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CONTENTS

PULSARS: A PROGRESS REPORT

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INTRODUCTION

More than five years have elapsed since the announcement of the discovery of the first pulsar early in 1968. Toward the end of the same year two pulsars were found in known sites of Supernovae explosions, the source PSR 0833 in the Vela Remnant and the source NP 0532 in the Crab Nebula. Because of well known reasons, this led to the general acceptance of the rotating neutron star hypothesis for the basic nature of these objects, as first suggested by T. Gold.

About the same time, accurate period measurements indicated that the Crab Nebula pulsar loses rotational energy at a rate $\sim 10^{38}$ ergs sec^{-1}. This is about as much as needed to compensate the radiation losses of the nebular particles: the pulsar is the energy source for the overall nebular activity and converts continuously its macroscopic rotational energy into a relativistic form (fast particles and the large scale magnetic field of the Nebula).

Much work on pulsar electrodynamics was done in the months immediately following these discoveries. As a result, we have probably understood the basic aspects of pulsar theory and the consensus on the rotating neutron star model appears well motivated. There have also been several attempts to adapt similar ideas to account for other phenomena in high energy astrophysics such as the origin of cosmic rays, the violent activity in the nuclei of galaxies, and the nature of galactic X-ray sources. Although some of the detailed processes on which people have speculated may turn out to be irrelevant, these general ideas seem to offer our best hope of understanding strange phenomena within the framework of conventional physics.

* Operated by Cornell University under contract with NSF.

In this Report we shall first, in Section 1, briefly review some observational data and then outline the theory of pulsars with the aim of stressing the remaining ambiguities and unsettled questions. It is indeed true that the theoretical understanding of these objects has progressed slowly in the last 2 or 3 years, probably because most of the problems to be solved are of increased complexity. The most important question can be summarized as follows:

a) What are the basic mechanisms by which a neutron star can convert the rotational energy into energy of relativistic particles and magnetic fields?

b) What is the origin of the highly coherent radioemission and what is the mechanism giving rise to sharp pulses?

c) What is the relation between pulsars and Supernovae Remnants?

Accordingly, Section 2 will deal with general aspects of pulsar electrodynamics; Section 3 will briefly review the quite controversial subject of radiation mechanisms; Section 4 will emphasize some possible features of the electrodynamic link between pulsars and Supernovae Remnants.

In general, we have not attempted to give here proper credit for all individual discoveries and ideas. We recall that three rather extensive review articles have recently appeared in print:

M. Ruderman, Ann. Rev. Astr. and Ap. **10,** 427, 1972.

F. G. Smith, Reports Progr. Physics **35,** 4, 1972.

D. ter Haar, Physics Reports **3,** 57, 1972.

These articles contain an extensive list of original references.

1. BASIC OBSERVATIONAL DATA

Pulsars are characterized primarily by the emission of sharp flashes of radiowaves at almost exactly maintained time intervals P. The typical duty cycle $\delta = \dfrac{\tau}{P}$ is in the range 1 — 10 %. About one hundred pulsars are now known, a large proportion of them concentrated along the galactic plane. The observational data available about these sources represent a vast amount of detailed facts, possibly more vast than for any other celestial object. In this section we shall only describe the basic characteristics: for more details we refer to the review article by F. G. Smith quoted above.

The following are the main characteristics of pulsars:

Periods' distribution and slowing down. The periods range from 33 msec for the Crab Nebula pulsar NP 0532 to 3.75 seconds for NP 0527. Most pulsars have periods in the range 0.5 — 1 sec. A tendency toward a secular lengthening of the period appears general for all pulsars whose period has been monitored with sufficient precision. It is found that the " apparent lifetime " $\frac{P}{\dot{P}}$ generally increases with period and ranges between $\sim 10^3$ years for NP 0532 up to about 10^8 years for some slow objects. Since pulsars spend only a small fraction of their life with a very short period, it is not surprising that only relatively few objects have been found with periods $P < 0.5$ sec. The lack of pulsars with very long period, (say $P >> 2$ sec) can only be explained in terms of a fairly steep dependency of the pulsar radio luminosity from the period. Two pulsars (NP 0532 and PSR 0833) have shown abrupt changes in their period with sudden speeding-up's in the range

$$\frac{\nabla P}{P} \sim 10^{-9} - 10^{-6}.$$

Distances. The same pulse appears systematically first at high and later at low radiofrequencies. This is due to the dispersive properties of the interstellar medium and the amount of delay depends on the number of electrons along the line of sight $\int n_e \, dl$. Since n_e is (roughly) independently known one can estimate the distance of pulsars from the observed amount of delay per unit frequency. The typical distances range between a few hundred and a few thousand parsecs.

Intrinsic power, spectra, brightness temperatures. The radio power emitted from pulsars ranges between 10^{27} ergs sec^{-1} up to 10^{31} ergs sec^{-1}. A basic uncertainty on the emitted power stems from our ignorance of the pulsar emission diagram. The radio spectra emphasize the low frequencies (say, around 100 MHz) and generally decrease rapidly toward the high frequencies. Several pulsars show also the presence of intrinsic cut-off's below 100 MHz.

The sharpness of the pulses indicates that the emitting region is not larger than $c\tau$ (τ is the pulse time-length): typically this gives emitting regions of the order of a few Kms up to a maximum ~ 100 Km. The brightness temperatures of the emitting regions are then found to range between $\sim 10^{20}$ °K up to $\sim 10^{28}$ °K for the Crab pulsar (occasionally even higher). This entails that the electric field radiated by NP 0532 can reach and exceed 10^{10} Volts m^{-1} at the source. Also, the power flux through the emitting surface can exceed 10^{12} Watts cm^{-2}. (F. Drake has pointed out that this is an amount of power per square centimeter equivalent to the electrical power produced by all plants on the earth). There is no doubt that such brightness temperature demand an extremely coherent radiation process at radiofrequencies.

Optical and X-ray emission. Only the fastest pulsar NP 0532 has been detected also at optical and X-ray frequencies (radiopower $\sim 10^{31}$ ergs sec^{-1}; optical power $\sim 10^{34}$ ergs sec^{-1}; X-ray power $\sim 10^{37}$ ergs sec^{-1}). The lack of optical emission from the second fastest pulsar PSR 0833 (which has a period only $\sim$ 3 times that of NP 0532) entails a period dependency of the optical luminosity $L \propto P^{-n}$ with $n > 8.4$. The brightness temperatures of the Crab pulsar at optical and X-ray frequencies are $< 10^{10}$ °K, perfectly compatible with an incoherent (but non-thermal) process. The optical and X-ray pulses are found to be simultaneous with the radiopulses when the effect of interstellar dispersion has been taken into account.

Pulse shapes and intensity fluctuations. The pulse shapes are rather complicated, often with separate components or " sub-pulses ". In some cases the sub-pulses are persistent features; in others, especially for small structures, no coherence exists from pulse to pulse. The pulses also show considerable intensity fluctuations, partly intrinsic, partly due to interstellar scintillation. No fluctuations exist for the optical, X-ray pulses from NP 0532.

Drifting subpulses. In several sources, the sub-pulses reappear in consecutive pulses. However, as first noted by Drake and Craft, a given subpulse can arrive somewhat earlier in each successive pulse. This phenomenon, called " the marching subpulses ", is probably fundamental for the understanding of the radiation process and seems to indicate the existence of organized motions in the emitting region.

Polarization. The polarization characteristics are different from pulsar to pulsar but are well marked in any individual source. It is interesting that the radiation in some cases can reach a degree of polarization close to 100 % and that the polarization can vary in a simple way across the pulse. Also, in presence of marching subpulses, each subpulse maintains its polarization characteristics.

Association with SN Remnants. Only the two fastest pulsars NP 0532 and PSR 0833 have been found inside SN Remnants. The absence of extended SN Remnants around slow pulsars is easy to understand since slow pulsars have " ages " $P/\dot{P} >>$ lifetime of a SN Remnant (a few 10^4 years at most). The lack of observed pulsars in some young Remnants such as Cas A can imply that not all SN Remnants leave behind a neutron star but could also be explained in terms of the unknown pulsar emission diagram. In addition, because of the high radio-luminosity of Cas A, it is unlikely that even a source like NP 0532 would have been discovered in it against the strong radio background.

As of today, it is very difficult to determine directly or indirectly (i.e. by comparing the number of pulsars in the galaxy with the frequency of SN explosions) what is the fraction of Supernovae explosions which leave behind a pulsar.

Pulsars in binary stars. No evidence exists for the presence of pulsars in binary stars. This can be due to either of the following (or to a combination of the two): (a) most binary systems are disrupted when one star becomes a supernova (b) the lifetime of pulsars in binary systems is rather short and very soon the energy output from the pulsar is replaced by accretion of matter from the companion (this would suffocate the pulsar itself).

2. GENERAL ELECTRODYNAMICS

The collapse of an ordinary star into a neutron star is likely to be accompanied by a large increase in the strength of the magnetic field. At least two processes can lead to this increase:

a) Differential rotation—if any—in the collapsing star. If the stellar core spins at a different rate than the envelope, a magnetic linking between the two regions results in the twisting of the field lines and the gradual building-up of large toroidal fields which in principle could reach values of order $10^{15} - 10^{17}$ gauss. These toroidal fields could remain submerged inside the star and have no effect on the external electrodynamics. Presently, there are no observational arguments for (or against) the presence of huge toroidal fields inside neutron stars.

b) Flux conservation during the collapse. Depending upon the initial conditions, this could lead to fields in the range $10^{10} - 10^{15}$ gauss.

Similarly, just because of conservation of angular momentum, one expects that the collapsing star will increase its rotation rate and become a fast rotating object.

The electrodynamics of a rotating neutron star closely resembles that of a rotating magnetized conductor.

Consider first an aligned dipole with coinciding rotation and magnetic axes $\underline{\Omega} \parallel \underline{B}$. The problem is time-independent and the induced electric field derives from the charge distribution in the star. Indeed inside a perfect conductor the total electric force experienced by a corotating charge should vanish

$$\underline{E} - \frac{(\underline{\Omega} \times \underline{r}) \times \underline{B}}{c} = 0 \tag{1}$$

and this is accomplished by an internal readjustment of the electric charges. From the Poisson equation one obtains

$$n_- - n_+ = \frac{2\underline{\Omega}\,.\,\underline{B}}{4\pi ec} \qquad (2)$$

Note that inside a conductor $\underline{E}.\underline{B} = 0$.

Outside the star a priori one expects a near-vacuum situation, just because the scale-height of any thermally supported neutron star atmosphere is extremely small (say, a few centimeters). The external problem is therefore a classical potential problem which is solved by choosing the proper boundary conditions on the star.

The electric field at the star surface is of order $E \sim \dfrac{\Omega aB}{c}$ (a is the stellar radius), and there is a component $E_{||}$ along the magnetic field lines. It is easy to see that the electric force on a proton or electron at the stellar surface is much greater than the gravitational pull: for the Crab pulsar $\dfrac{eE}{m_p g} \sim 10^9$. As a consequence, in real life neutron stars cannot be surrounded by a vacuum: electrons and ions are lifted electrostatically from the outer layers until some sort of conductive magnetosphere is formed. If we again neglect inertia and assume infinite conductivity along the field lines, the magnetosphere should largely behave like an extension of the star itself and is subject to the same basic equations (1) and (2) (the latter is valid only relatively close to the star). The magnetic field lines would be equipotentials but there is of course a difference of potential across the field lines: between poles and equator this can reach or even exceed 10^{16} eV for fast neutron stars*.

The magnetosphere can corotate with the star as long as the electromagnetic forces dominate the inertial forces. In case of relatively weak magnetic fields and rather dense magnetospheres, corotation could only be enforced up to the point where the tangential velocity approaches the local Alfven velocity v_A, *i.e.* up to $R_A = \dfrac{v_A}{\Omega}$. It is however conceivable that around neutron stars $v_A = c$ and that the magnetosphere corotates until its tangential speed approaches the speed of light.

The circumstellar space can be naturally divided in two different regions. In the first region (corotating or closed magnetosphere) the

* In a similar laboratory experiment (unipolar inductor) a sphere of size $\sim$ 10 cm., field $\sim 10^4$ gauss, spinning frequency $\Omega \sim 10^3$ sec^{-1} would produce a difference of potential of only a few volts.

field lines close within the speed of light cylinder and the charges corotate with the star. In the other region (open magnetosphere) the field lines cross the critical surface and there is a continuous, electrically driven, outflow of ions and electrons. It would be easy to see that close to the star the bundle of the open field lines has an angular extent given by $\sin\theta = \left(\frac{\Omega r}{c}\right)^{1/2}$. If the magnetic axis and the rotation axis are parallel, the magnetosphere consists of an excess of electronic plasma emanating from the poles and a contiguous region with positive charges (the opposite would be true if the two axes were antiparallel).

Apart from these generalities, the structure of the neutron star magnetosphere represents a major unsolved problem for the pulsar theory. Simple dimensional arguments show that the corotating charges deeply influence the structure of the magnetic field in proximity of the speed of light cylinder. It is therefore necessary to find a self-consistent solution where the space charge given by the field is the appropriate field source. The basic equations of this problem have been considered by various authors (Michel, 1973; Salvati, 1973a; Scharlemann and Wagoner, 1973) but their solution in realistic circumstances appears very difficult and, maybe, even impossible without additional assumptions. It would not be surprising if much of the magnetospheric structure depends on the details of the neutron star surface where the currents originate. Also, it is unclear whether the closed corotating magnetosphere is really inert or whether some sort of large instabilities could lead to its periodical emptying and refilling; if so, it has been speculated that this behavior could be connected to the period irregularities in pulsars.

Another major question which awaits an answer relates to the validity of the condition $\underline{E}.\underline{B} = 0$ along the open field lines. If equation (2) applies in the open magnetosphere, then the sign of the ambient charge excess should change along a given field line as soon as the line bends back toward the equator. However, the sign of the charges which escape along a given line does not change and is determined by the electric potential of the line itself. This entails a situation where a stream of charges of one sign can be surrounded by charges of the opposite sign, probably an unrealistic situation. Alternatively, it is possible that equations (1) and (2) do not apply in the open magnetosphere and that the open field lines are not equipotentials. In this case the particles would be accelerated electrostatically by an amount depending on the magnitude of the residual electric field. If the magnetospheric value of $\underline{E}.\underline{B}$ were comparable to the maximum vacuum level, then the particles could be accelerated in the Crab Nebula up to energies of order 10^{16} eV. The same particles would however suffer from heavy radiation losses since they would move along curved lines. It is unfortunate that no reliable estimate exists for the magnitude of the product $\underline{E}.\underline{B}$ along the open lines: this leads to a fundamental ignorance of the particles energy at various distances from the star. As we shall see in Section 2 this also affects our possibility of understanding the nature and the location of the process which leads to the emission of sharp pulses.

In any case, the particles should certainly be relativistic in proximity of the speed of light cylinder and their inertia cannot be neglected any more. Because of this inertia the field lines lag behind the star and acquire an azimuthal component: the resulting torque is responsible for braking the stellar rotation.

Changing the angle between $\underline{\Omega}$ and $\underline{B}$ from zero (aligned rotator) to a finite value (oblique rotator) does not alter the basic problem in the region $r < \frac{c}{\Omega}$. The problems just become more complicated because of the time dependency but the main results listed above still apply. The main difference between the two cases is that an aligned rotator would not slow down if it were in a real vacuum but an oblique rotator would still slow down because it emits electromagnetic waves at the basic rotation frequency (magnetic dipole radiation). Quite generally, the rate of energy loss through the light cylinder should be of order

$$\left(\frac{B_{cr}^{2}}{8\pi} + \rho_{plasma}\right) 4\pi c R_{c}^{2}$$

where B_{cr} is the magnetic strength at the critical distance R_c. For an oblique rotator emitting dipole radiation or for an unipolar inductor where $\frac{B_{cr}^2}{8\pi} >> \rho_{plasma}$ the corresponding loss of rotational energy

$w_R = \frac{1}{2} I\,\Omega^2$ would be

$$-I\,\Omega\,\dot{\Omega} \simeq B_{cr}^{2}\,c\,R_{c}^{2} \tag{3}$$

Slowing down measurements determine the strength of the magnetic field in proximity of the light cylinder: this ranges between 10^6 gauss for the Crab pulsar (where $R_c \sim 10^8$ cm $\sim 10^2$ star radii) down to one gauss or less for some slow pulsars (where $R_c \sim 10^{10}$ cm $\sim 10^4$ star radii). If the field obeys a dipole law in the near-zone $r < R_c$, then one obtains

$$-I\,\Omega\,\dot{\Omega} \simeq \frac{B_{o}^{2}\,a^{6}}{c^{3}}\,\Omega^{4} \tag{4}$$

i.e. $\dot{\Omega} \propto \Omega^3$. If thep lasma outflow were dominating the field would fall off with the radial law $B \propto r^{-2}$ and similar arguments would lead to a lower expected value for the braking index n in the relation $\dot{\Omega} \propto \Omega^n$.

There is observational support for a largely dipolar structure of the magnetic field in the near magnetosphere of NP 0532: accurate timing of this pulsar have revealed a relation $\dot{\Omega} \propto \Omega^n$ where $n \sim 2.5$ (possibly the braking index is subject to small semi-periodical changes but the evidence is not yet conclusive). The small difference between the dipolar value and the observed value could indicate a slight radial distortion of the field lines under the pressure of the outflowing plasma.

With a dipole geometry the above mentioned values for the magnetic strength at $r \sim R_c$ yield surface fields around 10^{12} gauss: note however that the real surface fields could be much stronger if higher multipole components dominated the dipole moment close to the star.

Beyond the speed of light cylinder the electromagnetic energy flux should be constant and the magnetic field (by now predominantly toroidal) should fall off as $\frac{1}{r}$. Again in case of the Crab Nebula, this entails that the large scale nebular field has a strength

$$B \sim \frac{10^{14}}{r_{cm}} \text{ gauss} \tag{5}$$

that is roughly 10^{-4} gauss over most of the nebular volume. This is not too far from the determined value (around 3×10^{-4} or 10^{-3} gauss) and appears to support the conjecture that the pulsar is the source of the extended nebular field. Note, however, that around an aligned rotator, B is a genuine static field produced by the escaping poloidal currents but for an oblique rotator it refers to the magnetic component of a low frequency wave. For a skew (but not perpendicular) orientation of the two axes, the circumstellar field will be a mixture of the two components. Observational arguments on which we shall return in Section 3 suggest that in the Crab Nebula the static field exceeds a possible wave component by a factor $\gtrsim 10$ in strength. These arguments, however, do not rule out completely the possibility of a low frequency radiation field beyond the light cylinder and it is well known that if such a radiation field were present, it would provide an extremely efficient means of accelerating particles to relativistic energies. In essence what happens is that a particle interacting with a large amplitude low frequency wave becomes immediately relativistic and its motion can remain in phase with the wave. If one defines the strength parameter of the wave $f = \frac{eB}{mc\Omega}$, then it can be shown that any electron exposed to a strong wave ($f \gtrsim 1$) acquires a Lorentz factor in the range $f \lesssim \gamma \lesssim f^2$ (plane wave) or $\gamma \sim f^{2/3}$ (spherical wave). For the Crab Nebula at the beginning of the wave zone $f = 10^{11}$.

