged 148 mass-number isobar exists which intercepts the β-decay chain. It seems that this process of interception is significant, but that some delayed neutron emission or neutron capture was sufficient to produce a considerable change in the abundances.

The lightest isotope of samarium (mass number 144) has an abundance of 3%, whereas theory predicts a negligibly small value. This is apparently due to the fact that Sm^{144} contains 82 neutrons. Our calculations have consistently given much too low values for the abundances in every case where 82 or 50 neutrons are present. This is strikingly shown in the abundance of isotopes of element 60, in which mass number 142 happens to be the most abundant one rather than the least abundant, as predicted by theory.

Another element in which considerable discrepancies are observed is element 78. The reason for discrepancies in this case seems to be unclear. Part of it may be due to the simplifying assumption of using constants rather than functions of Z in the Gaussian (Equation 1). The magnitude of these discrepancies indicates the considerable influence of further corrections which will have to be employed

(1) Calculations for this figure were performed by R. W. Christy at the University of Chicago.

before the calculations here initiated can be brought to a successful conclusion.

For odd mass numbers the chains of β-decay disintegrations may end with equal probability in even or odd-Z-values. Since in the region here considered no nucleus containing an odd number of neutrons and an odd number of protons is stable with restect to β-decay, all the even mass-number nuclei will end with an even charge. If we should assume that among the primary fission products odd and even mass numbers are found with equal probability we shall be led to the conclusion that among even Z-values odd mass numbers are half as probable as even mass numbers. The actual average abundance ratio of even and odd mass numbers for even Z-values seems to be closer to 3:1 when an average over all heavy nuclei is taken. This is indeed to be expected, since according to Formulae 1 and 2 it is more likely that the neutron evaporation processes should stop at an even-neutron nucleus than at an odd-neutron nucleus. Since all original nuclei have been assumed to have even Z-values even mass numbers are favoured, and the greater value of the ratio of even to odd isotopes in nuclei having Z-values can be understood.

VII.

THE POLYNEUTRON MODEL

In the previous section we have assumed a state of matter, the Ylem, from which heavy elements were formed by fission. In trying to make more detailed statements about the Ylem one necessarily enters the realm of speculation. We shall, nevertheless, try to discuss this question, knowing that the statements made here will be hard to verify and are not al likely to be correct in every detail.

As a starting point, we shall assume a neutron liquid of a size very great compared to the atomic nuclei. In order to avoid the necessity of discussing the effects of gravity, we assume that the mass of the neutron liquid considered is small compared to the mass of a star.

It will be convenient to assume that this neutron liquid is stable in the sense that it has a lower energy than the separated neutrons. This assumption is contrary to current ideas about nuclear structure. In fact, the di-neutron is, according to the present theories of nuclear forces, unstable by approximately 10^5 eV. The uncertainties of present nuclear theory however leave room for the possibility that the di-neutron might be stable. If the di-neutron is stable and has a small energy, its production and detection would be extremely hard. Furthermore, even the instability of the di-neutron does not prove that a collection of more than two neutrons is unstable. Thus two α particles do not form a stable compound, but three or more α particles do. Finally, the stability of the polyneutron might be due to gravitational effects.

The polyneutron will certainly be unstable with respect to β-decay. Transformation of a neutron into a proton will not only liberate an energy corresponding to the mass difference between these two particles, but may in a single step lead to the formation of a triton-like structure whose energy is further lowered by « solvation » effects due to surrounding neutrons. Thus, the transformation energy in the β-process may be as great as 10 MeV. In a second disintegration, a solvated α particle will be formed liberating an energy exceeding 20 MeV. If the electrons emitted in the β-process should leave the polyneutron, then the accumulating charge soon will make further β-processes energetically impossible. We have, however, assumed that the polyneutron is of considerable size. Since the intercation between electrons and neutrons is certainly small, there is no reason why the electrons should not stay inside the polyneutron and why they should not neutralize by their average charge the charge distribution due to the protons formed by β-decay.

As the process of β-decays continues, more and more electron orbits within the polyneutron will be filled up and the only remaining available orbits for electrons will be orbits of high kinetic energy. When this kinetic energy becomes equal to the energy liberated in the β-transformation that leads to a solvated triton, further β-formations will become energetically impossible.

The electron distribution will not remain completely confined to the polyneutron. Due to the high zero-point energy of the electrons the electron distribution will protrude from the polyneutron giving rise to an electron cloud near to the surface outside of the polyneutron. This electron distribution is a rudimentary form of the extra-nuclear electrons which we find in atoms.

The distribution of electrons near the surface of the polyneutron can be crudely described with the help of a relativistic Thomas-Fermi model

$$\Delta\varphi = \frac{1}{K^2 e^2}\varphi^3 \tag{3}$$

where φ is the average potential and K is a dimensionless number.

$$K = \left[\frac{3\pi}{4}\left(\frac{\hbar c}{e^2}\right)^3\right]^{\frac{1}{2}} = 2462 \tag{4}$$

Neglecting effects of curvature, one obtains

$$\varphi = \frac{\sqrt{2}\,Ke}{r + r_o} \tag{5}$$

where r is the distance of a point outside the polyneutron from the surface of the polyneutron and r_o is an integration constant which by conditions of continuity at the surface is connected to the potential φ_i inside the polyneutron by the equation

$$r_o = \frac{\sqrt{2}\,Ke}{\varphi_i} \tag{6}$$

The presence of the electrons outside the surface of the polyneutron is according to our theory the reason for the mechanical instability of the polyneutron. This can be seen by considering a surface wave on the polyneutron. Such a wave will increase the surface and will allow more neutrons to extrude from the body of the polyneutron. Therefore, the energy is lowered. This phenomenon can be described by a negative surface tension σ_e, which can be evaluated from the Thomas-Fermi model. One obtains

$$\sigma_e = -\frac{1}{6\pi}\,\frac{K^2 e^2}{r_o^3} \tag{7}$$

Attractive forces between the neutrons within the neutron liquid will give rise to a positive contribution of the surface tension. By assuming that two neutrons are found within each de Broglie wave length cube of the polyneutron and assuming that the zero point kinetic energy of the neutrons within the polyneutron is approximately equal to the binding energy of the neutrons, one can estimate the positive contribution to the surface tension in terms of the binding energy of the neutron within the polyneutron. One finds that the positive contribution to the surface tension becomes equal to the negative contribution due to the protrusion of the electrons if the binding energy of the neutron within the polyneutron is approximately equal to 1 MeV. It is, therefore, possible to obtain a model in which the neutrons are weakly bound and in which the total surface energy is negative. Under these conditions, surface waves on the boundary of the polyneutron will tend to grow exponentially with time and droplets will break off the polyneutron. It is this droplet-formation which we believe is responsible for the heavy nuclei. Thus, the word fission, introduced in the previous section, is a misnomer. The polyneutron does not divide into two approximately equal parts, but instead small droplets break off from its surface.

It is possible to estimate the charge which one of these droplets will carry. The velocity of growth of surface waves will depend on the wave length. It is easy to show that the exponential growth will occur according to a formula $e^{\omega t}$, where ω is given by the equation

$$\omega = \left(\frac{|\sigma|}{\lambda^3 \rho}\right)^{\frac{1}{2}} \tag{8}$$

Here $|\sigma|$ is the absolute value of the surface tension, ρ is the density of the liquid, and λ is the wave length of the surface waves. It is seen that waves of shorter length will grow faster and correspondingly, formation of smaller droplets is preferred.

The shortest wave lengths, however, will not grow at all. As soon as the wave length becomes small compared to the average distance to which electrons protrude from the polyneutron, the formation of the wave will not permit further extrusion of electrons. Thus, for such short wave lengths only the positive, or cohesion terms in the surface tension will be important and the short wave length surface perturbations will have no tencency to grow.

The average distance to which the electrons protude from the polyneutron is given by r_o in Equation (6). If the wave length is long compared, Formula (8) shows that the growth of the wave is slow. If, on the other hand, the wave length is short compared to r_o the surface perturbations will not grow at all. It is, therefore, plausible to assume that most droplets will be formed with an approximate diameter equal to r_o. Calculating the total charge contained in the polyneutron within a sphere of diameter r_o, one obtains for the charge Z_p carried by a primary fragment

$$Z_p = \frac{2^{\frac{1}{3}}}{12} K = \frac{1}{12}\left[\frac{3\pi}{2}\left(\frac{\hbar c}{e^2}\right)^3\right]^{\frac{1}{2}} = 290 \tag{9}$$

We see, therefore, that the charge carried by a droplet does not differ in order of magnitude from the charge of heavy nuclei.

Within this primary fragment there are still contained a great and indeterminate number of neutrons. For our theory the only important point is that most of these shall be lost by evaporation before the β-decay is started. At first it might seem surprising that starting from an excess of neutrons which may be as great as several hundred per cent, one will nevertheless end up rather regularly with nuclei carrying only approximately 5 excess neutrons. Actually, the quantitative situation is as follows: the evaporation of the first neutrons requires very little energy because these neutrons are so lightly bound. In the total evaporation process there will be involved a few hundred million volts. In order to get agreement with observed abundances, we must assume that on the average the energy available is 30 or 40 MeV less than would suffice to reach the stability line. Near the end of the evaporation process the remaining energy is distributed on a Gaussian whose half-width is about 20 or 30 MeV. These are rather specific requirements, and it seems almost hopeless to derive them as a necessary consequence from our polyneutron model. In a detailed discussion to be published in the *Physical Review*, we are showing that the quantitative situation described is not in contradiction with the properties and hydrodynamic behavior of our model.

VIII.

CONCLUSION

It will appear from the above discussion that there does not as yet exist a detailed and satisfactory explanation of the origin of elements. One of the statements made earlier in this paper shall be reemphazized here: At the time of formation of the heavy elements, a great excess of neutrons must have existed. This statement can be considered proved with reasonable certainty. The separation of elements into the class of light and heavy elements and the hypothesis of a completely different origin for these two classes may be worthy of serious consideration. The explanation given for the isotopic composition of heavy elements might be considered as a first step in elucidating a considerable body of well-established experimental measurements. The speculations on the polyneutron model might be excused as a radical attempt to clear up the non-equilibrium processes which must have played an essential part in the formation of elements.

Institute for Nuclear Studies,
The University of Chicago, October 1948.

Discussion du rapport de M. Teller

M. Klein. — As Prof. Teller has already mentioned we have in Stockholm tried to develop the thermo-dynamical theory of the abundance distribution of the elements by regarding the thermal and gravitational equilibrium of a star of very high temperature ($kT \simeq 1$ MeV). As is known from the classical work of Emden on gas spheres there is on ordinary assumptions no solution corresponding to an isothermal star model of finite mass, the density at large distances R from the center being $\sim 1/K^2$. A necessary attribute of a temperature equilibrium of the high temperature mentioned is, however, a radiation field (consisting of electromagnetic radiation, electron position pairs and neutrinos) the mass density of which is of the order of magnitude $10^6 g/cm^3$. A star embodded in a sea of such δ radiation would now have a finite mass due to the gravitational effect of the radiation.

In this connection the problem arises of how much dense radiation would be maintained in a state of approximate equilibrium, since an Einstein closed universe of such high density would only have a mass $\simeq 10^5$ cm. masses and a radius not very much larger than that of the star models considered in the work of Beskow and Treffenberg. On the other hand a universe considerably larger than this would expand extremely rapidly, the density falling to half its value in a time of the order of magnitude of a second.

The only way to accumulate the great amount of radiation mass required in order to keep the stars at a temperature of the order of magnitude of 1 MeV, would be to surround each star or small groups of stars separately by such radiation, i. e. to look for solutions of the Einstein gravitational equations corresponding to stars consisting of radiation kept in a state of approximate equilibrium by its own gravitational field.

As a preliminary to such an investigation a statical centrally symetrical solution has been studied, in which the radiation density, at the centre is given. While this solution, when the central temperature is taken to be 1 MeV, gives a strong concentration of the radiation within a sphere of radius 10^{10} cm., it suffers from the same disadvantage as the Emden solution for an isothermal gas sphere, the density approaching for large distances from the centre a singular solution proportional with $1/R^2$. It is not quite clear how this difficulty may be avoided, but we are now planing an investigation of solutions, which are not strictly statical and also such with several concentrations corresponding so to say to a system of radiation stars.

Turning now to the star models calculated by Beskow and Treffenberg in the basis of statical mechanics, taking account of the gravitational equilibrium, I shall stress that these models, the temperature given, are completely determined by a single parameter defining the conditions at the centre. Thus in same of their models they have fixed the value at the centre of the chemical potential of the neutrons, while in other models they have assumed a core of given radius consisting of condensed nuclear matter. In doing this they have used an extrapolation from the known mass defects from which follows that nuclear matter of arbitrary sized is formed at a given value of the neutron potential, which even at the lowest temperature would correspond to a positive pressure. In this respect the assumptions differ from those forming the basis of Teller's model. On the other hand the presence of such a core in the Beskow Treffenberg models may perhaps help to remove some of the difficulties of the thermodynamical theory pointed out by Prof. Teller.

Turning now to the question of the transition of the state of quasi-equilibrium considered in the thermodynamical theory to the present conditions of the universe we may perhaps be allowed to assume that its first stage consists of a slow breaking away of the radiation, which gradually will increase in rate when the radiation density and the corresponding gravitational fields are decreasing, the last part of this process becoming explosive in character since the radius of the star models is only about one light second. The disappearence of the radiation will make the star and especially its outer parts mechanically instable, a very rapide expansion being the result. At the same time the nuclear chemical equilibrium will be disturbed, above all through the disappearance of the neutrinos, whereby the neutron excess will soon be removed. In this way it would not seem excluded that reaction velocities will be sufficiently lowered to make the freezing in the main features of the equilibrium possible. If in order to maintain a quasi-equilibrium of the during a relatively long period we assume that the separate stars with their intense radiation are embedded in weaker radiation and the stellar systems again in still weaker radiation it is tempting to assume that not only each star but also the stellar systems were originally in a state of approximate equilibrium, and that new developments starded, when the radiation vanished. Possibly we may obtain in this way a close understanding of the cosmological problems connected with the redshift of the spiral nebulae.

Leaving these speculative features aside, it should be stressed that the precise conclusions to be drawç from the models of Beskow and Treffenberg will of course depend upon the changes produced after the disappearance of the radiation, a problem which we hope to attack in a near future, but that it is still rather interesting that with incomparatively wide limits for the state at the centre of the star the main features of the abundance distribution both for heavy and light nuclei come out within a reasonable approach of the empirical values, and that at the same time the masses of the corresponding star models are of the same order of magnitude as those of real stars.

M. Teller. — In a part of the star where there is not a very great neutron excess, anything like lead cannot be formed at all, and where there is a neutron excess a nucleus of the charge of lead will have many more neutrons than lead contains.

M. Klein. — I understand from your communication that there are two ways of building the heavy elements. One is to have very large neutron excess, the other is to have matter of nuclear density.

Now in the models of Beskow and Treffenberg quite enough of the heaviest elements is present under conditions where high density and neutrons excess are combined, in a somewhat more moderate way, the density beeing not quite as high as nuclear density but about 1/100 of it.

M. Teller. — The binding energy of the neutrons should not be too big otherwise the Ylem would not break up at all. If the density of neutrons is inversely proportional to the cube of their de Broglie wave length, and their kinetic energy is of the order of their binding energy, one finds for that the density of the Ylem is about 30 or 40 times smaller than the nuclear density, which is not very different from the value obtained by Klein at the centre of the stars.

M. Perrin. — En présence d'un champ electro-magnétique pur, selon les équations d'Einstein la courbure totale de l'espace — temps doit être nulle (la courbure tensorielle ne l'étant pas).

Comment se peut-il alors qu'une étoile formée seulement de lumière puisse être stable? Une solution statique, sans courbure dans la direction du temps, devrait correspondre à une courbure totale d'espace nulle.

M. Klein. — Tolman has treated similar cases in which the energy momentun tensor is given by pressure and energy.

There is a relation between pressure and energy, of the form $p=1/3$ E and that gives particular kind of solutions.

In this static cases, there is no curvature in the fourth dimension, but there is one in three dimensional space.

M. Peierls. — Teller's arguments about the fate of a sphere of Ylem seem to be conclusive once one accepts a small attraction which will make the surface tension negative.

The negative surface tension means however, but it is very difficult to imagine how such a compact body was produced since it is less stable than a large number of much smaller fragments.

Almost the only way of producing it will be to compress enough matter into a « bottle » and then to release it. Then however, a legitimate question is that of the origin of the bottle.

M. Teller. — There are many amusing possibilities, for instance, one can assume high nuclear density inside stars on a high density of matter out a very early stage in the development of the universe. For the time beeing I prefer to say what I hear beeing whispered here in my neibourhood, that the bottle was made by God.

M. Schrödinger. — At very high radiation temperature, one has to ask whether the Maxwell linear theory is still correct, I think that Born's non-linear theory would produce very great deviations from the usual Maxwell theory.

Artificial mesons,

by Mr. Serber

I should like to describe the present status of the work on mesons which is being carried on in Berkeley. In the interest of clarity, I shall divide my talk into two parts. First I shall summarize what appear at present to be the facts and explain our interpretation of them. Then I shall show the experimental procedure and results in greater detail.

FIRST PART

We write the process by which π mesons are produced when a target is bombarded by the high energy α particles accelerated in the Berkeley cyclotron.

(1) $$N \rightarrow P + \pi^-$$

We know, that μ mesons are not involved in this reaction, since none are seen to come from the target (other than μ^+'s from the decay of π^+'s stopped in the target). The production of π's in pairs is extremely unlikely on energetic grounds, since meson production has been observed with α particle energies as low as 275 MeV. To sharpen this point, we can compare the observed variation of meson yield from a carbon target as the α energy is changed with that computed in the way described by Teller and Mc. Millan, the momentum distribution of nucleons in the carbon nucleus and the α particle being taken as that of a degenerate Fermi gas. The calculations have been carried out by Horning and Lewis, taking into account the details of the experimental arrangement, such as the limited energy interval examined, the focusing effect of the magnetic field, and the finite thickness of the target. The threshold for meson production on this model is expected to be 260 MeV and the yield is expected to rise roughly as the fourth power of the energy above threshold. This leads to the comparison shown in the table.

Energy	calc.	obs.
380	1	1
340	1/5	1/3
305	1/50	1/20

The value of the calc. at 305 MeV, should be somewhat reduced for reasons having to do with the details of the experiment.

The sense of the deviation of the observed and calculated yields is just what would be expected because of the crudity of the assumed Fermi momentum distribution; higher momenta are undoubtetly present in the distribution, which would raise the curve at the lower energies and in fact give a finite yield below the 260 MeV « threshold ». As mentioned before, a small number of mesons have been observed with bombarding energies as low as 275 MeV.

Something can also be said about the absolute value of the cross section for the production of mesons. I have calculated this cross section from the observed number of mesons on the plates, and the measured of the α beam (it is necessary also to use a theoretical meson energy distribution, since only a limited energy range is observed). The result was $\sigma = 2 \times 10^{-31}$ cm^2 per nucleon — nucleon collision. This is for 380 MeV α's.

The theoretical value calculated by Horning turned out to be just the same. Neither of these numbers is to be taken too literally, but the agreement in order of magnitude is gratifying. It should be emphasized that in these calculations the details of meson theory are not very important, the dominating factor beeing the available phase-space volumes. The only features of meson theory which play a part are the order of magnitude of the meson nucleon interaction, which is determined from nuclear forces, and the further assumption that the matrix element of this interaction is not varying rapidely with energy, e. g. does not vanish for slow mesons.

We next turn to the question whether light particles (electrons, γ rays, neutrinos) can be involved in reaction (1). This is disproved by observations of the inverse reaction, the capture of π^-'s by nuclei. Such a capture characteristically leads to formation of a star. The frequency-size distribution of these stars is similar to that of the stars produced by bombarding the photographic plates with deutons of between 100 and 200 MeV energy. On the other hand if light particles were involved, there would be expected to carry off almost all the energy made available by meson capture, and stars would not be produced.

It is thus indicated, that (1) is a proper description of the production process, and an immediate conclusion is that the π meson has integral spin.

The next process to be considered is the decay of the π meson:

$$\pi \rightarrow \mu + \begin{cases} \nu \\ \gamma \end{cases} \tag{2}$$

The observations made by Gardner and Lattes agree with those of Lattes Ochialini, and Powell in showing that the μ meson has a well defined range, of a little over 600 microns, the only deviation being those to be expected due to straggling. Furthermore thirty π^+ mesons have been observed to stop in 100 micron plates, and in every case the decay meson was seen.

In writing (2) we have supposed the third particle involved to be a neutrino or gamma ray, since the evidence on the masses, of π and meson now seems compatible with a third particle of zero mass, and we should like to avoid the hypothesis of a new kind of neutral particle. Lattes and Gardner now give as their best value for the π mass $M\pi = 286 \pm 6$ electron masses. Prof. Brode finds for μ mesons of cosmic ray origin $M\mu = 212 \pm 4$.

The ratio of these two numbers is $R = 1.35 + 0.04$, which is to be compared to the ratio $R = 1.32$ which is to be expected for zero mass of the third particle. Lattes and Gardner are now carrying out a more direct measurement of the ratio. A very preliminary value is $R = 136$ which however, is based on the measurement of the masses of only two μ's, the values found beeing 209 and 211.

The rate of process (2) is the subject of another experiment carried out at Berkeley by Prof. Richardson. He finds for the meanlife of the π meson the value, which should also be regarded as preliminary, $\tau_{\pi} = 9 \times 10^{-9}$ sec.

It may be mentioned that a π lifetime of such a magnitude, together with the previously mentioned fact that of thirty π^+'s seen to stop all decayed, rules out the connection originally suggested by Yukawa between the β decay of nucleus and the β decay of mesons. For if β decay of the π meson is a process competing with (2), the above evidence shows that its lifetime must be greater than 3×10^{-9} sec. On the other hand, the observed nuclear β decay rate would demand a meson lifetime of about 3×10^{-9} sec., one hundred times shorter than is admissible.

The process by which a μ^- meson is captured by a nucleus can be described as taking place through the inverses of the two processes already considered :

$$(3) \qquad P + \mu^- \rightarrow P + \pi^- + \genfrac{}{}{0pt}{}{\nu}{\gamma} \rightarrow N + \genfrac{}{}{0pt}{}{\nu}{\gamma}$$

One consequence of (3) is that the light particle emitted will take off most of the available energy, so that the stars would not be expected to appear. Lattes and Gardner have identified, by grain counting, twelve cases in which μ^- mesons stop in the emulsion; in no case was there a visible star.

Since (3) is the inverse of (1) and (2), its rate is determined of those of the others. If we take the coupling struytt in (1) to agree with the magnitude of nuclear forces, we are left with a relation between the lifetime for π decay and the lifetime for μ^- capture. A number of people have independently worked this out; I will quote only my own result. Using the lifetime for μ^- capture as determined by Rossi, Schein and their students, found $\tau_\pi = 1.2 \times 10^{-8}$ sec.

In the calculation the scalar theory was used for the π, and the μ was supposed to have spin 1/2. Other choice give different numerical factors, but the same order of magnitude. This check on the order of magnitude of the π lifetime gives considerable confidence in the general picture we have of the processes which are taking place.

It should be mentioned that an experimental check on whether γ rays are emitted into the capture of μ^- mesons would serve to determine whether the μ meson has integral of half integral spin (at least unless we invent a new neutrino to replace the γ ray). This could also be told from the spectrum of the electron emitted in μ^+ decay. For integral spin we would expect decay into electron and neutrino, and monoenergetic electrons. However, for spin 1/2 a plausible hypothesis would be

$$(4) \qquad \mu \rightarrow e + 2\nu$$

An interesting feature of this suggestion is that the energy spectrum of the electrons may depart quite radically from an ordinary β spectrum. The predicted shape of the spectrum depends on the assumed coupling law, but it is possible to find couplings (e. g. a. scalar coupling) which actually give a maximum at the upper limit of electron energy. A spectrum with a large proportion of high energy electrons is of considerable help in reconciling the results of the measurements on decay electrons reported by Anderson, Thompson, and Steinberger.

An experiment is now being carried out by Prof. Alvarez which it is hoped will lead to a good determination of the spectrum of decay electrons. The α particle beam is pulsed by the cyclotron's electric deflector on to a target, the pulse of α's lasting about 0.2 microseconds. The π mesons produced in the target decay into μ^+ mesons, and these in turn into positive electrons. The electrons escaping from the target, are but in a semi circle by the magnetic field of the cyclotron, and pass through two anthracene counters which are operated in coincidence, and can be delayed any desired time after the π particle.

The energy of the electrons is determined by their radius of curvature in the field, and the reality of the effect is checked both by measuring the decay lifetime and by placing absorbers between the counters to make sure that coincidences are due to electrons of the proper energy. It is believed that decay electrons have already been detected by this apparatus, if so we should soon have an answer.

SECOND PART (EXPERIMENTAL)

I should first describe the work of Prof. Brode determining mass of μ mesons found in cosmic rays at sea level.

Prof. Brode's apparatus. There are three cloud chambers. The upper two are used to determine the deflection of the meson by the permanent magnet between them. The meson's range is then determined by the number of 1/4" lead plates it penetrates in the lower chamber.

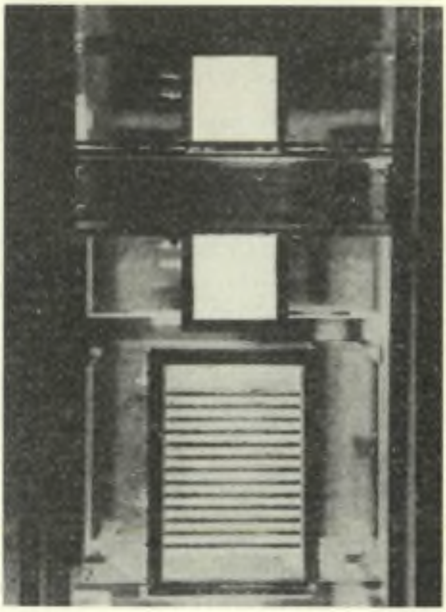


Fig. 1.

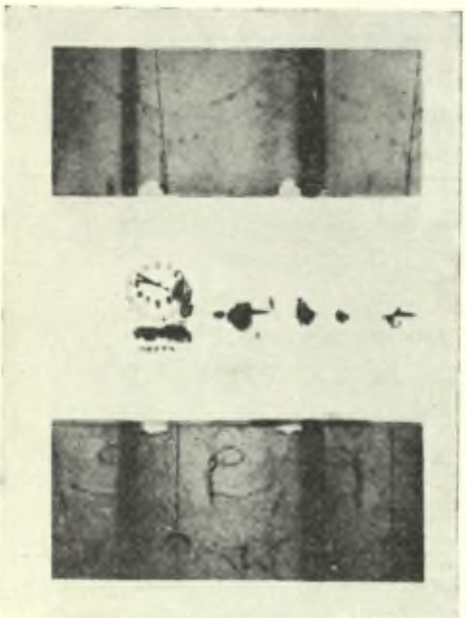


Fig. 2.

Track of meson in upper chamber.

Fig. 3.

Same meson in lower chamber.

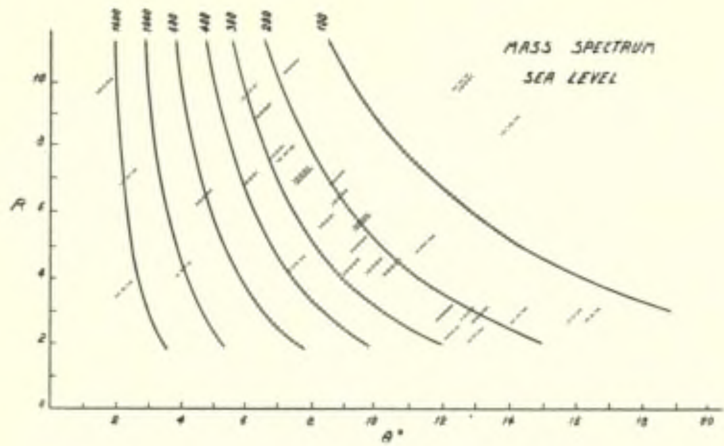


Fig. 4.

Range angle distribution of all measured tracks. It will be observed that the mass curves are nearly equally spaced on a logarithmic scale, so that random errors in R and O mean approximately random errors in log Mμ rather than in Mμ itself.

It was the realisation of this fact that led to a revision of the results noted for the earlier work by Retallack and Fretter, which when properly analysed give a higher mass.

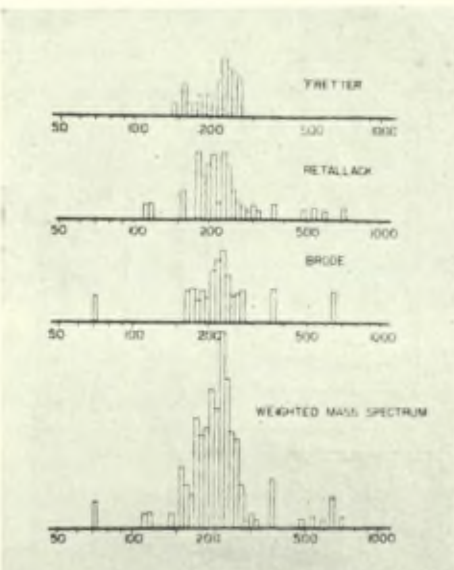


Fig. 5.

Results of the mass measurements. The average is $M\mu = 212 \pm 4$. Prof. Brode believe the conclusion that there is a particle of mass between 500 and 800 is difficult to avoid.

Meson Experiments on the Cyclotron.

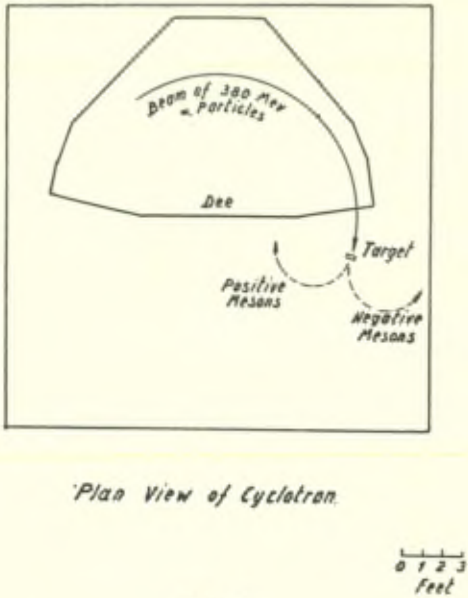


Fig. 6.

Shows position of target in cyclotron, and orbits of mesons.

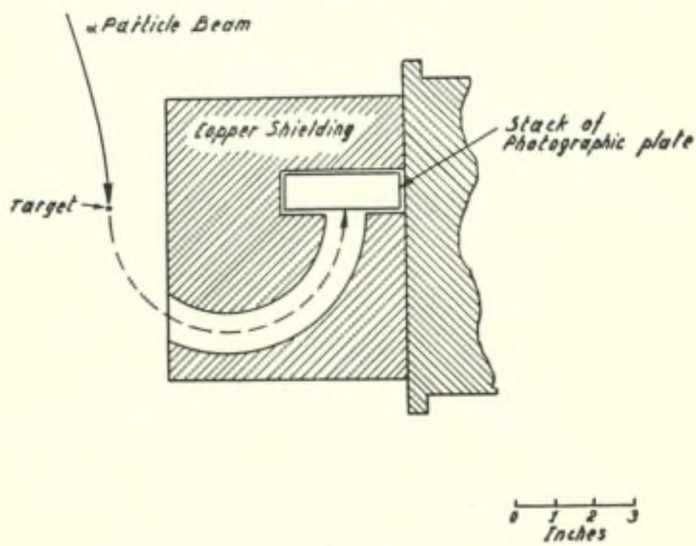


Fig. 7.

Apparatus with wich mesons where first detected. This experiment was designed by Prof. Mc. Millan. Both π^- and μ^- mesons are found on the plate. Energies are found between 2 and 16 MeV.

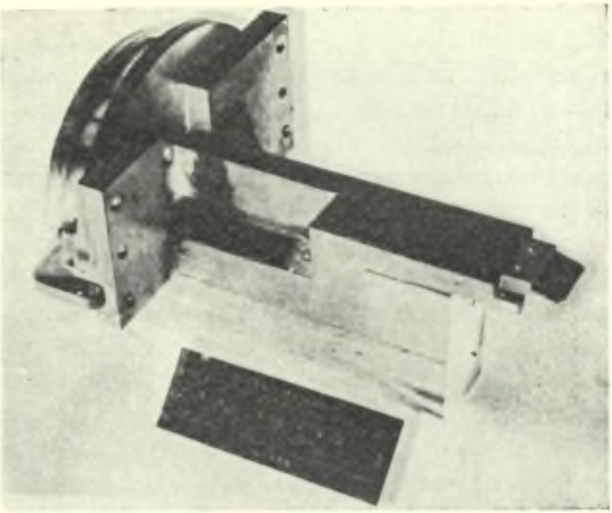


Fig. 8.

Photograph of apparatus described above.

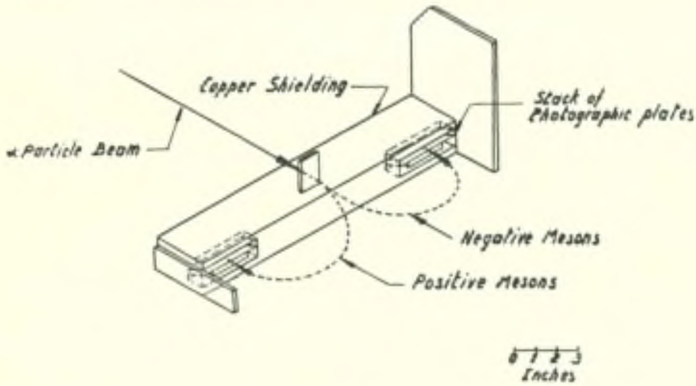


Fig. 9.

Arrangement for simultaneous measurement of positive and negative mesons. The ratio of π^{-}'s to M^{+}'s was found to be 4. A calculation of the effect to be expected because of the influence of the Coulomb field of the carbon nucleus in which the meson was produced in increasing the unification of the π^{-} and decreasing that of the π^{+} gave for the expected ratio 3.5, in good agreement of that observed.

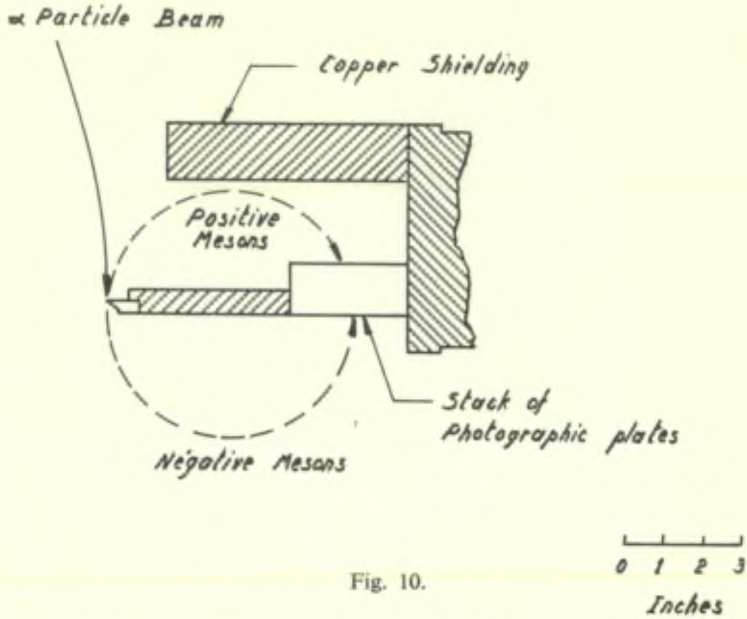


Fig. 10.

Arrangement for obtaining positive and negative mesons simultaneously on opposits edges of the same photographic plate. Comparison with the previous arrangement showed 16 mesons emitted backward for 24 forward. A slight forward assymetry is expected but the effect has not been calculated as yet.

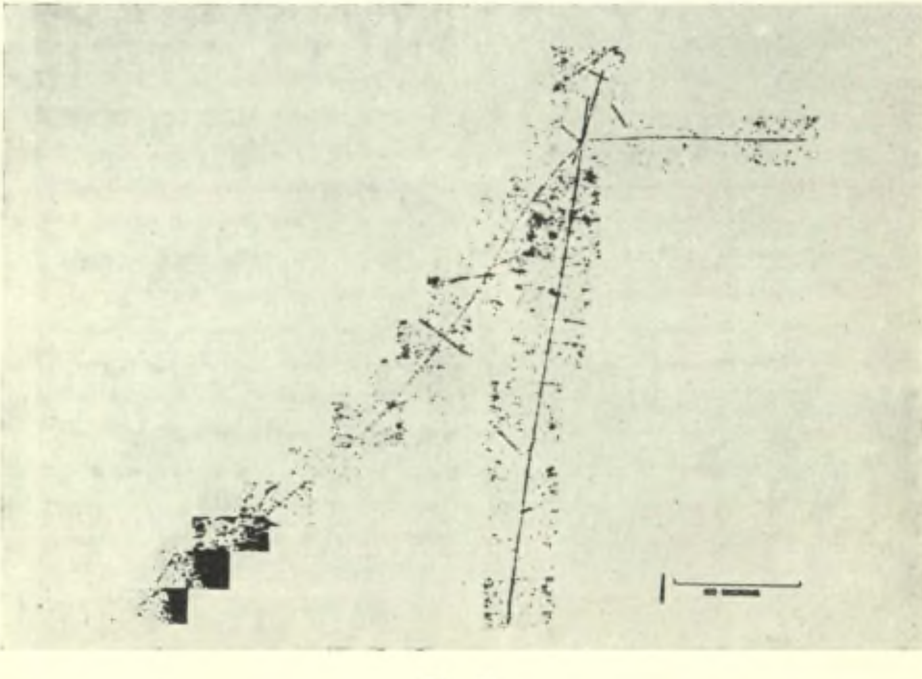


Fig. 11.

Star produced by π.

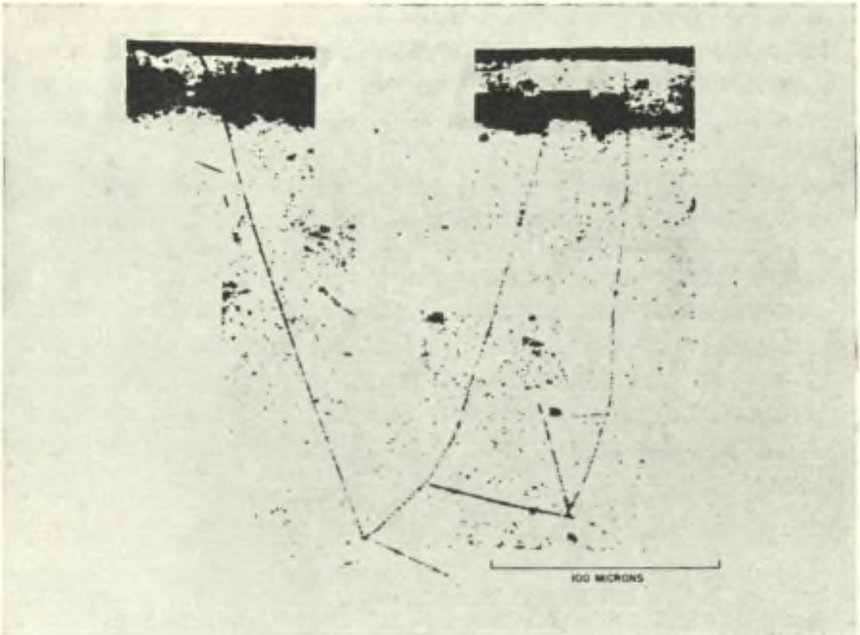


Fig. 12.

Two π^-'s appear in same field of view.

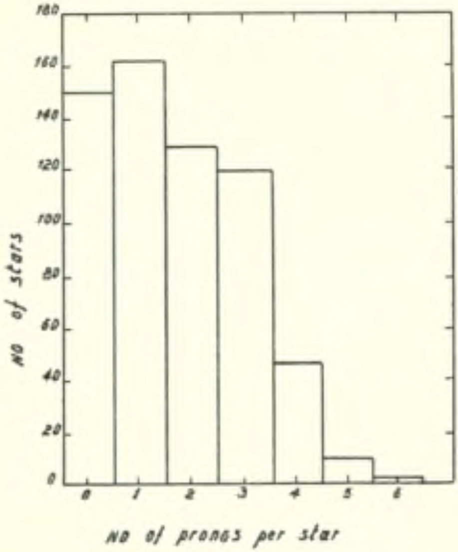


Fig. 13.

Size distribution of stars produced by π^- mesons. 75% produce visible stars. It is believed that only neutrons are emitted in the remaining cases.

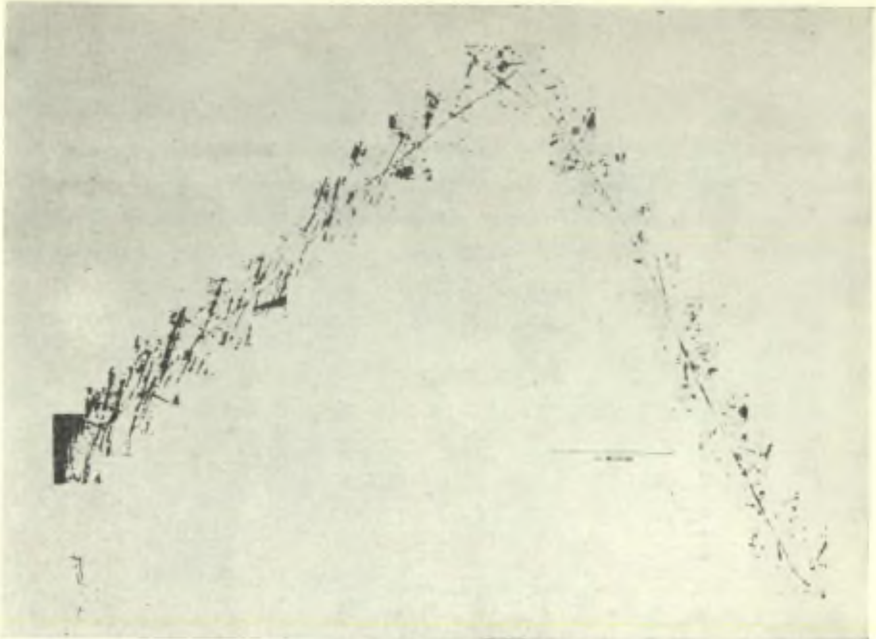


Fig. 14.

π^+ comes in on left, decays to μ^+.

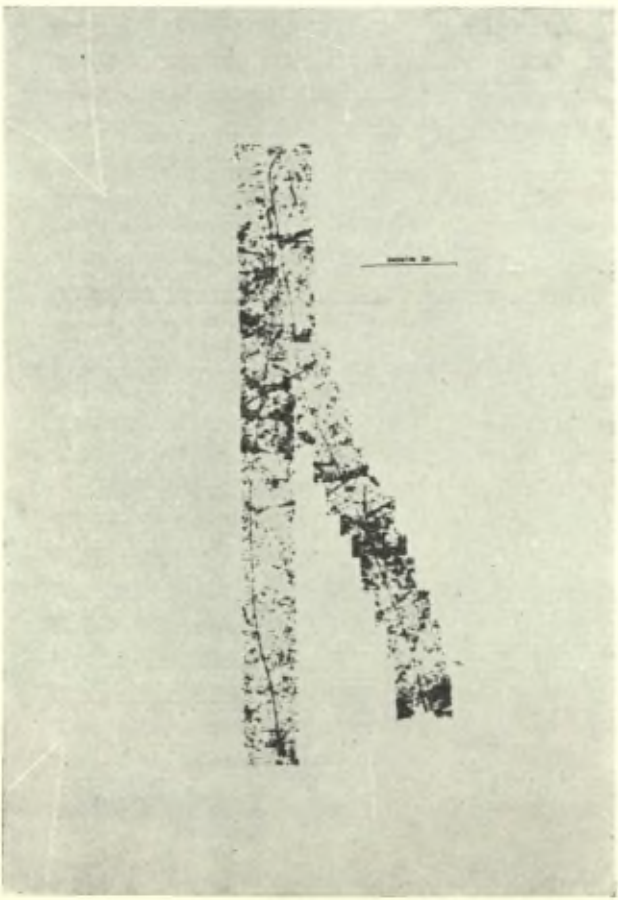


Fig. 15.

Believed to be a π^- decaying a μ^-. If the lifetime is 10^{-8} sec., about one π^- in 500 might be expected to decay in flight. About 3000 have been seen to stop, many in 50 micron plates, where the decay meson might be missed.

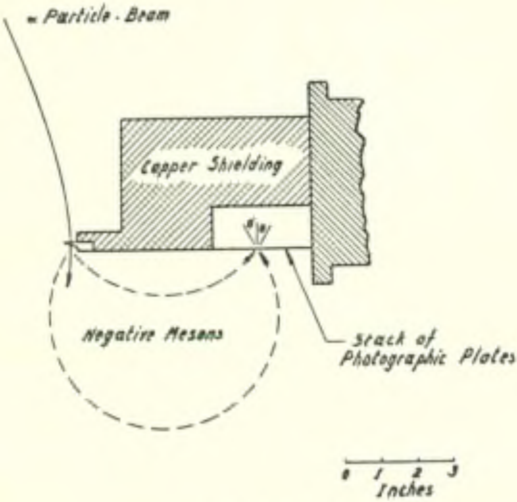


Fig. 16.

Apparatus used for measurement of mass of π^- meson. With such a channel pointing to target only one μ meson has been found over 100 π. μ's are not produced at target.

Direction of mesons on plate, and correlation of range and position on plate show mesons are not coming from walls of channel, or decay in flight. With a channel on opposite side of target it has been verified that positive and negative π's have the same mass.

The apparatus for measuring the π–μ mass ratio has two channels, one for π^- mesons, the other for μ^+ mesons produced by π^+ decay in the target. The mass ratio determined in this way is independent of the calibration of the magnetic field.

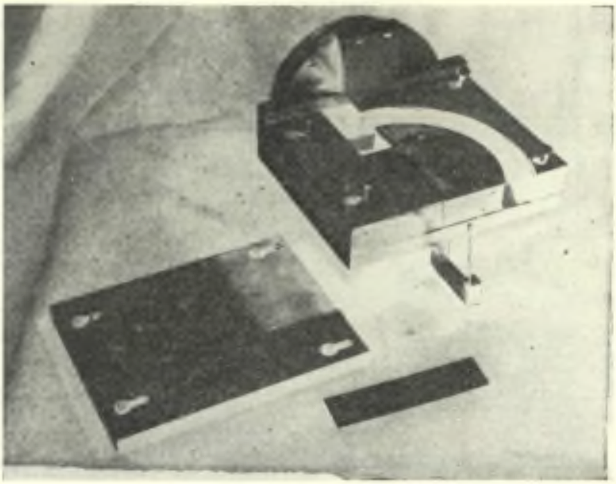


Fig. 17.

Photograph of preceding apparatus.

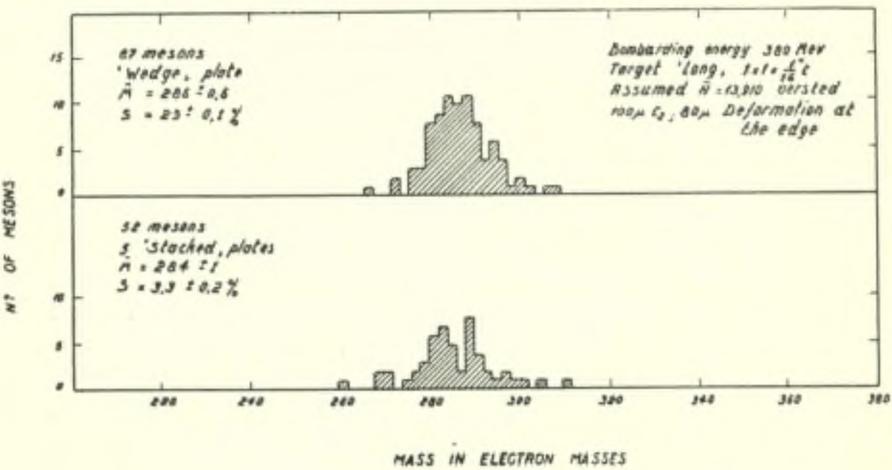


Fig. 18.

Result of mass measurements. The upper group were measured with the plates placed at an angle so the meson does not pass through the edge of the emulsion which is deformed on development. This is more accurate, but more tedious, since the area of the plate must be scanned rather than just the edge. The average M = 286+0.6

contain only the statistical error calculated from the distribution curve, and does not include an allowance for range-energy-curve and magnetic field uncertainly. Including a 2% range error and a 1% magnetic field error gives the previous quota 286±6. Although the magnetic field is supposedly known to within 1%, Dr. Lattes would like to make a reservation on his mass value until the field has been measured. The standard deviation of the measured distribution is 2.5%, only slightly larger than the 1.9% expected on the basis of range straggling. The lower distribution was taken with the plates edge on.

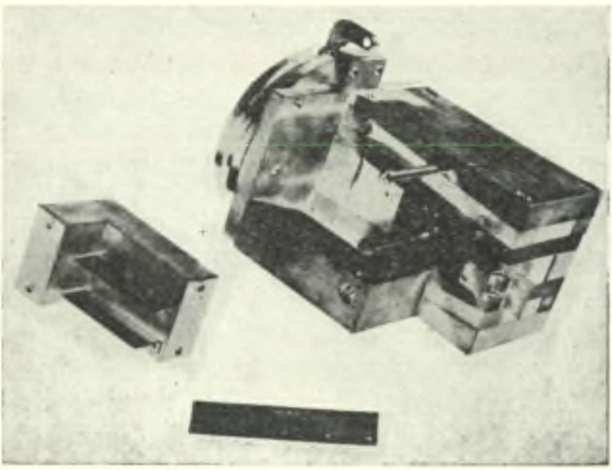


Fig. 19.

Apparatus used by Prof. Richardson to measure the π lifetime. Mesons travelling on an upwardly directed helix stricke a photographic plate after making half a circle, those spiralling downward after one-and-a-half circles. The observed density of mesons on the plate after 1 1/2 turns was 1/6, rather than 1/3, the density after 1/2 turn indicating a half-life equal to the period of rotation. We would feel happier about this experiment if the lifetime of the α particle were measured on the same apparatus. Prof. Parofsky plans to do this, replacing the target by a α source.

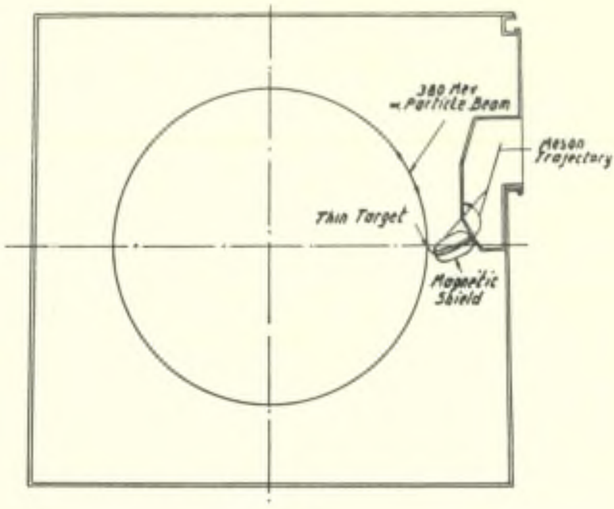


Fig. 20.

Magnetic deflector built by Prof. Parofsky which brings mesons out of vacuum chamber of cyclotron. Another measurement of the lifetime is being made by comparing mesons intensity at various points along the channel with α intensity produced by α source at position of the larget. Attempts to get cloud chamber pictures of mesons in outer region has so far not been successful.

Discussion du rapport de M. Serber

M. Heitler. — The calculation of the yield are made on a scalar theory. Would one expect a difference using a spin dependent theory for the nuclear forces.

M. Serber. — One would expect a factor of 10 smaller and a more rapidly falling curve.

We have used the most elementary theory not taking into account tensor forces.

M. Teller. — Do the experimental results exclude the other theories ?

M. Serber. — Under these conditions the calculations are so dependant on features having to do with details of nuclear structure that it is not a certain test of the assumptions of meson theory.

We will learn more about nuclear forces and mesons when the cyclotron will be able to accelerate protons up to 350 MeV. It should be ready for the end of october.

M. Teller. — Due to the small momentum these mesons have when emitted, we approximately would expect forward and backward mesons to be produced in equal numbers. Have the expected relative probabilities been calculated? Could a 50% difference be accounted for ?

M. Serber. — It has not been strictly calculated. We have only 16 and 24 tracks respectively, the statistic is very small.

M. Cockcroft. — Are there any measurements of the masses of the artifically produced μ mesons?

M. Serber. — The experiment has just been started by Gardner and Lattes. They have so far measured two μ mesons and found mass values 209 and 211.

M. Kramers. — How much is experimentally known about the angular distribution of mesons from the target ?

M. Serber. — We have only the 3 to 2 ratio for forward to backward direction.

M. Oppenheimer. — May I ask two questions.

You have quoted values of the mass of the meson obtained in Berkeley, and of the meson of cosmic rays measured by Brode. How far can we rely on a comparison of two such different types of experiment?

Also does the lifetime of the μ meson on the two neutrino hypothesis agree with the lifetime of the neutron ?

M. Serber. — The answer to the first question is that the preliminary values obtained by Lattes and Gardner do not desagree with Brode's value, but we expect soon to have a better comparison.

The answer to the second is that we have not finished the calculations, but I have seen no reason for a large change.

M. Peierls. — The experimental evidence we have seen shows a wide distribution of measured masses.

Does the usual method of averaging and taking the square root of the number of measurements give the real standard error. As Serber has pointed out, averaging on the logarithmic and linear scale does not give the same results. One has to take proper care to calculate the average and the standard error on the scale on which the distribution of errors is random.

M. Serber. — Prof. Brode realized this fact and therefore, since his errors are random in angle and range, changed to the linear scale from the logarithmic scale on which the masses were previously calculated. A more careful analytic treatment led to indistinguishable result.

M. Teller. — I should like to make a statement about experiments carried out in Chicago by J. Steinberger.

He has studied the electrons from the decay of cosmic ray mesons, presumably μ mesons. The results are preliminary, but certain statements can be made. His results are incompatible with the hypothesis of the decay of the μ meson into one electron and neutral particle of zero or of non-zero constant mass. They could be interpreted as due to a decay into an electron and a particle of variable mass, or into an electron and two neutrinos.

M. Bohr. — Having shown that the ranges of the mesons are consistent within the estimated straggling, and established that they are the same particles, one can now use these ranges to test the much discussed theory of straggling.

M. Serber. — Prof. Thornton has studied the straggling of 190 MeV deuterons. He finds a straggling slightly larger than predicted by the theory, which can readily be accounted for by an energy spread of the deuterons of 1 MeV and the shape of the curves fits exactly.

M. Kramers. — Has one studied the ionisation curve, analogous to the Bragg curve?

M. Serber. — It is assumed that the ionisation of the meson is the same as that of a proton of equal velocity. The range energy curve for protons has been extrapolated to energies higher than 13 MeV, but it has been checked at 32 MeV experimentally with protons from the linear accelerator.

M. Teller. — Is it verified that the meson cross section of a nucleus is proportional to its weight?

M. Serber. — Very little quantitative evidence is available, but it is known that the yield does not depend very much on the type of target used. The results are quite consistant with a production proportional to the atomic weight.

M. Kramers. — Have you used hydrogen — enriched targets?

M. Serber. — It is difficult to introduce enough hydrogen into the target.

M. Oppenheimer. — No mesons are to be expected with an hydrogen target.

M. Bohr. — We must remerber we are dealing with the threshold of the reaction. A reaction with a free proton would give insufficient energy. When the beam of protons is available, it will give sufficient excess energy for the study of the problem.

M. Dee. — What statistics do you have about the decay of π^+ mesons? Do you observe anything other than the π-μ decay?

M. Serber. — In 30 examples we have, the decay mesons are always observed.

M. Klein. — I understand there is less evidence now that a few months ago for the decay of the μ meson into an electron and a neutral meson of about 100 electron masses. I should like to point out, however, how well on this view the lifetimes of the π-meson would fit in with the lifetimes of ordinary β processes.

M. Oppenheimer. — As I mentioned before, if the μ meson did decay into a neutral meson, an electron and a neutrino, there would be a good check by the lifetime.

In the case of a decay into an electron and two neutrinos, there would be very little difference, because the recoil of the second neutrino whould carry pratically the same energy as the neutral meson in the other case.

I do not think it would be wrong by an order of magnitude.

M. Serber. — The simple relation of Yukawa between the β decay and the decay of the meson can no longer be maintained.

However, the lifetimes of a β decay and of decay of the μ meson seem to be given by couplings of the same order.

M. Bohr. — How does it stand with the comparison between the lifetime of the meson and the lifetime of the neutron which was previously based on a value of the mass of the neutral μ meson which we know now is certainly wrong?

M. Oppenheimer. — That is the point I wanted to make.

The second neutrino will on the average take one third of the total energy, which is not far from 30 MeV. What we consider is a parallelism between the reactions.

$$n \rightarrow e + p + \nu$$
$$\mu \rightarrow e + \nu + \nu$$

M. Rosenfeld. — The necessity of abandoning the conception of β-decay suggested by Yukawa, and involving the meson as intermediary, does not, at the present stage, imply any loss of simplicity.

In fact, we have in any case to asume four independent couplings between the various bonds of elementary particles; from this point of view it does not matter whether we assume a direct coupling between π mesons and leptons, as in Yukawa's conception, or between nucleons and leptons, as in Fermi's original theory. In the latter case, the resulting indirect coupling between μ mesons and leptons turns out to be weaker than that between π mesons and μ mesons, which agrees with the Berkeley findings.

Observations on the Properties of Mesons of the Cosmic Radiation

by C. F. Powell

Recent observations, made at Bristol, on the properties of the mesons of the cosmic radiation, have been described in a number of papers which have either been published, or which will appear in the press before the opening of the conference ([2, 4, 5, 8, 9]). This report therefore contains only a Summary of the main features of the results, and the conclusions which follow from them.

It has been found possible to account for all the main phenomena, involving slow charged mesons with masses in the interval from 100 — 400 m_e, observed in photographic plates exposed to the cosmic radiation, in terms of two types of particles, π— and μ—, which can be charged either positively or negatively ([1], [2]). A π-particle, of mass 310 m_e ([3]), decays spontaneously with the emission of a μ-particle of mass $\sim$ 200 m_e, and a neutron meson of mass $\sim$ 80 m_e. The mean lifetime, τ_π, of both the positive and negative π-particles, is of the order of 5×10^{-9} sec. The π-particle can be produced in processes which lead to the explosive disintegration of nuclei ([2]); and when brought to the end of their range in photographic emulsions they are captured by atoms, and produce disintegrations of both light and heavy nuclei ([4]). Their modes of creation and extinction are therefore consistent with the view that they have a strong interaction with nucleons. On the other hand, the μ -mesons are produced directly, either rarely, or not at all; and, when brought to rest in silver bromide, they never, or only rarely, produce nuclear disintegration ([5]) observable in the conditions of our experiment. The observed behaviour of the μ -mesons is therefore consistent with the view that they have a very weak interaction with nucleons.

In previous papers ([2], [4]), it was found convenient to employ a phenomological classification of the mesons recorded by photographic plates on the bases of the secondary effects observed at the end of their range. Thus a ρ-meson was defined as one which produces no secondary particles of which we distinguish the tracks. The majority of the ρ-mesons are μ^+ — and μ^--particles, but we may include among them π^+-particles of which we fail to distinguish the tracks of the associated secondary μ^+ -particle; and π^- -particles which lead to disintegrations with the emission of neutrons. Similarly, a σ-meson was defined as one which produces a nuclear disintegration with the emission of heavy charged particles of which we distinguish the tracks. Whilst it was known that most of these mesons must be negatively charged heavy particles, π^-, the possibility remained that some of them were μ^- -particles which had produced disintegrations after being captured by heavy atoms. As indicated in the previous paragraph, the new observations (5) show that the proportion of μ^--particles which produce stars is certainly very small, and it appears reasonable to assume that they never do so. In what follows, we shall therefore refer to the σ -mesons as π^- -particles. Finally, the π and μ -mesons, of which the sign of the electric charge has been established at Berkeley (6), are now described as π^+— and μ^+ -particles, respectively. Whenever it is possible to do so, a particle will be characterised as μ^+, μ^-, etc., the terms ρ-mesons and σ-mesons being reserved for those cases where the nature of the particles in question is ambiguous or under discussion.

The μ-decay. (1)

Eighty examples have now been found of the process in which a heavy meson π^+, at the end of its range, decays with the emission of a secondary meson, μ^+, which also stops in the emulsion. The sign of the charge of these particles has been established by magnetic deflection experiments (7), The distribution in range of the particles is consistent with the assumption that the velocity of emission of the μ^+ -particle is constant within narrow limits (8). It follows that the masses of the two types of particles, m_π and m_μ, are constant within narrow limits.

Masses of the π^+ - and μ^+ -particles. (2)

The masses of the π and μ -particles have been determined by two methods; (*a*), by grain-counting (8); and, (*b*), by a study of the scattering of the particles in their passage through the emulsion (9).

By grain counting, it is possible to determine the ratio of the masses, and the value thus obtained is $m_\pi/m_\mu = 1.65 \pm 0.15$. Assuming that the μ -mesons are identical with the particles of the penetrating component, of mass $\sim 200\ m_e$, it follows that $m_\pi = 330 \pm 30\ m_e$.

By studying the scattering of the particles, and from the known atomic composition of the emulsion, the absolute values of m_π and m_μ can be determined independently. Goldschmidt-Clermont, King, Muirhead and Ritson (9) have thus found that $m_\mu = 200 \pm 30\ m_e$; and $m_\pi = 270 \pm 40\ m_e$. See also Lattimore (10).

Production of mesons in processes taking place in the emulsion. (3)

Twenty-five examples of the production of mesons in processes occurring in the emulsion itself, in which the ejected meson reaches the end of its range in the emulsion, have now been observed (4). In 22 cases, these ejected mesons produce disintegrations in which different numbers of heavy charged particles are emitted. The distribution showing the relative frequency with which one, two, three, etc., heavy charged particles are emitted is identical within the limits of the statistical fluctuations, both with the corresponding curve for σ-mesons entering the emulsion from outside, and with that for the heavy mesons produced artificially at Berkeley. The observations are consistent with the assumption that (*a*) all the ejected mesons are negatively charged; and (*b*), that they are captured by both light and heavy nuclei and lead to their disintegration.

These observations suggest that in the very low energy region accessible to our observations only π^- -particles are produced directly; and that the absence of ejected π^+ -particles is due to the repulsion exerted on the outgoing meson by the charge of the parent nucleus. This ensures that such particles are emitted with considerable energy so that the probability of observing is greatly reduced.

Observations on the directions of motion of the mesons. (4)

By making observations on plates exposed at high altitudes with the emulsion in the vertical plane, and by determining the directions of motion of the mesons at their points of entry into the emulsion, it is possible to distinguish different streams of mesons which have originated in different processes (5). In plates exposed in this way under a light roof, and two meters above a concrete floor equivalent in stopping power to 10 cm. of lead, it can be shown that there is,

(a), a downward flux of ρ-mesons. The absolute intensity of this downward stream is equal, within 30% to the values obtained by Rossi, Sands and Sard (11) using a delayed coincidence method. The distribution in angle of the particles in this stream is consistent with the observations of Greisen (12) made with a counter « telescope ». It may therefore be regarded as established that thuis stream of mesons is identical with the particles constituting the low energy end of the spectrum of the penetrating component of the cosmic radiation. There is also observed,

(b), an upward flux of similar ρ-mesons. These may be regarded as due, mainly, to μ^+- and μ^--particles, formed by the decay in flight of upward moving π^+- and π^--particles which have been produced in the matter beneath the plate assembly. It is observed further,

(c), that both streams of ρ-mesons are accompanied by π^+- and π^--particles of much smaller intensity.

Support for the above explanation of the origin of the upward stream of ρ-mesons is provided by the following observations.

1. If lead blocks are placed close to the plates during exposure, an additional stream of π^+- and π^--particles can be distinguished, with directions of motion which indicate that they have originated in the lead.

2. If the plates, during exposure, are placed on the surface of the earth, instead of being supported above it, a backward stream of mesons of approximately the same intensity as in the first experiment is observed, but a much larger proportion of the particles are now π-particles.

We conclude that, in these two latter experiments, the additional matter is sufficiently close to the plates to ensure that a much smaller proportion of the π-particles generated in it have had time to decay in flight before being brought to rest in the emulsion.

Behaviour of the μ^--particles at the end of their range. (5)

The number of ρ-mesons in the downward stream may be compared with the number of π^+ and σ-mesons entering the plates in the same directions (5). Magnetic deflection experiments indicate that the ρ-mesons are made up of approximately equal numbers of positively and negatively charged particles (7). Assuming that 40% of the ρ-mesons are μ^--particles, and that *all* the σ-mesons are also μ^--particles, it follows that only 30% of the μ^--particles brought to rest in silver bromide crystals produced disintegrations observable in the conditions of the experiments. It is reasonable, however, to attribute a number of the σ-mesons, equal to the number of π^+-particles observed in the same angular range, to π^--particles. The estimated proportion of the μ^--particles which produce disintegrations in silver bromide is then reduced to $10 \pm 10\%$. This result suggests that the μ^--particles never produce disintegrations observable in the conditions of our experiments; and that the explanation for the failure to observe decay electrons in the delayed-coincidence experiments (13), — when negative mesons of mass $\sim 200\ m_e$, are brought to rest in materials of high atomic number — must be sought among processes of another type.

Lifetime of the π-particles (6)

By comparing the number of π^+- and π^--particles in the upward stream with the number of ρ-mesons in the same angular range, an estimate of the life-time, τ_π, can be determined (5). This involves a knowledge of the « range » of the « primary » radiation which leads to the production of the mesons, in the matter beneath the plate assembly. Preliminary estimates, which must be accepted with reserve, give values :

$$\tau_\pi = 6 \times 10^{-9}\ \text{sec.}$$

Details of the method and estimates made on the basis of more complete information will be given at the Conference.

The sign of the charge of the mesons of different types. (7)

Preliminary results obtained by the magnetic deflection experiments (7) show, (*a*), that the σ-mesons are negatively charged; (*b*) that the π-mesons and the μ-mesons into which they decay are positively charged; and (*c*), that the ρ-mesons are made up of positive and negative particles in approximately equal numbers.

The development of this method would appear to be of importance in connection with the possible existence of other types of mesons.

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Prof. Powell's Communication - Addenda

I.

THE LIFE-TIME OF THE π^- MESON

Dr. Serber has told us of one π^- meson having been observed which decayed into a μ^- meson. This observation appears to be of great interest in connection with the important problem of the time taken by a meson to reach its state of lowest energy round a nucleus. This problem, which arrised such great attention about a year ago in connection with the experiments of Piccioni and others, presents great theoretical difficulties. It would be very important if we could obtain direct experimental evidence.

This could be provided if we could measure the relative probability of decay and of nuclear interaction of particles; for both the mass and the life-times of these particles are now known; and the time for nuclear capture from a K-orbit is presumably very short.

II.

EVIDENCE FOR THE EXISTENCE OF MASS 800.

At the Bristol Symposium, Dr. Peters of Rochester described some further evidence for the existence of mesons of mass about 800 times that of the electron, in addition to that accumulated during the last few years, notably by Leprince-Ringuet and Rochester. The new results were obtained in experiments in collaboration with Dr. Bradt, Dr. Peters emphasised that the conclusions must, for the present, be accepted with some reserve, until certain technical objections have been met. Nevertheless, it is difficult to see what physical processes could account for the observations except on the assumption of particles of mass 800 m_e, and the conclusions are of such great interest that they should perhaps find some place in our discussions.

The mass spectrum of single tracks observed in photographic plates exposed for 4 hours at an altitude of 90,000 ft. was determined by grain counting. In such a short exposure, fading is negligible and a direct comparison between the tracks arising in events is possible. By using plates of 200 μ thickness they obtained a considerable number of tracks greater than 1000 μ in length. Further by using the new method of temperature development recently described by Dilworth Occhialini and Payne, it was ensured that the plates were developed uniformly in depth.

The grain counting curves showed a well defined group between that of the π mesons and that of the protons. The mean value of the mass of this group is about 800 electron masses, and they are referred to provisionally as τ particles.

In addition, comparison of the grain densities of the track forming the branches of some of the stars showed that particles of this intermediate mass were produced in nuclear explosions.

The reserve observed by Dr. Peters regarding the conclusions is based upon the fact that there where large changes of temperature during the exposures (up to 35^0 C) which may have introduced an accelerated rate of fading; and there may have been some pre-exposure of the plates during their transport by aircraft from England.

In every cases in which a τ particle came to rest in the emulsion it produced no observable disintegration. Some of the ejected particles must be negatively charged since they are of low energy. Indeed, by analogy with the ejected π particles we should expect them to have a strong interaction with nuclei.

These considerations appear to allow us to make some reasonable speculations about the magnitude of the life-time of the τ particles, if they exist. We know it must be greater than 10^{-11} seconds since this is the order of magnitude of the time taken by an individual particle to traverse the observed trajectory in the emulsion. Indeed, since the decay in flight of a particle would produce a track without the characteristic increase in the ionisation at the end of the range, and since such tracks are certainly very rare, it appears reasonable to place the lower limit to the life time at 10^{-10} second.

We may perhaps avoid the difficulty of the absence of observed nuclear disintegrations at the end of their range, of particles which are observed to be generated in nuclear explosions, and which therefore may be presumed to have a strong nuclear interaction, by assuming that they commonly decay before reaching a K-orbit round a nucleus. I understand, however, that although we now know that the time to reach this orbit, in the case of a π-meson is less than 5×10^{-9} sec., one cannot infer that the analogous process for a particle of mass 800 may not be much greater.

We cannot therefore set an upper limit to the life-time from these considerations, apart from the fact that the interaction of a τ meson with a nucleus may not produce a disintegration observable in the conditions of the experiments.

Discussion du rapport de M. Powell

M. Blackett. — Exhibited two photographs obtained in cloud chamber by Rochester and Butler. One of them shows a forked track originating in the gas of the chamber underneath a lead plate. It can be interpreted as disintegration of a neutral particle of mass between 700 and 1.000 electron masses into a positive and a negative particle of lighter mass, for instance π or μ meson.

The other photograph shows one track making a deflection of 18^0 in the gas, and only a subsequent 1^0 deflection in 3 cm. lead. This may indicate the disintegration of a charged particle which must have mass between 800 and 1.000, into a charged and a neutral particle. The lifetime of the process would seem to be in the region of 10^{-8} to 10^{-7} sec.

M. Bohr. — But for the momentum, is there any evidence for the nature of the decay particle which passes through the lead plate?

M. Blackett. — In the second photograph the secondary charged particle is certainly not an electron, since an electron of momentum 800 MeV would produce a cascade. It is probably mesonic either π or μ.

M. Bohr. — It is the same type of process as the π μ decay and is there any evidence for the mass of the neutral particle?