Several doubts have been voiced on whether calculations based upon the scheme of vacuum waves interacting with test particles are relevant to the description of the pulsar enviroment. The answer depends on the density of the circumstellar plasma. First, one wonders whether these low frequency waves can really propagate away from the pulsar. In principle one expects very little—if any—thermal plasma because the waves would exert enough pressure to expel this material (there is indeed observational evidence that the circum-pulsar region in the Crab has been cleared of ambient plasma). On the other hand the pulsar itself continuously ejects particles along the open field lines and one should worry at least about this amount of plasma. There are various aspects of this question which should be investigated and which are now receiving attention (Max 1973, Salvati 1973b). It is clear that at least the usual propagation condition $\omega > \omega_p$ (ω_p is the rest plasma frequency) should be modified in order to account for the increased mass of the relativistic electrons: the new condition is found to be $\omega > \dfrac{\omega_p}{\langle\gamma\rangle^{1/2}}$. Whether this condition is satisfied around the Crab pulsar depends on the assumed rate of particle outflow. If the number of escaping charges matches the charge density given by the Poisson equation, then the waves should be able to propagate. If however the charge separation is small and the plasma density by far exceeds the charge density (as some observational evidence seems to indicate) then whether low frequency waves can propagate becomes an open question.

The simple picture of accelerating particles in vacuum by means of pure electromagnetic waves could also fail on other grounds: for instance it is likely that a static field is superimposed on the wave field and this modifies the particle's dynamics. Also, even a small amount of plasma would introduce an index of refraction and change the ratio E/B in the wave.

In conclusion, the simple picture of low frequency, large amplitude waves accelerating particles deserves great attention and undoubtedly represents a very attractive possibility. The plasma physics of this problem is poorly known, but various groups have now tackled the question and one can reasonably expect some progress in the near future.

The first part of Table 1 summarizes the main consequences of the unipolar inductor mechanism and of the emission of low frequency magnetic dipole radiation.

Also, one can compare the predictions of these models with real life as this manifests itself in the Crab Nebula. This is done in the second part of Table 1, and one can see some good agreements as well as some puzzling discrepancies. Various explanations are possible for some of the existent discrepancies. For instance, the difference between the predicted braking index and the observed value can be due to several causes, including a decreasing moment of inertia for the star or maybe a disalignment of the rotation and magnetic axes. As mentioned earlier, the discrepancy could also simply be caused by the outflowing plasma which perturbs radially the geometry of the field lines.

TABLE 1

A) Basic Theoretical Expectations

UNIPOLAR INDUCTION	energy loss $B^2_{o\parallel}\ \Omega^4$ particles extraction → magnetosphere poloidal currents → extended toroidal field electrostatic acceleration
DIPOLE RADIATION	energy loss $B^2_{o\perp}\ \Omega^4$ large amplitude low frequency waves extended magnetic wave field acceleration in the wave

B) Theory Versus Real Life

Theory	Observation
$\dot{\Omega}\ \alpha\ \Omega^3$	$\dot{\Omega}\ \alpha\ \Omega^{2.5}$
$I\Omega\dot{\Omega} \sim 10^{38}$ ergs sec^{-1}	$L_{Crab} \sim 10^{38}$ ergs sec^{-1}
charges' outflow $\sim 10^{33}$ sec^{-1}	particles' outflow $\sim 10^{40}$ sec^{-1}
monochromatic energy spectrum	power law energy spectrum
$B_{static} \sim 10^{-4}$ gauss	$B_{static}/B_{wave} \geqslant 10$
$B_{wave} \sim 10^{-4}$ gauss	$B \sim 3 \times 10^{-4} - 10^{-3}$gauss

A more serious problem exists when one compares the expected charges' outflow $\sim 10^{33}$ sec^{-1} with the number of particles which are known to be continuously accelerated in the Crab Nebula, roughly 10^{40} sec^{-1} (average energy 10^{10} eV). There is no doubt that the pulsar is the source of energy for this acceleration but one wonders whether the acceleration can take place far from the pulsar, for instance when the low frequency waves interact with the thermal filaments. This possibility has been suggested by various authors but there are arguments which tend to make it unlikely. A reasonable alternative would be a very small charge separation in the outflowing plasma (one part in 10^7). In this case, as we have mentioned before, the simple vacuum picture for the acceleration mechanisms is likely to fail. Maybe this would automatically remove the other discrepancy, that is the expectation of a single value acceleration. Power laws can also be obtained by injecting particles at various distances from the star, either in the near zone or in the wave region; or maybe they are the consequence of more complicated random walks of the particles in the pulsar electromagnetic fields.

We shall return in Section 3 on the subject of the discrepancy between the predicted nebular field $\sim 10^{-4}$ gauss and the estimated, somewhat higher strength: for the moment we simply remark that this could be a consequence of the existence of a conductive shell at a finite distance from the pulsar which prevents the electromagnetic energy from propagating at the speed of light, thus making possible some storage of magnetic energy.

3. RADIATION THEORY

The pulsar radiation could in principle provide some direct information about the neutron star magnetosphere. Its origin, however, is extremely controversial, and there is no general agreement on the nature of the radiation process and on whether the pulses arise close to the stellar surface or in proximity of the speed of light cylinder.

We recall that the most striking features shown by pulsars are:

a) the periodical character of the received signals with a rather small duty cycle $\delta = \dfrac{\tau}{P}$ (typically $\delta \sim 1 - 10$ %),

b) the extremely high brightness temperature of the radioemission, in the range $10^{20} - 10^{28}$ °K (in the case of NP 0532 occasionally even higher),

c) the polarization characteristics, different from pulsar to pulsar but well marked in any individual source, *d*) the existence of organized motions in the emitting region (" drifting subpulses "),

e) why only the fastest pulsar emits an appreciable flux of optical and X-ray radiation?

Nobody has yet presented a fully deductive theory for the origin of the pulses in the neutron star magnetosphere, but various attempts have been made to understand why a pulsar pulsates. In general, the models proposed fall in two different categories. The first category considers genuine rotating lighthouses which imply the existence of privileged emitting regions around the star. The second category suggests either the possibility of converting low frequency magnetic dipole radiation into high harmonics in the wave zone (Michel 1971; Cabibbo 1973) or the possibility that in the wave zone the particles move with such a phase relationship that they all become periodically visible simultaneously from an assigned direction of space. The second category of models has the advantage that a pulsar could be visible over a large solid angle but most authors have considered " lighthouse type " models. In either case, the radiation of individual particles should be anisotropic: this is realized if the particles move relativistically, so that the cone of emission covers an angle of order γ^{-1} around the instantaneous velocity. Also, all particles seen by a given observer should be moving in the same direction within an angle of order γ^{-1}. Note that a duty cycle $\sim$ a few percent typically implies $\gamma \gtrsim 10^2$.

The high brightness temperature of the radiopulses can only be achieved by an extremely coherent radiation mechanism. Two general classes of coherent mechanisms have been considered and they involve either bunches of particles moving in phase (" antenna mechanisms ") or negative absorption (" maser mechanisms "). At the present time there seems to be no method of distinguishing between these alternatives, but the antenna mechanisms have been applied more frequently (perhaps, as noted by Ginzburg, because the physics involved is known to a larger segment of the scientific community). In case of bunches, the usual thermodynamic limitation $kT_b \lesssim$ (energy per radiating charge) refers to individual bunches and not to the single particles in the bunch: the brightness temperatures which can be achieved are therefore increased by a factor equal to the number of charges per bunch. If the radiation is due to electrons with energy, say, 100 MeV ($\gamma \sim 10^2$), then $T_b \sim 10^{26}$°K corresponds to about 10^{14} electrons per bunch. Also, in order to have coherent radiation at wavelengths $\lambda \sim 10 - 100$ cm, the size of the bunch along the visual line has to be less than one wavelength. In real life, one would certainly expect bunches of different size and the emitted spectrum should reflect essentially the distribution of sizes (as well known, the spectra of incoherent processes essentially reflect the energy distribution of the particles). The observed pulse-to-pulse radio intensity fluctuations would not be unexpected due to the instability and short lifetime of the individual bunches. Also, the overall process cannot afford any thermalization of the available energy since otherwise an intense flash of energetic radiation would arise.

As we have already mentioned, only the fastest pulsar NP 0532 has been detected at optical and X-ray frequencies. The optical and X-ray pulses are simultaneous to the radio pulses but are remarkably steady. This, together with the relatively low brightness temperature at high frequency (10^{10} °K in the optical band, 10^5 °K in the X-ray band), suggests that the emission is normal incoherent radiation, probably by the same particles which radiate coherently at radiowavelengths.

The simplest and historically first model of pulsar radiation was proposed by T. Gold immediately after the discovery of these objects. In this model, streams of plasma are ejected from selected hot spots on the star surface and corotate with the star up to the speed cylinder. At $r \sim R_c$, the plasma is relativistic and gives rise to an emission beamed in the direction of motion. The emitted spectrum is given by the usual theory of emission from particles in circular orbits. The critical emitted frequency is $\omega_c \sim \Omega\gamma^3$ and therefore the Lorentz factor would have to be $\gamma \gtrsim 10^2 - 10^3$ in order to produce radioemission. If $\gamma \gtrsim 10^5 - 10^6$, the spectrum could reach the optical and X-ray bands. Note that coherent amplification through the antenna mechanism could only operate at radiofrequencies since certainly (size bunch) $>> \lambda_{\text{optical}} >> \lambda_{\text{-rays}}$.

Alternative but somewhat similar processes have been analyzed by Smith (1972) who has considered particularly those models which can explain the pulsar polarization behavior. The spectral characteristics of Gold's process remain unchanged if the radiation does not arise in the corotating magnetosphere but derives from relativistic motions along the bundle of the open field lines since the radiation characteristics for a particle of assigned γ depend only on the local geometry (the open field lines at $r \sim R_c$ have a radius of curvature $\rho \sim R_c = \dfrac{c}{\Omega}$). The problem here is that the open lines diverge as they approach the critical distance and it is not clear what would define a preferred sector with small angular extent. The short duty cycle can be understood more easily if the emission arises when the particles slide along the open lines close to the star; we have seen in Section 2 that in a dipole field the bundle of the open lines close to the surface subtends a rather small angle $\theta \sim \left(\dfrac{\Omega a}{c}\right)^{1/2}$. This type of model considers the pulses as a by-product of the extraction and acceleration of particles very close to the star. Based on earlier ideas of Radhakrishnan and Komesaroff, Goldreich-Keeley and Sturrock (with various coworkers) have tried to spell out the basic physics of the model (for a recent summary see Roberts *et al.* 1973). As mentioned in Section 2, if in the proximity of the star $\underline{E}.\underline{B}$ is given by the vacuum value, the particles could reach extremely high energies. For the Crab Nebula these energies are around 10^{16} eV and the corresponding curvature radiation is peaked at a photon energy $\varepsilon_\gamma \sim 10^{12}$ eV. The gamma-ray photons would be moving with a finite angle with respect to the strong magnetic field and may annihilate into e^+e^- pairs if the condition $\varepsilon_\gamma B_\perp > 4 \times 10^{18}$ is satisfied (ε_γ in eV, B in gauss). It is assumed that the positrons will be turned around by the electric field and flow back to the surface of the star while the electrons would come out in bunches rather than in a steady flow. Since the electrons are produced with a small (but finite) pitch angle, in their motion one can distinguish a gyration as well as a relativistic sliding along the curved field lines. Their emission therefore results from two separate processes, the normal synchrotron radiation and in addition the curvature radiation. Since the sizes of the bunches are much larger than the Larmor radius, synchrotron radiation will be incoherent, but coherence effects could dominate the curvature radiation. An analysis of this combination of curvature radiation and synchrotron process has been carried out some time ago (Pacini and Rees, 1970) and we recall here the main arguments and results.

If the radioemission at frequency υ is attributed to bunches of particles moving along the field lines with velocity corresponding to a Lorentz factor γ_{bunch}, then we have seen before that the following condition should be satisfied

$$\frac{1}{2\pi}\,\frac{c}{\rho}\,\gamma^3{}_{\text{bunch}} \gtrsim \upsilon$$

The effective Lorentz factor of the bunch is related to the velocity v of the particles and to the pitch angle β by the relation

$$\gamma_{\text{bunch}} \lesssim \left(1 - \frac{v^2}{c^2}\cos^2\beta\right)^{1/2}$$

Close to the surface, say at 5 stellar radii, the radius of curvature would be $\rho \sim 2 \times 10^7$ cm and therefore $\gamma_{\text{bunch}} \gtrsim 200$. The brightness temperature of NP 0532 with an emitting region close to the star could reach 10^{28} °K; considerations similar to those made earlier then require 10^{16} electrons or 10^{13} protons per bunch.

The interpretation of the pulsar radiation in terms of curvature plus synchrotron processes near the star is, however, incompatible with the idea that the electrons are involved. The incompatibility lies in the position of the low frequency cut-off of the optical radiation from NP 0532. If this cut-off is due to the synchrotron reabsorption, its position and the corresponding spectral flux determine through the usual considerations the strength of the magnetic field in the emitting region, provided one knows the mass of the radiating particles. For electrons, one finds that $B_\perp \sim 10^4$ gauss, while for protons one obtains $B_\perp \sim 10^9$ gauss. Since the pitch angles are only a few degrees, the emission could arise near the star only if it were due to protons with no contribution by electrons. On the other hand, if the observed optical and X-ray emission is due to electrons, the position of the cut-off indicates that the pulses are emitted in proximity of the speed of light cylinder where one expects $B_\perp \sim 10^4$ gauss. As we have mentioned before, this would leave completely unclear the origin of the small duty cycle. On the other hand, the hypothesis that the pulses arise in a region surrounding R_c (either just inside or a few basic wavelengths away on the outside) would offer an immediate explanation of why only the Crab Nebula pulsar emits optical and X-ray radiation. Indeed, if the high frequency radiation is due to the synchrotron process, the total output should roughly scale like the product of the energy outflow in relativistic particles (proportional to Ω^4) multiplied by the square of the magnetic field in the emitting regions $B_c{}^2$ (in a dipole field $B_c \propto \Omega^3$ and therefore $B_c{}^2 \propto \Omega^6$). One would therefore expect a scaling-law $L_{syncro} \propto P^{-10}$. This would entail for the second fastest pulsar (PSR 0833 in Vela) an average optical magnitude $m > 25$ as well as an annual decrease of the optical luminosity of NP 0532 around 0.5 % per year. It is interesting to recall that the present observational limits (plus the—maybe dangerous—assumption that NP 0532 and PSR 0833 only differ in their period) entail a relation $L_{syncro} \alpha P^{-n}$ where $n \gtrsim 8.4$ (Kristian, 1970). Another point in favor of emission close to the critical distance is that one finds that it would imply particle densities in the range $10^{12} - 10^{14}\ \mathrm{cm}^{-3}$: this matches the value inferred from the rate of particles' input in the Crab Nebula $\sim 10^{40}\ \mathrm{sec}^{-1}$.

The above considerations can be summarized by saying that the characteristics of the incoherent high frequency radiation would favor an emission process taking place near the critical distance. On the other hand, if this were the case, the small duty cycle would have to be the consequence of a poorly understood pattern of the electromagnetic field and of the particles' motion in it.

4. SOME PROBLEMS RELATED TO THE LINK BETWEEN PULSARS AND SUPERNOVAE REMNANTS

The importance of the relation between a central pulsar and the surrounding Supernova Remnant is clearly exemplified by the Crab Nebula.

Electrons emitting optical and X-ray radiation in this object have a lifetime considerably shorter than the age of the Nebula. This high frequency emission would be absent if the pulsar was not acting as a huge, very efficient accelerator continuously injecting new particles in the energy range 10^8 eV $< E < 10^{14}$ eV. In addition, the nebular field is estimated to be in the range 3×10^{-4} to 10^{-3} gauss; over a volume $\sim$ (1 parsec)^3 this cannot result simply from the presence of a central dipole or from the radial expansion of a pre-existing stellar field since in both cases the sources would have an unreasonable strength and the observed field geometry would not match the theoretical expectations. It is now generally believed that the pulsar is also the source of the nebular field by the sort of mechanisms which were reviewed in Section 2. It becomes therefore of considerable interest to attempt a description of the overall relationship between the pulsar and the Remnant and the evolutionary history which this link implies (of course this will only apply to those Remnants which are energized by a rotating neutron star). In the following we shall report briefly on some aspects of this problem, largely referring to recent results of the Frascati group. This work can be divided in three different (but strictly related) aspects: (a) nature of the nebular magnetic field, (b) dynamics of the SN shell, (c) evolution of the luminosity of SN Remnants.

(a) Nature and evolution of the nebular magnetic field

As we have seen in Section 2, both the aligned unipolar inductor and the oblique rotator model lead to the expectation of a large scale toroidal magnetic field, decreasing like $\frac{1}{r}$ beyond the critical distance. In the first case the field is generated by the poloidal currents escaping from the open magnetosphere and the nebular particles radiate in it via the normal synchrotron process in a static field. In the second case, the magnetic field is a component of the low frequency waves emitted directly by the pulsar at the basic rotation frequency. A relativistic electron would radiate in this field in a way similar to synchrotron radiation in an equivalent static field, but there would be a substantial amount of circular polarization (Rees, 1971). For the Crab Nebula, a model where the large scale magnetic field is an oscillating wave field leads to the prediction of a few percent circular polarization. No evidence for it has been found to a level of order 0.1 % (Landstreet and Angel, 1971), but the measurements do not rule out the simultaneous presence of low frequency waves plus a static field stronger by at least a factor of 10. (If so, the details of the far-wave acceleration mechanisms would change and are currently under investigation).

The field strength predicted by both mechanisms is possibly one order of magnitude less than the lower limit inferred from the upper limit to the γ-ray flux (Fazio *et al.*, 1972). This discrepancy can only be understood if the toroidal field generated by the currents keeps accumulating inside the Remnant. In essence, one has to suppose that new magnetic lines are continuously produced by the pulsar and propagate outwards with the speed of light up to a certain critical distance R^*. The spacing of the field lines is $\frac{c}{\Omega}$ in the internal region (this region can be described in the way sketched by Goldreich and Julian, 1969). Beyond R^* the situation changes due to the existence of a conductive Supernova shell. The field lines can only propagate outwards at a velocity less than the velocity of the shell, which leads to a continuous increase in the nebular magnetic energy. See Fig. 1. The critical distance R^* is probably the distance at which the ambient energy density is of the same order as the energy density in the outflowing relativistic wind. (In the Crab Nebula $R^* \sim 10^{17}$ cm; several authors have noted that this is roughly the region where the wisps are located).