M. Blackett. — Presumably yes. There is however, no evidence for the mass of the neutral decay product.

M. Dee. — Why should not the photograph be explained by the nuclear disintegration by a proton or π meson producing a neutron?

M. Blackett. — Thought no proof can be given, it seems rather unplausible to assume that such energetic nucleons will normally collide with one neutron only out of a nucleus and give insufficient energy to any proton, or to the nucleus as a whole, to make a detectable track. One must remember, that the experimental evidence of George shows that the cross section for the production of 3 or more prongs by a fast nucleon roughly half the nuclear area. We would therefore expect in general to observe such stars originating at the point of collision in the gas, rather than a single track. Further, in the first slide shown, the two emergent particles are certainly lighter than protons.

M. Dee. — In photographic plates one does see stars with one particle.

M. Blackett. — That is in a much lower energy region. Also, this is in air where rather shorter ranges can be observed than in emulsion.

M. Teller. — Also such process as Dee mentioned would more likely occur in lead than in the gas.

M. Blackett. — The process is much more likely in the lead plate. It cannot be the same particle making a 18^0 deflection in the gas and 1^0 deflection in the lead plate.

M. Leprince-Ringuet. — Puisque le problème du méson très lourd τ se pose d'une manière aiguë, je crois utile de résumer les expériences et les indications obtenues par les différents physiciens et favorables à des degrés divers à l'existence d'une telle particule.

a) La première indication a été obtenue par Lhéritier et moi-même en 1944.

Un cliché de rayon cosmique à la chambre de Wilson montre une collision entre une particule rapide et un électron. Nous avons développé pendant plusieurs années la méthode de mesure de la masse du méson par l'étude des collisions de ce type. En dehors de celles qui donnent une masse de l'ordre de 200 pour le méson, nous avons observé un cas tout à fait particulier, qui au point de vue des erreurs se présente de façon favorable, et donne une masse de 990 ± 120.

La particule incidente est chargée positivement et la masse est calculée par les lois ordinaires de la mécanique en supposant le choc élastique.

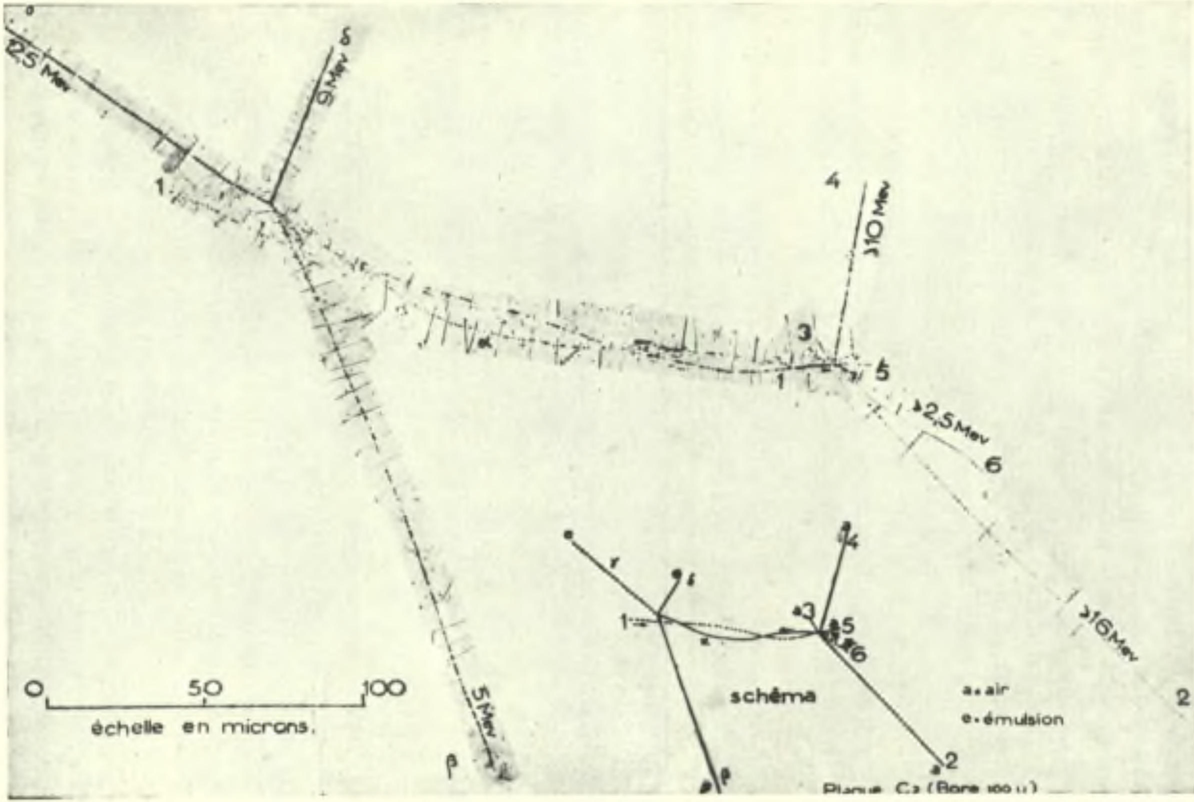
La possibilité, pour les rayons, de subir un scattering notable est un inconvénient de cette méthode. Ce point a été étudié très attentivement par H. Bethe; pour ce cliché, il faut tenir compte du scattering des deux particules dans le gaz, et aussi d'une déviation possible au départ de l'électron. Les conclusions sont que l'on ne peut pas descendre en dessous de 600 m_e. En revanche on peut atteindre à la rigueur la masse du proton en multipliant tous les écarts probables par deux et en les comptant tous dans le même sens, ce qui correspond à une probabilité d'environ 1%. Il n'est pas démontré non plus que la collision soit élastique ce qui est pourtant probable. Il y a aussi une probabilité de 1% que la secondaire soit un électron Compton ayant pris naissance au voisinage de la trajectoire principale. Ce ne peut être, en revanche, un électron, venant du dehors car la partie éclairée de la chambre est beaucoup plus grande que celle contrôlée par les compteurs. Enfin l'hypothèse d'un proton incident se heurte a une difficulté : un tel proton aurait une ionisation 2,3 fois plus grande que celle de l'électron secondaire; alors qu'aucune différence n'est visible sur le cliché.

b) Les indications suivantes ont été rappelées par Blackett, ce sont les expériences de Rochester et Butler (1947) qui sont favorables à l'existence d'un méson chargé et d'un méson neutre, instables et de masse très élevée. Sans entrer dans les détails qui viennent d'être exposés, on peut dire essentiellement que, si l'interprétation est correcte, la masse est plus grande que 700. La limite supérieure de la masse semble difficile à évaluer car la courbure des trajectoires est très faible, mais puiqu'on observe des phénomènes de désintégration radioactive, il ne peut pas s'agir de protons.

c) Une troisième indication apparaît sur une plaque photographique exposée à 4300 m. au-dessus du niveau de la mer, au Mont Blanc, par le groupe des physiciens qui utilise la technique des émulsions dans mon laboratoire, Hang Tchang Fong, Jauneau, Morellet et moi-même.

La photo montre deux étoiles. Celle de droite présente 5 ou 6 branches visibles. Du centre sort un méson normal, qui donne une désintégration nucléaire à trois branches de faible énergie (on observe un proton de 5 MeV). Il s'agit probablement d'un méson π puisque l'énergie des particules émises est faible et que le noyau désintégré est probablement léger (barrière de potentiel). Les conclusions générales ne seraient pas différentes d'ailleurs si cette particule était un méson léger.

La partie curieuse du phénomène est une trajectoire qui présente *deux caractères*, elle passe exactement par le centre de l'étoile de


2,5 Mev
δ
9 Mev
1
4
>10 Mev
3
5
1
>2,5 Mev
6
>16 Mev
2
0
50
100
échelle en microns.
5 Mev
β
schéma
a = air
e = émulsion
Plaque C2 (Bore 100 μ)

droite et se dirige vers elle. Sa direction peut être établie d'une manière assez sûre par le comptage des grains, (notamment l'accroissement notable des grains près de l'étoile) et par la variation du scattering multiple. D'autre part, la coïncidence dans l'espace entre cette trace et le centre de l'étoile de droite est aussi bonne que l'on peut l'établir expérimentalement, mais il n'est pas absolument certain que la trace s'arrête exactement au centre de l'étoile : on peut trouver une des traces de l'étoile à peu près dans son prolongement, mais l'ionisation de cette trace est plus faible. D'ailleurs, en supposant qu'il s'agit de la coïncidence fortuite entre le centre d'une étoile et un proton vers la fin de son parcours, on peut calculer la probabilité de cet événement : la probabilité pour qu'un proton dans les derniers 100 microns de sa trajectoire, soit passé fortuitement au centre de l'une quelconque de toutes les étoiles que nous avons observées est inférieure à 1/1.000. On est alors conduit à supposer que l'étoile est due à la désintégration de cette particule capturée par un noyau : si alors on fait le bilan d'énergie de l'étoile en tenant compte des neutrons émis, ainsi que de l'énergie de formation du méson π, l'on trouve une masse plus grande que 700.

d) On possède encore les indications données par Brode. Il a obtenu un petit nombre de mesures de masses de ces particules τ avec le dispositif décrit par Serber. Des mesures ont été faites à 10.000 m. et aussi à basse altitude. L'appareil semble précis, on peut craindre les difficultés dues au scattering dans la dernière chambre, mais je ne pense pas que cela soit très grave, car on y observe des trajectoires dont l'ionisation augmente entre les écrans lorsqu'elles approchent de leur fin de parcours.

Powell nous a parlé des expériences faites aux Etats-Unis par Bradt, F. Oppenheimer, Peters, etc..., qui ont été exposées au congrès de Bristol. Ces expériences donnent deux indications qui me semblent dissociables.

1. Elles montrent par comptage de grains sur rayons isolés l'existence d'un groupe important de particules entre les mésons et les protons. Powell a discuté précédemment les objections qui pouvaient être faites et le caractère assez convaincant de ces mesures.

2. Six ou huit étoiles reparquables (sur un total de 450 observées) dans lesquelles deux particules de masses différentes sont émises en même temps avec une grande longueur de trajectoire : on peut donc, puisqu'il s'agit de phénomènes simultanés, calculer dans chaque cas le rapport des masses des diverses particules émises. Dans deux

cas il semble y avoir émission d'un deutéron et d'un méson π très lourd. Le rapport des masses établi par « comptage de grains » a une valeur très voisine de 4. Si les étoiles étaient vieilles et dataient du transport des plaques aux Etats-Unis, on ne pourrait sans doute pas trouver un rapport 4 : seul un rapport 2 ou 3 pourrait s'expliquer en supposant que des traces de proton et deuteron (ou de triton) aient été prises pour des traces de méson et proton. S'il s'agissait de vieilles traces d'une particule α et d'un proton, on trouverait un rapport très supérieur à 4. Dans les 4 ou 5 autres cas où le rapport est de l'ordre de 2, on peut faire l'objection que ce sont des étoiles anciennes et du fading : pourtant les valeurs absolues de comptage des grains sont en bon accord avec celles obtenues sur les trajectoires isolées indiquant des mésons très lourds.

e) Il y a enfin des expériences faites par Alichanow et Coll, qui indiquent tout un spectre de masses, mais je ne les discuterai pas ici, faute d'avoir pu les étudier en détails.

M. Casimir. — How many cases have been observed by Brode?

M. Leprince-Ringuet. — Je pense que MM. Serber et Oppenheimer seront plus qualifiés que moi pour le dire.

M. Serber. — Between six and eight are shown in the histogram of the sea level results. I believe more have been observed in high altitude work.

M. Leprince-Ringuet. — Il a trouvé des particules positives et négatives. Pour les positives, on peut toujours craindre que ce ne soient des protons, mais la présence de négatives supprime cette crainte.

M. Bohr. — In these problems it seems relevant to discuss the stopping of fast particles.

At velocities less than 10^8 cm./sec., électronic collisions give place to nuclear collisions. This can be well seen in the case of fission fragments and α particles. It has been investigated in Copenhagen by Boggild, studying the difference in stopping power of hydrogen and deuterium.

Here the electronic structure is the same, but due to the heavier mass of the deuterium nucleus, the ranges are 7% longer in deuterium than in hydrogen.

Now let us consider the stopping of particles which are several times heavier than the electrons, such as the various mesons.

The pioneer experiments in Italy and Princeton on the capture of mesons in various materials raised the question as to whether the mesons could be captured into orbits and decay before beeing brought to rest on the nucleus. This appears now not to be the case. It still seems of interest to study the capture of particules of lifetime shorter than that of the μ-meson.

At the end of the range of the mesons the nuclear collisions become important. The frequencies of the atoms bound into the lattice in matter are small compared to the atomic frequencies, and therefore they act as if they were free. At each collision with a nucleus, exchanges of momentum and also of energy occurs, and this process brings the velocity down to values comparable to those at which the mesons can be born in the nuclei. Then arises the problem of interchange of the mesons with the atomic electrons.

The problem is one of higly adiabatic character. Another well known problem of this kind is the Auger effect. Here we have to deal with particles much heavier than electrons, and in comparable regions of the atom they have much smaller velocities. The probability of occurence of anything similar to Auger effect is much smaller. I do not mean that a particle could be kept for a time comparable with the lifetime of the μ meson.

The problem seems to be in the first approximation not one of collision of free particles; but an adiabatic one in a strong field of force.

If these particles can live in light materials for a time of 10^{-10} to 10^{-6}, and of course much shorter times in heavy material, the problem of the stopping of the mesons in silver bromide or in gelatine has to be examined. Powell supposes that the meson have equal chances to stop in gelatine or silver bromide because they occupy equal spaces. But the chances of stopping in lighter material is much larger because of these atomic phenomena.

In heavy materials, the mesons have more chances to be captured before decay.

The problem also applies for mesons of 600 electrons masses or more. If their properties are to be compared to those of the π mesons, it is an important point because the evidence is that some of them are stopped in the photographic plates without producing stars.

M. Teller. — I would like to recall the work Fermi and I have done on the capture in the orbits of μ mesons of mass 200. Here we are concerned with a meson at mass 800, which might make some difference.

For a positive particle of slow velocity, the energy loss to electrons is small, and the energy loss to nuclei is very strong.

Fermi and I have established that for negative particles, even of very small energies, the energy loss to electrons is always very important. When the particle is drawn towards the nucleus, it acquires high kinetic energy and again becomes able to interact with the electrons.

M. Bohr. — This is not only a problem of energies, but also of frequencies.

M. Teller. — For the 200 mass meson, the energy loss to nuclear vibrations has been found negligible.

The second point is the question of energies and frequencies. In the case when the electrons can only acquire a definite energy, as in insulators, it might be very difficult for the particle to loose energy.

We have found that, in the case of insulators, 200 masses particle, and nuclear charges less than 6, that there exist, orbits of very high angular momentum from which any further loss of energy is very difficult if not impossible. For the particles of 800 mass, and for gelatine which is an insulator, and of nucleus charge less than 6 it seems indeed possible that the particle might be captured in such orbit, and consequently it would not fall onto the nucleus and not give birth to a star. In fact, for 800 masses particles I believe that energy loss from circular orbits may become difficult up to $Z = 10$. Even for light elements there are other orbits sufficiently close to the nuclei for the capture to occur.

It all depends on the angular momentum of the particle in the last carbon on which it will stay.

It is probable that the capture by hydrogen has not to be considered, because the meson can form with hydrogen a close neutral body which can go further through matter and be eventually broken up by another nucleus, the meson is eventually captured.

The absence of stars in gelatine in a few cases is understandable, but in the general case is not.

In the case of silver bromide, the frequencies are such that capture always occurs.

The absence of stars can be explained if one assumes that in the capture of the meson by a nucleus, at least one particle is emitted, as suggested by Serber.

On the other hand, the evidence of Leprince-Ringuet indicates that in some cases mesons produce stars.

There is therefore, no contradiction if the lifetime of the π-meson is of the order of 10^{-8} or 10^{-10}. At the same time, the time of capture as calculated by Fermi and myself may be of the order of 10^{-13} sec.

M. Bohr. — The statistical methods might not be applicable in this case. I think it is in first approximation a problem of adiabacy.

Calculations already made do not agree with Teller's conclusions, and the problem clearly needs to be examined in much greater details.

The angular momentum at which the particle arrives is determined not purely by chance, but by the moments of the electrons.

M. Peierls. — Discussions at Birmingham and extensive work by Ferretti has shown that there is no possibility of this kind in the case of the μ-meson, in particular for metals.

The problem has to be re-opened in the case of dielectrics and of heavy mesons.

It is not excluded that τ mesons are predominantly positive. They might be in some way the transformation products of the incident protons. This, however, would be ruled out by the fact that Brode has found both positive and negative τ particles.

Propriétés des particules des grandes gerbes de l'atmosphère

P. Auger

Travail exécuté par R. Maze, A. Frésin, J. Dandin et P. Auger

INTRODUCTION

Nous avons cherché dans ce travail à obtenir des données précises sur les propriétés des particules contenues dans les grandes gerbes de l'atmosphère. Nous avons montré en 1938 que ces gerbes contenaient certaines particules de pouvoir pénétrant plus grand que celui qui peut être normalement attribué à des électrons. Ces études ont ont été faites par deux procédés.

1. Des mesures d'absorption faites avec des compteurs, mesures permettant d'étudier en même temps la distribution géométrique des trajectoires.

2. Des mesures du pouvoir gerbigène (production de secondaires multiples) des particules ayant traversé des écrans de plomb variables.

Dans les mesures d'absorption et de distribution géométriques, nous avons cherché à observer des différences de distribution en fonction de l'épaisseur d'écran protégeant chaque compteur. Nous avons disposé ces compteurs au nombre de 9 sur une distance horizontale de 15 mètres, au niveau de la mer, à l'air libre. Les coïcidences intéressant au moins 3 de ces compteurs étaient enregistrées, et dans chacun des cas d'indication individuelle des compteurs touchés était inscrite, permettant ainsi des conclusions au sujet de la répartition des particules.

GERBES ETROITES, LOCALES ET GRANDES GERBES

Depuis longtemps, des coïncidences assez fréquentes ont été observées entre compteurs placés à l'air libre à petite distance les uns des autres, soit moins de 50 cm. au total. Nous avons décrit des coïncidences comme dues à des gerbes atmosphériques locales, limitées en étendue. On sait en effet, qu'au-delà de 50 cm. d'envergure, le nombre de coïncidences enregistrées tombe à une valeur beaucoup moindre (au niveau de la mer surtout) valeur qui ne change ensuite que très lentement avec l'augmentation de la distance locale. Pour séparer nettement ces gerbes locales des grandes gerbes, nous avons disposé les 9 compteurs en 3 groupes de 3, dont le groupe central enregistrait les gerbes locales. Deux dispositions ont été utilisées :

$$
\begin{array}{cccc}
 & \overbrace{0\;\;0\;\;0}^{4\mathrm{m}} & \overset{25}{0\;\;0\;\;0} & \overbrace{0\;\;0\;\;0}^{4\mathrm{m}} \\
\mathrm{I} & & & \\
 & \mathrm{A} & \mathrm{B} & \mathrm{C} \\
\mathrm{II} & \underbrace{0\;\;0\;\;0}_{1\mathrm{m}} & \underset{50}{0\;\;0\;\;0} & \underbrace{0\;\;0\;\;0}_{4\mathrm{m}} \\
 & 1\;\;2\;\;3 & 4\;\;5\;\;6 & 7\;\;8\;\;9
\end{array}
$$

Avec I, les écrans individuels couvrant chaque compteur pouvaient atteindre l'épaisseur de 7,5 cm., avec II ils atteignaient 20 cm.

Sans écran, on observe dans les deux dispositifs un comptage de triples (456), c'est-à-dire du groupe B, très supérieur à ceux des groupes A ou C. On peut raisonner simplement comme suit pour distinguer les gerbes locales des grandes gerbes. Il y a 84 combinaisons de trois compteurs dans le système des 9. Or, avec I, le total des coïncidences triples autres que (456) est de 276, pour 149 coïncidences en (456). S'il ne se superposait pas en (456) l'effet des gerbes locales à l'effet des grandes gerbes, nous ne devions avoir en (456) que $\frac{276}{83} = 3{,}6$ dans le même temps. Il y a donc 145 coïncidences (456) dues à ces gerbes locales. Si nous faisons la même étude avec le dispositif II, nous trouvons que le groupe central B de 50 cm. d'envergure ne compte plus que 2 fois plus que les groupes A et C (1 m. et 4 m.) au lieu de 40 fois. L'effet des gerbes locales est presque supprimé. Entre A (1 m.) et C (4 m.) il n'y a plus aucune différence mesurable (sans écran de plomb).

GRANDES GERBES

Ayant fait cette séparation, nous pouvons éliminer l'effet des gerbes locales et étudier la répartition des particules de grandes gerbes parmi les 9 compteurs. Un calcul simple au sujet de l'abondance relative à attendre pour les triples, quadruples, quintuples, sextuples, septuples, octuples et nonuples, peut être fait en admettant une distribution des grandes gerbes en $\Delta^{-\gamma}$ où Δ est la densité des particules, et γ un exposant trouvé égal à 1,5 expérimentalement de 0 jusqu'à 5.000 m. d'altitude. Le tableau 1 montre la comparaison des valeurs trouvées à Paris et à 5.000 m. d'altitude.

TABLEAU 1

Multiplicité	Calculé	Mesuré à Paris		Mesuré à 5.000 m.		Remarque
		coïncidence	Pour ‰	coïncidence	Pour ‰	
N_3^{-6}	556	280	596	2052	530	On désigne par N_3^{-6} tout événement tel que : 3 Cpt touchés 6 non touchés N_6^{-1} = 8 Cpt touchés 1 non touché. N_9^0 = les 9 compteurs touchés. N_3^{-6} est le nombre total d'événements triples contenus dans l'enregistrement etc., porté dans les colonnes.
N_4^{-5}	203	85	199	834	210	
N_5^{-4}	98	41	87	436	110	
N_6^{-3}	56	23	50	243	62	
N_7^{2}	36	20	42	159	41	
N_8^{-1}	21	8	17	109	28	
N_9^{0}	30	4	9	68	19	
Total d'événements	1000	461	1000	3900	1000	

On remarque la valeur plus élevée des nonuples calculées, parce qu'elles contiennent les coïncidences d'ordre supérieur. Au contraire, expérimentalement, la valeur est plus petite. En effet, dans le calcul, le spectre de densité est supposé d'extension indéfinie vers les grandes densités, alors que ce n'est pas vrai expérimentalement et que γ varie avec Δ. Tel qu'il est, le tableau montre un bon accord avec les prévisions. Le rapport des triples à l'ensemble des coïncidences $\frac{N_3}{N}$ est une façon simple de définir cet accord. Il est égal à 0,53 à 5.000 m. d'altitude, et à 0,59 à Paris (pour 0,56 calculé).

Il est intéressant de considérer le nombre de fois que chaque compteur a été individuellement touché, dans toute combinaison. Ainsi, dans le dispositif I, on distingue une grande différence entre les comptages de 1, 2, 3 ou de 7, 8, 9 avec ceux des compteurs proches 4, 5, 6. Ceux-ci comptent deux fois plus environ. Dans le dispositif

II, cette différence s'efface presque complètement, et une faible décroissance s'observe seulement lorsqu'on va des compteurs à 50 cm. à ceux à 1 m., puis à ceux à 4 m.

TABLEAU II

Groupe	Compteur	Dispositif I		Dispositif II	
		Chocs	Total par groupe	Chocs	Total par groupe
A	1	160	558	318	940
	2	188		319	
	3	210		303	
B	4	349	1023	312	971
	5	344		342	
	6	330		312	
C	7	185	551	313	923
	8	188		322	
	9	178		288	

Cet ensemble d'observations montre que la distribution des particules de grande gerbe, sans écran, est uniforme. Il s'y superpose une distribution locale de nombreuses gerbes de petite surface et de densité comparable. Dans un travail précédent, nous avons mesuré la variation en altitude de ces deux types de gerbes, et montré que les gerbes locales croissent nettement moins vite que les grandes gerbes jusqu'à 3.000 m.; à partir de cette altitude, il n'y a plus de différence de comportement. Nous avons attribué cela à l'intervention des gerbes secondaires locales produites par les mésons dans la basse atmosphère, intervention qui devient négligeable au-dessus de 3.000 m. d'altitude.

ABSORPTION JUSQU'A 10 cm. Pb

Il n'était pas possible de placer des écrans de plomb supérieurs à 7,5 cm. sur les compteurs du dispositif I à cause de la proximité des compteurs centraux 4, 5, 6. Mais la mesure d'absorption faite sur ce dispositif a l'avantage de permettre de comparer la décroissance des gerbes locales avec celle des grandes gerbes. Le résultat est que les 7,5 cm. de plomb n'apportent aucun changement observable dans la distribution géométrique des coïncidences, ni dans le comptage relatif des compteurs individuels. Un faible changement est trouvé dans le rapport entre les gerbes locales 4, 5, 6 et les grandes gerbes : avec l'écran il est de 40% au lieu de 33% sans plomb. L'exposant γ calculé présente la même valeur 1,5. Le rapport des triples aux coïncidences totales est 0.57. Bien entendu, tous ces événements ont une fréquence décrue par un facteur d'absorption qui est égal à 5,8 environ. Tout se passe en somme comme si nous avions diminué la surface de tous les compteurs environ 10 fois.

La seule petite différence étant une décroissance un peu moindre des coïncidences locales, entre les trois compteurs proches, nous avons voulu nous assurer que ces coïncidences ont la même signification que sans plomb, et qu'elles sont indépendantes des événements touchant les autres compteurs. Nous avons alors enlevé le plomb de tous les compteurs sauf 4, 5, 6 de façon à augmenter considérablement le comptage de ces compteurs 1, 2, 3, 7, 8, 9, et transformer en quadruples et autres toutes les triples (4, 5, 6) qui ne seraient pas réellement dues à un phénomène local isolé. Il ne s'est produit aucun changement dans le comptage (4, 5, 6) triple isolé, dont le caractère local est ainsi démontré. La similitude du pouvoir pénétrant de ces gerbes locales avec celui des grandes gerbes est également démontré au moins jusqu'à 7,5 cm. Pb.

ABSORPTION JUSQU'A 20 cm. Pb

Avec le montage II nous avons pu étudier les mêmes phénomènes sous 11; 13.5; et 20 cm. de plomb. Dès 11 cm. une concentration dans l'espace pour les coïncidences de grandes gerbes, ainsi qu'une diminution de densité est observable. Elle est très marquée pour 13.5 cm., épaisseur avec laquelle nous obtenons une distribution très différente de celles décrites jusqu'ici. Si nous admettons que les gerbes locales n'ont que peu d'importance avec ce dispositif (puisqu'à l'air libre les compteurs à 50 cm. n'enregistrent que 2 fois plus de triples isolées que les compteurs à grande distance) et si nous comparons simplement les nombres de fois que les différents compteurs sont touchés, nous trouvons les compteurs à 50 cm., 1 m., 4 m. touchés respectivement avec les fréquences 3, 2, 1. D'autre part, les triples sont en nombre beaucoup plus élevé que les autres multiplicités. Même en ne comptant pas les coïncidences (4, 5, 6) isolées nous avons $\frac{N_3}{\text{total}} = 0{,}73$. Si les coïncidences triples sur 50 cm. étaient comptées, on aurait $\frac{N_3}{\text{total}} = 0.81$.

TABLEAU III

Dispositif I sous 7,5 cm. Pb

Groupe	Compteur	Chocs	Total par groupe	Coïncidence	Nombre	Total
	1	127				
A	2	125	384	N_3^{-6}	170	
	3	132				
				N_4^{-5}	70	
	4	228				
B	5	267	748	N_5^{-4}	22	292 grandes gerbes
	6	253				
				N_6^{-3}	15	
	7	130				
A	8	128	363	N_7^{-2}	8	
	9	105				
				N_8^{-1}	7	
				N_9	5	

TABLEAU IV

Groupe	Compteur	Chocs	Total par groupe	Coïncidence	Nombre	Total
	1	37		N_3^{-6}	46	
A	2	36	100			
	3	27		N_4^{-5}	11	
	4	55		N_5^{-4}	3	
B	5	53	152			63 grandes gerbes
	6	44		N_6^{-3}	2	
	7	18		N_7^{-2}	1	
C	8	18	51			
	9	15		N_8^{-1}	0	
				N_9	0	

Il est intéressant d'examiner ce qui se passe avec ce montage II pour de plus faibles épaisseurs. L'expérience a été faite avec 5,2 cm. et a montré une augmentation relative importante des triples à 50 cm., et même à 1 m., par rapport à l'expérience sans écran. L'effet de concentration décrit pour 13,5 cm. commence déjà, et avec les mêmes caractères. Si cet effet n'a été observé que de façon insignifiante avec le dispositif I, c'est que l'abondance des gerbes locales à 25 cm.

masquait fortement tout effet de ce genre. On pouvait cependant lui attribuer une partie de l'augmentation relative de coïncidences triples à 25 cm. par rapport au total.

Nous nous trouvons donc en présence de deux effets des écrans de Pb, concentration dans l'espace des trajectoires, changement du spectre en densité (augmentation de l'exposant γ). Ces deux effets pourraient être appliqués si l'on admet que les particules très pénétrantes ainsi observées (plus de 10 cm. de plomb) sont rassemblées dans une partie peu étendue de la gerbe (partie centrale?) et que l'abondance de ces gerbes varie plus rapidement en fonction de la densité de ces particules que pour les particules ordinaires.

EFFETS LOCAUX DU TYPE EXPLOSIF

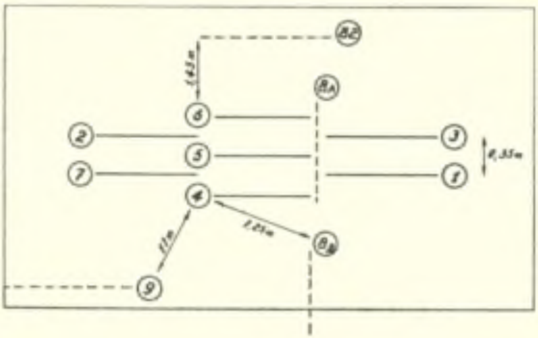
Notre attention a cependant été attirée sur la possibilité de l'intervention d'autres phénomènes que ceux des gerbes atmosphériques proprement dites. Nous avons observé depuis longtemps (1) des effets relativement rares donnant lieu à des coïncidences entre compteurs fortement protégés et séparés par du plomb. Ces effets qui semblent dus à des explosions locales, déterminés dans l'écran même, produisent une abondance de particules les unes très peu pénétrantes, les autres assez pénétrantes pour sortir des écrans et pouvoir être décelées au dehors, à la chambre à détentes. Les mesures décrites ici sous écran épais, correspondent à des rythmes de comptage assez faibles pour que de tels effets, mêmes rares, puissent intervenir. Il est alors possible d'attribuer une partie au moins des coïncidences entre compteurs proches (50 cm.) très protégés, à de tels effets qui ne sont pas alors nécessairement et étroitement liés à une grande gerbe ainsi que nous l'avons montré dans le travail cité. Evidemment, de tels effets ne peuvent servir à interpréter les coïncidences entre compteurs éloignés, ceux à 4 m. par exemple, pour lesquels seules des trajectoires séparées simultanées venant de l'air sont nécessaires.

Afin d'élucider ce point, nous avons groupé nos compteurs tout autrement, et de façon plus analogue au groupement employé dans l'étude ancienne des effets explosifs. Les 7 premiers compteurs, placés sous 20 cm. de plomb, formaient un ensemble serré, figuré en plan horizontal sur la figure (V). Le huitième pouvait être placé à l'air

(1) *Phys. Rev.*, **61**, p. 549 (May 1942).

libre (en fait sous 4mm. de Pb), soit très près du groupe central, (8A) soit à 1,50 m. (8C). Le neuvième et dernier était placé à l'air libre, à 15 mètres de distance horizontale. Avec ce dispositif, les coïncidences entre les 7 premiers compteurs peuvent être dues, soit à des particules pénétrantes venues de l'air, faisant partie d'une grande gerbe, soit à des phénomènes locaux (explosifs), accompagnés seulement rarement d'une gerbe atmosphérique. Dans le premier cas, il semble que les compteurs 8 et 9 doivent être touchés à peu près de même façon, et souvent ensemble en même temps que le ou les compteurs couverts. Dans le second, le compteur 8, (proche) doit être touché plus souvent que 9 (lointain) et d'autant plus souvent qu'il est plus proche.

TABLEAU V



Voici ce qu'indiquent les expériences. Tout d'abord, il n'arrive pratiquement jamais que le compteur nu lointain soit touché sans que le compteur nu proche le soit aussi. Cela montre qu'il s'agit alors toujours de grandes gerbes de forte densité. Je rappelle qu'au moins un des compteurs sous 20 cm. de plomb est toujours touché, pour donner une triple. Au contraire, il est très fréquent que 8 (proche) soit touché sans que 9 (lointain) le soit. Ce cas, à peu près aussi fréquent que les grandes gerbes touchant 8 et 9 ensemble est dû à un phénomène local, très sensible à la distance de ce compteur 8 proche depuis le centre du groupe sous plomb. Ainsi quand ce compteur 8 nu est situé directement sur le plomb écran mais non directement sur un des compteurs pour éviter les coïncidences doubles dues à des particules verticales pénétrantes, ces effets locaux sont presque deux fois plus fréquents que les grandes gerbes, tandis qu'ils tombent au-dessous de celles-ci dès que le compteur 8 est déplacé à 1 m. 50 de distance du groupe sous plomb.

TABLEAU VI

Nombres relatifs de coïncidences dans chaque série.

	8 sans 9	9 (et 8)	9 sans 8	ni 8 ni 9
8 proche (A)	18	11	(1)	3
8 à 1,50 (C)	6,8	8	(1)	6

Enfin il arrive que ni l'un ni l'autre des compteurs nus ne soit touché, alors il s'agit d'un effet local non accompagné de gerbe de l'air (grande ou petite), peut être d'une gerbe pénétrante de nature explosive (trajectoires très divergentes). L'étude de la répartition des chocs sur les compteurs couverts montre en effet une très forte concentration, les compteurs centraux étant plus souvent touchés que les périphériques. Cette concentration ne se montre pas dans les chocs dus à des grandes gerbes. Il paraît donc possible de séparer nettement les grandes gerbes avec partie pénétrante des effets locaux intérieurs à l'écran, non accompagnés. Il sera nécessaire de faire de longues statistiques sur ces différents phénomènes afin d'en examiner la structure géométrique, ainsi que les clichés de chambre à détentes en continuant le travail déjà cité.

TABLEAU VII

Fréquence des chocs pour les coïncidences comprenant le compteur éloigné (9)		Fréquence des chocs pour les coïncidences ne comprenant ni le compteur 9 ni le compteur 8 (effets locaux)	
Compteur	Nombre de chocs	Compteur	Nombre de chocs
1	23	1	7
2	9	2	8
3	22	3	15
4	15	4	32
5	16	5	39
6	22	6	40
7	11	7	10

Une première statistique a permis de montrer que le point de divergence des effets locaux se situe bien dans l'écran de plomb.

Les compteurs situés du côté où est placé le compteur proche nu (N^0 8) comptent nettement plus que ceux qui sont situés sur l'autre côté du groupe sous écran. Il semble bien qu'il faille attribuer une partie importante des chocs qui déclenchent le compteur 8 et pas le compteur 9 à de tels effets produits dans l'écran, et dont certaines particules secondaires sortent pour atteindre des compteurs placés à petite distance. La question de savoir en quelle proportion il s'y ajoute des gerbes très étroites d'origine atmosphérique est en cours d'étude.

CARACTÈRES DES PORTIONS PÉNÉTRANTES. ABSORPTION

Nous avons vu que des portions notables des grandes gerbes donnaient encore des chocs dans des compteurs protégés par 20 cm. de plomb. Nous avons voulu tout d'abord mesurer aussi correctement que possible l'absorption de ces parties pénétrantes dans le plomb, puis examiner leurs propriétés, surtout en ce qui concerne le pouvoir de production de secondaires (pouvoir gerbigène).

En ce qui concerne l'absorption, les mesures faites avec 9 compteurs au niveau de la mer, permettent de calculer un coefficient si l'on admet une certaine distribution spectrale des gerbes donnant leur fréquence en fonction de leur densité (Densité de particules par unité de surface horizontale). Avec une loi en $\Delta^{-\gamma}$ et un exposant de 1,5, les mesures que nous avons faites montrent que le coefficient massique $1/\alpha$ entre 5 et 15 cm. de plomb est égal à environ 0,025 cm. gr.$^{-1}$. Ce coefficient n'est pas constant, et décroit notablement lorsque la mesure est portée jusqu'à 20 cm.

Nous avons utilisé également (Daudin) une autre méthode qui consiste à faire intervenir dans la mesure les gerbes même très peu denses (les plus nombreuses) et à ne protéger par l'écran de plomb qu'un seul compteur de petite surface parmi les 3 qui déterminent la coïncidence mesurée. On peut alors admettre que la décroissance du nombre de chocs observés quand l'écran augmente est due à l'absorption seule et non à la sélection. Ces intensités ont été mesurées, à basse altitude, avec un appareil comportant deux groupes A et B composés chacun de 15 compteurs en parallèle (surface environ 0,20 m^2) pouvant être placés à 5 m. ou 70 m. l'un de l'autre, et un troisième compteur C de petite surface sous écran variable.

TABLEAU VIII

Epaisseur de l'ecran en centimètres de plomb.	Nombre relatif de coïncidences triples. Basses altitudes.	Remarques
0	100	
0,6	122	
2	100	
4	50	Effet des transformation des photons en électrons.
8	21	
14	4,5	
19	3,5	

Les valeurs de l'intensité ne décroissent plus que très lentement après 14 cm. d'écran. De 4 à 14 cm., il y a une réduction d'un facteur 11 pour 10 cm. de plomb : cela correspond à un coefficient massique de $\alpha = 40$ gr./cm^2 environ, en accord raisonnable avec le coefficient obtenu par la méthode précédente, qui est si différente dans son principe. La valeur de ce coefficient ne concorde bien ni avec les propriétés des électrons ni avec celles des mésons.

PROPRIÉTÉS GERBIGÈNES.

En vue d'une étude des propriétés de ces particules pénétrantes, le compteur de petite surface C a été remplacé par deux compteurs CC' en coïncidence, et les coïncidences triples ABC et quadruples ABCC' ont été comparées. Rappelons que les groupes A et B pouvaient être placés à 5 m. ou à 70 m. l'un de l'autre. Dans ces conditions le rapport Q/T des quadruples aux triples donne une indication des propriétés des particules pénétrantes de produire des effets secondaires sous les écrans qui protègent la paire CC'. Le tableau IX montre que ce rapport qui est de 0,28 à l'air libre, s'accroît jusqu'à 0,53 sous un demi-centimètre de plomb à cause des gerbes de Rossi qu'y produisent les électrons de grande gerbe. Mais lorsque l'écran est augmenté à 8 cm., il reste encore égal à 0,34. Ceci pour la base 70 m. comme pour la base 5 m.

TABLEAU IX

Epaisseurs de l'écran en centimètres	Triples (nombres relatifs)	Quadruples (nombres relatifs)	Quadruples / Triples
0	100	100	0,28
0,6	132	250	0,53
2	92	166	0,51
4	73	133	0,54
8	20	31	0,45
14	4,3	5,2	0,35
19	3,3	3,9	0,33

Cet effet correspond bien à la production de gerbes d'électrons, comme on peut le montrer en interposant une paroi de plomb de 0,8 cm. entre C et C'. La décroissance des quadruples est alors de 40%. Une telle absorption de près de moitié dans 0,8 cm. de plomb est bien caractéristique de gerbes d'énergie assez faible. Je rappelle qu'ici il s'agit de phénomènes liés à une grande gerbe et non d'effets locaux produits dans l'écran de plomb par des particules solitaires. Ici encore les propriétés des particules pénétrantes de ces grandes gerbes ne sont ni celles des mésons, ni celles des électrons. Des mésons ne pourraient être accompagnés que de moins de 10% d'électrons secondaires, et ce pourcentage tomberait encore par interposition de la cloison. Des mésons doubles venus de l'air ne seraient pas sensibles à la cloison. Des électrons qui auraient réussi à traverser 19 cm. de plomb seraient accompagnés d'une gerbe de très grande densité, insensible à la cloison.

CONCLUSION

Pour autant que des conclusions nettes peuvent être déduites des mesures, dans leur état actuel, il semble que ce soient les suivantes :

1. Il y a deux types de groupements de particules dans les gerbes qui se produisent au sein de l'atmosphère. L'un est très local puisque l'écartement des branches ne dépasse pas 50 centimètres au niveau de la mer. L'autre est très étendu puisque des particules simultanées peuvent être décelées à plusieurs centaines de mètres les unes des autres. Ces deux types de groupes de particules sont indépendants. Ils ne varie pas exactement de la même façon avec l'altitude dans la basse atmosphère.

2. Sous des écrans de plomb, ces deux groupes de particules subissent une absorption du même ordre, mais dans le cas du groupe de petite extension, les mesures sont compliquées par l'intervention de phénomènes secondaires produits dans l'écran lui-même. Les particules du groupe étendu (grandes gerbes) sont absorbées de façon que leur nombre tombe d'un facteur 10 pour 10 cm. de plomb dans la région entre 5 et 12 cm. de plomb. Cela correspond à un coefficient massique de l'ordre de 40 gr./cm^2.

Pour des épaisseurs plus grandes telles que 20 cm. de plomb, le coefficient d'absorption décroît notablement, indiquant un durcissement du rayonnement qui subsiste sous ces écrans. Le coefficient d'absorption de 15 à 20 cm. dépasse certainement 100 gr./cm^2.

3. En même temps que ce changement de coefficient d'absorption, une différence de distribution des particules se fait sentir, l'extension horizontale des parties très pénétrantes des grandes gerbes paraissant beaucoup moindre que celle de leurs parties très absorbables.

4. Les parties très pénétrantes des grandes gerbes (au-delà de 20 cm. de plomb) sont productrices de nombreux électrons secondaires dans le plomb.

5. Dans l'étude des coïncidences entre compteurs proches, sous écran épais de plomb (10-20 cm.) un phénomène local produit dans le plomb se substitue au phénomène local atmosphérique sans que l'on puisse être assuré que l'origine en soit la même. Le coefficient d'absorption de ce phénomène secondaire local est de l'ordre de 100 à 200 gr. par cm^2, c'est-à-dire du même ordre que le coefficient de décroissance dans l'atmosphère des grandes gerbes (et aussi des gerbes locales au-dessus de 3.000 m. d'altitude).

Une explication de cet ensemble ne peut être donnée dans l'hypothèse de la présence seulement de mésons et d'électrons dans le rayonnement cosmique. Il n'est possible d'attribuer à des mésons les effets secondaires si importants observés même sous 20 cm. de plomb dans les parties pénétrantes des grandes gerbes, ni sans doute les effets « explosifs » observés pour des particules pénétrantes qui ne font pas partie des grandes gerbes. L'absorption des particules de grandes gerbes entre 5 et 15 cm. de plomb ne peut guère être attribuée à des électrons seuls.

Discussion du rapport de M. Auger

M. Ferretti. — I would like to know the exact definition of the absorption coefficient.

M. Auger. — The one that has been observed with two trays of unprotected counters, and one small protected counter is an exponential absorption coefficient.

M. Ferretti. — I find rather difficult to explain what exactly is an exponential coefficient in such an experiment. You have observed penetrating particles from showers of very different densities, and penetrating particles from showers of various sizes are perhaps not absorbed exactly in the same way.

M. Auger. — We have tried to eliminate the density effects by using very large trays.

M. Ferretti. — I am not sure that such an arrangement does eliminate the density effects. In fact, the frequency of the showers is rapidly decreasing as the density increases and therefore you are recording a large amount of showers of small density, which may be discriminated also by large trays, and a smaller amount of showers of higher density which are not discriminated.

If I have correctly understood, you are trying to explain your experimental results assuming two different components which are both exponentially absorbed. I want to remark that also assuming that this hypothesis is correct for showers of a given density, it should not be any more correct measuring the absorption for showers of different densities without discrimination, if the absorption coefficient do depend from the density; therefore I suspect that your arrangement should give a kind of average of rather difficult interpretation. It appears that you are thinking that events observed under 15 cm. of lead are due to the more penetrating component, and events observed under 4 cm. of lead correspond to the less penetrating one. I would like to make the remark that perhaps the number and the precision of your data relating to the absorption of air extensive showers are not sufficient to establish the existence of two components exponentially absorbed and for measuring the exponential absorption coefficient. Do you have some data for other thiknesses of lead?

M. Auger. — On table VIII of the report you have the values for 0.6, 2, 4, 8, 14, 19 cm. of lead. Between 4 and 14 the absorption coefficient remains the same.

M. Ferretti. — From some of your experiments it appears that at least a considerable amount of the penetrating particles in the air extensive showers are produced in the absorber of the detecting apparatus. There are some others experiments on the subject by Cocconi, Jànossy, Salvini and Tagliaferri and others which give some evidence in the same way as ours. Therefore it seems quite safe to assume that there is really a considerable production of penetrating particles in the absorber. If this assumption is correct it might be of interest to quote here some experiments by Cocconi and Festa that have been confirmed by Jànossy.

Two trays of counters forming a vertical telescope with 10 cm. of lead in between, were protected everywhere with lead (in the vertical direction with 25 cm. of lead). Multifold coincidence were recorded between the two trays, and also three other unprotected trays of counters 4 meters apart from the central arrangement. Two experiments have been made : one by adding 10 cm. of lead on top of the absorber, the other by replacing the 10 cm. of lead by a thikness of bricks of equivalent mass per unit of surface. It was observed that the rate of coincidences was not changed appreciably. This result seems to show that the penetrating particles are produced with a cross section that depends on Z, the atomic number of the absorbing nuclei, according to the same law as the absorption cross section of the radiation by which they are produced. For instance, if the radiation producing the penetrating component were photons, the cross section for production of the penetrating particles should be proportional to Z^2. As the number of pairs of ordinary mesons which could be produced by the materialisation of gamma rays is much too small for accounting for the number of the « penetrating events » we may be quite sure that the penetrating particles in the air extensive showers are not all ordinary mesons produced by photons or by electrons. I thought indeed that the Cocconi and Festa results could give a rather strong support to the hypotesis that in the air extensive showers there are nucleons that may produce mesons both in the air and in the absorber, giving rise to the « penetrating events ».

This interpretation cannot give an account of certain recent experimental results by Jànossy. However I would like to point out that it seems in agreement with the finding by Rochester, Fretter and Rossi that the nucleons by some processes may give rise not only to mesons but also to a soft component. If this happens in the atmosphere, we may observe an air extensive shower with nucleons and mesons.

In any case it seems completely excluded that the phenomenon involved in extensive and penetrating showers can be explained by photons, electrons and mesons only.

M. Oppenheimer. — I would like to suggest that we go back to consider the origin of these great showers. I think it is not settled, but believe they are not made by primary electrons. There is a great mass of evidence that when primary nucleis collide in the atmosphere they produce three components, mesons of various kinds, nucleons and soft radiation. On purely theorical grounds it is easier to understand the soft radiation as a decay phenomenon rather than a collision phenomenon. So in an Auger shower, there may be, together with the cascades, nucleons and mesons.

The mesons will be of such high energy that a part of them will not have decayed into « cold » mesons, but remain « hot » i. e. still capable of nucleus intercations.

One must expect therefore that there is a component in the Auger shower carrying at least 50% of the energy, which is capable of nuclear interaction and is penetrating. This does not mean that the penetrating particles observed are not secondary, but it does mean that they belong in part to another component. If so a 10^{15} MeV proton produces a shower, the protons and « hot » mesons coming fom that event will produce other smaller showers.

This seems to me one explanation of the fact that however much you filter you still get cascade radiation. Further evidence may sustain this view, but if it is so, I feel it would explain both Auger's experiments and also the observations on the way the showers are initiated and the kind of interactions involved. This is only to start the discussion; perhaps it can be disproved.

M. Heitler. — Perhaps we can discuss this point later on.

M. Bhabha. — I would like to know on what assumptions are based the results given in table I. Are there any assumptions introduced other than those of the usual cascade theory.

M. Auger. — The calculations are given in a paper in *Journal de Physique.* There is an assumption introduced on the spectral distribution of the showers. It is supposed to be a power distribution in density with an exponent of 1.5, and an ordinary Poisson distribution of the showers. This assumption has no theorical basis, but it fits with the experiments on the surface distribution as studied by the Italian and French schools. It is found that the number of counts when the surface of the counters was changed could be explained by such a spectrum.

M. Bahbah. — The assumption then is that the density decreases uniformly with distance from the centre of the shower, and that these are no local concentrations and not localized. This is of interest in connexion with the point made by Oppenheimer, that there are hot mesons present in the showers. One might then expect areas of locally greater density for the soft component in an extensive shower.

M. Oppenheimer. — This has been looked for but needs evidently a specific theory. Rossi's work shows that the theory we proposed two years ago for the distribution in energy and angle of the mesons does not fit the facts, but it is by no means a closed subject.

M. Ferretti. — It seems to be that unless the showers are produced very close to the counters, which rarely occurs, the scattering of the electrons will be sufficient to level out such localized effects.

M. Heitler. — Recently Miss Chowdhuri, and Jànossy and McCusker, have carried out an experiment that may throw light on the nature of the penetrating particles in extensive air showers. A set of counter trays was arranged in coincidence to record extensive showers. A central tray was covered with 15 cm. Pb so that only showers are recorded which contain at least one penetrating particle reaching the central tray. Now a roof was placed 50 cm. above the central tray consisting either of 1.7 cm. Pb or an equivalent thickness of bricks of the same weight/cm^2. If this roof is placed immediately above the 15 cm. Pb of the central tray it has no influence at all. However, if the roof is placed at a distance of 50 cm. above the tray it is found that : (i) the number of coincidences decreases by 25% with the lead roof whereas (ii) the brick roof did not reduce the number of coincidences.

The experiment shows that : (i) the penetrating particles are produced locally by a radiation which is absorbed in 1.7 cm. Pb, but not in an equivalent thickness of bricks. This radiation can hardly be anything else but electrons and photons. (ii) The penetrating particles produced in the roof do not reach the bottom tray, the simplest explanation of which is that they decay partly during the passage of the 50 cm. distance.

It had previously been shown that the number of penetrating particles in a similar experiment (without roof) is 2% of the number of electrons and independent of the material of the absorber above the central tray. It follows, since the electron-photon component is absorbed according to a Z^2-law, that the penetrating particles are also produced according to a Z^2-law. The simplest explanation is that we have to deal with ordinary pair creation, of a new type of particle. Since pair creation is proportional to $1/m^2$ it follows that the mass of the penetrating particles is of the order $\sqrt{50}$, or, say, 3^{-10} electron masses.

Naturally, such a far-reaching conclusion will hardly be accepted before much more experimental material is available. It may, however, be pointed out that the above experiment cannot be understood by any of the familiar processes or particles such as : mesons coming from the air; nucleons accompanying the extensive shower produce mesons in the absorber; photons produce mesons in the absorber in nuclear collisions. In none of these cases a Z^2-law would hold. It may also be that the phenomenon is more complex and cannot be explained by a single type of particle and process.

M. Bhabha. — Is it possible that such particles of about 3 to 5 electron masses have been missed so far in Wilwon chamber photographs?

If particles of mass equal to three electron masses exist we should expect there to be about 10% of these particles among the electrons in cascades.

The ionization curve for such particles must lie close to the corresponding curve for electrons, and the two could be differentiated only in a very narrow momentum range. The ionisation method is rather insensitive for distinguishing such particles unless a special effort is made. One would rather expect to detect them more easily by absorption measurements since the variation and pair-creation cross sections vary as the square of the mass.

M. Blackett. — As far as I am aware no systematic search for such particles in cloud chambers has been made, Such experiments are certainly possible but not particularly easy, so it is not unlikely that if such particles exists they would have been missed hitherto.

Mr. Oppenheimer. — No miss of energy has been observed in numerous pair production experiments, at 17 MeV, this was done in a laboratory Pasadena.

Mr. Heitler. — At an energy as low as 17 MeV, the cross-section for such pair production would be very small.

Mr. Auger. — I think that the λ—mesons could have been missed in cloud chamber photographs just owing to the argument put forward by Bhabha. Scattering experiments of extensive showers in lead seem to bring some evidence in favor of those particles.

Mr. Bloch. — It would perhaps not be so easy to detect particles of 3 or 5 electron masses, even in pair creation, because the measurements were made using the curvature of paths, and thus the momenta of the particles. Since the energy of the γ—rays used is very large compared to the self energy of the electron, most of these heavier particles would still be highly relativistic and, for the same energy, exhibit practically the same curvature as ordinary electrons.

Mr. Teller. — In some of the pairs, one electron had a low energy, and in this case Bloch's remark does not apply.

Mr. Blackett. — Is it not possible to attribute the effect to a neutral particle (heavy photon) rather than to a heavy electron?

Mr. Heitler. — Insofar the present experiment and the Z^2—law are concerned, the answer is affirmative. But a discussion of Janossy and Ferretti in Bristol pointed out an argument which seems to exclude it.

Mr. Ferretti. — In some previous experiments Janossy used a different arrangement, in which manyfold coincidences between protected counters together with an unprotected tray were recorded. The rate of coincidences which was recorded seems incompatible with the assumption of neutral particles producing electrons, because too many neutral particles should be necessary to give rise to a coincidence between several counters separated by thick layers of lead.

I would like to make a remark about the hypothseis of the mesons in connection with this experiment. It seems to me that a mass of 3 m_0 is perhaps hardly compatible with the observed rate of the « penetrating » events. In fact a particle of mass 3 m_0 should have, in average, a considerable loss of energy by radiation in 15 cm. of lead. Therefore the average initial energy should be too high to account for the observed rate of production.

Maybe a mass of 5 — 6 m_o could be better.

One may remark that as the phenomenons of pair production and bremsstrahlung are strictly related, the higher is supposed to be the production by gamma rays, the higher will be the radiation loss of the λ_m mesons. On the other hand, there is a kind of competition between the rate of production and the rate of absorption. Therefore for a certain value of the cross section for production, there will be a maximum of the number of events which can be observed with the Janossy's arrangement. I wonder whether this maximum is compatible wlth the observed rate. I think that one should look at this point to test the internal consistency of the explanation of the Janossy's results by the λm meson hypothesis.

I should like to remark that the results of Powell and Leprince-Ringuet about slow π mesons which come out from the stars, may give a valuable information relating to the interaction between π mesons and nucleons.

The total number of stars observed by Powell was 20 — 30.000. Taking into account that for geometrical reasons a fraction only of the total number of these mesons could be absorbed it seems to me quite safe to deduce from these data that there is in average one meson of energy less than 3MeV produced in a star out of 100 — 200 stars.

One may try to use this information to get an average value of the cross section for production of slow mesons. This value will be an average on the energy of the incoming nucleons producing the stars and the mesons.

We may remark that most of the stars which are observed in the emulsion are very likely produced by nucleons that have not an energy sufficient to produce the meson and the star together. This minimum energy is probably about 250 MeV.

Therefore to get a more useful value of the average cross section we have to consider those incoming nucleons only which have an energy greater than 250 MeV.

Using the available data on the energy spectrum of the nucleons and on the rate of star production (Rossi, *Rev. Mod. Phys.*, Vol. 20, pp 537, 1948, Tab. II, page 562, Tab. III, page 565) one may estimate that about 80 % of the stars are produced by particles (chiefly neutrons) which have an energy less than 250 MeV. Then, the average partial cross section *per nucleon* for production of mesons having an energy less than 3 MeV appears to be not less than $2 - 3 \; 10^{-28}$ cm^2. It seems to me that this partial cross section is rather high. In fact the total cross section per nucleon for production of mesons can't probably be much higher than about $5 \; 10^{-26}$ cm^2 and is unlikely that this maximum value is reached when the kinetical energy of the meson is much less than the rest energy.

Now, the ratio between the volume of the phase space relating to mesons heaving a kinetic energy not greater than the rest mass and the corresponding volume for mesons of an energy smaller than 3 MeV is 2—300, i. e. not smaller and may be greater than the ratio between the maximum total cross section for productionof mesons and the partial cross section for production of mesons having an energy smaller than 3 MeV.

It seems to me that this fact is an indication that the matrix element relating to the interaction between π mesons and nucleons cannot probably increase strongly with the energy. This indication points in the same way that Serber's calculation, about the dependance of the yield of mesons from the energy of the particles in Berkeley experiments.

Detailed calculations are of course necessary before drawing any sure conclusion on this point. However if my guess is right I think that it may be difficult to explain the production of slow π mesons with the conventional theory of nuclear meson field.

In connection with the point that I have discussed just now I should like to make a remark about the hypothetical τ meson observed by Peter. The number of these τ mesons seems to me much greater (by a factor 10) than the number of the ordinary mesons ending their range in the plate.

Following certain results of Bernardini, the number of slow mesons in the plate increasses more rapidly than their number of stars with the height. This means that at a great height most of the mesons which end their range in the plate are created locally. But the τ mesons are supposed to have a life time shorter than the ordinary mesons : therefore they too must be created locally, and the ratio between the number of τ mesons and π mesons can give directly the ratio of the cross sections for production of slow τ mesons and slow π mesons.

As the cross section for production of slow π mesons is rather great, the cross section for creation of slow τ mesons should be extremely great : this seems rather unlikely.

Mr. Bhabha. — The ratio of the cross section for creation of λ mesons and electrons by a 17 MeV photon will be different from the ratio of the square of their masses. The cross section is a function of the energy of the photon measured in terms of the rest mass of the particle. This cross section starts at $h\nu = 2mc^2$, where m is the mass of the particle, and increases at first slowly. Thus the ratio of the cross sections for the creation of λ mesons and electrons may be as small as 1 % for 17 MeV photons. Thus these experiments cannot certainly exclude the existence of such particles yet.

Mr. Ferretti. — I want to modify my preceding statement about τ mesons. Dr. Occhialini pointed out to me that Dr. Peter did underdevelop his plates. In these conditions one cannot compare any more the number of τ and π mesons which end their range in the plate. Therefore the great number of the supposed τ mesons tracks which have been observed is not any more a difficulty against the τ mesons hypothesis.

M. Blackett. — Recently Mitra and Rosser in my laboratory have studied air showers by means of a cloud chamber. They have found that the integral energy spectrum of the electrons is given by

$$N\,(>E) \propto (E + E_c)^{-(1.1 \pm 0.3)}$$

when E_c is the critical energy in air. The result is in agreement with the predictions of cascade theory. In addition they find that some of the air cascades contain penetrating particles with energies over 10^9 ev, the ratio of penetrating particles to electrons being $(0.8 \pm 0.4)\,\%$. It seems likely that these particles are either protons, σ, μ or τ mesons.

We have not looked carefully for very light particles, which therefore might well have escaped our notice.

M. Auger. — The particles you observe are penetrating and non-shower producing and they might be responsible for the tail of the absorption curve.

M. Blackett. — I will show some slides of cosmic ray showers with the object of revealing the complexity of the phenomena which are observed by counters. These photographs have been taken by my colleagues, Rochester, Butler, Mitra and Rosser. The magnetic field in all cases is about 7000 gauss.

Photo 1. An incident positive particle of $p = 5 + 10^9$ ev/c, initiates an explosive type shower in the middle of 3 cm. lead plate. Four of the emitted particles have momenta over 10^9 ev/c; of these 3 are positive and 1 negative. Two particles of momenta $+\ 6.0 \times 10^8$ and $+5.0 \times 10^8$ ev/c show appreciably heavier ionisation and are almost certainly protons. The two fast particles moving to the right form a positive and negative pair, each of momentum extent 1.5×10^9 ev/c.

The relatively small number of low energy electrons emersing from the plate shows that no high energy electrons can have been formed in the collision.

The wide angle of projection, up to 45^0, of the energetic particles is noteworthy. Taking particles with momenta over 5×10^8 ev/c, we find six spread fairly evenly over a solid angle of about 2 square radians, that is the rays have an angular density of about 3 rays per square radian.

Photo 2. An incident positive particle of momentum $+5 \times 10^9$ ev/c initiates an explosive type shower in which 4 high energy particles are emitted over a solid angle of about 2 square radians. The two particles on the left form a positive and negative pair of momentum $+1.5 \times 10^9$ ev/c — 1.1×10^9 ev/c respectively.

Clearly Photos 1 and 2 represent closely related phenomena; it will be convenient to refer to them as wide angle penetrating showers.

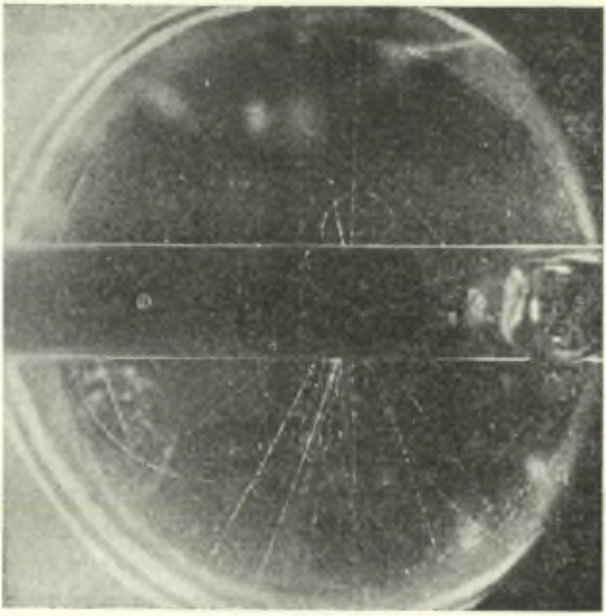


Photo 1.

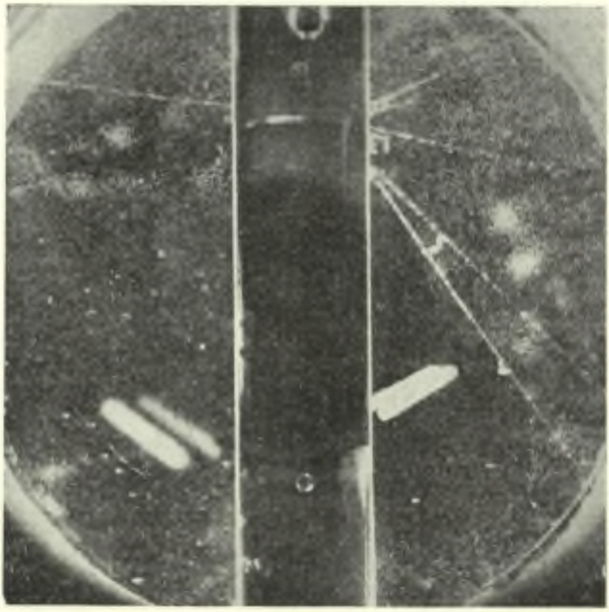


Photo 2.

Photo 3. A shower of four penetrating particles of momenta from 1 to 3×10^9 ev/c. All four are positive. The one of lowest momentum is anomalously scattered through 13^0. All four rays lie within a solid angle of about 0.02 square radians giving an angular density of some 200 rays per square radian. Such a shower will be referred to as a narrow angle penetrating shower.

Photo 4. A complex penetrating shower associated with an extensive shower. One particle, a positive particle with $\rho = 0.63 \times 10^9$ ev/c, is clearly penetrating, but at least three other particles in the narrow angle core seem to pass through the plate without multiplication. The shower is coming forward in the chamber at a rather steep angle. Accurate measurement of the tracks is difficult because of the confusion and the rather low technical quality. Most of the particles seem to be positive and some are lightly ionising with momenta about 5×10^8 ev/c. suggesting that they are not protons. The wide-angle pair at the lower right-hand side of the photograph seem to be protons.

The central narrow angle core of penetrating particles closely resembles the narrow group in Photo 3. The main difference between the showers lies in the considerable number of electrons of relatively low energy in Photo 4 but not in Photo 3. Showers such as Photo 4 will be called narrow penetrating showers with an electronic component.

Photo 5. A penetrating shower consisting of three penetrating particles. One is a negative particle of momentum 1.1×10^9 ev/c. and is anomalously scattered through 12.0^0 in the lead plate. Another is positive with a momentum of 4.5×10^9 ev/c. above the plate and 3.0×10^8 ev/c. below the plate an is scattered through 28.0^0. The particle below the plate is a proton. The heavily ionising particle which appears to come from the same region in the lead plate as the middle incident particle is actually in a plane 1.8 cm. behind it. Thus if the heavily ionising particle is connected with this particle it must be through an intermediate link. The heavily ionising particle is positive and has a momentum of 1.6×10^8 ev/c. A proton of this momentum ionising $15 \times$ minimum whereas the ionisation is estimated as $7 \times$ minimum. The difference may be due to fluctuation or indicate a particle of intermediate mass.

Photo 6. This photo can be interpreted as a cascade shower initiated by a penetrating particle of 10^{11} ev. about 1 cm. above the lower surface of the 5 cm. lead block over the chamber.

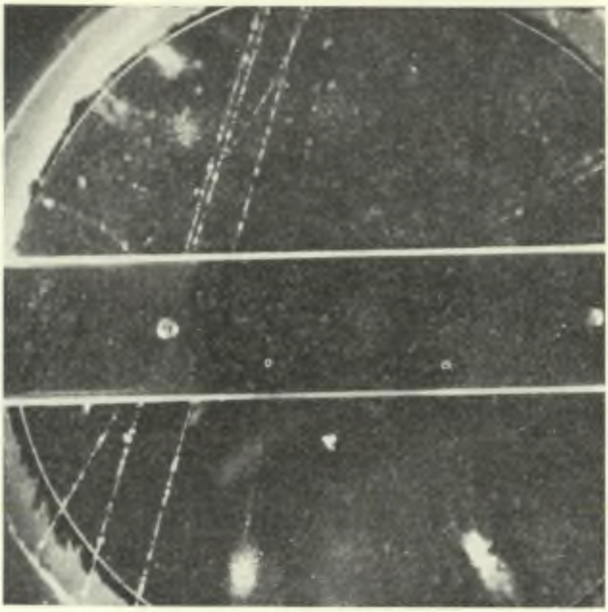


Photo 3.

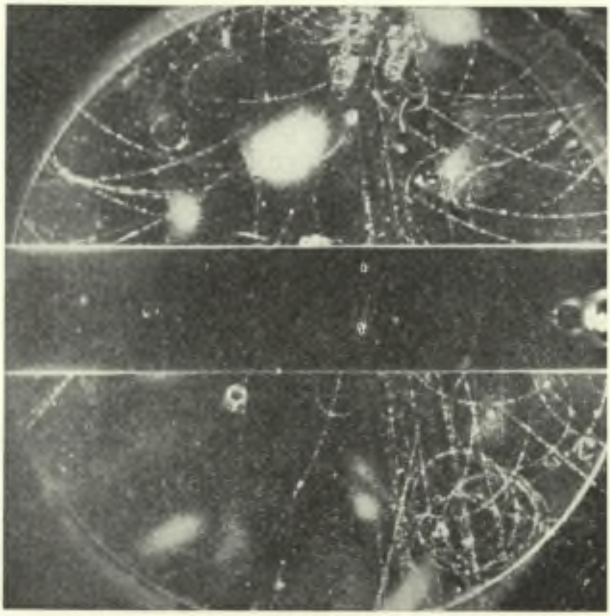


Photo 4.

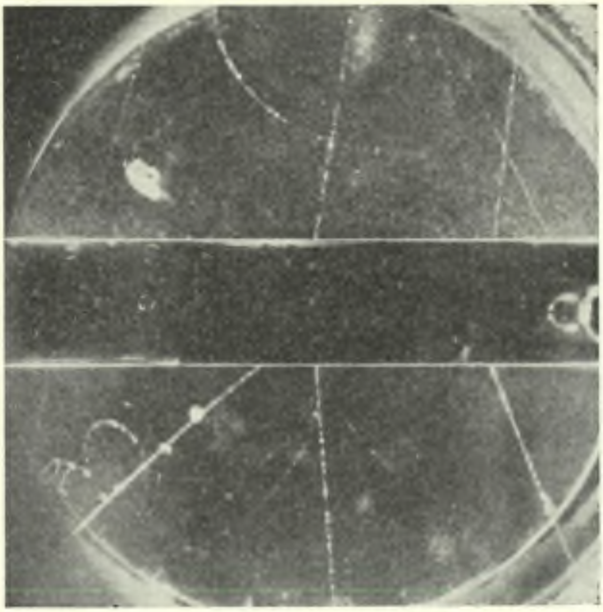


Photo 5.

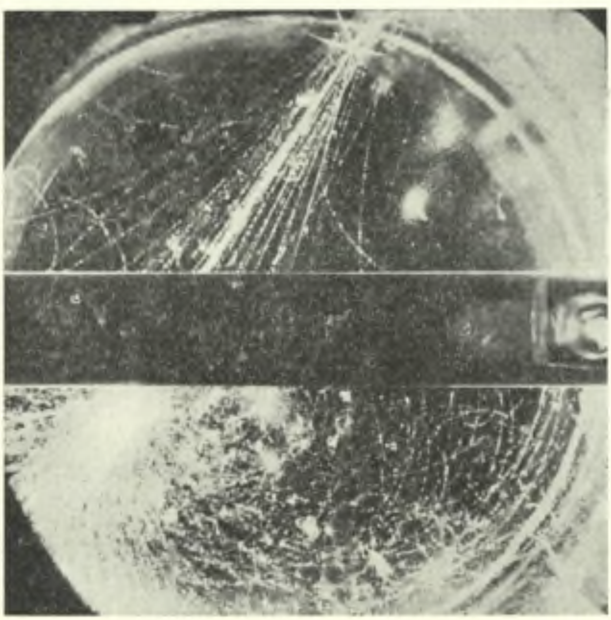


Photo 6.

The initial process could be a knock-on collision or more probably a radiative collision.

About half the 36 electrons in the top of the chamber are over 10^8 ev. The electrons below the plate are mostly of relatively low energy and number some 500.

Near the top of the photo a 9 pronged star originates in the gas exactly on the central core of the shower. One of the proton tracks is measurable and has a momentum of 5×10^8 ev/c. The total energy of the star particles including the unseen neutrons must approach 10^9 ev.

It is very difficult to explain the origin of this star. If the rest of the shower is a pure cascade, the core will contain only a few dozen high energy electrons and photons at the most and these are expected to have a very small chance of disintegrating a nucleus. Even if the core is supposed to contain a few heavier particles with a strong nuclear interaction, e. g. nucleons, σ or ρ mesons, still the chance of a star being formed is small. For the mean range, of a nuclear collision of a particle, in argon at 1.5 atmosphere, is some 300 meters, and this collision has occured within a few centimeters of the origin of the shower. It seems that one is almost forced to the conclusion, either that the core of the shower must contain a large number (several hundred) of particles with a strong nuclear interaction, or that the occurrence of the star is due to an extreme statistical fluctuation, and so unlikely to be found again until many hundreds of such photos have been obtained.

Photos 7 and 8. These are the two photographs already published and fully described by Rochester and Butler, (*Nature*, vol. 160, p. 855, 1947) which show the existence of particles of mass about 900 (τ mesons). The first shows a forked track which is interpreted as due to a neutral τ meson disintegrating spontaneously into a positive and negative particle of lower mass. The second shows an apparent scattering in the gas which is interpreted as a positive τ meson disintegrating spontaneously into a lighter positive and a lighter neutral particle. A very rough estimate of the lifetime of the particles gives 5×10^{-8} secs.