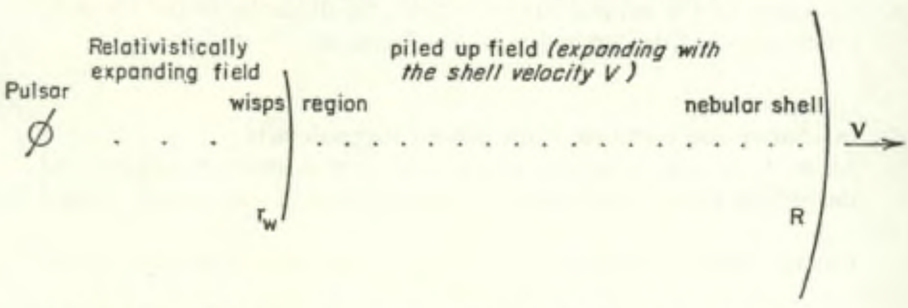

Fig. 1 A schematic drawing of the magnetic field behavior in a SN Remnant.

If this conjecture is correct, a certain fraction of the stellar rotational energy is gradually converted into magnetic energy of the nebula. Assuming that the magnetic energy increases at a rate given by the pulsar input minus the adiabatic losses (due to the expansion of the shell), one obtains the time evolution of the magnetic field inside a SN Remnant. Figure 2 depicts this evolution in the case of the Crab Nebula, assuming an initial pulsar period P $\sim$ 16 msec (this results from an extrapolation back in time of the present period) and an expansion velocity $\sim$ 1000 km sec^{-1}. One can see the nebular magnetic field did increase during the first few hours and reached a maximum strength of several hundred gauss. Only after about 3 years the field dropped below 1 gauss. The present-day expected value is close to

10^{-3} gauss, if all the pulsar energy goes into magnetic energy. (It is probably more realistic to assume that no more than 50 % of the energy goes into magnetic fields). Note that in this picture the wave field should be less than the static field since the first certainly cannot accumulate.

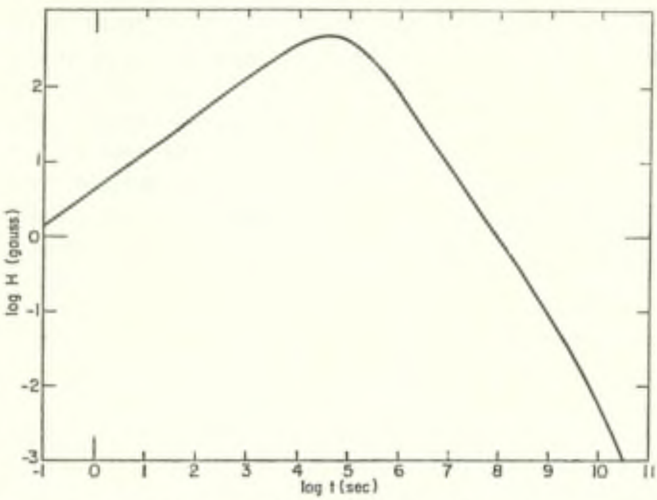

Fig. 2. Time evolution of the magnetic field in the Crab Nebula based upon a uniform expansion velocity.

(b) Expansion of the Supernova shells

It is well known that the expansion velocity of the Crab Nebula has increased during its lifetime. The question arises on whether the entire kinetic energy of the nebular shell can have derived from the electromagnetic pressure of a central pulsar. This possibility was mentioned (in a slightly different form) by F. Hoyle during the 13th Solvay Conference (Hoyle, 1965). In general one has discussed simplified models, assuming that a newly formed rotating neutron star is surrounded by a thin shell. If the initial velocity is negligible (which of course could be not true in real life), the shell will implode under the influence of the gravitational forces. Nevertheless, if the energy lost by the pulsar can accumulate between the star and the shell, the resulting pressure can slow down the collapse. It is intuitive that an explosion will follow if the energy released by the pulsar during the free-fall time excedes the initial binding energy of the shell. Any form of relativistic energy that can accumulate would have similar effects. Ostriker and Gunn (1971) have assumed that low frequency waves can be trapped; recently Maceroni, Salvati, and Pacini (1973) have developed this by considering the dynamical effects of an accumulated toroidal magnetic field. It should be stressed, however, that the SN explosion itself could hardly derive from the pulsar pressure if the initial slowing-down time scale is relatively long. On the other hand, this time scale can be very short, say as short as a few seconds, if there is twisting of the field lines under the influence of differential rotation between the collapsing stellar core and the envelope. In the case of the Crab Nebula, the numerical results show that the expansion velocity of the shell could entirely derive from the internal pressure of the electromagnetic fields, provided that the initial radius was larger than 3×10^{13} cm (if so, the present-day expectations depend very little upon the initial radius). Also, since in a self-consistent calculation the radius of the nebula is always less than (or at most equal to) what one gets from uniform expansion at the present velocity, the magnetic field strength reaches values $\simeq$ thousand gauss, higher than in the case of uniform expansion. Today's expected value for the nebular field is again $\sim 10^{-3}$ gauss, a value identical to the one derived in the uniform expansion case. Figures 3 and 4 show the behavior of the nebular radius and field based on the self-consistent calculation.

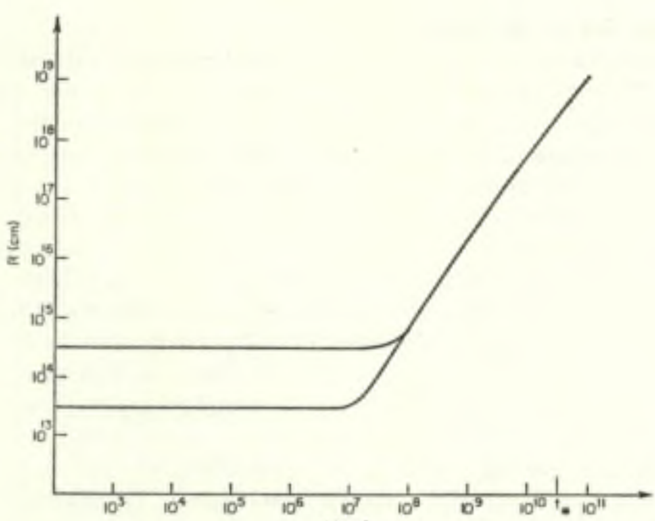

Fig. 3. Time evolution of the radius of the Crab Nebula assuming zero initial velocity and two different values for the initial radius (t* is the present-day age of the Crab).

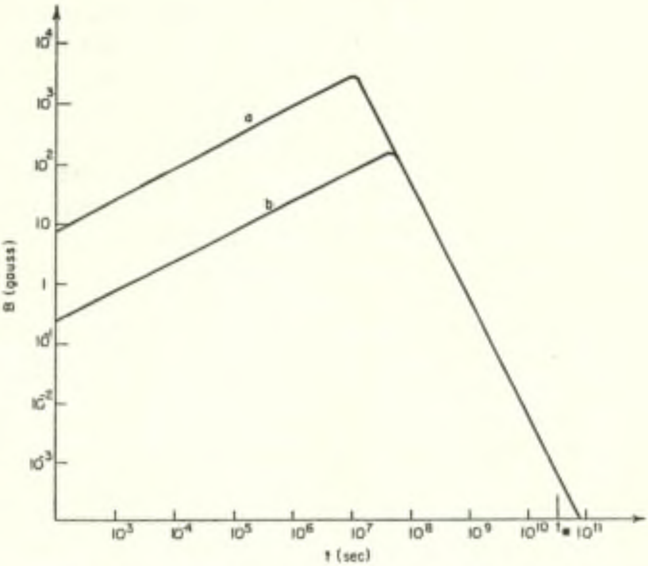

Fig. 4. Time evolution of the magnetic field in the Crab Nebula assuming zero initial velocity and two different values for the initial radius (t* is the present-day age of the Crab).

(c) The evolution of the luminosity of Supernovae Remnants

Knowing the evolution of the magnetic field and assuming a continuous (but decreasing with time) injection of particles, one can investigate the whole electromagnetic history of Supernovae Remnants. In particular one can investigate the evolution of the nebular particle's spectrum and the changes in the emitted luminosity at various frequencies. In a recent paper (Pacini and Salvati, 1973), we have assumed that the magnetic field evolves in the way sketched before in (a) and that the injection follows a single power law with a fairly flat spectrum $N(E) = E^{-\gamma}$, with $\gamma < 2$. This case is of considerable physical interest since it applies to the Crab Nebula and to several other non-thermal sources. The main results of this investigation can be summarized as follows:

1) the observed nebular electron content does essentially derive from injection long after the initial explosion. This is due to heavy radiation losses in the early, strongly magnetized, nebula. It is therefore very unlikely that an object like the Crab Nebula is resupplied continuously only with optical and X-ray electrons, the (relatively low energy) radioemitting ones being possible remnants of the original event.

2) the shape of the injected particles' spectrum should match the present-day energy spectrum.

3) the spectral luminosity will reach a maximum first at low and then at higher frequencies (note however that one still expects most of the total output to be in the high frequency band). See Fig. 5. Radioemission would therefore be strong in the early life of a SN Remnant, at least in absence of various absorption phenomena. Recent negative results on the radioemission of young SN Remnants (De Bruyn, 1973) imply either a failure of the model for the observed objects (possibly because the explosion did not leave behind a pulsar) or important absorption effects. The latter possibility is currently being investigated.

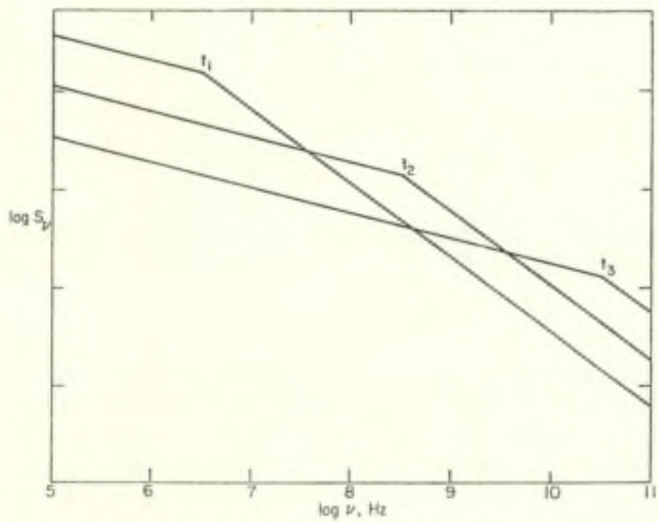

Fig. 5. A schematic drawing of the evolution of the spectral luminosity of SN Remnants. $(t_3 > t_2 > t_1)$

4) equipartition of energy between electrons and magnetic fields in general should not exist in SN Remnants, even if it exists at the source. The basic reason is that magnetic fields only lose energy because of expansion losses while electrons lose energy both because of expansion losses and radiation losses. A possible proton component would also lose energy mainly because of expansion; therefore the ratio (proton energy/magnetic energy) could remain constant until the SN Remnant merges with the interstellar medium. This leads to the interesting speculation that—if pulsars account for the production of cosmic rays in the galaxy—they might also account for the energy density of galactic magnetic fields.

CONCLUSION

Many important aspects of pulsar theory are undoubtedly either controversial or unclear. On the other hand, in the first five years since their discovery, we have probably understood the basic nature of these objects as well as some of the relevant physical processes. Above all, the basic process operating in pulsars, that is the gradual conversion of macroscopic rotational energy into magnetic fields and relativistic particles, seems to offer a good starting point for understanding other phenomena in high energy astrophysics.

ACKNOWLEDGEMENTS

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Discussion of the report of F. Pacini.

F. G. Smith: I have two comments:

1) There are often strong selection effects in observational data on pulsars. The detection processes are usually less sensitive to short periodicities, so that the population statistics are here in some doubt. At the long periods there is no selection effect and the fall for periods above 1 second is well established.

The recent pulsar search at Jodrell Bank has a uniform sensitivity from 160 milliseconds up to about 2 seconds. The present statistics show a smooth peak at 1.2 — 1.4 sec. This is easily accounted for as a statistical fluctuation rather than as a separate type of pulsar.

2) There is an additional reason for not finding a close association between pulsars and supernova remnants, i.e. the high velocity which we now believe is characteristic of pulsars. This velocity has been established by direct measurement, using the movement of scintillation patterns across the earth's surface; it is now confirmed by the pulsar distribution formed in the recent Jodrell Bank survey. We find that the distribution perpendicular to the galactic plane has a thickness of about 250 pc between half density points. This is consistent with an origin close to the plane, with a velocity of about 100 km sec^{-1} and a lifetime of about 3.10^6 years. The direct measurements of velocity give values around 100 km sec^{-1}; one pulsar has a velocity of about 350 km sec^{-1}.

J. Bahcall: There is a preprint by M. Elitzur who suggests that the high frequency radiation (X-rays and optical) arises from electron scattering of coherent radio emission near the neutrons stars. In his opinion this gives a consistent picture of the spectrum, polarization, and relative X-ray intensities from different pulsars.

D. Pines: I would like to recall the possibility of another radiation model which has been considered especially by Tsytovich: as a result of physical processes which give rise, say, to strong plasma turbulence, one has an extremely active region near the magnetic poles of the star (let us say ~ 1 km^3) which is responsible for both the coherent emission of radio waves and the acceleration of particles to ultrarelativistic energies. The latter then produce by synchroton radiation the pulsed optical X, and γ radiation observed for the Crab pulsar. I wonder if Dr. Pacini would comment on the merits of such a phenomenological model, and the problem it raises.

F.G. Smith: The idea of a common origin for the radio, optical, and X-ray emission from the Crab pulsar can be supported by some simple theory as recently published (Smith F. G., Nature **243,** 207, 1973).

I think we should assume that the pulses are formed by relativistic beam compression, which is the only explanation for the close similarity of pulse shape over more than 50 octaves of the electromagnetic spectrum. It is then allowable for the radio and optical radiation mechanisms to differ somewhat and even to have different emission polar diagrams. I have therefore used the observational evidence in proposing the following model:

i) the emitting region is located close to the velocity of light circle, so that it moves with a speed of the order of 0.8 to 0.9 c.

ii) the radio emission is coherent radiation at the fundamental gyrofrequency from electrons moving in bunches orbiting roughly perpendicularly to the magnetic field.

iii) the coherence only applies to the fundamental, while synchroton radiation at the harmonics is incoherent. This is the optical and X-ray radiation.

In this model the particle energies are in the region of 10^{10} volts. The beaming is determined not by the particle energy but by the speed of the whole source, which corresponds to a γ of only 2 or 3.

A. Hewish: I would like to stress one difficulty concerning the emission of radiation near the velocity of light cylinder. Since the particles must receive most of their acceleration near the light cylinder (since they are not observed to radiate on the way out) one would expect a relativistic beaming process to produce two beams, due to each intersection of the light cylinder with the magnetic axis. These beams would be directed normal to the rotation axis and would give two observed pulses per revolution.

Most pulsars, however, give only a single pulse per revolution.

F. Pacini: I feel that your remark is model dependent and it applies only to the early, fairly naive views where an emitting region lies above the poles.

Most proponents of the idea that pulses arise close to the critical distance would now think in terms of more complicated models or particle's orbits.

M. J. Rees: I would like to comment on some recent work by J.E. Gunn and myself concerning the origin of the magnetic field and relativistic electrons in the Crab Nebula. This work is somewhat similar to that of Pacini and Salvati. We have assumed that the power output from the Crab pulsar has three ingredients: 30 HZ waves, a relativistic wind, and a non-oscillatory toroidal field frozen into the wind. If we interpret the interpulse as evidence that the angle between the rotational and magnetic axes is close to $\pi/2$, we might naively expect that *most* of the energy emerges as 30 HZ waves. There will be a shock discontinuity at $\sim 10\,\%$ of the radius of the nebula. Outside of this shock, magnetic flux and relativistic particles accumulate. The magnetic energy rises relative to the particle energy (a phenomenon emphasized by Piddington 15 years ago), and an equipartition field strength can be obtained. (However the field can never *exceed* equipartition, because once magnetic stresses become comparable with particle pressures, the field will readjust so that the magnetic energy decreases at the expense of particle pressure). The 30 HZ waves will be absorbed by the synchroton process just outside the shock, their associated energy going into relativistic electrons. The continuum emission from the nebula would thus be due entirely to the synchrotron process, rather than being synchrocompton radiation.

M. A. Ruderman: An argument for strong accelerating electric fields near pulsar surfaces may be the difficulty in pulling electrons from the surface without them. The huge magnetic field causes the surface to have a sharp edge and an important work function.

Copious electron emission may, therefore, need a very much higher surface temperature than old pulsars would have or an electric field which is a substantial fraction of the vacuum one.

F. Pacini: In the case of the Crab pulsar, the large difference between the expected charge outflow $\sim 10^{33}$ charges sec^{-1} and the estimated number of particles accelerated per unit time $\sim 10^{40}$ sec^{-1} could imply the possibility of extracting particles without using the electric field generated by the unipolar inductor. We cannot exclude other mechanisms, for instance something like a continuous small scale " flare " activity on the star's surface.

M. A. Ruderman: The great excess of electrons accelerated near the pulsar, enormously beyond the minimum needed for charge separation in the corotating magnetosphere, may come from the copious pair production by gamma rays in the huge magnetic field of the neutron star magnetosphere. As emphasized by Sturrock such electron-positron showers must follow but only if the accelerating electric fields near the neutron star are not small relative to their vacuum values.

J. Bahcall: I have a naive question. You have described some fairly complicated electromagnetic situations on which there seems to be a good bit of controversy. What in your opinion are the areas that are ripe for future development?

F. Pacini: I feel that the three major theoretical questions are:

1) self-consistent model for the magnetosphere.

2) propagation of low frequency, large amplitude waves in the circumpulsar medium.

3) radiation theory.

M. J. Rees: It seems to me that it might be especially worthwhile to attempt to understand the optical and x-ray pulses from NP 0531, rather than the radio pulses. This is because the former, being presumably incoherent, provide rather more direct evidence on the particle flux and the particle energies.

OBSERVATIONAL RESULTS ON COMPACT GALACTIC X-RAY SOURCES

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INTRODUCTION

With the advent of orbiting X-ray observatories, such as UHURU, X-ray astronomy has entered an era of new astronomical discoveries.

The long time available for observation, the sensitivity of the instrumentation and the ability to orient the spacecraft on command to any desired location in the sky have resulted in a qualitative improvement in our understanding of the X-ray sky.

A catalog has been compiled which includes some 160 sources ranging in apparent luminosity from Sco X-1 (the first observed X-ray star and the most intense in the 1-10 keV energy region) to 10^{-4} Sco X-1. Of these sources, about 60 appear to be extragalactic and about 100 galactic.