Both these novel processes occur in penetrating showers under lead absorbers.

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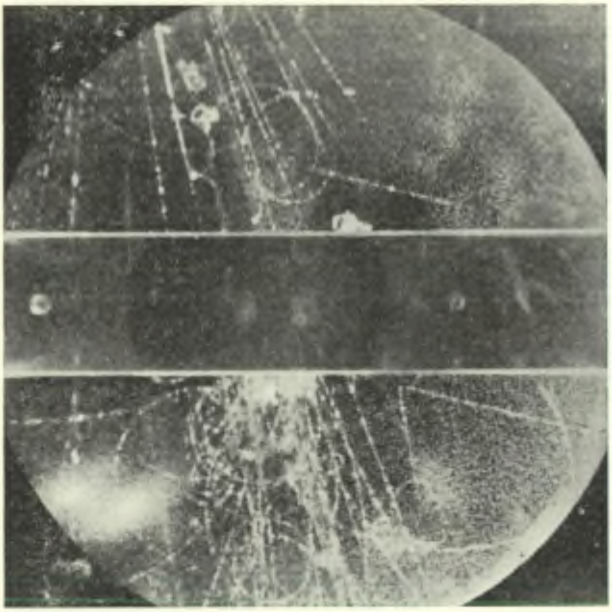

Photo 7.

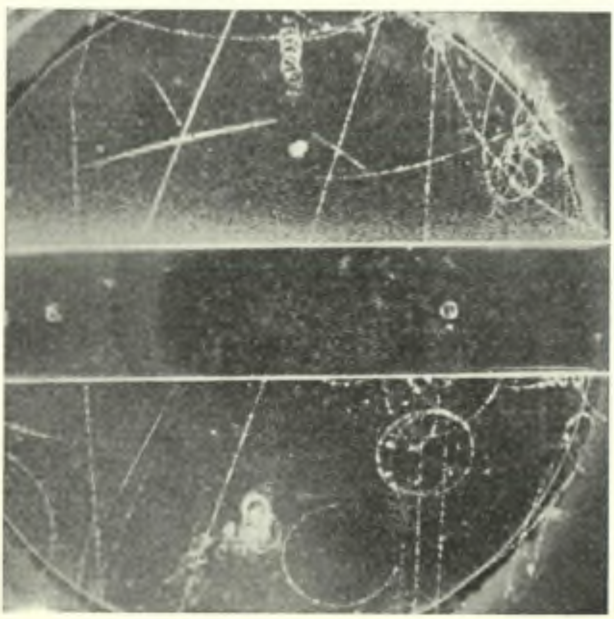

Photo 8.

The study of photographs such as these reproduced here serve to show the extraordinary complexity of shower phenomena, and to emphasize the difficulties of making clear cut and unique deductions from counter experiments alone. The study of these shower photographs must surely begin with their classification into distinct types, before any very detailed quantitative measurements are made. The first approach must thus be more « botanical » than quantitative.

Quantum Theory of Damping and Collisions of free Mesons

by W. Heitler

(Dublin Institute for Advanced Studies)

The theory of damping (1) is a heuristic attempt at eliminating the infinities from the present quantum theory of fields for the purpose of singling out the useful finite parts of the theory and thus of obtaining a workable theory that can be applied to a large class of physical phenomena. It is a generalization of and embodies the earlier theory of Wigner and Weisskopf applicable to the emission of light by atoms and allied phenomena but is more general in that it includes also a theory of collisions between free particles. In the latter case the theory also appears as a generalization of the expansion method used previously. It goes, however, far beyond this first approximation through the inclusion of the finite parts of the radiation damping which, in processes involving mesons, is very large and necessary in order to obtain at all reasonable results.

A particular advantage of this theory is its relativistic invariance.

There are physical phenomena which rest on the parts of the theory which are at present infinite. These cannot be treated by the present theory. They will be discussed in § 3.

Very recently further progress in the tendency of splitting off finite parts from the actually diverging formalism has been made by Tomonaga, and co-workers, Feynman, Schwinger and others (2). These authors have shown that in the electromagnetic case, all the divergencies can be reduced to three quantities: an infinite self-energy of the electron, an infinite self-energy of the photon, and an infinite self-charge of the electron. By separating off these quantities and subtracting them (i. e. by replacing them by the observed masses and charges) it could be shown that everything else becomes finite. At the time of writing a general formulation has not been given yet, and it is not known yet whether it will also work for meson processes.

We shall therefore not embody this development in this report although it would be the logical continuation of our present attempt.

§ 1. GENERAL THEORY

Consider a system with a Hamiltonian $H_o + H'$ where H_o is the Hamiltonian of a system of non-interacting particles, quanta, atoms, etc., and H' their interaction. Let n, m., be the eigenstates of H_o and $b_n(t)$ their probability amplitudes. Then ($\hbar = 1$)

$$i\, b_n(t) = \Sigma_m H_{n|m}\, b_m(t)\, e^{\,i\,(E_n - E_m)t} \tag{1}$$

$H_{n|m}$ are the matrix elements of H'. For stationary solutions,

$$b_n = \Psi_m\, e^{\,i\,(E_m - E)\,t} \tag{2}$$

where E is the total energy including the interaction, (1) becomes

$$(E - E_n)\,\Psi_m = \Sigma_m H_{nlm}\,\Psi_m. \tag{3}$$

If O is the initial state (incident wave) the solution of (3) is of the form

$$\left.\begin{aligned} \Psi_n &= \delta_{nO} + X_{nO},\; X_{OO} = O \\ (E - E_n)\, X_{nO} &= \Sigma_m H_{nlm}\,\Psi_m = U_{nlO} \end{aligned}\right\} \tag{4}$$

and hence

$$X_{nO} = p(E - E_n)\, U_{nlO} \tag{5}$$

with

$$p(x) = \frac{P}{x} - i\,\pi\,\delta\,(x) \tag{6}$$

The form of $p(x)$ ensures that all states other than O are final states (outgoing waves). P/x means the principal value of $1/x$ when an integration over x occurs. From (4) and (5) we get

$$U_{nlO} = H_{nlO} + \Sigma_{m \neq O} H_{nlm}\, p(E - E_m)\, U_{mlO} \tag{7}$$

The Σ_m includes an integration over the energy. The p-function is an operational quantity defining the path of integration over x. This path is along the real axis but deviates into the upper half plane by a small semicircle at the singularity $x = O$. Alternative useful representations of $p(x)$ are

$$\left.\begin{aligned} p(x) &= \lim_{\sigma \to O} \frac{1}{x + i\sigma} \\ p(x) &= \lim_{\tau \to \infty} \frac{1 - e^{ix\tau}}{x} \end{aligned}\right\} \tag{8}$$

The limits $\sigma \rightarrow O$ and $\tau \rightarrow \infty$ are to be taken after the integration. From (7) we can obtain an iterated equation. We substitute for each term $U_{mlO}/(E-E_m)$ occurring on the right of (7) what is obtained by using the equation (7) for U_{mlO}, but leave the terms $\delta(E-E_m)$ unaltered (*).

We find :

$$U_{nlO} = \overline{H}_{nlO} - i\pi \Sigma_A \overline{H}_{nlA} U_{AlO} \delta(E-E_A) \tag{9}$$

$$\overline{H}_{nlA} = H_{nlA} + \Sigma_m \frac{H_{nlm} H_{mlA}}{E - E_m} + \Sigma_{m,r} \frac{H_{nlm} H_{mlr} H_{rlA}}{(E-E_m)(E-E_r)} + \ldots \equiv H^0_{nlA} + H^1_{nlA} + \ldots \tag{10}$$

The $\overline{H}_{nlA}$ are represented by an infinite series of compound matrix-elements.

In the quantum theory of physical fields the series (10) does not only not converge but, apart from a non-vanishing first term, the remaining terms are infinite individually. The first non-vanishing term of (10) H^o_{nlA}, say, depends on the nature of the states n,A and is in general not the first term H_{nlA} of (10), but some higher term of (10). For all collision processes H^o_{nlA} is finite but the subsequent terms H^1_{nlA}, H^2_{nlA}, say, are infinite.

There are reasons to expect that these finite terms of (10) are in reality small. The series (10) progresses according to a power series in the coupling parameter g^2, say, because H is proportional to g (and (10) contains in fact only either odd or even powers of H). In a future correct theory (10) must converge and it is therefore plausible to assume that H^1 and the subsequent terms are small compared with H^o, as g^2, even in meson theory, is small compared with 1.—For the electromagnetic field interacting with electrons it is an experimental fact that $U = H^o$ (Born approximation) is a very good approximation (wich means that also the second term of (9) is small). We therefore can expect to obtain an approximate, divergence-free theory by neglecting the divergent terms H^1_{nlA} etc. and replacing H_{nlA} by H^0_{nlA}. We assume that the neglected terms are small compared with H^0_{nlA}, but this, of course means that our theory is only approximate and it is not the idea that the infinite parts should be zero exactly.

(9) becomes then :

$$U_{nlO} = H^0_{nlO} - i\pi\ \Sigma_A H^0_{nlA} U_{AlO}\ \delta(E-E_A) \tag{11}$$

(*) This way of carrying out the iteration (due to Pauli) is simpler than the one originally used by Peng and the present author.

This equation has always finite, and as will be seen, reasonable solutions. The elimination of the divergent parts of the theory is an *invariant* procedure.

For a number of problems also the first non-vanishing term of (10) diverges. (§ 3) These cannot be treated by the present theory. We call the divergent terms matrix elements for round-about transitions.

When dealing with collisions between free particles the variable energy E is identical with the energy E_0 of the initial state. U will then also be needed only for states for which $E_n = E_0$.

(11) connects then states with the same energy only. When systems with finite line widths are involved, E is a variable energy which is no longer identical with any E_A or E_O. (11) must then be solved for each value of E. It is shown in a paper by M*a* and the present author [3] that (11) is then equivalent to the theory of Weisskopf and Wigner (for the problems considered by these authors). In particular one obtains the correct formulae for the emission and absorption of light by atoms, resonance, fluorescence, etc., and also the correct velocity of light for the exchange of excitation energy by two like atoms (contrary to a recent suggestion by Ferretti and Peierls [4]).

From (2)—(5) we can go back to time representation by a Fourier transformation. To obtain non-stationary solutions satisfying the proper initial conditions, we put :

$$b_{AO}(t) = -\frac{1}{2\pi i}\int_{-\infty}^{+\infty} \frac{e^{i(E_A-E)t}}{E-E_O+\frac{i\gamma}{2}}\,\Psi_A dE = -\frac{1}{2\pi i}\int_{-\infty}^{+\infty} \frac{e^{i(E_A-E)t}}{E-E_O+\frac{i\gamma}{2}} \times$$

$$p(E-E_A)U_{AIO}\,dE = U_{AIO}\frac{e^{\frac{-\gamma}{2}t}\,e^{i(E_A-E_O)t}-1}{E_O-E_A-\frac{i\gamma}{2}} \qquad (12)$$

$$b_{OO}(t) = -\frac{1}{2\pi i}\int_{-\infty}^{+\infty} \frac{e^{i(E_O-E)t}}{E-E_O+\frac{i\gamma}{2}}\,dE = e^{\frac{-\gamma}{2}t}$$

Here U is considered as independant of E which in view of the narrow range of E needed is a good approximation.

The probability amplitudes b satisfy the correct initial conditions $b_{OO}(t=O)=1$, $b_{Ao}(t=O)=O$. The transition probability per unit time is

$$\gamma_{Ao} = 2\pi \left|U_{AlO}\right|^2 \rho_A \text{ and } \gamma = \Sigma_A \gamma_{Ao} \tag{13}$$

where ρ_A is the density function. From (13) the cross sections are obtained in the well known way.

The following applications refer to collisions between free particles and we can then put $E = E_O$. The integration of (11) over dE_A gives then

$$U_{nlO} = H^o_{nlO} - i\pi \Sigma_A H^o_{nlA} \rho_A U_{AlO} \tag{14}$$

where the sum Σ_A now extends only over states with energy $E_A = E_O$. We call the second term of (14) the damping term.

The neglect of H^1 against H^o does not mean that the second term of (14) is also small. In fact this term can only be neglected if

$$H^o_{nlA} \rho_A << 1 \tag{15}$$

For photon-electron-processes this is always the case. It can be shown, for instance for the scattering of light by a free electron, that the second term of (14) describes the *classical damping force* $\sim \dddot{x}$. For the non-relativistic case $h\nu << mc^2$ we obtain then the Thomson formulae corrected by the damping viz. :

$$\Phi = \frac{8\pi}{3} r_o^2 \frac{1}{1+K^2}, \qquad K = \frac{2}{3}\frac{e^2\nu}{mc^3} \tag{16}$$

and $K<<1$ when $\hbar\nu<<mc^2$. (14) would suggest a large departure from the Klein-Nishina formula when $\hbar\nu > 137\ mc^2$. However, this is not the case. A relativistic treatment (5) shows that the influence of the « damping term » is also here small. It should be remarked that whilst both conditions

$$H^1 << H^0, \qquad H^0\rho << 1$$

are satisfied for processes involving the electromagnetic field and electrons, and $U = H^0$ is a good approximation, small corrections will arise both from H^1 and the damping term $i\pi H^0 \rho U$. It cannot then be stated beforehand which of the two corrections will be the larger one.

For meson processes the situation is very different. In contrast to the photon-electron interaction which decreases with increasing energy such that (15) is satisfied, the meson nucleon interaction increases with energy as a result of the spin and isotopic spin dependent interaction. We have here, for sufficiently high energies $H^0 \rho >> 1$, and also $|H^0|^2\rho >> 1$. Consequently, if we were to use perturbation theory, the cross sections (13) would increase indefinitely with increasing energy. It is precisely for the purpose of avoiding this difficulty that the present theory has been devised. We can therefore not neglect the damping term in (14).

One can easily see what happens, by regarding (14) as a matrix equation for U. (Rows and colums are the states, *n*, A, O, etc.). The solution of (14), or indeed also of (9), is

$$U = \frac{\overline{H}}{1 + i\pi \overline{H} \rho} \tag{17}$$

where $\overline{H} \rightarrow H^0$ if the neglect of H^1 is accepted. Now for simple problems $H^0\rho$ can be regarded as a function of energy rather than a matrix. If now $H^0\rho >> 1$, we get

$$U \sim 1/\rho \tag{18}$$

Now ρ increases with energy. For instance for the scattering of a meson by a nucleon $\rho \sim \varepsilon^2$. Consequently $U \sim \varepsilon^{-2}$ and, by (13), the cross section

$$\Phi \sim |U|^2\rho \sim 1/\rho \sim \varepsilon^{-2} \tag{19}$$

We therefore see that the cross section decreases again for high energies in contrast to the result obtained from perturbation theory, which gives $|H^0|^2\rho \sim \varepsilon^2$. The reversal of the situation is entirely a result of the damping.

The result (19) appears even to be independent to a large extent of the neglect of H^1. Obviously (19) will hold always provided that $\overline{H}\rho >> 1$. Since now $H^0\rho >> 1$, it would be necessary that the neglected terms H^1, H^2 ... etc., completely alter the order of magnitude of $\overline{H}$ (and are therefore large compared with H^0) in order to invalidate the condition $\overline{H}\rho >> 1$. This seems very unlikely. There is therefore good reason to believe that the following results will at least have approximate validity in the future correct theory.

The subtraction methods of Tomonaga and Lewis, mentioned in the introduction, may, when further developed, lead to a determination of the neglected terms H^1, H^2 .... If the method works also in the meson case, it will be possible to decide to what extent the neglect of H^1 is justified.

The present theory can also be regarded as the limiting case of the so-called strong coupling theory where a finite size of the sources is introduced and where a characteristic quantity, the spin inertia, occurs. The present theory is the limiting case spin inertia $\rightarrow$ O, or very small.

§ 2. APPLICATIONS TO COLLISIONS INVOLVING FREE MESONS

(14) represents a set of integral equations for U and from U the cross section for the collision in question follows from (13). When U can be replaced by H^0 we obtain the results of the expansion method used in earlier theories. The second term of (14) on the right describes the damping. In meson collisions this cannot be neglected as it changes the whole order of magnitude of the cross section, as was shown above.

The following applies to mesons which are strongly coupled with nucleons in the sense of the Yukawa theory. We call these mesons nuclear forces mesons (N. F. mesons).

It is now practically certain that the π—meson is an N. F. meson in this sense, and that its spin is integral. It is also certain that the μ—meson is not an N. F. meson and is very weakly coupled with nucleons. Probably, also the mesons of mass 700 — 1000 m., if they exist, (τ—mesons) are strongly coupled with nucleons but little is as yet known about the nature of this coupling. We shall therefore not include τ — mesons in the following. Naturally, all the following results depend to some extent on the particular form of meson theory we choose to describe N. F. mesons. Throughout, a spin and charge dependent interaction is assumed, (pseudoscalar and/or vector mesons). Unless otherwise stated we assume the charge-symmetrical form of meson theory. The quantitative results will be subject to modification when more is known about the number, masses, spins, etc., of the N. F. mesons (in particular when τ —mesons are included) but the main features of the results are largely independent of the particular form of meson theory.

The mass of the π—meson is now known to be (286 ± 15 m) but most of the results (where quantitative figures are given) have been evaluated for the earlier mass value 315 m. We put throughout $\hbar = c = 1$.

(i) *Scattering of meson by a free nucleon* (6). For low energies ε of the meson (momentum k) the cross section Φ increases $\sim k^4/\varepsilon^2$. This is the result of perturbation theory. Φ reaches a maximum at about $\varepsilon \sim 3\mu c^2$ (the factor 3 is roughly $\sqrt{hc/g^2}$ where g stands for the coupling constant) and decreases then again. It is asymptotically

$$\Phi = \frac{4\,\pi}{\varepsilon^2} \qquad (\varepsilon << Mc^2) \qquad (20)$$

independent of the mass μ of the meson. The decrease is entirely due to the damping. In the relativistic region $\varepsilon > Mc^2$ the decrease continues further, but this depends on the spin of the meson. For pseudoscalar mesons it is $8\pi\ /\ \varepsilon^2$ in the centre of gravity system, or $16\ \pi/\varepsilon\ M$, when the nucleons are initially at rest.

A particular feature of the present theory is the occurrence of selection rules. In the charge-symmetrical theory also neutrettos occur, and if the perturbation method is used, the transformation of a charged meson into a neutretto is equally probable as the ordinary scattering. This, however, applies only to very low energies. Owing to the damping the cross section for this transformation is exceedingly small at high energies and decreases $\sim \varepsilon^{-6}$.

Measurements of the scattering of π —mesons hardly exist so far. Rochester (*) has observed five cases of large angle scattering of particles in local penetrating showers. These particles may well be partly protons and partly π —mesons. (One of the five particles was negative, two positive and for two the charge was not determined). If two of five particles are π—mesons, the cross section for scattering would be 2.5×10^{-27} cm^2 per nucleon. The average energy was of the order of 10^9 ev. For this energy (20) gives a cross section of 4×10^{-27} cm^2. Further experiments are needed to show that this agreement is genuine.

No case of a transformation of a meson into a neutretto (stoppage of a meson in a lead plate) has been observed so far and this is understandable according to what was said above.

(ii) M*ultiple processes* (*explosions*). (1) The meson theory has

(*) Reported at the Bristol conference, September 1948.

all the features which, according to the earlier theory of Heisenberg would lead to the occurrence of multiple processes. In a collision of a free meson with a nucleon, the meson can split up into several mesons. This is due to the increase of the meson-nucleon interaction with energy. In fact, if one applies perturbation theory, one finds that from a certain energy upwards the multiple process is more probable, and increases even more rapidly with energy, than the single process. The rapid increase is, of course, entirelty unreasonable. The result is radically changed throught he damping. In the present theory multiple processes can be treated *quantitatively*. It turns out that the cross section reaches a maximum, but at the maximum it is still 10 — 20 times smaller than the cross section for the single scattering. After the maximum it decreases again rapidly, like ε^{-6}, and thus becomes soon entirely negligible. The result can be understood immediately from (19). If this formula is applied, we have only to take into account that, for two mesons in the final state, ρ is proportional to ε^5. The solution of (13) gives really ε^{-6} which is due to the more complicated interplay of single and multiple scattering, as it occurs in (13).

We can conclude that actual multiple processes are comparatively rare, according to the present theory. However, this has been shown only for small multiplicities, up to 3 or 4 say. Some caution is required when very high energies are considered, where the meson could split up into many secondaries, and a large number of different multiplicities may occur. This case is very difficult to treat and it may be that the sum of the probabilities for all possible multiplicities is appreciable.

The multiple processes considered here are sharply to be distinguished from the multiple processes occurring in electrodynamics. It is known that in every electromagnetic process, for instance the scattering of an electron in a potential, a large number of very soft photons is emitted (infrared-problem). This is a result of the fact that the electron-photon interaction increases with decreasing energy. The treatment of the infrared problem requires special methods (first given by Bloch and Nordsieck and later by Pauli and Fierz). The new subtraction methods (2) lead to a solution of this problem which rests essentially on the inclusion of the low energy end of H^1... These terms we have neglected so far. An extension of the present theory is required to give an account of the small radiative corrections (*). This has not been worked out yet. On the other hand, in meson theory no infrared problem exists, owing to the decrease of the intercation with decreasing energy, and that would even be so if the meson mass were zero. The theory is adequate then to deal with the divergencies occurring at high energies where the damping is important. Perturbation theory can be applied at high energies (excluding very soft quanta) in the electromagnetic case, but at low energies in the meson case.

The competence of the various methods in the two cases is as follows :

	low energies	high energies
electromagnetic case	infrared problem	perturbation theory
meson case	perturbation theory	damping

(iii) *Production of negative protons* (7). Every relativistic theory of particles known at present predicts the existence of antiparticles with opposite charge, and therefore also of negative protons. These have not been found yet although cosmic ray energies are quite sufficient to produce them. The most probable process is by a collision of a meson with a nucleon. Even assuming that the ordinary μ—meson is an N. F. meson, the calculation by Mc. Connell shows that the process is a very rare one and that selection rules in the sense of (i) hold. A closer examination shows that the cross section is so small that it is extremely unlikely that the negative proton can be observed by the present type of experiment. One finds that about 1 : 100,000 particles at sea level should be a negative proton slow enough to be identified as such. The small rate of production is intirely a result of the damping. This figure is further decreased by the fact that only π—mesons (and perhaps τ—mesons) can produce negative protons and these have a short life time.

(*) This modification must lie in the following direction : In the present theory the state of a free electron is that with no photons present and transitions are considered into states with and without photons present. The treatment of the infrared problem requires now a re-definition of the state of a free electron in such a way that the virtual photons accompanying the electron are included in the definition of the free electron state (bound photons) and only a change in the number of bound photons is regarded as a real emission or absorption. The inclusion of the virtual photons changes only the higher order corrections (terms H^1, etc.). For the carrying out of the subtraction procedure (2) this change is also essential.

We can therefore conclude that the fact that no negative protons have been observed so far is no argument against their existence, or against the use of the Dirac equation for the proton or neutron. On the other hand, it may be that the negative proton really does not exist, and that fundamental changes in the wave equation of the nucleons will have to be made.

(iv) *Meson production in nucleon-nucleon collisions.* This is probably the process by which mesons are produced in the atmosphere, and also in penetrating showers. Although the experimental material is still very scanty, the following facts seem to be well established.

1. Most mesons are produced very near the top of the atmosphere in a layer about 120 g./cm^2 thick, and this means that the cross section per nucleus must be of the order of the geometrical cross section or, per nucleon, of the order of the area occupied by a nucleon in a nucleus.

2. Mesons are produced in small groups of several particles at each collision.

Calculations have been carried out both in the non-relativistic and the extreme relativistic regions, i. e. for E (energy of the incident nucleon) $<< Mc^2$ and $>> Mc^2$.

$E << Mc^2$ (8) A single meson mass of 315 m. is assumed and the Moller-Rosenfeld combination of pseudoscalar and vector mesons. The cross section for production of a positive meson in a P-P collision is at all energies in this region about three times larger than that for the production of a (positive or negative) meson in a P-N collision. In a P-N collision the rates for production of a positive or negative meson are about equal and are the same whether the fast nucleon is a proton or neutron. Neutrettos are, of course, also produced in both P-P and P-N collisions. We denote by Φ_{+1PP}, Φ_{o1PP} ... the cross sections for production of a positive meson in a P-P collision, a neutretto in a P-P collision, etc. Then approximately :

$$\Phi_{o1PP} = 1/3\,\Phi_{+1PP}, \qquad \Phi_{o1PN} = 2\Phi_{+1PN} = 2\Phi_{-1PN}$$

Φ_{+1PP} and $\Phi_{+1PN} + \Phi_{-1PN}$ are given in the following table :

E	3.2 4.8 6.4 8.0	10^8 ev.
Φ_{+1PP}	0 1.5 7.1 17	10^{-27} cm^2
$\Phi_{+1PN} + \Phi_{-1PN}$	0 0.5 2.7 5.7	

The large difference between Φ_{+1PP} and Φ_{+1PN} is due to the use of the charge symmetrical theory. It will have a bearing on the positive excess if it persists in the high energy region $E > Mc^2$.

The cross sections rise rapidly in the whole energy region considered up to values of the order of 10^{-26} cm^2. The damping has only a minor influence in this region.

$E >> Mc^2$. Owing to extreme mathematical difficulties, the method of Weizsäcker-Williams has been used for the calculation [6] [9]. This method is very crude and inaccurate in the meson case, chiefly because of its strong dependence on the lower limit of the impact parameter of which it is only known that it is of the order of $\hbar/Mc$. Therefore only the order of magnitude of the cross section and its energy dependence could be worked out, but the numerical coefficient is somewhat uncertain. Also, only the average of P-P and P-N collisions is obtained. An attempt at a direct calculation has been made by Morette (unpublished) and the essential results of the Weizsäcker-Williams method confirmed, but the exact solution has not been found yet.

The result is as follow's : For $E >> Mc^2$ the cross section becomes constant, independant of E. The value of the cross section is

$$\Phi = k\left(\frac{\hbar}{\mu c}\right)^2,\ k = 4 - 18,\ \text{or}\ \Phi \sim 10^{-25}\ \text{cm}^2. \qquad (21)$$

$$\left(\text{for}\ \mu = 315\ \text{m.},\ \left(\frac{\hbar}{\mu c}\right)^2 = 1.5 \times 10^{-26}\ \text{cm}^2\right)$$

This very large value fits well with the strong increase found in the region $E < Mc^2$. Probably the lower value of k is nearer the truth because Φ can hardly be much larger than $\pi\,(\hbar/\mu c)^2$.

The energy distribution of the emitted mesons is a broad distribution ranging from $\varepsilon = O$ to $E - Mc^2$ and depending mainly on ε/E on which a low energy distribution with a maximum at about $3\ \mu\,c^2$ independent of E is superimposed. From the energy distribution and the total cross section the energy loss of a fast nucleon passing through nuclear matter can be obtained :

$$-\frac{\delta E}{\delta x} = Nk'\left(\frac{\hbar}{\mu c}\right)^2 E \qquad k' = 1 - 4 \qquad (22)$$

where N is the number of nucleons per cm^3. We see that a fast nucleon passing through a nucleus loses a large fraction of its energy.

If x is the lenght of path in the nucleus the ratio of the energies before and after the collision is given by :

$$log \frac{E_1}{E_2} = k'N (\frac{\hbar}{\mu c})^2 x. \qquad (23)$$

In a diametrical passage through a nucleus $N(\frac{\hbar}{\mu c})^2 x$ is roughly of the order 1 in nitrogen or oxygen and larger in heavy nuclei, like lead. Therefore, in such a passage through a nitrogen nucleus, the energy of the fast nucleon should decrease roughly by a factor of at least e.

The cross section (21) is of the order of the area occupied by a nucleon in a nucleus. It follows that the total cross section of a nucleus for meson production must be the geometrical cross section. It follows further [10] that in a passage of a fast nucleon through a nucleus *several* mesons are produced at a time, (plural production). It is also clear that a fairly large amount of energy is communicated to the nucleus in form of recoil energy which leads to the production of a *star*.

All this is substantially in agreement with the facts : The fast primary protons entering the atmosphere have an average energy of say 6×10^9 ev. We see from (23) that these protons are on the average capable of penetrating about 1 — 2 nitrogen nuclei until their energy has decreased to the order Mc^2. We have seen that then meson production decreases rapidly when the energy decreases further and can be regarded as unimportant. This is in agreement with the observed thickness of the meson producing layer quoted above (120 gr./cm^2), wich just corresponds to twice the mean free path in air.

That mesons are produced in small groups is a well established fact (penetrating showers). Bernardini gas shown (*) that star production in the high atmosphere follows a $A^{2/3-}$ law, which means that the cross section is the geometrical one. In lead, a fast nucleon will only be capable of penetrating through one nucleus, unless its energy is extremely high, or if it passes somewhat tangentially through the nucleus. The penetrating shower experiments show that the transition curve reaches saturation after 10 cm. Pb, which is just the mean free path in lead. Photographs exist in which two penetrating

(*) Reported at the Bristol Conference, September 1948.

showers are produced in succession, separated by about 10 cm Pb.

Meson production is often accompanied by small cascade showers. Since nucleons can hardly produce photons or electrons directly, it is very probable that these cascades arise through the decay of a short lived meson into an electron or photon. Possibly it is the neutretto which is responsible in this way for the cascades. It has been shown (11) that the bulk of the soft component in the high atmosphere can be accounted for in this way.

In the high atmosphere as well as in penetrating showers, a large positive excess has been found. Although this may largely be due to the presence of protons, a certain positive excess may also have to be attributed to mesons. This may be connected with the difference of the cross sections for P — P and P — N collisions found for $E < Mc^2$. To calculate the positive excess, we would require a more detailed knowledge of the energy distribution of the protons and neutrons after a P — P collision (one of the two particles has necessarily become a neutron, but it is not known yet whether the faster particle is a neutron or proton). This has not been worked out yet.

The meson groups are here interpreted as due to the compound structure of the nucleus (plural production) and not as genuine multiple processes (cf. (ii)). Experimentally it is difficult to decide whether the groups are due to plural or multiple production. It would be characteristic of plural production if the multiplicity would increase on the average with the atomic weight (in particular in hydrogen only single mesons should be produced) whereas for genuine multiple processes the multiplicity should be independent of the material. However, with the cross sections as large as is observed, one should in any case expect that some of the multiplicities are plural. Possibly the meson groups in penetrating showers are partly due to plural and partly to multiple production. Experimental evidence for genuine multiple processes does not exist so far. On the basis of the present theory one should expect that they are comparatively rare (cf. (ii)), except perhaps at extremely high energies.

(v). *Production of mesons by photons* (12). A photon colliding with a nucleon can produce a meson with a corresponding change of charge of the nucleon. The cross section has been worked out for a purely charged meson theory only, and only for $\hbar\nu << Mc^2$. The cross section is given in the following table for $\mu = 315$ m. and for pseudoscalar and vector mesons :

$\hbar\nu$	1.6	4	6	10	10^8 ev.
pseudoscalar	0	2.8	3.2	2	10^{-28} cm^2
vector	0	6	11.5	16.5	

For vector mesons the cross section tends to a constant at high energies whereas for pseudoscalar mesons it decreases $\sim (h\nu)^{-2}$ The cross sections are very much smaller than for meson production by nucleon-nucleon collisions.

Relevant experiments do not exist yet. One should expect to find mesons in large cascade showers. Indeed, penetrating particles are found there, and their number is about 2% of the number of electrons. However, if one works out the number of mesons to be expected from the cascade theory and the above figures, one finds that in the main body of the cascade only about 0.1% of the particles should be mesons. One reason why this figure is so small is that the mesons are produced at large angles and are not found in the comparatively narrow body of the cascade. (This reduces the number of mesons by a factor 5 at least). It is therefore not likely that the bulk of the penetrating particles in big cascades are produced by photons. (cf. also the discussion in connection with Auger's report.)

(vi). *Further applications to electromagnetic processes.* We mention finally some further applications of the theory to electromagnetic processes. For mesons with spin 1 perturbation theory leads also to unreasonable results. It is, for instance found that the formulae for the Compton effect and Bremsstrahlung of spin 1 mesons increase indefinitely with energy. This is due to the large magnetic moment of spin 1 mesons, which also increases with energy. The damping has also here the effect of cutting down the cross sections at high energies. In the centre of gravity system, the damping becomes appreciable for the Compton effect at $\hbar\,\nu \sim \sqrt{137}\,\mu\,c^2$. Apart from numerical constants the cross section is

$$\Phi \sim (\frac{e^2}{\mu\,c^2})^2\,(\frac{\hbar\,\nu}{\mu\,c^2})^2 \text{ for } h\nu << \sqrt{137}\,\mu c^2$$

$$\Phi \sim \quad 1/(\hbar\nu)^2 \quad \text{for } \hbar\nu >> \sqrt{137}\,\mu\,c^2$$

Again the cross section decreases with energy at high energies. Similar results apply to the cross section for Bremsstrahlung. The results are important for the well known discussion of the burst production by mesons (cf. the paper by Wilson [1]).

A further case of purely academic interest is the scattering of light by light, which is a result of the vacuum polarization in the positron theory. Also here the cross section appears to increase with energy if perturbation theory is used but this increase is over-compensated by the damping effects.

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Quite generally we can state that the difficulty of cross sections increasing indefinitely with energy is an apparent one and only due to the faulty use of perturbation theory. In particular, the damping always corrects the cross sections in a reasonable way. On the other hand the present attempt to overcome the difficulty can only be regarded as preliminary and should be further developed. So far we have seen that the results are at least roughly in agreement with the few facts which are known, but the agreement cannot be close owing to the neglect of the terms $H^1, \ldots$, quite apart from the uncertainty as regards meson theory as a whole. Not only are the problems discussed in the next section outside the reach of the present theory, but a proper treatment of the infrared problems should also be embodied in the theory. Attempts in this direction are in progress.

§ 3. UNSOLVED STATIC PROBLEMS

The neglect of the diverging terms of the series (10) can be expected to be a good approximation if a first finite term exists against which the following terms are small. This is usually the case for collisions. There are physical phenomena which rest entirely on such divergent terms. These cannot be treated as long as the terms H^1... are neglected.

The most notable examples for such phenomena in meson theory are :

(i) *The mesonic charge cloud surrounding a nucleon.* If the effect is calculated in first approximation it is found that the total charge contained in a sphere outside the radius R increases indefinitely as $R \rightarrow O$ [13]. The recently discovered attractive neutron-electron interaction [14] shows that the effect exists. If the present very inaccurate data are taken for granted it appears that the contribution from mesons with momentum $p < \mu c$ or so to the charge cloud must be cut off. A quantitative treatment is not yet possible.

(ii) *The anomalous magnetic moments of the proton and neutron.* These are due to the current cloud surrounding the nucleons ([15]). The qualitative explanation of these effects is no doubt a great success of the meson theory, but since the current density increases in the same way as the charge density for $R \rightarrow O$ the expressions obtained for the magnetic moments are infinite. One obtains again the right order of magnitude by cutting off in momentum space at $p \sim \mu c$.

(iii) *In Quantumelectrodynamics* also small effects exist which rest on the divergent parts of the theory. The line shift of the 2S-level of hydrogen is due to the change of the self-energy of the electron in the bound state. Whilst the self-energy in both the free and the bound states is infinite the difference is finite ([16]). It seems, according to Schwinger, that the subtraction of the infinite self-energy in the free state can be carried out in a relativistically invariant manner. Similarly, the anomalous magnetic moment of the electron of the value $1/2\pi.137$ of one magneton, rests on round-about transitions which are neglected in the present theory. Also here the effect appears as a finite difference of two infinite self-energies, with and without magnetic field ([16]).

Finally, as was mentioned before, the proper treatment of the infrared problem ([17]) requires the inclusion of the low-energy end of the neglected terms H^1, etc.

It is therefore evident that the neglected divergent terms of the theory are not zero but are in reality finite and probably small.

In particular the low energy end of the divergent integrals has physical reality. A consistent and satisfactory theory that would make these divergent terms finite has not been found yet, but possibly some substantial advance can be expected from the new subtraction procedure ref. 2.

Added in proof (October 1949). While this report was in press the subtraction methods ([2]) have been further developed with the following results :

1) For electrons interacting with the electromagnetic field the subtraction of the infinite self-mass and self-charge of the electron and the removal of certain non gauge-invariant parts (like the self-

energy of the photon) suffice to make the formalism finite and unambiguous. The theory of § 1 remains, of course, unaltered only the explicit expressions of H^1.. are modified by the said subtraction. These terms can now be evaluated in principle.

2) For the meson-nucleon interaction the method works only partially and only for certain forms of meson theory. For some forms of meson theory finite expressions for the static problems of § 3 can be obtained which, however, are not in agreement with the experimental facts. It may be that this is due to the fact that the subtraction method amounts to a virtual cut-off at meson energies Mc^2 whereas a cut-off at μc^2 is required to reach agreement with the experiments. It is not known whether the fault lies in the subtraction method or in the foundation of meson theory or in the expansion method or in the fact that the correct superposition of meson fields has not been found yet.

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Discussion du rapport de M. Heitler

Mr. Peierls. — About the basic ideas in Heitlers' report, one must be aware that already equation (9) may contain some assumption, namely that initially the particles are in a well defined state with a definite value for the energy. The energy is here taken as the unperturbed energy, which does not take into account the coupling of the particle with the field. This may invalidate the conclusion of Heitler that the cross section at very high energies does not depend very much on the interaction matrix.

Mr. Heitler. — Although the importance of this point was recognized, it could not be taken into account owing to the well-known divergences.

An attempt of this kind is presently done by Miss Morette and Dr. N. Hu.

Problems of Nuclear Forces

by L. Rosenfeld

> Such welcome and unwelcome things at once
> 'Tis hard to reconcile.
> (*Macbeth*, act IV, sc. III.)

Nuclear interactions can either be described in a purely phenomenological way by means of the concept of nuclear potential, or interpreted as due to a nuclear field, related to some kind of particles of intermediate mass. At the present stage, both descriptions involve too many uncertainties or even contradictions to allow a consistent picture of nuclear forces to be drawn. The present report will therefore be confined to a discussion of some rather disconnected aspects of the problem, which have been the subject of recent study. We shall first examine how far the field description of nuclear interactions is in harmony with the experimental facts concerning nuclear processes and with the various kinds of mesons lately discovered. We shall then enquire what inferences can be drawn about the properties of the mesons responsible for the nuclear field from the results of a more phenomenological analysis of the relevant empirical evidence.

1. NUCLEAR FIELD

1.1. *General features.* The concept of a field is eminently suited to provide an invariant description of interactions, taking account of their finite velocity of propagation. In a field theory, a coupling energy is assumed between the field and its sources, as a result of which these sources are able to receive any action transmitted by the field. By associating a field with every kind of elementary particle, quantum theory has considerably deepened and extended the scope of the field description of the coupling between such particles. In fact, the existence of a coupling between any two kinds of particles implies that each of them, by its field properties, establishes an interaction between particles of the other kind. The range R of such an interaction is connected with the mass M of the particle transmitting it by a general relation

$$R = \hbar/Mc, \tag{1}$$

which is a direct consequence of the fundamental uncertainty relation for time and energy.

According to this general picture, we must expect the nuclear field, of range $R \approx 10^{-13}$ cm., to be associated with particles of mass $M \approx 300\ldots 400$ times the mass of the electron. The order of magnitude of the constants g (analogous to the elementary electric charge in the theory of electromagnetic interactions) which give a measure of the intensity of the sources of the nuclear field can be estimated from the strength of the nuclear potential; it is expressed by the dimensionless ratio

$$g^2/\hbar c \approx 0{,}1. \tag{2}$$

The field theory of nuclear interactions yields a natural explanation of the anomalous magnetic moments of the proton and the neutron, as being due to the « clouds » of charged nuclear fields virtually surrounding every nucleon. In the present state of the quantum theory of fields, however, such an explanation must unfortunately remain qualitative. One may enquire whether there are other phenomena implied by a field theory of nuclear forces but not accountable by means of forces due to a nuclear potential, and from which, therefore, the existence of a nuclear field might be inferred. All such effects, however, whose order of magnitude can be estimated from the general properties of the nuclear field mentioned above, turn out to be too small for experimental verification at the present time.

As an example of nuclear field effect, which has been recently discussed, I shall mention the contribution to the magnetic moment of a complex nucleus arising from the virtual exchange of charge between the constituent nucleons through the intermediary of the nuclear field. This « exchange magnetic moment » has been calculated in the case of ^3H and ^3He, and compared with the deviation of the empirical values of the magnetic moments of these nuclei from the values to be expected on a simple model of their structure, treated from the phenomenological point of view. The result is too small to account for this deviation, which must probably be attributed, at least partially, to the effect of spin-orbit couplings not taken into consideration in the model mentioned (**2.2**, NF **A2.251**) ([1]).

1.2. *Nuclear field and cosmic ray particles.* The relationship of the nuclear field to the particles of intermediate mass, or mesons, observed in cosmic radiation, is seen in a new light since the discovery of the unsuspected complexity of the mass spectrum of these particles. We may now regard as firmly established the existence of mesons of three different masses, with a genetic relation between them. The π -mesons, of mass $M_\pi \gtrsim 300m$, decay within about 10^{-8} sec. into a pair consisting of a charged μ -meson of mass $M_\mu \gtrsim 200m$ and a neutral « μ_0 -meson » of mass $0 \leqslant M\mu_0 < 90\ m$.

Obviously the π -mesons ([2]) are strongly coupled to the nucleons and must essentially contribute to the nuclear interactions; their mass gives a quite acceptable value for the range of the forces (**2.3**). Precisely how the nuclear forces are brought about by the π -mesons will primarily depend on their spin, about which we have no experimental indication whatever. Much the simpler course, for the time being, is to *assume that they have integral spin* (0 or 1). It is then possible to take over the usual meson theory of nuclear interactions, as it has been developed until now. In particular, the charge independence of the nuclear interactions will then require the existence of neutral π-mesons (NF **8.31**), which would presumably be highly unstable with multiple photon emission (NF **8.311**).

The assumption that π -mesons have integral spin does not commit us to any definite value for the spin of the μ and μ_0 -mesons. Indeed, there is nothing in the known properties of these mesons to give the preference either to an integral or to a half-integral spin; cosmic ray evidence (NF **1.333**) excludes only spin 1, and is compatible with either 0 or 1/2. In the following, we shall treat the case of half-integral spin; but the whole discussion would not be materially altered if we started from the alternative assumption. We shall thus consider the μ and μ_0 -mesons as two states (of different charges and masses) of a new kind of particle of spin 1/2, for which we shall use

([1]) The letters NF indicate that the following references are to subsections of the author's book *Nuclear forces* (1948).

([2]) We shall call π-mesons all those of the kind to which both π and σ-mesons belong, whether they be positively or negatively charged, or possibly neutral.

the name « μ -meson » (just as we use « leptons » for both electrons and neutrinos). In this case, the decay process of a π -meson into a pair of μ -mesons can be treated in much the same way as its decay into a pair of leptons (NF **15.35**); we have to introduce a coupling between the π -meson and the pair consisting of a μ and a μ_0 -meson, with a set of coupling constants whose order of magnitude we shall represent by $\hat{g}$. The life-time of the π -meson is given by formulae somewhat more complicated than those corresponding to its decay into leptons, owing to the possible occurrence of a non-vanishing mass of the μ_0 -meson; they have been established for a pseudoscalar and a vector π - meson by Marty and Prentki (1948).

Now, the order of magnitude of the life-time of the π -mesons, as already stated, can be estimated at 10^{-8} sec. It is remarkable that this implies for the coupling constants $\hat{g}$ roughly the same order of magnitude as that which had to be assumed for the coupling constants $\check{g}$ between mesons and leptons in order to explain the β -decay by a mechanism involving the meson field as an intermediary between nucleons and leptons (NF **1.322**). In view of this coincidence, it is tempting to imagine that the π -mesons are coupled to pairs of μ -mesons and to pairs of leptons in *exactly* the same way (in fact, as if μ -mesons and leptons were different states of a single species of elementary particles of half integral spin). The mechanism just mentioned for the β -decay could then be maintained. On the other hand, it is just as likely, for all we know [1], that there is no direct coupling at all between π -mesons and leptons, and that the β -decay results from a direct coupling between nucleons and leptons, as assumed in Fermi's original theory, and symbolized, as regards the order of magnitude, by a « coupling constant » [2] $g_F \quad g \gtrsim \check{g}/\varkappa^2$. As to the decay of the μ -meson, it raises a problem by itself, which will be discussed below.

According to the above scheme, there is an indirect coupling between a nucleon and a pair of μ-mesons, through the intermediary of a π-meson; on account of $\hat{g} \gtrsim \check{g}$, the corresponding coupling constant is of the same order of magnitude g_F as that between nucleons and leptons. Such a weak coupling is just what we should

(1) The decay of a π-meson into *leptons* would not be observable with the present emulsion technique.

(2) We use the notation $\kappa = M_\pi/\hbar c$.

expect from the fact that a negative meson slowed down to the « K-orbit » around a light nucleus has time to decay before being captured by the nucleus; a straightforward estimate (Lodge, 1948) of the capture probability on the basis of the theory just outlined agrees even quantitatively with the latest data on negative meson decay. If, as the above scheme implies, the μ-mesons interact with nucleons only in pairs, the capture process of a charged μ-meson should be accompanied by the emission of a (neutral) μ_0-meson, carrying away a considerable amount of energy. This mechanism would explain the empirical fact that the μ-meson capture does not give rise to nuclear excitation and explosion.

A few words may be added concerning the decay of the μ-meson. The latest available evidence would seem to indicate that the balance of energy and momentum not accounted for by the observed decay electron is not simply carried away by a neutrino of small or vanishing mass, but that we have a process involving more than two particles. In view of the close association of μ and μ_0-mesons in the decay of the π-meson, it is tempting to assume that besides an electron and a neutrino, the decaying μ-meson produces a neutral μ_0-meson; the so-called decay of the charged μ-meson might thus more appropriately be called a transmutation into a neutral μ_0-meson with emission of a lepton pair. Theoretically, this process would be described by introducing a direct coupling between the μ - μ_0 -meson pair and the electron-neutrino pair; the coupling parameter, determined by the life-time t_0 of about 2 μsec, would be of the order of magnitude g_F. The complete net of interactions between the various kinds of particles might then be symbolized by either of the following schemes, in which dotted arrows represent indirect couplings :

$$\begin{tikzcd}
\text{Nucleons} \arrow[r, "g"] \arrow[d, dashed, "g_F"'] \arrow[dr, dotted, "g_F" near start] & \pi\text{-mesons} \arrow[d, "\check{g}"] \arrow[dl, "\check{g}"' near start] \\
\text{Leptons} & \mu\text{-mesons} \arrow[l, "g_F"]
\end{tikzcd}
\quad \text{or} \quad
\begin{tikzcd}
\text{Nucleons} \arrow[r, "g"] \arrow[d, "g_F"'] \arrow[dr, dotted, "g_F" near start] & \pi\text{-mesons} \arrow[d, "\check{g}"] \arrow[dl, dotted, "<<\check{g}"' near start] \\
\text{Leptons} & \mu\text{-mesons} \arrow[l, "g_F"]
\end{tikzcd}$$

2. PHENOMENOLOGICAL DESCRIPTION OF NUCLEAR FORCES AND MESON PROPERTIES

2.0 Assuming that, in a general way, π-mesons of integral spin are responsible for the nuclear interactions, what can we learn from the features of these interactions about the specific properties of the mesons involved? In the following sections, devoted to this question, we shall successively discuss the concept of nuclear potential underlying the phenomenological description of the interaction between nucleons, and the possible relations of some of its properties to the spins, masses and charges of the mesons associated with the nuclear field. It will appear that the present evidence cannot give us more than some indications of varying weight in these respects.

2.1. *Limitations of concept of nuclear potential.* The phenomenological approach is based on the essentially non-relativistic concept of a nuclear potential, which defines the interaction between a pair of nucleons. The total potential energy of a nuclear system is then assumed to be the sum of the interactions between all pairs of constituent nuclei. The nuclear potential is predominantly static, but may include terms of the first order in the nucleon velocities. The inclusion of higher order terms, and especially of the many-body interactions, would require a relativistic treatment. In the phenomena hitherto studied, the need for such a treatment did not arise, but with the expansion of nuclear research into domains of higher and higher energies, this problem is likely to become acute fairly soon. For instance, SNYDER and MARSHAK (1947) have drawn attention to the fact that in the analysis of the scattering of nucleons of about 100 MeV energy (**2.41**) the velocity-dependent nuclear interactions might begin to become significant; at any rate, no precise comparison of theoretical and experimental results should be attempted without taking them into consideration. An estimate of the scattering cross-section including the effect of the non-static forces may be obtained by applying Møller's relativistic treatment to the meson field description of the interaction between nucleons : this corresponds to a relativistic extension of Born's method. In a special example, the authors quoted show that the result, for the energies considered, may differ by as much as a factor 2 from that of Born's approximation applied to the static interaction.

As regards the many-body interactions, or direct interactions between more than two nucleons, there is only some slight evidence of their occurrence from the impossibility of accounting in a consistent way for binding energies of the lightest nuclei ^2H, ^3H and ^4He on the assumption of pair interactions only (NF **14.22**, **17.2**). Convergence difficulties have hitherto prevented a quantitative discussion of the types of many-body interactions which can be derived from nuclear field theories.

2.2. *Non-central interactions and meson spin.* As is well known, the electromagnetic properties (magnetic dipole and electric quadrupole moment) of the ground state of the deuteron reveal the presence of an appreciable amount of non-central coupling between the proton and the neutron constituting this nucleus. In fact, the existence of an electric quadrupole moment means that the ground state is not a pure ^3S level, but must contain some admixture of $^3D_1$ state. The numerical value of the magnetic moment points to an amount of D state admixture of about 4%; there is, however, some uncertainty about this figure owing to the neglect of the relativistic correction to the magnetic moment (NF **A2.27**). From the two main types of non-central interactions that may be expected on general invariance grounds, viz. spin-orbit couplings and axial dipole coupling (NF **15.21**), only the latter can effect the required admixture of D state to the ground state of the deuteron. In heavier nuclei, on the other hand, spin-orbit couplings probably play an important role also; but this is a question greatly in need of further elucidation, both from the empirical and the theoretical point of view (NF **17.43**, **A2.251**).

To return to the deuteron ground state, we may write the effective nuclear potential in this state in the general form

$$V = -J(r) - F(r)\, D^{(12)}; \tag{1}$$

in this formula

$$D^{(12)} \equiv (\sigma^{(1)} x_0)\,(\sigma^{(2)} x_0) - \tfrac{1}{3}\,\sigma^{(1)}\,\sigma^{(2)} \tag{2}$$

is the operator defining the dependence of the axial dipole interaction on the spins $\sigma^{(1)}$, $\sigma^{(2)}$ of the two nucleons and the unit vector x_0 giving the direction of the line joining them; the functions $J(r)$ and $F(r)$ describe the distance dependence of the central and non-central part of the potential. For these functions, meson field theory yields expressions of the form

$$\mathcal{J}(r) = \mathcal{J}\,\varphi(r),\; F(r) = F.\left(1 + \frac{3}{\varkappa r} + \frac{3}{\varkappa^2 r^2}\right).\,\varphi(r), \qquad (3)$$

$$\text{with } \varphi(r) \equiv e^{-\varkappa r}/r,$$

in which $\varkappa$ is the inverse of the range R (**1.1**—1) and the « strength » constants $\mathcal{J}, F$ depend on the type of meson field adopted, i. e. essentially on the spin of the mesons. The non-central potential $F(r)$ exhibits a third order pole at small distances, which must be « cut-off » at some critical distance.

A direct solution of the deuteron problem on the assumption of the potential (3) has been carried out by Mr. Grosjean for different values of the range and of the strengths $\mathcal{J}, F$. These calculations show that in order to account for the deuteron properties the ratio $F/\mathcal{J}$ must have values (depending on the choice of the meson mass) of the order 1/2. This means, in the first place, that the axial dipole part of the potential is of considerable importance, being in fact largely responsible for the singlet-triplet separation. Moreover, the value just quoted of the ratio $F/\mathcal{J}$ does not correspond to any single type of meson field, but indicates a mixture of fields of spins 0 and 1. In contrast to the simple combination of pseudoscalar and vector fields proposed by Møller and Rosenfeld, this mixture must be such as to give rise to an axial dipole coupling even in the static approximation.

2.3. *Range of nuclear potential and meson mass.* The most accurate evidence concerning the range of the nuclear potential is that given by the study of the scattering of protons; assuming the meson type of potential, these experiments are in good agreement with a value of about 300 m for the meson mass (NF **7.13**). Other evidence, derived from the properties of the proton-neutron system (NF **8.33**, **8.34**) or from the binding energies of light nuclei (NF **14.22**), when analysed on the assumption of a central meson potential corresponding to a *single* meson mass, would seem to favour a lower value for this mass ($M_\pi \approx 220\, m$); but it is certainly not incompatible with the higher value suggested by the proton-proton scattering data.

Nevertheless it appeared worth while to enquire whether this discrepancy, if taken at its face value, could not be brought into harmony with the point of view of Schwinger's mixed theory, in which different masses are assumed for the different (pseudoscalar and vector) types of mesons contributing to the nuclear interactions. The closer examination of this point, carried out by RAMSEY (1948) leads, however, to a negative conclusion. If $\varkappa$, $\mu\varkappa$ represent the inverse ranges of the pseudo-scalar and vector meson fields, and γ^2 the ratio of the strengths of the two possible kinds of central couplings provided by the vector meson field, the effective potential in ^{1}S states given by Schwinger's theory is proportional to

$$-\frac{1}{r}\left[e^{-\varkappa r} + (2\,\mu^2 - \gamma^2)\, e^{-\varkappa\mu r}\right]. \tag{4}$$

The sign of the electric quadrupole moment of the deuteron requires $\mu > 1$. Now, let us compare the expression (4) with a simple meson potential of the form

$$e^{-\mathrm{K}r}/r. \tag{5}$$

If we identify these two expressions and their derivatives for some value r_0 of r, we readily find that the range parameter K is a monotonic function of r_0; the sign of $d\mathrm{K}/dr_0$ being that of $(\gamma^2 - 2\,\mu^2)$. Now, since one would expect that the proton-proton scattering is mainly determined by the behaviour of the nuclear interaction at larger distances than the neutron-proton scattering or binding, the trend of empirical evidence, pointing to a larger value of the range parameter for the proton-proton than for the neutron-proton potential, would be interpreted, from the present point of view, to require that K must be an *increasing* function of r_0, i. e., that $\gamma^2 > 2\,\mu^2$. It is easily seen, however, that this condition, together with $\mu > 1$, would lead to such high values of the coupling constants that the static approximation would break down altogether.

2.31. *Slow neutron diffraction and range of nuclear potential.* New evidence which may lead to an improved determination of the ranges of the effective potentials in the S states of the proton-neutron system is afforded by recent diffraction experiments with slow neutrons. There are two independent sets of experiments bearing on this subject : *a*) the scattering of extremely slow neutrons by para- and ortho-hydrogen; *b*) the diffraction of slow neutrons by crystals.

The scattering of neutrons cooled to very low temperatures by gaseous ortho- and para-hydrogen at 19,5° K has been investigated, with improved technique, by SUTTON *et al.* (1947). The scattering cross-sections have been determined for the whole range of neutron temperatures 10 .... 30° K (E = 0,8673 ... 2,386 MeV); the results are markedly different from the provisional values obtained earlier (NF **6.31**) by Alvarez and Pitzer. For 20° K neutrons, e. g., Sutton *et al.* find

$$\text{for} \left\{ \begin{array}{l} oH_2 : 124 \\ \\ pH_2 : \quad 3{,}97 \end{array} \right\} .10^{-24} \text{ cm}^2.$$

The theory of the effect, on the schematic assumption of a *well type* of potential, leads to simple relations between the cross-sections S_{ortho}, S_{para} and the scattering amplitudes ^3a, ^1a associated with the ^{3}S and the ^{1}S level of the proton-neutron system (NF **6.32**). From the scattering amplitudes it is further possible to derive the widths of the corresponding potential wells, as well as the scattering cross-section $S(0)$ of protons for neutrons of vanishing kinetic energy.

When subjected to this analysis, Sutton *et al.* 's results yield a value of the proton scattering cross-section for slow neutrons $S(0) = 19{,}7.10^{-24}$ cm^2, in excellent agreement with the direct measurements of this quantity. Since $S(0)$ depends essentially on S_{ortho}, the agreement just mentioned therefore confers additional reliability to the determination of S_{ortho}. The measurement of S_{para}, however, is more difficult and any error in its determination would mainly affect the value of the relatively small scattering radius ^3a. From the data, one finds

$$^3a = 0{,}522.10^{-12} \text{ cm}, \qquad {}^1a = 2{,}34.10^{-12} \text{ cm}.$$

The singlet scattering radius ^1a is compatible with a width $^1D = 2{,}8.10^{-13}$ cm for the corresponding potential well, such as one would expect, assuming charge independence of the nuclear interactions (**2.4**), from the proton-proton scattering data. But the triplet scattering radius ^3a yields a much smaller width value for the ^{3}S effective potential, viz.

$$^3D = (1{,}5 \pm 0{,}4)\ .10^{-13} \text{ cm}.$$

One would have to assume a one to two percent contamination of the para-hydrogen sample with orthohydrogen to raise the triplet width derived from the experiments to the same value as ^1D.

However, the low ^3D value has received a striking confirmation from quite independent experiments of Wollan, Shull and their collaborators (1948) on the diffraction of slow neutrons by crystalline powders. The distribution of the diffracted neutrons will depend on the *scattering amplitudes* characteristic of the nuclei of the atoms constituting the crystal; for hydrogen this scattering amplitude is clearly proportional to $(3.\,{}^3a + {}^1a)$. Now, by combining the results obtained with different suitable crystals, it is possible to assign definite scattering amplitudes to the various nuclei involved. Thus, experiments with sodium-containing crystals, including metallic Na and NaH, yield among others the hydrogen scattering amplitude. The resulting value of $(3.\,{}^3a + {}^1a)$ is in complete agreement with that derived from the molecular hydrogen data; combining it with the known value of $S\,(0)$, one finally gets ${}^3D = (1{,}6 \pm 0{,}2).10^{-13}$ cm.

This small value for the width of the ^{3}S potential appears to be in contradiction with the conclusions derived from an analysis of the deuteron properties based on similar assumptions about the shape of the nuclear potential. As is well-known, Rarita and Schwinger, using a potential of the form (1), (2) and taking for $\mathcal{J}\,(r)$ and $F\,(r)$ potential wells of the same width D, were able to show that it is possible to account quantitatively for all the properties of the deuteron ground state by suitably fixing the depths $\mathcal{J}$ and F of the two wells. In fact, the numerical values of the potential parameters are determined by the known values of the binding energy and the quadrupole moment and by the amount of D state admixture. Assuming the latter to be 4%, one finds

$$D = 2{,}8.10^{-13} \text{ cm}, \qquad \mathcal{J} = 13{,}9 \text{ MeV}, \qquad F = 2{,}325\, \mathcal{J}; \qquad (6)$$

thus, the central part of the ^{3}S potential determined in this way has *the same width* (and about the same depth) as the ^{1}S potential derived (on the assumption of charge independence) from the analysis of the proton-proton scattering data. This equality of the widths of the two effective potentials is only established, of course, within the margin of uncertainty affecting especially the electromagnetic data (magnetic and quadrupole moments). However, a simple semi-qualitative argument, due to SCHWINGER (1941) (NF **6.12**), shows that these data impose upon the width of the ground state potential a *lower limit* not much smaller that the above value (6).

It might appear, however, that the assumptions underlying Rarita and Schwinger's theory give but a poor representation of the nuclear potential. In fact, in view of the very dissimilar expressions (3) for $\mathcal{J}(r)$ and $F(r)$ given by meson theory, one might think that the assumption of equal ranges for the central and the non-central potential well involves an arbitrary restriction. This point has been investigated at my request by Mr. Demeur [1]. For this purpose, he introduced different ranges for the two kinds of potentials just mentioned and tried to adjust all the parameters of the theory thus extended so as to account for the properties of the ground state of the deuteron. His conclusion is that Rarita and Schwinger's original choice is essentially unique [2]; of course, some slight variations (including unequal ranges) are allowed by the margin of uncertainty of the empirical data, the most sensitive one being the electric quadrupole moment. In particular, a value of the range of the central part of the ground state potential much lower than that given by (6) would, in conformity with Schwinger's argument, be entirely excluded by the accepted value of the quadrupole moment.

Before concluding, however, that there is a serious discrepancy between the two sets of data just discussed, — diffraction effects on the one hand, electromagnetic properties of the deuteron on the other, — it remains to be seen whether the contradiction is not peculiar to the well shape assumed for the potentials and does not disappear if the diffraction problem is treated on the basis of meson theory.

2.4. *Isotopic factor of nuclear potential and charge of mesons.* The general operator of potential energy of a pair of unspecified nucleons contains a factor depending only on the isotopic variables $\tau^{(1)}$, $\tau^{(2)}$ of these nucleons and expressing how the interaction energy varies according to the proton or neutron states of the nucleons. The simplest possible forms for such an isotopic factor are the following :

neutral type :	1
symmetrical type :	$\tau^{(1)}\,\tau^{(2)}$
charged type :	$\frac{1}{2}\,(\tau_1^{(1)}\,\tau_1^{(2)} + \tau_2^{(1)}\,\tau_2^{(2)})$;

the names attached to them refer to their derivation by means of a field theory of the interaction : the neutral and charged types of interaction are brought about by neutral and charged meson fields, respectively, while the symmetrical type arises from a combination of both neutral and charged fields operating in a symmetrical way. From a more

(1) Similar results have recently been published by GUINDON (1948).

(2) At any rate so long as the width of the non-central well is assumed to be smaller than that of the central one, as suggested by the expressions (3). If this further condition is not imposed, other solutions exist, as pointed out by Miss PADFIELD (1948).

phenomenological point of view, the neutral type corresponds to « ordinary » forces, that are independent of the charges of the nucleons; the charged type to « exchange » forces between neutron and proton only, resulting from a virtual exchange of charge between them; the symmetrical type to a combination of ordinary and exchange forces, which is also independent of the charges of the interacting nucleons.

The property of charge independence of the nuclear potential, partially established on empirical grounds (NF **8.1**), still leaves a choice open between the neutral and the symmetrical type of isotopic factor. From the empirical evidence, only a few arguments, none of them entirely decisive, can be found in favour of a symmetrical theory :

a) the behaviour of nucleons scattered by deuterons (NF **14.12, 14.13**);
b) the saturation properties of nuclear bindings (NF **11.33**);
c) indications from scattering of fast protons by protons that the effective ^3P potential is repulsive (NF **7.131**).

It must be observed that the analysis of the above evidence has been based on the assumption of purely central forces; the effect of non-central interactions on these phenomena, however, is likely to be small, except on the saturation properties of the nuclear forces, which require closer examination (NF **17.1**).

It is somewhat paradoxical that the phenomenon which would be expected to provide the most direct evidence about this question, viz., the scattering of fast neutrons by protons, cannot yet be included in the above list. The important results obtained with 90 MeV neutrons from the large Berkeley cyclotron are in fact not yet sufficiently complete to allow an unambiguous analysis. All one can say at the moment is that they very probably exclude the purely neutral form of theory but do not seem to be in quantitative agreement with any simple type of symmetrical potential. The consideration of the non-central part of the nuclear interaction (according to Rarita and Schwinger's theory) would seem to reduce the discrepancy between calculated and observed angular distribution of the scattered neutrons. Moreover, it should be borne in mind that velocity-dependent forces (**2.1**) might play a far from negligible part in such a quantitative comparison. The theoretical angular distribution generally has a minimum in the neighbourhood of $\Theta \gtrsim \frac{1}{2}\pi$ (Θ being the scattering angle in the barycentric system) and a maximum as well in the forward as in the backward direction. In a neutral theory, the forward maximum is much larger than the other, while the situation is reversed in a symmetrical or charged theory, in conformity with the « exchange » character of the interaction in these cases; there is no great difference between charged and symmetrical theory. It is just the presence in all cases of a « secondary » maximum in the theoretical distribution that complicates the interpretation of the experimental results; so long as the behaviour of the differential cross section at small angles is not better elucidated, it will be premature to draw any definite conclusions.

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Discussion du rapport de M. Rosenfeld

Mr. Oppenheimer. It is dangerous to conclude from the work on the dipole forces that there are two kinds of mesons. There is no way of dealing with the tensor force.

Mr. Peierls. There is no need for explaining the apparent difference between proton-proton and proton-neutron forces by an effect due to the importance of different regions. The only evidence for the hypothesis of « charge-independence » of the forces comes from the comparison of the neutron-proton and proton-proton interaction. If it is not borne out exactly by better data it must be abandoned or modified.

Mr. Pauli. The explanation of the magnetic moments of ^{3}He and ^{3}H by exchange currents requires only that the proton-proton and neutron-neutron forces are equal; the neutron-proton force could be different.

Mr. Rosenfeld. The approximate equality of proton-proton and neutron-proton forces in ^{1}S-states is obtained for various shapes of the potential; there is a difference of about 2 % which is approximately constant.

Mr. Serber. *High energy Neutron Proton Scattering.*

Experiments have been done at Berkeley by Prof. Segrè and Wilson Powell and their students on the scattering of neutrons by protons. The neutrons energies used were 90 and 40 MeV. Segrè's experiments were done using coincidence counters to detect the scattered protons, Powell used a hydrogen-filled cloud chamber and measured proton recoils in the gas.

Fig. (R1) shows the apparatus used to measure the total cross-section. The monitor and detector foils are polystyrene or paraffin; the protons they scatter are detected by sets of three proportional counters in line. An absorber is placed between the second and third counter whose thickness is chosen so that, in the 90 MeV experiments,

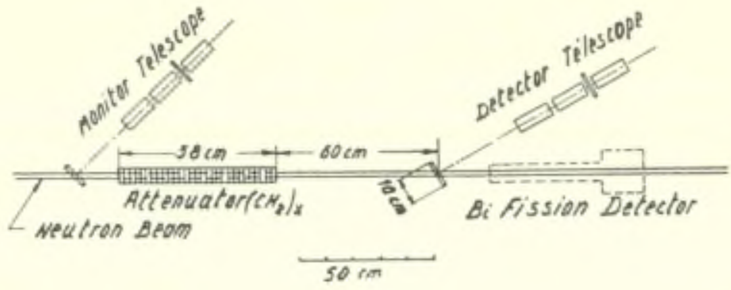

Fig. 1.

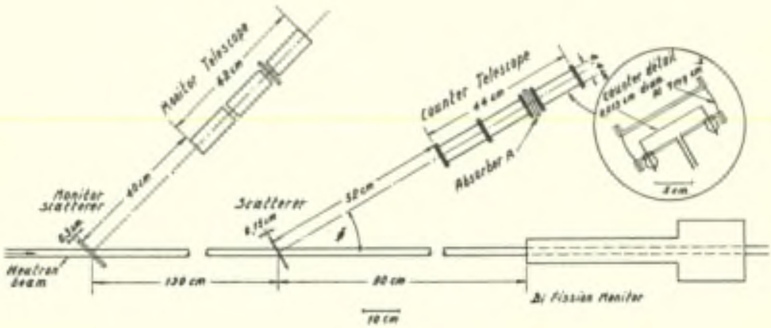

Fig. 2.

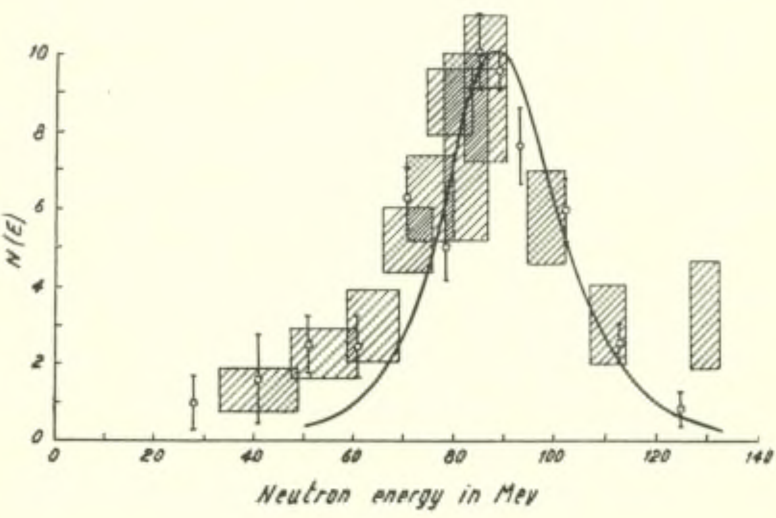

Fig. 3.

only neutrons of energy greater than 66 MeV can give detectable proton recoils. The cross-section was determined by placing absorbers of various hydrocarbons in the position shown, and measuring the charge in the relative rates of the two counter sets. The correction for the scattering by carbon was determined using carbon absorbers. A similar experiment has previously been done by Prof. Mc. Miller using the radioactivity induced in a carbon disc to measure neutron intensity.

Fig. (R 2) shows the apparatus used in the measurements of the angular distribution of the scattered protons. The second counter telescope is placed at various angles with the neutron beam. Again only neutrons of energy greater than 66 MeV are measured.

For the large scattering angles, where the energy of the recoil protons is low, a different apparatus is used, designed to detect lower protons.

Fig. (R 3) shows the energy distribution of « 90 MeV » neutrons. The solid curve is the theorical distribution expected on the basis of stripping the deuteron. The experimentally measured points agree well, but with indications of more slow neutrons. It should be remembered that only the region above 66 MeV is used. In determining the effective energy the detector efficiency is taken into account, and also the variation of cross section with energy, which is supposed to be $\sigma \sim 1/E$. The cross-sections quoted are corrected to 90 MeV neutron energy.

Fig. (R 4) gives the result of the counter measurements. The upper points are for 40 MeV neutron energy, the lower for 90 MeV. The cross section per unit solid angle is given on an absolute scale, in the center of mass system. Zero degrees means neutron forward, 180° proton forward. The points show the result of various runs, with various equipment. The stars give the average value at each angle. The total cross section at 90 MeV is 0.075×10^{-24} cm^2, at 40 MeV it is 0.177×10^{-24} cm^2. The error in the numbers is given as 10 %.