With respect to galactic sources, the most important aspects of the observations have been:

1. The discovery that large variations in intensity of the X-ray sources are the rule rather than the exception. Several nova-like sources have been detected in which the intensity changes by factors of as much as 100 in a few days. Variability of factors of 2 in time scales of months to minutes is present for almost all sources. Several sources emit pulses (Cyg X-1, Circ X-1) or trains of pulses in time scales of 100 milliseconds or less. Some sources (Cen X-3, Herc X-1) are emitting periodic pulsations with characteristic periods of seconds.

2. The discovery that individual stars can emit X-rays with very large intrinsic luminosities of order 10^{38} erg/sec making them among the brightest stars in the sky. This became apparent with the first detection of individual stars in external galaxies of the local group (LMC and SMC).

3. The discovery of the binary nature of several galactic X-ray sources which will constitute the main subject of today's discussion.

4. The existence of fast time variations in several X-ray sources and the measured parameters of the components of the binary systems strongly indicate that galactic X-ray sources correspond to stars near the end point of their evolution. In most cases, one can surmise that the X-ray emitting object must be a White Dwarf or a neutron star. In one case, Cyg X-1, the evidence strongly suggests that we might be dealing with a very massive, compact object, possibly the first black hole ever detected.

With respect to extragalactic sources, the number of X-ray sources which we believe to be extragalactic objects has increased by tenfold.

All galaxies are found to be X-ray emitters with intrinsic luminosity spanning a range from 10^{39} to 10^{46} erg/sec. In normal galaxies, such as our own, the emission is predominantly the result of the integrated contribution from individual stellar sources.

Seyfert galaxies, N-type galaxies, Quasars, and radio galaxies have much greater intrinsic luminosities spanning the range from 10^{42} to 10^{46} erg/sec. The emission appears to be due to high energy events taking place in the nucleus of the galaxy.

In addition to this general improvement of our understanding of extragalactic sources, the most significant contributions to extragalactic astronomy have been:

1. The discovery of yet another type of X-ray emission associated with clusters of galaxies. In the case of the Virgo, Coma, Perseus and several other clusters, an extended region of emission of about 1° diameter has been detected. The emission may be due to a tenuous, hot gas having a mass about equal to the one contained in the galaxies and emitting X-rays via the thermal bremsstrahlung process. There is some evidence that X-ray emission is related to the cluster kinematic parameters. The intrinsic luminosity is of the order 10^{42} to 10^{44} erg/sec.

2. The discovery of a large number of objects which are believed to be extragalactic, but are as yet unidentified with any known class of galaxies. These objects are not simply the next brightest member in each of the classes of galaxies which have been identified as X-ray emitters. The absence of peculiar visible or radio counterparts at their location seems to indicate that the X-ray emission is the predominant form of electromagnetic energy loss. In addition to the intrinsic interest in the physical conditions which might lead to such abundant X-ray emission in a galaxy, this class of objects has great interest in that they might be responsible for a significant fraction of the diffuse X-ray background.

In Figure 1 a plot of the intensity of the X-ray sources versus their galactic latitude is given as a convenient way to show in summary all the sources from the third UHURU Catalog. Today's discussion* will be limited to the strongest galactic sources of intensity greater than about 40 cts/sec. Further, I will restrict my remarks to recent work on binary X-ray sources, thereby ignoring the very interesting results from recent studies of X-ray emission from supernova remnants.

This approach seems particularly fruitful to set the observational stage for several of the theoretical discussions that will follow in this Conference.

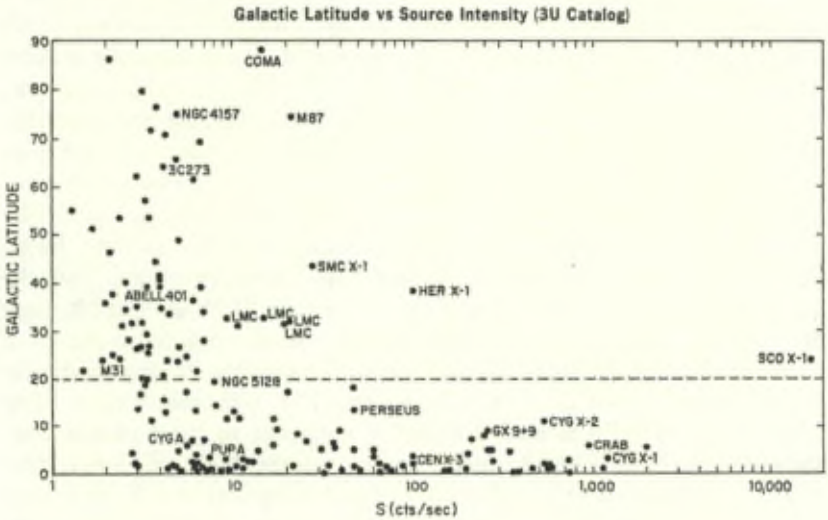


Fig. 1

X-RAY SOURCES IN BINARY SYSTEMS

Long before many of the results, which I will be discussing, had been obtained, Zel'dovich and Novikov (1964) had suggested that condensed stars could be found as X-ray sources accreting matter from binary companions.

The first indication that some X-ray sources might be associated with collapsed stars was provided by the first optical studies of compact galactic sources. The spectrum of Sco X-1 (Sandage et al, 1966) was

*Many of the X-ray results that I will be discussing in this paper have been obtained by the UHURU Group at American Science and Engineering, now at the Center for Astrophysics (SAO/HCO) at Harvard. Among this group are Harvey Tananbaum, Ethan Schreier, William Forman, and Christine Jones Forman, to whom I would like to express my gratitude for their assistance in the preparation of this material. I would also like to thank Wallace Tucker and George Field for their helpful discussions.

found to be similar to that of an old nova. It is well known that such objects are close mass exchange binaries containing a collapsed (probably white dwarf) companion (Kraft, 1964). Shortly thereafter, Burbidge et al (1967), reported evidence for the binary nature of Cyg X-2. These observations suggested an attractive model for X-ray sources: matter lost from the primary star in a close binary would accrete onto a compact object, releasing $\sim 1 - 10$ keV of gravitational potential energy per particle in the case of a white dwarf, ~ 100 MeV per particle in the case of neutron stars and black holes. The thermalization of this energy, either by shock waves or by viscous heating in a disk, would produce a hot gas which would emit X-rays. In this way, X-ray luminosities in the range $10^{36} - 10^{38}$ erg/sec could be explained by modest accretion rates in the range $10^{-8} - 10^{-10}$ $M_\odot$/yr. This idea was pursued by Cameron and Mock, Shklovskii, Prendergast and Burbidge in papers published in 1967-1968. However, further observations of Sco X-1 and Cyg X-2 failed to show definite evidence of binary motion. Thus, for a time the possibility that compact X-ray sources could be binaries decreased in popularity. A number of other models, based for the most part on analogy with pulsars, were proposed, and the picture became somewhat muddled. Then in 1971 a breakthrough in our understanding of compact X-ray sources was achieved as a result of observations by the X-ray satellite, UHURU. Two periodically pulsating X-ray sources, Cen X-3 and Her X-1, were discovered. The observation of eclipses and a Doppler variation in their period conclusively established their binary nature on the basis of X-ray data alone. Today, as a result of combined radio, optical and X-ray observations, we can make a fairly convincing case that all compact X-ray sources not associated with supernova remnants are associated with mass transfer binaries containing a collapsed star. Perhaps, most significant of all, in the case of Cyg X-1, X-ray astronomy has furnished the strongest evidence yet for the existence of a new class of objects, black holes.

The work by Kippenhahn and his associates, and by Paczynski, has shed considerable light on the evolutionary tracks that may lead to the formation of the type of binary systems which the X-ray observations seem to require (Paczynski, B., 1971). It was shown that in the evolution of short period binary systems, there exists the possibility of obtaining systems in which a collapsed star is accompanied by a massive companion which overflows its lobe of the zero velocity surface (Roche lobe). Detailed computations were carried out by van den Heuvel and Heise for Cen X-3 (van den Heuvel, 1972). They follow the evolution of a system containing stars of mass $16M_\odot$ and $3M_\odot$ and $\tau = 3$ days. Using the evolutionary track of Paczynski, they find a descendant system containing $15M_\odot$ and $4M_\odot$ with a period of 1.53 days. Assuming a supernova explosion ejecting $3.5M_{\odot}$, and assuming that the system remains bound, they are left with a $15M_{\odot}$ star accompanied by a $0.5M_{\odot}$ neutron star with a period of about 2 days. It should be pointed out that several assumptions on the conservation of angular momentum and total mass for the system, as well as on the details of the explosion are made in the computation. Colgate has pointed out the difficulties of retaining a bound system through the supernova event and has questioned whether neutron stars can be found in binary systems at all (Colgate, 1968). It is not my intention to deal in detail with evolutionary questions except to point out that the existence of an X-ray emitting phase appears to be a required stage in the evolution of massive stars in short period binaries, rather than an oddity.

Apart from the insight that the discovery of such systems gives us on the evolution of stars, the finding of compact objects in binaries presents us for the first time with the opportunity to investigate their properties in detail. Thus, the possibility exists in the case of Her X-1 and Cen X-3 that a precise determination of mass could be obtained, as in the case of double line spectroscopic binaries. In this case, the velocity of one component is derived by X-ray measurements and of the other by conventional spectroscopic techniques in the optical. If these objects are indeed neutron stars, a statistical analysis on several such systems will give us an important indication of the possible upper limits on their mass. Also, the detailed study of the changes in orbital period and pulsation periods which are observed in X-rays will greatly improve our understanding of matter loss processes from the system, changes in moments of inertia of neutron stars and the nature of their physical state (Lamb, 1972; Baker, 1973). The detailed analysis of X-ray absorption features, will permit us to understand the gas dynamics in the system. Finally, the discovery and subsequent study of black holes is of great significance for general relativity, a point vigorously made by Wheeler and Ruffini (1971).

In this survey of X-ray binaries, I will not attempt to expand on the points above. I will simply endeavor to bring to your attention the wealth of information which is rapidly being uncovered by X-ray observations.

HERCULES X-1

The best studied of the binary X-ray sources is Hercules X-1. As we will see, many of the observational phenomena for Hercules X-1 can be clearly explained in terms of a rotating neutron star orbiting in a binary system, although some of the complex features of this system are not yet fully understood. The view that Her X-1 is a neutron star is by no means generally accepted. Models based on differentially rotating degenerate dwarfs have recently been suggested; and the proponents of vibrating degenerate dwarfs have not yet conceded. Without going into the merits of the models, I have adopted the neutron star point of view as a means of more conveniently describing the data. If, indeed, degenerate dwarfs with masses of $1.4M_\odot$ can rotate or pulsate at 1.24 seconds, then the alternate models are at least plausible although no detailed computation to explain the details of the observations has, to my knowledge, yet been carried out.

If we start from the simplest X-ray observations, we find that Hercules X-1 shows periodic occultations with a 1.7 day period. Figure 2 shows the 2-6 keV X-ray intensity, as observed by UHURU, varying from a high of 100 counts/second to a level below the limit of UHURU's detectability of a few counts/second. The transitions between the high intensity state and eclipse take a time less than 12 minutes. Data for 3 adjacent occultation cycles in July 1972 are shown in the figure, which also shows that the X-ray eclipse lasts for 0.24 days. Figure 3 from Forman et al, (1972), shows the optical behavior of Hz Herculis in the summer of 1972 and on plates from the 1940's. The 1.7 day optical variations in phase with the schematically represented X-ray eclipses of Hercules X-1 make certain the identification of the star with the X-ray source. The optical variations are interpreted in terms of a binary in which the central star has a hot side facing the X-ray source and heated by the X-ray emission with temperatures of order 10,000 °K for the hot side and 6-7,000 °K for the cooler side. Recent papers, such as those of Wilson, Joss et al, and Rucinski, have considered the various detailed models which are required to reproduce the shape of the observed optical variations.

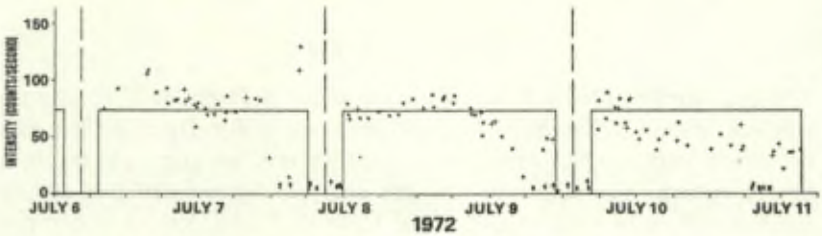


Fig. 2

HZ HERCULIS OPTICAL OBSERVATIONS FOLDED 1.70015 DAYS

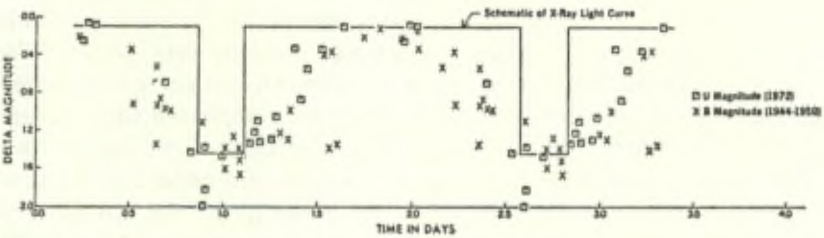


Fig. 3

The first X-ray observations of Hercules X-1 also showed another regular feature. The source pulses periodically with a 1.24 second period. This behavior is illustrated in Figure 4 where the lighter histogram shows 30 seconds of 2-6 keV counting rate data accumulated in 0.096 second bins, the highest time resolution available with UHURU. The heavier curve is a minimum χ^2 fit to the pulsations of a sine function plus two harmonics.

SOURCE IN HERCULES (2U1702+35)
November 6, 1971

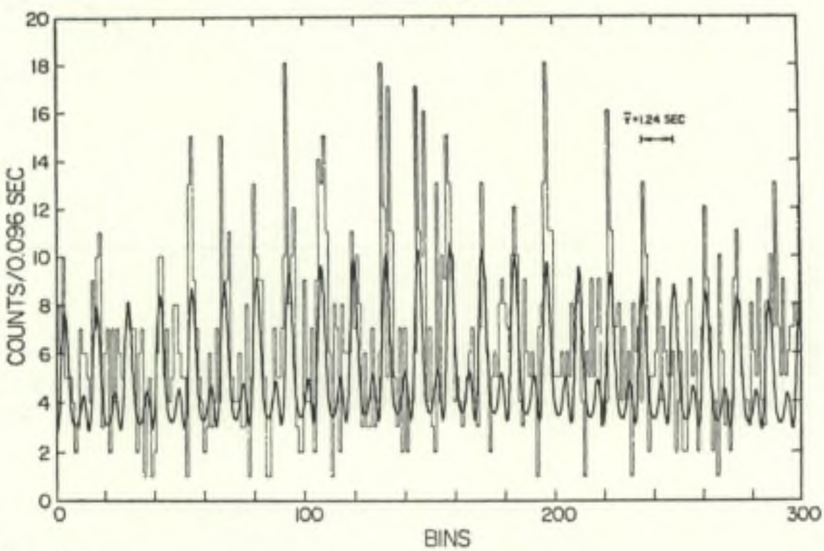


Fig. 4

Since Hercules X-1 undergoes regular eclipses, it should not be surprising that its 1.24 second pulsing shows a regular Doppler pattern.

As the X-ray source orbits its binary companion, the pulses appear closer together (shorter period) as the source comes out of eclipse and approaches us and the pulses appear farther apart (longer period) as the source heads towards eclipse and moves away from us. This behavior allows us to measure directly and precisely very many of the parameters of the Hercules X-1 system. The Doppler curve is indicated in Figure 5 where in the bottom half, we have replotted the intensity data from the first Hercules figure and in the top half, we have shown the time difference between the time of arrival of a pulse and the time predicted for a constant period plotted versus time. As can be seen from the figure, the arrival time of the pulse is delayed 13.2 seconds at the center of the occultation and is 13.2 seconds early at the center of the high state.

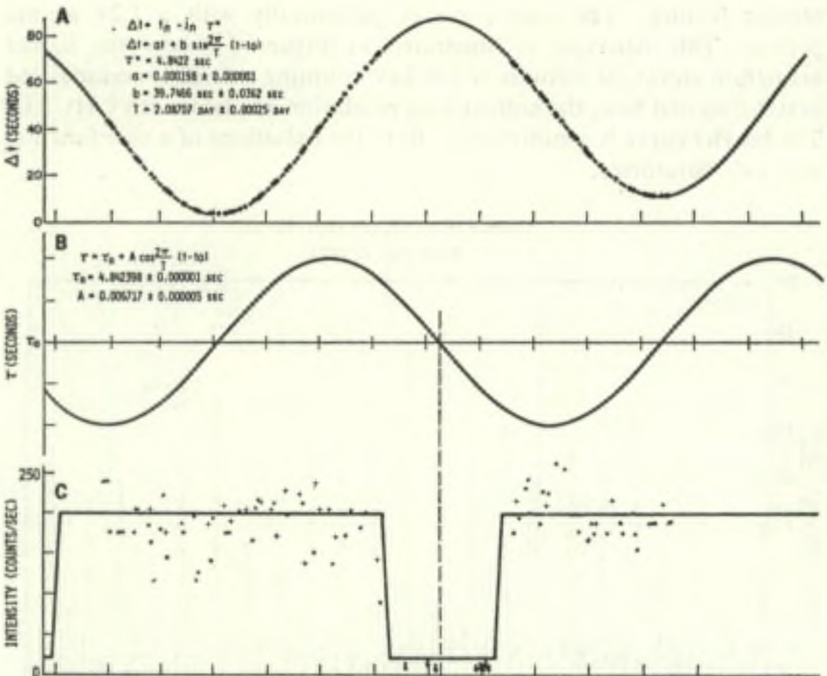


Fig. 5

The delay time of 13.2 seconds is a direct measure in light seconds of the radius of the orbit of the X-ray star about the center of mass of the binary system as projected into the observing plane. Most importantly, the presence of the sinusoidal Doppler curve confirms the picture of the X-ray source orbiting in a binary system as first suggested by the regular eclipse.

Figure 6 contains a summary of the observed parameters of the Hercules X-ray system: the average pulsation period τ, the half-amplitude of the period variation $\Delta\tau$, the orbital period T, and the phase of the eclipse center φ_T. In addition, four parameters are derived from the measured quantities under the model of a binary system in an orbit with an inclination angle i to observer ($i = 90°$ corresponds to the observer being in the plane of the orbit of the binary system). The first derived parameter is the eccentricity of the orbit of the binary and from the good quality of the sine fit to the pulse arrival times is determined to be less than 0.05. This result justifies the assumption of a circular orbit which can be used to derive the projected orbital velocity *vsini*, the projected orbital radius about the center of mass *rsini*, and the mass function of the system $(M_2^3 \sin^3 i)/(M_1 + M_2)^2$ where M_1 is the mass of the X-ray star and M_2 the mass of the occulting star.