We now come to the question of the interpretation of these results. Before speaking of the 90 MeV experiments a word must be said about possible relativistic corrections. These are not predictable on a priori grounds. However calculations based on specific meson theories by Marsha and Snyder, and other calculations made in Berkeley with different assumptions about the type of meson theory lead to corrections to the total cross section of less than 10% (the sign varying with the theory) and to corrections in the angular distri-

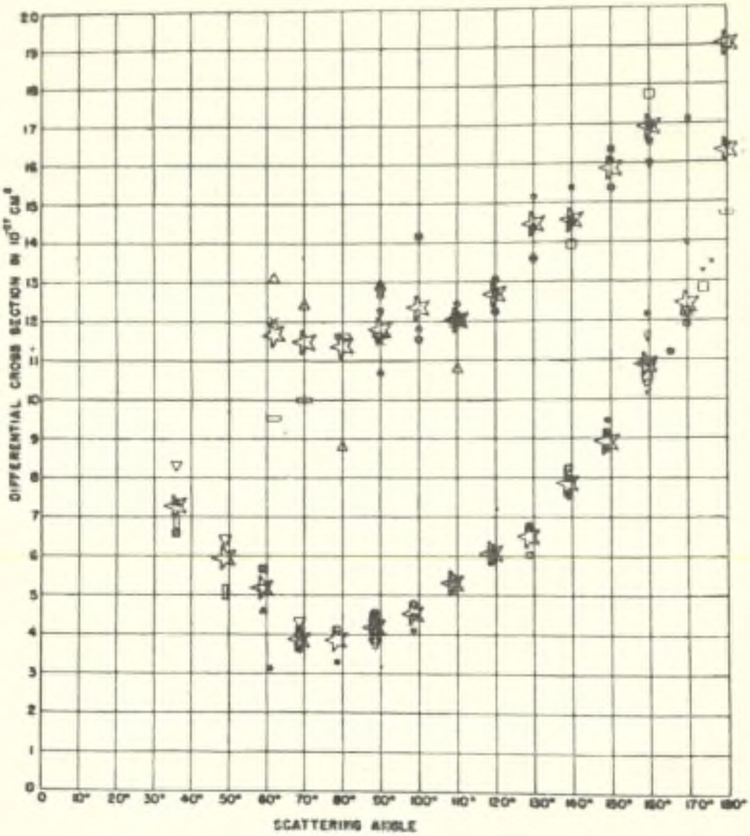
DIFFERENTIAL CROSS SECTION IN 10^{-27} CM^2
SCATTERING ANGLE

Fig. 4.

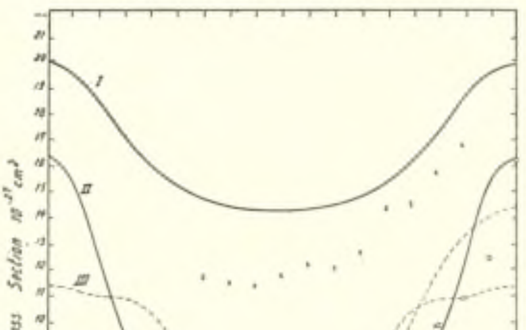
Differential Cross Section 10^{-27} cm^2
Scattering Angle

Fig. 5.

bution also of the order of 10%. So no gross discrepancy with non-relativistic theory is expected, although too high precision in attempting to fit the experimental curves is certainly unwarranted.

Fig. (R 5) shows the results of calculations carried out by the theoritical group in Berkeley, particulary by Mr. Christian and Mr. Hart. The circles are the experimental points, corresponding to the stars in the preceding figure. Let us look first at the 90 MeV results. The curve marked II was calculated with a simple central force, given by a Yukawa-potential with a range corresponding to a meson mass 320, the mass found by Breit from analysis of the *p-p* scattering (reduction of the mass to 286 would not greatly alter the results). The force is supposed to be half an exchange force, half an ordinary force, i. e. there is no force in states of odd angular momentum, and the scattering is symmetrical in the center of mass system. We do not mean to imply that the observed scattering is actually symmetric, indeed the observation indicate a somewhat smaller scattering at small angles than at large. This potential fits all the facts about low energy *n-p* scattering, including the crystal scattering. It may be mentioned that the Yukawa potential is the only one tried with which it is possible to fit both *p-p* and *n-p* scattering with forces of the same range. It will be seen that the general features of the scattering calculated with this potential agree reasonably well with experiment, although the total cross section is somewhat too high. Calculations have also been made with other forms of potential, e. g. exponential wells, and gauss wells; if the ranges are suitably chosen the results are not too different. Only a square well gives an appreciable difference, it being possible to get smaller cross sections using square wells.

A question of considerable interest is whether charge symmetric theories give the right scattering.

For the Yukawa potential, the charge symmetric theory would give an unacceptable result, the cross section at 180°, for example, being 40×10^{-24} cm². The curve marked IV has been calculated for charge symmetric theory using a square well of range 1.8×10^{-13} cm. It is seen to be considerably more asymmetric than the experimental curve.

We now come to the strangest feature of the results, which has to dowith the expected influence of the tensor force. Curve III is calculated again with the previously described Yukawa potential, but this time with the tensor force necessary to give the observed quadripole moment of the deuteron. It will be seen that the curve is considerably flattened and widened near the ends of the angle range. This behaviour is characteristic of a tensor force, calculations have been carried out, for example, supposing the tensor force has a long range, and while it is possible to make the effect less pronounced, it is always present.

The experimental curve seems to behave in just the opposite way, rising very steeply near 180°; in fact its behaviour suggests that to be expected from a central force of very long range. The scattering evidence thus gives no confirmative evidence for the existence of a tensor force.

The 40 MeV curve labeled I is calculated with the same potential as II. The form of the angular distribution agrees fairly well, through the absolute value is appreciably too high. This illustrates the point that the observed cross sections are rather remarkably low, considerably lower than would be anticipated in advance, and it is a matter of considerable difficulty to find potentials that give at the same time low enough cross sections, and angular distributions that are not grossly in error at one or both energies.

The final figures (Fig. (R 6)) gives the cloud chamber data for the scattering of 80 MeV neutrons. The curve is the one labeled II in

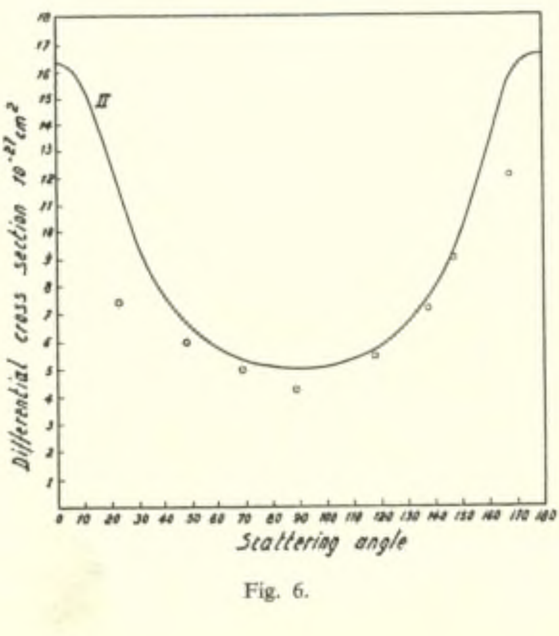

Fig. 6.

the previous figure. The cloud chamber data agree very well with the counter data in the region in which they overlap. It has proved possible to extend the cloud chamber data to smaller angles, and a clear indication of an asymmetry of the scattering curve is now evident. An attempt will be made to extend the measurements to still smaller angles.

It seems to me that the above results point the moral that we know very little in fact about the nuclear forces, and that meson theories predicated on the supposed properties of such forces must be taken with appropriate caution.

Electric and Magnetic Nuclear Moments(*)

by F. Bloch

I.

INTRODUCTION

It has been recognized for a considerable time that one can ascribe to the atomic nucleus a definite value of the spin as well as of its electric and magnetic moments, and the investigation of these quantities constituates an essential part of nuclear physics. The determination of spin values, obtained from spectroscopy, was originally both of greatest interest and accuracy. More recently, quantitative and partly very accurate results have also been obtained for the values of nuclear moments and these results are likewise significant because of their relation to the structure of nuclei and the forces between their constituents It must be remarked that the existence of moments is closely related to the spin which determines the rotational symmetry properties of the nucleus. The complete spherical symmetry of a nuclear state with spin 0 excludes, for example, the existence of any nuclear moments. A nucleus with spin 1/2 can have no other than a magnetic dipole moment; with spin 1 both a magnetic dipole and an electric quadrupole moment are possible, etc. Nuclear electric dipole moments have never been observed; in fact, they can be generally excluded if the ground state is not degenerate and if the expression for the total energy does not change upon a change of sign of all relative position coordinates in the nucleus («Mirror symmetry»). The existence of magnetic quadrupole moments can be excluded for the same reasons. Introducing an integer n, one can have in general for a nucleus with spin I :

$$\left.\begin{array}{l}\text{Magnetic poles of order } 2^{2n+1} \text{ with } 0 \leqslant n < I \text{ and} \\ \text{Electric poles of order } 2^{2n} \quad \text{with } 0 \leqslant n \leqslant I\,.\end{array}\right\} \quad \mathrm{I}$$

(*) This manuscript has been prepared for the originally planned earlier date of the Solvay Congress in the spring of 1948. Footnotes have been inserted to take into account results which have since been obtained.

We shall here mainly discuss the results, obtained for the moments of the neutron and proton themselves and of their simplest combinations in the isotopes of hydrogen, since they promise a more direct insight for the investigation of elementary particles than the moments of the rather complex heavier nuclei. The study of these heavier nuclei has been and continues to be important, both from the point of view of method and of general significance. In fact, it was largely the work on heavier nuclei which has e tablished the rule that those with even or odd atomic weight have a spin which is integer or integer plus one-half, respectively; the significance of this rule has become evident through the proton-neutron model of the nucleus, postulated by Heisenberg and Majorana.

At present there exist data for spin and moments of about 90 stable and a few radioactive nuclei. We shall give an account of the various methods by which they have been obtained. In spite of its importance for the investigation of nuclear spins, band spectroscopy will not be discussed in this account, since it is not used for the determination of nuclear moments. The results of the recent interesting method for the study of molecules by the absorption of microwaves concern also mainly nuclear spins and they will be omitted for the same reason. Several of the techniques for the determination of nuclear moments are applicable both to heavy and light nuclei, and our emphasis on the lightest nuclei will result mostly in a strict selection of he numerous results rather than of the available methods.

II.

METHODS FOR THE DETERMINATION OF NUCLEAR MOMENTS

1. *Spectroscopic measurement of hyperfine structures.* The first indication of nuclear magnetic moments has been obtained from the hyperfine structure of spectral lines. The observed fact that many atomic energy levels consist of a very narrow multiplett has been explained by Pauli (1) as due to a coupling of the nuclear moment with the electrons. If one denotes by H(0) the magnetic field produced by the electrons at the place of the nucleus with magnetic moment μ and by I and J the angular momentum of the nucleus and of the electrons, respectively, both in units of $\hbar = h/2\pi$, one finds that the hyperfine multiplett consists of

$2I+1$ components with a total splitting $\mu H(0) \frac{2J+1}{J}$ if $I < J$ and of $2J+1$ components with a total splitting $\mu H(0) \frac{2I+1}{I}$ if $I > J$.

A further splitting of the levels has been observed under the influence of an external magnetic field ([2]). It has been shown to be in complete analogy to the Zeeman and Paschen-Back effect of ordinary multipletts with the coupling of the electronic spin to the orbital motion replaced by that of the nucleus to the total atomic moment. The corresponding transition from weak to strong fields has been treated ([3]) and the study of these effects represents an important part in the general investigation of hyperfine structures. The number of hyperfine structure components, together with their interval rules, has led to the spin value for many of the heavier nuclei. The application of an external field has here also proven to be very helpful, since it leads to a separation into the total of $(2I+1)(2J+1)$ components.

Besides the existence of nuclear magnetic moments, that of electric quadrupole moments has also been recognized. Schueler ([4]) first observed that the separation of hyperfine structure components of E*u* deviates from that to be expected from the interaction of a pure magnetic dipole moment of the nucleus with the electrons. While such an interaction should lead to a splitting proportional to the eigen values of cos (IJ) with (IJ) being the angle between the angular momenta of the nucleus and the electrons, it was found necessary to introduce another interaction term, proportional to $\cos^2$ (IJ). Such an interaction is indeed to be expected if one assumes that the charge distribution in the nucleus is not spherically symmetrical, but that it has a finite quadrupole moment ([5]). The appearance of electric quadrupole moments is a rather general phenomenon for nuclei with higher spin.

The spectroscopic investigation of hyperfine structures has not only given the first information on nuclear moments, but many of the existing data, particularly on spin values, are still due to this method; nevertheless, it has characteristic and rather serious limitations. While the spin can often be directly obtained from the number of components in suitable multipletts, it is far more difficult to determine from hyperfine structure the value of nuclear moments. In the first place, such a determination requires the exact measurement of very small line separations, which is greatly limited by the finite line widths. In the second place, even if the levels in a multiplett are quantitatively established, they still do not contain all the information which is needed to know the moment. In the case of a pure magnetic dipole moment, there enters in the splitting the moment, multiplied with the magnetic field H (0) at the place of the nucleus; likewise, the electric quadrupole moment appears multiplied with the gradient of the electric field. A determination of the moments presupposes, therefore, a knowledge of these fields, which is usually rather inaccurate, since it involves the electron configuration of atoms with many electrons. The combination of measured hyperfine structure splittings and calculated values of H (0) ([6]) has led to values of nuclear magnetic moments with an accuracy of a few per cent at best.

The magnetic moments of all nuclei have been found to be of the order of the « nuclear magneton » $\mu_n = \frac{e\hbar}{2Mc} = 5.02 \times 10^{-24}$ erg/Gauss, which is obtained from the Bohr magneton by replacing the mass of the electron by the mass M of the proton. This can be expected from moments arising from the orbital motion of protons within the nucleus; for a more detailed account, one must also consider the contribution due to the intrinsic magnetic moments of the nuclear protons and neutrons, which, however, are likewise of the order of μ_n. Similarly, one can explain the observed order of magnitude of 10^{-24} cm^2 of nuclear quadrupole moments, defined as the charge-weighted average of $3z^2 - r^2$ over the nucleus, by the known linear dimensions of the order of 10^{-12} cm.

Not only the value but also the sign of nuclear moments can be defined and ascertained from the order in which the components appear in a hyperfine structure multiplett. The sign of a magnetic moment refers to its orientation with respect to the angular momentum of the nucleus, in the sense that for a relative orientation like that occurring from the rotation of a positive or negative charge, the moment is called positive or negative, respectively. From the definition of the quadrupole moment it follows that positive or negative values refer to a charge distribution in the nucleus corresponding to the shape of a prolate or oblate spheroid, respectively. Both magnetic dipole and electric quadrupole moments have been observed with either sign.

Besides its limitations in accuracy, two of the most important moments, those of H^1 and H^2, could not be obtained from the spectroscopy of hyperfine structures. While the splitting increases generally with increasing atomic number, and thus makes the method more suitable for heavier nuclei, it is too small for these lightest nuclei to be observed by spectroscopic means. The more recent methods to be described below, are not based on optical transitions and are, therefore, not affected by this drawback.

2. *Molecular and atomic beams.* The investigation of nuclear moments by molecular and atomic beams differs essentially from the spectroscopic method insofar as it is not the emitted light but the deflection in an inhomogeneous magnetic field, which is affected by the presence of these moments. There are various factors modifying the deflection, and they lead to different methods and results which shall be discussed separately.

a) *Deflection of hydrogen molecules.* The first direct measurement of nuclear moments by molecular beams has been achieved by Stern and his collaborators in their determination of the magnetic moments of the proton and the deuteron (7). Hydrogen molecules, containing either of the two nuclei, have a magnetic moment, originating from the nuclear moments and from the rotation of the molecule around its center of gravity. The contribution due to the latter which is likewise of the order of the nuclear magneton μ_n can be ascertained from the deflection of para-molecules where the total nuclear moment is zero. The deflection of ortho-molecules allows then the determination of the nuclear moments.

Since one is dealing here with moments of the order of μ_n, the deflection is under the same conditions approximately 1000 times smaller than that obtained in the Stern-Gerlach experiment which is caused by atomic moments of the order of the Bohr magneton $\mu_o = \dfrac{eh}{2m_oc}$ (m_o = restmass of the electron). Actually it has not been possible to resolve the molecular beam into the components resulting from the different orientations of the rotational and nuclear moments, with respect to the inhomogeneous magnetic field; the application of this field results, however, in a changed intensity distribution of the beam which leads to the desired, although not very accurate, determination of the nuclear moments.

One of the most important results on nuclear moments has been established by this method, i. e., the fact that the magnetic moment μ_P of the proton is not equal to the nuclear magneton μ_n but approximately 2.5 times larger. It was shown by Dirac that the combination of relativistic and quantum-effects leads to a natural explanation of the value μ_n for the magnetic moment of the electron. By the same argument, applied to the proton, one would obtain μ_n for the magnetic moment and the observed large deviation from this value points, therefore, to a very different nature of its origin which has not yet been explained satisfactorily.

While the magnetic moment of the proton has been established by this method to within about 10 %, the moment μ_D of the deuteron could merely be stated to lie between 0.5 and 1 nuclear magnetons. Nevertheless, this result was of similar importance, since it gave the first indication of the neutron possessing likewise a magnetic moment. The simplest assumption of a $^3S_1$ state for the deuteron and additivity of the magnetic moments of the proton and the neutron, leads for the latter to the value $\mu_N = \mu_D - \mu_P$ of approximately two nuclear magnetons and of negative sign. With the magnetic moment of the proton being at least partly explicable by the theory of Dirac, that of the neutron as particle without charge, requires a different explanation in its entirety.

b) *Measurement of spin and hyperfine structure by the deflection of atomic beams.* Although nuclear moments are about 1000 times smaller, one can see from the familiar classical consideration of the Zeeman effect that they have a major influence upon the magnetic moment of atoms in a weak external field. The slight coupling to the nuclear moment causes here a slow precession around the resultant angular momentum of nucleus and electrons with the result that the effective average value of the atomic moments can greatly differ from the full value attained in a sufficiently strong field where it precesses around the field direction.

Breit and Rabi [5] have treated the variation of the atomic moment, belonging to the different Zeeman levels of hyperfine structure, in the transition from weak to strong fields. They have pointed out that one obtains in an external field $(2I + 1)(2J + 1)$ different values for this moment so that for an atom with known angular momentum J of the electrons, the number of the Stern-Gerlach components in the atomic beam determines directly the nuclear spin. A simple relation also allows to obtain the hyperfine structure splitting from the magnitude of the atomic moment, measured for different values of the external field.

Based upon these ideas, Rabi and his collaborators have developped various ingenious methods for the determination of nuclear spins and the measurement of hyperfine structures. Their original method, where the atomic moment is directly obtained from the deflection in a known inhomogeneous field, has been applied to measure for the first time the hyperfine structure splitting of the light as well as the heavy hydrogen atom in its ground state (9). The field H (0), produced by the electron, is here, according to Fermi (10), simply related to its probability density at the place of the nucleus and hence determined by the well-known wave function, describing the ground state of the hydrogen atom. The measured hyperfine structure splittings allow therefore a determination of the magnetic moments of the proton and the deuteron and their values have thus been verified to be in qualitative agreement with those obtained from the deflection of hydrogen molecules.

A remarkable improvement of the technique was introduced by the method of zero moments (11), where the deflecting field is adjusted to such a value that the magnetic moment of certain Zeeman components vanishes, resulting in a zero deflection of the corresponding atoms, independent of their velocities. The number of field settings, for which zero deflection is observed, gives directly the nuclear spin and the corresponding values of the field determine the hyperfine structure splitting. While this method has been successfully applied for the alkalis, it could not be used for the proton, since a zero moment appears here only for vanishing external field.

The introduction of a second important null method allowed the further investigation of the proton and the deuteron (12). The beam of hydrogen atoms passes here first through a weaker inhomogeneous field A, and subsequently through a stronger field B, the latter with a gradient opposite to that of A and strong enough so that the atomic moment has practically its full value of a Bohr magneton. With the path length in A being larger than in B, one obtains, independent of their velocity, zero deflection and focusing on the detector of those atoms, for which the A field is chosen to cause the appropriate deflection. The knowledge of the gradient and strength of both fields leads then to the corresponding value of the atomic moment and to the hyperfine structure splitting. From the observed splitting, the magnetic moments of the proton and deuteron could be calculated to be $\mu_P = (2.85 \pm 0.15)\ \mu_n$ and $\mu_D = (0.85 \pm 0.03)\ \mu_n$, representing both a variation and an improvement in accuracy of the earlier results. With the spin of the deuteron and proton known from band spectroscopy to be 1 and 1/2 respectively, the latter was also verified by the occurrence of two different focusing settings of the A field, indicating the existence of four Stern-Gerlach components in the beam, and thus with $J = 1/2$ for the electronic angular momentum, a value $I = 1/2$ for the proton spin.

A significant and far reaching feature was simultaneously incorporated in the arrangement of double deflection of the atomic beam by a weak third field C, acting between A and B, and with a geometry such as to cause by its variation non-adiabatic transitions between the Zeeman levels of a passing atom. It had been pointed ou previously by Rabi ([13]) that such transitions would manifest themselves in a decrease of the focussed beam intensity and that their occurrence or non-occurrence for certain components depends upon the order of the hyperfine multiplett. It was thus possible to establish the fact that the order in the hyperfine structures for light and heavy hydrogen is normal, corresponding to a positive sign of the magnetic moments of both proton and deuteron.

c) *Molecular Beam magnetic resonance method.* While the previously described methods to determine nuclear moments by the deflection of molecular and atomic beams require a knowledge of the deflecting field, Rabi and his collaborators have developped a new technique where the deflection is merely used as an indicator and its quantitative value becomes irrelevant. The principle of the arrangement is the same as that mentioned at the end of the previous section, where non-adiabatic transitions are induced in a third magnetic field C, between the two deflecting fields A and B. However, instead of using a stationary field C, where the variation in time exists merely in the frame of reference of the moving atom, this necessary variation is provided here by one component of the C field itself and occurs with an externally controlled frequency.

The principle of the method can readily be understood from classical mechanics : the torque, produced by a constant magnetic field H_o, upon a nucleus with a magnetic moment μ, results in a precession with constant angle Θ around H_o, with a circular frequency

$$\omega_o = \gamma\ H_o \qquad (1)$$

$$\text{where } \gamma = \frac{\mu}{a} \tag{2}$$

is the « gyromagnetic » ratio of the magnetic moment μ and the angular momentum a of the nucleus. By measuring μ in Bohr magnetons and a in units $\hbar$, one can also write

$$\gamma = \frac{e}{2m_0c}\, g \tag{3}$$

where g is the familiar Landé factor. A field with circular frequency ω and amplitude H_1, at right angles to H_o, causes an additional torque which will tend to change the angle Θ. With $H_1 \ll H_o$ this change in angle will be small in one period but it can become accumulative if $\omega = \omega_o$, resulting under this resonance condition in a large change of Θ. By establissing the occurrence of resonance, the gyromagnetic ratio is immediately given by the corresponding values of the field H_o, and the frequency ω, and determines the magnetic moment of a nucleus with known spin.

It is sometimes more convenient to use the concepts of quantum mechanics in the description of the same phenomenon :

The energy levels of a nucleus with magnetic moment μ and angular momentum $a = I\hbar$, in the field H_o, have an equidistant separation $\Delta E = H_o\, \mu/I$ where this energy change corresponds to a change of the magnetic quantum number m by unity. The weak field with amplitude H_1, at right angles to H_o, will induce such transitions if its quantum energy $\hbar\, \omega$ is equal to ΔE so that the condition of energy conservation requires

$$\omega = \frac{\mu}{I\hbar}\, H_o = \frac{\mu}{a}\, H_o \tag{4}$$

This is equivalent with the classical resonance condition $\omega = \omega_o = \gamma\, H_o$ with ω_o and γ having the same significance as in (1) and (2). For a nucleus with spin 1 and a magnetic moment, equal to the nuclear magneton μ_n one would obtain a resonance frequency of one megacycle in a field H_o of 1300 Gauss and one can generally expect that resonance in the convenient range of radio frequencies requires field strenghts which can easily be obtained.

The first experiment to detect this magnetic resonance of nuclei was made by Gorter (14) in an attempt to detect the resonance absorption of radio quanta, through heating of L*i*F and A1K crystals. While the results of this experiment were negative, Rabi (15) has treated the transitions in a rotating field and has pointed out their use in molecular beam experiments, particulary for the determination of the sign of nuclear moments. An independent suggestion to use magnetic resonance in oscillating fields for the dertermination of the neutron moment has been made by Bloch. (See section 4.)

A successful measurement of nuclear moments, through magnetic resonance, was first carried out by Rabi and his collaborators ([16]) who observed transition between the magnetic levels of L*i* and F in a molecular beam. The C field has two components, one of constant magnitude H_o and provided by an electromagnet, the other oscillating at right angles and produced by the radio frequency current of opposite direction in two parallel wires. The fields A and B were adjusted to give compensating deflections to a molecule with fixed magnetic quantum number m. The change of this quantum number in the C field has the result that the field B no longer compensates the previous deflection in the field A. Resonance conditions could therefore be established by the resultant drop in intensity of molecules with zero deflection at the detector.

The use of an oscillating, instead of a rotating component of the C field, although perfectly adequate to determine the magnitude of the moment, prevents the direct dependence of the effect upon its sign. In the classical description of the phenomenon, a change in sign of the magnetic moment would invert the torque upon the nucleus and thereby its sense of rotation around the field H_o. Resolving the oscillating field into two components, with opposite sense of rotation, it will be essentially only the component, rotating in the sense of the nuclear precession, which will be effective in causing transitions. In a purely oscillating field however, both components will be present with the same strength, and the experiment does not reveal to which of the two the observed transitions are due. Millman ([17]) has observed that end effects from the wires which produce oscillating field will, nevertheless, over a short lenght of the path, cause a rotating field in the frame of reference of the moving molecules. This results in a slight asymmetry of the resonance curve from which the sign of the moment can be deduced.

Among the many moments which have been determined through magnetic resonance, we shall mention particularly those of the proton and the deuteron, obtained from experiments with hydrogen molecules ([18]), It is not sufficient for the analysis of the observations to consider here merely the effect of the external C field upon a single nucleus. Just as in the deflection of hydrogen molecules, mentioned in section 2a, one has to take into account that there exists also a rotational moment which is coupled to the nuclear moment, and furthermore that the moments of the two nuclei in the same molecule interact through their magnetic dipoles. This results in a rather complex structure of the observed resonance lines which, however, could be resolved with the effects from nuclear moments clearly separated from those originating due to molecular rotation ([19]). The moments of the proton and the deuteron could thus be established with considerable accuracy to be $\mu_P = (2.785 \pm .02)\ \mu_n$ and $\mu_D = (0.855 \pm .006)\ \mu_n$. The limitation in accuracy is determined by the measurement of the magnetic field; its value does not enter, however, in the ratio of the two moments which has been determined with higher accuracy to be $\frac{\mu_P}{\mu_D} = 3.2570 \pm .001$.

While the structure, obtained from H_2 molecules, could be completely and quantitatively explained by the magnetic moments of the two nuclei, interacting both with each other and with the rotational moments of the molecule, there appeared strong quantitative discrepancies in the case of HD— and D_2— molecules. The observed structures were, however, fully explained by an additional interaction with the molecular rotation due to a finite electric quadrupole moment of the deuteron ([20]). Contrary to the magnetic moment, which is determined by its energy in the external and measurable field H_o, the observations do not yield directly the value of the quadrupole moment. As in the original discovery of nuclear quadrupole moments from hyperfine spectroscopy (see section1), the measurements refer merely to the interaction energy of the elec-quadrupole with the gradient of the electric field, existing at the place of the deuteron. This quantity has been calculated by Nordsieck ([21]) for the hydrogen molecule; based upon his result, the quadrupole moment of the deuteron was found to be $Q_D = 2.73 \times 10^{-27}\ cm^2$. Although this value is about a thousand times smaller than the normal magnitude, observed for other nuclei, it is of great significance in its bearing upon nuclear forces. (See Chapter III, section 2.)

Besides the determination of nuclear moments in molecular beams, magnetic resonance has also been successfully applied to the measurement of atomic hyperfine structures ([22]). The principle difference lies in the fact that the energy of orientation of the nuclear moment is here not primarily due to the external field H_o but arises largely from the interaction with the atomic moment. The transitions in the C field occur here between Zeeman levels of the hyperfine structure, and the quantum energy at resonance determines the energy difference between these levels. Because of its close relation to the spectroscopic phenomena, observed in the optical region, this method has been properly labeled as « atomic radio frequency spectroscopy ». As for molecular beams, the resonance manifests itself again in a drop of intensity at the detector, adjusted for zero deflection, except that it is here the much larger atomic moment which undergoes a change by transitions in the C field. Radio frequency spectra have led to highly precise results for the hyperfine structure of the alkalis. In connection with the magnetic moment of the proton, they deserve special attention since they have led to the, at present, most accurate calibration of its value.

It has been mentioned above, that the determination of μ_P and μ_D through magnetic resonance is limited by the accuracy with which the field strenght H_0 can be measured. While the use of flipcoils limits this accuracy to little less than one per cent, a far higher precision can be obtained from radio frequency spectra ([23]). Through the Zeeman effect of the hyperfine structure, some of the transition frequencies depend strongly on H_0 and they determine its value uniquely in terms of the hyperfine structure splitting which exists in the absence of an external field, and of the electronic moment of the atom. By measuring in the same field H_0 the magnetic resonance frequency for protons in the H_2-molecule and a transition frequency in the radio spectrum of an alkali atom, one obtains the magnetic moment of the proton μ_P directly in terms of the electronic moment μ_A of the atom, with the ratio of the two expressed in nothing but the ratio of measured frequencies. Taking μ_A as the Bohr magneton and 1836.6 for the ratio of the masses of proton and electron, the most accurate value of the proton moment has thus been found to be $\mu_P = (2.7896 \pm .0008)\ \mu_n$. With the previously measured ratio of μ_D/μ_P this give for the magnetic moment of the deuteron $\mu_D = (0.8565 \pm .0004)\ \mu_n$.

In connection with these values for μ_P and μ_D, recently obtained accurate values for the hyperfine structure from the radio frequency spectra of H — and D — atoms ([24]) are of considerable interest. The values for μ_P and μ_D which one obtains by applying the formula of Fermi ([10]) to the measured hyperfine structure separation are both significantly greater by about 0.3 per cent than those which have been measured directly. A much closer, although not perfect, agreement is obtained for the ratio μ_P/μ_D obtained from both methods. This indicates that the discrepancy is mainly due to an inaccurate knowledge of the field, produced by the electron at the place of the nucleus, since a correction factor in this quantity would affect the calculated hyperfine structure splitting of both isotopes, leaving their ratio unchanged (*).

3. *Nuclear induction.* Besides their effects upon the emission of light and the deflection of atomic and molecular beams, nuclear moments can also be observed through their manifestation in purely electromagnetic phenomena.

The most direct phenomenon of this type is the contribution to the magnetic susceptibility of a substance by nuclear magnetic moments. It is given by the familiar Curie formula

$$\chi_n = \frac{I+1}{3I}\,\frac{n\,\mu^2}{kT} \tag{5}$$

where n is the number per unit volume of nuclei with magnetic moment μ and spin I. With matter of normal density and μ of the order of μ_n one obtains at room temperature $\chi_n \simeq 10^{-10}$ so that this nuclear susceptibility adds only a very small fraction to the atomic susceptibility which is normally of the order $\chi_a \simeq 10^{-6}$. Although for protons in liquid hydrogen ([25]) it could be separated from the diamagnetic contribution of the molecules, one cannot expect from direct measurements of the total susceptibility to obtain more than a qualitative result for the nuclear moment.

(*) Recent experimental and theoretical developments (P. Kusch and H. M. Foley, *Phys. Rev.*, **72**, p. 1256, 1947; **73**, p. 412, 1948; J. Schwinger, *Phys. Rev.*, **73**, p. 416, 1948) support the fact that this correction arises from that of the magnetic moment μ_e of the electron which is not given by the Bohr magneton μ_0 but by

$$\mu_e = \left(1 + \frac{\alpha}{2\pi}\right)\mu_0 = 1.00116\ \mu_0$$

(α = finestructure constant). The agreement with the measured hyperfinestructure separation is thereby greatly improved (see Chapter III, section 4). The above mentioned calibration of μ_P has consequently to be modified by the same correction factor, giving

$$\mu_P = (2.7928 \pm .0008)\ \mu_n \qquad \text{and} \qquad \mu_D = (0.8575 \pm .0004)\ \mu_n$$

Instead of this static method, a far more powerful dynamic method has been developed where the electromotive force, induced by nuclear magnetic moments in their precession around an external field, is utilized. This nuclear induction is observed through its action upon a suitable electric circuit. The original, although unsuccessful, attempt in this direction was carried out by Gorter and Broer ([26]) in an experiment to establish the effect in L*i*C*l* and KF through the slight change in frequency of an electric oscillator.

The first successful experiments to detect nuclear induction signals were carried out simultaneously and independently by Purcell, Torrey, Pound ([25]), and Bloch, Hansen and Packard ([28]). The original method of detection, used by Purcell and his collaborators, was based upon the absorption of energy by the nuclei and was called « magnetic resonance absorption ». It was somewhat different from the one which is now used in the work of both groups, and where absorption is not the sole factor determining the observed effects. The term « nuclear induction » seems more suitable to characterize the method in its general present aspect, where the observed signals are always caused by the electromotive force, induced by the nuclei. Its essential features can be understood from a brief description of the original arrangement by Bloch, Hansen and Packard and its underlying theory ([29]).

Like magnetic resonance, nuclear induction is based upon the precession of the nuclear magnetic moment in an external field, which was described in classical terms, in section 2c. Actually, this classical description is here sufficient since the simultaneous observation of many nuclei results always in that of expectation values for which classical mechanics is valid. With the nuclear susceptibility, given by (5), the application of an external field H_o leads to an induced nuclear polarization.

$$M_n = H_o \chi_n \qquad (6$$

in the field direction, after establishment of the thermal equilibrium, A perpendicular oscillating field of radio frequency ω and amplitude H_1 will cause a deviation of the polarization vector $\vec{M}_n$ from the field direction and a stationary situation will result where $\vec{M}_n$ precesses around H_o with an angle Θ, and with the frequency ω of the oscillating field. With $H_1 \ll H_o$, this angle will be small unless ω is close to the Larmor frequency ω_o given by eq. (1). It can actually become 90°

for $\omega = \omega_o$ so that the total polarization vector, with magnitude (6), will precess with the Larmor frequency around H_o. In any direction, perpendicular to H_o, there will thus appar a magnetic induction

$$B_n = 4 \pi M_n \tag{7}$$

oscillating with the frequency ω_o and it will induce an electromotive force of the same frequency across the terminals of a wire, surrounding the sample. Although the nuclear polarization M_n, in a field H_o of several thousand Gauss, is normally only of the order of 10^{-6} Gauss or less, it leads to induced voltages, well above the thermal noise and easily detectable after amplification. Resonance conditions can be periodicaly established by a slight modulation of H_o with audio frequency so that the electromotive force induced by the nuclei appears after rectification as an audio signal which can be amplified and presented on an oscillograph screen. In the original arrangement of Bloch, Hansen and Packard, two separate coils at right angles to each other and to H_o were used, the one to produce the oscillating field, the other to receive the induced signal. In the system, used by Purcell and his collaborators, the signal appears as a small change in voltage across the same coil, which produces the oscillating field, and is detected through the use of a radio frequency bridge. While it is the same phenomenon which is observed in either case, specific technical advantages may be found in both systems.

Just as in the magnetic resonance method, the establishment of resonance at a frequency $\omega = \omega_o$ in a field H_o determines through (1) the gyromagnetic ratio γ of the nuclei under investigation. A more detailed analysis ([29]) shows that besides the frequency at which it occurs, the magnitude and phase of the nuclear induction signal is also significant, the former leading to the value of the magnetic moment and therefore through γ to the spin, the latter giving the sign of the magnetic moment. Important features in the observation are related to the relaxation time, characteristic for the exchange of energy of a nucleus with its surroundings. It affects not only the magnitude and shape of the signal, but particularly also its line width and thereby the ultimate limit of accuracy in the determination of nuclear moments.

The original investigations were carried out on protons, with Purcell and his collaborators observing the absorption in paraffin, while nuclear induction signals were obtained by Bloch and his collaborators from water with a qualitative change of relaxation time and consequent signal shape observed through the use of a dissolved paramagnetic salt. A far more complete and quantitative study of the various factors, determining relaxation time and signal shape, has recently been concluded by Bloembergen, Purcell and Pound (30) who have also included gases and solids in their investigation. Of their many interesting results we shall mention particulary the observation of extraordinary sharp lines in liquids with a half-width down to as little as 1/70000 of the applied field H_o.

Protons are particulary convenient for nuclear induction because of their high gyromagnetic ratio $\gamma_P = 2.66 \times 10^4$ sec^{-1}/Gauss and their appearance in matter under widely varying physical and chemical conditions; it is for this reason that the basic development of the method was carried out in its application to protons. During the past year it has already been applied to various other nuclei, including those of the heavier elements Li, F (30) and Tb (31). Because of their special significance, we shall restrict the description to the investigations, concerning the isotopes of hydrogen.

Through observation on a small sample of H^2O, containing the isotopes H^1 and H^3 the spin of the triton (H^3) and the sign of its magnetic moment could be determined (32). This was done by comparing the nuclear induction signals from protons and tritons, obtained for two different values of the field H_o but under otherwise identical conditions. The magnitude of the two signals was very closely in proportion to the amount of the respective isotopes in the sample and they had the same sign; this establishes the result that, like the proton, the triton has a spin 1/2 and a positive magnetic moment.

Besides the spin of the triton and the sign of its magnetic moment, the magnitude of the latter was also compared to that of the proton with the highest accuracy, obtained so far in the determination of nuclear moments (33). Both isotopes in the sample were here exposed to the same modulated field H_o; superimposing two frequencies ω_P and ω_T in the radio frequency field, the signals from protons and tritons could be observed simultaneous on the screen. By adjusting ω_P to coincidence of the two signals, the resonance value of H_o for the protons could be ascertained to have the same value as that for the tritons so that the ratio μ_T/μ_P of the moments of proton and triton was here equal to that of their respective resonance frequencies ω_P and ω_T and measurable with high accuracy. It was found to be

$$\mu_T/\mu_P = 1.066636 \pm .00001 \quad .$$

With lower accuracy, the same ratio was simultaneously obtained by other observers (34).

As mentionned in section 2c, a highly accurate measurement of the ratio μ_P/μ_D of the magnetic moments of the proton and the deuteron is of considerable interest in its comparison with the ratio of the corresponding hyperfine structure splittings. The nuclear induction method with superimposed radio frequencies in the same sample has also been applied recently (35) to determine this ratio and a percentage accuracy, similar to that obtained for the triton, has been reached, giving

$$\mu_P/\mu_D = 3.257195 \pm .00002 \quad .$$

With lower accuracy, the same ratio was simultaneously obtained by other observers (31).

4. *Magnetic scattering and resonance depolarization of neutrons.* Already the first measurement of the magnetic moments of the proton and the deuteron (see section 2a) have shown that the neutron gives a negative contribution of about two nuclear magnetons to the latter. There arose thus the problem to ascertain the existence of a magnetic moment for the free neutron and to measure its value.

With the neutrons of thermal velocity, known since the work of Fermi and his collaborators (37), this could be achieved in principle by deflecting a beam of such neutrons in an inhomogeneous field and to determine thus their magnetic moment in analogy to that of protons and deuterons, obtained from the deflection of hydrogen molecules. Such an experiment is rendered difficult through the necessity of working with neutron beams of a very high collimation, comparable to that used for the beams of hydrogen molecules. It has become at all feasible only verry recently with slow neutron reactors as extremely powerful sources, but has not been carried out yet.

The existing experiments are not based upon the deflection but upon the polarization of neutron beams, introduced by Bloch (38), through the process of magnetic scattering. These processes occur in a ferromagnetic substance as a consequence of the magnetic interaction of the atomic moments with the neutron moment and consist in a resulting contribution to the scattering cross section. The total scattering of neutrons by an atom is obtained from the interference of two scattered waves, one originating at the nucleus through nuclear forces acting on the neutron, the other caused by the magnetic interaction of the neutron with the electrons. For thermal neutrons, these two amplitudes are of comparable magnitude with the phase of the magnetically scattered wave depending upon the relative orientation of the neutron moment to the atomic moment. In a magnetized substance, neutrons with opposite orientation of their moments, with respect to the direction of magnetization, have a different scattering cross section, due to the constructive or destructive interference of the two scattered waves. An originally unpolarized neutron beam, after passing through the substance, will thus emerge polarized with neutrons of one orientation of their moment more numerous than those with a moment of opposite orientation. It is one of the consequences of the magnetic scattering that the total transmission of iron, for slow neutrons, should increase upon magnetization if the free neutron has a magnetic moment. This has been experimentally verified by Dunning and is collaborators ([39]), who have also shown that the effect increases rapidly near magnetic saturation. The strong variation of the transmission, in the immediate vicinity of saturation, has been explained by Halpern and Holstein ([40]); quantitative investigations of the phenomenon have first led to the prediction and later to the realization of greatly increased effects, by the use of very strong magnetizing fields ([41]).

While the magnetic scattering proved that the free neutron possesses a magnetic moment, it gave no information about the value of the moment beyond the fact that the magnitude of the observed effect was compatible with a value of the order of the nuclear magneton. The main uncertainty arises from the magnetic field, produced by the atomic moment; a rather detailed knowledge of this field would be required to arrive at more definite conclusions.

In a more promising approach to the problem, magnetic scattering is used merely as an indicating device to detect the action of well-defined external magnetic fields upon neutrons. The first experiment of this type by Frisch, Halban and Koch ([42]), was aimed at a determination of the sign of the neutron moment through its sense of precession in an external magnetic field. A second arrangement by Powers ([43]), very similar in principle to that mentioned at the end of section 2b, to determine the sign of the proton and deuteron moment through non-adiabatic transitions, indicated a negative sign for the magnetic moment of the neutron. In both cases a beam of slow neutrons passes through two plates of magnetized iron. The first plate serves as a polarizer; the second is used as analyser and indicates a change of orientation of the neutron moments, caused by the action of an external static field between the two plates, through a change of the total transmitted intensity of the beam. The geometry of the external fields was chosen so that a neutron of thermal velocity would pass them during a time, comparable to the period of the Larmor precession.

A quantitative determination of the magnetic moment of the neutron was made possible through the suggestion of replacing the rather indefined time scale, based upon the neutron velocity, by the period of an external oscillating field. The fundamental considerations, from which the method was derived by Bloch, are very similar to those which have led Rabi to the magnetic resonance in molecular beam and have been described at the beginning of section 2c. The magnetic moment of a neutron will change its component in the direction of a constant field H_0 if the frequency ω of a relatively weak perpendicular field is close to the value

$$\omega = \omega_0 = \gamma_N H_0 \tag{8}$$

where γ_N is the gyromagnetic ratio of the neutron. While the occurrence of the changes of orientation depends merely upon this frequency, the magnitude of the change varies with the time during which the neutron is exposed to the oscillating field and thereby with its velocity. An originally polarized beam of neutrons, with a rather wide velocity distribution undergoes, therefore, essentially a resonance depolarization at the Larmor frequency and this depolarization results in a change of transmission through a magnetized analyser plate.

This method, to determine the magnetic moment of the free neutron through the observation of resonance depolarization in a neutron beam, has first been applied in an experiment, carried out by Alvarez and Bloch ([44]). The field H_0 was produced in the gap of an electromagnet, placed between polarizer and analyser; the neutron beam passed through a region of the gap where an oscillating current in a coil provided the weak field at right angles. The counting rate of α-particles, in a BF_3 filled ionization chamber behind the analyzer, gave the measure for the total transmitted intensity of the beam.

With fixed frequency of the oscillating field, the constant field H_o was varied and the value H_o^*, at which resonance depolarization occured, was ascertained through a drop in intensity at the detecting ionization chamber. Although the change of intensity at resonance amounted only to 1.5% of the total intensity, it was possible to clearly establish the effect for different frequencies and proportionally different values of the resonance field

Assuming the spin of the neutron to be 1/2, the magnetic moment μ_N of the neutron was thus measured in terms of the values H_o and ω for resonance field and frequency. Indeed, one has then

$$\gamma_N = \frac{2\,\mu_N}{\hbar} \tag{9}$$

and from (8)

$$\mu_N = \frac{\hbar\,\omega}{2\,H_o^*} \tag{10}$$

One of the principal limitations in accuracy was again set (see section 2c) by the measurement of the field value H_o^*. In order to check the result, obtained with a flip coil, an independent method was used where the field H_o was compared to the field H_c in a cyclotron, accelerating protons with a frequency ω_c. With the cyclotron relation

$$\frac{e}{Mc} = \frac{\omega_c}{H_c} \tag{11}$$

one obtains by division of (10) by (11)

$$\mu_N = \mu_n \frac{\omega}{\omega_c} \frac{H_c}{H_o^*} \tag{12}$$

where $\mu_n = \dfrac{e\hbar}{2\,M_c}$ is the nuclear magneton. While the determination of μ_N from (10) requires the absolute measurement of a frequency and a magnetic field, the value obtained from (12) requires only the relative measurement of two frequencies and two fields. Although this latter method served merely as a check on the former, without increase of accuracy, it has the remarkable advantage of giving the neutron moment directly in units of the nuclear magneton.

Taking the negative sign, the neutron moment was thus determined to be $\mu_N = -1.93 \pm .02$, a value which, within the errors, is compatible with the previously determined values μ_P and μ_D for the moments of deuteron and proton and the simple law of additivity $\mu_D = \mu_P + \mu_N$.

A more accurate check of this relation was desirable in view of its connection with the quadrupole moment of the deuteron. (See section 2c and Chapter III). With μ_P and μ_D and particularly their ratio already known with sufficient accuracy, this required a more precise determination of μ_N; such a determination has just been completed by Bloch, Nicodemus and Staub ([45]). While the essential features of the arrangement are the same as in the original experiment of Alvarez and Bloch, a much higher accuracy was made possible through two major improvements:

When nuclear induction was introduced, its use for the determination of the neutron moment was immediately suggested ([25]). It has been stated before that the previous measurement of the neutron moment was seriously limited through inaccuracies in the determination of the resonance field. As a first and most essential innovation, this limitation was completely avoided by applying nuclear induction as an ideal field-stabiliting device. Through observation of the proton resonance in a small sample of water, at different positions in the gap, it was ascertained first, that within 1/2 Gauss the field H_o of approximately 10000 Gauss had the same value, not only throughout the region in which the neutron beam was exposed to the oscillating field, but also at the final position of the sample just outside of the beam. With the frequency ω_P for the protons fixed, the induction signal in this final position was then used to control electronically the field H_o so as to hold it constantly during the measurements of the neutron intensity as its resonance value, corresponding to ω_P so that one has automatically

$$H_o = \frac{\omega_P}{\gamma_P} \tag{13}$$

with γ_P denoting the gyromagnetic ratio of the proton. The frequency of the oscillating field, acting on the neutrons in the gap, was varied until that value ω_N was reached at which resonance depolarizarion of the neutrons was manifested by a maximum drop in intensity at the detector. At this value one has then

$$H_o = \frac{\omega_N}{\gamma_N} \tag{14}$$

with γ_N as the gyromagnetic ratio of the neutron and by division of (13) and (14)

$$\gamma_N/\gamma_P = \omega_N/\omega_P \quad . \tag{15}$$

With the equal value 1/2 for the spin of the proton and the neutron, their gyromagnetic ratios are proportional to the magnetic moments so that one has also

$$\mu_N/\mu_P = \omega_N/\omega_P \quad . \tag{16}$$

By measuring merely the ratio of the two frequencies ω_N and ω_P, which can be achieved with very high accuracy, one obtains thus the magnetic moment of the neutron in terms of the proton moment.

As a second improvement over the previous measurement, the magnetization in the polarizing and analyzing plate of iron was held extremely close to saturation, resulting in a greatly increased transmission effect ([41]). Instead of the earlier observed intensity drop at resonance of merely 1.5%, resonance polarization could now be established by effects of as much as 12%. This had not only the advantage that the maximum in the resonance depolarization could be established with higher precision, but it also permitted a quantitative and complete investigation of the effect in its dependence on amplitude and frequency of the oscillating field. With this investigation, necessary for the reliability of the result, the ratio of the magnetic moments of neutron and proton was thus found to be

$$\frac{\mu_N}{\mu_P} = 0.68494 \pm .00007 \quad .$$

Adding this figure to the value $\dfrac{\mu_D}{\mu_P} = 0.307013 \pm .000002$, obtained from the ratio of the proton and deuteron moments (section 3), one observes a significant deviation from unity by 8.05 ± 07 permille as a measure for the non-additivity of the constituent moments in the deuteron. (See Chapter III.) While these measurements were in progress, other observers ([45]) have also used nuclear induction to determine this ratio and have obtained the same result with lower accuracy.

***

Before concluding this description of the various methods by which nuclear moments have been determined, we shall briefly discuss the characteristic ultimate limitations in accuracy which are inherent in these methods. Although instrumental difficulties are often of more serious practical concern and may be prohibitive in reaching this ultimate limit, they shall be considered as incidental. As a common basis, the characteristic inaccuracy shall be measured through the inaccuracy $\Delta\ \nu$ of a frequency, particularly since most methods are based directly upon frequency measurements.

In the spectroscopic measurements of hyperfine structure (section 1), the finite width of spectral lines, either due to their natural width or to Doppler effect, leads to an inaccuracy $\Delta\nu$ of the observed optical frequency. While this may not amount to more than one part in 10^9, it must not be compared with the optical frequency itself but with the frequency differences in the hyperfine structure which are about 10^6 times smaller, leading thus to a percentage accuray of the order of one permille for the splitting of hyperfine structure levels. It is true that the optical determination of nuclear moments does not suffer seriously from this limitation, since greater errors are usually involved in their calculation.

The measurement of nuclear moments and hyperfine structures by the deflection of molecular and atomic beams (sections 2a, 2b) is not directly based upon a frequency measurement but, through the deflection force, rather to that of an energy E of orientation in an external field. Through the relation $E = h\nu$ it is, however, connected with a precession frequency ν of the observed moments; in molecular beams it is that of the nuclei itself, while in atomic beams it is the precession of the atomic moment in the external field. The ultimate inaccuracy $\Delta\nu$, in the determination of this frequency, is given by the time t during which the precession can take place while the beam particles passes through the apparatus and is of the order $\Delta\nu = \frac{1}{t}$.

The same limit in accuracy appears and has actually been reached in the direct frequency measurements used in the magnetic resonance method (Section 2c), and the resonance depolarization of neutron beams (section 4). With the precession frequency of nuclei and neutrons in normal fields of the order of $\nu = 10^7$ cycles/sec. and passage times for thermal velocities and normal dimensions of the apparatus of the order of $t = 10^{-3}$ seconds this leads to a limiting relative accuracy of the order $\frac{\Delta\nu}{\nu} = \frac{1}{t\nu} = 10^{-4}$.

Although nuclear induction experiments lead to frequencies of the same order, the limit with which they can be determined is of rather different origin. The nuclei remain in the sample and can therefore be subjected to observation during an unlimited time. The ultime limitation is here provided by the finite relaxation time and manifested in the natural width of the observed signals. One obtains thus for the limiting relative accuray again $\frac{\Delta\nu}{\nu} = \frac{1}{\nu t}$ where t stands here, however, for a time of the order of the relaxation time. Relaxation times have been observed to be as large as several seconds; with frequencies of the order of 10^7 cycles/sec. this would lead to a limiting accuracy of 10^{-7}. In actual measurements the highest accuracy, obtained so far, is of the order 10^{-5}.

It should be added to the previous discussion that the mentioned accuracies do not always represent the ultimate limit lent that a good experimental and theoretical knowledge of the line shape often allows to go beyond these limits. However, the difficulties which are met in practice, when ascertaining details within the line width, usually prevent a very large improvement of accuracy by this procedure.

III.

RESULTS FOR THE LIGHTEST NUCLEI

Before entering the discussion of the experimental results obtained for the neutron and the isotopes of hydrogen, we shall summarize them in the following table.

	I	$\|\mu\|/\mu_P$	Σ	μ/μ_n	$Q/10^{-27}$ cm^2	$\Delta\nu/10^6$ sec^{-1}
P	(47) (12) 1/2	1	(12) +	(23) 2.7928 $\pm$.0008		(24) 1420.410 $\pm$.006
N	(1/2)	(45) 0.68494 $\pm$.00007	(43) —	—1.9129 $\pm$ .0006		
D	(48) 1	(35) 0.30713 $\pm$.000002	(12) +	0.85744 $\pm$.00025	(20) (21) 2.73 $\pm$.05	(24) 327.384 $\pm$.003
T	(32) 1/2	(33) 1.066636 $\pm$.00001	(32) +	2.9789 $\pm$.0008		

The rows refer to the different nuclei with :

P = proton (H^1); N = neutron; D = deuteron (H^2); T = triton (H^3).

In the columns we have listed the corresponding values of :

1. I = spin = angular momentum in units $\hbar$.
2. $|\mu|/\mu_P$ = magnitude of the magnetic moment in units of the magnetic moment of the proton μ_P.
3. Σ = sign of the magnetic moment.
4. μ/μ_n = magnetic moment μ in units of the nuclear magneton μ_n where μ_P/μ_n (23) is used as a standard.
5. Q = quandrupole moment, defined as the charge weighted average of $3z^2 - r^2$, measured from the center of gravity.
6. $\Delta\nu$ = frequency difference between the two hyperfine structure levels in the ground state of the atom.

The work from which the various data have been obtained is indicated by the corresponding references given in the list of publications. For the numerical values we have used those with the highest present accuracy [1].

The neutron spin $I_N = 1/2$ has been given in brackets, since its value is based on indirect conclusions rather than on a direct measurement. It is only this measurement and that of the frequency difference $\Delta\nu_T$, for the hyperfine structure of the H^3 atom, which are missing to complete the table. In connection with these data for the neutron and the isotopes of hydrogen, corresponding and similarly accurate information for He^3 would also be of interest and should be forthcoming soon [2].

The discussion of the experimental results leads under various angles to problems of considerable basic significance. Many of the questions are directly connected with the inherent difficulties in the

[1] In the column for μ/μ_n the value for the proton, given in reference (23) as 2.7896 ± .0008, has been corrected by the factor 1.001 16, taking into account the changed value of the magnetic moment of the electron (see footnote on p. 00). With the fixed ratios, given in the previous column for $|\mu|/\mu_P$, this same correction is consequently taken into account in the figures, giving μ/μ_n for the other nuclei.

[2] Some results, concerning the magnetic moment of He^3 have since been reported by H.L. Anderson and A. Novick (*Phys. Rev.*, **73**, p. 919, 1948). Assuming the spin value 1/2 for He^3, their data give

$$|\mu He^3|/\mu_P = 0.763 \pm .007.$$

Choosing further the negative sign, one obtains then

$$\mu He^3/\mu_n = 2.131 \pm .02.$$

Neither the sign nor the spin of He^3 have, however, been ascertained yet.

present theory of elementary particles and cannot yet be answered satisfactorily. Noteworthy attempts have been made, nevertheless, to interpret the existing data and they will be briefly discussed.

1. *Magnetic moments of the proton and the neutron.* The most outstanding information which has been gained from the investigation of nuclear moments is still contained in the result that the neutron has a finite magnetic moment and that the pro on moment differs from the nuclear magneton. It clearly signifies that neither of the two particles is of the same nature as the electron, which has been successfully described by the famous relativistic wave equation of Dirac. The same theory, applied to the neutron and the proton, would give $\mu_N = 0$; $\mu_P = \mu_n$ and it seems plausible that the different values, which have been observed for both moments, are of common origin. Using the values for $|\mu_N|/\mu_P$ and μ_P/μ_n, given in the table one finds,

$$\begin{aligned} \frac{\mu_P + \mu_N}{\mu_n} &= (1 - |\mu_N|/\mu_P)\,(\mu_P/\mu_n) \qquad (17) \\ &= 0.8799 \pm .0003 \end{aligned}$$

The fact that this number is close to unity suggests that the mechanism, which provides the magnetic moment of the neutron, adds, in the case of the proton, an approximately equal and opposite excess to the magnetic moment μ_n which one would expect to find as a direct consequence of relativity. Frohlich, Heitler and Kemmer ([54]) have shown that such a mechanism is rather naturally found if one accepts Yukawa's hypothesis of the meson field for the nuclear forces. Both the negative neutron moment and the positive excess moment of the proton are here explained by the magnetic moment of negative and positive mesons respectively, which exist with a finite probability within the range of nuclear forces from the heavy particle. The calculations are based upon a weak coupling between the heavy particle and the meson field and the relatively small difference of the value ([17]) from unity appears here as an effect, due to the small but finite ratio of the masses of meson and heavy particles. Pauli and Dancoff ([50]) have shown, however, that in a theory with strong coupling the magnetic moments of proton and neutron would appear as equal and opposite so that a vanishing result would be obtained instead of the value (17); the fact that this actual value lies between zero and unity might thus call for a theory with intermediate coupling.

The typical failure of present field theories manifests itself in the calculations through a divergent result for the magnetic moment, due to the meson field. With these divergencies removed through rather artificial procedures, the actual value of these theories must be seen in their suggestive qualities rather than in their numerical accuracy.

2. *Magnetic moment and quadrupole moment of the deuteron.* Another interesting feature of the value (17) is found in the fact that it is very closely equal to the magnetic moment of the deuteron, measured in units μ_n. Using here also the value $|\mu_D|/\mu_P$ from the table, one has

$$\frac{\mu_P + \mu_N - \mu_D}{\mu_n} = \left(1 - \frac{|\mu_N|}{\mu_P} - \frac{|\mu_D|}{\mu_P}\right)\frac{\mu_P}{\mu_n} = .0225 \pm .0002. \quad (18)$$

A vanishing result instead of this small number would indicate that the magnetic moments of the proton and neutron are additive in the deuteron and the first approximate data for the neutron moment (see II, section 2a) were actually derived under this assumption. The value (18) represents the measure of a small but well established deviation from additivity which can be ascribed to various causes.

A first and, in fact, very accurate prediction of the non-addivity has been given by Rarita and Schwinger (51) in connection with the quadrupole moment Q_D of the deuteron. Although the observed finite value of Q_D is compatible with the $I_D = 1$ of the deuteron, it would vanish, if the orbital angular momentum of the proton and the neutron were zero, i. e., if the ground state of the deuteron were a pure S—state. Such a vanishing result would not be accidental but it would necessarily follow if the interaction potential between proton and neutron were spherically symmetrical. Through the assumption of a spin dependent spherical symmetry, it is found, however, that the ground state of the deuteron is not a pure S-state but rather a mixture of an S- and a D-state. With the simplest assumptions about the form of the interaction, Rarita and Schwinger were able, from the observed value of Q_D, to estimate the relative strength of the spin-dependent part.

Through the orbital contribution to the magnetic moment by the D- state, there enters a correction to the total magnetic moment of the deuteron. It is directly related to the probability of finding the deuteron in the D-state. With the form of interaction, which leads to the observed value of Q_D, this probability was found to be 0.039; introducing the observed values for μ_P and μ_N one obtains then from the theory of Rarita and Schwinger $(\mu_P + \mu_N - \mu)_D/\mu_n = .022$. This excellent agreement with the observed value (18) is, however, largely accidental; it could be easily spoiled by altering the simple but implausible assumption that the radial dependence of the interaction energy has the form of a square well.

The agreement seems even more accidental if one considers that another major cause of non-additivity has been entirely omitted in the calculations of Schwinger and Rarita. It has been pointed out by several authors ([52]) that relativistic effects, due to the motion of the constituents in the deuteron, could likewise cause deviation from additivity in the same direction and of comparable magnitude as those introduced through the mere presence of the D-state. The estimates vary widely, depending on the different assumptions about the type of interaction between the proton and the neutron. While no definite answer can thus be given at the present state of the theory, it is not justified either to overlook these relativistic corrections.

Finally, deviations from additivity for the magnetic moments can appear for a third independent reason. If one assumes (see III, section 1) that the excess moment of the proton and the moment of the neutron are due to the meson field, one is led to the possibility that the modification of this field, which causes their interaction, affects also, the resultant magnetic moment of proton and neutron in the deuteron. The existing theories contain indeed features which lead in general to a modification of the intrinsic moments through that of the meson field ([53]). In the special case of the deuteron, however, this mechanism contributes no correction in virtue of the particular symmetry properties with respect to an interchange of proton and neutron.

It is evident that the present knowledge of nuclear forces is insufficient to give a satisfactory explanation of the observed value (18). With its considerable accuracy this experimental number may serve as an important and rather severe test for future theories.

3. *Magnetic moment of the triton.* Considerations, similar to those for the deuteron, have also been applied to the magnetic moment of the triton (H^3). As a three body problem it presents even greater difficulties, particulary since the spin value $I_T = 1/2$ precludes here the existence of a quadrupole moment and thereby

information about the presence of higher angular momenta. Assuming a pure S-state, in which the two neutron moments cancel each other because of the exclusion principle and neglecting all other corrections, one is led to the expectation that the magnetic moment μ_T of the triton is equal to the proton moment μ_p. Sachs and Schwinger ([54]) have concluded that under simple and plausible assumptions the ground state of the triton should consist of a combination of a ^{2}S- and ^{4}D-state and that the magnetic moment μ_T should be smaller then the proton moment μ_p. With an estimated probability for the ^{4}D-state, they predicted

$$\mu_T = 2.71\ \mu_n, \tag{19}$$

a value wich is 3.5 percent smaller than the proton moment μ_T.

In striking contrast to this prediction, the observed value of μ_T (see table) has been found to be about 6.7 percent larger than μ_p. Efforts have been made to explain this discrepancy through the assumption that besides the ^{2}S- and ^{4}D-state there are also ^{2}P- and ^{4}P—states present in the triton ([55]). The conditions under which interference between ^{2}S and ^{2}P and between ^{4}P and ^{4}D would give the desired result are, however, too artificial to make the argument convincing.

A more natural explanation has been offered by Villars ([56]) who pointed out that the symmetry conditions which prevent modifications of the intrinsic moments of the proton and neutron in the meson field theory of the deuteron, are absent in the case of the triton. Whereas a substitution of the proton by the neutron and vice versa reproduces the ground state of the deuteron so that the contributions from positive and negative mesons cancel each other, this same substitution would lead from the triton to the different nucleus He^3. As a consequence, there remains a finite correction; the fact that it is quite naturally obtained in the right direction and of the observed order of magnitude makes it seem plausible that the observed triton moment offers indeed the first evidence for a change of the intrinsic moments of nuclear constituents.

With the inverted role of protons and neutrons in the nucleus He^3, the same correction with the opposite sign, should here be applied to the neutron moment, and the observation of the magnetic moment of He^3 would serve as a valuable test (*).

4. *Hyperfine structure and magnetic moments of H^1 and H^2.* The highly accurate data which now exist for the hyperfine structure frequencies $(\Delta\nu)_P$, $(\Delta\nu)_D$ and for the magnetic moments μ_P, μ_D of light and heavy hydrogen allow an interesting comparison ([24]).

In the first place, it should be possible to calculate the hyperfine structure splitting from the well known eigenfunction of the electron in the ground state of the hydrogen atom, and from the spin and magnetic moment of the nucleus. Using the formula of Fermi ([10]) and the original value for μ_P and μ_D, obtained by Millman and Kusch (23), one finds

$$(\Delta\nu)_P = (1416.9 \pm .5)10^6/\text{sec}^{-1} \qquad (20)$$
$$(\Delta\nu)_D = (\ 326.5 \pm .1)10^6/\text{sec}^{-1}$$

for the frequency difference of two hyperfine structure levels in light and heavy hydrogen respectively. The observed values are both about $1.0026 \pm .0005$ times larger than the calculated values (20) and this descrepancy is well outside the error involved in the natural constants which enter in the calculation.

The fact that approximately the same percentage correction must be applied to obtain the actual hyperfine structure for both isotopes suggests immediately that the discrepancy is not primarily of nuclear origin but due to common features concerning the electron which have not been taken into account in the applied formula. Other

(*) The results for He^3, mentioned in the footnote on page 27 seem indeed to confirm the prediction of Villars. Assuming the spin of He^3 to be 1/2 and with the negative sign of the moment, one has

$$(\mu He^3 - \mu_N)/\mu_n = -0.22 + .02.$$

Using the values, given on the table of page 26, one has on the other hand

$$(\mu_T - \mu_P)/\mu_n = +0.18610 + .00005.$$

These two numbers would have to be exactly equal and opposite if the deviation of μ_P from μ_T and of μHe^3 from μ_N were entirely due to meson effects. The difference of $.04 + .02$ in their magnitude must be interpreted, according to E. Gerjnoy and J. Schwinger (*Phys. Rev.*, **61**, p. 138, 1942) as indicating a small lent finite orbital correction. With the simplest assumption, originally made by Sachs and Schwinger which allows only for the presence of a ^2S and ^4D state, the probability for the latter can be derived from the above mentioned difference (H. L. Anderson, *Phys. Rev.*, **73**, p. 919, 1948) to be approximately 4 percent in satisfactory agreement with the theoretical expectations. The contribution due to the pure meson effect amounts then to about 0.27 μ_n with the positive and negative sign for H^3 and He^3 respectively.

discrepancies from the accepted theory of the hydrogen atom have been found in an important experiment by Lamb and Retherford [57] who observed the fine structure of the first excited state. They have been related by Bethe [58] to the interaction of the electron with the radiation field, and Schwinger [59] has recently pointed out that this interaction modifies also the effective magnetic moment of the electron and thereby the hyperfine structure splitting in qualitative agreement with the observation (*).

There remains, however, another discrepancy which is apparently not related to the one just mentioned and which has not been explained yet. Except for the small difference in the effective mass of the electron, which has been taken into account in calculating the values (20), the orbital motion of the electron in light and heavy hydrogen is the same. Any common correction, concerning the electron, should therefore cancel out to very high order in forming the ratio $\Delta\nu_D/\Delta\nu_P$ for the hyperfine structures of the two isotopes. With the reduced mass m_1 and m_2 of the electron for H^1 and H^2 respectively, and with the observed value for the ratio μ_P/μ_D of the magnetic moments, one obtains

$$(\Delta\nu_P/\Delta\nu_D)_{\text{calc.}} = \frac{2I_P+1}{I_P} \cdot \frac{I_D}{2I_D+1}\left(\frac{m_1}{m_2}\right)^3 \frac{\mu_P}{\mu_D} = 4.33937 \pm .00003 \qquad (21)$$

The observed ration is instead

$$(\Delta\nu_P/\Delta\nu_D)_{\text{obs.}} = 4.33867 \pm .00004 \qquad (22)$$

i. e. $1.00016 \pm .00001$ times smaller than the calculated value (21). It has been suggested by Halpern [60] that relativistic effects may modify the appearance of the effective masses m_1 and m_2 in (21) and, in particular, that the third power of their ratio should be replaced by the exponent 3/2; such a correction, however, would not explain the discrepancy but would even accentuate it, increasing the calculated value to 1.0005 times the observed one. However,

(*) The correction factor 1.00116, mentioned in the footnote on page 15, appears here quadratically, since the expression for the hyperfinestructure separation contains the product of the magnetic moment μ_e of the electron and that of the nucleus and since the calibration by Millman and Kusch gives the latter likewise in terms of μ_e. One obtains thus a correction factor 1.0023 to the values (20) for the hyperfinestructure separation and sufficient to explain most of the experimentally observed discrepancy.

Breit and Meyerott [61] have given arguments which show that any such correction would actually be of much higher order and, within the experimental error, would not affect the result (21).

The discrepancy between the two values (21) and (22) may call for short range interaction between electrons and nuclei of hitherto unknown origin. Assuming that it can be described by an energy of the form

$$U(r) = J(r) + K(r) \cos(\vec{I}\vec{S}) \quad , \tag{23}$$

one would be lead, through the second term, to a correction to the hyperfine structure splitting which could be entirely different for two nuclei of different structure like the proton and the deuteron and not merely proportional to their magnetic moments.

$\vec{I}$ and $\vec{S}$ stand here for the spin of nucleus and electron respectively, and r for the distance of the electron from the center of the nucleus. To explain the order of the observed discrepancy of 1.6×10^{-4} one would have to assume for the magnitude of the energy K (r)

$$|K| \cong 1.6 \times 10^{-4} . h\Delta\nu \left(\frac{a_B}{\rho}\right)^3 \cong 10^{-5} \text{ volts} \tag{24}$$

where $\Delta\nu$ is of the order 10^9 sec^{-1} of the observed splitting, a_B the Bohr radius and ρ a distance over which K is essentially different from zero and which has been assumed to be $\rho = 2 \times 10^{-13}$ cm., i. e., of the order of the range of ordinary nuclaer forces.

An interaction, similar to (23) has been considered for the interaction of electrons and neutrons and experiments have been performed to investigate the influence of the first term J (r) in (23) upon the scattering cross section of slow neutrons [62]. The results allow merely the conclusion that, if at all different from zero, the magnitude, of J is less than 5000 Volts and an estimate of the electrostatic interaction between the electron and the meson field [63] has lead to a magnitude which is even 100 times less than this upper limit of the energy J. The experiments give no information, concerning the second term in (23), but its presence might be revealed through the observation of the angle dependence in the magnetic scattering of slow neutrons [64]. To conclude from the present evidence of the hyperfine structures that there exists such a new type of spin dependent interaction seems premature until it is shown beyond doubt that the small correction cannot be due to more familiar causes (*).

***

It may be seen from the previous discussion that the results on the moments of the lightest nuclei are directly connected with several fundamental problems concerning the nature of elementary particles. While the interpretation of these results is certainly still far from being satisfactory, they represent an important quantitative basis for further developments.

(*) A satisfactory explanation has recently been given by Dr. Aage Bohr (reported at the meeting of the *Ann. Phys. Soc.*, May 29-31, 1948). He has pointed out that the finite radius of the deuteron has a small effect upon the motion of the electron which must be taken into account and leads to a more rigorous expression for the hyperfinestructure separation of H^2. The correction has the proper sign and magnitude to account for the discrepancy in the two ratios (21) and (22).

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Discussion du rapport de M. Bloch

Mr. Heitler. — If you take into account the Schwinger correction for the magnetic moment of the electron, there is still a discrepancy with the theoretical fine structure and experimental value as observed by Rabi 1 : 0.9997 $\pm$?.

Mr. Oppenheimer. — How sure is the fact that the fine structure constant that appears in the square is well known.

Mr. Heitler. — There is a similar discrepancy, but in the opposite direction for the deuteron, and that effect has been explained successfully by Aage Bohr. If now the Bohr-effect is taken into account there is still an unexplained discrepancy left for the deuteron and that is of the same sign and magnitude as for the proton. What this discrepancy is exactly due to we do not know, but eventually the finiteness of the cloud of the proton must come in. This correction, calculated with the meson theory appears with the right sign, but is numerically too small. If the observed discrepancy should be explained by the finite size of the proton we should assume for the proton and neutron a size twice as great as the size of the deuteron which is unreasonable. Another explanation is that suggested by Oppenheimer, namely errors in the estimation of the fundamental constants. Another possibility is that radiation effects which would be the same for the deuteron and for hydrogen.

Mr. Oppenheimer. — The magnetic moment is defined usually in an homogeneous field. There are corrections of 1 % on the Schwinger effect in an inhomogeneous field.

M. Teller. — I think that the explanation proposed by A. Bohr is not so sure, and would propose another effect.

Mr. Bhabha. — Teller was referring to the Coulomb field of the meson, but I am refering to the Coulomb interaction between two protons and I think that this effect would be larger.

Mr. Oppeneimer. — I think that it should not be effect.
(cf 2S states of the nucleous.)

Mr. Peierls. — For the magnetic moment of the deuteron calculated by Rarita and Schwinger, it is not surprising this calculated value is so close to experimental moment since the important contribution of the D state is given by the part of the wave fonction outside the range of nuclear forces and is thus very insensitive to the model of nuclear forces, depending only on a mean range (cf Hepner and Peierls, *Proc. Roy. Soc.* A. 181, p. 43, 1942).

Mr. Bloch. — The calculations of the relativistic effect have been made with assumption of surral mesons or also with a square well, could you say if the predicted magnetic field moment is affected by 1% or 10%.

Mr. Peierls. — It is rather insensitive, but I cannot give a precise limit. The uncertainty in the correction is more likely to be 10% than 1%.

M. Casimir. — The electric quadrupole moment of the deuteron also manifests itself in microwave absorption spectra, especially in the case of ammonia. How accurately can the value be derived from those spectra or is the evaluation of the electronic wave functions too difficult?

Mr. Teller. — It is one very nice case, that of alcalies, but that does not apply here.

Mr. Casimir. — In polar crystals containing deuterium the nuclear induction spectra will presumably show a fine structure. Has this question been studied. Although it is doubtful whether cristalline field can be calculated with a higher degree of accuracy than the field in a hydrogen molecule. Such experiments would certainly be interesting.

Mr. Bloch. — It has not been done for lattices containing deuterium.

Mr. Casimir. — The electrical quadrupole moment has also an influence on relaxation phenomena. An interesting case is the ortho-para conversion of deuterium dissolved in heavy water. Unfortunately the calculations by which I tried some years ago to arrive at quantitative conclusions where shown by Hammermesh to contain a number of errors and to be too naive. A very rough order of magnitude is all one can hope to find from such methods.

Mr. Bloch. — This relaxation effect because of the quadrupol moment has also been observed in deuterium and there was found by Pound a relaxation time not shorter than for pure water.

Mr. Teller. — I don't think that in more complicated molecules or in lattice it is easy to find exact relationships between the inhomogeneity of the electric field and other quantities that one can measure. With the use of high speed computing equipments it is possible to go much further in the calculations.

Oppenheimer proposed a very nice method based on the following idea. In a formal way you replace in the time-dependent Schrödinger equation $\frac{\hbar}{i}\frac{\partial}{\partial t}$ by $\hbar\frac{\partial}{\partial t}$. You obtain so a diffusion equation, and when you follow this equation sufficiently you get a space dependence which treated in the right way gives the lowest wave function.

Now the diffusion equation can be treated by a statistical method in which you go back from a differential method to the theory of a particle going along and making collisions in a statistical manner. The elementary steps are very simple and are just what the high speed computing machines can do so very well.

Mr. Casimir. — It is interesting that in the course of the development of this field of physics the idea that there exist a specific non-electromagnetic interaction between electrons and protons has frequently been introduced. However so far a more accurate theoritical analysis has always shown that one could dispense with such an interaction term. Appearent discrepancies in hyperfine structure spectra were explained by Fermi and Segre, in the case of internal conversion, the introduction of quadrupole and magnetic dipole and octupole radiation removed the difficulties. Also in the case of the hyperfine structure of hydrogen the influence of such non-electromagnetic interaction terms is certainly smaller than was at first suspected by Rabi. It remains to be seen, whether we have now finally arrived at a case where the non-electromagnetic terms really enter.

Mr. Bloch. — If such a specific interaction exists between neutrons and electrons it would manifest itself in the magnetic scattering of neutrons by electrons. It turns out that the azimuthal dependence of the scattered electron would be essentially different when such a spin dependance exists. The experiments have not been feasible until the use of very fine collimators and intensive beams from piles. It is possible now but the effect is probably very small.

Non-relativistic Quantum-Electrodynamics and correspondence Principle

by H. A. Kramers

I.

INTRODUCTION

In Lorentz'theory of electrons we start by introducing some hypothesis regarding the structure of the electron; at some later stage it is then found that many features of the interaction between electrons and radiation field involve only two constants characterizing the electron, viz. its charge e and its mass m. This result is the fundament for the numerous classical applications of electron theory to actual problems and experiments. In all of these the electronic structure plays no explicit part; only e and m appear in the formulae and m which in the theory appears as the sum of the inertial and the electromagnetic mass, may therefore also be denoted as the experimental mass. It is of importance A : to find the simplest way in which the structure-independent part of the assertions of the electron theory can be mathematically expressed, and B : to bring these expressions in the form of canonical equations of motion. In fact, we will thereby have obtained a trustworthy basis for a quantum-theory of interaction between charged particles and field, i. e. for a quantum theory which shows the necessary correspondence with that part of the classical electron theory which is of physical importance.

In literature, the structure-independent assertions of the electron theory are usually presented in a way which involves the application of retarded potentials (compare the well known expression $2e^2/3c^3.\dddot{x}$ for the radiation reaction), whereas for the solution of problem A mentioned above it is preferable that no notions should be introduced which conceal the symmetry of time. This requirement can be fulfilled, however, in a natural way if we introduce the « proper » field of an electron in the way in which it was done in my monograph on quantum theory [1] and in my paper delivered at the Galvani congress in Bologna 1937 [2]. A definite solution of problem A has been proposed in these papers (in non-relativistic approximation), but the equations of motion were only discussed in some detail as far as the field equations are concerned

The solution in question has been taken as a basis for two papers of 1941. In the first of these, Mr. Serpe [3] shows how it can be used for correcting certain points in the quantum theory of a harmonic oscillator emitting a light quantum. In the second Mr. Opechowski [4] succeeds in establishing to a certain approximation a Hamiltonian for the equations of motion, i. e. he gives an approximate solution for problem B mentioned above.

In this report we will present an approximate treatment of A and B, i. e. of the problem of structure elimination and discuss, on this basis, certain problems of quantum electrodynamics. The approximations involved refer first of all to the use of Lorentz' old model of the rigid electron. Our treatment will therefore be non-relativistic in this sense that our formulae will only be valid in the region where the velocities of the electron are small compared to the velocity of light. A relativistic treatment would indeed hardly seem possible or promising : on the one hand there exists no relativistic classical electron theory which provides us with a precise and simple model (*) of the contractile electron and its interaction with the radiation field, on the other hand the spin property of the electron and Dirac's theory of 1928 warn us that the requirements of relativity are met in nature in a way which is hardly properly reflected in any classical theory. Another approximation in our treatment will be that we restrict ourselves mostly to the interaction of the electron with the electric dipole radiation only; this means for instance the neglect of radiation pressure on the electron, which may be said to be due to the magnetic dipole radiation and which in quantum theory is reflected in the recoil of the electron from the incident and emitted light quanta.

In several respects our treatment will show similarity with a well known paper by Bloch and Nordsieck [5]. There, the « bound » light quanta carried along by the moving electron were eliminated by a proper transformation; they are practically identical with what we call the proper field of the electron. It will be seen, however,

(*) The only precise model I know of would necessitate the conception of the electron as a relativistic elastic body along the lines of Herglotz' theory of 1911, which would endow the electron with an infinite number of degrees of freedom.

that our discussion and results move along lines rather different from those followed by the mentioned authors.

It would seem as if our treatment first of all allows an analysis of emission, absorption and scattering of light which supplements that hitherto presented in literature. In the second place it also throws some light on the problem of the divergencies in quantum electrodynamics and on the questions connected with the Lamb-shift, although of course these problems, for their exact treatment require an analysis which goes beyond the approximations which we have imposed upon ourselves.

II.

ELIMINATION OF THE STRUCTURE FROM THE EQUATIONS OF MOTION

Consider one electron in the E, H field, the latter satisfying everywhere the Lorentz equations (light velocity = 1) :

$$\mathrm{rot}\,\overline{\mathrm{H}} = 4\,\pi\,\overline{j} + \dot{\overline{\mathrm{E}}}\,, \qquad \mathrm{div}\,\overline{\mathrm{H}} = 0$$
$$\mathrm{rot}\,\overline{\mathrm{E}} = -\,\dot{\overline{\mathrm{H}}} \quad , \qquad \mathrm{div}\,\overline{\mathrm{E}} = 4\,\pi\,\rho$$

Take for the electron a rigid body of mass m_0 and charge e. The mass distribution has central symmetry and extends over a region the linear dimensions of which are of the order of magnitude a (« electron radius »). If the wave lengths caracterizing the fields which affect the electron are large compared to a, a structure-independent behaviour of the electron can be expected if its velocity stays small compared to that of light, and we may also assume that the electron does not rotate.

The electron equations of motion are

$$m_0\,\ddot{\overline{\mathrm{R}}} = e\,\overline{\overline{\mathrm{E}}} + e\,\dot{\overline{\mathrm{R}}} \wedge \overline{\overline{\mathrm{H}}} - \frac{\partial\,\mathrm{U}}{\partial\,\overline{\mathrm{R}}} \tag{1}$$

$\overline{\mathrm{R}}$ denotes the position of the electron centre and U the potential energy of the electron in a fixed field of force such as might arise from a fixed distribution of electric charges. If the latter is the case, our E does not include the field strenght due to these charges. The symbol $\widetilde{Q}$ is a mean value explained by

$$e\,\widetilde{Q} = \int Q\,\rho\,dV$$

With our model we have

$$\dot{\vec{j}} = \rho\,\dot{\overline{R}}$$

We introduce field potentials by

$$\overline{H} = \mathrm{rot}\,\overline{A}, \qquad \overline{E} = -\nabla\,\varphi - \dot{\overline{A}}, \qquad \mathrm{div}\,\overline{A} = 0$$

Thus

$$\Delta\,\varphi = -4\,\pi\,\rho$$

$$\Delta\,\overline{A} - \ddot{\overline{A}} = \square\,\overline{A} - = 4\,\pi Tr\,\overline{j} = -4\,\pi Tr\,\rho\,\dot{\overline{R}} = -4\,\pi\,\rho\,\dot{\overline{R}} + \nabla\,\dot{\varphi} \quad (2)$$

where $Tr\,\overline{B}$ stands as a symbol for the transversal or solenoidal part of a vector field $\overline{B}$.

The Coulomb potential φ is given by

$$\varphi_P = \int (\rho_Q/r_{PQ})\,dV_Q$$

At a distance r from the electronic centre large compared to a we have

$$\varphi \cong e/r$$

We now divide A into two parts

$$\overline{A} = \overline{A}_1 + \overline{A}_0 \quad (3)$$

where $\overline{A}_0$ is the divergence-free solution of

$$\mathrm{rot}\ \mathrm{rot}\ \overline{A}_0 = 4\,\pi\ Tr\ \rho\,\dot{\overline{R}} \quad (4)$$

Thus with neglect of terms of the relative order $\dot{R}^2$, $\overline{A}_0$ is the divergence-free vector potential of a uniformly moving electron in radiation-free space, which at time t has the same $\overline{R}$ and $\dot{\overline{R}}$ as the actual electron. By definition we call $\overline{A}_0$ the vector potential of the « proper » field of the electron; similarly we call $\overline{A}_1$ the vector potential of the « external » field.

The solution of (4) is

$$\overline{A}_{oP} = \int Tr\,(\overline{j}_Q/r_{PQ})dV_Q = \int (\overline{j}_Q/2r_{PQ} + r_{PQ}(\overline{r}_{PQ}\,\overline{j}_Q)/2r^3_{PQ})dV_Q \quad (5)$$

At distances $r \gg a$ this field reduces to

$$\overline{A}_0 \cong e\,(\dot{\overline{R}}/2r + \overline{r}(\overline{r}\;\dot{\overline{R}})\;/2r^3) = Tr\;(e\dot{\overline{R}}/r)$$

For $\overline{\overline{A}}_0$ one finds

$$\overline{\overline{A}}_0 = \Theta \frac{e}{a} \dot{\overline{R}} \qquad (6)$$

where the numerical factor Θ depends on the ρ distribution (for a surface-charged sphere of radius a we have $\Theta = 2/3$). The same factor appears in the expression for the electromagnetic mass μ :

$$\mu = \Theta \frac{e^2}{a} \qquad (7)$$

The electric field strength can be written

$$\overline{E} = \dot{\overline{A}}_1 - \dot{\overline{A}}_0 - \nabla \varphi$$

Now $\overline{A}_0$ depends on t both through $\overline{R}$ and $\dot{\overline{R}}$ .Hence

$$-\dot{\overline{A}}_0 = -(\dot{\overline{R}}\;\partial/\partial\,\overline{R})\overline{A}_0 + \overline{F}$$

$$\overline{F} = -(\ddot{\overline{R}}\;\partial/\partial\,\dot{\overline{R}})\overline{A}_0,\;\; \overline{F}_P = -\int (\ddot{\overline{R}}/2r_{PQ} + \overline{r}_{PQ}(\overline{r}_{PQ}.\ddot{\overline{R}})/2r^3_{PQ})\rho_Q\, dV_Q$$

At distances $r \gg a$, $\overline{F}$ reduces to

$$\overline{F} \cong -Tr(e\ddot{\overline{R}}/r)$$

whereas for $\overline{\overline{F}}$ we find

$$\overline{\overline{F}} = -\Theta \frac{e}{a}\ddot{\overline{R}} = -\frac{\mu}{e}\ddot{\overline{R}} \qquad (8)$$

We now define the « proper » electric field as the sum of the Coulomb field $-\nabla\varphi$ and a transversal part $\overline{E}_0$ defined by

$$\overline{E}_0 = -(\dot{\overline{R}}\;\partial/\partial\;\overline{R})\;\overline{A}_0,$$

which at distances $r \gg a$ reduces to

$$\overline{E}_0 \cong e\,\overline{r}\,(\dot{\overline{R}}^2/2r^3 - 3(\dot{\overline{R}}.\overline{r})^2/2r^5)$$

The proper magnetic field is defined by

$$\overline{H}_0 = \mathrm{rot}\;\overline{A}_0$$

and reduces for distances $r \gg a$ to

$$\overline{H}_0 \cong e\,\dot{\overline{R}} \wedge \overline{r}/r^3$$

Apart from terms of the relative order $\dot{R}^2$, $\overline{E}_0$ and $\overline{H}_0$ are the (transversal) electric and magnetic field of the uniformly moving electron mentionned above. The averages $\overline{\overline{E}}_0$ and $\overline{\overline{H}}_0$ both vanish; the same holds for $\widetilde{-\nabla\varphi}$.

The « external » electric and magnetic field are defined by

$$\overline{E}_1 = \overline{E} + \nabla\varphi - \overline{E}_0 = -\dot{\overline{A}}_1 + \overline{F} \tag{10}$$

$$\overline{H}_1 = \overline{H} - \overline{H}_0 \qquad = \text{rot}\,\overline{A}_1 \tag{11}$$

Through $\overline{F}$, $\overline{E}_1$ will behave like $\ddot{\overline{R}}/r$ in the neighbourhood of the electron.

Introducing (10) and (11) into (1) we get

$$m_0\,\ddot{\overline{R}} = -e\,\widetilde{\dot{\overline{A}}_1} + e\,\widetilde{\overline{F}} + e\,\dot{\overline{R}} \wedge \widetilde{\text{rot}\,\overline{A}_1} - \partial\,U/\partial\,\overline{R}$$

Using (8) and introducing the experimental mass m of the electron by

$$m = m_0 + \mu = m_0 + \Theta e^2/a \tag{12}$$

the equations of motion take the simple form

$$m\,\ddot{\overline{R}} = -e\,\widetilde{\dot{\overline{A}}_1} + e\,\dot{\overline{R}} \wedge \widetilde{\text{rot}\,\overline{A}_1} - \partial\,U/\partial\,\overline{R} \tag{13}$$

For the field equation satisfied by $\overline{A}_1$ we get from (2) and (3)

$$\text{rot rot}\,\overline{A}_1 + \text{rot rot}\,\overline{A}_0 = 4\,\pi\,\rho\,\dot{\overline{R}} - \ddot{\overline{A}}_1 - \ddot{\overline{A}}_0 - \partial\nabla\varphi/\partial\,t$$

The second term on the left hand cancels against the first and the last term on the right, so we are left with

$$\Box\,\overline{A}_1 = \ddot{\overline{A}}_0 = \Big\{ (\dddot{\overline{R}}\,\partial/\partial\,\dot{\overline{R}}) + 2(\ddot{\overline{R}}\,\partial/\partial\,\overline{R})\,(\dot{\overline{R}}\,\partial/\partial\,R) + (\dot{\overline{R}}\,\partial/\partial\,\overline{R}) + (\dot{\overline{R}}\,\partial/\partial\,\overline{R})^2 \Big\}\,\overline{A}_0 \tag{14}$$

where the symbolic notation in the last member will be easily understood. It shows that there are three types of terms in $\ddot{\overline{A}}_0$, viz. terms linear in $\dddot{R}$, terms linear in $\ddot{R}$ and linear in $\dot{R}$, and terms cubic in $\dot{R}$, the coefficients being still functions of R (and x, y, z).

Until now everything was purely formal. The physical question which arises is whether or — if not — to what approximation (13) and (14) together give us a structure-independent description of the system. Since in (13) the experimental mass itself occurs, the question refers clearly to the behaviour of $\overline{A}_1$. If $\overline{A}_1$ and $\dot{\overline{A}}_1$ are prescribed at $t = 0$ in such a way as to present the necessary singularities (required by the form of $\ddot{\overline{A}}_0$) but no infinities at the electrons position will they — at later times — behave in a way which does not depend on the particularities of the charge distribution, such that the motion of the electron and the evolution of the field can be said to depend only on e and m? First of all it is clear that $\overline{A}_1$ and $\dot{\overline{A}}_1$ should be prescribed to be sufficiently smooth functions (characteristic wavelengths all large compared to a), so that the mean values $\dot{\overline{A}}$ and $\widetilde{\mathrm{rot}\,\overline{A}}$ are equal to the values of $\dot{\overline{A}}$ and $\mathrm{rot}\,\overline{A}$ at the centre of the electron. But even so, we can be sure that — rigorously — $\overline{A}$ will in the course of time develop a structure-dependent singularity near the electron. In fact we know that — rigourously — the electromagnetic momentum of the electron apart from the main term $\mu\dot{\overline{R}}$ will contain structure-dependent terms proportional to the higher odd powers of $\dot{\overline{R}}$. This means that $\overline{A}_1$ and $\dot{\overline{A}}_1$ will in short time take such values that (13) accounts at least for this (from relativity point of view erroneous) dependency of momentum on velocity

Still, within a certain approximation, (14) will certainly possess solutions with the structure-independent behaviour which we require. In fact, denoting by *Hom.* a smooth singularity-free solution of the homogeneous equation $\Box\,\overline{A}_1 = 0$ and by *Inh.* the well known time-symmetrical solution of the inhomogeneous equation (5) (half sum of retarded and accelerated potential)

$$Inh._P = \tfrac{1}{2}\int \left([\rho_Q\dot{\overline{R}}/r_{PQ}]_{t'=t-r} + [\rho_Q\dot{\overline{R}}/r_{PQ}]_{t'=t+r}\right) dV_Q \quad (15)$$

the transversal part of which, apart from relativistic corrections, behaves for near the electron like

$$Tr\ Inh. \cong Tr\ [e\ \dot{\overline{R}}/r]_{t'=t} = \overline{A}_0$$

the required solutions of (14) will be

$$\overline{A}_1 = -\overline{A}_0 + Tr\ Inh. + Tr\ Hom. \quad (16)$$

For later use it will be convenient here to introduce the (transversal) Hertz vector $\overline{Z}_1$ of the external field, defined by

$$\dot{\overline{Z}}_1 = \overline{A}_1 \quad (17)$$

Comparison with (14) gives

$$\Box\,\overline{Z} = \dot{\overline{A}}_0$$

From this we find

$$\text{rot rot}\,\overline{\mathrm{Z}}_1 = -\Delta\,\overline{\mathrm{Z}}_1 = -\ddot{\overline{\mathrm{Z}}}_1 - \dot{\overline{\mathrm{A}}}_0 = -\frac{\partial}{\partial t}(\overline{\mathrm{A}}_1 + \overline{\mathrm{A}}_0)$$
$$= Tr\,\overline{\mathrm{E}} = \overline{\mathrm{E}}_1 + \overline{\mathrm{E}}_0 \qquad (18)$$

This shows that $\overline{\mathrm{Z}}_1$, and thereby $\overline{\mathrm{A}}_1$ will necessarily show singularities at the electron's position since $\overline{\mathrm{E}}_1$ and $\overline{\mathrm{E}}_0$ become infinite like $1/r$ and $1/r^2$ respectively; at the same time we see that $\overline{\mathrm{Z}}_1$ and $\overline{\mathrm{A}}_1$ stay finite and may be sufficiently smooth that $\overline{\mathrm{Z}}_{1\,r=o} = \overline{\overline{\mathrm{Z}}}_1$ and $\overline{\mathrm{A}}_{1\,r=o} = \overline{\overline{\mathrm{A}}}_1$ are fulfilled. In the electric dipole radiation approximation discussed in the next paragraph $\overline{\mathrm{E}}_0$ may be neglected.

III.

ELECTRIC DIPOLE RADIATION ONLY

Consider in particular the case where the influence of retardation and acceleration on ρ in (15) may be neglected, i. e. where as far as r is concerned we may reckon with a fixed position of the electron in space. In that case an analysis of *Hom.* in multipole radiations is indicated and we need only retain the electric dipole radiation. Then (16) assumes the form

$$\overline{\mathrm{A}}_1 = e\,Tr\,(\frac{1}{2}\,\dot{\overline{\mathrm{R}}}_{t'=t+r} + \frac{1}{2}\,\dot{\overline{\mathrm{R}}}_{t'=t-r} - \dot{\overline{\mathrm{R}}}_{t'=t} - \frac{1}{2}\,\dot{\overline{\mathrm{S}}}_{t'=t+r} + \dot{\overline{\mathrm{S}}}_{t'=t-r})/r \qquad (19)$$

where $\dot{\overline{\mathrm{S}}}$ is the derivative of a vector the components of which are arbitrary functions of an argument t'. We see that (19) is a solution of (14) in which $\overline{\mathrm{A}}_0$ is considered independent of $\overline{\mathrm{R}}$. Developing in terms of powers of r we get

$$\overline{\mathrm{A}}_1 = e\,Tr(\frac{1}{2}\,\ddot{\overline{\mathrm{R}}}\,r + \ldots - \ddot{\overline{\mathrm{S}}} - \frac{1}{6}\,\ddddot{\overline{\mathrm{S}}}\,r^2 - \ldots) \qquad (20)$$

In our particular case we may in the equations of motion (13) neglect the Lorentz force since the magnetic dipole radiation was neglected and we have to use (20) only in order to find the quantity $\dot{\overline{\overline{\mathrm{A}}}}$ which — on our theory — should reduce to the value at the origin $r = 0$. Thus only the term $\ddot{\overline{\mathrm{S}}}$ counts in (20), and we find for what from a simplistic point of view may be called the external electric field acting on the electron (*)

$$-\dot{\bar{\bar{A}}}_1 = -\dot{\bar{A}}_{1 r=o} = \frac{2}{3}\, e\, \ddot{\bar{\bar{S}}}$$

Thus the equation of motion for the electron becomes

$$m\, \ddot{\bar{R}} = \frac{2}{3}\, e^2\, \dddot{\bar{S}} - \partial\, U/\partial\, \bar{R} \tag{21}$$

where $\dddot{\bar{S}}$ is an arbitrary function of its argument t.

As a first application consider the case where the E.H field does not contain a component of ingoing radiation [represented by the terms $\dot{\bar{R}}_{t+r}$ and $\dot{\bar{S}}_{t+r}$ in (19)]. Then we have clearly $\bar{S}=\bar{R}$ and the terms $\frac{2}{3} e^2\, \dddot{\bar{S}}$ in (21) becomes the well known radiation reaction term $\frac{2}{3} e^2\, \dddot{\bar{R}}$. More precisely, the condition of no ingoing radiation only requires that $\bar{R}$ equals $\bar{S}$ for all values of the argument above a certain minimum value; the $\dddot{\bar{R}}$ term will then be correct for all t-values larger than this minimum. We can also put the following problem : at $t = 0$ the quantities $\bar{R}$ and $\dot{\bar{R}}$ are prescribed and moreover no radiation field is present outside a sphere with radius $r = r_0$. Then we must clearly put

$\bar{S}\,(t') = \bar{R}(t')$ for $t' > r_0$ and $\bar{R}(t') = -\bar{S}(t')$ for $t' < -r_0$. In the interval $-r_0 < t' < 0$ we can still choose $\bar{R}(t')$ and in the interval $-r_0 < t' < r_0$ still choose $\bar{S}(t')$ as we like. From $t > r_0$ on the $\dddot{\bar{S}}$ will anyhow be reduced to the $\dddot{\bar{R}}$ value.

Next ask for solutions of our equations of motion which are purely harmonic in time. This will clearly require that $-\partial\, U/\partial\, \bar{R}$ in (21) is linear in $\bar{R}$, i. e. that we have a harmonic oscillator or even a free electron. Considering for simplicity an isotropic oscillator with natural frequency k_1, we put

$$\bar{R} = \bar{R}_0 \cos kt, \qquad \bar{S} = \bar{S}_0 \sin kt, \qquad -\partial\, U/\partial\, \bar{R} = -m k_1^2\, \bar{R}$$

(*) We have made use of $Tr\, \dot{\bar{\bar{S}}} = \frac{2}{3} \ddot{\bar{S}}$. The following formulae allows us to find the transversal part of any vector field $\bar{f}\,(r)$ the components of which depend only on r. Introducing another vector field $\bar{g}\,(r)$ the second derivative of which with respect to r is equal to $r\bar{f}$ the formulae runs

$$Tr\, \bar{f} = Tr\, \bar{g}''\,(r)/r = \bar{g}/r^3 - \bar{g}'/r^2 + \bar{g}''/r + \bar{r}(\bar{r}, -3\bar{g}/r^5 + 3g'/r^4 - g''/r^3)$$
$$= (\bar{g}''/r - \bar{r}(\bar{r}\,\bar{g}'')/r^3) - (r^2 \frac{d}{dr}(\bar{g}/r) - 3\,\bar{r}(\bar{r}, \frac{d}{dr}(\bar{g}/r))/r^3$$

Then (19) reduces to

$$\overline{A}_1 = k\, e \sin kt\; Tr\{\overline{R}_0 (1 - \cos kr) + \overline{S}_0 \sin kr\}/r \tag{22}$$

and (21) gives us the ratio between $\overline{R}_0$ and $\overline{S}_0$, $-mk^2\overline{R}_0 = -\frac{2}{3} e^2k^3\overline{S}_0 - mk_1^2\overline{R}$.

Introducing a new vector $\overline{T}_0$ and a phase angle η by putting

$$\left.\begin{array}{l} \overline{S}_0 = \overline{T}_0 \cos\eta \qquad \overline{R}_0 = \overline{T}_0 \sin\eta \\ \operatorname{tg}\eta = \dfrac{k^3/K}{k^2 - k_1^2} \text{ harm. osc.} \\ \qquad\qquad\qquad\qquad K^{-1} = \dfrac{2e^2}{3m} \\ \operatorname{tg}\eta = k/K \qquad \text{free el.} \end{array}\right\} \tag{23}$$

where K^{-1} is a length of the order of the conventional electronic radius, (22) takes the form

$$\overline{A}^1 = e\, k \sin kt\; Tr\, \overline{T}_0 \frac{\sin(kr - \eta) + \sin\eta}{r} \tag{24}$$

The electromagnetic field thus behaves like a system of ingoing waves which are reflected at the electron and give rise to outgoing waves. In the wave zone the resulting standing waves are, due to the presence of the electron shifted by the angle η from the standing dipole waves in free space. Fig. 1 gives a sketch of the behaviour of η (k) for a free electron and for a harmonic oscillator.

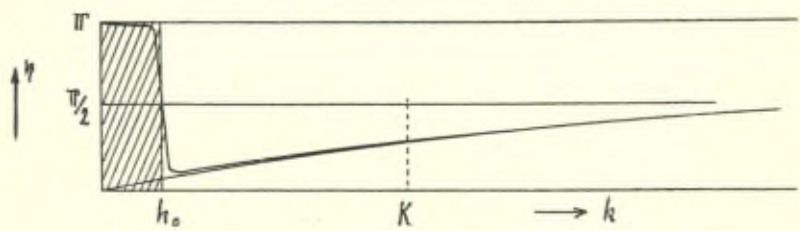

Fig. 1

According to the assumptions in our theory, (24) can only be trusted as long as k is small compared to K (and as long as the amplitude T_0 is sufficiently small). For any particular charge distribution standing oscillations described by (24) will be possible, but the η function will in general differ from (23) as soon as k is no longer small compared to K. Only in the particular case where ρ is given by

$$\rho = e\, K^2\, e^{-Kr}/4\, \eta\, r$$

the η - function happens to remain unchanged.

We haves till to investigate whether $\overline{R}$ and $\overline{S}$ can change exponentially with time :

$$\overline{R} = \overline{R}_0\, e^{\pm \kappa t}, \qquad \overline{S} = \overline{S}_0\, e^{\pm \kappa t}, \qquad -\partial\, U/\partial\, \overline{R} = -mk_0^2\, \overline{R}$$

Then (19) reduces to

$$\overline{A}_1 = \kappa\, e\, e^{\pm \kappa t}\, Tr \,\{\pm\, \overline{R}_0\, (Ch\, \kappa\, r - 1) - \overline{S}_0\, \overline{S}h\, \kappa\, r\}/r$$

Since for physical reasons the field should not increase exponentially with r we have clearly

$$\overline{R}_0 = \pm\, \overline{S}_0$$

On the other hand (21) gives us

$$m\, \kappa^2\, \overline{R}_0 = \pm\, \frac{2}{3}\, e^2\, \kappa^3\, \overline{S}_0 - mk_1^2\, \overline{R}_0$$

Therefore

$$K\, (\kappa^2 + k_1^2) = \kappa^3$$

This equation has one real root which for the free electron is $\kappa = K$ and for the harmonic oscillator still larger. This means that the field (25)

$$\overline{A}^1 = -\, \kappa\, e\, e^{\pm \kappa t}\, Tr\, \overline{S}_0\, (1 - e^{-\kappa r})/r \tag{26}$$

corresponds to a radiation field confined within a region of linear extension 1/K and cannot, from our point of view, be reckoned to belong to the features of the structure-independent theory looked for. It is of course connected with those well known solutions of the equations of motion with radiation reaction

$$m\, \ddot{\overline{R}} = \frac{2}{3}\, e^2\, \dddot{\overline{R}} - \partial\, U/\partial\, \overline{R}$$

which give rise to an exponential behaviour of $\overline{R}$ with time and which are commonly rejected as non-physical. If in the model we let the electron radius go to zero, μ to $+\infty$ and m_0 to $-\infty$, $\mu + m_0$ remaining constant, we create an infinite source of electric energy and solution (26) becomes understandable.

In this connection the following point has some mathematical interest. For the free electron, tg $\eta = k/K$, the functions sin $(kr - \eta)$ form an orthogonal set but it is not complete, there being one function, viz. e^{-Kr} orthogonal to them all.

Having thus learnt that, in electric dipole approximation, the classical system : elastically bound electron + field, can be considered as a sum of structure-independent harmonic oscillators, at any rate if we restrict ourselves to states in which the frequencies characterizing the system are small compared to K, it is tempting to quantize the system by assigning to each of these three-dimensional oscillators an energy $(\eta_{lk} + 3/2)\, h$. In the ground state we find for the energy of the system

$$\varepsilon = \sum_{K} \frac{3}{2} \hbar\, k \tag{27}$$

We will surround the electron by a reflecting spherical shell of large radius L. The condition that at the wall the tangential component of $\overline{\mathrm{E}}$ vanishes leads to

$$k\mathrm{L} - \eta\,(k) = \pi\,\mathrm{N} \qquad (\mathrm{N} = 1, 2, 3 \ldots) \tag{28}$$

The allowed k-values are characterized by the integer N and, since

$$\Delta\,\mathrm{N} = \frac{1}{\pi}\,(\mathrm{L} - \frac{d\eta}{dk})\,\Delta\, k$$

the sum (27) reduces to the integral.

$$\varepsilon = \frac{3\hbar}{2\pi}\int_{0} (\mathrm{L} - \frac{d\eta}{dk})\, dk$$

For free space, $\eta = 0$, and we get as contribution of the electric dipole radiation to the zero point radiation energy in space

$$\varepsilon = \frac{3\,\hbar\,\mathrm{L}}{2\,\pi}\, k_{max}$$

which of course diverges for $k_{\max} \rightarrow \infty$.

The influence of the electron on ε adds the contribution

$$\varepsilon_{\mathrm{el.}} = \frac{3\hbar}{2\pi}\,(-\int_{o}^{k_m} k\,\frac{d\eta}{dk}\, dk + \eta_m k_m) = \frac{3\hbar}{2\pi}\int_{o}^{k_m} \eta\, dk \tag{29}$$

if in the two cases — electron absent and present — we sum over the same number of oscillators. This procedure is of course arbitrary, the difference between two divergent sums not being well defined, but it corresponds to the quantum mechanical formalism by which contributions of this kind are customarily calculated (adiabatic increase of e from zero to its actual value).

For the « fluctuation » energy of a free electron we therefore get

$$\varepsilon_{\text{el. free}} = \frac{3\hbar}{2\pi}\int_o^{k_m} \operatorname{arctg} k/\mathrm{K}\, dk = \frac{3\hbar}{2\pi}\Big\{ k.\operatorname{arctg} k/\mathrm{K} - \frac{1}{2} lg\,(1 + k^2/\mathrm{K}^2)\Big\}_{k=k_m} \tag{30}$$

For large k_m this expression diverges as $\frac{3\hbar}{4}\, k_m$ whereas for $k_m \ll \mathrm{K}$ it behaves like

$$\varepsilon_{\text{el. free}} = \frac{9}{8}\, m\, \frac{\hbar}{e^2}\,(k_m/\mathrm{K})^2 \tag{31}$$

This is the well known result (6) if, with customary perturbation methods, one asks for the contribution proportional to e^2. There is no reason to assign structure-independent significance to the exact expression (30) for k-values of the order K or larger. Due to the smallness of $e^2/\hbar$ we see from (31) that on quantum theory we would even have to take k/K small compared to $e/\sqrt{\hbar}$. Indeed, if for k we choose the inverse Compton wave lenght : $\lambda_C^{-1} = \hbar/m$, $\varepsilon_{\text{el.}}$ becomes $\frac{1}{2}\frac{e^2}{\hbar}\; m$ and from relativity considerations one clearly hesitates to go far beyond this k value. The same caution is indicated by a consideration of the amplitude of the motion of the electron in a classical picture where the oscillators are each given the energy $\frac{3}{2}\,\hbar k$. One finds that the mean square of the amplitude in a given direction is given by the formal integral

$$\overline{\text{ampl}^2} = \frac{2}{\pi}\frac{e^2}{\hbar}\,\lambda_C^2\int\frac{dk}{k}$$

which, on closer investigation, shows that the electric dipole approximation will start to fail when the wave lengths of the radiation field become smaller than the Compton wave length λ_C

A consideration of the harmonic oscillator yields some points of interest. First of all, the application of (29) leads formally to the energy increase

$$\varepsilon_{\text{harm. osc.}} = \frac{3\hbar}{2\pi}\int_o^{k_m} \operatorname{arctg} k^3/\mathrm{K}(k^2 - k_1^2)\,dk$$

which for $k_m \gg K$ diverges in the same way as (30). The difference between (31) and (29) however converges. Indeed one finds

$$\Delta \varepsilon = \frac{3\hbar}{2\pi} \int_o^{k_m} (\eta_{\text{harm. osc.}} - \eta_{\text{free}})\, dk = \frac{3}{2} \hbar k_1 \left\{ 1 + \frac{1}{\pi} \frac{k_1}{K} (lg \frac{K}{k_1} + C) + \dots \right\}$$

where the terms neglected contain only the second and higher powers of k_1, and where the correction term depends on k_m and vanishes for $k_m \rightarrow \infty$:

$$C = -\frac{1}{2}\, lg(1 + K^2/k_m^2)$$

The main term $\frac{3}{2} \hbar\, k$, is just what should be expected, namely the energy in the ground state of a harmonic oscillator of natural frequence k_1. It corresponds to the area of the shaded rectangle in Fig. 1, multiplied by $3\hbar/2\pi$. The second term has almost the form which Bethe's original formula (7) gives for the Lamb shift of the levels of a harmonic oscillator (it is the same for all levels), the difference between that in Bethe's formula the K in the numerator of the argument of the logarithm is replaced by the inverse Compton wave length, which we will call k_C. The terms left out are even small compared to the natural line breadth k_1^2/K of the harmonic oscillator. The correction term C vanishes for $k_m \rightarrow \infty$ but, if we dare not go beyond $k_m \doteq k_C$ in the integration, it becomes pratically $lg\ (k_C/K)$ and we get precisely Bethe's result. In this connection we note that Bethe's formula for the Lamb shift in an arbitrary atom can be written as the sum

$$\Delta\, \varepsilon_{\text{Lamb}} = \sum_j f_j \frac{3}{2\pi} \frac{\hbar k_j^2}{K}\, lg(k_C/k_j)$$

extended over all absorption and emission frequencies k_j and where f_j if the corresponding oscillator strength.

Another point which, this time, lies quite within the domain where our formulae are significant refers to the scattering of light by an elastically bound electron. Consider again our assembly of harmonic oscillators of frequencies k which we may imagine to take the discrete but very finely distributed values determined by (28). Let all of them be in the ground state (energy $\frac{3}{2} \hbar\, k$) with the exception of one (frequency k') which has the energy $\frac{5}{2} \hbar\, k'$, the vibration parallel to — say — the x-axis being excited by one quantum $\hbar\, k'$. In this situation light of frequency k' coming from all directions is continuously scattered by the electron. By the well known devices of collision theory we are then also able to describe the scattering of a plane wave (with its $\overline{\mathrm{E}}$-vector in the x-direction) by the electron. The phase shift η appears from the outset in our formulæ; it is the same as that in classical theory but it is now naturally incorporated in a pure quantum theoretical treatment which — in contrast to the customary Dirac treatment however — corresponds exactly to the classical method where scattering is described as a steady state and no longer as a generation of light in other directions than the direction of incidence. In the theory of particle collisions these steady states never gave difficulties; in the quantum theory of light scattering it seemed difficult to handle them in a rigorous way.

We will explain this steady-state quantum description in some detail. Describe an arbitrary situation of the classical system as a superposition of harmonic oscillations of frequencies by means of the Hertz vector and the vector potential of the external field [compare (24) and (17)]

$$\left.\begin{aligned} \overline{\mathrm{Z}}_1 &= -\, e \sum_k Tr\, \overline{\mathrm{T}}_{ok} \frac{\sin(kr-\eta) + \sin\eta}{r} \cos kt \\ \overline{\mathrm{A}}_1 &= e\, k \sum_k Tr\, \overline{\mathrm{T}}_{ok} \frac{\sin(kr-\eta) + \sin\eta}{r} \sin kt \end{aligned}\right\} \tag{33}$$

From this it will be possible to introduce pairs of canonical variables $\bar{q}_k$ and $\bar{p}_k$ belonging to the x, y and z direction of every oscillator. Normalizing them in such a way that the Hamiltonian becomes

$$\left.\begin{aligned} H &= \frac{1}{2} \Sigma\, k(\bar{p}_k^2 + \bar{q}_k^2) \\ \dot{\bar{q}}_k &= k\, \bar{p}_k\,, \qquad \dot{\bar{p}} = -\, k\, \bar{q}_k \end{aligned}\right\} \tag{34}$$

we infer from (33) that we may put

$$\overline{\mathrm{T}}_{ok} \cos kt = -f(k)\, \bar{q}_k\,, \qquad \overline{\mathrm{T}}_{ok} \sin kt = f(k)\, \bar{p}_k$$

The factor f can be found by considering the total energy of the system

$$\begin{aligned} \varepsilon &= \frac{1}{2} m_0\, \dot{\overline{\mathrm{R}}}{}^2 + \frac{1}{2} m\, k_1^2\, \overline{\mathrm{R}}{}^2 + \frac{1}{8\pi} \int (\mathrm{H}^2 + \mathrm{E}^2) dV \\ &= \frac{1}{2} m_0\, \dot{\overline{\mathrm{R}}}{}^2 + \frac{1}{2} m\, k_1^2\, \overline{\mathrm{R}}{}^2 + e\, \dot{\overline{\mathrm{R}}} \cdot \overline{\mathrm{A}}_1 \cdot + \frac{1}{8\pi} \int (\mathrm{H}_1^2 + \mathrm{E}_1^2) dV \end{aligned}$$

In deriving this structure-independent expression we have made use of (11), (9), (4) and — as is permitted in electric dipole approximation—neglected $\overline{\mathrm{E}}_0 = \overline{\mathrm{E}} - \overline{\mathrm{E}}'$. From [compare (23)]

$$\left.\begin{aligned} \dot{\overline{\mathrm{R}}} &= -\sum_k k\,\overline{\mathrm{T}}_{ok} \sin\eta \sin kt = -\sum_k k\,f(k) \sin\eta\,\bar{p}_k \\ \overline{\mathrm{R}} &= -\sum_k f(k) \sin\eta\,\bar{q}_k \end{aligned}\right\} \tag{35}$$

and expressing also $\overline{\mathrm{H}}_1$ and $\overline{\mathrm{E}}_1$ in the $\bar{p}$'s and $\bar{q}$'s respectively by means of (33), (11), (32), a calculation shows that (34) is justified and that moreover

$$f = \frac{1}{e}\sqrt{\frac{3}{k^3\mathrm{L}}}$$

The standard formulæ for the field become therefore

$$\left.\begin{aligned} \overline{\mathrm{Z}}_1 &= Tr \sum_k \sqrt{\frac{3}{k^3\mathrm{L}}}\,\bar{q}_k \frac{\sin(kr-\eta) + \sin\eta}{r}, \quad \overline{\mathrm{E}}_1 = \mathrm{rot\,rot}\,\overline{\mathrm{Z}}_1 \\ \overline{\mathrm{A}}_1 &= Tr \sum_k \sqrt{\frac{3}{k^3\mathrm{L}}}\,\bar{p}_k \frac{\sin(kr-\eta) + \sin\eta}{r}, \quad \overline{\mathrm{H}}_1 = \mathrm{rot}\,\overline{\mathrm{A}}_1 \end{aligned}\right\} \tag{36}$$

If now we promote p_k and $\bar{q}_k$ to q-numbers satisfying the well known commutation properties, the formalism has become purely quantum mechanical. The $\bar{q}$'s and $\bar{p}$'s describe the light quanta in the external field, i. e. the free light quanta, which are affected by the presence of the electron through the appearance of the phase shift η. We might also call them *phase shifted light quanta*; the free electric dipole light quanta in empty space correspond to $\eta=0$. In the next paragraph we will give a more general quantum mechanical treatment which does not primarily rest on an analysis of the system in harmonic components.

Returning now to the state considered above where one $\bar{p}, \bar{q}$ oscillator (frequency k') is in the first excited state, all the others in the ground state, we will consider what happens if k' is equal to the natural frequency k_1 or differs from it by an amount small compared to the natural line with k_1^2/K. The phase is then nearly $\pi/2$ and from (23) or (35) we see that the amplitude of the electronic motion (classically or quantum mechanically considered) is much larger than when k' lies outside the line breadth. We have now, indeed, what in customary language would be called an electronic oscillator in its first excited state. It does not jump to the lowest because it is sustained so to say by an incoming free space radiation. By an appropriate quantum mechanical superposition of states of this kind, all with η very near to $\pi/2$ we could, however, construct a situation in which at $t = 0$ the incoming radiation is zero beyond some distance from the electron, analogous to the choice of $\overline{\mathrm{S}}$ discussed on p. 9 and where in the course of time the amplitude of the electronic motion falls exponentially off to zero. This description would be nearly related to the Weisskopf-Wigner treatment of a spontaneous Bohr transition.

If we imagine k' to take successively increasing values and to pass through the region where η changes from π to 0, the customary language which is so well adapted to most physical processes fails in the regions where η is neither near to π or 0, nor to $\pi/2$; there we can neither speak of light which is scattered from the ground state nor of an oscillator in its first excited state. The situation is of course exactly analogous to that in particle collisions but the present rigorous formalism in the case of light quanta which does not rest on development in powers of e^2 fills a gap in the customary methods.

IV.

THE STRUCTURE-INDEPENDENT HAMILTONIAN

It will clearly be of interest to bring the structure-independent classical equations of motion derived in § 2 in Hamiltonian form. We begin with the variational principle which leads to the equations of motion (1) and (2) from which we started in § 2 :

$$\delta \int \mathrm{L}\, dt = 0 \tag{37}$$

$$\mathrm{L}\left(\overline{\mathrm{R}},\dot{\overline{\mathrm{R}}},\overline{\mathrm{A}},\dot{\overline{\mathrm{A}}}\right) = \frac{1}{2}\, m_0\, \dot{\overline{\mathrm{R}}}^2 + e(\dot{\overline{\mathrm{R}}}\, \overline{\overline{\mathrm{A}}}) - \mathrm{U}(\overline{\mathrm{R}})$$
$$- \frac{1}{8\pi} \int (\overline{\mathrm{H}}^2 - (Tr\, \overline{\mathrm{E}})^2) d\mathrm{V} \tag{38}$$

where $\overline{\mathrm{R}}$ and $\overline{\mathrm{A}}$ are arbitrarily varied subject to δ div $\mathrm{A} = 0$. The introduction (3) of a new field $\overline{\mathrm{A}}_1$ is not immediately permitted since $\overline{\mathrm{A}}_0$ depends both on $\overline{\mathrm{R}}$ and $\dot{\overline{\mathrm{R}}}$. This difficulty can be overcome if we replace L by a new Lagrangian

$$\mathrm{M} = \mathrm{L} - \overline{\mathrm{P}}.\overline{\mathrm{V}} - \dot{\overline{\mathrm{P}}}.\overline{\mathrm{R}} \tag{39}$$

$$\overline{\mathrm{P}} = \partial\, \mathrm{L}/\partial\, \dot{\overline{\mathrm{R}}} \tag{40}$$

where $\overline{\mathrm{V}}$ is numerically equal to $\dot{\overline{\mathrm{R}}}$ but is expressed, just as M itself, in terms of $\overline{\mathrm{R}}$, $\overline{\mathrm{P}}$, $\dot{\overline{\mathrm{P}}}$, $\overline{\mathrm{A}}$, $\dot{\overline{\mathrm{A}}}$ by means of (40). It is easily proved that

$$\delta \int \mathrm{M}\, dt = 0$$

leads to the same equations of motion as (37). Of course there now appears a redundant variable, the generalized momentum $\partial \mathrm{L} / \partial \dot{\overline{\mathrm{R}}}$ being identically zero. If $\overline{\mathrm{A}}_0$ is now considered as a function of $\overline{\mathrm{R}}$, $\overline{\mathrm{P}}$, $\overline{\mathrm{A}}$ instead of $\overline{\mathrm{R}}$, $\dot{\overline{\mathrm{R}}}$ the substitution $\overline{\mathrm{A}} = \overline{\mathrm{A}}_1 + \overline{\mathrm{A}}_0$ in (39) is permitted.

From (40) and (38) we have

$$\overline{\mathrm{P}} = m_0\, \dot{\overline{\mathrm{R}}} + e\, \overline{\overline{\mathrm{A}}}$$

and therefore

$$m_0\, \overline{\mathrm{V}} = \overline{\mathrm{P}} - e\, \overline{\overline{\mathrm{A}}} \tag{41}$$

The new Lagrangian takes the form

$$\mathrm{M} = -\frac{1}{2} m_0\, \overline{\mathrm{V}}^2 - \mathrm{U} - \dot{\overline{\mathrm{P}}}.\overline{\mathrm{R}} - \frac{1}{8\pi} \int (\overline{\mathrm{H}}^2 - (Tr\, \overline{\mathrm{E}})^2) d\mathrm{V} \tag{42}$$

$\overline{\mathrm{V}}$ being explained by (41).

Introducing (6), (7), (12) in (42) we find

$$m\, \overline{\mathrm{V}} = \overline{\mathrm{P}} - e\, \overline{\overline{\mathrm{A}}}_1 \tag{43}$$

Moreover we have from (11), (10)

$$\overline{\mathrm{H}} = \overline{\mathrm{H}}_1 + \overline{\mathrm{H}}_0 = \mathrm{rot}\, \overline{\mathrm{A}}_1 + \mathrm{rot}\, \overline{\mathrm{A}}_0, \quad Tr\, \overline{\mathrm{E}} = \overline{\mathrm{E}}_1 + \overline{\mathrm{E}}_0 = -\dot{\overline{\mathrm{A}}}_1 + \overline{\mathrm{F}} + \overline{\mathrm{E}}_0$$

For magnetic and electric field energy we thus get

$$\frac{1}{8\pi} \int \mathrm{H}^2 d\mathrm{V} = \frac{1}{8\pi} \int \overline{\mathrm{H}}_1^2 d\mathrm{V} + \frac{1}{4\pi} \int \overline{\mathrm{H}}_1.\overline{\mathrm{H}}_0 d\mathrm{V} + \frac{1}{8\pi} \int \overline{\mathrm{H}}_0^2 d\mathrm{V}$$

$$= \frac{1}{8\pi} \int (\mathrm{rot}\, \overline{\mathrm{A}}_1)^2 d\mathrm{V} + e\, \overline{\overline{\mathrm{A}}}._1 \overline{\mathrm{V}} + \frac{1}{2} \mu\, \mathrm{V}^2$$

$$\frac{1}{8\pi} \int (Tr\, \overline{\mathrm{E}})^2 d\mathrm{V} = \frac{1}{8\pi} \int \overline{\mathrm{E}}_1^2\, d\mathrm{V} + \frac{1}{4\pi} \int \overline{\mathrm{E}}_1.\overline{\mathrm{E}}_0\, d\mathrm{V} + \frac{1}{8\pi} \int \overline{\mathrm{E}}_0^2\, d\mathrm{V}$$

As regards $\overline{\mathrm{F}}$ and $\overline{\mathrm{E}}_0$ we must be cautious. They are obtained by differentiating $\overline{\mathrm{A}}_0$ with respect to time but $\overline{\mathrm{A}}_0$ will now (outside the electron) be expressed by

$$\overline{\mathrm{A}}_0 = Tr\, e\, \overline{\mathrm{V}} / r$$

$\bar{V}$ being explained by (43) as a function of $\bar{P}$, A_1 and R. Thus we have

$$\bar{F} = Tr\ \dot{\bar{V}}/r, \qquad \dot{\bar{V}} = \frac{1}{m}\left(\dot{\bar{P}} - e\,\dot{\bar{\bar{A}}}_1 - e\left(\dot{\bar{R}}\,\partial/\partial\,\bar{R}\right)\bar{\bar{\bar{A}}}_1\right) \tag{44}$$

and $\bar{F}$ depends on $\dot{\bar{P}}$, $\bar{A}_1$, $\dot{\bar{A}}_1$, $\bar{R}$, $\dot{\bar{R}}$, being linear in all but $\bar{R}$. Similarly we have outside the electron

$$\bar{E}_0 = e\ Tr\ \bar{V}\left(\dot{\bar{R}}\,\frac{\partial}{\partial\,\bar{R}}\right)\frac{1}{r}$$

showing that $\bar{E}_0$ depends on $\bar{P}$, $\bar{A}_1$, $\bar{R}$, $\dot{\bar{R}}$, linearly in all but $\bar{R}$.

The integral $\int \bar{E}_1.\bar{E}_0\ dV$ will be structure-independent in spite of the fact that $\bar{E}_1$ through $\bar{F}$ behaves like $1/r$ and $\bar{E}_0$ like $1/r^2$. On the other hand $\int E_0^2\ dV$ diverges if (44) should hold down to $r = 0$ and becomes finite but structure-dependent and proportional to the fourth power of the velocity if the charge-distribution is taken into account. This term must clearly be neglected; it refers to effects which anyhow only a true relativistic theory might describe rightly.

We are thus left with the structure-independent Lagrangian

$$\begin{aligned} M &= -\frac{1}{2}mV^2 - e\,\bar{\bar{A}}_1.\bar{V} - U - \dot{\bar{P}}.\bar{R} - \frac{1}{8\pi}\int(H_1^2 - E_1^2 - 2\bar{E}_1.\bar{E}_0)dV \\ &= -P^2/2m + e^2\,\bar{\bar{A}}_1^2/2m - U - \dot{\bar{P}}.\bar{R} - \frac{1}{8\pi}\int(H_1^2 - E_1^2 - 2\bar{E}_1.\bar{E}_0)dV \end{aligned} \tag{45}$$

M depends on $\bar{P}$, $\dot{\bar{P}}$, $\bar{A}_1$, $\dot{\bar{A}}_1$, $\bar{R}$, $\dot{\bar{R}}$, but because of the redundant variables we cannot use it immediately for bringing the equations of motion in canonical form since there exists an identical relation between the moments.

From now on we will restrict ourselves to the electric dipole approximation. This means that we can take for $\bar{A}_1$ and $\bar{F}$ (in $\bar{E}_1$) their values for $r = 0$, i. e. neglect their dependency on $\bar{R}_1$, and leave out $\bar{E}_0$ entirely. This means that M no longer depends on $\dot{\bar{R}}$:

$$\frac{\partial\,M}{\partial\,\dot{\bar{R}}} = 0$$

and the canonical form is easily established. Indeed with Lagrangian $M(x_o;\ x_1,\ \dot{x}_1;\ x_2,\ \dot{x}_2\ \ldots\ x_n,\ \dot{x}_n)$ the equations of motion are :

$$\frac{\partial\,M}{\partial\,x_o} = 0, \qquad \dot{p}_k = \frac{\partial\,M}{\partial\,x_k}\,(k = 1, 2\ \ldots\ n),\, p_k = \frac{\partial\,L}{\partial\,\dot{x}_k}$$

Therefore

$$\delta\,(\sum_1^n p_k \dot{x}_k - \mathrm{M}) = \sum_1^n \dot{x}_k\,\delta p_k - \sum_1^n \frac{\partial \mathrm{M}}{\partial x_k}\,\delta x_k = \sum_1^n (\dot{x}_k\,\delta p_k - \dot{p}_k\,\delta x_k)$$

If x_o, $\dot{x}_1 \ldots \dot{x}_n$ can be expressed in terms of $x_1 \ldots x_n, p_1 \ldots p_n$ by means of $\partial \mathrm{L}/\partial x_o = 0$, $\partial \mathrm{L}/\partial \dot{x}_k = p_k$, the p_k's and x_k's $(k = 1,2 \ldots n)$ should satisfy the canonical equations with the Hamiltonian

$$H = \sum_1^n p_k \dot{x}_k - \mathrm{L} = H\,(p_1 \ldots p_n, x_1 \ldots x_n)$$

Applying this to our case we have first of all to calculate the vector conjugate to $\bar{\mathrm{P}}$ and the vector field conjugate to the vector field $\bar{\mathrm{A}}_1$. From (44) and (45) we find :

$$\gamma_{\bar{\mathrm{P}}} = \frac{\partial \mathrm{M}}{\partial \dot{\bar{\mathrm{P}}}} = -\bar{\mathrm{R}} + \frac{1}{4\pi}\int \bar{\mathrm{E}}_1 \frac{\partial \bar{\mathrm{F}}}{\partial \dot{\bar{\mathrm{P}}}}\, d\mathrm{V} = -\bar{\mathrm{R}} - \frac{e}{4\pi m}\int Tr\,\frac{\bar{\mathrm{E}}_1}{r}\, d\mathrm{V}$$

Since rot rot $\bar{\mathrm{Z}}_1 = \bar{\mathrm{E}}_1$ and div $\bar{\mathrm{Z}}_1 = 0$ ($\bar{\mathrm{Z}}_1$ Hertz potential) the value of $\bar{\mathrm{Z}}_1$ in the origin is just given by the integral appearing in the fourth member, divided by 4π. Denoting the momentum conjugate to $\bar{\mathrm{P}}$ by $-\bar{\mathrm{R}}_1$ we have therefore

$$-\gamma_{\bar{\mathrm{P}}} \equiv \bar{\mathrm{R}}_1 = \bar{\mathrm{R}} + \frac{e}{m}\bar{\mathrm{Z}}_1 \tag{46}$$

In order to find the momentum conjugate to $\bar{\mathrm{A}}_1$ in a certain point of space, we have to vary M with respect to $\dot{\bar{\mathrm{A}}}_1$. There is a contribution both from $\dot{\bar{\mathrm{A}}}_1$ and from $\bar{\mathrm{F}}$ in $\bar{\mathrm{E}}_1$:

$$\delta \mathrm{M} = \frac{1}{4\pi}\int \bar{\mathrm{E}}_1\,\delta \bar{\mathrm{E}}_1 d\mathrm{V} = \frac{1}{4\pi}\int \bar{\mathrm{E}}_{1\mathrm{P}} \Big\{ -\delta \dot{\bar{\mathrm{A}}}_1$$

$$+ \frac{e}{m} Tr \int (\rho_{\mathrm{Q}}\,\delta \dot{\bar{\mathrm{A}}}_{1\mathrm{Q}}/r_{\mathrm{PQ}}) d\mathrm{V}_{\mathrm{Q}} \Big\}\, d\mathrm{V}_{\mathrm{P}} = \int \gamma_{\bar{\mathrm{A}}_1}\, d\mathrm{V}$$

Representing the field $\gamma_{\bar{\mathrm{A}}}$ by $-\bar{\mathrm{E}}_2/4\pi$ we have thus

$$-\frac{\bar{\mathrm{E}}_2}{4\pi}(\equiv \gamma_{\bar{\mathrm{A}}_1}) = -\frac{\bar{\mathrm{E}}_1}{4\pi} + \frac{e\rho}{4\pi m}\int Tr(\bar{\mathrm{E}}_1/r) d\mathrm{V} = -\frac{\bar{\mathrm{E}}_2}{4\pi} + \Delta\frac{e^2}{m}\bar{\bar{\mathrm{Z}}}_1 \tag{47}$$

where we have denoted the scalar field ρ by Δ/e. In structure-independent applications we may take for Δ the Dirac function $\delta\,(x-\mathrm{X})\;\delta\,(y-\mathrm{Y})\;\delta\,(z-\mathrm{Z})$ where X, Y, Z is the position of the electron and x, y, z the position where $\bar{\mathrm{E}}_2$ is required. The field $\bar{\mathrm{E}}_2$ is determined by the field $\overline{E}_1$ and reversely; $\overline{E}_2$ exibits both a $1/r$ and a Δ singularity at the origin.

The Hamiltonian is easily found from (45) by remarking that terms in M quadratic in $\dot{\overline{P}}$ and $\dot{\overline{A}}_1$ will stay the same, that the term linear in $\dot{\overline{P}}$ will vanish and that the terms independent of $\dot{\overline{P}}$, $\dot{\overline{A}}_1$ will change sign :

$$H = \frac{P' - e^2 \overline{\overline{A}}_1^2}{2m} + U(\overline{R}) + \frac{1}{8\pi}\int (H_1^2 + E_1^2) dV \qquad (48)$$

$$\text{canonical } p-q \text{ pairs} \rightarrow \overline{P} - \overline{R}_1 \left(= \overline{R} + \frac{e}{m}\overline{\overline{Z}}_1\right)$$

$$\text{and } \overline{A}_1 - E_2/4\pi \left(= \overline{E}_1/4\pi - \Delta\frac{e^2}{m}\overline{\overline{Z}}_1\right)$$

The only canonical coordinates figuring explicitely in this expression are $\overline{P}$ and $\overline{A}_1$ ($= \text{rot}^{-1}\,\overline{H}_1$). E_1 is connected with the canonical field $\overline{E}_2$ through (47) and $\overline{R}$ with the canonical variable $\overline{R}_1$ through (46). We still remark that $\overline{P}, -\overline{R}_1$ is here treated as a $q-p$ pair. Analysing the field in multipole components, only the electric dipole component appears in $\overline{\overline{A}}_1$ and $\overline{\overline{Z}}_1$. For the other components we have so to say the ordinary A — E conjugation known from radiation-free space. A direct check shows that the canonical equations resulting from (48) are identical with (14) and (13) (with Lorentz force neglected).

We will now explicitely introduce the canonical variables $\overline{R}_1$ and $\overline{E}_2$ into (48). We will thereby formally develop in powers of e and reject all powers higher than the second :

$$H = \frac{P^2 - e^2 \overline{\overline{A}}_1^2}{2m} + U\left(\overline{R}_1 - \frac{e}{m}\overline{\overline{Z}}_2\right) + \frac{1}{8\pi}\int \Big(H_1^2 + E_2^2 + \frac{8\pi e^2}{m}\Delta\,\overline{E}_2 . \overline{\overline{Z}}_2\Big) dV$$

Omitting, for simplicity, all indices this gives

$$H = \frac{P^2}{2m} - \frac{e}{m}\Big(\overline{\overline{Z}}\frac{\partial}{\partial\,\overline{R}}\Big)U + \frac{1}{2}\frac{e^2}{m}\Big(\overline{\overline{Z}}\frac{\partial}{\partial\,\overline{R}}\Big)^2 U + \frac{e^2}{2m}\left(-\overline{\overline{A}}^2 + 2\overline{\overline{E}}.\overline{\overline{Z}}\right) + \frac{1}{8\pi}\int (\text{rot}\,\overline{A}^2 + \overline{E}^2) dV \qquad (49)$$

$$\text{Canonical } p-q \text{ pairs} \rightarrow \overline{P} - \overline{R} \text{ and } \frac{\overline{E}}{4\pi} - \overline{A}$$

Let us compare this with the customary non-relativistic Hamiltonian in literature :

$$H = \frac{P^2}{2m} - \frac{e}{m}\bar{\bar{A}}.\bar{P} \qquad + \frac{e^2}{2m}\bar{\bar{A}}^2 \qquad + \frac{1}{8\pi}\int(\text{rot }\bar{A}^2 + \bar{E}^2)dV \quad (50)$$

The term in (49) proportional to e was found by Opechowski in his 1941 paper mentioned on page 1. If we consider it as a perturbation term, proceed in the usual Dirac way and ask for perturbation effects proportional to the first power of e, such as are sufficient for the calculation of emission and absorption of spectral lines (Einstein's A's and B's), it is easily seen that it gives exactly the same result as the corresponding term in (50). In fact, their matrix elements for energy-change zero are the same. This follows for instance from the fact that $\frac{1}{m}\frac{\partial U}{\partial R}$ in the unperturbed motion is equal to $-\ddot{\bar{R}}$, $\frac{\bar{P}}{m}$ to $\dot{\bar{R}}$ and $\bar{A}$ to $\dot{\bar{Z}}$. Therefore the e-proportional term in (49) equals $e\,\bar{\bar{Z}}.\ddot{\bar{R}}$ and that in (50) $-e\,\dot{\bar{\bar{Z}}}.\bar{R}$. The difference is a pure time derivative $\frac{d}{dt}(\bar{\bar{Z}}.\bar{R}.)$ which has no matrix elements for energy change zero. As a third possibility for an e-proportional term with the same matrix elements we mention $e\,\ddot{\bar{\bar{Z}}}.\bar{R} = -e\,\bar{E}.\bar{R}$ ($\bar{E} = -\dot{\bar{A}}$ is the electric field in the unperturbed system) which was used by Kramers and Heisenberg in their 1925 work.

For the scattering of light the Dirac perturbation calculus has to be extended to the second power of e^2. The scattering of the free electron is on (50) described by $\frac{e^2}{2m}\bar{\bar{A}}^2$ only, on (49) by $\frac{e^2}{2m}(-\bar{\bar{A}}^2 + 2\bar{\bar{E}}.\bar{\bar{Z}})$ and again the no-energy-change matrix elements are required. They are clearly equal in both cases since the difference $\bar{\bar{A}}^2 - \bar{\bar{E}}.\bar{\bar{Z}} = \dot{\bar{\bar{Z}}}^2 + \ddot{\bar{\bar{Z}}}.\bar{\bar{Z}}$ is a pure time differential in the unperturbed state.

For the scattering from a bound electron a second order calculation must be carried through as regards the term proportional to e. The results are of course not the same on (49) and on (50) but the difference is, as can easily be shown, just compensated by the first order effect of the e^2 term in (49) which has no analogon in (50). Thus the Kramers-Heisenberg formulae result in both cases.

The result of the second order calculation which would formally lead to the (Lamb) shift of the energy levels is, however, different with the two Hamiltonians. Here we find that the use of ours gives a correction to the use of the customary one which is precisely the correction proposed by Bethe in his (first) derivation of the Lamb shift formula in order to eliminate the divergence introduced by the point model of the electron. This can easily be seen as follows. With (50) the shift of the level m wou'd be (we suppress certain trivial details) :

$$\Delta\,\varepsilon = -\frac{e^2}{m^2}\sum_b\sum_m \frac{(\overline{\mathrm{A}}_{ab}\,.\,\overline{\mathrm{P}}_{mn})\,(\overline{\mathrm{A}}_{ba}\,.\,\overline{\mathrm{P}}_{nm})}{(\mathrm{E}_b+\mathrm{E}_n)\text{-}(\mathrm{E}_a+\mathrm{E}_m)} \cong$$

$$\cong -\text{ const. }\sum_b\sum_m \frac{\mathrm{P}_{mn}\,\mathrm{P}_{nm}/k}{k+k_{nm}} = -\text{ const. }\sum_n \mathrm{P}_{mn}\,\mathrm{P}_{nm}\int\frac{kdk}{k+k_{nm}}$$

Here a denotes the state of no light quantum in empty space, b the state of one light quantum (momentum $\bar{k}$). The summation is over all states b and all other states n of the atom, hk_{nm} being the energy difference between states n and m. Bethe corrects by subtracting the same expression but with k_{nm} everywhere zero and gets

$$\Delta\,\varepsilon = \ldots\int(-\frac{k}{k+k_{nm}}+1)dk = \ldots\int\frac{k_{nm}}{k+k_{nm}}\,dk$$

It was difficult to make his argument quite rigorous but it had certainly physical plausability.

With our Hamiltonian we get from the e-term

$$\Delta_1\,\varepsilon = -\frac{e^2}{m^2}\sum_b\sum_m \frac{(\overline{\mathrm{Z}}_{ab}\cdot\dot{\overline{\mathrm{P}}}_{mn})\,(\overline{\mathrm{Z}}_{ba}\cdot\overline{\mathrm{P}}_{nm})}{(\quad)-(\quad)} \cong$$

$$\cong -\text{ const. }\sum_b\sum_n \frac{\mathrm{P}_{mn}\mathrm{P}_{nm}k_{nm}^2/k^3}{k+k_{nm}} = -\text{ const. }\sum_n \mathrm{P}_{mn}\,\mathrm{P}_{nm}$$

$$\int\frac{k_{nm}^2}{k(k+k_{nm})}\,dk$$

From the e^2-term we must ald

$$\Delta_2\,\varepsilon = \left(\frac{1}{2}\frac{e^2}{m}\sum_{xy}\mathrm{Z}_x\mathrm{Z}_y\mathrm{U}_{xy}\right)_{am,am} = \left(\frac{1}{2}e^2\sum_{xy}\mathrm{Z}_x\mathrm{Z}_y\frac{\mathrm{P}_x\dot{\mathrm{P}}_y-\dot{\mathrm{P}}_y\mathrm{P}_x}{\mathrm{i}\hbar}\right)_{am,am}$$

$$\cong \text{const. }\sum_b\sum_n \mathrm{P}_{mn}\mathrm{P}_{nm}k_{nm}/k^3 = \text{const }\sum_n \mathrm{P}_{mn}\mathrm{P}_{nm}\int\frac{k_{nm}}{k}\,dk$$

Together

$$\Delta\varepsilon = \text{const} \sum_n \mathrm{P}_{mn}\mathrm{P}_{nm}\int\left(-\frac{k_{nm}^2}{k(k+k_{nm})}+\frac{k_{nm}}{k}\right)dk = \ldots\int\frac{k_{nm}}{k+k_{nm}}\,dk$$

On the question of the upper limit of the logarithmically divergent integral this treatment throws of course as little light as Bethe's (first) treatment refered to, the theory being restricted to electric dipole radiation.

V.

FINAL REMARKS

The theory given in the preceding paragraphs has not yet been developed much further. The extension to a system containing more than one electron (always in electric dipole approximation) is straightforward and consists chiefly in the addition of the Coulomb terms and Darwin terms describing the electric and magnetic interactions respectively of the electrons due to the mutual influence of their proper fields. One can say, therefore, that a promising basis is given for a really rigorous scattering theory of atoms in the electric dipole radiation domain. This theory should tell us the exact behaviour of the atoms towards incident light, also within the natural breadths of the absorption and emission lines. We have seen how the notion of the phase shifted light quanta afforded the natural means for the solution of this problem in case of the harmonic oscillator. The problem of the states of steady scattering in arbitrary atoms is a good deal more complicated, of course, not at least due to the Raman effect, but an appropriate choice of phase-shifted light quanta will presumably be helpful also here. There should of course no longer be question of development in powers of e and thus an approximation of the kind involved in (49) will be no help.

A second question is that of the extension of the theory to include also magnetic dipole radiation. Thereby the Compton recoil would be taken care of, be it only in non-relativistic approximation.

In many problems the development of radiation in multipole components will certainly not be appropriate. An example is afforded by the question of a rigorous quantum mechanical dispersion theory of extended matter Here the mathematical handling of the free light quanta will certainly be complicated, even if we restrict ourselves to the model of an assembly of harmonic oscillators.

The present theory shows how the spurious divergencies due to the point model of the electron may be removed to a certain extent. With respect to those due to the zero point fluctuation in the radiation field the situation lies different; with Weisskopf one might say that this problem lies outside the domain of correspondence. The treatment of this question, as well as the establishment of a relativistic theory of charged particles has hitherto been, and will perhaps remain so for some time, a story of artful and ingenious guessing.

The results obtained in this way, from Dirac's theory of 1928 to the beautiful results of the very last year are certainly impressive. Still the fundaments on which they rest are naturally for a large deal derived from « correspondence », so first of all the device of introducing the interaction of radiation with a particle by adding to the momentum p the term $-eA$. Now, we have seen that, in non-relativistic approximation, the elimination of the structure by introducing a « proper » field of bound light quanta does not simply consist in changing this A in A_1, where A_1 represents the free radiation, but involves a much profounder change in the form of the Hamiltonian. Thus one might say that the famous $e\,\alpha\,A$ interaction in Dirac's theory does not even ensure that certain simple, unrelativistic effects will be well rendered by the theory, although it is known to work for the very simplest effects like low energy scattering.

It may of course be that point-model and fluctuation-divergencies can and must ultimately be traced to the same source from the point of view of a future complete theory and that one therefore should not think to hard of the device : first quantizing a wrong Hamiltonian and trying to make amends later on. Still something might be learned from a theory of the sort to which the present paper aims and in which it is tried to analyse the structure-independent features of classical theory first and to see whether and to what extent quantization can learn us something new.

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Discussion du rapport de M. Kramers

Mr. Dirac. — Can one get a connexion between your new Hamiltonian and the usual one by a contact transformation?

Mr. Kramers. — To work with contact transformation is complicated, it is easy when you use a power development of e, but what I try to do is a theory where it is of no importance if the electromagnetic mass is small or not compared with mechanical mass.

Mr. Oppenheimer. — One can work with a simple contact transformation exact to all orders of e, but only in dipole approximation and in the non-relativistic case.