TABLE I

Parameters of Hercules X-1

Observed Parameters	
τ (average heliocentric pulsation period in seconds)	1.2378206 ± 0.0000001 (Jan. '72)
$\Delta\tau$ (half-amplitude of period variation in seconds)	0.0006979 ± 0.0000003
T (orbital period in days)	1.70017 ± 0.00001
Φ_T (phase of eclipse: low center)	UT 1972 Jan. 13.0772 ± 0.0003 UT 1972 Jul. 6.1897 ± 0.0004
Derived Parameters	
ε (eccentricity)	$\leqslant 0.05$
$v \sin i = (\Delta\tau/\tau)\, c$ (km s^{-1})	169.0 ± 0.1
$r \sin i = (T/2\pi)\, v \sin i$ (cm)	$(3.95 \pm 0.001) \times 10^{11}$
$M_2^3 \sin^3 i/(M_1 + M_2)^2 = (2\pi/T)^2 (r \sin i)^3/G$ (grams)	$(1.69 \pm 0.01) \times 10^{33}$

Fig. 6

We should point out that the value of the period T given in the table is the heliocentric value determined in January 1972. We have now determined the pulsation period for 14 times from December 1971 through March 1973. These data are shown in Figure 7 with corrections for all significant motions applied. The figure shows the heliocentric period versus time. Significant changes are clearly occurring but the picture is quite complex. From January 1972 to August 1972 the period decreased by 5-1/2 microseconds, from September 1972 to October 1972, the period increased by about 3 microseconds, and then from October 1972 to March 1973, the period decreased by 3-1/2 microseconds.

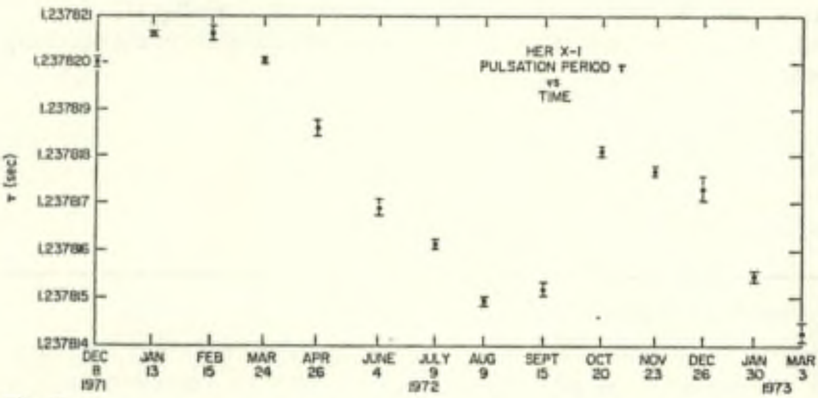


Fig. 7

The fact that the X-ray period shows a net decrease of 6 microseconds over 1-1/4 years rules out models, such as have been applied to radio pulsars and the Crab Nebula pulsar, in which a rotating neutron star slows down (period increases) at a rate approximately equal to that required to supply the energy emitted as radiation.

Since Hercules X-1 is a member of a binary system, we turn to accretion as the most likely source with the rotation of a neutron star still providing the clock mechanism. Although accretion models are extremely complicated, a few general statements may be appropriate. Material can be accreted radially if it has insufficient angular momentum for centrifugal forces to halt its free fall. However, for close binary systems accreting matter is likely to have so much angular momentum that it cannot fall directly onto the compact object. In this case, the matter will form a disk composed of material which gradually spirals inward and transfers its angular momentum to the compact star. The accretion problem has been studied by Zel'dovich and Novikov (ZY 71), Prendergast and Burbidge (PK 68), Schwartzman (SV 71), Pringle and Rees (PJ 72), Ostriker and Davidson (OJ 72), and Lamb, Pethick, and Pines (LF 72), Shakura and Sunyaev (SN 73). Most relevant to the X-ray results on the speeding up of Hercules X-1, both Pringle and Rees, and Lamb et al, predict that matter being accreted will transfer angular momentum to the star thereby speeding it up with the predicted changes being of the same order that we have observed for Hercules X-1. The models can explain the more complicated picture now presented on the change of the period with time on the basis of changes in the rates of matter transfer.

Distances determined from optical data imply an X-ray luminosity from 10^{36} to 10^{37} ergs/second although this could decrease by a factor of 10 by accounting for the limited solid angle that the pulsed source fills. The accretion process is capable of producing at least 10^{38} ergs/second and therefore is capable of producing the observed luminosity. As already described in the last table, the mass function of Hercules X-1 is 0.85 solar masses. One of the most important quantities to determine is the mass of the X-ray source itself which might yield the first direct measurement of the mass of a neutron star. The mass function derived from the Doppler shift of the X-ray pulsation period gives one equation involving M_1, M_2 and sini. There are several approaches that can be followed at this point. The simplest conceptually is to determine the velocity of the central star about the system center of mass from visible light and spectroscopic observations. Further the spectral typing of the central star allows its mass to be determined, in principle. These two additional results then determine with the X-ray data M_1, M_2 and i. Alternatively, the detailed shape of the optical eclipse can be used to determine the system parameters. These areas of effort are still the subject of much discussion and debate on the interpretation of the observed radial velocities, although preliminary results, such as those of Crampton and Hutchings, who obtain $1.4 \pm 0.4 M_\odot$ for the X-ray star, agree with the numbers obtained below. I understand that Bahcall will treat this subject in considerable detail.

Another technique that can be used to estimate the masses involved uses the X-ray data alone. One equation relates the size of the Roche lobe of the primary to the separation of the stars and their masses. A second expression relates the radius of the occulting object, the separation of the binary components, the inclination angle i, and the phase angle of the occultation duration, a measured quantity. Using these equations requires us to relate the radius of the occulting region to the size of the Roche lobe.

If we assume, as is likely, that $R=R_L$, we obtain a value of M_1 and M_2 for each possible i, and some results of such a calculation are tabulated in Figure 8. For example, at an inclination of 85°, assuming the radius of the occulting region is equal to the size of the Roche lobe, determines a mass of 1.99 solar masses for the central star and $1.04M_\odot$ for the X-ray source.

HERCULES X-1 MASS ESTIMATES — X-RAY DATA ONLY

Assumes $R_{\text{occulting}} = R_{\text{Roche lobe}}$

Inclination Angle	Mass X-Ray Star	Mass Central Star
90°	1.19	2.09
85°	1.04	1.99
80°	0.74	1.78
75°	0.45	1.56
60°	0.09	1.47
45°	0.03	2.46
30°	0.04	6.8

Fig. 8

The observations of Hercules X-1 show further structure in the X-ray intensity versus time. For 10 or 11 days the source is intense and pulsing and can be seen following the 1.7 day occultation cycle. Then for 24 days the source is too weak to be observed.

Figure 9 shows the detailed X-ray intensity data, corrected for aspect, observed during the high states in January, March and July, 1972. Each dot represents a single spin of the satellite across the source. Two types of error bars are shown in the figure; the statistical error bar is determined from counting statistics and is the appropriate error bar to apply locally when considering variability. The larger error bar is dominated by the aspect correction and must be applied when comparing data taken on different days since the satellite is normally maneuvered once per day.

In addition to the fluctuations observed within a day, there are other features that stand out in this figure. First, we see that the source turns on rather abruptly. The intensity then increases rapidly to a maximum level and stays near this level for several days. The intensity then decreases rather smoothly for several days until the source is not detectable. Recently, the X-ray experiment on Copernicus has detected X-ray emission from Hercules X-1 during a time in which it was predicted to be in the off part of the cycle. We first established the 35 day cycle from observations of 5 cycles from November 1971 until March 1972; 4 of which the source was monitored with coverage on 67 out of 96 days occurring between high states. On all of these 67 days, as well as a number of additional scattered days of observation, the source was not detected above background, thereby indicating the rarity of events, such as detected by Copernicus and possibly by the Livermore group in a May 1971 rocket flight.

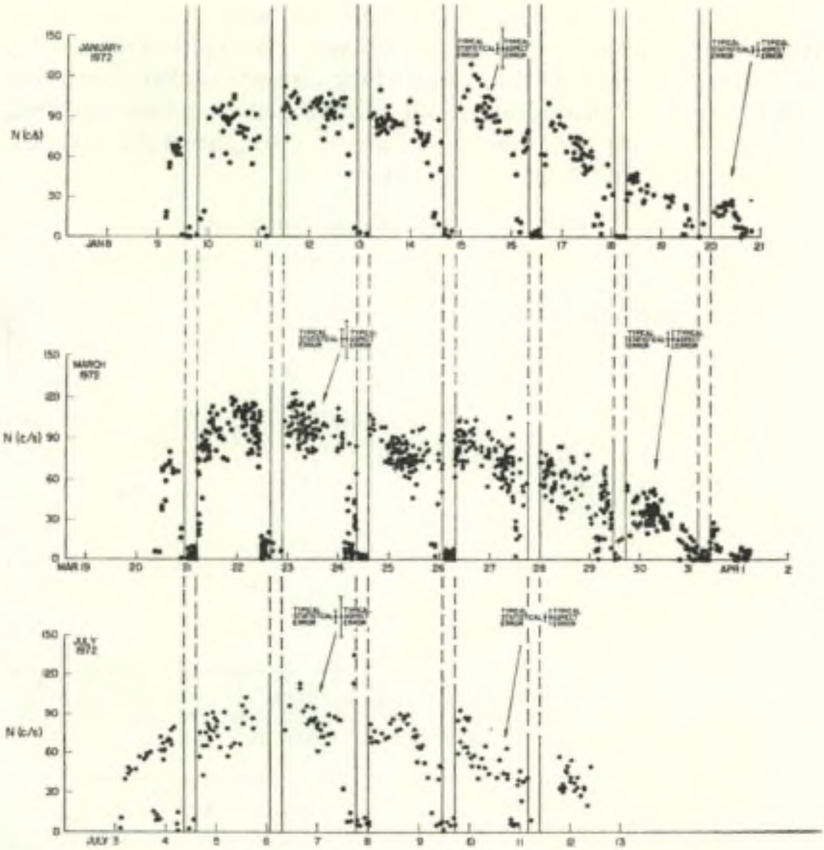


Fig. 9

In Figure 10 (top half) we have used the sharpness of the high state turn-on to look at the time of occurrence of the 35-day cycle. We have plotted the time of the turn-on minus an integer *34.85. The first several points lie on a straight line whose slope indicates the 35.7 day periodicity first reported. Other points lie on lines with slopes corresponding to 34.85 day and 34.0 day periods, while a few points are scattered off the lines. While the turn-on data definitely do not show a strict periodicity, we should note that all of the points are within one eclipse cycle of the average 34.85 day cycle chosen. Thus, while there appears to be an underlying clock, it just does not seem to keep time precisely except on the average.

This may be understood, perhaps, by considering the bottom half of Figure 10 where we have plotted the turn-on times as a function of 1.7 day orbital phase. We should note that the range of phase indicated in this plot, as well as the error bars in the upper half of the figure, represent absolute limits on the uncertainty in turn-on times. We see that the turn-ons cluster at two phases: around phases 0.2 and 0.7. An earlier analysis suggested that all of the turn-ons could be consistent with two very well defined times, but as more data have been analyzed, we find that while the turn-ons do cluster near phases 0.2 and 0.7, there are some that definitely do not overlap.

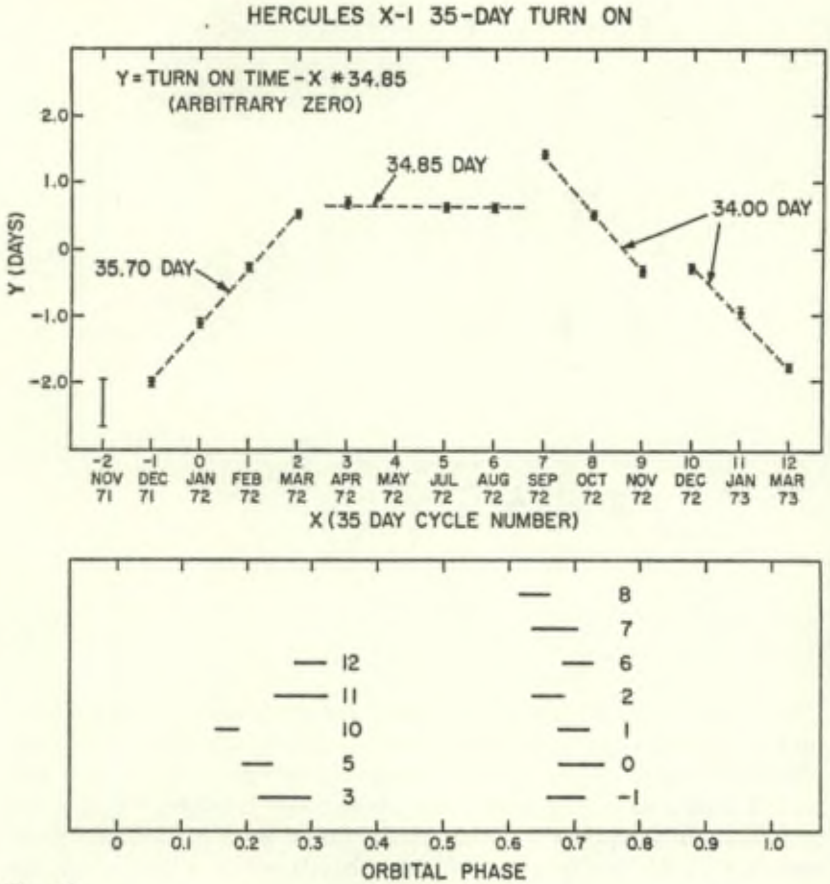


Fig. 10

The simplest model for the 35 day cycle is torque-free precession of an oblate spheroid neutron star first described by Brecher. If this is the case, then the 35 day cycle of Her X-1 is a reflection of conditions in the interior of the neutron star. For example, Pines, Pethick, Lamb and Shaham have shown that the model works only if the neutron star has a solid core. This model has been modified by Pines and co-workers to explain the sudden onset of the " on " state by a triggering effect of the accreting material piling up at the Alfven surface until the precession angle becomes such that accretion onto one or the other of the magnetic pole regions is no longer prevented. By assuming that the accumulated disk of matter accreted during the " off " state is thickest near the inner and outer Lagrange points, Pines and co-workers explain the turn-ons at phases near 0.25 and 0.75 as occurring at the locations at which the X-rays can most easily " burn " through the disk and first be seen. Further detailed work is, of course, necessary to fully evaluate this model, but it appears that further details of X-ray absorption by streaming gas, such as the dips described by Giacconi et al (1973a), and the complicated optical behavior can be explained.

We now turn our attention to Cen X-3. Figure 11 shows this source pulsing with a 4.8 second period. The lighter histogram is again the 2-6 keV intensity observed with UHURU in 0.096 second bins and the heavier curve is a fit to the data. Figure 12 shows that, as for Hercules X-1, this source undergoes regular occultations and that the 4.8 second pulse period is Doppler shifted in phase with the eclipses. One difference from Hercules X-1 is that Cen X-3 does not exhibit a regular 35-day cycle, but as is shown in Figure 13 has times when it is intense and pulsing, and other times when it is seen weakly (if at all), and still other times when it alternates erratically between a high state and a low state. The data in the figure cover the 2-1/2 years' time over which UHURU has operated. There are no clear cut regularities in the highs and lows - - no single period comes close to fitting all the observations. The data suggest that typical high states may last for around 4 months and typical low states for around 2 months; but exceptions are almost the rule.

CENTAURUS X-3 (2ASE 1119 -60)
May 7, 1971

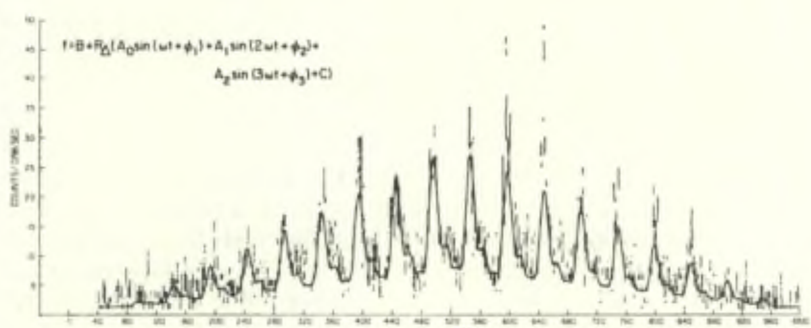


Fig. 11

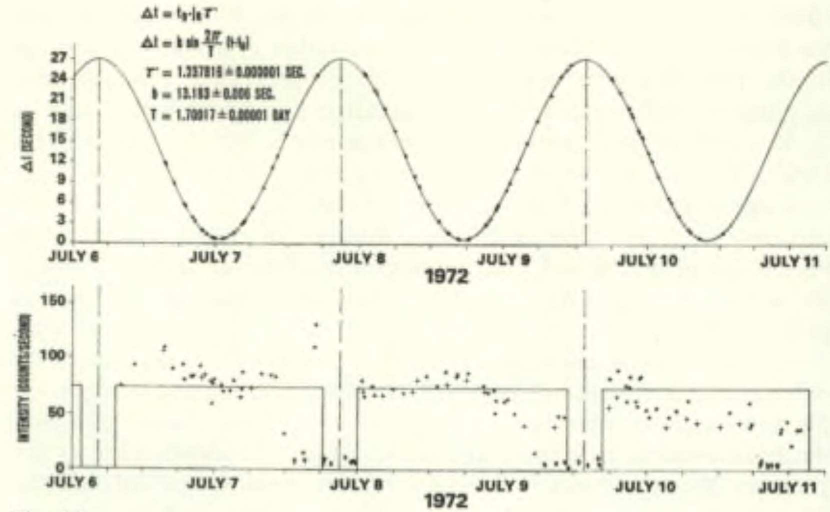


Fig. 12

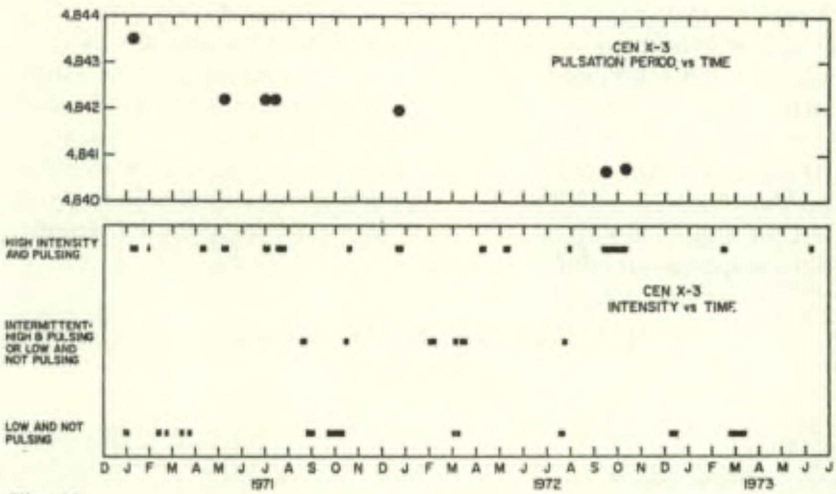


Fig. 13

In the top half of the figure, we have plotted several heliocentric 4.8 second periods observed for Cen X-3. The changes in period are so great that error bars are smaller than the dots representing the data points. The period decreased by 1.1 millisecond from January to May 1971, decreased by 0.2 milliseconds from May to July, decreased by 0.25 milliseconds from July to December 1971, decreased by 1.3 milliseconds from December 1971 to September 1972 and increased by about 50 microseconds from September to October 1972.

One additional piece of evidence in the Cen X-3 picture is shown in Figure 14 where we plot the two day orbital period of Cen X-3 versus time. The first few data points indicate that the orbital period is decreasing at a rate consistent with mass loss from the primary at a rate of $\sim 10^{-5}$ solar masses per year. However, additional data have shown that the orbital period is increasing with time. Liller has recently found a 14th magnitude optical counterpart for Cen X-3, although the identification requires small changes in the presently observed X-ray period to fit the data on old plates. Changes that are required are of the order of the changes observed in the X-ray orbital period so at present the identification seems likely to be correct.

The dissimilarities of their detailed behavior notwithstanding, Her X-1 and Cen X-3 seem to fit well in a model where accretion occurs on a magnetic, rotating neutron star of small mass 0.2 to 1.4$M_\odot$. The observational data regarding Cyg X-1 seem, however, to require a different model.

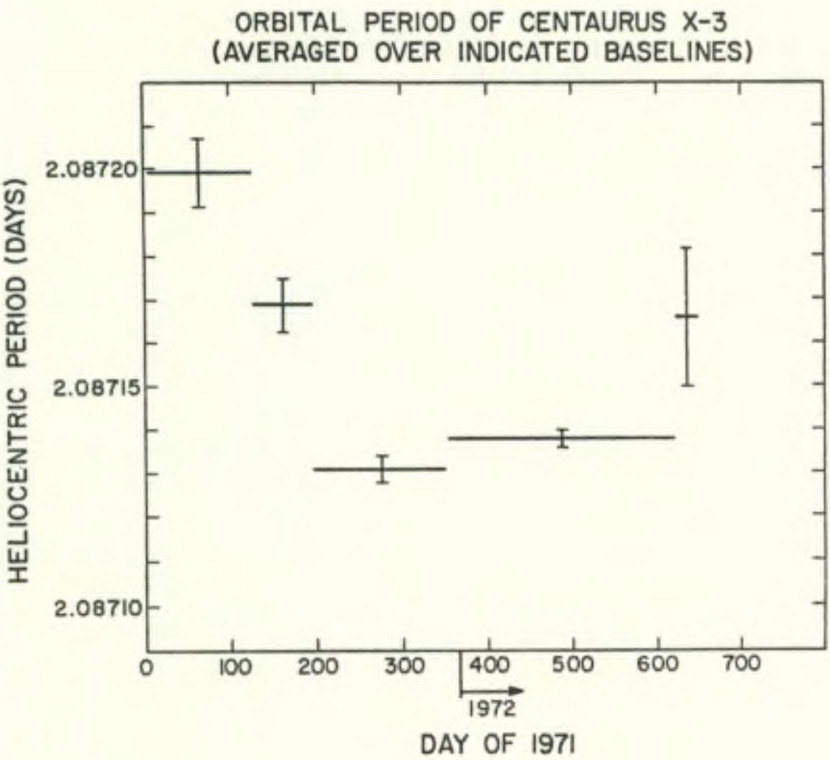


Fig. 14

CYG X-1

The discovery of short X-ray pulsations from Cyg X-1 has been certainly one of the most significant achievements of the UHURU orbiting observatory with regard to galactic X-ray sources. This discovery stimulated a wealth of X-ray observations on periodic and non-periodic pulsating X-ray sources which resulted in the discovery of Herc X-1, Cen X-3, and many of the binary sources we are presently studying. It also stimulated a concentrated effort in identifying the radio and optical counterparts of the object, whose detailed study has led to the conclusion that the Cyg X-1 system contains a black hole. Due to the importance of this finding and to the rapidly accumulating evidence strengthening this conclusion, I believe it is useful to review some of the experimental results and discuss some of the arguments involved in reaching it.

Cyg X-1 has been observed in some of the earliest surveys in X-ray astronomy. In 1966, a survey of the Cygnus region (GR 67a) resulted in the first accurate location determinations for Cyg X-1, Cyg X-2 and Cyg X-3. As a result of this survey, the optical candidate for Cyg X-2 was found. No candidate objects could be found for Cyg X-3 or Cyg X-1. The energy spectra of the Cygnus sources were also measured. The spectrum of Cygnus X-1 covering the range from 1 to 80 keV was measured by many observers from balloons and rockets. It appeared to have a flat power law spectral shape ($E^{-\alpha}$) with $\alpha = 0.7$, thus rather similar to the one found in the Crab Nebula. It was considered puzzling that while the source was not too different in intensity and spectral form from the Crab Nebula source, no evidence of radio emission could be found with a limit of 1/500 of Crab radio flux. Since at the time we only knew of 2 types of X-ray sources, Sco X-1-like and supernovas, this was considered evidence for a different type of X-ray emitter. How truly different was not revealed to us until December, 1971, when the UHURU X-ray observatory detected the existence of X-ray pulsations from Cyg X-1 (OM 71), Figure 15.

Figure 16 contains data already reported in the literature (Schreier et al, 1971) showing substantial variations in X-ray intensity on time scales from 100 milliseconds to 10's of seconds. Some 80 seconds of data are shown here summed on 4 time scales from 100 msec up to 14 sec. I should point out that similar X-ray variability also reported by scientists at MIT (Rappaport et al, 1971a), Goddard Space Flight Center (Holt et al, 1971) and NRL (Shulman et al, 1971) compels us to consider a source region of 10^9 cm or less. Figure 17 shows the X-ray location obtained from an MIT rocket flight and from UHURU which led to the discovery of a radio source by Braes and Miley (1971) and by Hjellming and Wade (1971). It is this precise radio location that led to the optical identification by Webster and Murdin (1972)

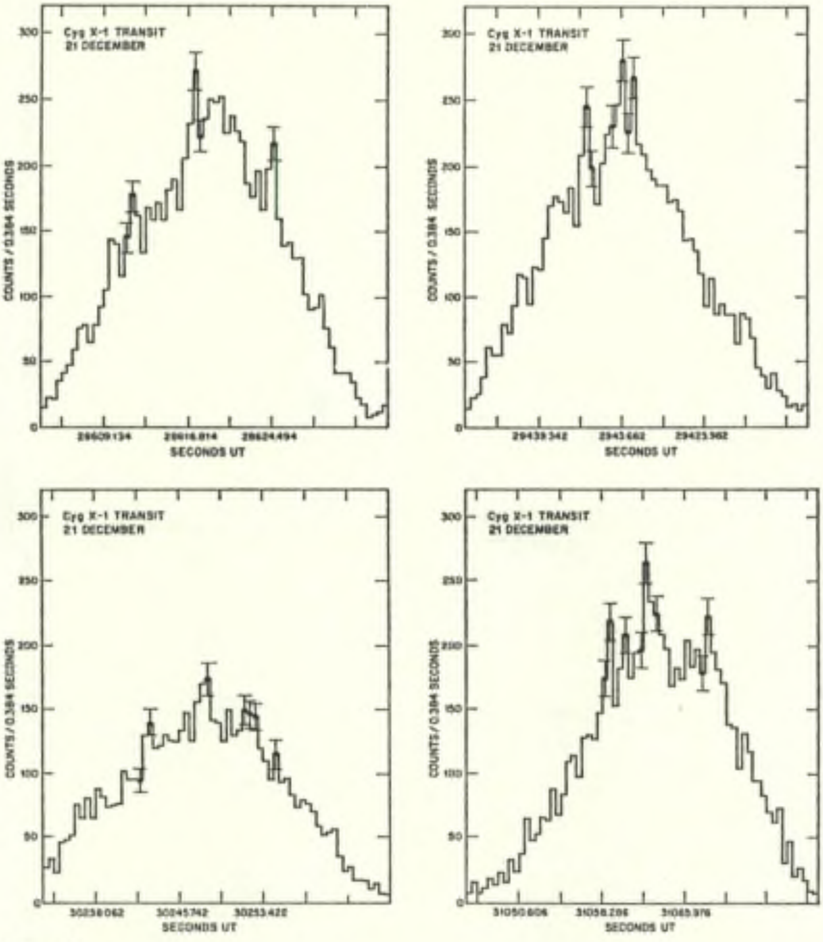

Cyg X-1 TRANSIT
21 DECEMBER
COUNTS / 0.384 SECONDS
SECONDS UT
Cyg X-1 TRANSIT
21 DECEMBER
COUNTS / 0.384 SECONDS
SECONDS UT
Cyg X-1 TRANSIT
21 DECEMBER
COUNTS / 0.384 SECONDS
SECONDS UT
Cyg X-1 TRANSIT
21 DECEMBER
COUNTS / 0.384 SECONDS
SECONDS UT

Fig. 15

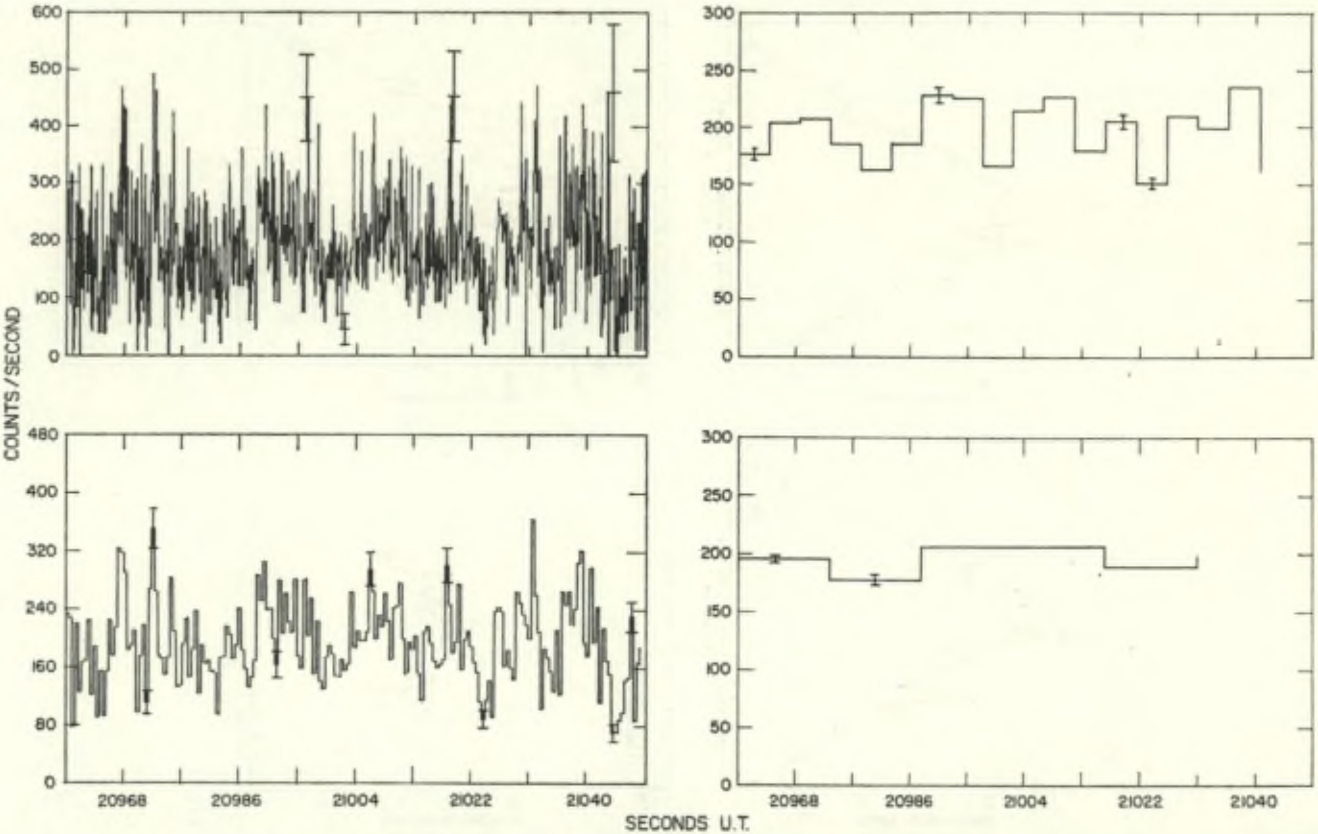

COUNTS/SECOND
SECONDS U.T.
600 500 400 300 200 100 0
480 400 320 240 160 80 0
300 250 200 150 100 50 0
20968 20986 21004 21022 21040

Fig. 16

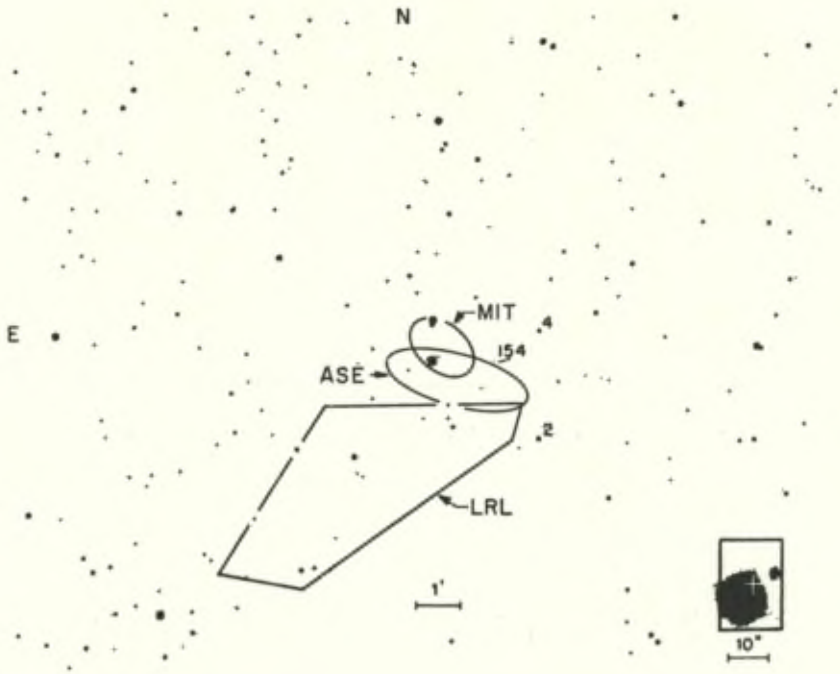


Fig. 17

and by Bolton (1972) of Cygnus X-1 with the 5.6 day spectroscopic binary system HDE 226868. The central object of this system is most likely a 9th magnitude B0 supergiant and conservative mass estimates for the primary lead to a mass in excess of several $M_\odot$ for the unseen companion. If the companion is the compact X-ray source, then it could be a black hole.

The conclusion that Cyg X-1 consists of a binary system in which one of the stars is a black hole is based then upon three main points:

1. HDE 226868 is the optical counterpart of Cyg X-1,
2. The mass of HDE 226868 $\geqslant 20M_\odot$, and
3. The X-ray emitting object is compact.

I will consider the three points in order.

Identification

The identification is based on first order of the positional coincidence between X-ray and optical object (1′), Figure 17. No evidence, however, could be found in the UHURU data at high X-ray energies (2-10 keV) of the binary nature of the system. Since the positional coincidence between radio and optical yields much greater accuracy (1″), we have attempted to establish a correlated behavior between radio and X-ray emission.

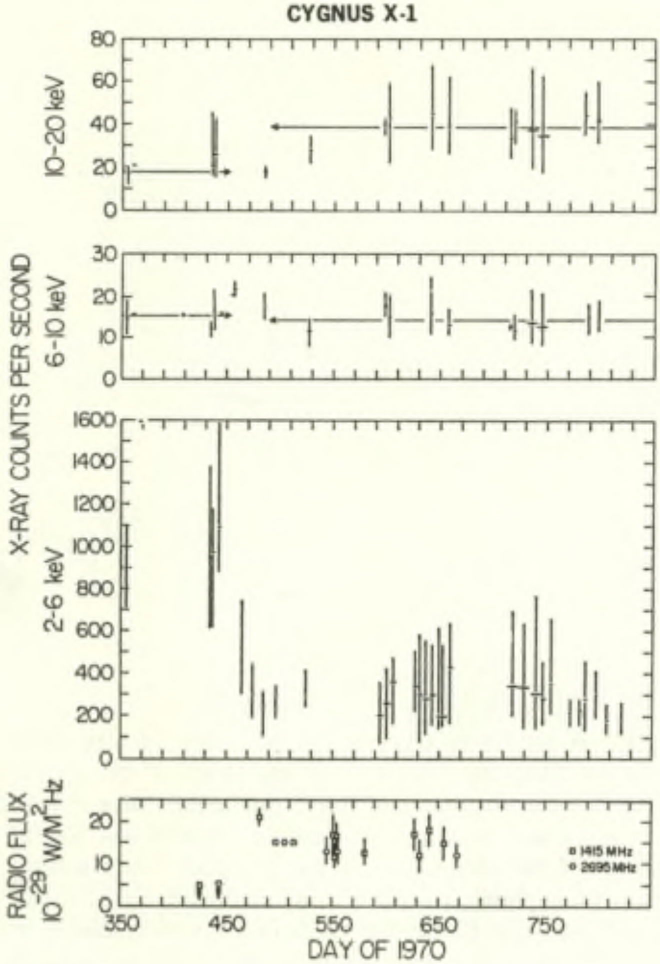


Fig. 18

With the use of UHURU as an observatory, we have analyzed 16 months of data on Cygnus X-1, which are shown in Figure 18. We have plotted the 2-6 keV intensity versus day of 1970. The vertical lines for a given day show the range of variability observed on that day. For some days we have only the average intensity shown by a dash available in our analyzed results. We see that a remarkable transition occurred in March and April, 1971, with the source changing its average 2-6 keV intensity level by a factor of 4. We have also indicated in the figure the 6-10 keV and 10-20 keV X-ray intensities and see that the average level of 10-20 keV flux increased by a factor of 2. The figure also shows that at the same time the X-ray intensity changed, a weak radio source appeared at the Cyg X-1 location and was detected by the Westerbork and NRAO groups. Hjellming has reported analysis of additional radio data which shows the radio source first appeared sometime between March 22 and March 31, essentially the time during which the 2-6 keV X-ray intensity first headed downward. This correlated X-ray radio behavior is strong evidence, in addition to the positional data, that Cyg X-1 is, in fact, identified with the optical and radio object.