Mr. Dirac. — I think that it should be possible to set up an exact contact transformation which will make Kramer's Hamiltonian more comparable with the usual one.

Mr. Oppenheimer. — You cannot do it, — as things stand to day — taking into account all relativistic terms, you cannot do it if the wave lenght of the radiation is comparable with the size of the material system. But in the dipole approximation you can make an exact contact transformation and then the only difference between Kramers hamiltonian and the usual one is an additional electromagnetic mass.

Mr. Dirac. — I would like to ask Kramers if his scheme allows non-physical solutions.

Mr. Kramers. — It is outside of the spirit of customary electron theory to take this non-physical solutions called by Peierls run-away solutions too seriously in account. Any formula which I derive holds no longer as soon as the wave-length of the electron becomes too small.

In the second part of my report where I made precise calculations I show that you get also run-away solutions, and the field connected with such solutions is confined to a small region (10^{-13} cm.) near the electron.

Some people ask sometimes why I did not treat the theory relativistically. The first reason is that in a relativistic theory there are so many things which are not correspondence-like in the ordinary way. Secondly, I think there does not exist a really consistent classical relativity theory?

It would be a desillusionement for Lorentz, who liked to work with precise models that the work he did with rigid electron could not be done in a simple way with a contractible electron.

If you developed my formula in Fourier series I would not believe in the terms with too short wave-length.

Mr. Oppenheimer. — It is possible to consider the contact transformation in a slightly different way. One can seek to make an electron not subject to an external force behave as an electron does. There we must not expect to find the equation of motion which corresponds to a Lorentz-equation

$$m\,\ddot{x} + 2e^2/3c^3.\,\dddot{x} = \mathrm{F}\,(x) \tag{1}$$

The terms $2e^2/3c^3.\dddot{x}$ is not present with the new hamiltonian if the force is zero. It the force is not zero you get the following equation

$$m\ddot{x} = \mathrm{F}\left(x + \frac{e\check{\mathrm{Z}}}{m}\right)$$

the force is at a displaced position; this is just in accord with Bethe's report. $\check{\mathrm{Z}}$ is the Hertz vector that contains the external field and all of the electron's field but its quasi-static field; it does contain the radiation field

If you take for $\check{\mathrm{Z}}$, as in classical physics you may, no light quantum fluctuations, but just the radiation field given by Lorentz, then you come back to an equivalent of the physically admissible solutions of equation above (1).

If on the other end you put for $\check{\mathrm{Z}}$ the Hertz-vector of the light quantum fluctuations (these are the dominant term for $e^2 \ll \hbar c$), then this equation gives Berthe's expression for the Lamb-shift and the Lewis formula for radiation correction. But here it is necessary to allow for the relativistic convergence effects.

Mr. Bhabha. — The run-away solutions of the first equation are highly singular functions of the electric charge as can be seen by the fact that when $e = 0$, the order of the differential equation drops, the second derivation of the velocity no longer appearing in the equation. The equation then does not have any run-away solutions.

As might be expected from this, the run-away solutions have an essential singularity for $e = 0$, and no expansion in powers of e is possible.

On the other hand the physical solutions can have no such essential singularity, since they must satisfy the requirement that as $e \rightarrow 0$ in them, the solution becomes that of a particle moving with uniform velocity. Hence the physical solutions can be characterised as those which can be expanded in powers of e.

Electron Theory

by J. R. Oppenheimer

In this report I shall try to give an account of the developments of the last year in electrodynamics. It will not be useful to give a complete presentation of the formalism; rather I shall try to pick out the essential logical points of the development, and raise at least some of the questions which may be open, and which bear on an evaluation of the scope of the recent developments, and their place in physical theory. I shall divide the report into three sections : (1) a brief historical summary of related past work in electrodynamics; (2) an account of the logical and procedural aspects of the recent developments; and (3) a series remarks and questions on applications of these developments to nuclear problems and on the question of the closure of electrodynamics.

I.

HISTORY

The problems with which we are concerned go back to the very beginnings of the quantum electrodynamics of Dirac, of Heisenberg and Pauli [1]. This theory, which strove to explore the consequences of complementarity for the electromagnetic field and its interaction, with matter, led to great success in the understanding of emission, absorption and scattering processes, and led as well to a harmonious synthesis of the description of static fields and of light quantum phenomena. But it also led, as was almost at once recognized [2], to paradoxical results, of wich the infinite displacement of spectral terms and lines was an example. One recognized an analogy between these results and the infinite electromagnetic inertia of a point electron in classical theory, according to which electrons moving with

(1) Heisenberg and Pauli, *Zeits. f. Physik.*, **56**, p. 1 (1929).
(2) J. R., Oppenheimer *Phys. Rev.*, **35**, p. 461 (1930).

different mean velocity should have energies infinitely displaced. Yet no attempt at a quantitative interpretation was made, nor was the question raised in a serious way of isolating from the infinite displacements new and typical finite parts clearly separable from the inertial effects. In fact such a program could hardly have been carried through before the discovery of pair production, and an understanding of the far reaching differences in the actual problem of the singularities of quantum electrodynamics from the classical analogue of a point electron interacting with is field. In the former, the field and charge fluctuations of the vacuum — which clearly have no such classical counterpart — play a decisive part; whereas on the other hand the very phenomena of pair production, which so seriously limit the usefulness of a point model of the electron for distances small compared to its Compton wave lenght $\hbar/mc$, in some measure ameliorate, though they do not resolve, the problems of the infinite electromagnetic inertia and of the instability of the electron's charge distribution. These last points first were made clear by the self-energy calculations of Weisskopf (3), and were still further hempasized by the finding, by Pais (4), and by Sakata (5), that to the order e^2 (and to this limitation we shall have repeatedly to return) the electron's self-energy could be made finite, and indeed small, and its stability insured, by introducing forces of small magnitude and essentially *arbitrarily* small range, corresponding to a new field, and quanta of arbitrarily high rest mass (6).

On the other hand the decisive, if classically unfamiliar, role of vacuum fluctuations was perhaps first shown — albeit in a highly academic situation — by Rosenfeld's calculation (7) of the (infinite) gravitational energy of the light quantum, and came prominently into view with the discovery of the problem of the self-energy of the photon due tot the current fluctuations of the electron-positron field, and the related problems of the (infinite) polarizability of that field. Here for the first time the notion of renormalization was introduced. The infinite polarization of vacuum refers in fact just to situations in which a classical definition of charge should be possible (weak, slowly varying fields); if the polarization were finite, the linear

(3) V. Weisskopf, *Zeits. f. Physik.*, **90**, p. 817 (1934).

(4) A. Pais, *Verhandelingen Roy. Ac.*, Amsterdam, **19**, p. 1, (1946).

(5) Sakata and Hara, *Progr. Theor. Phys.*, **2**, p. 30 (1947).

(6) For a recent summary of the state of theory, see A. Pais, *Developments in the Theory of the Electron*, Princeton University Press (1948).

(7) L. Rosenfeld, *Zeits. f. Physik.*, **65**, p. 489 (1930).

constant term could not be measured directly, nor measured in any classically interpretable experiment; only the sum of « true » and induced charge could be measured. Thus it seemed natural to ignore the infinite linear constant polarizability of vacuum, but to attach significance to the finite deviations from this polarization in rapidly varying and in strong fields (8). Direct attempts to measure these deviations were not successful; they are in any case intimately related to these which do descripe the Lamb-Retherford level shift (9), but are too small and of wrong sign to account for the bulk of this observation (10). But the renormalization procedure and philosophy here applied to charges was to prove, in its obvious extension to the electron's mass, the starting point for new developments.

In their application to level shifts, these developments, which could have been carried out at any time during the last fifteen years, required the impetus of experiment to stimulate and verify. Nevertheless, in other closely related problems, results were obtained essentially identical with those required to understand the Lamb-Retherford shift and the Schwinger corrections to the electron's gyromagnetic ratio.

Thus there is the problem — first studied by Bloch, Nordsieck (11), Pauli and Fierz (12), of the radiative corrections to the scattering of a slow electron (of velocity v) by a static potential V. The contribution of electromagnetic inertia is readily eliminated in non-relativistic calculations, and involves some subtelty in relativistic treatment only in the case of spin 1/2 (rather than spin zero) charges (13). It was even pointed out (14) that the new effects of radiation could be summarized by a small supplementary potential.

$$\sim 2/3\,\pi\, e^2/\hbar c(\hbar/_{mc})^2\ \Delta\, \mathrm{V}\ 1n\ c/v \qquad (1)$$

(where e, h, m, c, have their customary meaning). This of course gives the essential explanation of the Lamb shift.

On the other hand the anomalous g-value of the electron was

(8) General treatments : R. Serber, *Phys. Rev.*, **48**, p. 49 (1938) and V. Weisskopf, *Kgl. Dansk. Vidensk. Selskab. Math.-fys. Medd.*, **14**, p. 6 (1936).
(9) Lamb and Retherford, *Phys. Rev.*, **72**, p. 241 (1947).
(10) E. Uehling, *Phys. Rev.*, **48**, p. 55 (1935).
(11) Bloch and Nordsieck, *Phys. Rev.*, **52**, p. 54 (1937).
(12) Pauli and Fierz, *Il Nuovo Cimento*, **15**, p. 167 (1938).
(13) S. Dancoff, *Phys. Rev.*, **55**, p. 959 (1939); H. Lewis, *Phys. Rev.*, **73**, p. 173 (1948).
(14) Shelter Island Conference, June, 1947.

foreshadowed by the remark (15), that in meson theory, and even for neutral mesons, the coupling of nucleon spin and meson fluctuations would give to the sum of neutron and proton moments a value different from (and in non-relativistic estimate less than) the nuclear magneton.

Yet until the advent of reliable experiments on the electron's interaction, these points hardly attracted serious attention; and interest attached rather to exploring the possibilities of a consistent and reasonable modification of electrodynamics, which should preserve its agreement with experience, and yet, for high fields or short wave lengths, introduce such alterations as to make self-energies finite and the electron stable. In this it has proved decisive that it is *not* sufficient to develop a satisfactory classical analogue; rather one must cope directly with the specific quantum phenomena of fluctuation and pair production (6). Within the framework of a continuum theory, with the point interactions of what Dirac (16) calls a « localizable » theory — no such satisfactory theory has been found; one may doubt whether, *within* this framework, such a theory can be formed that is expansible in powers of the electron's charge e. On the other hand, as mentionned earlier, many families of theories are possible which give satisfactory and consistent results to the order e^2.

A further general point which emerged from the study of electrodynamics is that — although the singularities occuring in solutions indicate that it is not a completed consistent theory, the structure of the theory itself gives no indication of a field strength, a maximum frequency of minimum length, beyond which it can no longer consistently be supposed to apply. This last remark holds in particular for the actual electron—for the theory of the Dirac electron-positron field coupled to the Maxwell field. For particles of lower and higher spin, some rough and necessarily ambiguous indications of limiting frequencies and fields do occur.

To these purely theoretical findings, there is a counterpart in experience. No credible evidence, despite much searching, indicates any departure, in the behaviour of electrons and gamma rays, from the expectations of theory. There are, it is true, the extremely weak couplings of β decay; there are the weak electromagnetic interactions of gamma rays, and electrons, with the mesons and nuclear matter.

(15) Fröhlich, Heitler and Kemmer, *Proc. Roy. Soc.*, A **166**, p. 154 (1938).
(16) P. Dirac, *Phys. Rev.*, **78**, p. 1092 (1948).

Yet none of these should give appreciable corrections to the present theory in its characteristic domains of application; they serve merely to suggest that for very small (nuclear) distances, and very high energies, electron theory and electrodynamics will no longer be so clearly separable from other atomic phenomena. In the theory of the electron and the electromagnetic field, we have to do with an almost closed, almost complete system, in which however we look precisely to the absence of complete closure to brings us away from the paradoxes that still inhere in it.

II.

PROCEDURES

The problem then is to see to what extent one can isolate, recognize and postpone the consideration of those quantities, like the electron's mass and charge, for which the present theory gives infinite results — results which, if finite, could hardly be compared with experience in a world in which arbitrary values of the ratio $e^2/\hbar c$ cannot occur. What one can hope to compare with experience is the totality of other consequences of the coupling of charge and field, consequences of which we need to ask : does theory give for them results which are finite, unambiguous and in agreement with experiment?

Judged by these criteria the earliest methods must be characterized as encouraging but inadequate. They rested, as have to date *all* treatments not severely limited throughout by the neglect of relativity, recoil, and pair formation, on an expansion in powers of e, going characteristically to the order e^2. One carried out the calculation of the problem in question; (for radiative scattering corrections, Lewis (17), for the Lamb shift, Lamb and Kroll (18), Weisskopf and French (19), Bethe (25); for the electron's g-value, Luttinger (21); one also calculated to the same order the electron's electromagnetic mass, its charge, and the charge induced by external fields, and the light quantum mass; finally one asked for the effect of these changes in charge and mass on the problem in question,

(17) H. Lewis, *Phys. Rev.*, **73**, p. 173 (1948).
(18) Lamb and Kroll, *Phys. Rev.* (in press).
(19) Weisskopf and French, *Phys. Rev.* (in press).
(20) H. Bethe, *Phys. Rev.*, **72**, p. 339 (1947).
(21) P. Luttinger, *Phys. Rev.*, **74**, p. 893 (1948).

and sought to delete the corresponding terms from the direct calculation. Such a procedure would no doubt be satisfactory — if cumbersome — were all quantities involved finite and unambiguous. In fact, since mass and charge corrections are in general represented by logarithmically divergent integrals, the above outlined procedure serves to obtain finite, but not necessarily unique or correct, reactive corrections for the behaviour of an electron in an external field; and a special tact is necessary, such as that implicit in Luttinger's derivation of the electron's anomalous gyromagnetic ratio, if results are to be, not merely plausible, but unambiguous and sound. Since, in more complex problems, and in calculations carried to higher order in e, this straightforward procedure becomes more and more ambiguous, and the results more dependent on the choice of Lorentz frame and of gauge, more powerful methods are required. Their development has accurred in two steps, the first largely, the second almost wholly, due to Schwinger (22).

The first step is to introduce a change in representation, a contact transformation, which seeks, for a single electron not subject to external fields, and in the absence of light quanta, to describe the electron in terms of classically measurable charge e and mass m, and eliminate entirely all « virtual » interaction with the fluctuations of electromagnetic and pair fields. In the non-relativistic limit, as was discussed in connection with Kramer's report (23), and as is more fully described in Bethe's (24), this transformation can be carried out rigorously to all powers of e, without expansion; in fact, the unitary transformation is given by

$$\mathrm{U} = \exp \frac{e}{mc}(\dot{\mathrm{Z}} \cdot \nabla) \tag{2}$$

where Z is the (transverse) Hertz vector of the electromagnetic field minus the quasi-static field of the electron. When this formalism is applied to the problem of an electron in an external field, it yields reactive corrections which do not converge for frequences $\mathrm{V} < mc^2/\hbar$, thus indicating the need for a fuller consideration of typical relativistic effects.

This generalization is in fact straightforward; yet here it would appear essential that the power series expansion in e is no longer

(22) J. Schwinger, *Pys. Rev.*, **74**, p. 1439 (1948) and in press.
(23) Report to the 8th Solvay Conference.
(24) Report to the 8th Solvay Conference.

avoidable, not only because no such simple solution as (2) now exists, but because, owing to the possibilities of pair creation and annihilation, and of interactions of light quanta with each other, the very definition of states of single electrons or single photons depends essentially on the expansion in question ([25]). However that may be, the work has so far been carried out only by treating $e^2/\hbar c$ as small, and essentially only to include corrections of the first order in that quantity.

In this form, the contact transformation clearly yields :

(*a*) an infinite term in the electron's electromagnetic inertia;

(*b*) an ambiguous light quantum self-energy;

(*c*) no other effects for a single electron or photon;

(*d*) interactions of order e^2 between electrons, positrons, and photons, which in this order, correspond to the familiar Moeller interactions and Compton effect and pair production probabilities;

(*e*) an infinite vacuum polarizability;

(*f*) the familiar frequency-dependent finite polarizability for external electromagnetic fields;

(*g*) emission and absorption probabilities equivalent to these of the Dirac theory for an electron in an external e. m. field;

(*h*) now reactive corrections of order e^2 to the effective charge and distribution of an electron correspond to vanishing total supplementary charge, and to currents of the order $\frac{e^3}{\hbar c}$ distributed over dimensions of the order $\hbar/mc$, and which include the supplementary potential (1), and the supplementary magnetic moment $\frac{e^2}{2\pi\hbar c}\frac{e\hbar}{2mc}\vec{\sigma}$ as special (non-relativistic) limiting cases.

Were such calculations to be carried further, to higher order in e, they would lead to still further renormalizations of charge and mass,

([25]) This may be seen very strikingly in writing down an explicit solution for the Tomonaga equation (3) below. Formally it is :

$$\Psi(\sigma) = \text{« exp »}\left[\frac{1}{\hbar c}\int_{\sigma_0}^{\sigma} j\mu\, A^{\mu}\, d_4x\right] \Psi(\sigma_0)$$

In order to define the « exp », we have at present no other resort than to approximate by a power series, where the ordering of the non-commuting factors $j\mu\ A\mu$ at different points of space-time can be simply prescribed (e. g., the later factor to the left). Cf. especially F. J. Dyson, *Phys. Rev.* (in press).

to the successive elimination of all « virtual » interactions, and to reactive corrections, in the form of an expansion in powers of $e^2/\hbar c$, to the probabilities of transitions : pair production, collisions, scattering, etc. Nevertheless, before such a program could be undertaken, or the physically interesting new terms (*h*) above be taken ad correct, a new development is required. The reason for this is the following : the results (*h*) are not in general independent of gauge and Lorentz frame. Historically this was first discovered by comparison of the supplementary magnetic interaction energy in a uniform magnetostatic field H

$$e^2/2\,\pi\,\hbar c\,\frac{e\hbar}{2mc}\;(\vec{\sigma}\cdot\vec{H})$$

with the supplementary (imaginary) electric dipole interaction which appeared with an electron in a homogeneous electric field E derived from a static scalar potential.

$$e^2/6\,\pi\,\hbar c\,\frac{e\hbar}{2mc}\,i\,\rho_2\,(\vec{\sigma}\cdot\vec{E})$$

a manifestly non-covariant result.

Now it is true that the fundamental equations of quantum-electrodynamics are gauge and Lorentz covariant. But they have in a strict sense no solutions expansible in powers of *e*. If one wishes to explore these solutions, bearing in mind that certain infinite terms will, in a later theory, no longer be infinite, one needs a covariant way of identifying these terms; and for that, not merely the field equations themselves, but the whole method of approximation and solution must at all stages preserve covariance. This means that the familiar Hamiltonian methods, which imply a fixed Lorentz frame t = constant, must be renounced; neither Lorentz frame nor gauge can be specified until after, in a given order in *e*, all terms have been identified, and those bearing on the definition of charge and mass recognized and relegated; then of course, in the actual calculation of transition probabilities and the reactive corrections to them, or in the determination of stationary states in fields which can be treated as static, and in the reactive corrections thereto, the introduction of a definite coordinate system and gauge for these no longer singular and completely well defined terms can lead to no difficulty.

It is probable that, at least to order e^2, more than one covariant formalism can be developped. Thus Stueckelberg's 4-dimensional perturbation theory (26) would seem to offer a suitable starting point, as also do the related algorithms of Feynman (27). But a method originally suggested by Tomogana (28), and independently developed and applied by Schwinger (22), would seem, apart from its practicality, to have the advantage of very great generality and a complete conceptual consistency. It has been shown by Dyson (29) how Feynman's algorithms can be derived from the Tomogana equations.

The easiest way to come to this is to start with the equations of motion of the coupled Dirac and Maxwell field. These are gauge and Lorentz covariants. The commutation laws, through which the typical quantum features are introduced, can readily be rewritten in covariant form to shown: (1) at points outside the light cone from each other, all field quantities commute; and (2) the integral over an *arbitrary space-like* hypersurface yields a simple finite value for the commutator of a field variable at a variable point on the hypersurface, and that of another field variable at a fixed point on the hypersurface.

In this Heisenberg representation, the state vector is of course constant; commutators of field quantities separated by time-like intervals, depending on the solution of the coupled equation of motion, can not be known a priori; and no direct progress at either a rigorous or an approximate solution in powers of e has been made. But a simple change to a mixed representation, that introduced by Tomonaga and called by Schwinger the « interaction representation », makes it possible to carry out the covariant analogue of the power series contact transformation of the Hamiltonian theory.

The change of representation involved is a contact transformation to a system in which the state vector is no longer constant, but in which it would be constant if there were no coupling between the fields, i. e., if the elementary charge $e = 0$. The basis of this representation is the solution of the uncomplex field equation, which, together with their commutators at all relative positions, are of course well known. This transformation leads directly to the Tomogana equation for the variation of the state vector :

$$\frac{\delta \psi}{\delta \sigma} = -\frac{1}{c} j^{\mu}(\mathrm{P})\, \mathrm{A}_{\mu}(\mathrm{P})\, \psi \tag{3}$$

(26) Stueckelberg, *Am. der Phys.*, **21**, p. 367 (1934).
(27) R. Feynman, *Phys. Rev.*, **74**, p. 1430 (1948).
(28) S. Tomonaga, *Progr. Theor. Phys.*, **1**, pp. 27 and 109 (1946).
(29) F. Dyson, *Phys, Rev.* (in press).

Here σ is an arbitrary space-like surface through the point P. $\delta \psi$ is the variation in ψ when a small variation is made in σ, localized near the point P; $\delta \sigma$ is the 4-volume between varied and unvaried surface; $A_m(P)$ is the operator of the 4-vector electromagnetic potential at P; $j_\mu(P)$ is the (charge-symmetrized) operator of electron-positron 4-vector current density at the same point.

It may be of interest, in judging the range of applicability of these methods, to note that in the theory of the charged particle of zero spin (the scalar and not Dirac pair field), the Tomonaga equation does not have the simple form (3); the operator on ψ on the right involves explicity an arbitrary time-like unit vector ([30]).

Schwinger's program is then to eliminate the terms of order e, e^2, and so, in so far as possible, from the right hand side of (3). As before, only the « virtual » transitions can be eliminated by contact transformation; the real transitions of course remain, but with transition amplitudes eventually themselves modified by reactive corrections.

Apart from the obvious resulting covariance of mass and charge corrections, a new point appears for the light quantum self-energy, which now appears in the form of a product of a factor which must be zero on invariance grounds, and an infinite factor. As long as this term is identifiable, it must of course be zero in any gauge and Lorentz invariant formulation; in these calculations for the first time it is possible to make it zero. Yet even here, if one attempts to evaluate directly the product of zero factor and infinite integral, indeterminate, infinite, or even finite ([31]) values may result. A somewhat similar situation obtains in the problem, so much studied by Pais, of the direct evaluation of the stress in the electron's rest system, where a direct calculation yields the value $\left(-e^2/2\pi hc\right) mc^2$, instead of the value zero which follows at once as the limit of the zero value holding uniformly, in this order e^2, for the theory rendered convergent by the f-quantum hypothesis, even for arbitrarily high f-quantum mass. These examples, far from casting doubt on the usefulness of the formalism, may just serve to emphasize the importance of identifying and evaluating such terms without any specialization of coordinate system, and utilizing throughout the covariance of the theory.

([30]) Kanesawa and Tomonaga, *Progr. Theor. Phys.*, **3**, 1, p. 107 (1948).
([31]) G. Wentzel, *Phys. Rev.*, **74**, p. 1070 (1948).

To order e^2, one again finds the terms (*a*) to (*h*) listed above; the covariance of the new reactive terms is now apparent; and they exhibit themselves again but more clearly as supplementary currents, corresponding to charge distribution of order $e^3/\hbar c$ (but vanishing total charge) extended throughout the interior of the light cones about the electron's position, and of special dimensions $\sim \hbar/mc$; inversely, they may also be interpreted as corrections of relative order $e^2/\hbar c$ and static range $\hbar/mc$ to the external fields. The supplementary currents immediately make possible simple treatments of the electron in external fields (where neither the electron's velocity, nor the derivatives of the fields need be treated small), and so give corrections for emission, absorption and scattering processes to the extent at least in which the fields may be classically described ([32]), the reactive corrections to the Møller interaction and to pair production can probably not be derived without carrying the contact transformation to order e^4, since for these typical exchange effects, not included in the classical description of fields, must be expected to appear.

At the moment, to my best present knowledge, the reactive corrections agree with the S level displacements of H to about 1%, the present limit of experimental accuracy. For ionized helium, and for the correction to the electron's g-value, the agreement is again within experimental precision, which in this case, however, is not yet so high.

III.

QUESTIONS

Even this brief summery of developments will lead us to ask a number of questions :

(1) Can the development be carried further, to higher powers of e, (*a*) with finite results, (*b*) with unique results, (*c*) with results in agreement with experiment?

(2) Can the procedure be freed of the expansion in e, and carried out rigorously?

([32]) See for instance results reported to this conference by Pauli on corrections to the Compton effect for long wave lengths.

(3) How general is the circumstance that the only quantities which are not, in this theory, finite, are those like the electromagnetic inertia of electrons, and the polarizarion effects of charge, which cannot directly be measured within the framework of the theory? Will this hold for charged particles of other spin?

(4) Can these methods be applied to the Yukawa-meson fields of nucleons? Does the resulting power series in the coupling constant converge at all? Do the corrections improve agreement with experience? Can one expect that when the coupling is large there is any valid content to the Maxwell-Yukawa analogy?

(5) In what sense, or to what extent, is electrodynamics — the theory of Dirac pairs and the e. m. field — « closed »?

There is very little experience to draw on for answering this battery of questions. So far there has not yet been a complete treatment of the electron problem in order higher than e^2, although preliminary study [33] indicates that here too the physically interesting corrections will be finite.

The experience in the meson fields is still very limited. With the pseudoscalar theory, Case [34] has indeed shown that the magnetic moment of the neutron is finite (this has nothing to do with the present technical developments), and that the sum of neutron and proton moments, minus the nuclear magneton (which is the analogue of the electrons anomalous-g value) is of the same order as the neutron moment, finite, and in disagreement with experience. The proton-neutron mass difference is infinite and of the wrong sign; the reactive corrections to nuclear forces, formally analogous to the corrections to the Møller interaction, have not been evaluated. Despite these discouragements, it would seem premature to evaluate the prospects without further evidence.

Yet it is tempting to suppose that these new successes of electrodynamics, which extend its range very considerably beyond what had earlier been believed possible, can themselves be traced to a rather simple general feature. As we have noted, both from the formal and from the physical side, electrodynamics is an almost closed subject; changes limited to very small distances, and having little effect even in the typical relativistic domain E-mc^2, could suffice

[33] F. Dyson, *Phys. Rev.* (in press).
[34] K. Case, *Phys. Rev.*, **74**, p. 1884 (1948).

to make a consistent theory; in fact, only weak and remote interactions appear to carry us out of the domain of electrodynamics, into that of the mesons, the nuclei, and the other elementary particles. Similar successes could perhaps be expected for those mesons (which may well also be described by Dirac-fields), which also show only weak non-electromagnetic interactions. But for mesons and nucleons generally, we are in a quite new world, where the special features of almost complete closure that characterizes electrodynamics is quite absent. That electrodynamics is also not quite closed is indicated, not alone by the fact that for finite $e^2/\hbar c$ the present theory is not after all-consistent, but equally by the existence of those small interactions with other forms of matter to which we must in the end look for a clue, both for consistency, and for the actual value of the electron's charge.

I hope that even those speculations may suffice as a stimulus and an introduction to further discussion.

Discussion du rapport de Mr. Oppenheimer

Mr. Dirac. — Oppenheimer has given us an account of the present situation of the theory of electrons, but there is one aspect he did not mention and I would like to bring up. All the infinities that are continually bothering us arise when we use a perturbation method, when we try to expand the solution of the wave equation as a power series in the electron charge. Suppose we look at the equations without using a perturbation method, then there is no reason to believe that infinities would occur. The problem, to solve the equations without using perturbation methods, is of course very difficult mathematically, but it can be done in some simple cases. For example, for a single electron by itself one can work out very easily the solutions without using perturbation methods and one gets solutions without infinities. I think it is true also for several electrons, and probably it is true generally : we would not get infinities if we solve the wave equations without using a perturbation method.

If we look at the solutions which we obtain in this way, we meet another difficulty : namely we have the run-away electrons appearing. Most of the terms in our wave functions will correspond to electrons which are running away, in the sense we have discussed yesterday and cannot correspond to anything physical. Thus nearly all the terms in the wave functions have to be discarded, according to present ideas. Only a small part of the wave function has a physical meaning. We now have the problem of picking out the very small physical part of the exact solution of the wave equation. That is a problem which has not been solved yet. I hope to look into it in greater detail, in the future

If one takes the trivial example of a single electron all alone not perturbed by any field, then it is a simple matter to separate out the run-away part of the wave function from the other terms. One is then left with just what one wants, the electron moving with a constant velocity, and having a mass equal to the original mass parameter appearing in the equation.

Handling the problem by this method, there is thus no mass renormalisation. This is perhaps a general result, and it may be that all the terms which in the treatment Oppenheimer gave are to be counted as a mass renormalisation or charge renormalisation would appear if one did not use a perturbation method as terms which should be discarded as corresponding to run-away motions. One would then have more a satisfactory reasons for discarding these terms. One could hope in this way to get a better basis for founding the separation of the important terms from the terms that have to be discarded.

Mr. Oppenheimer. — I think these remarks of Dirac raise points to which more than one person may want to respond. In the first place, there is a real difference, I think, in the electrodynamics that you are solving and the electrodynamics we have been talking about. The λ — limiting process is a real change, and does alter the situation. I am doubstful that the unmodificed equations are rigorously soluble.

Mr. Bohr. — This whole question of solving by approximations is a problem that I quite understand, and also for instance the points raised by Dirac. But I think we must look upon the physical problem before us. In that way, in very many points, I must deeply sympathise with the approach by Oppenheimer. I think first of all we must think how it has stood in physics. There is hardly any practical way, and that has been through ages, to attack a problem, except by approximations. One has had to find a possibility to eliminate smaller terms to get on at all. Now it has been found that it was possible to transform it often into a more and more general treatment. And the way in which this was made into a far more closed system by the relativity theory... But nowadays we must realize that these ideas do not correspond to the actual state of physics. This was all done before one appreciated the problem of the quantum of action. But we now know that we have an idealisation of force which actually does not correspond in very essential points to the actual situation of physics.

The problem next is the quantisation.

There it depends how far the electron in electrodynamical problem is a closed problem by itself, in the way that has been talked upon by Oppenheimer. I must say that I think on the whole situation on that way that we have in quantum electrodynamics really gradually been able to see a kind of treatment of force which counts a very large field of application, but which we at the same time have neither reason to ask nor to expect that it is anything of a complete description, as we know, as Oppenheimer has said that there are other aspects of the natural phenomena as fundamental for our whole physical picture, as the electron and the radiation, which ought to have a connexion with other problems. And therefore I think that on the whole, when one looks upon the problem of proceeding by approximation, this must be looked at into the case from the very beginning. And there is to my mind just very clear indications that the whole succes of the present attack rests upon its procedure by approximation, while we cannot otherwise separate from the other features that will come in.

All I want to say, is that it is clear to me that we need a new departure, the way has not yet been indicated, but it will probably become clear when we consider other problems.

Mr. Oppenheimer. — I agree, but there is probably one point I did not stress enough. In electrodynamics there are no great questions of a practical nature. But in the meson-nucleon problems, everything is wrong. The crucial problem is, whether, as Schwinger and I at one time believed, the real importance of his work lay in the fact that one had an entirely new way of dealing with the Maxwell — Yukawa analogy, or whether this analogy is rubbish, and that is a big question. And I only wanted to be sure that I made it clear that is was that question, and not whether one can calculate to one part to 10^8 th, the Lamb shift. That seems to be worth discussing and above all worth working upon, even if one thinks the results would be negative. But one has to find that out.

Mr. Pauli. — I just want to make some remarks on Dirac. I have the impression that Dirac's statement about the existence of rigourous solutions in quantum electrodynamics is incorrect, because this rigorous solutions for one electron alone in a quantised electrodynamics is there obtained only if negative energy photons are introduced. But then the theory has no connexion with nature. I never saw any proof that even if you admit the λ-limiting processes, a rigorous solution for one electron alone exists in a quantised theory. There is no paper where such a proof has been given. I admit there is no proof either of the contrary. I also wish to emphasize that by discarding « run a way parts » in a solution the residue is not any longer a solution of the original equation.

Mr. Dirac. — A solution without negative photons is given in a paper of mine published in the Communications of the Dublin Institute for Advance Studie, n° 3. I suggested an approximate treatment, based on a one-dimensional solution of the wave equation, but this approximation is not needed for seeing that an exact solution exists in the three-dimensional case

Mr. Bhabha. — In the interaction between mesons and nucleons, there are several changes which have been introduced from the Maxwell case, for example one has introduced the spin interaction, then the mesons are taken with charges in some cases and so on. Then of course there is a principle difference, namely that there is a mass attached to the particle. Now, I was wondering if the question had been considered, in which only one such difference is taken into account, and the essential one, namely the interaction of a neutral meson field with a nucleon, without spin coupling. This is the real analogous case of the Maxwell interaction with the electron. In that case I would be very surprised if the theory gave something essentially different from the Maxwell case. I know that the general hamiltonian has terms which look very radically different, but if that was taken properly, probably the same answer would be obtained.

Mr. Oppenheimer. — This as been looked at in a sort of easy-going way, enough to know that for a neutral scalar meson field, there is no analog of the Lamb shift.

Mr. Bhabha. — Does not the finite mass of the meson make a difference?

Mr. Oppenheimer. — Then there is of course the essential difference that there is an (infinite) renormalization of meson mass required.

Mr. Casimir. — So far no reference has been made to the zero point energy of free space. If we expand the electromagnetic field into plane waves and write down $1/2 \Sigma h\nu$ this expression is wildly divergent. I know of course that there have been proposed certain formal procedures by means of which one might get rid of this infinity more easily than of other infinities. And I think that Schwinger has argued as follows: since this energy is a property of empty space it has to be zero in a relativistically covariant and gauge invariant field theory. But on the other hand if we take a resonant cavity and look at the standing waves inside this cavity they are just as real physical entities as for instance the elastic waves in a crystal and as far as I can see each of their waves should certainly have the energy $1/2\, h\nu$. I should like to add that the *change* of the quantity $\Sigma\, 1/2\, h\nu$ introduced by changing the distance between two walls or between two polarisable particles has certainly a physical meaning. As a matter of fact the calculation of such changes is a convenient mathematical procedure for calculating Van der Waals forces.

Mr. Heitler. — There is a certain danger in accepting the renormalization of the charge due to the polarization of the vacuum too freely as a physical fact. The charges of the fundamental particles are all the same within an exceedingly high accuracy. Unless we are prepared to assume that the original charges are different, the renormalization correction δe must be the same for all fundamental particles and independent of their mass and spin at least to within the first few orders of e^2/hc. In the present formalism δe diverges, but if one cuts off somehow it is not obvious — at least I dont see any obvious reason — that δe should really be independent of the spins and masses. The conservation of total charge is no argument, because δe is always compensated by a charge $-\delta e$ removed to infinity (like in any polarized medium; the total charge of the created pairs is always zero).

Communication de M. W. Pauli

I shall speak about some applications of the formalisms due to Weisskopf, Schwinger and Tomonaga about which Oppenheimer has already reported. The example treated in Zürich is the correction to the cross section for scattering of photons by free charged particles (Compton-effect) of the relative order of magnitude $e^2/\hbar c$.

In order to get rid of divergent terms one has to separate in the result of present quantum electrodynamics the effects due to the selfenergy of both the charged particles and the photons, and also of those polarisation effects which appear as multiplication of the electric charges by an infinite constant (renormalisation of the charge). For charged particles with Spin 0 (Bosons) the calculations have been made by *R. Jost* and *E. Corinaldesi*, for electrons (Spin 1/2) by *R. Schafroth*. For a first orientation the calculations were made using a procedure which is not Lorentz invariant in itself (mostly because of the elimination of the longitudinal photons). Also the question of uniqueness needs a further investigation. As a consequence of the interaction of the real mesons with other particles than photons (nucleons) the results for Bosons may have only an academic interest but it seems to me nevertheless instructive to compare them with the results for electrons. I shall restrict myself to writing down the correction terms for the light scattering cross section only for small frequencies of the incident light precisely for $\varkappa \ll \mu$ (see explanation of notations below). Moreover terms of the order $(\varkappa/\mu)^2$ without a logarithmic factor are not written down here, which is indicated by $+ \dots$ at the end of the formulas.

R. Schafroth (electron Spin 1/2).

$$d\sigma_6 = \frac{r_0^2}{137} \; \frac{d\Omega}{4\pi} \left(\frac{\varkappa}{\mu}\right)^2 \left\{ \left(1 + \cos^2 \theta\right)\left(2 \log \frac{\varkappa}{\mu} + \frac{8}{3} \log \frac{\omega}{\mu}\right) - \frac{8}{3}(1 + \cos^2 \theta) \cos \theta \log \frac{\omega}{\mu} - \frac{4}{3}(1 - \cos^2 \theta) \cos \theta \log \frac{\varkappa}{\mu} \right\} + \dots$$

$\theta =$ Scattering angle, $r_o = \dfrac{e^2}{mc^2}$

$\varkappa =$ energy of photon

$\mu =$ rest energy of electron

$\omega =$ upper bound for low frequency photons

R. *Jost and E. Corinaldesi* (Spin 0 particles)

$$d\sigma_6 = \frac{r_o^2}{137}\ \frac{d\Omega}{4\pi}\left(\frac{\varkappa}{\mu}\right)^2\left\{\left(1+\cos^2\theta\right)\left(-8\log\frac{\varkappa}{\mu}+\frac{8}{3}\log\frac{\omega}{\mu}\right)\right.$$
$$-\frac{8}{3}\left(1+\cos^2\theta\right)\cos\theta\left(\log\frac{\varkappa}{\mu}+\log\frac{\omega}{\mu}\right)$$
$$\left.-\frac{16}{3}\left(1-\cos^2\theta\right)\cos\theta\log\frac{\varkappa}{\mu}\right\}+\ldots$$

Some comment is needed regarding the significance of the frequency ω. This upper bound for low frequency photons is not a mathematical cut off but a property of the processes for which the cross section is computed. One adds the corrections for the ordinary scattering of a single photon to the cross section for the scattering of two photons one of which is assumed to have a frequency smaller than ω. In the considered order of magnitude in $e^2/\hbar c$ a cancellation of two well known infinities with opposite sign occurs then for small frequencies so that for this question, as first shown by Lewis, the socalled infrared difficulties are removed. In this connection it may be noted that the numerical factor of $\log\dfrac{\omega}{\mu}$ is the same in the two cases of spin 0 and spin 1/2 particles while the factor of $\log\dfrac{\varkappa}{\mu}$ is different. The problem of the shape of the Compton line needs further investigation.

I shall also speak about a method of *Luttinger* to compute the corrections to the magnetic moments of a particle due to the change of its self energy in an external magnetic field. For particles with spin 1/2 and a gyromagnetic coefficient $g = 2$ Luttinger could avoid more complicated rules of substraction by using simple properties of the ground state of such particles in a homogenous magnetic field. Due to the cancellation of the magnetic energy of the orbital moment and the spin moment, the energy value of this particular state is independent of the magnetic field strengths, whatever the value of charge and mass of the particles may be, if only its g value is exactly equal 2. By developing the self energy of the considered particle in its ground state in presence of an external homogenous magnetic field in powers of the field strengths, the term linear in this field strength, which is finite in the first approximation of the perturbation theory, can directly interpreted as due to a change of the g value and therefore of the magnetic moment of the particle.

The application of the method to electrons in interaction with the electromagnetic field (1) gives exactly Schwinger's result for the correction of the electrons magnetic moment of the order of magnitude $e^2/\hbar c$.

The method can also be applied to the magnetic moments of protons and neutrons. Taking all relativistic effects for the nucleons into account, Luttinger obtained a finite result for charged and neutral pseudoscalar mesons which, however, is in disagreement with experiment (2). For the ratio of the magnetic moments of proton and neutron this disagreement is even independent of the assumed value of the coupling constant. The reason for this failure of the theory can either be due to the application of perturbation theory with respect to the coupling constant or the entire model of the interaction of nucleons with mesons needs essential modifications. For vector mesons *F. Villars* has shown that the method in question gives an infinite magnetic moment of the nucleons.

Mr. Heitler. — An effect similar to the magnetic moments of the proton and neutron is the charge-distribution due to the meson field around the proton and neutron. Non-relativistically this had been calculated by Fröhlich, Kahn and myself many years ago and it was found that the total charges diverges. If one cuts off at a distance of the order $\hbar/\mu c$, one gets the right order of the meutron-electron interaction. Relativistic calculations have been begun by Mr. Stotnick, they are completed yet, but it looks as if the total charge was still diverging.

(1) J. M. Luttinger, *Phys. Rev.*, **74**, p. 893 (1948).
(2) *Helv. Phys. Acta* (in press).

Self-energy problems

by R. E. Peierls

I.

INTRODUCTION

It is well known that the infinite self-energy resulting from almost all forms of field theory that have been studied is one of the most important obstacles to the progress of fundamental theory. The position has remained virtually unchanged for about fifteen years in so far as there still exists no formalism which is free from singularities, compatible with the requirements of Lorentz invariance and other general laws, and which, in the domain to which ordinary non-relativistic quantum theory is applicable, represents the earlier results already achieved there and confirmed by experiment.

Nevertheless the subject has developped in several ways. Firstly, a number of attempts to remove the difficulties have been investigated and while a few of them lead to mathematical problems of such complexity that no conclusions have been drawn with certainty, most of them have been shown to fail. The reasons for these failures have become appreciated more clearly and, as a result, we can now discriminate more easily between possible hypotheses and thus limit the kind of theories that are still worth investigating. Secondly, it is now known that physics is much richer in elementary particles (or at least in particles which we have not yet succeeded to deriv from fewer elementary units) than was believed some fifteen years argo. At that time it appeared as if the proton and electron with their necessary counterparts, the positron and negative proton, and possibly the neutrino, might comprise all the fundamental units of physics. It was then expected that it remained merely to clear up the theory of their interactions nad that, therefore, this theory ought to be in the form of some very simple basic laws. Since then, the discovery of the neutron and of a variety of new particles, all referred to as mesons, has shown nature to be much more complex, and no-one would expect today that fundamental theory can be completed in one final stop. It is known today whether a problem like the electromagnetic self-energy can be solved on its own, or whether only a formalism which accounts also for all other interactions could have a chance of success. But at any rate a satisfactory theory of this self-energy, if it is possible, would not represent the last word and one is, therefore, more inclined to accept formalisms which will contain arbitrary parameters or arbitrary functions which at this stage do not come out of the theory but have to be taken from experiment. Equally it would not be surprising if the formalism would not finally settle the value of the fine structure constant or the ratio of proton to electron mass.

Lastly, progress has been made in adding to the list of problems to which one can obtain finite answers (if with procedures of doubtful legitimacy) which can be checked against experiments. The fact that these answers seem to be right further adds to the condition that have to be satisfied by any complete formalism and may, therefore, be a useful guide in further developments to the theory. This last aspect is being covered in the report by Bethe and I do not, therefore, propose to deal with it in this report.

Section II below lists a number of attempts that have been made to modify classical field theory in a way which removes the infinities. Section III then sketches the present position as regards the conventional quantum field theory and its relation to « hole theory ». Section IV deals with the extent to which the various modified classical theories can be translated into quantum formalisms and also with those modifications of the theory which are possible only in quantum theory and have no classical counterparts. It will not be possible in a report of this nature to deal with every single attempt that has been made; the choice will be governed partly by the limits of my knowledge, and partly by the extent to which any such attempts appears to teach a lesson about the nature of the problem.

Section V contains some remarks about the conditions in which new developments of field theory might become accessible to experimental test.

For brevity, I shall throughout this article refer to the interaction of an electron with its own electromagnetic field. Very similar problems arise from the interaction of a proton or a meson with its own electromagnetic field, or a proton or neutron with the nuclear field. Even a photon has a self-energy due to the virtual production of pairs.

II.

CLASSICAL THEORY. (1) **Conventional Theory**

The question is raised from time to time whether the singularities usually obtained are inherent in classical theory, or whether they arise from misuse of the concepts of field theory. I shall, therefore, begin by describing in a few words the origin of these difficulties in the classical case.

The most satisfactory derivation of the field equations and the one which is most convenient for translation into quantum theory rests on a formulation of the problem in terms of a Hamiltonian. Alternatively, one can start from an action principle (Lagrangian) which shows up the Lorentz invariance more clearly and which usually, with exceptions to which we shall refer later, can easily be translated into Hamiltonian form. In that case the total energy of the system is a sum of the kinetic energies and rest energies of all particles plus the field energy, which is the integral of a positive energy density. In this form, the ordinary Coulomb attraction between two particles of opposite charge is due to the reduction of the field energy as the particles approach and thereby partly cancel each other's field. In classical theory the work available as the particles approach indefinitely closely is infinitely large. Since all this work represents the reduction of the field energy (which is zero when the two opposite and equal point charges coincide) it is evident that the energy stored when the particles are at any finite distance must be infinitely large. In relativistic theory infinite energy brings with it infinite mass and infinite inertia.

In order to elucidate this well-known difficulty Lorentz and others considered equations in which the point charge was replaced by a charge distribution of finite extent, (radius a). This procedure is in any case helpful as a limiting process in which one may ultimately consider the limit of an infinitely small radius. The original investigations of this were carried out at a time when the theory of relativity itself was in an early stage of development and when it was neither evident that a physical theory should be Lorentz invariant in content (if not necessarily in form), nor what such invariance implied. We now known that the motion of a rigid body is incompatible with relativity even if one attempts to allow for Lorentz contraction. The reason is roughly that the rigid body contains only a small number of degrees of freedom. Hence disturbing forces acting on one end of it must either have no effect at all, or an effect which is instantaneously felt over the whole body so that it would propagate through the body with infinite velocity contrary to the basic concepts of relativity. One way out of this would, of course, be to postulate a charge distribution which is not rigid but elastic, stabilized by some unknown force of great strength so that a particle would be capable of internal vibration of high frequency. While there is no rigorous argument to exclude such a possibility, such an idea is generally considered unattractive and it is also likely that it would merely postpone the solution of the difficulty, since the internal vibrations would again have to be described by field equations which on closer analysis would, no doubt, again lead to singularities.

Lorentz himself studied the equations of a finite charge distribution without postulating strict invariance. He showed that for motions in which the velocities and their time derivatives of first and higher order varied little over times of the order of the characteristic time τ of the particle ($\tau = a/c$, a being the radius), the leading terms in the equation were an inertia term and radiation damping term. The damping term is independent of the details of the charge distribution and remains finite as a tends to zero. The inertia term would become infinitely large in the limit, and for finite a is not, in general, Lorentz invariant. It can be made invariant if one postulates that the charge distribution is subject to Lorentz contraction, although, as I have pointed out, this postulate leads to difficulties in describing accelerated motion.

The Lorentz equation is

$$m\ddot{x} + \frac{2}{3}\frac{e^2}{c^3}\left(\dddot{x} - \frac{\ddot{x}^2\,\dot{x}}{1+\dot{x}^2}\right) = \mathrm{F} \qquad \text{(II.1)}$$

where x is the coordinate of the electron, s the length along the world line, m the total (electromagnetic plus mechanical) mass and F the external force. The dots stand for differentiation with respect to s.

The terms which are neglected are indeed small if the Fourier transform of the velocity as function of s contains only frequencies small compared to $1/\tau$. This is the case in most practical problems, if a is taken to be the classical electron radius e^2/mc^2, so that the electromagnetic mass is comparable to (and may even be equal to) the actual electron mass. Then

$$\left.\begin{aligned} a &= \frac{e^2}{mc^2} \sim 2.10^{-13}\ \text{cm.} \\ \tau &= \frac{e^2}{mc^3} \sim 10^{-23}\ \text{sec.} \end{aligned}\right\} \qquad \text{(II.2)}$$

The force F in (II.1) contains only the electromagnetic field due to external causes, but not that due to the electron itself since the reaction of the self-field is already taken account of on the left-hand side. The definition of this « external » field has been discussed particularly carefully by Dirac(1).

The damping term in (II.1) is uniquely connected with the law of emission of radiation by the electron, and it represents merely the loss of energy which compensates for the energy carried away in the form of radiation. Conservation of energy requires that (II.1) must hold as long as the wavelength of the emitted radiation is long compared to a so that the radiation is practically that from a point charge. Since both energy conservation and the laws of emission of radiation are well confirmed within the range of applicability of (II.1) the same is evidently true of this equation itself.

We have thus the difficulty in the form that infinitely small a makes the inertia term infinite, whereas a finite a makes it impossible to describe (II.1) as an approximation to a Lorentz invariant equation.

It is true that one need not necessarily regard the Hamiltonian formalism as the starting-point, and that, for instance, one may start from the equations of motion instead; while such a view in some sense may be regarded as merely a different interpretation of the usual classical equations, it will be convenient to discuss it in connection with attempts to modify the theory.

(II.2) *Dirac's equations and allied methods.* Dirac(2) has proposed an interpretation of classical theory in which the field at the position of the electron is re-defined in a way different from that obtained by making the electron radius in the Lorentz picture tend to zero.

In this way it is possible to eliminate the effect of the self-field on the inertia term in equation (II.1) while keeping the finite radiation damping term unchanged. The electron is on this view still regarded as a point charge, and the higher-order terms go out since they contain positive powers of the electron radius.

Dirac did this by re-defining the « field intensity » at the place of the electron as one-half the difference of retarded and advanced field. An alternative method leading to the same result had been given previously by Wentzel(3) on the basis of a « two-times » formalism. Just as for an electron with coordinates x, y, z one can calculate the field variables at a different point x', y', z', one can also introduce an independent time t' for the field which does not necessarily equal the time t for the electron. For $t' = t$ one obtains the usual field equation. Wentzel proposes, however, to define the field for a world point for which $x - x'$, $y - y'$, $z - z'$, $t - t'$ is a time-like vector and then let this vector tend to zero. While the approach from a space-like vector defines a frame of reference, the equations become Lorentz invariant only after the limiting process has been completed. On the other hand, even before going to the limit, the equations can be written in Hamiltonian form, and are, therefore, convenient for quantization.

Another, equivalent, formalism is due to Riesz (4). In this the self-field is defined as the solution of an equation differing from the usual d'Alembert equation, by a variable parameter. The equation is solved for such a value of the parameter that there is no singularity and then extended by analytical continuation to the value representing the d'Alembert equation. Hence the theory is Lorentz invariant even before proceeding to the limit, but it does not take Hamiltonian form until the limiting process has been carried out.

Lastly (5), the same result can be obtained by taking a finite electron radius yielding an electromagnetic mass m_e, and assuming a mechanical mass m_m so that $m_e + m_m = m$ equals the actual electron mass. If one then proceeds to the limit $a \to 0$, m_m must tend to ∞ but the total inertia will, of course, remain finite.

All these methods lead to equation (II.1) without terms of higher order. It then represents on equation of third order and the solution must contain three arbitrary constants, so that an electron with given initial position and velocity can still have arbitrary acceleration. For a free electron (F = 0) all solutions in which the initial acceleration does not vanish, are violently accelerated, the velocity rising to about light velocity in a time of the order of a/c, with the acceleration referred to an observer moving with the electron increasing indefinitely. These « runaway solutions » are physically unreasonable and must be omitted. This amounts to imposing the condition that at $t = + \infty$ the electron must have a finite acceleration. If classical theory were to be taken quite seriously one would then get into conceptual difficulties, since if we arbitrarily decided to change the conditions of an experiment when the electron is already moving, we would have to revise slightly the exact values of its past velocity and acceleration as a result of altering the forces that are going to act on it in the future. Other odd features of this theory were discussed by Eliezer [1].

All this, however, is hardly a valid objection in itself since in all such cases quantum phenomena would play a non-negligible part. One would, however, expect that after transition to quantum theory it will be impossible to avoid the runaway solutions since there is, in general, a finite probability of transition to any possible state of motion which is not excluded by rigid conservation laws. This is analogous to the impossibility of avoiding transitions to states of negative energy if we are dealing with the Dirac equation for a single electron.

A new paper by Eliezer [1] generalizes Dirac's method, leading to an equation involving higher-order terms and free from runaway solutions. Its relation to field theory, however, is not yet clear.

(II.3). *The Born-Infeld Theory.* While the above theories make (II.1) an exact equation, it is possible to take the view (which corresponds much more to that of Lorentz) that it represents only an approximation. In particular Born [1] has shown that it is possible to write Lorentz invariant field equations which are non-linear, and which are identical with the usual ones only for not too intense fields. For this purpose one can replace the integrand of the action integral, which must be a scalar, and which in the usual theory is

$$\underline{E}^2 - \underline{H}^2$$

by an arbitrary function of the two scalars

$$\underline{E}^2 - \underline{H}^2 \text{ and } \underline{E} \cdot \underline{H}$$

If this action function is chosen suitably, the field will saturate near the electron, instead of becoming infinitely large, and the self-energy remains finite. In this theory the primary quantity is a « critical field intensity » at which the deviations from Maxwell's equations become appreciable. The «electron radius» is a derived quantity: being that distance at which the field is of the order of the critical field. (For a point charge of varying strength the radius would vary as the square root of the charge.)

In this theory the self-energy is finite, and can, for example, be chosen to equal the actual mass, so that no mechanical mass need be assumed. While the non-linear equations are not easy to handle mathematically, the classical consequences for the classical Born-Infeld equations appear in every respect reasonable. The equations can immediately be written in Hamiltonian form, and it is therefore easy to write, in general terms, quantum equations. However, the problem of solving these quantum equations is extremely difficult and nobody has yet succeded in developing mathematical techniques adequate to this task. Hence the consequences of the quantum form of the Born-Infeld theory are not known.

(II.4) *Subtraction Theories.* I have previously referred to the connection of the infinite self-energy with the infinite amount of work done of two opposite charges approaching indefinitely closely. It is reasonable to try to reduce the latter by using a law of force which increases less strongly at close approach. One way of doing this is to subtract from the Coulomb force between two particles another force which is equal to the Coulomb force at close approach, but decreases more rapidly with distance. The most suitable force of this description is the « meson force » which is derived from the potential

$$V(r) = A \frac{e^{-kr}}{r} \tag{II.3}$$

A and k being constants. Such a force is obtained of the two particles interact with a field of the meson type in which the wave equation

$$\Box \Phi = O \qquad\qquad \Box \equiv \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2$$

is replaced by

$$(\Box + \varkappa^2)\,\Phi = O \tag{II.4}$$

It is, therefore, easy to obtain the interaction (II.3) from a suitable field theory. It is also possible to assume the electrons to interact with both electromagnetic and the new field, but this would result in an *addition* of the two types of force. In order to obtain the difference, rather than the sum, it is necessary to make the ad hoc assumption that all the effects of the new field are to be reversed. Such subtraction formalisms were constructed by Bopp (9) and by Stueckelberg (10).

They involve the assumption that the new field makes a negative contribution to the energy density. This is, in fact, essential to enable the new field to compensate the possible electrostatic self-energy. But this also means that of any radiation emitted by a moving particle the part which consists of the new field has negative energy. A particle radiating such waves will not lose, but gain energy. Such a state of affairs is likely to be unstable.

This scheme of using two compensatory fields will take a rather wider aspect in quantum theory, see section (IV.3) below.

(II.5) *Four-dimensional form function.* Another possible way of implementing the programme of Lorentz was suggested by Peierls and worked out by Mc. Manus (11). If one regards particles as extended in space, the field equations at a point x will be influenced by the presence of an electron at a neighbouring point x'. This is possible in a relativistic scheme only if the equation at a time t may also depend on the presence of a particle in the neighbourhood at a slightly different time t'. In other words the field equation

$$\Box \Phi_i (\underline{x}, t) = 4 \pi s_i (x, t) \qquad \text{(II.5)}$$

where Φ_i is the four-potential and s_i the current density, is replaced by

$$\Box \Phi_i (\underline{x}, t) = 4 \pi \int F (\underline{x}\text{-}\underline{x}', t\text{-}t') s_i (\underline{x}', t') \, dx' \, dt' \qquad \text{(II.6)}$$

where F $(x\text{-}x', t\text{-}t')$ is a given function. If F is independent of t and t', one obtains the usual equations for a spatially extended electron, which give a finite energy but are not Lorentz invariant. In order to make the equation invariant, we must evidently make F depend only on

$$R^2 = c^2 (t - t')^2 - (\underline{x}\text{-}\underline{x}')^2 \qquad \text{(II.7)}$$

One would choses for F a function which decreases rapidly for (negative or positive) values of R^2 exceeding the «electron radius» a. By studying the Fourier transform of F one can see that one must, in fact, choose a function which changes sign at small arguments and whose integral vanishes in order to ensure that the field of an electron reduce to the ordinary Coulomb field at distances large compared to a.

With a function of this type, one can derive a consistent set of wave equations and equations of motion from a common action principle. The equations cannot, however, be brought into Hamiltonian form, since they contain the physical variables in the form of an integral over time, and therefore may be regarded as differential equations of infinitely high order in the time. The problem of how to quantize equations of this kind is at present unsolved.

As far as the classical equations go, they are free from infinities, appear to have no runaway solutions (though a rigorous proof of this is still lacking) and reduce to the Lorentz equations when all quantities vary little over distances of the order a and time of the order a/c.

They are in these respects similar to the Born-Infeld equations. They differ, however, in leaving the equations of the field completely unaltered in the absence of charges, the modifications being contained entirely in the interaction of field and matter.

(II.6) *Action at a distance*. Another attempt to avoid the infinite self-energy in the classical equations is a method due to Wheeler and Feynman ([12]). In order to be able to separate the affect on a particle due to the field of other particles (which is physically significant) from that due to the particle itself (which diverges) they give up regarding the field intensity as a physical variable and express tho force on each particle as functions of the world lines of all other particles. At first sight one might expect that the field due to each particle should be the usual retarded field, so that radiation is *emitted* by any disturbance in the motion of the particle. In this case, however, the radiation damping term in (II.1) disappears and the equations violate the principle of conservation of energy. This is not suprising, since the radiation damping results from the effect of the radiation emitted by the particle on the particle itself.

Feynman and Wheeler show, however, that one gets more reasonable answers if one uses, in place of the retarded field the average of retarded and advanced field, the latter being the formal solution of the equations corresponding to radiation being *absorbed* by the particle if it changes its motion. The description arrived at in this way seems to rearrange the usual causal description in a curious manner. The energy lost by an electron upon being deflected by a force appears here as the absorption, by the electron, of advanced radiation emitted by the surrounding matter at a later time, where in the conventional picture the signal from the electron would have reached the surrounding matter to ensure that any radiation must ultimately be absorbed, the formalism gives results in complete agreement with conventional theory, including radiation damping, but without self-energy. It is not clear whether it will also contain runaway solutions of the type discussed in (II.6) but since the causal description has here disappeared altogether and one has to discuss in one step the whole of the world lines of all particles concerned, one must in any case make some assumption about the behaviour of the particles in the distant future and this can probably be used to get rid of runaway solutions, in which at $t = \infty$ the velocity is light velocity.

The equations obtained by Feynman and Wheeler are Lorentz invariant but not Hamiltonian in form and a quantum version is not at present known.

III.

QUANTUM THEORY (1) Conventional Theory

Before discussing possible modifications of the existing theory, it is worth remembering just how the difficulty arises. The simplest case for discussion (and the one with the worst singularity) is that of a single electron, without introducing pair theory.

A free electron is, in current one-particle theory, regarded as a particle of mechanical mass m_m, coupled with the surrounding radiation field. Its interaction with this field consists of two parts, the Coulomb (or longitudinal) energy and the interaction with the transverse part of the field. The Coulomb part is directly given by the charge density of the electron by the equation.

$$\operatorname{div} E = 4 \pi \rho \tag{III.1}$$

in which all quantities commute and which can thus be regarded as a classical equation, and the longitudinal field thus has, as in classical theory, the energy

$$E_{long} = \int \frac{\rho(x)\, \rho(x')}{R}\, dv dv' \tag{III.2}$$

where dv, dv' are the volume elements referring to the points x and x', and R their distance.

If the charge density $\rho(x)$ of the electron is taken to be that of a point charge, (II.2) diverges. If, instead, we take the electron charge as distributed over a sphere of radius a, the energy becomes, in order of magnitude,

$$E_{\text{long}} = e^2/a \tag{III.3}$$

The interaction with the transverse part of the field is not quite as easy to ascertain, since the coupling between the electron and the transverse electromagnetic waves does not commute either with the mechanical kinetic energy of the electron, not with the energy of the field. One must, therefore, solve the quantum equations for the stationary state of the system consisting of field plus electron. This is usually done with perturbation theory; regarding the electronic charge e, and hence the coupling term as small.

The first-order perturbation energy (which would consist of terms proportional to e) is easily seen to vanish. The general expression for the second-order perturbation to the energy is

$$E_2 = \Sigma' \frac{|V_{on}|^2}{E_o - E_n} \tag{III.4}$$

where V_{on} is the matrix element of the coupling energy between the state 0 of which the perturbed energy is to be found, and any other state n. E_o and E_n are the unperturbed energies of these two states, and the apostrophe indicates that the term $n = 0$ be omitted. Here the coupling term has matrix elements only for such states as differ by one photon, and with identical momentum. Hence if we are describing an electron with momentum zero the only states n occuring in the summation are those in which, besides the electron, a photon of momentum P is present, while the electron momentum has changed to —P. Hence the denominator of (III.4) has the value

$$E_o - E_n = c \times \pm \left\{ mc \mp \sqrt{m^2c^2 + P^2} - P \right\} \tag{III.5}$$

The signs depend on whether the electron states referred to are the positive or negative-energy solutions of Dirac's equation. If the state n belongs to negative energy, the value of (III.5) for large P is asymptotically constant equal to mc^2. For large P the matrix element in the numerator of (III.4) is, apart from numerical factors

$$|V_{on}|^2 \sim \frac{e^2 \hbar^2 c}{P}$$

and bearing in mind that the number of photon states per unit volume is

$$\frac{8\pi}{\hbar 3} \mathrm{P}^2 \, d\mathrm{P}$$

one finds that the sum diverges as the integral

$$\frac{e^2}{\hbar c} \int \frac{\mathrm{P} \, d\,\mathrm{P}}{m\,c^2} \tag{III.6}$$

This result was first derived by Waller ([13]).

The corresponding calculations for the Dirac hole theory was carried out by Weisskopf ([14]). In this theory we regard the vacuum as the state in which all levels of negative energy are occupied; a real electron is additional.

Since the energy of empty space is our natural reference point we are concerned with the difference of energy between the state of one electron and none. Considering first the case of one electron (in addition to the hypothetical negative-energy ones) no states can occur in (III.4) in which this electron has changed to a state of negative energy, since these are all occupied. If the extra electron has moved to another state of positive energy, the denominator (III.5) is

$$c \left\{ mc - \sqrt{m^2 c^2 + \mathrm{P}^2} - \mathrm{P} \right\} \tag{III.7}$$

and the integral would still diverge as

$$\frac{e^2}{\hbar c} \int d\,\mathrm{P} \tag{III.8}$$

However, we must also bear in mind states in which one of the other electrons has moved. Most of these transitions occur also in the expression for the self-energy of empty space and therefore drop out of the difference which we require, with one exception. In the vacuum each of the negative-energy electrons could have made a transition to the state of momentum zero and positive energy. When this state is occupied such transition become impossible, and hence the difference contains just those transition with the opposite sign. The denominator becomes

$$c \left\{ -\sqrt{m^2 c^2 + \mathrm{P}^2} - mc - \mathrm{P} \right\} \tag{III.9}$$

which is for large P of the order of $-2c\mathrm{P}$, the same as (III.7). Since the numerators can also be shown to be the same, the two contributions cancel as far as their leading terms go, and the energy diverges only as

$$\frac{e^2}{\hbar c}\; m c^2 \int \frac{d\mathrm{P}}{\mathrm{P}} \tag{III.10}$$

The divergence is now only logarithmic. If we again use the (non-Lorentz invariant) model in which the electron has a finite radius, we would have to limit the integration at an upper limit

$$\mathrm{P}_o = \hbar / a \tag{III.11}$$

and we would find the leading term in the transverse energy

$$\frac{e^2}{\hbar c}\; m c^2 \log\,(\hbar / m c a) \tag{III.12}$$

The electrostatic term becomes modified in a similar way, since the coordinate of the particle in Dirac's equation is an operator which causes transitions from positive to negative energies and vice versa. For this reason, the density $\rho\,(x)$ which occurs in (III.2) contains transitions which will be excluded by Pauli's principle, and one obtains just the same kind of cancellation so that the longitudinal energy is of the same order as (III.12).

This energy would be negligible in comparison with mc^2 even if a was very much smaller than any dimension to which we can apply present theory with much justification. The objection remains however, that any such cut-off procedure destroys the Lorentz invariance.

(III.2) *Part played by perturbation theory.* The use of perturbation theory in the discussion of the previous section is, of course, without justification, since the result shows that the change in energy caused by the coupling is infinitely large. One might thus suspect that, in spite of these results, the exact solutions of the problem might have finite energy ([15]). This has been disproved in the one-electron problem by Salpeter ([16]). If one goes to higher than the first approximation, one finds that no stationary solution exists for an electron in a state of positive energy, since it can always, by the emission of two photons, change to a state of negative energy (this is the situation that made it necessary to invent device of having the negative energy states occupied). One can, however, define the problem for a single electron which is in a state of negative energy from the start, since energy and momentum conservation would not permit it to sink any lower, even with emission of any number of photons. For this case it can be proved that, if we solve the equations rigorously for finite a, and then go to the limit $a \to 0$, the energy goes to $-\infty$, almost certainly as $-\sqrt{e^2/\hbar c}.\hbar/a$.

It would be interesting to generalize this proof for the case of hole theory but that is difficult since there exists no form of hole theory which is mathematically consistent beyond first approximation, so that the problem is not very well defined. It may not be possible to elucidate this further until the difficulties of hole theory have been removed, and it may well be that the difficulties of hole theory and of self-energy are intimately connected.

(III.3) *The « infra-red catastrophe »*. Quite apart from the divergence at the short-wave end, the application of perturbation theory leads to difficulties in that, in a simple scattering process, the probability of emitting a soft photon of momentum between P and P + dP appears in first-order approximation as proportional to dP/P so that the total probability of emitting a photon of *any* frequency would diverge. In this form the result does not make sense at all since a probability must always be less than unity, and it merely shows again that perturbation theory is not applicable. This difficulty disappears if perturbation theory is replaced by a more exact calculation. This was shown by Bloch and Nordsieck (17) and a more careful analysis was made by Pauli and Fierz (18). For this purpose one imagines the problem of an electron with its own field solved exactly. Because of the divergence of the self-energy term for hard photons, this is consistently possible only if the singularity is eliminated, for example by cutting off the effect of quanta above a certain frequency. This destroys Lorentz invariance but is not likely to affect qualitative results for a slow-moving particle. The stationary states of an electron with its own field and with or without added « free » photons are then used as the basis of the description. The potential which causes the scattering is treated as a perturbation causing transitions between these basic states.

The description of these states becomes easy if one chooses a cut-off wavelength so long as to make the self-energy of the one-body problems negligible compared to the rest energy of the particle.

Such a particle would not be very similar to a real electron, but it is nevertheless instructive to study its behaviour. To avoid the transition to negative-energy states, it is also preferable to neglect these states altogether and start from the non-relativistic form of the quantum equations.

It is then found that the number of soft photons accompanying the particle with momenta between P and P $+ d$P is proportional to dP/P. If we imagine the particle to move in a volume of linear dimensions L with reflecting walls, the largest possible wavelength for a photon is L, and the number of photons per mode of vibration is, apart from numerical factors

$$\frac{c^2}{\hbar c} \frac{h^3}{L^3 P^3} (v/c)^2 \tag{III.13}$$

where P is the momentum of the photon in the mode of vibration concerned and v the velocity of the particle. Since the lowest value of P is h/L the number of photons per mode of vibration is always small, the probability of having two photons in the same state being quite negligible.

The number of possible photon modes is

$$8 \pi (L/h)^3 P^2 dP$$

so that the total number of photons is of the order of

$$\frac{e^2}{\hbar c} \left(\frac{v}{c}\right)^2 \int \frac{dP}{P} \tag{III.14}$$

The lower limit of integration is h/L, the upper limit (near which the expression (III.13) is not, in fact, adequate) is of the order of the cut-off h/a hence the total number is

$$\frac{e^2}{\hbar c} \left(\frac{v}{c}\right)^2 \log (L/a) \tag{III.15}$$

This is still a small number unless

$$\frac{L}{a} > e^{137 (c/v)^2} \tag{III.16}$$

which would probably make L greater than the universe. If we are therefore content to describe the phenomena inside a very large but not infinite box, we can neglect the possibility of more than one photon being present and in that case first-order perturbation theory gives an adequate description of the electron with its own field.

One can then apply the calculation of Pauli and Fierz to the scattering of the particle by a centre of force and it is found that (*a*) the probability of emitting more than one photon is negligible, (*b*) the probability of emitting one photon contains the factor (III.15) and, because of our assumption about L, is less than unity, (*c*) the probability of scattering *without* emission of light is reduced from the value for an unchanged particle by an amount equal to (*b*) except for small corrections independent of L. Hence the total probability of scattering (with or without radiation) is not influenced by the coupling with the radiation field except for small corrections. This concellation depends on assuming negligible self-energy. If a cut-off were chosen which made the self-energy appreciable, it would influence the inertia, and thereby the scattering process. Even then, however, the logarithmic terms of the type (III.15) would still cancel so that one would not except a logarithmic dependence of the scattering cross section one the size of the box.

As Bethe and Oppenheimer ([19]) have pointed out, for this cancellation it is essential, on the one hand, that there exist states in which there is a « free » photon besides the electron and its own field; transitions to this state mean emission of a photon. On the other hand, the probability of the electron *not* being accompanied by a « bound » photon must be less than unity, in order to reduce the probability of radiation-less scattering. This comes out of the theory by virtue of the fact that there is a finite probability of the electron being accompanied by one « bound » photon.

It seems in some way artificial and complicated to describe a phenomenon as simple physically as one free electron in terms of a picture in which there may or not be « bound » photons. Yet, at least in a formalism of the present type, the probability of finding these « bound » photons which are not directly observable in practice (though they should be observable in principle) is related to the possibility of emitting a free photon, which is observable.

If we try to omit « bound » photon from our description, the logarithmic contributions no longer cancel each other. The result is a logarithmic dependence of the cross section on the size of the box. While this dependence is never very strong, Ferretti and Peierls ([20]) have pointed out that it is observable within a time insufficient for the radiation to reach the walls and return, so that the system cannot be influenced by whether or not the walls are actually present and by their position. Hence omission of the « bound » photons must either lead to a theory in which the cross section depends on time, the final value being reached only after a time L/c, and very much longer than the duration of the collision process in the ordinary description, or else the theory must contradict the usual ideas of the localization in space, and the propagation with light velocity, of radiant energy. This does not exclude the possibility of a theory based on an entirely different formalism in which the cancellation is achieved in other ways.

For a theory of the present type, however, one would conclude that the « bound » photons of long wavelength must not be omitted. On the other hand, since the infinite self-energy arises from bound photons of short wavelength, one would expect to need a formalism in which the effect of such bound photons is reduced by a factor depending on their wavelength. Such a theory is similar to those classical theories in which the self-energy is finite, and in which in the Lorentz equation (II.1) for the motion the terms of higher order are present, though usually negligible.

IV.

MODIFIED FORMS OF QUANTUM THEORY (1)

The theory of « radiation damping ». In spite of the singularities which have been mentioned, a large number of problems can be treated by means of the present theory. In fact, in such problems as the absorption, omission or scattering of one or more photons by atoms or by free electrons, the equations give reasonable answers if one uses expansion in powers of the charge e, and retains only the terms of lowest order in which the described effect occurs. Since one expects for dimensional reasons that the convergence of the series depends on the dimensionless number $e^2/\hbar c = 1/137$, one would expect the omission of higher terms to cause errors of the order of 1% which are usually unimportant. In practically all cases in which such transition probabilities could be checked by experiment they were found to be correct to within the experimental error. Actually, of course, the neglected terms are not small, but infinitely large, hence there is reason to believe the results of lowest significant order to be more reliable than the more rigorous solution.

In some cases we can get a little further by the use of conservation theorems. Take, for example, the case of an atom in an excited state, and denote by n the state of the system in which the atom is excited, and no light is present, by P the state in which the atom is in the ground state, and a photon of momentum P has been produced. Then, without radiation ($e = 0$) the probability amplitude of state n is unity, that of state P zero. The state P occurs first in the first-order calculation, and hence the emission probability as a function of time is finite to first order. The probability of the atom remaining in state n is large in zero order, and hence cannot be worked out to any higher order without trouble. Yet one knows, of course, that this probability must decrease at the same rate as that of all states P increases, and this decrease is one particular form of radiation damping.

In the usual formal description of this process one obtains together those transitions which lead to a physically possible photon state and to actual emission (free photon) with those referring only to virtual states (« bound photons »). Those of the first type are required to maintain the probability normalized, those of the second kind diverge and are connected with the infinite self energy.

Heitler has given a rule which can be used to separate the two kinds of terms, and which, if consistently applied, leads to a formalism with which it is possible to calculate the probability of any process, in principle, to unlimited accuracy. This rule, in substance, amounts to the omission of the transitions involving bound photons.

There exist several forms of Heitler's equations. A particulary clear exposition has been given by Pauli ([21]), dealing only with stationnary solutions. Heitler himself has written equations in a time-dependent form ([22]), but these are somewhat less general, since in their formulation approximations have been made which are valid for the scattering problems to which the theory is usually applied, but which would need modification to become a perfectly general theory. As applied to stationary problems, the two methods are equivalent. They have, in particular, the advantage over ordinary perturbation theory that for the scattering of all kinds of particles at very high energies, whey give a finite cross section, whereas the simple perturbation theory would give a cross section which may increase indefinitely with increasing energy.

Bethe and Oppenheimer have applied the theory to the infra-red limit and, as was to be expected from the discussion of the preceding section, they find that the logarithmic terms arising from very long waves do not cancel. As we have seen this must mean either that the cross section is slowly time-dependent or that the theory is not in agreement with the ordinary laws of propagation of light.

The latter possibility is not ruled out by the Lorentz invariance of the theory, since, after applying the modifications proposed by Heitler, the theory no longer uses field variables in space-time in the ordinary sense. It is not at present possible to decide which of the two alternatives would result from the theory, since to find the time-dependent cross section or to treat prorogation of light one has to elaborate the time-dependent equations for a case not covered by Heitler's formalism. One fairly natural way of applying it leads to the result that light is not propagated with light velocity.

As Bethe and Oppenheimer suggest, these difficulties must be common to all theories in which the self-energy is completely eliminated since these amount to omitting the effect of all « bound » photons.

(IV.2) *The « λ limiting » process and allied methods.* Dirac ([23]) and others have applied quantum theory to the equations without self-energy which were described in section (II.2) and it is then found that there divergent terms, since the effect of field fluctuations, which are a typical quantum phenomenon, has not yet been eliminated. In order to avoid this difficulty, Dirac has proposed a formalism in which some of the usual reality conditions are abandoned so that one obtains in certain cases negative probabilities. Since these cannot have any direct physical meaning, one needs a new rule to translate the results of this theory into terms of observable quantities.

I do not propose to enter into a detailed description of this theory, since I have not studied it sufficiently to be able to present it in a satisfactory form. Since this is again a theory in which the existence of « bound » photons is ignored, one would expect difficulty at the infra-red end. Pauli ([24]) has, indeed, shown that this is the case. A somewhat different theory has been proposed by Gustafson ([25]). He starts from the Riesz process like the λ limiting process in sufficient to eliminate the terms in the self-energy problem which are analogous to the classical self-energy, but leaves again the fluctuation terms diverging.

Ma ([22]) has, in fact, shown that to second order of perturbation theory the Riesz process is strictly identical with the λ-limiting process.

Gustafson proposes to alter the fluctuation terms in the expression for the second-order perturbation theory by applying to the electron wave functions which enter into them a limiting process of the same kind as is applied in the Riesz method to the field equations themselves. In this way one can make the self-energy finit.

However, by modifying the *result* of perturbation theory rather than the basic equations, one has no longer a consistent wave mechanical description and, for example, the result of higher approximations and the internal consistency of the theory as regards the transformation properties of operators is thrown in doubt.

(IV.3) *Subtraction theories.* A careful study of a quantum theory analogous to the classical methods of Bopp and Stueckelberg (see section II.4) has been made by Pais ([27]). The situation is different from classical theory, since in quantum theory the self-energy can be negative even tough the field energy itself may be positive definite. This again is due to the fluctuations of the field which depend on quantum effects.

It is, therefore, possible in principle to assume, besides the electromagnetic field, a second field due to particles of non-vanishing rest mass and of spin zero which would modify the effects of the field only at small distance and which would give a self-energy of opposite sign. It is then possible to choose the constants in such a manner that to first approximation the divergent terms cancel. However, since the new field had particles of a spin different from that of the photons, which to all intents and purpose have spin one, one cannot expect the concellation of the singular terms to be an identity, and if the singularity is removed to first order of approximation, it does not follow (and is, in fact, unlikely) that the concellation would still remain in higher approximations. So far the problem has not as yet been solved beyond the first approximation, but there is little hope that the theory can be freed from difficulties.

This theory in which the long waves are treated in the same manner as in ordinary theory, would not, of course, meet with difficulties at the infra-red end.

(IV.4) *Quantization of space.* A completely different approach has been proposed by Snyder ([28]). He uses a scheme in which the coordinates of a particle are capable of discrete values only. Nevertheless, the equations are Lorentz invariant. This is achieved by assuming the three coordinates and the time to be operators which do not commute with each other, and the situation is analogous to the invariance of the three components of angular momentum under rotations of ordinary space, even though each component has discrete eigenvalues. The chief objection to Snyder's porposal appears to be the question of the physical significance to attach to his operators.

If in ordinary quantum theory the coordinate is regarded as an operator and hence connected with a possible observation, this can only have the meaning « the position x at which a particle is found at a certain time t ». Alternatively, if the time is regarded as an operator, this can only mean the time at which a certain event takes place, for instance, the time at which a particle passes a certain position. In several dimensions one can define also an operator representing the x coordinate of the particle which goes together with a fixed y or z coordinate. This operator is the proper one to use when selecting beams of particles by means of a slit system.

If, however, x does not commute with the other coordinates or the time, its relation to possible observations becomes questionable and for that reason it is also not clear how to interpret wave equations which these operators take the place of the coordinates.

At present no complete formalism based on Snyder's ideas has been constructed.

(V) *Non-Hamiltonian Theories.*

We have not discussed the quantum equivalent of two of the classical schemes, namely the relativistic form function (II.5) and the action at a distance (II.6). The reason is that in these two cases the classical equations do not appear in Hamiltonian form, even though in both cases they follow from an action principle. In this connection it should also be remembered that there is a similar difficulty in ordinary pair theory if one attempts to formulate it in a manner in which the infinite charge density and the infinite fluctuations of current density due to the hypothetical electrons in negative energy states to not appear. Such a form of the theory had been proposed by Dirac ([29]) and Heisenberg ([30]), and Serber ([31]) showed that in their theory the commutation laws were not compatible with the equations of motion. This is again due to the fact that the equations are not of Hamiltonian form.

The Hamiltonian is an essential part of quantum theory in order to determine a time variation of the wave function. It is usually assumed that all the knowledge which the uncertainly principle may allow us to possess of a system, can be available at one instant and summarised in a wave function. It then follows that the time variation of this wave function must be determined by a linear operator which has all the properties required for the Hamiltonian. It would not be unreasonable, however, to suppose that we could never define the state of an electron and its intrinsic field accurately at an instant defined to within better than the characteristic time of 10^{-23} seconds, or even to predict from this the results of a second experiment carried out 10^{-23} seconds later.

The equations of the form function theory explained in section (II.5) are integral equations in time and they refer, strictly speaking, to the whole space-time development of the phenomenon in one set of coupled equations. The major contribution to the integral, however, comes from a region of the order of magnitude of the characteristic time, which is 10^{-23} seconds if the critical radius is chosen to equal the classical electron radius. One would, therefore, expect that these equations define in good approximation the further development of a system if its behaviour over a time interval of the order of the characteristic time is given. It seems an attractive idea to replace the Hamiltonian description by one in which one has necessarily to use regions of finite extent in space-time and to avoid the use of a wave function referring to an exact instant in time which, in any case, results in a very unsymmetrical treatment of a particular time coordinate compared to other possible time-like directions, so that even if the content of the theory is relativistic, it is expressed in quite unrelativistic variables.

The theory of Feynman and Wheeler (section II.6) is similar in character, except that large time intervals occur in the equations and in that way one could not avoid using one set of equations describing together the whole space-time development over a period large enough for all radiation emitted by any of the particles to be absorbed again.

Lastly, in order to remedy the difficulties of pair theory to which reference has been made, it is likely again that one has to introduce a description in which time intervals of finite lenght occur. One might, in this connection, expect the times to be larger than e^2/mc^3 though this question has not been explored very far.

In this connection it is of interest to mention an attempt by Wataghin [32], who has modified the ordinary perturbation theory by introducing a cut-off factor depending on the invariant combination of the energy difference of the two electron states connected by a given matrix element, and their momentum difference. In other words, a matrix element linking a state of momentum p and energy E of the electron with a state p'E' is reduced by a factor which depends on

$$(\mathrm{E} - \mathrm{E}')^2 - c^2\,(p - p')^2$$

This procedure, if carried in perturbation theory to terms proportional to e^2, gives finite and Lorentz invariant results. It is not, however, the result of applying consistant basic equations and it is not known at present how to generalise it to terms of higher order. It seems rather likely that the form function theory of section (II.5), if it could be expressed in quantum form, would lead to equations to which Wataghin's results would represent a first approximation. A theory of that kind would seem to be in agreement with all evidence so far available. Even if one could overcome the difficulties of quantizing a non-Hamiltonian theory, it would be incomplete in so far as it will contain an arbitrary function which would have to be fixed by experiment, or by some future more fundamental theory.

Some hope of obtaining a method for quantizing theories which have an action principle, though not necessarily a Hamiltonian, is raised by recent work by Feynman [33] starting from a formulation of the principles of quantum theory given first by Dirac [34]. This method is applicable to theories in which the action function contains terms which depend on the dynamic variables at two different times. However, so far it has not been possible to apply this method to relativistic problems and to obtain with it the Klein-Gordon, or even the Dirac, wave equation.

V.

POSSIBLE OBSERVABLE EFFECTS

The effects of the interaction of a charged particle with its own electromagnetic field may, in certain circumstances, be observable, apart, of course, from the ordinary effects of radiation damping, which follow from the laws of emission and absorption of light by virtue of conservation of energy. In the first place even in processes which involve only particles of low energies, the self-field may be responsible for changes in the equations, which, in general, will only be of the nature of very small corrections. Such effects and the experimental evidence for them, belong to the subject of Bethe's report.

It is also possible, however, that at high energies when the wavelengths of the particles or radiation concerned are comparable to the critical radius which has been introduced in some of the discussions of the preceding sections, the results may be radically different from those expected from a theory in which this critical radius was negligibly small. Until a consistent theory is available, it is, of course, difficult to foresee in what particular circumstances such effects should arise, but it is evident that, in any case, they must be Lorentz invariant. If we consider, therefore, a collision involving an electron iniatially in a state of energy E and momentum p, and in which the electron is finally found in a state of energy E' and momentum p', the quantity which measures the importance of these new correction terms must be an invariant function of the quantities E, E', p, p' and apart from universal constants must, therefore, be a function of

$$\sqrt{(E - E')^2 - c^2 (p - p')^2} \qquad (V.1)$$

This would evidently be the case in the kind of scheme discussed in section IV, but it must hold equally in any other theory in which there are deviations.

Deviations will become important when the quantity (V.1) reaches a certain value, say $\eta\, mc^2$. Then, for a collision of any fast particle with an electron at rest, we have $p = 0, E = mc^2$. After the collision the electron has high energy E', with $E'^2 = c^2p'^2 + m^2c^4$, so that

$$(E - E')^2 - c^2 (p - p')^2 = - 2mc^2E' + 2m^2c^4 = \eta^2 m^2 c^2$$

and hence approximately

$$E' = \frac{\eta^2}{2} m\, c^2 \qquad (V.2)$$

We do not, of course, know the value of η. It is not unreasonable to suppose that the self-energy of the electron may be computed by a cut-off procedure which eliminates transitions in which (V.1) exceeds η in magnitude. This is, for example, the case in the theory of Wataghin, section IV.

If one applies such a cut-off to the one-body problem, one finds that this cuts out intermediate states of energy E' greater than (V.2). In order that the electrostatic self-energy be of the order of the rest energy of the electron one must cut off at

$$E' \sim 137\, mc^2 \,,\ \eta \sim \sqrt{137} \tag{V.3}$$

On the other hand, in pair theory it appears that we can make E', and hence η, much larger, and the self-energy will still be negligible.

On the other hand, if we consider the collision of a fast electron with a stationary proton, conditions are more favourable for observing the deviations. In that case E and E' are both large and very nearly equal. p' is of the same order of magnitude as the initial momentum p provided the electron is deflected through an appreciable angle. Therefore, in this case $E. \sim \eta\, mc^2$ and, if the correct cut-off were given by (V.2) we should have deviations for $E = \sqrt{137}\; mc^2$. A critical energy of this order was mentioned by Bohr in Copenhagen last year.

On the other hand, if anomalies were found in the scattering of electrons by protons, one would be in doubt whether they ought not in part to be ascribed to the interaction of the electron with the nuclear field surrounding the proton. Such an interaction arises, for example, in the charged meson theory of nuclear forces in which the proton spends part of its time as a neutron surrounded by a positively charged meson field. In that event, however, also the neutron should spend some fraction of its time as a proton surrounded by negative mesons and there should, therefore, be an interaction between electrons and neutrons.

I believe that experiments are now in progress to detect a weak interaction of this kind. If their result is negative, they would rule out this particular type of meson theory, but it would still not follow that there might not be some specific anomaly in the proton case resulting, may be, in a spreading of the proton charge over space which had no direct connection with the self-energy problem.

Nevertheless, deviations found in the scattering of fast electrons by protons would give some support to theories of the kind that appear probably now, and they could serve as a test of any specific theory which makes predictions about the magnitude of the critical lenght.

In writing this report, the author was materially assisted by discussions with Dr. K. Bleuler and Mr. E. E. Salpeter.

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Report

by Mr. Bhabba

I have been asked to report on the present state of the theory of relativistic wave equations, and then to say something about the question of spin and statistics.

This work started with a paper by Dirac (1936) which gave equations for particles of higher spin. The work was first done for the force-free case and then the interaction with e. m. field was taken into account by the usual procedures,

$$p_k \rightarrow p_k - \frac{e}{c}\,\Phi_k$$

but it was shown by Pauli and Fierz that this gave an inconsistency and the correct generalisation was given by Fierz and Pauli.

Since then other equations have been given and the question is to know how far it is possible, or not, to set in such equations or to exclude them in the framework of present quantum mechanics.

The result is that in this framework, it is to say with linear equations and bounding non linear terms, it is not possible to exclude equations other than those of Dirac, Scalar equations or Proca equations. The result is thus that if there are no particles of spin higher than one, the reason will appear only in a theory were the interaction is not taken into account in the simple way of present theory.

Let us say what are the assumptions upon which those equations are based.

The first one is relativistic invariance. The second assumption is that all what you know about the system is contrained in a wave function which is given at all points of a space-like surface.

The third assumption is the requirement of predictability, it means that if the wave function is known on a surface π_0 it is possible to deduce the wave function on an other space-like surface π_1.

In practice one specializes further saying that the equations are partial differential equations and it is always possible to write down these equations in the form of first order differential equations introducing if necessary new functions. One works so with equations which are linear in the wave function for the free particle case and for the interaction part, it is possible to say that however complicated they are, there must be conditions under which the interaction must be negligible. If it is not so, you would not be able to identify a particle at all. For example, however strong the interaction of the π meson with the nuclei may be, this π meson can be identified in some cases with a particle (giving tracks in cloud chambers and so on) and so it must exist some contact transformation which absorbs those interaction terms so that the π meson behaves like a free particle.

Let us now consider the free force case. The equation must be of the first order and linear, it can be written thus in the following form

$$(\alpha^k p_k + \varkappa \beta) \psi = 0$$

where α^k and β are matrices and $\varkappa$ some constant.

This equation must be invariant, thus for a Lorentz linear transformation we have :

$$p_k^l = \tau_k^l p_l$$

and there must be some matrix T so that

$$T \alpha^k T^{-1} = \tau_l^k \alpha^l$$

From this property we can deduce immediately some commutation relations between the α^k and the infinitesimal transformation matrices I^{lm} of the ψ, which are the spin matrices. These relations are well known :

$$[\alpha^k, I^{lm}] = e^{klm}_{\ n} \ \alpha^n \quad [e^{klm}_{\ n} = \text{constants}]$$

There exist, too, commutation relations between the spin matrices alone :

$$[I^{kl}, I^{mn}] = e^{l\,klmn}_{\ op} I^{op}$$

We must deduce from the wave function ψ, which is complex some real quantities. This can be done by using the complex conjuguate to the ψ. This new function ψ^+ satisfies some equations connected with the first one and therefore certain conditions must be satisfied.

These conditions are well known. They are that a hermitian matrix D must exist which has the property

$$\alpha_k^+ \mathrm{D} = \mathrm{D}\, \alpha_k.$$

With this matrix one can build conservative real quantities, the first one transforms like a current and has the form

$$s^k = \psi^+ \mathrm{D}\, \alpha^k\, \psi.$$

The second transforms like an energy and has the form :

$$\mathrm{T}^{00} = \psi^+ \mathrm{D}\, \alpha^0 p^0\, \psi.$$

We can deduce from this result certain facts concerning the antiparticles. For example : let us consider a static state where the wave function depends of the time by a factor of the form :

$$e^{iEt}\ .$$

When you make a transformation reversing the direction of the time you shall obtain another solution with the factor e^{-iEt}.

The important thing is to see how behave the quantities s^k and T^{00} with regard to this transformation.

One can prove a general property. For the half integral spin, s^0 (the charge) conserves its sign and the sign of T^{00}(energy) is reversed by such a transformation; and for integral spin the reverse is found.

So that if you want to avoid these antiparticles you shall work differently with the two cases of spin. Thus we are lead to consider the connexion between spin and statistics.

For half integral spin, because the energy is negative for the antiparticles, one must assume that the particle satisfies the exclusion principle and follows thus the Fermi-Dirac statistics.

For the integral spin quantization following the Fermi-Dirac statistics is impossible because of mathematic inconsistency and we must use Bose statistics.

The position is now that for physical interpretation one must quantize the half-integral spin particles with Fermi-Dirac statistics and for mathematical reasons, the integral spin particles following Bose statistics.

From the experimental point of view the situation is that we have informations only in three cases, proton, neutron and electron, and in all three cases the particles have one half spin and obey Fermi-Dirac statistics. I don't think that we know at the present time that any elementary particle has certainly an integral spin.

About the photon I must say that the theory presented here concerns only particles with finite mass, the particles of vanishing mass need a somewhat different treatment.

Mr. Serber. — I think that the argument for a spin 1 for π mesons are very strong.

Mr. Bhabha. — Yes but I think that this is not definitive and does not give connexion between spin and statistics.

There is also other properties of the wave equations that we can deduce in a general way. For example : the equation satisfied by one component only of the wave function ψ.

We shall write P for the operator $\alpha^k p_k$. Since the α^k have a finite number of rows and colomns we know that they must satisfy some algebraic equation this is true also for P. We obtain

$$P^n + a_{n-2} P^{n-2} + \ldots = 0$$

We can see by some elementary considerations that if the highest power is n then a_{n-1}, a_{n-3} ... are equal to zero, and since the P contains explicitly the p's, we might expect that the a will also contain the p's but this equation is invariant, it is to say that it transforms into itself by a Lorentz transformation, thus the momenta might only appear in the a's in an invariant combination. But the only invariant combination of the p's is

$$p_k p^k = p^2.$$

We shall restrict ourselves to equations where $\beta = 1$. Then $(P + \varkappa) \psi = 0$ and we obtain finally

$$[P^n + b_{n-2} p^2 P^{n-2} + \ldots] \psi = 0$$

In the particular case where $p_1 = p_2 = p_3 = 0$, $p^2 = p_0^2$ we have $P = p \alpha^0$ and

$$(\alpha^0)^n + b_{n-2} (\alpha^0)^{n-2} = \ldots = 0$$

Thus the eigenvalues of α^0 are connected with the value of p^2 acting on ψ.

Then if we assume that the rest mass has one single value then all the b's must be equal to 0 except the first one. We obtain

$$(\varkappa^2 - p^2) \psi = 0 \text{ and } \alpha_o^{n-2} (\alpha_0^2 - 1) = 0$$

As the contrary when α^0 has the eigenvalues $\pm n$, $\pm (n\text{-}2)$... then immediately we must have more than one value for the rest mass.

We shall now see which are the equations that we can built in this framework. The first criterium is that the matrices will be algebraically irreducible. The simplest set of such irreducible matrices gives the Dirac equation. In this case the spin matrices are connected with the matrices α by the simplest possible relation.

$$I^{kl} \div \alpha^k \alpha^l - \alpha^l \alpha^k$$

The next simplest case corresponds to an equation in α^0 of the type

$$\alpha^0 [(\alpha^0)^2 - 1] = 0$$

with the same relation between I^{kl} and α^k. These are two irreducible representations, one of five rows (Klein-Gordon equations) one of ten rows (Proca equations).

The α are still hermitian and so α can be put in a diagonal form and you get a matrix like

$$\begin{pmatrix} 1 & & & & & & & & & & \\ & 1 & & & & & & & & & \\ & & \cdot & & & & & & & & \\ & & & \cdot & & & & & & & \\ & & & & \cdot & & & & & & \\ & & & & & 0 & & & & & \\ & & & & & & 0 & & & & \\ & & & & & & & 0 & & & \\ & & & & & & & & \cdot & & \\ & & & & & & & & & \cdot & \\ & & & & & & & & & & \cdot \\ & & & & & & & & & & & -1 \\ & & & & & & & & & & & & -1 \end{pmatrix}$$

Because of these o's there exist equations in the system in which the time differential does not appear. There are thus subsidiairy conditions and an important thing is that these conditions are contained in the wave equation in this frame work.

If you assume that either the total charge or total energy should be positive, then you will have the following property that D α_o^{n+1} should be a non-negative matrix, if the charge must be positive and D $\alpha_o^n \geqslant 0$, if the energy must be positive.

I shall say in passing that the equation of a particle of spin 3/2 given by Pauli and Fierz and the equation of Harrish-Chandra for Bose particles are included in this scheme. One can see that in the two cases mentioned above the subsidiary conditions get more and more complicated when the spin increases, but it is one distinction between the Pauli-Fierz equation and the Harrish-Chandra equation.

In the case of spin 3/2 for the rest state, the Pauli-Fierz equation contains only the spin 3/2 and have thus 4 independant solutions, in contrast with this. The Harrish-Chandra equations contain the spin 3/2 and 1/2 states and have 6 independant solutions.

If we conserve the relations between the spin matrices and the α matrices :

$$[\alpha^k, \alpha^l] \div I^{kl}$$

one can write all the commutation relations in one form :

$$[I^{kl}, I^{mn}] = C^{klmn}_{op} \, I^{op}$$

where $k = 0,1...4$ and $\alpha^k = I^{k4}$.

And then we get the ψ gives a representation of the Lorentz group in 5 dimensions. We know all these representations and we can label them by 2 numbers (n, m), and α^0 has all the eigenvalues $n, n-1, \ldots -n$, and we must also have several values of the rest mass. For example : a particle of spin 3/2 has 2 masses m_0 et $3m_0$.

One can prove that the second number m is the value of the spin in the state of lowest rest mass.

When we try to quantize this equation we meet some difficulties. If for the lowest value of the rest mass the charge is positive then the charge will be negative in the following case and so on.

If we quantize the lowest mass states by the commutation rule

$$[\varphi, \varphi^+] = +1$$

in the next state, of higher mass value, we shall have

$$[\varphi, \varphi^+] = -1$$

and this would, of course, lead for an inconsistency.

So that the only way of quantizing is by introducing a value of the metric which is not definite-positive. Then the state of lowest restmass will be stable. But there the transition probabilities are not positive. But one can use a procedure which is interesting.

This procedure is due to Le Couteur and Rosenfeld. The present theory is based on the fact that the wave function at the time t is defined by the wave function at the time t_0 but the matrix of transformation is no more unitary because of the indefiniteness of the metric.

But you can always introduce a matrix $\varkappa$

$$S = \frac{1-\varkappa}{1+\varkappa}$$

in such a way that $\mu\varkappa$ is hermitian and thus

$$T = \frac{1-\mu\,\varkappa}{1+\mu\,\varkappa}$$

is unitary and can be used instead of S.

In doing so you are getting away from the original scheme, but T commutes' with the unperturbed Hamiltonian, and so the energy and momentum is conserved as long as the interaction is not operative.

To summarize : as long as we introduce the interaction in the naive way we cannot exclude particles of higher spin.

And if those are to be excluded we have to work with a more general form of the interaction.

One can ask why the electron, meson, proton, all have the same value of the charge.

This indicates that the charge is a property of the field rather than of the particle, whereas the mass is a property of the particle.

Etude de la théorie générale des particules à spin par la méthode de fusion

par M. Louis de Broglie

INTRODUCTION

La méthode de fusion est une méthode qui permet d'obtenir les équations des particules de spin supérieur à $h/4\,\pi$ et de prévoir leurs propriétés en considérant ces particules comme complexes et formées par la fusion intime de corpuscules élémentaires de spin $h/4\,\pi$ obéissant aux équations de Dirac.

Cette méthode a été développée par l'auteur du présent rapport à partir de 1932 en vue d'obtenir une nouvelle théorie du champ électromagnétique dite « Mécanique ondulatoire du photon » qui permette de retrouver les résultats essentiels de la théorie quantique des champs sous une forme qui soit plus voisine que la forme habituelle des conceptions générales de la Mécanique ondulatoire.

Secondé par certains de ses jeunes collaborateurs, l'auteur du présent rapport est ainsi parvenu à construire la Mécanique ondulatoire du photon d'abord sous sa forme non quantifiée, puis sous la forme quantifiée (c'est-à-dire avec seconde quantification). Généralisant ensuite cette méthode de fusion pour l'étude de toutes les particules de spin supérieur à $h/4\,\pi$, il est parvenu à obtenir de cette manière la théorie générale des particules à spin et à retrouver ainsi les résultats obtenus presque simultanément par d'autres auteurs. Reprenant une idée de MM. Pauli et Fierz, Mme M.-A. Tonnelat est parvenue à préciser la relation entre la théorie de la gravitation et celle des particules de spin 2 (en unités $h/2\,\pi$).

L'étude de la théorie du champ électromagnétique (c'est-à-dire des photons de spin $h/2\,\pi$) et celle de la théorie générale des particules à spin par la méthode de fusion a permis de retrouver, souvent sous une forme nouvelle, les résultats obtenus indépendamment par d'autres auteurs. Elles ont permis de préciser certains points demeurés obscurs. Dans la théorie des interactions entre particules électrisées et le champ électromagnétique, la Mécanique ondulatoire du photon nous a conduit, dès janvier 1935, à suggérer une manière d'éviter la difficulté si souvent discutée des énergies infinies d'interaction rencontrée par la théorie quantique des champs sous sa forme usuelle. Une hypothèse analogue a été développée récemment par M. A.March, puis sous une forme presque identique à notre idée initiale par M.N. Rosen. Nous reviendrons plus loin sur cette question.

La méthode de fusion consiste essentiellement à considérer les particules de spin supérieur à $h/4\pi$ comme complexes et dès le début du développement de la Mécanique ondulatoire du photon, M. J.L. Destouches a proposé de considérer les équations du photon comme décrivant le mouvement d'ensemble (mouvement du centre de gravité) de la particule *complexe* « photon ». L'auteur de ce rapport a cherché à développer cette idée dans une note en 1936 : bien que les résultats de ce travail ne soient pas entièrement satisfaisants, la voie qu'elle indique paraît bonne et le même genre de considérations doit pouvoir s'appliquer à toutes les particules de spin supérieur à $h/4\pi$. Tout récemment, M. Frenkel qui ignorait certainement notre note de 1936 est arrivé à une idée analogue à la nôtre. Nous étudierons cette question dans la dernière partie de ce rapport.

I.

LA MÉCANIQUE ONDULATOIRE DU PHOTON

Nous avons développé la Mécanique ondulatoire du photon dans une série de notes parues dans les Comptes rendus de l'Académie des Sciences de Paris à partir de 1932, puis dans une série d'opuscules et d'ouvrages (1), (2), (3).

Nous avons précisé les rapports de cette théorie avec la théorie quantique des champs dans un ouvrage actuellement sous presse (4). Divers jeunes savants travaillant autour de l'auteur à l'Institut Henri Poincaré, Mme Tonnelat, MM. Jacques Winter, Jean-Louis Destouches, J. Géhéniau, Gérard Petiau ont contribué au développement de la théorie : je citerai particulièrement les travaux étendus de M. Géhéniau (5).

Nous avions fait un rapport sur la Mécanique ondulatoire du photon pour le Conseil de Physique Solvay qui devait avoir lieu à Bruxelles en octobre 1939. Les circonstances n'ayant pas permis la réunion de ce Conseil, ce rapport a été inséréré sous le titre « le Photon » dans un ouvrage ultérieur (6).

Toutes ces publications nous dispensent d'insister très longuement sur la Mécanique ondulatoire du photon et nous allons en présenter seulement un résumé succinct.

Conceptions générales de la Mécanique ondulatoire du Photon.

Nous prendrons désormais pour unité de spin, la grandeur $h/2\pi$. Les corpuscules élémentaires (électron, proton, neutron) paraissent être tous doués du spin 1/2 et avoir des équations d'onde du type de Dirac. Au contraire, les photons doivent être des particules de spin 1. C'est donc une hypothèse assez naturelle d'admettre que les photons sont des particules complexes formées par la fusion intime de deux corpuscules élémentaires de spin 1/2. Suivant la façon dont se disposeront les spins des deux constituants, on obtiendra, soit une particule constituante de spin total 1 qui sera le photon usuel, le photon de la lumière, soit une particule de spin total 0 qui sera un « photon scalaire » actuellement encore inconnu. Le photon ordinaire de spin 1 pourra se trouver dans trois états de spin distincts suivant que la composante de son spin suivant la direction de propagation sera $+1$, -1 ou 0. Les deux premières hypothèses correspondront aux ondes transversales avec leurs deux états possibles de polarisation (circulaire droite et circulaire gauche), la troisième hypothèse correspondrait aux ondes longitudinales qui existent, même en théorie classique de Maxwell pour les potentiels.

Un des avantages de cette conception du photon est d'expliquer pourquoi la statistique de Bose-Einstein s'applique à une assemblée de photons comme le prouve indubitablement la forme de la loi de Planck pour le rayonnement noir. En effet, si l'on admet, ce qui paraît vraisemblable, que tous les corpuscules élémentaires ont un spin 1/2 et obéissent à la statistique de Fermi-Dirac, un théorème général de Mécanique ondulatoire nous apprend que toute particule complexe formée d'un nombre *pair* de corpuscules élémentaires doit obéir à la statistique de Bose. L'hypothèse que le photon est une particule complexe formée de 2 corpuscules élémentaires de spin 1/2 entraîne donc bien que les photons en assemblée doivent obéir à la statistique de Bose.

Pour développer la Mécanique ondulatoire du photon, une fois admise l'hypothèse que tout se passe comme si le photon était formé de deux corpusculaires élémentaires de spin 1/2, il faut parvenir à définir le champ électromagnétique associé à un photon dont l'état est représenté par une certaine fonction d'onde que nous désignerons suivant l'usage par ψ. Diverses considérations sur lesquelles nous ne pouvons insister ici nous ont conduit à la conclusion suivante : à toute grandeur électromagnétique associée au photon (composante de potentiel ou composante de champ), on doit faire correspondre un certain opérateur linéaire, disons F_{op}, et la valeur de la grandeur considérée, quand l'état initial du photon est représenté par la fonction d'onde ψ, est donnée par une expression de la forme $\psi^0 F_{op} \psi$, où ψ^0 est une fonction d'onde représentant l'état d'annihilation du photon quand il a cédé toute son énergie à la matière. Cet état d'annihilation a été introduit naguère par M. Dirac, dans ses premiers travaux sur la théorie quantique des champs. On voit que les grandeurs électromagnétiques sont ainsi définies comme des densités d'éléments de matière attachée à la transition qui fait passer le photon de son état initial à l'état d'annihilation, c'est-à-dire à l'absorption du photon par la matière. La grandeur conjuguée $\psi^* F_{op} \psi^0$ se rapporte au contraire à l'émission.

Avec les définitions adoptées pour les grandeurs électromagnétiques, le principe de superposition est visiblement satisfait et, comme il y a lieu d'admettre que la fonction ψ^0 est indépendante des coordonnées d'espace et de temps, les expressions $\psi^0 F_{op} \psi$ représentent certaines combinaisons linéaires des composantes de la fonction d'onde du photon. Ainsi à chaque état du photon caractérisé par une fonction correspondante des potentiels et des champs qui sont définis linéairement à partir des composantes du ψ. La fonction d'onde ψ du photon, comme toutes les fonctions d'onde de la Mécanique ondulatoire doit être complexe : les grandeurs $F = \psi^0 F_{op} \psi$ seront donc elles-mêmes complexes ce qui signifie, contrairement à l'opinion de certains auteurs, que les champs électromagnétiques décrivant les phénomènes *élémentaires* d'interaction entre la matière et le rayonnement sont complexes. A partir des F complexes, on pourra définir les quantités réelles

$$F_r = F + F^*$$

(F^* étant la quantité complexe conjuguée de F). Nous avons montré que les grandeurs électromagnétiques *réelles* F_r sont celles qui interviennent dans les phénomènes macroscopiques à grande échelle où entrent en jeu un grand nombre de photons. (Voir [4] et [5]). Cette distinction entre les grandeurs électromagnétiques qui sont complexes et les grandeurs électromagnétiques macroscopiques qui sont évidemment réelles nous paraît très importante.

Equations d'ondes du photon dans le vide.

Quand on a admis l'ensemble des idées qui viennent d'être exposées, il est naturel de chercher à représenter le mouvement global de la particule « photon » par une certaine fonction d'onde obéissant à des équations aux dérivées partielles analogues à celles qu'on rencontre en théorie de Dirac. En d'autres termes, on représentera à l'aide d'une fonction d'onde $\psi\,(x, y, z, t)$ le mouvement d'ensemble du photon considéré comme une unité. Les coordonnées x, y, z peuvent alors être considérées comme se rapportant au centre de gravité. Diverses considérations que nous ne pouvons reprendre ici en détail nous ont conduit à des équations d'onde du photon dont nous allons rappeler la forme.

Tout d'abord, le photon étant par hypothèse formé de deux corpuscules de spin 1/2 (corpuscules de Dirac), on est conduit à admettre que le mouvement d'ensemble du photon doit être décrit par une fonction d'ionde ψ à 16 composantes ψ_{ik}, les indices i et k qui se rapportent respectivement aux deux constituants variant de 1 à 4. On sait qu'en théorie de Dirac, on introduit 4 matrices hermitiennes anticommutantes que l'on désigne habituellement par α_r ($r = 1, 2, 3, 4$). Ici, nous avons besoin de 4 matrices agissant sur le premier indice des α_r et de 4 matrices agissant sur le second indice. Nous avons été amenés à définir dans la représentation que nous employons, ces 8 matrices à 16 lignes et 16 colonnes de la façon suivante (*) :

$$(A_r)_{ik,\,lm} = (\alpha_r)_{il}\,\delta_{km}; \qquad (B_r)_{ik,\,lm} = \begin{cases} (\alpha_r)_{km}\,\delta_{il}\ (r = 1,3) \\ (-\alpha_r)_{km}\,\delta_{il}\ (r=2,4) \end{cases}$$

Enfin, en s'inspirant d'une notation classique dans la théorie de Dirac, on est amené à poser

$$A_r\,\psi_{ik} = \Sigma_{lm}\,(A_r)_{ik,\,lm}\,\psi_{lm}; \qquad B_r\,\psi_{ik} = \Sigma_{lm}\,(B_r)_{ik,\,lm}\,\psi_{lm}$$

(*) δ est le symbole de Kronecker.

Ces préliminaires une fois posées, nous pouvons maintenant écrire les deux groupes de 16 équations que nous considérons comme les équations de base de la Mécanique ondulatoire du photon dans le vide.

Ce sont :

$$\text{(A)} \quad \frac{1}{C}\frac{\partial\,\psi_{ik}}{\partial t}=\left[\frac{\partial}{\partial x}\,\frac{A_1+B_1}{2}+\frac{\partial}{\partial y}\,\frac{A_2+B_2}{2}+\frac{\partial}{\partial z}\,\frac{A_3+B_3}{2}+\mathrm{X}\,\mu_0 C\,\frac{A_4+B_4}{2}\right]\psi_{ik}$$

$$\text{(B)} \qquad 0=\left[\frac{\partial}{\partial x}\,\frac{A_1-B_1}{2}+\frac{\partial}{\partial y}\,\frac{A_2-B_2}{2}+\frac{\partial}{\partial z}\,\frac{A_3-B_3}{2}+\mathrm{X}\,\mu_0 C\,\frac{A_4-B_4}{2}\right]\psi_{ik}$$

où l'on a posé $\mathrm{X} = \frac{2\pi}{h}\sqrt{-1}$. Dans ces équations, μ_0 représente la masse propre de la particule « photon » que l'on pourra ensuite poser égale à 0 si l'on le juge nécessaire.

Les équations (A) sont des équations d'évolution règlant l'évolution des ψ_{ik} dans le temps tandis que les équations (N) sont des équations de condition qui imposent certaines conditions à la valeur des ψ_{ik} à chaque instant. On peut démontrer que les équations (B) sont compatibles avec les équations (A) et même que, pour les composantes spectrales de fréquences non nulles, les équations (B) sont des conséquences des équations (A). Il existe d'ailleurs d'autres manières intéressantes de répartir les 32 équations (A) et (B) en deux groupes de 16 équations.

On peut enfin démontrer qu'en vertu des équations (A) et (B), chacun des seize ψ_{ik} obéit à l'équation du second ordre

$$\text{(C)} \qquad \frac{1}{C^2}\frac{\partial^2\psi_{ik}}{\partial t^2} - \Delta\ \psi_{ik} = \mathrm{X}^2\mu_0^2 C^2\,\psi_{ik}$$

qui se réduit à $\Box\,\psi_{ik} = 0$ si le terme en μ_0^2 est considéré comme nul ou négligeable. Le passage des équations du premier ordre (A) et (B) aux équations du second ordre (C) est tout à fait comparable à celui que l'on effectue en théorie de Maxwell quand on démontre que les composantes du champ électromagnétique vérifiant les équations du premier ordre de Maxwell pour le vide satisfont chacune à une équation du second ordre du type $\Box f = 0$. Nous voyons ainsi que la théorie classique de Maxwell correspond en Mécanique ondulatoire du photon à l'hypothèse $\mu_0 = 0$.

Indiquons maintenant rapidement comment les équations (A) et (B) permettent de retrouver le champ électromagnétique Maxwellien de la lumière. Nous avons déjà indiqué que les grandeurs électromagnétiques liées au photon sont données par certaines combinaisons linéaires des fonctions ψ_{ik}. Pour représenter les six composantes du champ électromagnétique et les quatre composantes du potentiel électromagnétique, il est nécessaire d'utiliser 10 combinaisons linéaires des ψ_{ik} et, comme il y a seize ψ_{ik}, il restera encore 6 combinaisons linéaires indépendantes de ces ψ_{ik}. Nous obtiendrons ainsi 16 combinaisons linéaires indépendantes des ψ_{ik} dont 10 seulement auront un sens en théorie électromagnétique de la lumière de Maxwell (nous les appellerons les grandeurs Maxwelliennes) et dont 6 n'auront pas de sens connu dans cette théorie (nous les appellerons les grandeurs non-Maxwelliennes). Ces 16 grandeurs obéissent à 32 équations que nous pouvons obtenir aisément par des combinaisons linéaires des équations (A) et (B) et dont l'une se réduit d'ailleurs à une identité. Sur les 31 équations non identiques ainsi obtenues, 15 contiennent uniquement les grandeurs Maxwelliennes et forment un groupe autonome : elles ont la forme suivante :

$$-\frac{1}{C}\frac{\partial \vec{H}}{\partial t} = \mathrm{rot}\,\vec{E};\ \mathrm{div}\,\vec{H} = 0;\ \vec{E} = -\,\overrightarrow{\mathrm{grad}}\,V - \frac{1}{C}\frac{\partial A}{\partial t}$$

$$\text{(D)}\quad \frac{1}{C}\frac{\partial \vec{E}}{\partial t} = \mathrm{rot}\,\vec{H} - X^2\mu_0^2C^2\vec{A};\ \mathrm{div}\,\vec{E} = X^2\mu_0^2C^2V;\ \vec{H} = \mathrm{rot}\,\vec{A}$$

$$\frac{1}{C}\frac{\partial V}{\partial t} + \mathrm{div}\,\vec{A} = 0$$

Ces équations ont la forme des équations données ultérieurement par M. Proca : elles se réduisent aux équations de Maxwell en négligeant les termes en μ_0^2.

Il est facile de vérifier que les équations (D) décrivent la particule de spin 1 obtenue quand les spins des corpuscules élémentaires constituants s'ajoutent : cette particule est le « photon vectoriel » tel qu'il se révèle dans la lumière et les autres rayonnements.

Par contre, les 16 autres équations non identiques obtenues par combinaisons linéaires des équations (A) et (B) ne contiennent que les grandeurs non-Maxwelliennes. Elles décrivent les unes l'état d'annihilation du photon, les autres la particule complexe de spin nul, obtenue quand les spins des deux corpuscules constituants se retranchent. Cette particule est un « photon scalaire » (analogue au méson scalaire) qui est actuellement inconnu dans la nature.

Revenons aux équations du photon vectoriel. Il est aisé de montrer qu'elles admettent comme solutions des ondes planes monochromatiques pouvant présenter deux états de polarisation transversale (circulaire droite et circulaire gauche) et un état de polarisation longitudinale (qui, dans le cas où l'on admet l'hypothèse $\mu_0 = 0$, est représenté uniquement par des potentiels, les champs étant nuls). Ces trois états de polarisation correspondent aux trois états de spin possible du photon vectoriel. En effet, celui-ci étant une particule de spin 1, la composante de son spin dans la direction de propagation peut avoir les 3 valeurs $+ 1, — 1, 0$; les deux premières correspondent aux deux états de polarisation transversale circulaire de sens inverse et la troisième à l'état de polarisation longitudinale. Naturellement dans le cas général, il y a superposition de ces trois cas simples. La corrélation ainsi établie entre la polarisation d'une onde lumineuse plane et les états de spin du photon est tout à fait satisfaisante.

Partant toujours des équations (A) et (B), il est possible de construire un formalisme analogue à celui que l'on rencontre dans les autres formes de la Mécanique ondulatoire, notamment dans la théorie de Dirac. Mais, en Mécanique ondulatoire du photon, on rencontre une circonstance tout à fait particulière qui est connue depuis longtemps en théorie quantique des champs et qui a, nous le verrons, une portée très générale : il n'est pas possible de trouver pour la densité de probabilité de présence de la particule photon une expression partout définie positive comme l'expression $\rho = |\psi|^2$ des autres formes de la Mécanique ondulatoire. Néanmoins, en Mécanique ondulatoire du photon, on peut introduire l'expression (non définie positive)

$$\rho = \psi^* \frac{A_4+B_4}{2} \psi$$

qui peut dans une certaine mesure jouer le rôle de densité de probabilité de présence et permettre de normaliser l'onde.

La quantité $\mu_0 C^2 |\psi|^2 = \mu_0 C^2 \Sigma_{ik} \psi^*_{ik} \psi_{ik}$ joue en Mécanique ondulatoire du photon le rôle d'une densité d'énergie. En accord avec ce résultat, on trouve que, pour une onde plane monochromatique de fréquence la quantité ρ ci-dessus définie à la valeur

$$\rho = \frac{\mu_0 C^2}{h\nu} |\psi|^2$$

qui traduit bien l'existence dans l'onde des quanta d'énergie $h\nu$.

Si l'onde ψ est une superposition d'ondes monochromatiques de la orme $\psi = \Sigma_\nu \psi\nu$, on a encore la formule intégrale $\int_D \rho d\tau = \Sigma^\nu \int_D \frac{\mu_0 C^2}{h\nu} |\psi|^2 \, d\tau$ mais on n'a plus *localement* de relation simple entre ρ et la densité d'énergie de sorte que la définition adoptée pour ρ n'a plus de signification locale nette.

En adoptant pour ρ la définition ci-dessus, on peut, non sans quelques difficultés, constituer un formalisme général de la Mécanique ondulatoire du photon analogue à celui qu'on utilise dans les autres branches de la Mécanique ondulatoire. A chaque grandeur attachée au photon, on fait correspondre un opérateur linéaire et hermitien, les valeurs propres de cet opérateur donnant les valeurs possibles de la grandeur considérée : dans un état du photon défini par une certaine fonction d'onde ψ, les carrés des modules des coefficients figurant dans le développement de la fonction d'onde ψ suivant les fonctions propres de l'opérateur donnent les probabilités des diverses valeurs possibles de la grandeur dans l'état envisagé, etc... On peut ainsi définir pour chaque grandeur des éléments de matrice, une valeur moyenne et des densités d'éléments de matrice et de valeur moyenne. Par exemple, on définit aisément les densités de valeur moyenne pour les composantes du spin pour un état déterminé.

On peut aisément définir un tenseur symétrique du second rang qui représente les densités et les flux de l'énergie et de la quantité du mouvement par la particule photon. On peut même définir 2 tenseurs susceptibles de jouer ce rôle (Géhéniau). L'un a la forme habituelle du tenseur énergie-impulsion en Mécanique ondulatoire, tandis que l'autre correspondant au tenseur de Maxwell. La distinction de ces deux tenseurs est l'un des résultats intéressants de la Mécanique ondulatoire du photon.

Masse propre du photon et invariance de jauge.

La Mécanique ondulatoire du photon conduit à attribuer à un photon représenté par une certaine fonction d'onde ψ bien déterminée des valeurs parfaitement définies des potentiels A et V. Au premier abord, cela peut étonner parce qu'on est habitué à admettre pour les potentiels électromagnétiques une certaine indétermination qui s'exprime par « l'invariance de jauge ». La raison qui conduit à admettre l'invariance de jauge est la suivante : si l'on suppose que les propriétés de la lumière ne peuvent être connues que par ses actions sur les particules électrisées et si l'on admet que ces actions ne dépendent que des champs et non des potentiels, seuls les champs paraissent avoir un sens physique et alors les valeurs des potentiels ne paraissent déterminées qu'au gradient près d'une fonction des variables d'espace-temps. Naturellement en Mécanique ondulatoire du photon, si l'on ne s'intéresse qu'aux actions de la lumière sur la matière, ces actions étant supposées ne dépendre que des champs, on peut ajouter aux potentiels A et V définis à partir des ψ_{ik} les composantes d'un gradiant d'Univers. Mais cela ne nous paraît pas entraîner qu'il n'y a pas pour chaque état du photon une *véritable* valeur des potentiels A et V et qu'une description complète du photon ne doive pas faire intervenir ces valeurs.

A la question des valeurs des potentiels et de l'invariance de jauge, se rattache celle de la masse propre μ_0 du photon. Il est classique d'admettre que cette masse propre est rigoureusement nulle et nous avons vu que pour retrouver exactement les équations de Maxwell, en Mécanique ondulatoire du photon, il faut faire cette hypothèse. Mais comme en Mécanique ondulatoire *non superquantifiée* toutes les propriétés du photon sont des fonctions continues de la valeur μ_0, on peut retrouver les équations de Maxwell avec une approximation aussi grande que l'on veut en supposant μ_0 assez petit. Des raisons que nous avons exposées ailleurs (voir [3] et [6]) nous font penser qu'il n'est pas absolument nécessaire de poser $\mu_0 = 0$ et qu'il suffit de prendre μ_0 suffisamment petit (certainement inférieur à 10^{-45} grammes). Nous indiquerons plus loin une des raisons qui peuvent porter à ne pas prendre μ_0 rigoureusement nulle.

Seconde quantification de l'onde ψ du photon et quantification du champ électromagnétique.

Jusqu'ici nous avons considéré l'onde ψ du photon dans le vide. Cette hypothèse a quelque chose d'artificiel parce qu'on a pratiquement toujours affaire à des assemblées de photons en présence de matière et que le nombre des photons est alors constamment variable par suite des phénomènes d'émission et d'absorption. Il est alors tout indiqué d'introduire en Mécanique ondulatoire du photon les méthodes de la seconde quantification qui s'appliquent d'une façon très aisée pour les particules obéissant, comme c'est le cas des photons, à la statistique de Bose-Einstein. Nous avons effectué cette seconde quantification par des méthodes classiques (voir [3] et [6]) : on retrouve exactement les résultats de la théorie quantique des champs, mais on a ainsi l'avantage de faire clairement ressortir (ce qui n'apparaît pas toujours très bien dans les exposés habituels de la quantification des champs) que la quantification des champs résulte automatiquement de la seconde quantification de l'onde ψ du photon.

Au cours des calculs que l'on est ainsi amené à effectuer, on s'aperçoit que si la masse propre μ_0 du photon n'est pas rigoureusement nulle, quelque petite que soit sa valeur, la quantification des champs s'opère sans aucune difficulté. Si, au contraire, on pose $\mu_0 = 0$, on rencontre une difficulté qui est classique en théorie quantique des champs et qui consiste en une incompatibilité entre les diverses formules de non-commutation obtenues (*). On a proposé différentes manières de lever cette difficulté. Celle qui est généralement accepté consiste à considérer que les opérateurs div $\vec{H}$ et $\frac{1}{C}\frac{\partial \vec{H}}{\partial t} +$ rot $\vec{E}$ sont nuls en tous points de l'espace-temps, tandis que les opérateurs $\frac{1}{C}\frac{\partial V}{\partial t} + \operatorname{div} \vec{A}$, div $\vec{E}$ et $\frac{1}{C}\frac{\partial \vec{E}}{\partial t} - \operatorname{rot} \vec{H}$ ne sont pas nuls, mais donnent seulement un résultat nul quand on les applique à la fonction de répartition des photons entre leurs divers états. Les équations de Maxwell se diviseraient donc en deux groupes ayant des significations tout à fait différentes. Si, au contraire, l'on admet que μ_0 est différent de zéro, si petit que soit sa valeur, une telle hypothèse n'est pas nécessaire et l'on peut considérer les 5 opérateurs écrits plus haut complétés pour les termes en μ^2 comme nuls en tout point de l'espace-temps.

Dans la théorie générale des particules de spin 1 de masse non nulle (par exemple théorie du méson), on admet, comme nous le rappellerons plus loin, qu'après la seconde quantification, les 5 opérateurs en question sont nuls en tout point de l'espace-temps. Si l'on admet le point de vue usuel, l'interprétation des équations Maxwelliennes superquantifiées serait donc tout à fait différente pour le photon et pour les autres particules de spin 1. Il n'en est pas de même

(*) Il n'y a plus ici continuité quand μ_0 tend vers 0.

si l'on admet que la masse propre μ_0 du photon n'est pas rigoureusement nulle. Il est donc certain que l'hypothèse $\mu_0 = 0$ pour le photon a pour résultat de rompre l'unité de la théorie générale des particules de spin 1 et d'obliger à adopter pour le cas du photon une interprétation particulière assez arbitraire. C'est là, pensons-nous, un argument en faveur de l'hypothèse $\mu_0 \neq 0$.

Interaction entre matiere et rayonnement en Mécanique ondulatoire du photon.

Pour calculer les phénomènes résultant d'interaction entre matière et rayonnement, la théorie quantique des champs usuelle considère le système formé par le rayonnement quantifié et la particule électrisée (électron de Dirac). Pour ce système, elle emploie un Hamiltonien obtenu en faisant la somme de l'Hamiltonien du rayonnement quantifié, de l'Hamiltonien de la particule (hamiltonien de la théorie de Dirac) et d'un terme d'interaction choisi de façon à être d'accord par « correspondance » avec l'expression classique de la force de Lorentz. On peut alors, par des procédés d'approximations successives, calculer les probabilités des transitions quantiques que peut subir le système rayonnement + particule et obtenir une théorie dans l'ensemble satisfaisante des phénomènes d'émission, d'absorption, de diffusion, etc... Néanmoins, comme il est bien connu, cette théorie quantique des interactions entre matière et rayonnement se heurte à des difficultés essentielles parce qu'elle conduit à trouver des valeurs infinies pour l'énergie des particules électrisées et que la convergence des approximations successives y est incertaine.

Il est très aisé de transposer la théorie précédente en Mécanique ondulatoire du photon où elle prend un aspect plus symétrique parce que le photon et l'électron y interviennent de la même façon. La Mécanique ondulatoire du photon fournissant un Hamiltonien pour le photon, on formera l'Hamiltonien du système photon + électron en ajoutant à l'Hamiltonien du photon, celui de l'électron augmenté d'un terme d'interaction. Si nous désignons par $\vec{r}$ l'ensemble des coordonnées $x\ y\ z$ du photon et par $\vec{R}$ l'ensemble des coordonnées X Y Z de l'électron, l'opérateur d'interaction $H^{(1)}_{op}$, qui a ici la forme d'un opérateur agissant à la fois sur les variables de spin du photon et de l'électron, sera

$$H^{(1)}_{op} = -e\,[1.V_{op} + (\vec{\alpha}.\vec{A}_{op})]\ \delta\,(\vec{R}-\vec{r})$$

Dans cette formule, V_{op} et $\vec{A}_{op}$ sont les opérateurs qui, en théorie du photon, correspondent au potentiel scalaire et au potentiel vecteur : 1 et $\vec{\alpha}$ représentent respectivement la matrice unité à 4 lignes et 4 colonnes et la matrice vecteur dont les composantes sont les matrices α_1, α_2, α_3, de la théorie de Dirac, l'ensemble des matrices 1 et $\vec{\alpha}$, multipliées par la charge — e de l'électron, représentant le quadrivecteur densité — flux de l'électricité en théorie de Dirac. Quant aux facteurs $\delta\,(\vec{R}-\vec{r})$, il sert à exprimer que le champ électromagnétique existant au point $\vec{r}$ agit sur la charge électrique qui se trouve au même point $\vec{R} = \vec{r}$ et l'introduction du facteur δ exprime que ceci a lieu avec une précision rigoureuse puisque la fonction δ de Dirac est une fonction en aiguille infiniment fine. Nous reviendrons bientôt sur ce point.

La théorie des phénomènes d'émission, d'absorption, de diffusion, etc..., fondée sur l'étude du système photon + électron, quand on a adopté la forme ci-dessus précisée du terme d'interaction et introduit la seconde quantification, conduit exactement aux mêmes résultats que la théorie quantique des champs, tels qu'ils sont exposés par exemple dans le beau livre classique de M. Heitler (7). La raison de cet accord est que ces phénomènes ne font intervenir que les ondes transversales : or, pour ces ondes, la Mécanique ondulatoire du photon et la théorie quantique des champs utilisent les même formules, que la masse propre μ_0 du photon soit ou non supposée rigoureusement nulle.

L'on sait que la théorie quantique des champs interprète l'existence des interactions Coulombiennes et Laplaciennes entre particules électrisées par des échanges virtuels de photons entre les particules s'opérant par l'intermédiaire des ondes longitudinales. Cette interprétation a quelque chose d'un peu paradoxal. En effet, la théorie quantique des champs admet l'invariance de jauge, ce qui dénie tout sens physique aux potentiels, et pose implicitement $\mu_0 = 0$. Or, si $\mu_0 = 0$, les champs des ondes longitudinales sont nuls et celles-ci se réduisent à des ondes de potentiel. Or, si les potentiels n'ont aucune réalité physique, ces ondes doivent être considérées comme inexistantes et il est paradoxal de les faire intervenir pour expliquer quoi que ce soit. Au contraire, en Mécanique ondulatoire du photon, si l'on admet que μ^0 n'est pas nul et par suite que les potentiels ont un sens physique, les ondes longitudinales comportent à la fois, des potentiels et un champ électrique : elles ont donc une existence physique et leur intervention pour expliquer un phénomène paraît plus justifiée. D'ailleurs, les calculs se présentent alors sous une forme un peu différente de la forme usuelle; nous renvoyons pour leur étude à d'autres exposés ([3], [6]).

L'énergie propre des particules.

La Mécanique ondulatoire du photon ramène, du moins chaque fois qu'il s'agit d'ondes transversales, aux conclusions connues de la théorie quantique des champs. En calculant les interactions par échange virtuel de photons sur ondes longitudinales, elle retrouve aussi la conclusion fâcheuse que les charges électriques ont une énergie propre infinie. De plus, comme en théorie quantique des champs, si les calculs d'approximations successives donnent souvent en première approximation de bons résultats, par contre les approximations supérieures donnent en général des intégrales divergentes. Par exemple, si l'on évalue l'énergie propre d'un électron résultant de son interaction avec les ondes transversales, on trouve zéro en première approximation, ce qui est satisfaisant, mais en seconde approximation on trouve une intégrale divergente donnant une valeur infinie.

Bref, la Mécanique ondulatoire conduit pour les énergies des particules aux mêmes difficultés que la théorie quantique des champs, mais il semble qu'elle permet d'en préciser un peu l'origine. On sait que les valeurs infinies de l'énergie résultent de l'hypothèse implicitement admise suivant laquelle il peut y avoir des interactions entre l'électron et toutes les composantes du rayonnement, si élevée que soit leur fréquence. Or, d'après une formule bien connue due à Jeans, le nombre de ces composants croît indéfiniment avec la fréquence et de là résulte la divergence des intégrales auxquelles conduisent les calculs d'approximations successives. Mais la Mécanique ondulatoire du photon, en écrivant l'expression précise donnée plus haut de l'opérateur d'interaction entre électron et rayonnement, permet de se rendre compte que la difficulté provient essentiellement du terme $\delta(\vec{R}-\vec{r})$ qui, dans le terme d'interaction, traduit le caractère rigoureusement ponctuel de l'électron.

Cette constatation a suggéré à l'auteur de ce rapport une idée qu'il a exprimée dans une note aux Comptes rendus de l'Académie des

Sciences, dès janvier 1935 (8). Cette idée consiste à remplacer dans le terme d'interaction la fonction singulière $\delta(\vec{R}-\vec{r})$ qui est nulle pour toute valeur de $\vec{R}$ autre de $\vec{r}$ (aiguille infiniment fine) par une fonction qui serait à peu près nulle partout sauf en voisinage immédiat de $\vec{R} = \vec{r}$ (aiguille très fine). A titre d'essai, nous suggérions de de remplacer $\delta(\vec{R}-\vec{r})$ par $e^{-\frac{(\vec{R}-\vec{r})^2}{\sigma^2}}$ où σ serait une longueur très petite jouant à peu près le rôle du « rayon classique » r_0 de l'électron. On éviterait ainsi la plupart des divergences fâcheuses signalées plus haut. Il est facile de comprendre pourquoi il en est ainsi. Ces divergences résultent habituellement, nous l'avons vu, du fait que les ondes réagissent sur l'électron, quelque petite que soit leur longueur d'onde, et ceci en raison du caractère strictement « ponctuel » de la fonction δ : mais si l'on substitue $e^{-\frac{(\vec{R}-\vec{r})^2}{\sigma^2}}$ à δ, dès que la longueur d'onde descendra sensiblement au-dessous de la valeur σ, les grandeurs électromagnétiques de l'onde subiront plusieurs oscillations à l'intérieur de la sphère de rayon σ et par suite d'une compensation d'effets l'action de l'onde sur l'électron sera nulle.

Avec cette hypothèse, les ondes en nombre indéfiniment croissant qui forment l'extrémité du spectre du rayonnement du côté des grandes fréquences n'agiraient plus sur l'électron et les divergences gênantes seraient évitées. On ne reviendrait pas ainsi, à proprement parler, à l'idée classique d'un électron ayant une structure et occupant une région finie de l'espace avec des dimensions de l'ordre de σ : on définirait, grâce à la longueur σ, un rayon de l'électron qui correspondrait à une sorte *d'incertitude* sur le point d'application exact du champ électromagnétique sur la charge et cette définition, qui éviterait toute image structurale, paraît plus conforme aux conceptions générales des théories quantiques actuelles.

Telle est l'idée que nous avions développée dans notre note de 1935. Mais nous avons reconnu que cette idée se heurte à des difficultés au point de vue de l'invariance relativiste. Ces difficultés sont reliées au caractère « spatial » du rayon de l'électron et de la grandeur σ. On ne rencontre pas ces difficultés si l'on garde dans le terme d'interaction le facteur δ, mais alors on admet implicitement le caractère ponctuel de l'électron et les difficultés d'énergie infinies apparaissent.

Ne sachant comment sortir de ce dilemme, nous n'avions pas poursuivi dans cette voie. Mais récemment deux auteurs, qui ne connaissaient pas notre note de 1935, ont repris des idées analogues.

Dans une série de très intéressants mémoires ou exposés (9), M. Arthur March, après avoir approfondi la notion de « plus petite longueur » introduite par M. Heisenberg, a proposé une nouvelle manière de tenir compte, dans les termes d'interactions entre matière et rayonnements, du rayon de l'électron. Pour éviter les difficultés d'invariance relativiste, M. March réintroduit sous une forme nouvelle la « contraction de Lorentz » de l'électron et montre qu'on parvient ainsi à écarter un grand nombre des obstacles rencontrés par la théorie quantique des champs. Bien que cette théorie ne soit pas à l'abri de toute objection et que son auteur (dans un nouveau mémoire dont il nous a aimablement communiqué le manuscrit) ait dû en modifier quelques points, il y a là dans l'ensemble une tentative intéressante qu'il ne faudra pas perdre de vue.

Dans un travail tout récent (10), M. Nathan Rosen, qui n'avait pas connaissance de notre note de 1935, a, comme nous l'avions fait, introduit dans le terme d'interaction entre matière et rayonnement une exponentielle de forme Gaussienne. Il a rattaché l'introduction de cette fonction à une intéressante distinction entre « l'espace abstrait » et « l'espace observable » et il a cherché à se débarrasser des difficultés d'invariance relativiste en admettant que cette invariance n'est valable que dans l'espace abstrait. Ce mémoire contient des remarques très ingénieuses et il y aurait lieu de réfléchir, pour la préciser, sur l'analogie que présentent les idées qu'elle contient avec celles exprimées dans notre note de 1935.

Les travaux de MM. March et Rosen n'apportent sans doute pas la solution définitive du problème des énergies infinies, mais pour atteindre cette solution, elles indiquent d'intéressantes voies à suivre et ces voies sont analogues à celles que paraît suggérer la Mécanique ondulatoire du photon.

II.

THÉORIE GÉNÉRALE DES PARTICULES A SPIN PAR LA MÉTHODE DE FUSION

Nous avons vu comment la Mécanique ondulatoire du photon avait pu se développer en partant de l'idée que le photon est formé par 2 constituants étroitement unis : on parvient ainsi aux équations d'onde du photon par une méthode que nous avons nommée la « méthode de fusion ». Cette fusion des deux constituants supposés de spin 1/2 donne naissance, soit à une particule de spin 1 si les spins des constituants s'ajoutent (cas du photon vectoriel de la lumière), soit à une particule de spin 0 si le spin des constituants se neutralise (photon scalaire actuellement inconnu).

Il est évident que ce schéma peut se généraliser et que l'on peut chercher à former par la méthode de fusion les équations d'onde de particules complexes résultant de l'union de n constituants de spin 1/2. Si n est pair, on obtiendra toute une série de types de particules ayant les spins entiers $\frac{n}{2}, \frac{n}{2}-1, \frac{n}{2}-2 \ldots 1, 0$. Si n est impair, on obtiendra toute une série de types de particules de spin demi-entier $\frac{n}{2}, \frac{n}{2}-1 \ldots \frac{3}{2}, \frac{1}{2}$.

Nous avons effectué l'étude générale des diverses particules à spin ainsi obtenues par la méthode de fusion dans un ouvrage assez récent (10). Nous allons résumer quelques points des résultats obtenus en renvoyant pour le détail à l'ouvrage cité.

Les particules de spin maximum 1. Meson-équations de Proca.

La fusion des deux corpuscules élémentaires de spin 1/2 fournit, nous l'avons vu, une « particule de spin maximum », c'est-à-dire deux types de particules de spin 1 et de spin 0 respectivement. Comme en développant cette idée en Mécanique ondulatoire du photon, nous n'avons pas à priori, supposée nulle la masse propre μ_0, les équations que nous avons obtenues en théorie du photon, doivent pouvoir représenter toute particule de spin maximum 1, du moins quand on néglige sa charge électrique et l'action éventuelle de champs électromagnétiques extérieurs sur cette charge. Si donc, on admet (ce qui peut d'ailleur être discuté) que les mésons sont des particules de spin maximum 1, les équations d'onde des mésons seront celles qu'on obtient en supposant le méson formé par la fusion de 2 corpuscules de spin 1/2. Ces équations seront donc identiques à celles que nous avons rencontrées en Mécanique ondulatoire du photon avec cette seule différence qu'ici nous sommes surs que la masse propre μ_0 est différente de zéro. Nous obtiendrons donc ainsi deux groupes d'équations indépendantes représentant l'une un méson

« vectoriel » analogue au photon de la lumière, l'autre, un méson « scalaire » analogue au photon scalaire. On sait qu'à la suite de difficultés éprouvées en théorie du Méson pour faire cadrer la vie moyenne calculée du méson avec sa vie moyenne observée, M. Rozental (11) a émis l'intéressante hypothèse que les mésons vectoriels à courte vie moyenne ne se rencontreraient habituellement que dans les hautes couches de l'atmosphère, les mésons à vie moyenne relativement longue observés à basse altitude étant des mésons scalaires. Bien que la question des diverses sortes de mésons soit encore en pleine évolution, l'hypothèse de M. Rozental paraît dans l'ensemble confirmée par les faits et les mésons usuellement observés à la surface de la terre semblant bien être des mésons scalaires. S'il est vrai qu'à côté des mésons vectoriels, il existe des mésons scalaires, l'idée qu'a côté des photons vectoriels de la lumière, il puisse exister des photons scalaires n'a plus rien d'extraordinaire.

Les équations d'onde de la Mécanique ondulatoire du photon s'appliquent donc au Méson. Elles ont été retrouvées, indépendamment par M. Alexandre Proca. Les équations de Proca ne diffèrent de celles que nous avons données en théorie du photon que par la présence de termes traduisant l'action d'un champ electromagnétique extérieur sur la charge des mésons, termes qui n'existent pas pour le photon dont on suppose la charge électrique nulle.

La masse propre du méson étant certainement différente de zéro, on peut effectuer la seconde quantification des équations du Méson sans difficulté en considérant les opérateurs $\operatorname{div} \vec{H}$, $\frac{1}{C}\frac{\partial \vec{H}}{\partial t} + \operatorname{rot} \vec{E}$,

$\frac{1}{C}\frac{\partial V}{\partial t} + \operatorname{div} \vec{A}$, $\operatorname{div} \vec{E} - x^2\mu_0^2c^2V$, $\frac{1}{C}\frac{\partial \vec{E}}{\partial t} - \operatorname{rot} \vec{H} + x^2\mu_0^2c^2\vec{A}$ comme nuls en tout point de l'espace-temps. Pour le photon, si l'on admet que sa masse propre est nulle, il faut, nous l'avons dit, procéder différemment en supposant que seuls les 2 premiers opérateurs sont nuls en tout point de l'espace-temps, les 3 derniers donnant seulement un résultat nul quand on les applique à la fonction de répartition des photons entre leurs divers états. L'unité de la Mécanique ondulatoire superquantifiée des particules de spin 1 se trouve ainsi rompue d'une manière qui paraît assez artificielle et c'est là une des raisons pour lesquelles nous préférons ne pas condidérer la masse propre du photon comme rigoureusement nulle.

Particule de spin supérieur à 1.

Comme nous l'avons dit, la méthode de fusion conduit à considérer la fusion de n corpuscules élémentaires de spin 1/2 comme fournissant une particule de spin maximum $\frac{n}{2}$, ou si l'on préfère, une série de particules de spin $\frac{n}{2}, \frac{n}{2} - 1, \ldots$ qui constituent comme divers états de spin de la particule de spin maximum $\frac{n}{2}$.

La Mécanique ondulatoire du photon, en nous fournissant le modèle d'une théorie générale d'une particule de spin maximum 1, nous a indiqué la voie à suivre pour obtenir la théorie de la particule de spin maximum $\frac{n}{2}$. Dans les chapitres IX et X de notre ouvrage sur les particules de spin (10), nous avons par cette méthode obtenu, sous forme non superquantifiée, les équations générales d'évolution et de condition pour les particules de spin maximum quelconque obtenues par fusion. Il est aisé d'écrire pour ces particules, l'expression des ondes planes monochromatiques, de définir un quadrivecteur densité-flux ainsi qu'un tenseur énergie-impulsion du type corpusculaire et *plusieurs* tenseurs énergie-impulsion du type Maxwellien dont le nombre croît avec n.

Au cours de cette étude, on parvient à des conclusions identiques à celles que plusieurs auteurs, notamment M. Pauli, et en particulier, M. Fierz dans un mémoire fondamental (12), avaient déjà énoncées. C'est ainsi qu'on trouve une différence essentielle entre les particules de spin entier obtenues par fusion d'un nombre pair de constituants et les particules de spin demi-entier obtenues par fusion d'un nombre impair de constituants. Pour les particules de spin entier, on peut trouver des grandeurs tensorielles que l'on peut substituer aux composantes du ψ et qui obéissent à des équations à caractère tensoriel; ce que nous avons vu déjà pour les particules de spin 1 (par exemple grandeurs et équations Maxwelliennes et non Maxwelliennes dans le cas du photon). Il en est tout autrement pour les particules de spin maximum demi-entier : pour elles il n'existe pas de grandeurs tensorielles pouvant remplacer les composantes de la fonction d'onde et les équations d'onde gardent nécessairement une forme « spinorielle » comme dans la théorie de la particule de spin 1/2 (théorie de Dirac). Ce résultat est en accord avec ceux de M. Fierz.