In Figure 19 we see the Cygnus X-1 " average " spectrum before the March 1971 transition and after. The spectrum before the transition has a low energy excess which can be fit by either a power law with energy index of 4 or an exponential with a temperature of 11×10^6K. The disappearance of this low energy component at the same time the radio source appeared, could be related to decrease in the plasma density which reduced the X-ray emission measure and also reduced the plasma frequency or the free-free absorption of the radio emission.

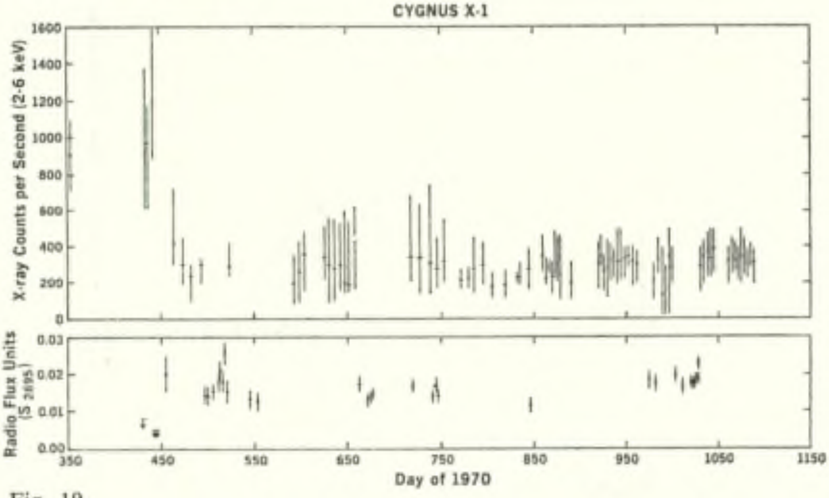


Fig. 19

We have extended the study to December 1972. We find no major change in the X-ray emission, such as observed in March 1970. However in connection with the increase in radio emission, reported by Hjellming, there is some tentative evidence for a decrease of X-ray emission in October 1972 (Figure 20). Additional evidence for the identification has been reported by Sanford (1973) on the basis of Copernicus satellite data. He reports that a decrease of soft X-ray emission from Cyg X-1 was detected at Phase 0 of the 5.6 period on several occasions. Such low energy X-ray behavior would prove conclusively the identification.

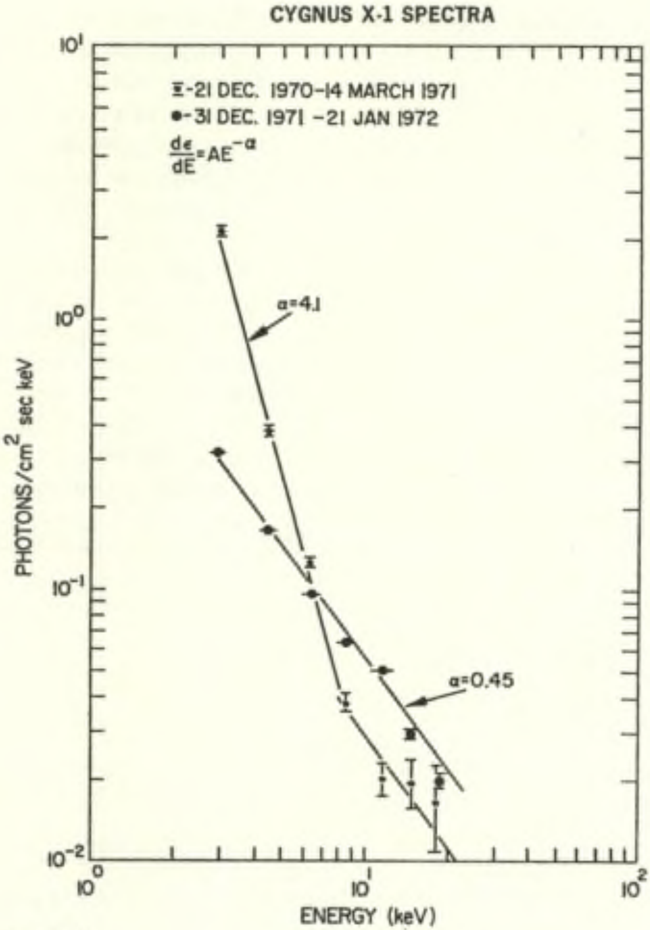


Fig. 20

Mass of HDE 226868

On the basis of the arguments given above, the HDE 226868 system is believed to be the optical counterpart of Cyg X-1. Bolton (1972) and Brucato and Kristian (1972) derived spectroscopically a mass function which leads to a minimum mass of about 5M⊙ for the X-ray emitting secondary star. This conclusion was based on the assumption that the primary star of HDE 226868 is a normal B0Iab supergiant of more than 20M⊙ according to the spectral characteristics given, for instance, by Wolburn (1973). Paczynski (1972) and Trimble et al (1973) criticized this assumption. Indeed a spectrum gives only information on the effective temperature and the surface gravity of a star. Trimble et al (1973) gave a model of a low mass star which is in full agreement with the observed spectral type. Paczynski pointed out that the strong X-ray flux from the secondary might alter the appearance of the observable spectrum of a star. Four recent papers have addressed these criticisms. One paper by Bolton (1973) used mass function as determined from the absorption line velocities, the HE II 4686 emission line velocities and the distance derived from interstellar reddening and the equivalent width and velocity of the interstellar K line, to determine values of $20M_{\odot}$ for the primary, $13M_{\odot}$ for the secondary, 2.2 kpc for the distance, and an inclination angle of 26°. Cherepashchuk, Lyuity, and Sunyaev (1972) used the absorption line velocities, the HE II emission line velocities with some allowance that the emission region may not belong to the X-ray star, but may lie between the two stars, and the photoelectric observations showing 0.07 magnitude changes due to a tidally distorted system. Taking into account limb darkening and gravity darkening, and assuming that the primary fills its Roche lobe, they determined a primary mass between 10.7 and $22M_{\odot}$, a secondary mass between 7.8 and $17M_{\odot}$, and a distance as large as 5 kpc. They appear to have neglected any interstellar absorption effects and have, therefore, overestimated the distance. H. Mauder (1973) used the absorption line velocities, the possible distances allowed by the observed reddening (and absence of a bright infrared source which could be produced by an absorbing circumstellar shell), the absence of any substantial reflection effects as demonstrated by the photoelectric observations, and the X-ray to visible light energy ratio, and the photoelectric observations. Assuming that the star cannot be any larger than its Roche lobe, he determines a self-consistent set of parameters that gives a distance of 2 kpc, a primary mass of 25 $M_{\odot}$, and a secondary mass between 6.0 and 7.3 $M_{\odot}$. Most importantly, Margon et al (1973) have recently determined the extinction of 50 stars in the field immediately surrounding Cyg X-1 and find $Av/d = (1.3 \pm 0.2)\ \text{kpc}^{-1}$. Since $Av = 3.3$ for HDE 226868, this leads to a distance estimate for Cyg X-1 of 2.5 ± 0.4 kpc, in agreement with the spectroscopic modulus for a B09 star. Kraft has reported at the recent IAU Symposium ※ 64 in Warsaw on a refined determination of distance which agrees with the above result.

In addition, the recent study of Sanduleak 160, reported by Liller and by Hiltner et al (1973), seems to me to lend much strength to these arguments. Sanduleak No. 160 (13.2 mag star) had been suggested as the optical counterpart of SMC X-1, the occulting binary X-ray source in the Small Magellanic Clouds. No doubt exists about the identification of this source since the period of the X-ray occultations and of the visible light emission coincide. The spectral type of the visible light star is reported as B01 according to Webster et al (1972). Liller (1972) has pointed out that if one derives a value for the absolute visual magnitude of the star following Keenan (1963), one finds -6.2 and -5.6 for the M_v of stars of spectral type B0Ib and B0II. This is in excellent agreement with the value of M_v derived using the distance modulus of SMC of 18.8 mag and the observed visual magnitude. The absolute value computed in this manner is of 6.0. Thus, for at least this case of SMC X-1, the presence of a strong X-ray source in the system has not altered the spectral appearance of the companion sufficiently to cause substantial errors in deriving the mass from spectroscopic data alone. It should be noted that in SMC X-1 the ratio between X-ray and visible light emission is considerably greater than in Cyg X-1. In fact, the heating effects of SK 160 by SMC X-1 are clearly seen in the light curves obtained by Petro et al (1973) for SK 160 while no such effect has been clearly established for Cyg X-1. Therefore, it appears that the effect of the X-ray flux on the primary is not important for the system containing SMC X-1 and should, therefore, be even less important for the system containing Cyg X-1.

It appears to me that the distance of Cyg X-1 has been clearly established to be greater than 2 kpc, that its luminosity and its mass are those consistent with a B0 supergiant and that, therefore, the value of a mass greater than $6M_\odot$ for the X-ray source has also been firmly established.

Compact Nature of the Source

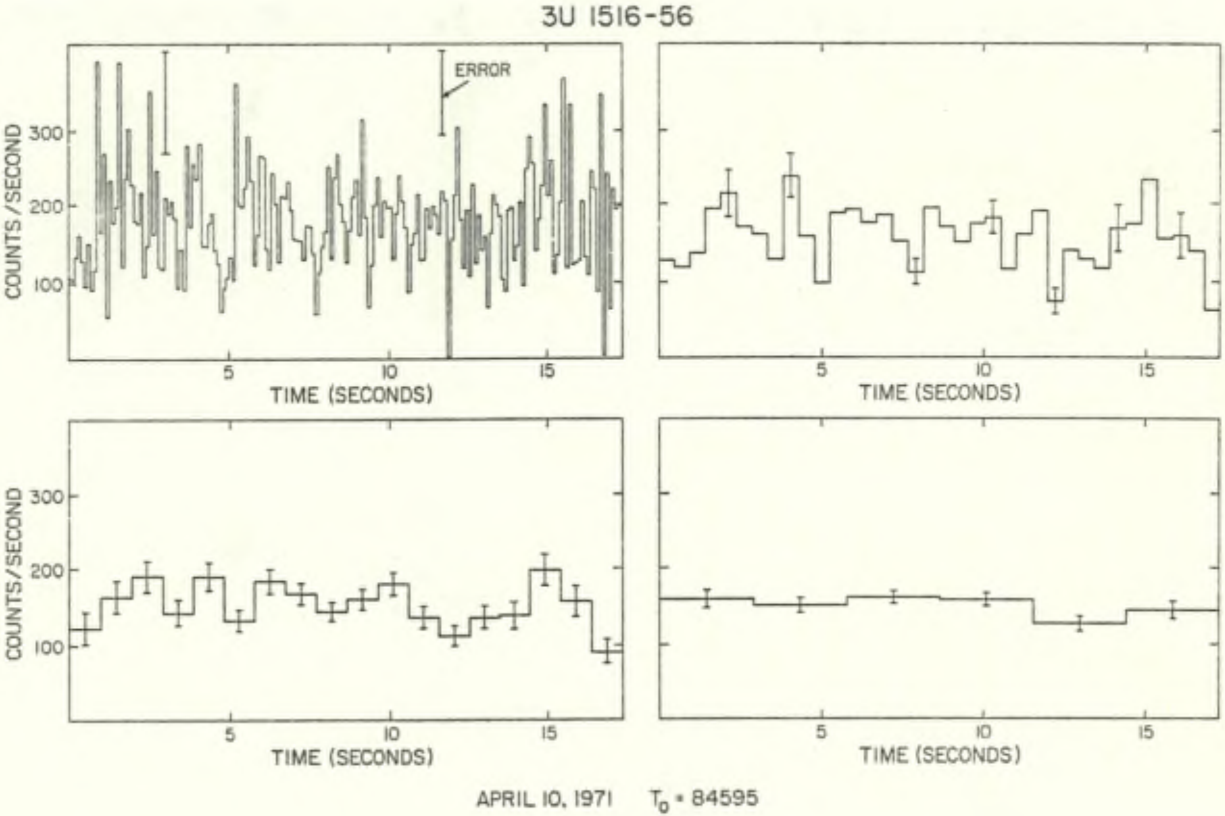
The rapidity of X-ray intensity variations leaves little doubt that the size of the X-ray emitting region is less than 10^9 cm. This can be used as an argument to conclude that the entire star on which the X-ray emission is taking place is compact. This argument depends only on the assumption of an accretion model, and the validity of the Eddington limit.

Additional evidence for compactness comes from the lack of any observable contribution to the visible light emitted by the system from the $6M_\odot$ star. For a main sequence star one could predict that approximately 3 % of the light from the system should arise from the object due to its intrinsic luminosity. A similar value would be due to reflection of the light from the primary.

No evidence is observed in the light curve for the presence of such a star and Bolton (1973) has placed a limit of 1 % of the total luminosity on its continuum contribution. A weaker upper limit on any line emission arising from the secondary is of order of 15 %.

In conclusion, there appears to be strong evidence that Cyg X-1 consists of a binary system containing a B0 supergiant of 20 or $30M_\odot$, and a compact source of $6M_\odot$ or greater. This is the strongest evidence to date for the existence of black holes. I would like to mention that

Fig. 21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200
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TIME (SECONDS)
APRIL 10, 1971 T_0 = 84595

perhaps even more convincing evidence for the existence of such systems will come from the discovery and study of other similar objects. Two, in particular, appear for different reasons as candidates for this search. The first is, of course, the SMC X-1 source. In this case, there appears to be no doubt about the identification and distance of the source and consequently about the mass of the primary. Petro et al point out, in a recent paper, that from an analysis of the optical photometry and X-ray eclipse duration, the mass ratio is found to be 0.28, $i = 79^{\circ}$, and $P = 3.^{d}89206$. On the basis of a mass of $20M_{\odot}$ for SK 160, the mass of SMC X-1 is $5.6M_{\odot}$. Confirmation of this result requires precise radial velocity measurements which unfortunately have not yet been made, although Webster has reported that he has been unable to observe any radial velocity shift in the lines. The source is too weak to allow determination of short time variability (0.1 - 10 keV). However, variability in the time scale of minutes to months is observed.

The source already mentioned previously, Circinus X-1, is potentially quite interesting from this point of view. It appears to exhibit the same rapid non-periodic pulsations as Cyg X-1 (Figure 21). Its position

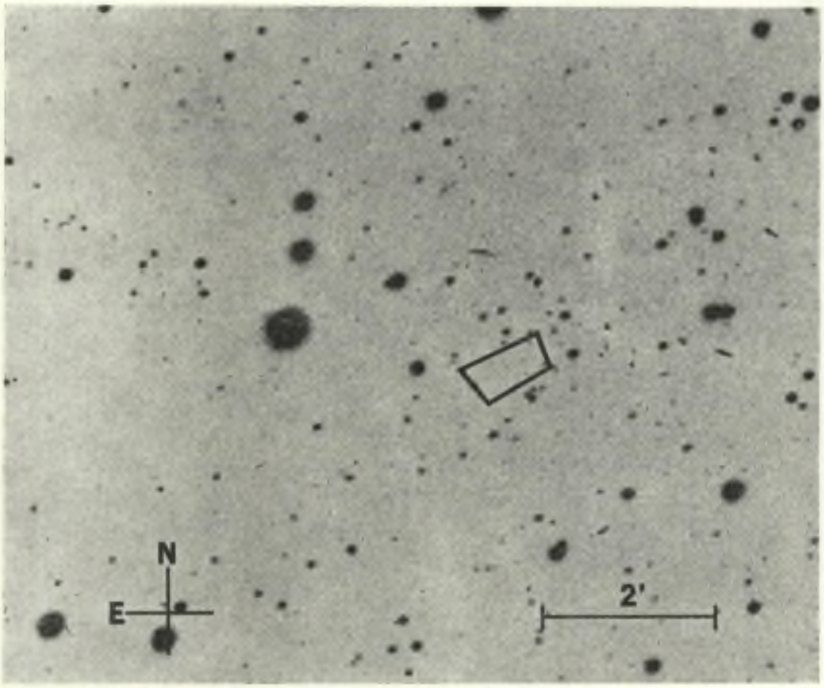


Fig. 22

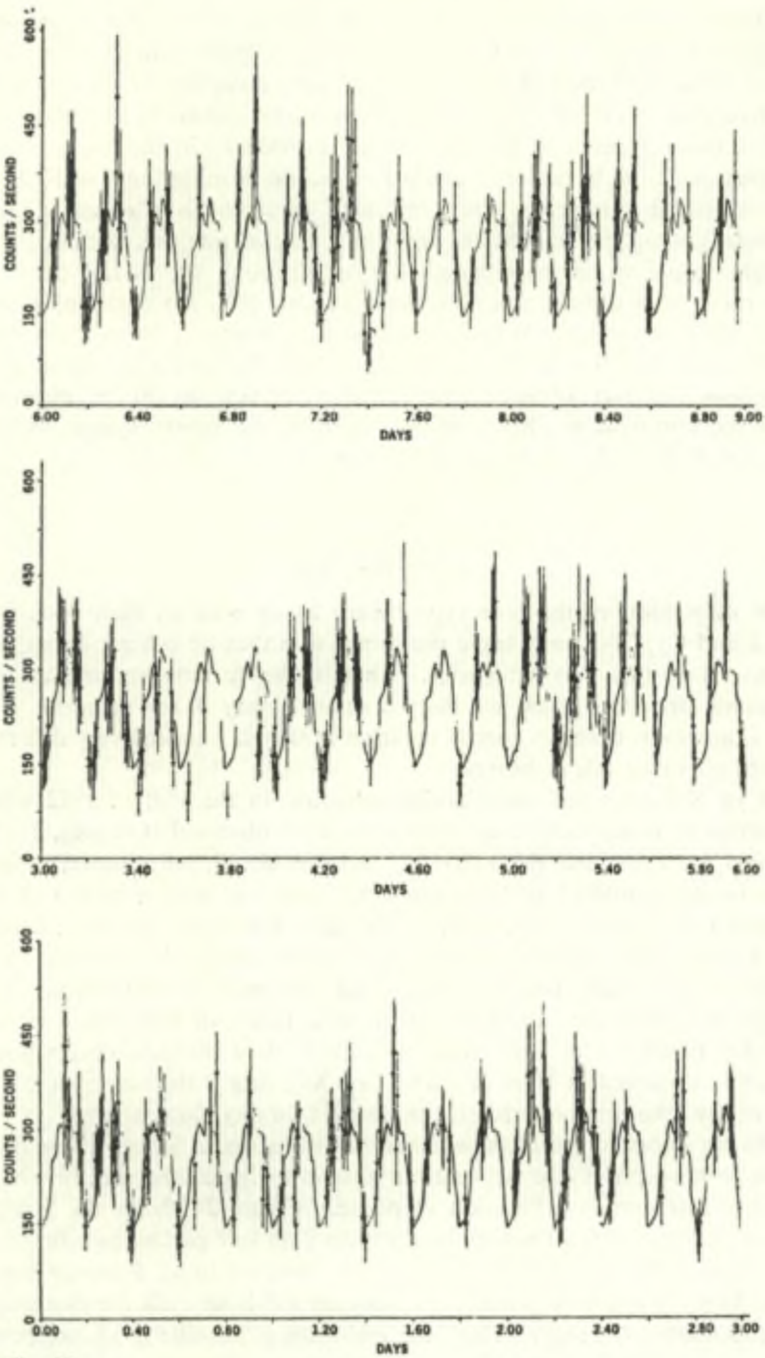
is quite well known (0.0002 sq. degrees) (Figure 22). No bright star appears to be present within the error box brighter than 15 mag. We have some indication of high and low states, however, which may have corresponding visible light variations and might aid in the identification. Also, there appears to be some tentative evidence in the X-rays for a binary occulting behavior. The period would be quite long, $\sim$ 12 days. If an optical counterpart could be found on the basis of either of these effects, among the stars in the error box, further analysis of this system might prove quite interesting, in that although the X-ray emission characteristics closely resemble those of Cyg X-1, the companion star appears to be quite dim and therefore of significantly lower mass than HDE 226868. Under these conditions, if the accreting object is a massive compact object, large radial velocities should be observed for the companion. Most of the mass of the binary system in this case would reside in the compact object.