La densité de probabilité de présence que l'on peut introduire dans la théorie générale des particules à spin n'est définie positive que pour $n = 1$ (cas de l'électron de Dirac). Pour $n \geqslant 2$, elle n'est plus définie positive. La densité d'énergie (*) est définie positive pour $n = 1$; pour $n = 2$, elle l'est encore si l'on emploie le tenseur Maxwellien. Mais pour $n > 2$, cette densité n'est j'amais définie positive. La probabilité totale de présence $\int \rho d\tau$ est définie positive pour n impair (spin demi-entier), mais ne l'est pas pour n pair (spin entier) à cause de l'existence des ondes à énergie négative. Enfin la valeur moyenne de l'énergie fournie par l'intégrale spatiale de la composante 44 de l'un des tenseurs impulsion-énergie n'est définie positive que pour n pair (spin entier) : elle ne l'est pas pour n impair (spin demi-entier) à cause de l'existence des ondes à énergie négative. Tous ces résultats sont conformes à ceux des autres auteurs et à ce que l'on pouvait attendre. .

On peut aisément définir les opérateurs de spin et les moyennes correspondantes pour la particule de spin maximum $\frac{n}{2}$ obtenue par fusion de n constituants de spin 1/2. On en tire aisément la nomenclature des états de spin correspondant aux diverses valeurs de spin dues à la façon dont les spins des constituants s'ajoutent ou se retranchent. Pour n pair, on obtient ainsi les valeur du spin $\frac{n}{2}, \frac{n}{2} - 1, \ldots, 1, 0$ qui peuvent être considérées comme correspondant à des types indépendants de particules : de même pour n impair, on obtient pour le spin les valeurs demi-entières $\frac{n}{2}, \frac{n}{2} - 1, \ldots\ 3/2, 1/2$ qu'on peut considérer comme correspondant à des types indépendants de particules.

Ainsi, par des raisonnements relativement simples, la méthode de fusion permet d'obtenir une claire vue d'ensemble de la théorie des particules de spin quelconque.

La théorie de la particule de spin maximum 2.

La théorie de la particule de spin maximum 2 obtenue par fusion de 4 corpuscules de spin 1/2 offre une belle illustration de la théorie générale esquissée plus haut. Nous l'avons résumée au chapitre XI

(*) Composante 44 du tenseur impulsion-énergie.

de notre livre sur les particules à spin (10) d'après les travaux de Mme Tonnelat (13). La particule de spin maximum 2 peut se présenter sous trois états de spin correspondant aux valeurs 2, 1 et 0 du spin. La théorie conduit à distinguer trois réalisations différentes de l'état de spin 1 et deux de l'état de spin 0, tandis qu'on obtient une seule réalisation de l'état de spin 2. La séparation exacte de ces divers états de spin (en particulier de l'état de spin 2 et de l'une des représentations des états de spin 0) est assez délicate : elle n'a été complètement réalisée d'une façon satisfaisante que dans un mémoire de M. Van Isacker (14). Finalement les états décrits par les diverses représentations sont indépendantes et peuvent être considérées comme décrivant des particules distinctes.

Ce qui rend particulièrement intéressante la théorie de la particule de spin maximum 2, c'est qu'elle présente une certaine analogie avec la théorie Einsteinienne de la Gravitation en Relativité généralisée. Cette analogie avait été signalée par MM. Pauli et Fierz (15) : alle a été approfondie par Mme Tonnelat dans ses travaux cités plus haut. En établissant une relation convenable entre les g_{uv} de la théorie de la Gravitation et certaines combinaisons linéaires des ψ qui décrivent les particules de spin 2, Mme Tonnelat parvient à identifier les équations de la Gravitation à *l'approximation linéaire* avec les équations (également linéaires) de la particule de spin 2. Cette identification lui permet de calculer la masse propre du graviton (peut-être égale à celle du photon) à partir de la constante cosmologique, c'est-à-dire du rayon de l'Univers de de Sitter : elle trouve ainsi pour cette masse propre une valeur voisine de 10^{-66} gramme.

Malgré ses aspects séduisants, le parallélisme ainsi établi entre la théorie de la Gravitation et celle de la particule de spin 2 comporte des difficultés parce que, partant d'équations d'ondes linéaires valables dans un espace-temps euclidien, elle cherche à en déduire des g_{uv} décrivant un espace-temps non euclidien. Peut-être cette difficulté se trouverait-elle levée si l'on pouvait construire une Mécanique ondulatoire non linéaire dont la Mécanique ondulatoire ordinaire ne serait qu'une approximation.

On doit remarquer que la théorie de la Gravitation de Mme Tonnelat est une théorie miscroscopique faisant usage de $g^{\mu\nu}$ complexes, tout comme en Mécanique ondulatoire du photon les champs électromagnétiques sont complexes. C'est en passant au point de vue macroscopique statistique que l'on doit retrouver les valeurs réelles des g^{uv} employées par la théorie usuelle de la Gravitation, tout comme en théorie de la lumière, c'est le passage au point de vue macroscopique qui permet de s'élever des grandeurs électromagnétiques complexes de la théorie du photon aux grandeurs electromagnétiques réelles qui, dans les théories classiques, servent à décrire l'action du rayonnement sur la matière.

III.

REMARQUE SUR LE SENS DE LA MÉTHODE DE FUSION

La méthode de fusion repose sur l'idée que les particules de spin 1/2 sont élémentaires, c'est-à-dire indissociables, et que les particules de spin supérieur à 1/2 sont complexes. Partant de cette idée, nous sommes parvenus, comme il a été dit plus haut, à obtenir des équations d'ondes pour le photon considéré comme une unité résultant de la fusion de deux constituantes de spin 1/2 et la même méthode fournit aussi des équations d'onde qui paraissent satisfaisantes pour les particules de spin supérieur à 1. Mais quel sens cela a-t-il de considérer une particule complexe comme une unité et que représentent les coordonnées x, y, z attachées à cette unité? Dès le début du développement de la Mécanique ondulatoire du photon, M. Jean Louis Destouches a suggéré que les équations d'onde du photon décrivant le mouvement du *centre de gravité* de cette particule complexe, les coordonnées x, y, z étant celles de ce centre de gravité. Cette idée peut naturellement se généraliser pour toutes les particules de spin supérieur à 1/2.

Les équations d'ondes des particules de spin supérieur à 1/2 considérées comme décrivant l'ensemble d'un système de constituants de spin 1/2.

Si l'on admet l'hypothèse de M. Destouches, il est naturel que l'on cherche à passer de l'équation d'ondes décrivant le système formé par les corpuscules constituant la particule complexe à l'équation d'ondes décrivant le mouvement du centre de gravité. Cette tentative soulève à priori de nombreuses difficultés. Il faudrait d'abord savoir écrire l'équation d'ondes d'un ensemble de corpuscules en tenant compte de la Relativité et en particulier savoir exprimer leurs interactions : si les corpuscules sont en interaction très étroite, ce qui doit être le cas pour les systèmes « fondus », il n'est même pas certain que cela ait un sens d'attribuer aux divers corpuscules une masse propre bien définie et même des coordonnées, car l'individualité des corpuscules est de plus en plus absorbée dans l'unité du système quand l'interaction devient plus étroite. Enfin, il faudrait arriver à reporter les propriétés de spin des constituants sur le système lui-même conçu comme une unité ou, si l'on préfère, sur le centre de gravité.

Bien que très conscient de la gravité de ces difficultés, nous avons fait une tentative de ce genre pour les particules de spin 1 à deux constituants (le photon par exemple) dans une note publiée en 1936 dans les Comptes rendus de l'Académie des Sciences (16). La méthode employée dans cette note est certainement défectueuse à divers points de vue. C'est ainsi que nous n'avons pas introduit explicitement dans notre texte le potentiel d'interaction entre les deux constituants (*), ce qui revient à admettre implicitement que ce potentiel a la forme singulière $\delta\,(\vec{R}-\vec{r})$; nous avons ainsi été amenés à prendre pour masse propre globale du système la somme des masses propres (supposées égales) des constituants, ce qui ne saurait être exact. De plus, nous avons adopté, pour décrire l'état interne de la particule complexe, une fonction à singularité polaire et nous avons reconnu depuis qu'il serait préférable de prendre une solution de forme dipolaire. Malgré ces graves imperfections, la méthode suivie nous a amené à effectuer le report sur le centre de gravité des propriétés de spin des deux constituants d'une manière qui nous paraît intéressante et instructive. Aussi pensons-nous que, malgré son caractère peu satisfaisant, cet essai donne une certaine indication sur la voie à suivre pour traiter la question de la fusion des corpuscules de spin 1/2 dans l'unité d'un système de spin plus élevé. Et le problème est certainement très important car il est lié aux questions les plus délicates concernant la Mécanique ondulatoire *relativiste* des systèmes de corpuscules, la définition du centre de gravité dans la théorie de la Relativité et la fusion progressive des constituants dans l'unité du système, fusion elle-même en relation étroite avec le principe de l'inertie de l'énergie.

Dans un important mémoire récent (17), M. Frenkel a développé de très intéressantes considérations sur les systèmes de corpuscules. Il a insisté sur le fait que, quand l'interaction augmente, il y a une sorte de fusion des constituants dans l'unité supérieure du système et a donné sur ce point quelques indications quantitatives. En faisant

(*) D'ailleurs, en raison du caractère « retardé » des interactions, la représentation des interactions par un potentiel ne peut avoir qu'une validité limitée.

accessoirement plusieurs remarques très pertinentes, il est arrivé au sujet des équations d'ondes des particules de spin supérieur à 1/2 à la conclusion suivante : « I believe that generalised equation of this type will give an adequate description of complex particles treated as a material point with certain inner degrees of freedom ». Cette conclusion est en parfait accord avec les idées que nous venons d'exposer et il est tout à fait intéressant de constater que M. Frenkel est parvenu à un point de vue tout à fait analogue au nôtre sans avoir eu connaissance de nos travaux.

Dans le même ordre d'idées, nous signalerons encore les travaux de M. Slansky sur la définition relativiste du centre de gravité (18) et la thèse de Doctorat (encore inédite) de M. Robert Murard « Sur la théorie générale des corpuscules et des systèmes de corpuscules ».

Particules complexes et probabilités de présence.

Nous avons vu que, dans la théorie de la particule de spin 1 formée par la fusion de 2 constituants, la quantité $|\psi|^2 = \Sigma_{ik} \, |\psi_{ik}|^2$ représente une densité d'énergie et non une densité de probabilité de présence. Pour trouver une quantité pouvant dans une certaine mesure jouer le rôle de densité de probabilité de présence, il faut considérer la grandeur $\rho = \Sigma_{ik} \, \psi^*_{ik} \frac{A_4 + B_4}{2} \psi_{ik}$. Pour une onde ψ plane et monochromatique correspondant à un mouvement rectiligne et uniforme de vitesse BC, on trouve $\rho = \Sigma_{ik} \, |\psi_{ik}|^2 \cdot \sqrt{1-B^2}$, expression où l'on voit apparaître un facteur de contraction de Lorentz $\sqrt{1-B^2}$.

Plus généralement, si l'on considère une particule de spin $\frac{n}{2}$ formée par la fusion de n corpuscules élémentaires de spin 1/2, on peut définir une grandeur ρ ayant la variance voulue en introduisant dans sa définition les produits $n - 1$ à $n - 1$ des n facteurs du type A_4 relatif aux n constituants (*). On obtient ainsi une densité ρ qui, pour une onde plane monochromatique, est égale à $(1-B^2)^{\frac{n-1}{2}} |\psi|^2$.

Dans le cas général des superpositions d'ondes planes monochromatiques, la quantité ρ peut n'être pas partout non négative, mais l'intégrale $\int \rho \, d\tau$ peut toujours servir à la normalisation, par ce qu'elle est positive (tout au moins pour les ondes à énergie positive).

(*) Voir (10), chapitre IX.

Le fait que, pour les particules de spin supérieures à 1/2, la densité ρ que l'on est amené à envisager ne soit pas égale à $|\psi|^2$ et ne soit pas partout définie positive, complique le formalisme et le rend moins satisfaisant que dans le cas du spin 1/2 (électron de Dirac). D'où vient cette complication? Nous soupçonnons depuis longtemps qu'elle a pour origine précisément le caratère complexe des particules de spin supérieur à 1/2 et que l'intervention des facteurs $\sqrt{1-B^2}$ dans le cas des ondes planes provient de la contraction de Lorentz subie par la structure interne de la particule. Pour préciser cette idée, nous allons traiter un problème un peu simplifié.

Considérons une particule complexe formée par deux constituants élémentaires de natures identiques (ayant par conséquent même masse propre) et faisons abstraction des spins. Si nous désignons pas x, y, z, et $x_2 y_2 z_2$ les coordonnées des deux constituants, la fonction d'onde $\Phi\,(x, y, z, x_2, y_2, z_2, t)$ du système est définie dans un espace de configuration à 6 dimensions et la probabilité de présence du point figuratif du système en un point de cet espace de configuration sera $|\Phi(x, y, z, x_2, y_2, z_2, t)|^2$.

Les deux constituants étant supposés identiques, il est naturel de définir les coordonnées du centre de gravité du système par les équations $x = \dfrac{x_1 + x_2}{2}\; y = \dfrac{y_1 + y_2}{2}\; z = \dfrac{z_1 + z_2}{2}$. On introduira alors les coordonnées internes :

$$u = x_1 - x = \frac{x_1 - x_2}{2};\; v = y_1 - y\,\frac{y_1 - y_2}{2};\; w = z_1 - z\,\frac{z_1 - z_2}{2}.$$

Plaçons-nous d'abord dans un système de référence Galiléen où le centre de gravité est immobile (système propre de la particule complexe) la fonction d'onde du système y aura la forme

$$\Phi\,(x_{10}, y_{10}, z_{10}, x_{20}, y_{20}, z_{20}, t_0) = \psi\,(x_0\, y_0\, z_0\, t_0)\,.\,\varphi\,(u_0\, v_0\, w_0)$$

avec

$$\psi\,(x_0\, y_0\, z_0\, t_0) = \mathrm{A}e^{\frac{2\pi i}{h}\,\mathrm{W}_0\, t_0}$$

Les indices zéro indiquent que les quantités sont mesurées dans le système propre. Nous normerons la fonction φ en posant

$$\int |\varphi\,(u_0, v_0, w_0)|^2\; du_0\, dv_0\, dw_0 = \frac{1}{|\Delta|}$$

Δ étant la valeur (égale à -8) du déterminant Jacobien des variables $x_{10} \ldots z_{20}$ par rapport aux variables $x_0 \ldots w_0$.

La probabilité de présence du point figuratif du système en un point de l'espace de configuration sera

$$|\Phi|^2\, dx_{10} \ldots dz_{20} = |\psi(x_0 y_0 z_0 t_0)|^2 . |\varphi\,(u_0 v_0 w_0)|^2\, |\Delta|\, dx_0 \ldots dw_0$$

La probabilité de présence du centre de gravité dans l'élément de volume $dx_0\ dy_0\ dz_0$ s'obtiendra par intégration de l'expression précédente sur $u_0\ v_0\ w_0$ et sera

$$|\psi\,(x_0 y_0 z_0 t_0)|^2\, dx_0 dy_0 dz_0$$

Nous allons maintenant nous placer dans un autre système de référence Galiléen animé de la vitesse relative $\overrightarrow{BC}$ par rapport au premier. Comme nous négligeons l'existence du spin, nous pouvons considérer les fonctions Φ et ψ comme des scalaires (nous reviendrons plus loin sur ce point), et nous aurons dans notre nouveau système de référence

$$\Phi\,(x_1 \ldots z_2, t) = \psi\,(x,y,z,t)\ .\ \varphi\,(u,v,w,t)$$

où $\psi\,(x, y, z, t)$ et $\varphi\,(u, v, w, t)$ sont ce que deviennent les fonctions $\psi\,(x_0\, y_0\, z_0\, t_0)$ et $\varphi\,(u_0\, v_0\, w_0)$ quand on y remplace $x_0\, y_0\, z_0\, t_0\, u_0\, v_0\, w_0$ par x, y, z, t, u, v, w à l'aide de la transformation de Lorentz correspondant à la vitesse relative $\overrightarrow{BC}$. Nous regarderons la fonction ψ (x, y, z, t) comme étant la fonction d'onde de la particule complexe envisagée comme une unité.

La probabilité de présence du point représentatif dans l'espace de configuration $x_1 \ldots z_2$ est alors

$$|\Phi\,(x_1 \ldots z_2 t)|^2 dx_1 \ldots dz_2 = |\psi\ (x,y,z,t)|^2 . |\varphi\,(u,v,w,t)|^2 . |\Delta|\ dx \ldots dw$$

La probabilité de présence du centre de gravité dans l'élément de volume $dx\ dy\ dz$ s'obtient par intégration sur u, v, w, ce qui donne

$$\rho\ (x, y, z, t) = |\Psi\,(x, y, z, t)|^2 \int |\varphi\ (u,v,w,t)|^2\ |\Delta|\ du,\ dv,\ dw$$

Pour calculer l'intégrale, on peut revenir aux variables $u_0\ v_0\ w_0$ et l'on obtient en raison de la contraction de Lorentz

$$\int |\varphi\,(u,v,w,t)|^2 du\, dv\, dw = \int |\varphi\,(u_0 v_0 w_0)|^2\ \sqrt{1-B^2}\, du_0\, dv_0\, dw_0 = \frac{\sqrt{1-B^2}}{|\Delta|}$$

d'où

$$\rho\,(x,y,z,t) = |\psi\,(x,y,z,t)|^2 . \sqrt{1-B^2}$$

La densité de probabilité de présence pour le centre de gravité de la particule complexe n'est donc pas égale à $|\psi|^2$, mais à cette quantité multipliée par le facteur de contraction de Lorentz $\sqrt{1-B^2}$. L'introduction dans la théorie de la fusion des facteurs $\frac{A_4+B_4}{2}$ dans l'expression de ρ a en somme pour but de tenir compte de cette contraction de Lorentz.

Nous avons jusqu'ici supposé que la particule est animée d'un mouvement d'ensemble rectiligne et uniforme dans le second système Galiléen envisagé. Mais en Mécanique ondulatoire, nous devons aussi pouvoir considérer une « superposition » de divers mouvements rectilignes et uniformes correspondant à diverses vitesses $\overrightarrow{B_i C}$.

L'onde Φ a alors la forme

$$\Phi\,(x_1\,\ldots\,z_2t) = \Sigma_i\, C_i\psi_i\,(x, y, z, t)\,\varphi_i\,(u, v, w, t)$$

avec $\Sigma_i\,|\,C_i\,|^2 = 1$. Les fonctions $\psi_i\,(x, y, z, t)$ et $\varphi_i\,(u, v, w, t)$ sont ce que deviennent $\psi\,(x_0y_0z_0t_0)$ et $\varphi\,(u_0v_0w_0)$ quand on y remplace les variables $x_0y_0z_0t_0u_0v_0w_0$ en fonction de x, y, z, t, u, v, w, à l'aide de la transformation de Lorentz relative à la vitesse $\overrightarrow{B_i C}$.

La probabilité de présence du point figuratif dans l'espace de configuration est donnée par

$$|\Phi(x_1\ldots z_2t)^2dx_1\ldots dz_2 = \Sigma_i|C_i|^2\psi_i(xyzt)|^2\,(dxdydz|\Delta|\varphi_i\,(uvwt)|^2\,dudvdw$$
$$+\sum_{ij}^{i\neq j}\, C^*_iC_j\,\psi^*_i\,(xyzt)\;\psi_j\,(xyzt)\,dxdydz\,|\,\Delta\,|\;\varphi^*_i\,(uvwt)\;\varphi_j\,(uvwt)\;dudvdw$$

La probabilité de présence du centre de gravité dans l'élément de volume $dx\;dy\;dz$ s'obtient par intégration sur u, v, w et s'écrit

$$\rho(xyzt) = \Sigma_i|C_i|^2|\,\psi_i\,(xyzt)\,|^2\sqrt{1-B_i^2} + \sum_{ij}^{i\neq j}\;C^*_iC_j\,\psi^*_i\,\psi_j\;\;|\Delta|\;\int\;\varphi^*_i\,(uvwt)\;\;\varphi_j\;\;(uvwt)\;\;dudvdw$$

Les termes de la seconde somme ne sont pas nuls et dépendent de la forme de $\varphi\,(u_0\;v_0\;w_0)$. La densité ρ ne peut donc plus ici s'exprimer à l'aide du ψ seulement. Cette densité ρ qui, d'après sa définition par $|\Phi|^2$, est définie positive, ne coïncide pas avec la densité ρ définie par la théorie de la fusion, laquelle s'exprime uniquement à l'aide du ψ et n'est pas définie positive. Donc, il y a une superposition d'ondes planes, la définition de ρ par $\Sigma_{ik}\,\psi^*_{ik}\,\frac{A_0+B_0}{2}\;\psi_{ik}$ n'est sans doute plus exacte, mais si l'on intègre la véritable densité ρ donnée par la dernière équation sur toutes les valeurs de $x\ y\ z$, l'on obtient

$$\int \rho\,(x, y, z, t)\,dx, dy, dz = \Sigma_i \mid C_i \mid^2 \int \mid \psi_i \mid_2 \sqrt{1-B_i^2}\,dx\,dy\,dz$$

et cette formule montre que la densité adoptée par la théorie de la fusion est « intégralement équivalente » à la véritable densité et peut par suite, servir pour la normalisation.

Les considérations précédentes paraissent bien expliquer pourquoi l'expression adoptée en théorie de la fusion doit être exacte pour les ondes planes et pourquoi elle est encore utilisable pour la normalisation dans le cas général.

Néanmoins, les raisonnements précédents n'ont qu'une valeur d'indication parce que nous avons négligé le spin. En réalité pour les particules à spin, la fonction ψ a plusieurs composantes et il faut tenir compte de la façon dont ces composantes se transforment lors d'une transformation de Lorentz. Dans notre note de 1936 (16) nous avons été amenés à écrire *en reportant le spin sur le centre de gravité*

$$\Phi_{kl}\,(x_1\ \ldots\ z_2 t) = \psi_{kl}\,(x, y, z, t)\ .\varphi\,(u, v, w, t)$$

les fonctions φ décrivant la structure interne de la particule complexe et nous avons retrouvé ainsi les résultats du calcul précédent avec substitution de $\Sigma_{ik} \left|\psi_{ik}\right|^2$ à $\left|\psi\right|^2$. Le même genre de raisonnement peut être appliqué aux particules complexes de spin $\frac{n}{2}$ formées de n constituants.

Même si des critiques peuvent être adressées aux calculs qui précèdent, il nous semble que l'idée générale suivant laquelle l'impossibilité de trouver une densité de probabilité de présence définie positive pour les particules de spin supérieur à 1/2 doit être rattachée à la nature complexe de ces particules et à la contraction de Lorentz qu'elles subissent dans leur structure interne quand elles sont en mouvement, doit être en principe exacte.

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(3) Louis de Broglie, *Une nouvelle théorie de la lumière : la mécanique ondulatoire du photon*, Hermann, Paris, 2 volumes (1940 et 1942).

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(15) W. Pauli et M. Fierz, *Physica Helvetica Acata*, vol. XII, fasc. **4**.

(16) Louis de Broglie, *Comptes rendus Acad. Sciences*, t. **203**, p. 473 (1936).

(17) J. Frenkel, « Relativistic quantum theory », *Journal of Physics* (U.R.S.S.), vol. **9**, p. 1943 (1945).

(18) Serge Slansky, *Journal de Physique*, t. VIII, p. 56 (1947).

ADDENDUM

par Marie-Antoinette Tonnelat

Ajoutons à ce rapport les quelques remarques suivantes :

La Théorie générale des particules à spin peut relever d'un formalisme qui ne fait appel à aucune hypothèse physique sur la constitution des particules.

Considérons, par exemple, le cas de la particule de spin 1. Nous pouvons partir d'une équation de Dirac et d'une équation adjointe relative non à un spineur mais à une matrice $|\Psi|$ à 4 lignes et à 4 colonnes. Les équations :

$$|\gamma^\mu \partial_\mu - k|\,|\Psi| = 0$$
$$\partial_\mu |\Psi|\,|\gamma^{\mu+}| + k|\Psi| = 0$$

représentent alors un produit de matrices. La fonction d'ondes, comme toute matrice à 4 lignes et à 4 colonnes, peut être développée suivant les 16 matrices de la Théorie de Dirac qui forment un système complet

$$|\Psi| = \gamma^o \varphi_o + \gamma^\mu \varphi_\mu + 1/2\, \gamma^\mu \gamma^\nu \varphi_{\mu\upsilon} + 1/6\, \gamma^\mu \gamma^\nu \gamma^\rho \varphi_{\mu\upsilon\rho}$$
$$+ 1/24\, \gamma^\mu \gamma^\upsilon \gamma^\rho \rho^\sigma \varphi_{\mu\upsilon\rho\sigma}$$

Les φ coefficients du développement du Ψ suivant les γ sont des grandeurs complexes qui représentent les composantes des champs maxwellien (φ_μ, $\varphi_{\mu\upsilon}$) et non maxwellien ($\varphi_{\mu\upsilon\rho}$, $\varphi_{\mu\upsilon\rho\sigma}$). Ce développement a l'avantage de mettre en évidence les relations linéaires entre chaque élément Ψ_{ik} et les grandeurs φ. La forme même des γ montre que chaque Ψ_{ik} s'exprime en fonction de 4 composantes φ et réciproquement.

D'autre part, la substitution du $|\Psi|$ dans les équations initiales met en évidence l'équivalence entre équations d'ondes et équations du champ.

La substitution d'un développement dual conduirait aux équations d'une particule scalaire et pseudo-vectorielle. On a donc, sous une autre forme, les quatre représentations prévues ultérieurement par Kemmer (1).

(1) M. A. Tonnelat, *Comptes rendus Acad. Sciences*, t. 208, p. 790 (1939).

D'autre part la substitution au facteur k d'une matrice k β remplissant certaines conditions permet de prévoir plusieurs états de masse. Dans le cas de la particule de spin 1, il est donc possible d'introduire deux masses différentes pour le groupe vectoriel et pour le groupe pseudo-scalaire (1).

Les matrices appelées A + B et A — B dans le rapport ci-dessus satisfont certaines relations établies par Petiau [2] et reprises par Duffin. En posant $1/2\ (A_\mu + B_\mu) = \Gamma_\mu$ et $1/2\ (A_\mu - B_\mu) = I_\mu$ on a

$$\Gamma^3_\mu = \Gamma_\mu$$

$$\Gamma^2_\mu \Gamma_\nu + \Gamma_\nu \Gamma^2_\mu = \Gamma_\nu$$

$$\Gamma_\mu \Gamma_\nu \Gamma_\lambda + \Gamma_\lambda \Gamma_\nu \Gamma_\mu = 0$$

Bien entendu ce formalisme permet toujours — sans l'imposer — l'interprétation des grandeurs de champ comme résultats des transitions de la particule, l'intervention d'un état d'annihilation, l'hypothèse d'une structure complexe de la particule.

Mais il en reste totalement indépendant.

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La comparaison entre la théorie de la particule de spin 2 et la Relativité générale soulève les difficultés indiquées dans le rapport de M. Louis de Broglie : d'une part, le rapprochement entre les grandeurs réelles de la Relativité générale oblige à donner à celle-ci une interprétation statistique. D'autre part, le parallélisme entre équations corpusculaires (du 1er ordre) et équations d'Einstein (du second ordre) oblige à se limiter à une approximation linéaire de ces dernières. Cela tant que l'on ne saura pas construire une mécanique quantique non linéaire satisfaisante.

Mais il est une difficulté de principe qui porte sur le sens même du rapprochement que l'on peut imaginer entre une théorie corpusculaire et la Relativité générale. Ce rapprochement peut en effet présenter deux sens très différents :

Ou bien on peut lui attribuer une interprétation réaliste en supposant que le mécanisme corpusculaire constitue la genèse microscopique des lois classiques que propose la Relativité générale. Dans ce cas, les deux théories forment les deux stades différents d'une même explication.

Ou bien on peut rapprocher ces théories d'une façon purement

(1) J. Van Isacker, *Comptes rendus Acad. Sciences*, t. **224**, p. 1758 (1947).

(2) G. Petiau, *Contribution à la théorie des équations d'ondes corpusculaires*, thèse de doct., p. 5 (1936).

formelle, c'est-à-dire établir une sorte de correspondance entre les définitions et les lois qu'elles supposent. On obtient ainsi deux explications formellement équivalentes mais physiquement incompatibles. Dans le cas présent, il semble que, seule, cette deuxième interprétation puisse être adoptée (1).

Ajoutons que l'interaction entre deux corpuscules doués de masses au moyen de la particule de spin 2 se présente d'une façon tout à fait analogue à interaction entre deux corpuscules chargés au moyen de la particule de spin 1. Loi de Newton et Loi de Coulomb s'obtiennent, par cette voie, d'une façon semblable et ce parallélisme est assez remarquable (2).

***

Terminons par quelques indications au sujet des possibilités pratiques de calcul qu'offre cette théorie des particules à spin. On peut l'utiliser avec succès pour la prévision des sections efficaces de diffusion simple. MM. Géhéniau et Van Isacker ont réussi à généraliser la formule de Rutherford et à calculer la section efficace dans le cas d'une particule de spin maximum quelconque $n/2$. Si R (0) est la section efficace prévue par Rutherford on obtient dans le cas d'une particule de spin $n/2$, une expression de la formule R (0) D_n Le facteur D_n prend une forme particulièrement simple dans les cas $n=1$ et $n=2$ où l'on retrouve les valeurs prévues par la Théorie de Dirac et la formule de Laporte. Mais le calcul peut être ici effectué dans le cas général et la section efficace explicitée quel que soit le spin. (3)

(1) Cf. M. A. Tonnelat, *Ann. de Phys.*, **19**, p. 408 (1944).
(2) Louis de Broglie et M. A. Tonnelat, *Comptes rendus Acad. Sciences*, t. **218**, p. 673 (1944).
(3) J. Géhéniau et J. Van Isacker, *Comptes rendus Acad. Sciences*, t. **222**, p. 377 (1946).

Discussion

M. Schroedinger. — Comment obtient-on la limite inférieure 10^{-44} pour la masse?

Mme Tonnelat. — On l'obtient de la façon suivante.

Si le photon a une masse nulle son impulsion est $p = h\, \nu/c$. Au contraire pour un photon de masse μ_0, il faut revenir aux formules

$$W = \frac{\mu_0\, c^2}{\sqrt{1-\beta^2}} \qquad \vec{p} = \frac{\mu_0\, \vec{v}}{\sqrt{1-\beta^2}}$$

La seconde de ces formules laisse prévoir une dispersion par le vide, un photon rouge se propageant moins vite qu'un photon violet.

En posant $W = h\, \nu$ et $\mu_0\, c^2$ petit par rapport à W, on tire de la première

$$v = c\ (1 - \frac{\mu_0^2\ c^4}{2\, h^2\, \nu^2})$$

Or l'expérience nous conduit à la conclusion suivante :

Aucune dispersion par le vide n'est effectivement observée. Même dans le cas des ondes hertziennes ($\lambda \sim 10^5$ cm), v est égal à c à moins de $\frac{1}{10.000}$ ce qui entraîne :

$$\frac{\mu_0^2\ c^5}{2\, h^2\, \nu^2} < 10^{-4} \quad \text{pour } v = \frac{c}{\lambda} = 3.10^5$$

D'où la limite supérieure

$$\mu_0 < 10^{-44} \text{ gr.}$$

D'une manière analogue, l'occultation d'un astre éloigné ne manifeste aucun phénomène de coloration.

Si v_r et v_v sont les vitesses des photons rouges et violets et L la distance de la terre à l'étoile, la différence des temps de parcours est inappréciable, inférieure, par exemple, 10^{-3} seconde. Il faut donc

$$\delta\, t = (v_v - v_r)\frac{L}{C^2} = \frac{1}{2}\ \frac{\mu_0^2\ c^3}{h^2}\ \frac{h}{c^2}\ (\lambda_r^2 - \lambda_v^2) < 10^{-3}$$

Si L est de l'ordre de 1000 ans-lumière, on obtient encore :

$$\mu_0 < 10^{-44} \text{ gr.}$$

M. Schroedinger. — Je crois que cette masse devrait être beaucoup plus petite, parce que si elle ne l'était pas, les lois du champ magnétique terrestre seraient profondément modifiées. Il me semble qu'il faudrait que $\hbar/\mu_0 c$ soit de l'ordre de 10.000 km. tout au moins, et même cela modifierait déjà l'influence du champ magnétique sur les particules cosmiques.

Le champ magnétique terrestre est un test beaucoup plus sensible que la vitesse de la lumière et il faudrait réduire la masse de quelques puissances de 10.

M. Casimir. — Quelle serait l'influence de la masse du photon sur les phénomènes statiques?

Est-ce que la valeur de C obtenue en mesurant la capacité d'un condensateur en unités électrostatiques et en unités électromagnétiques serait changée?

Mme Tonnelat. — Cette influence serait négligeable. La masse du photon ne modifie la loi du Coulomb que par un facteur e^{-kr} avec $k = \frac{2\pi}{h} \mu_0 c$.

Si $\mu_0 < 10^{-45}$ gr., la longueur $h/\mu_0 c$ est au moins de l'ordre de 10^3 km. Un écart par rapport aux valeurs prévues en se basant sur le potentiel coulombien ne pourrait s'observer qu'à des distances de cet ordre. La possibilité d'une perturbation effectivement observable sur les phénomènes statiques doit donc, actuellement, être éliminée.

M. Dirac. — La fonction d'onde a 16 composantes dont 10 sont relatives au champ maxwellien; les 6 autres composantes ont-elles une signification physique?

Mme Tonnelat. — Les composantes satisfont aux équations pseudo-scalaires, elles ne jouent pour le moment aucun rôle dans la théorie de la lumière, mais dans la théorie du méson, il en va, comme on sait, autrement. On peut en outre choisir des masses différentes pour ces deux particules; c'est ce qu'a fait M. Van Isacker en introduisant une matrice au lieu de la constante.

M. Kramers. — Il me semble qu'il y a deux aspects à la méthode de fusion. Un aspect mathématique, puis un aspect physique où se pose la question de la structure interne des particules.

Comment la structure interne des particules, est-elle liée au formalisme?

Mme Tonnelat. — On peut admettre les résultats mathématiques en rejetant l'idée de la structure interne.

M. Kramers. — C'est clair et on peut développer la théorie des ondeurs comme l'a fait mon élève Belinfante. Mais il semble que M. de Broglie et vous-même ayez quelque chose derrière la tête.

Mme Tonnelat. — M. de Broglie a notamment été guidé par la possibilité qu'a le photon de s'annihiler. Mais on peut développer complètement la théorie en faisant abstraction de la fonction d'annihilation.

M. Kramers. — Dans la forme de la théorie que vous avez présentée ici, les deux particules correspondant aux indices i et k sont intimement liées.

Ne pourrait-on imaginer en transformant cette théorie la possibilité d'une séparation effective de ces 2 particules?

Mme Tonnelat. — La théorie vaut plus par ses développements ultérieurs et sa généralité que par sa base qui repose sur la fusion de deux particules, point qui reste difficile et litigieux.

M. Perrin. — de Broglie avait imaginé de considérer le demi-photon comme un neutrino.

Mme Tonnelat. — Une possibilité intéressante est la création de paires par un photon lourd.

M. Bloch. — Il n'est pas nécessaire d'utiliser un photon lourd, les créations de paires sont aussi produites par des photons de masse nulle.

Mme Tonnelat. — Mais cette théorie répond à l'image intuitive de la création de paires.

Dernière discussion

Mr. Oppenheimer. — Amongst the most specific points that we are in a position to discuss, I shall take first the question of a zero mass for the neutral meson.

The interpretation of the experimental facts is that the meson decays into an electron and two neutrinos

$$\mu^+ \rightarrow e^+ + \nu + \nu$$

Two questions are connected with the interpertation first, the energy spectrum of the decay-electrons, and second, the relation between the life-time of the μ meson and the life-times of the β decay. On the first matter the results do not seem to be in disagreement with Serber's, but admit of a considerable variety amongst them. Om the second I believe that there is confirmation that such agreement can be found and we might ask Prof. Moeller to report on the calculation made by three of his Danish colleagues who have the general formulation for all values of the mass of the neutral meson for various couplings.

A question that was left in the air is that of the experiments and interpretations on the life-time and decay of the very heavy τ mesons.

Some of the evidence that has been presented is in favour of a long life-time and perhaps one will have to come back to the conclusion that perhaps all the particles that have been called τ mesons are not of the same species and that there is more complexity in that field.

Perhaps we can discuss the questions of productions of the mesons in connection with the enormous cross-section for this process as indicated by Peters and in connection with the value of the life-time as given by Powell.

I believe that the experimental indications of the spin of the τ meson as shown by Serber could be stressed somewhat more since at least two speakers at this conference said that they did not have any evidence on the subject.

On more theoretical side we might discuss whether the Maxwell-Yukawa analogy seems to be still a reasonable starting point for a theory of nuclear forces, and if so, whether Schwinger's generalization might be appropriate in this case and be a step in the right direction.

There are certainly many other points that might be discussed but these four occur to me as good starting point.

M. Moeller. — I would like to make a few remarks on the analogy between the process of decay of the μ meson and the β decay' i. e. between the two following processes

$$\mu \rightarrow e + \nu + \nu \tag{1}$$

$$N \rightarrow P + e + \nu \tag{2}$$

If one assumes in each case a coupling of the Fermi type between the four particles one can calculate the energy distribution of the electrons in the both cases, For case (2) the ordinary Fermi distribution is obtain

$$F = E^2 \sqrt{E^2 - 1}\,(E - E_0)^2 \tag{3}$$

when

$$E_0 = M_N - M_P$$

is the maximum value of the energy of the electron. All energies are measured in units of the proper energy of the electron and the velocity of light is put equal to 1. In case (1) the distribution may be obtained from a paper by Horowitz, Kofoed-Hausen and Lindgard (*Phys. Rev.* **74**, 713, 1948) who have calculated the distribution for the decay of the meson into an electron, a neutrino and a neutral particle of arbitrary mass. If the mass of the neutral particle is considerably smaller than the mass of a nucleon one has to take the recoil of this particle into account. The energy distribution of the electron will then depend on the type of coupling assumed. If the mass of the neutral particle is put equal to zero one obtains in the case of the original Fermi coupling the following spectrum for the distribution of the electrons

$$F' = \frac{E^2}{6}\,(3\,E_0 - E)\,(E_0 - E) \text{ for } E \gg 1 \tag{4}$$

where

$$E_0 = M\mu/2.$$

This formula holds only for where energies of the electrons which are large compared to the rest mass of the electrons.

When integrating the distributions given by (3) and (4) over all values of the energy, one obtains functions $f = f(E_0)$ and $f' = f'(E_0)$ respectively. The leading term is in both cases

$$E_0^5/30.$$

If the coupling constants in the two processes are the same we should find that the product of f by the life-time τ is a constant.

Now taking for the life-time of the neutron 20 minutes and $E_0 = 2.48$ we get

$$f.\tau = 1.700$$

and if we take for the life-time of the μ meson 2.1×10^{-6} sec. and for E_0 half of the mass of the μ meson which is 212, we find

$$f' \tau' = 937$$

These values should also be compared with the values obtained for the β decay of the light elements as ^3H and ^6H_e. In these cases one gets for $f.\tau$ the values 850 and 800 respectively, using for ^3H recently obtained values : $E_0 = 17{,}1$ keV and $\tau = 12$ years and for ^6H_e : $E_0 = 7{,}85$ Me V and $\tau = 0{,}85$ seconds. The uncertainties in the values for E_0 and τ may very well product an uncertainty in of a factor 2. If we assume that the interaction potential is a product of two scalars instead of the product of two four vectors as in the case of the original Fermi interaction we obtain for the distribution function

$$F'(E) = 1/6\, E_0\, E^2\, (3E_0 - 2E), \quad E \gg 1 \qquad (5)$$

The shape of this curve differs very much from the Fermi curve (3) and from the curve (4), in particular the function (5) has a finite value at the upper limit $E = E_0$. The leading term of the integral $f' = \int_1^{E_0} F'(E)\, dE$ is $E_0^5/12$, consequently we get in this case $f' \tau' = 2{,}342$. The general agreement between the values is good considering the strong variation of f with E_0 and the wide range of lifetimes and energies in the compared cases. This indicates that the decay of the μ meson is due to a similar process as the β decay and that we have a kind of Fermi interaction between all particles of spin 1/2.

Mr. Rosenfeld. — Is it all right to apply the formula for a zero mass of the neutral meson?

Mr. Moeller. — The fact that the neutrinos obey the Pauli principle will not make any change in the distribution of the electrons even if the neutrino and the antineutrino are supposed to be identical entities.

Mr. Ferretti. — Do we assume that neutrino and antineutrino are two different things?

Mr. Bbabha. — Why not couple the μ meson and the electron?

Mr. Peierls. — In the formulae for the life-time, does E_0 stand for the maximum energy of the spectrum or for the energy of the reaction.

Mr. Moeller. — It is the maximum energy of the electron.

Mr. Teller. — I would like to say that in the experiment on the life-time of the neutrons, performed by Snell, only the protons were counted and not the electrons. He has not measured coincidences but has tried to eliminate a quite considerable back-ground by taking differencies between two kinds of experimental arrangements. From what I last heard the life-time seems to be rather 15 minutes than 20 minutes as obtained previously.

M. Serber. — I think that the experimental results are still tentative.

Mr. Rosenfeld. — From the other evidence on β decay one would calculate for the neutron a life-time of about 25 minutes.

Mr. Teller. — That is the evidence coming from β decay of the light elements, except ^3H. The lifetime of ^3H is not in very good agreement with the other so called Wigner elements. Customarily the life-time of the neutron has been calculated only from the Wigner elements assuming that the matrix element there, is one. The ^3H would indicate a twice times shorter life-time and would necessitate quite suprisingly a reduction of the matrix elements of the Wigner elements to about $\sqrt{1/2}$.

Mr. Heitler. — May I ask what is the evidence for the β decay of the μ meson? I understood that but for one exception, and that is the photograph obtained by Anderson, there is no incompatibility between the experimental facts and the hypothesis that the meson decays into one electron and one neutrino.

Mr. Oppenheimer. — The question is whether one knows anything about the energy-spectrum of the electron emitted in the μ decay. The experiments that are known to me are :

1) An experiment by Rossi and his collaborators with a cloud chamber and a delayed coincidence to trigger the cloud chamber.

The measurements favor high energies because high energy electrons are more likely to trip the cloud chamber. All measurements reported by Rossi in Pasadena were compatible with 42 ± 15 MeV.

There are two photographs of Anderson both of which give 25 MeV for the energy of the position. In one of them the identification is difficult because the decay takes place within the glass.

2) There are also experiments described by Teller, carried out by Steinberger in Chicago measuring the linear absorption of the decay-electrons of cosmic ray mesons by an ordinary counter-arrangement. This gives the distribution in energy, which is not compatible with the assumption that a monoenergetic electron is emitted but is well compatible with any of the spectra described by Moeller.

Mr. Dee. — We have measured in Glasgow the energy of the decay-electron from ^{3}H. We find the value of 18, $1 \pm 0{,}2$ keV. and I think that result is in fair agreement with the β decays of the ^{6}H$_e$ and of the neutron. I am surprised that Teller thinks it to disagree with those spectra.

Mr. Teller. — It is in disagreement for the old ideas about the elements in which β decay interchanges the numbers of neutrons and protons.

It is in agreement with ^{6}H$_e$ which however is supposed to be outside of this series because there is a change of spin.

Mr. Oppenheimer. — In addition, the american measurements on ^{3}H were in much greater disagreement than yours.

Mr. Bhabha. — I understand that Steinberger's measurements were made on the absorption of the electrons. With these high energy electron doesn't radiation loss come in.

Mr. Teller. — Measurements were made in paraffin to minimize scattering and absorption. The effects of these two were calculated and essentially give a large straggling of the results.

A further uncertainty was introduced by the fact that in order to get sufficient intensity it was not possible to use a well collimated beam.

After all this has been accounted for, the experiment is still good enough to eliminate the possibility of a monocinetic group of electrons, but it is not good enough to distinguish between the type spectra indicated by the various theories of the decay. They gave essentially the same results at Chicago and Pike's Peak.

Mr. Kramers. — Is there any good reasons why one should expect agreement between the μ decay and the Fermi theory of β decay.

Mr. Moeller. — It seems to show that there are similar couplings between those four 1/2 spin particles.

Mr. Oppenheimer. — In the present ambiguous state of theory we are glad to find a few regularities of this kind. It seems an important fact that there is such close agreement between decay processes whose lifetimes differ by factors of 10^{10}.

Mr. Teller. — Serber has stated that of 500 negative π mesons, one was observed to decay into a negative μ. This is reasonable if the meson is assumed to have decayed in flight, since its time of flight into the emulsion is 10^{-11} sec.

If however it decays at the end of its range, the expected probability of finding a π meson is only 1/50.000, according to the calculations by Fermi and myself. The observation corresponds to a probability of 1% and is therefore what Fermi calls a dubble miracle.

How close of the end of its range was the π meson?

Mr. Serber. — The actual figure was one in 3.000. I gave on in 500 as the expected value from time of flight and lifetime.

There were two cases in which a π meson did not come to the end of its range but no μ meson was detected. As it happened in 50 microns plates, it would have been difficult to detect the μ meson. In the one case observed the π meson did seem to end close to the end of its range.

Mr. Teller. — May I press you a little further : how close to the end of its range was it. Was it 90% of the range?

Mr. Serber. — I should say it was within about 2 microns from the end.

Mr. Powell. — From the range of the μ meson, one gets a very precise indication of how close of the end of its range the μ meson was. Decay in flight enormously influences the range of the secondary particle. The point is that you have to add up the velocities in the center of gravity system. In the potograph I have seen of this decay, the meson is ejected backwards. If the decay occured more than 3 microns from the end of the range of the π meson, the range of the μ meson would be less than 500 microns.

On the very poor statistical evidence given by one particle, one can deduce that the time of capture is not very short compared to the time of flight (10^{-11} sec.). Certainly a decay in flight would be difficult to observe, and might be taken for a Colomb scattering.

Mr. Peierls. — The time of capture will depend om whether the particle is stopped in gelatine or silver bromide, on the character of the grain in which it stops. In dielectrics the particle might be stopped inside an atom in which it is difficult to reach the K-shell; hence one certainly cannot rule out the possibility that for some particles the capture time might be very long.

Mr. Teller. — My calculations were carried out for μ mesons and $Z > 6$. For π mesons it would correspond to $Z > 7$. Therefore the capture in a small circular orbit is possible in gelatine and could increase the life-time by a factor 100. In silver bromide, even beeing a dielectric, I don't think it possible.

Mr. Ferretti. — I should like to remark that the results of Powell and Leprince-Ringuet about slow π mesons which come out from the stars, may give a valuable information relating to the interaction between π mesons and nucleons.

The total number of stars observed by Powell was 20 — 30000. Taking into account that for geometrical reasons a fraction only of the total number of these mesons could be absorbed, it seems to me quite safe to deduce from these data that there is in average one meson of energy less than 3 MeV produced in a star out of 100—200 stars.

One may try to use this information to get an average value of the cross section for production of slow mesons. This value will be an average on the energy of the incoming nucleons producing the stars and the mesons.

We may remark that most of the stars which are observed in the emulsion are very likely produced by nucleons that gave not an energy sufficient to produce the meson and the star toghether. This minimum energy is probably about 250 MeV.

Therefore to get a more useful value of the average cross section we have to consider those incoming nucleons only which have an energy greater than 250 MeV.

Using the available data on the energy spectrum of the nucleons and on the rate of star production (Rossi, *Rev. Mod. Phys.* Vol 20, p. 537, 1948, Table II, p. 562, Tabl. III, p. 565) one may estimate that about 80% of the stars are produced by particles (chiefly neutrons) which have an energy less than 250 MeV. Then the average partial cross-section *per nucleon* for production of mesons having an energy less than 3 MeV appears to be not less than $2 \div 3\ 10^{-28}$ cm^2. It seems to me that this partial cross section is rather high. In fact the total cross section per nucleons for production of mesons can't probably be much higher than about $5\ 10^{-26}$ cm^2 and it is unlikely that this maximum value is reached when the kinetical energy of the meson is much less than the rest energy.

Now, the ratio between the volume of the phase space relating to mesons having a kinetic energy not greater than the rest mass and the corresponding volume for mesons of an energy smaller than 3 MeV is 2-300, i. e. not smaller and may be greater than the ratio between the maximum total cross section for production of mesons and the partial cross section for production of mesons having an energy smaller than 3 MeV.

It seems to me that this fact is an indication that the matrix element relating to the interaction between π mesons and nucleons cannot probably increase strongly with the energy. This indication points in the same way that Serber's calculation, about the dipendence of the yield of mesons from the energy of the particles in Berkeley experiments.

Detailled calculations are of course necessary before drawing any sure conclusion on this point. However if my guess is right I think that it may be difficult to explain the production of slow π mesons with the conventional theory of nuclear meson fleld.

In connection with the point that I have discussed just now, I should like to make a remark about the hypothetical τ meson observed by Peter. The number of these τ mesons seems to me much greater (by a factor 10) than the number of the ordinary mesons ending their range in the plate.

Following certain results of Bernardini, the number of slow mesons in the plate increases more rapidly than the number of stars with the hight. This means, that at a great hight most of the mesons which end their range in the plate are created locally. But the τ mesons are supposed to have a life time shorter than the ordinary mesons : therefore they too must be created locally and the ratio between the number of τ mesons and π mesons can give directly the ratio of the cross section for production of slow τ mesons and slow π mesons.

As the cross section for production of slow π mesons is rather great, the cross section for creation of slow τ mesons should be extremely great : this seems rather unlikely.

Mr. Occhialini. — Are the results of Bernardini in line with those of Peter's or not?

Mr. Powell. — I understand from the Dr. Peters that the ratio of mesons to stars decreases considerably with the altitude.

Mr. Ferretti. — This is in contradiction with Bernardini's evidence.

Mr. Oppenheimer. — The results are not necessarily in desagreement, because many of the mesons observed by Bernardini might in fact be τ mesons. Can one not make it simpler and say : the life is 10^{-10} so and so many are seen. We known the flux of star producing radiation that can produce observable τ mesons, therefore we can calculate the cross section for observable τ mesons, and this gives a value which is already hard to swallow.

Mr. Ferretti. — The life-time does not make much difference. There was only 10 gr./cm^2 of air above the plates, and about 10 gr./cm^2 of condensed material.

Mr. Powell. — The π and τ mesons seem both to be locally produced. If the probability for decay in flight is small, as Ferretti has indicated, we must assume for the τ mesons a production cross section at 30.000 m. about ten times larger than for the π mesons.

Mr. Ferretti. — The experimental facts are not in contradiction with a life-time of 10^{-9}; if we take for instance Blackett's evidence.

Mr. Oppenheimer. — This is certainly not compatible with the fact that Brode ioned find these particles in a machine several meters long.

Mr. Ferretti. — Let us say then 10^{-8} sec.

Mr. Blackett. — Our photograph enables us to compare very roughly the frequency of production of τ mesons to the production of protons or other penetrating particles in the showers. In the penetrating showers we have photographed the total number of penetrating particles cannot be more than 50 or 100. We do not know the number of τ mesons that crossed the chamber and did not decay in flight, but it is probable that then were, say 5 or 10 of them. The number of τ mesons can therefore not be much less than 1/10 of the total number of penetrating particles.

As regards the life-time, again the estimate can be only very rough. Since the chamber was 25 cm. high, and the velocity of the particle was not very far from c, the time of passage was of the order of 10^{-9} sec. If one supposes that 10% of the τ mesons decayed in the chamber, the life-time is then of the order of 10^{-8} sec.

Mr. Serber. — I might come back to the first point made by Dr. Ferretti that is, the evidence that the coupling of the mesons does not change rapidly with the energy.

There is a very indirect connexion which comes from the data on the scattering of neutrons of 90 MeV. by protons. We have seen from the results obtained by Segre that there is no evidence for the influence of any tensor forces on the protons scattered forwards. One possible explanation of this is that the tensor forces are non-exchange forces which affect the neutrons going forward but not the protons going forward.

It is perhaps still unreliable to try to connect this to the meson theory of nuclear forces, but one might suggest that the kind of mesons that give rise to tensor forces are neutral mesons rather charged ones, and therefore there is no reason to expect that their interaction should increase with energy.

Mr. Oppenheimer. — There is not yet sufficient experimental evidence to attribute a life-time to the τ meson, but it is certainly not shorter than 10^{-10} and probably not shorter than 10^{-9}, and possibly longer.

The evidence is that the τ mesons are obviously produced in nuclear collisions.

This makes a certain difficulty in understanding, namely, if the production of such a meson from nucleons is possible, and if there is any truth to the postulate of an anti-particle corresponding to the nucleon, then it should be possible to associate with the τ meson a charge and current due to proton pairs. It is doubtful whether a good calculation of this charge and current can be made, but one can estimate it. Then this charge and current is also coupled to the lighter meson, the π meson, and in fact very strongly. Since energy is available and there is a high frequency current associated with it, a spontaneous transition from τ meson to π meson would be expected. This was calculated, just assuming that one can treat the proton rather like a Dirac particle, by Finkelstein and leads to a lifetime of 10^{-18} sec., very short indeed.

No one would expect that we know enough to make such a calculation. But the physical arguments that there should be a current leading to this transition if there are materialization processes for protons or neutrons are very strong. It would be difficult to explain how one might be wrong by a factor 10^{-10} in such an estimate.

My question arises from the following facts : strongly coupled τ meson, strongly coupled π meson, stable or almost stable τ meson.

We have here some argument that materializations do not in fact take place, not to an extent that would be compatible either with present theory or a minor reformulation of that theory.

Mr. Bhabha. — That is true if the same sort of coupling exists.

But suppose that in the creation of one of these particles, another particle is involved, for example that in the creation of the τ meson a neutral particle is also emitted. Then the argument does not hold, because in the reserve process of the absorption of the τ meson and apparent decay into a π meson you want a neutral particle. That would save the situation.

Mr. Powell. — I would like to ask a question related to Mr. Serber's paper. One point he discussed is the question of the direct β decay of the π meson a process in competition with the μ decay.

I think the evidence is that in 95 % of the cases π particles brought to rest in the emulsion, decay into μ particles. If direct β decay occurs, it seems to be in less than 5 % of the cases.

I am rather surprised that it is possible to make such a firm statement on this, for the following reasons. We of course can distinguish the π meson decaying into the emulsion and producing a recognizable μ meson. But we always have to put in rather large corrections to calculate the actual numbers of π mesons into the emulsion, because when for instance a π meson ends its range near the surface, and gives a μ meson going vertically upwards, it is very difficult to distinguish that track. And how we do this is to measure the ranges in the emulsion of the μ mesons, arising from this process, that we actually observe, and then since the direction of ejection of a μ meson from π meson at rest is random, we can conclude how many π mesons have in fact stopped in the emulsion. The correction calculated in this way is usually large, there are in the emulsion two or three times as many π mesons as we actually observe.

I think that we have probably considerably worse conditions than in the artificial case, because there every π meson coming to rest into the emulsion can be subject to a scrutiny, whereas we have many thousands of mesons, most of which are μ particles giving no secondary tracks.

So, although the conditions are much more favourable, I am a litte suprised that it is possible to say so definitely, that the competitive β decay process is less than 5%.

Mr. Serber. — I am not in a position to discuss the exact experimental condition, and perhaps an element of good luck is involved. Buth the facts are these.

Lattes has used recently emulsions of greater thickness (100) and greater sensitivity. He has observed 30 positive π mesons which stop in the emulsion, and in every case a decay μ meson was visible.

In the earlier plates, about 50 microns only, and less sensitive, they were able to see only about 50% of the μ mesons.

Mr. Bhabha. — The remark Oppenheimer has raised and the answer I made does allow one to come to a definite conclusion. That is, the connexion between the τ meson and the nucleons cannot be of the Yukawa type involving only three particles, because if it was, then the objection on the lifetime could not be answered.

Mr. Oppenheimer. — I would not take it quite so because in the first place it would probably be a spin 1/2 situation, so that a neutral particle could not participate in the decay. Then I would think what we did not know for sure, that the structure of theory in this domain was such that materialisation of protons processes had to occur. The conclusion goes way beyond the statement that there are only certain representations of the Lorentz group.

I am inclined first of all to doubt all we heard so far; but if we take the τ mesons seriously, then I am inclined to doubt the materialisation premiss as much as to doubt that the spin of the τ meson is integral.

Mr. Heitler. — One point about these materialisation processes.

When one calculates the lifetime for any spontaneous decay in such virtual states, one can, and I admit this is very artificial, one can always introduce also a direct interaction with an arbitrary constant, whose sign is also quite arbitrary. And if one wishes, one can always find a way out by fixing this constant so as to obtain any desired lifetime. It is certainly artificial, I admit, but perhaps we should be not quite so definite in making statements about theories of which we obviously don't understand much yet.

Mr. Oppenheimer. — Not so definite as that : I would say that we don't understand them at all.

Mr. Bhabha. — What we understand is exceedingly small.

Break inthe discussion.

Mr. Ferretti. — In the question of the discrepancy between the results of Bernardini and Peters on the variation of the ratio of number of mesons to number of stars with altitude I have heard from Occhialini that the plates of Peters were underdevelopped in order to facilitate grain counting, and therefore will show on smaller number of mesons compared with stars.

This disposes of one of the objections to the τ mesons.

Mr. Bragg. — The President asks Mr. Oppenheimer to sum up.

Mr. Oppenheimer. — One point that has been satisfactorily cleared up is that of Moeller of the relation between β decay and μ decay. The two neutrinos hypothesis of the decay implies that the μ meson has spin 1/2. The assumptions made in the calculations are crude, but the results are satisfactory. The question of the spin of the meson can be argued on the evidence from Berkeley that in the production of the π meson no other particles is emitted. The spin must therefore be integral. In addition, the very different nuclear coupling of π and μ shows that one must have integral and the other half-integral spin.

Mr. Ferretti has pointed out that there is a large probability for production of slow mesons. It shows there is no selection rule forbidding slow mesons. This is not expected on vector or pseudo-scalar meson theory, only on scalar theory.

The evidence on the μ mesons is not consistent. It appears to decay rapidly and yet it has been picked up by Brode. It stability could be explained, if it has spin 1/2 and its production is accompanied by emission of another equally heavy particle, or by the failure or absence of nucleon materialisation processes.

One main point that has not yet been discussed is the validity of the Maxwell-Yukawa analogy.

Some General Comments on the Present Situation in Atomic Physics.

Mr. Bohr.— In connection with the great progress as regards the accumulation of new experimental evidence and the development of theoretical ideas, discussed during this Conference, it may be of interest, as a continuation of the elementary considerations presented at the first session, to make a few comments upon the situation in atomic physics in relation to our conceptional framework.

A question which has often been raised is to what extent the difficulties met with in present theories may have their origin in the application of classical concepts beyond their appropriate scope. In this connection, it should be remembered how useful considerations of idealized experiments, which might serve to measure physical quantities, have been for the clarification of essential aspects of the situation in relativity theory as well as in quantum theory. It must be stressed, however, that such considerations primarily aim at making us familiar with the foundations of the theories and, in general, do not allow us to investigate the correctness of the theoretical expectations which can, of course, only be tested by actual experiment. In fact, the question as to the results to be expected from an imagined experiment can only be judged from a purely theoretical standpoint and, as far as the theory presents a mat6ematically consistent scheme, no conclusions as regards limits of its scope can be derived in this way.

An instructive example is offered by the quantum theory of electromagnetic fields with its apparently paradoxical features as fluctuations of electric and magnetic intensities in empty space. In fact, attempts of tracing the origin of such paradoxes to an inherent limitation in the applicability of field concepts have proved misdirected on closer examination of the logical interpretation of the results which may be obtained by conceivable arrangements for measuring averages of field intensities over definite space-time extensions. In the treatment of this problem we are in the first place justified in disregarding the atomic constitution of the measuring agencies, like test bodies, since the quantum theory of electromagnetic fields contains only two fundamental constants, the velocity of light c and the quantum of action h, which are in themselves not sufficient to specify quantities of the dimension of a length or a time interval. Consequently, the characteristic features of the theory are essentially independent of the space-time scale, and like in proper quantum mechanics all paradoxes find their straightforward explanation in the complementary relationship between phenomena described in terms of space-time coordination and phenomena accounted for by means of dynamical conservation laws.

In the problems encountered in the theory of elementary atomic particles, the situation is of course quite different, since the introduction of the notion of intrinsic charge and rest mass, together with c and h, implies the possibility of specifying space-time quantities, like atomic diameters and periods. On account, however, of the appearance of the non-dimensional constant

$$\alpha = \frac{2\,\pi\,e^2}{hc}\;(\sim 1/137) \tag{1}$$

we cannot, without examining special problems, beforehand trace ultimate limitations as regards space-time coordination. Incidentally, it may be noted that all attempts to deduce the value of α by arguments resting upon theories which are presumed to be consistent independently of the value of e and h would seem futile, and that any further elucidation of this problem can only be expected from an examination of the limitation of the theories.

In looking for such limitations, it must be remembered that the present approach to atomic problems on correspondence lines is essentially an approximation procedure in which, as a first step, the constant α is considered to be vanishingly small. At the same time, however, the actual smallness of α is fundamentally connected with the finite value of h, and we must therefore be prepared for a radical departure from such lines of approach. Still, whatever shape the theoretical edifice for comprehending the atomic phenomena may take, the experimental evidence must always be described in classical terms and, since this description must conform with the demands of relativity theory, it will hardly be possible to obtain a consistent scheme, unless the whole formalism, including aspects which defy classical interpretation, exhibits relativistic invariance.

In the treatment of problems involving particles of rest mass m, the length

$$\lambda = \frac{h}{2\,\pi\, mc} \tag{2}$$

presents, as is well known, a measure for the spatial extensions where, in quantum theory, relativistic effects become of decisive importance. Thus, in problems involving lengths and time intervals comparable with or smaller than λ and λ/c, respectively, we are confronted with peculiar features the recognition of which has led to remarkable developments. Above all, in electron theory, Dirac's ideas and their confirmation by the discovery of the phenomena of creation and annihilation of electron pairs have radically changed the situation. Even if certain features like the filling up of phase space by particles of negative energies presents provisional difficulties for our world picture, the eventual removal of which may demand a compensation by means of the zero-point energy of quantum fields, at present also disregarded, our actual possibilities of treating relativistic electron problems have been most decisively augmented.

An especially instructive lesson we have received, as is well known, in connection with the problem of electromagnetic self-energy of charged particles, which already in classical electron theory presents characteristic divergencies. Introducing, for preliminary convergency, a so-called cut-off length a, we get in classical electrodynamics as a measure of the self-energy

$$W \sim \frac{e^2}{a}, \tag{3}$$

provided a is greater than $e^2/mc^2 (= \alpha\lambda)$. By means of (1) and (2), the expression (3) may be written

$$W \sim \alpha mc^2\, (\lambda/a). \tag{4}$$

In quantum theory, the situation is essentially changed if $a < \lambda$, in which case the self-energy as regards order of magnitude is found to be

$$W \sim \alpha\, mc^2 \log\, (\lambda/a) \tag{5}$$

for particles subjected to the Pauli principle and obeying the Dirac equation, and

$$W \sim \alpha\, mc^2\, (\lambda/a)^2 \tag{6}$$

for particles obeying Bose statistics. For $a \sim \lambda$, the expressions (5) and (6) are small compared with the rest energy mc^2 and comparable with (4), but if a is taken to be small compared with λ the character of the singularities for vanishing a is quite different in the three cases. In particular, it is significant that the degree of the singularities in (λ/a) which, by such a rough cut-off device, appear in the first stage of the approximation procedure may be smaller as well as larger in quantum mechanics than in classical theory according to the statistics of the particles, a notion which defies any interpretation on classical pictures.

As regards electron theory, the situation has been essentially clarified by the ingenious development of the mathematical formalism which, as we have learnt, has also allowed a quantitative explanation of the recent discoveries regarding finer spectral regularities obtained by new powerful experimental methods. As often stressed during the preceding discussions, the state of electron theory is in various respects more complete than has sometimes been assumed and it would seem excluded to specify its limitations on the basis of the theory itself. In fact, decisive progress in electron theory can hardly be obtained without introducing new fundamental features derived from the accumulation of experimental evidence concerning the interaction between the different kinds of elementary particles. In this connection, it is significant that the value of λ for the electron, owing to the comparatively small mass, is considerably larger than the λ-values for other particles and that, therefore, any change which new evidence may entail will not in the first place impede the application of electron theory in the region in which relativistic effects are predominant.

As regards the outlook to further developments, it must be stressed that quite a new stage in atomic theory has been initiated by the recognition that nuclear constitution demands force fields foreign to electromagnetic theory. The conception of short-range forces means indeed the explicit introduction of a microscopic feature in the foundations of the theory, while in the previous description all such features were considered as traceable, on correspondence lines, to the existence of the quantum of action. Notwithstanding the confirmation of Yukawa's ideas by the observation in cosmic radiation of particles with a rest mass intermediate between the masses of the electron and of the nucleons, meson theory is as yet in a most preliminary stage and new viewpoints will obviously be demanded for the theoretical comprehension of the rapidly increasing experimental evidence about the various types of mesons and the conditions for their production.

Even the simple estimate (6) indicates that essential modifications will be demanded for a consistent treatment of particles obeying Bose statistics in problems involving spatial dimensions comparable with the value of λ. In fact, for decreasing values of a this expression for the self-energy increases very quickly and approaches mc^2 for values of a of the same order as the λ corresponding to nucleonic mass. How much stress may be put on such a comparison is difficult to estimate, but further indication that the value of λ for nucleonic masses might constitute a critical limit of spatial coordination is suggested, as emphasized by Heisenberg, by the occurrence of explosive effects in high energy collisions. More over, it must be remembered that it is the position of the heaviest elementary particles which in the first place will define the reference frame in any conceivable measuring arrangement.

In a future, more comprehensive theory of elementary particles, the relation between the elementary unit of electric charge and the universal quantum of action may play a more fundamental rôle than in present theories, as is also indicated by the fact, often commented upon, that the value of α is of the same order as the empirical mass ratios. In conclusion, attention may once more be called to the necessity of removing by compensation effects obvious inconsistencies inherent in present theories. Here we meet new aspects of the duality between the corpuscle and field concepts originating in the very circumstance that, on the one hand, the definition of fields ultimately rests on their action on material corpuscles while, on the other hand, the properties of corpuscles are essentially defined by their field actions. Notwithstanding all additional features of complementarity this duality is in quantum theory as fundamental as in classical physics.

VARIA

A l'issue du Banquet qui a réuni les participants le vendredi 1er octobre, trois nouvelles communications ont été faites.

Il a été jugé utile de les publier ici dans le but de refléter de façon très complète l'activité du huitième Conseil.

THE MESON SONG

(Solvay Version)

The following song is an adaptation of a piece of classical American literature by Dr. and Mrs. H. C. Childs, Dr. and Mrs. R.E.Marshak, Dr. and Mrs. R. L. Mc. Creary, Dr. and Mrs. J. B. Platt, Dr. and Mrs. S. N. Voorhis, all of the University of Rochester, and Georges E. Valley of the Massachussetts Institute of Technology.

There are mesons pi, there are mesons mu
The former ones serve as nuclear glue
There are mesons tau, or so we suspect
And many more mesons which we can't yet detect
    Can't you see them at all?
    Well, hardly at all
    For their lifetimes are short
    And their ranges are small.
The mass may be small, the mass may be large,
We may find a positive or negative charge,
And some mesons never will show on a plate
For their charge is zero, though their mass is quite great
    What, no charge at all?
    No, no charge at all!
    Or, if Blackett is right
    It's exceedingly small
Some beautiful pictures are thrown on the screen,
Though the tracks of the mesons can hardly be seen,
Our desire for knodledge is most deeply stirred
When the statements of Serber can never be heard.
    What, not heard at all?
    No, not heard at all!
    Very dimly seen
    And not heard at all!
There are mesons lambda at the end of our list
Which are hard to be found but are easily missed,
In cosmic-ray showers they live and they die
But you can't get a picture, they are camera-shy.
    Well, do they exist?
    Or don't they exist?
    They are on our list
    But are easily missed.
From mesons all manner of forces you get,
The infinite part you simply forget,
The divergence is large, the divergence, is small,
In the meson field quanta there is no sense at all.
    What, no sense at all?
    No, no sense at all!
    Or, if there is some sense
    It's exceedingly small

Edward TELLER.

SOMMAIRE EN FORME DE BALLADE

Les électrons et positons
Mesons et forces nucléaires
Deuterons, protons, photons
Particules permanentes ou très éphémères
Particules bien connues ou trouvées naguère
Ou bien particules pas encore observées
Tout cela a été notre affaire
Et c'est là le but du Conseil Solvay.

Les grands corps en rotation
Le magnétisme de la terre
Et des étoiles; les explosions,
Gerbes nées dans l'atmosphère
Et la doctrine complémentaire
Qui a clairement montré
Qu'il faut renoncer aux images vulgaires
Et c'est là le but du Conseil Solvay.

On a vu que le triton
N'est point un être légendaire.
Quant au problème de l'électron,
La situation théorique était claire
Quand Oppi a dit qu'il pourra se défaire
De toute divergence et de l'infinité,
Une fois qu'il sera devenu grand-père;
Et c'est là le but du Conseil Solvay.

Envoi.

LEPRINCE Ringuet quoiqu'on ne sache guère
Si la τ particule est une réalité
Nous voulons bien le croire, seulement pour vous plaire
Et c'est là le but du Conseil Solvay.

H.-B-.G. CASIMIR.

THE LION AND THE UNICORN

The Lion and the Unicorn
Where fighting for the crown
Of Nature's secrets, leading down
To matter's innermost recesses,
The depth of which they searched in vain.
The Unicorn with hanging mane
Peered down the bottomless abysses,
The Lion with his mighty paw
In vain did scratch his lofty brow.
For, like a meadow dug by moles,
The universe seemed full of holes :
Every electron in its course
Bored such a hole through space,
And so did each proton, of course;
But, oh what a disgrace,
Each infant meson newly born
Of the same mischief was suspected,
And this was still more unexpected.
« Let's try our hand, quoth Unicorn,
At those holes we know best;
And later on cope with the rest. »
The Lion, sunk in deep concern,
Received the hint with scorn :
« Nature's great lesson we must learn »,
He said unto the Unicorn;
« Half-hearted work Dame Nature shuns :
Whatever holes she has to fill,
She'll fill them all at once. »
This speech was but of little use :
The Unicorn he sat quite still
And did not change his views.
So on they fight for Nature's crown
A fight that brings them wide renown,
And praise of ages yet unborn
To Lion and the Unicorn.

L. ROSENFELD.

TABLE DES MATIÈRES