CYGNUS X-3

The discussion of the prototype binary X-ray sources, Herc X-1, Cen X-3 and Cyg X-1, may leave the impression that all other binaries will neatly fall into two categories. This is clearly not the case as will become apparent when we discuss other binary X-ray sources. Cyg X-3, however, deserves special mention in that it is indeed very different from all other X-ray binaries.

Cyg X-3 attracted considerable attention in the Fall of 1972 when a series of exceptional radio outbursts were observed (Gregory, 1972). When we examined the UHURU data in detail, we discovered that the source exhibited periodic intensity variations with a period of 4.8 hours (Parsignault et al, 1972). The data for 9 days of observations of Cyg X-3 in May 1972 are shown in Figure 23 (2-6 keV range). The error bars contain both statistical and systematic uncertainties. The data show intensity variations of about a factor of two with a period of 4.8 hours. The light curve does not show definite occultations, such as observed in Herc X-1 and Cen X-3, and if the variation is due to eclipse, the orbital period is the shortest for any X-ray source.

In an effort to establish whether the variation in intensity could be due to absorption effects, we have now investigated the spectral shape of the emission as a function of phase. Figure 24 shows the 9 days' data in three different energy bands from 2-10 keV plotted as a function of phase. We see that the minimum is observed in all 3 energy bands and that the intensity variations across the 4.8 hour cycle are essentially independent of energy. Thus, the minimum is not due to photoelectric absorption which would show a strong energy dependence.


COUNTS / SECOND
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6.00
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6.80
7.20
7.60
8.00
8.40
8.80
9.00
DAYS
COUNTS / SECOND
0
150
300
450
600
3.00
3.40
3.80
4.20
4.60
5.00
5.40
5.80
6.00
DAYS
COUNTS / SECOND
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1.20
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2.80
3.00
DAYS

Fig. 23

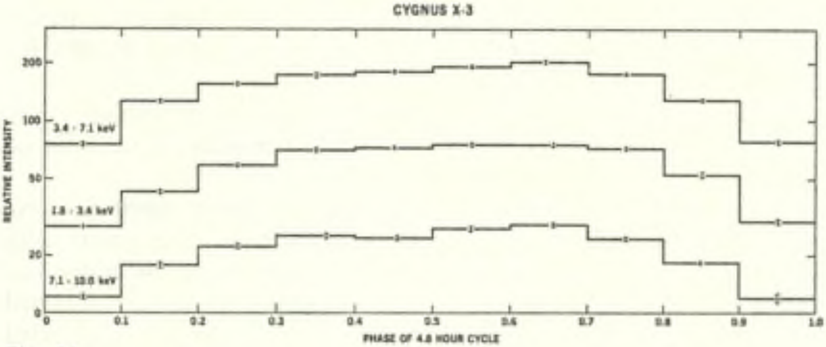


Fig. 24

Recently, observations of Cyg X-3 were carried out by Neugebauer in the infrared at 2.2 mu and correlated observations in the X-rays were carried out by the Copernicus Satellite (Becklin et al, 1973). These observations have led to the discovery of a synchronous 4.8 periodicity in the flux density of the infrared source. As pointed out by the authors, this uniquely associates the infrared source, the X-ray source and the radio object. They interpret the data as giving evidence that the variations in intensity are due to eclipses in a binary system.

We have found it difficult to reconcile the very shallow nature of the occultation, and the apparent coincidence between IR and X-ray emitting regions which, in the view of the authors, indicates an emission region of the order of 10^{11} cm, with fast transients in the X-ray emission which exhibits from time to time large changes in intensity on time scales of 0.1 seconds.

Alternative explanations, such as the possibility that the intensity variations could be due to energy independent Compton scattering in a hot, ionized cloud of different optical depths in different viewing direction, as suggested by Gursky, have been investigated by Tucker. The conclusion is that the emitting region ought to be quite compact (10^8 cm) in order for Compton Scattering to dominate with respect to photoelectric absorption. This seems in conflict with the view that the IR and X-ray emission originates from the same region as suggested by the IR observations, because it would require an unreasonably high effective temperature in the IR.

Other possibilities for the 4.8 hour cycle which, in my opinion, have been not sufficiently explored are the possibility that pulsations or rotation of a single star may give rise to the observed effects. In this connection, we have been examining our UHURU data for possible evidence of change in the 4.8 hour period. We find no compelling evidence for any variation. The average period between December 1970 and May 1972 is of 0.19967 ± 0.00004. It should be noted that these limits are still very coarse and more refined determinations of the period may still prove very helpful in choosing among various models for the 4.8 hour cycle.

If we now turn our attention to the variations of the average X-ray flux from Cyg X-3 and its possible relation to the observed radio flares, we find as follows:

Figure 25 shows several points over a 2 year period where we have determined the average intensity of Cygnus X-3. The points with the smallest error bars are data for which the average intensity was determined by the sine fitting technique. Other points are obtained as daily averages of randomly selected points or as averages of selected quick-look data points at various phases of the 4.8 hour cycle. The data suggest that Cyg X-3 may have average intensity levels which may persist for times of months with transitions between levels sometimes observed. Average intensity levels of 60 cts, 125 cts, and 230 cts are seen at various times. This picture is somewhat similar to that observed for Cygnus X-1 and suggests that the radio data be checked against the X-ray observations, if possible, for possible long term correlated changes in average intensity.

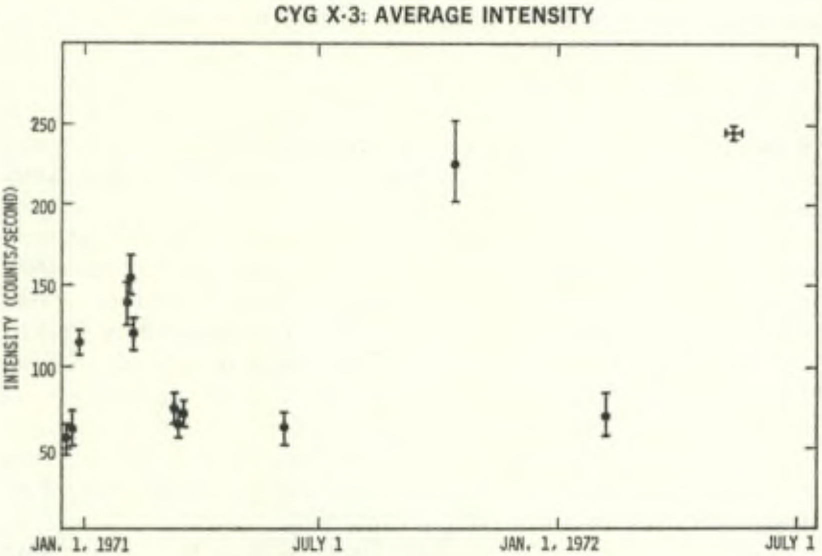


Fig. 25

Figure 26 shows new results for the X-ray intensity for 6 days at the time of the first September 1972 radio flare. We now include production data whereas our earlier report of no significant X-ray changes was based on quick-look data only. We see that there are several points of high intensity on September 1, the day before the radio flare was first reported, although the radio data allow for the possibility of an earlier startup since the flare was first observed already in progress. The presence of X-ray intensities of at least 600 cts/sec, an intensity greater than ever previously observed for Cygnus X-3, and at least a factor of 2 higher than any intensities measured on August 30 and 31, may indicate a connection between the X-ray and radio behavior. This view is supported by preliminary observations by Christine Jones Forman of steeper energy spectra for the high intensity data points, Figure 27. This observation thus confirms the X-ray - radio identification previously only suggested by positional coincidence of a few arc minutes and recently confirmed by the observation by Beklin et al (1973), of a 4.8 hour period in the infrared object identified with the Cyg X-3 radio source with the infrared minimum in phase with the X-ray minimum, as previously discussed.

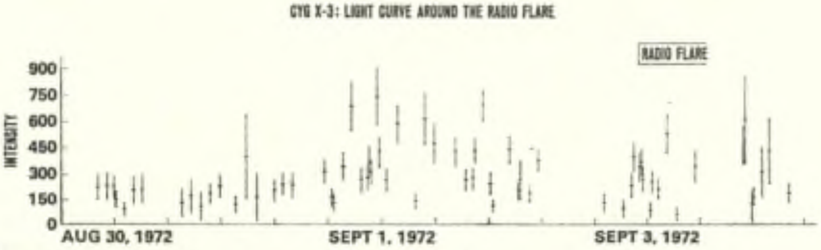


Fig. 26

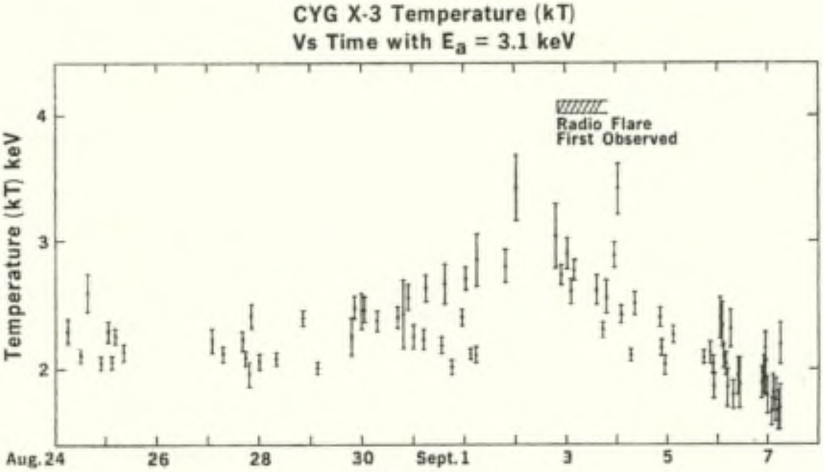


Fig. 27

Wallace Tucker and Harvey Tananbaum have performed some rough calculations to see what the parameters of a system might be in which 10^{38} ergs/sec is produced in a synchrotron process X-ray flare and 10^{33} ergs/sec is produced in a radio flare in the same event. The model assumes that the electrons are injected and first produce X-rays in a small region with an intense magnetic field and then expand into a larger region with smaller B and produce the radio emission. They assume an X-ray emitting region of order 10^9 cm and a radio emitting region of order 10^{15} cm. Assuming a radio lifetime of 10 days, they find, as already determined by Gregory and co-workers (1972), a magnetic field of $\lesssim 5$ gauss in the radio emitting region. Then they can determine a magnetic field of 5×10^9 gauss in the X-ray region, electron energies of ~ 10 MeV, and number densities of $\sim 5 \times 10^{-2}/\text{cm}^3$ for the radio emitting region and $\sim 5 \times 10^3/\text{cm}^3$ for the X-ray emitting region.

It is clear that much more work is required to explain the nature of this very unusual object. Even from the meagre data presently available, it appears, however, difficult to reconcile the Cyg X-3 observations with a Herc X-1 or Cyg X-1-like system.

OTHER BINARY SOURCES

The discovery of Herc X-1, Cyg X-1 and Cen X-3 raises naturally the question of whether most other galactic sources also can be associated with binaries.

At present we have some 6 X-ray sources we believe are associated with binaries on firm grounds. They are: Herc X-1, Cen X-3, Cyg X-1, 2U1700-37, 2U0900-40, and SMX-1. With the exception of Cyg X-1, all other sources exhibit an eclipsing behavior. Cyg X-3 was discussed separately.

The next several figures show some of the data on known eclipsing X-ray binaries. In particular, Figure 28 shows the data used to demonstrate that 2U1700-37 eclipses with a 3.4 day period. Data in the upper portion show seven days of data in May 1972 from which the approximate period was obtained. Data in the bottom half of the figure are all of the UHURU data from December 1970 to May 1972 folded with a 3.412 day period. Figure 29 shows the UHURU observations of 2U0900-40 folded with an 8.95 day period. The determination of this period was considerably complicated by the complex, erratic variability that the source undergoes. In particular, times of low intensity at phases other than eclipse are observed, although with no regular pattern. This type of behavior obviously greatly complicates the search for new eclipsing X-ray binaries. Figure 30 shows the

UHURU data on the X-ray source in the Small Magellanic Cloud obtained in January and June 1971, showing the source eclipses with a 3.9 day period.

Figure 31 summarizes much of the data on established X-ray binaries. The first column names the 6 sources; the second column gives the binary period (the period for Cygnus X-1 is deduced from the behavior of its optical counterpart), the third column describes the short term variability observed in all 6 sources - - (SMC X-1 is too weak to detect 0.1 to 1 second pulsations with UHURU), the fourth column names the optical candidate to which the candidate for Cen X-3 should now perhaps be added; the fifth column estimates the distance from the

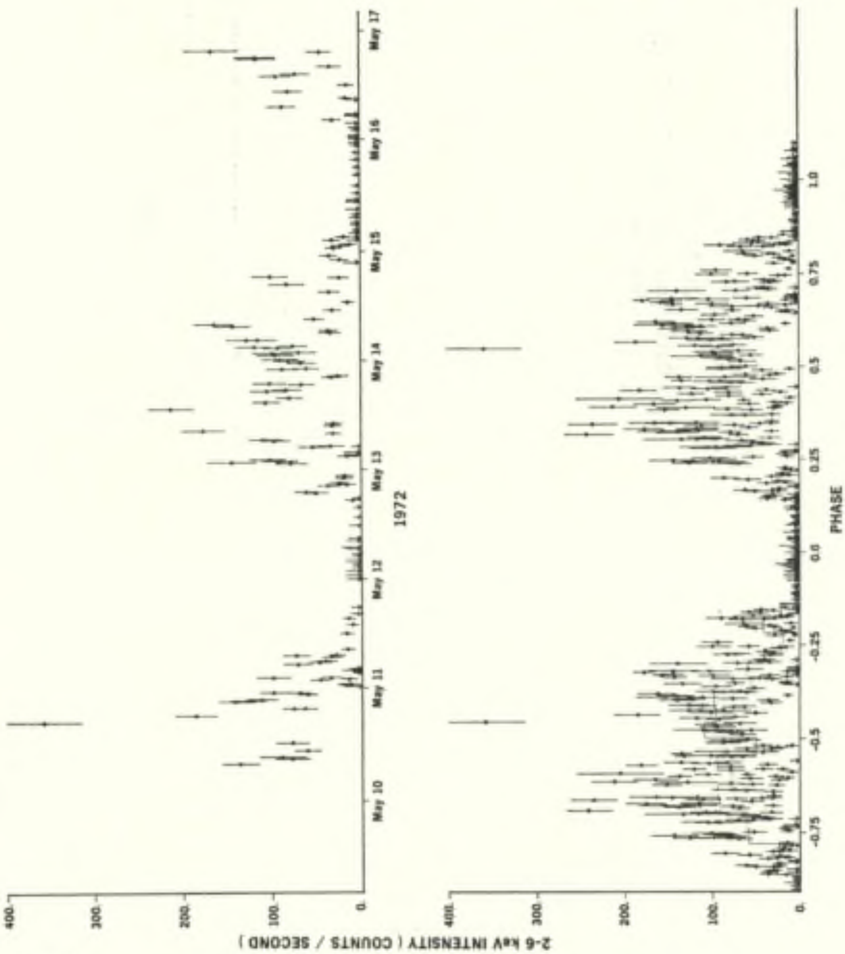


Fig. 28

Fig. 29

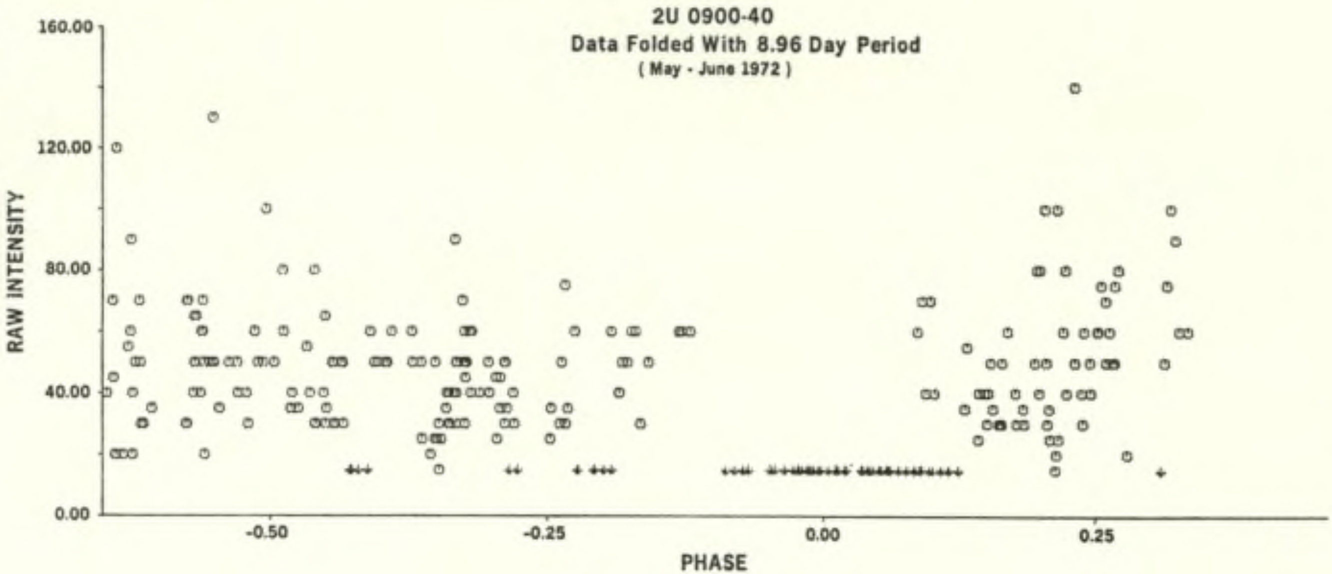
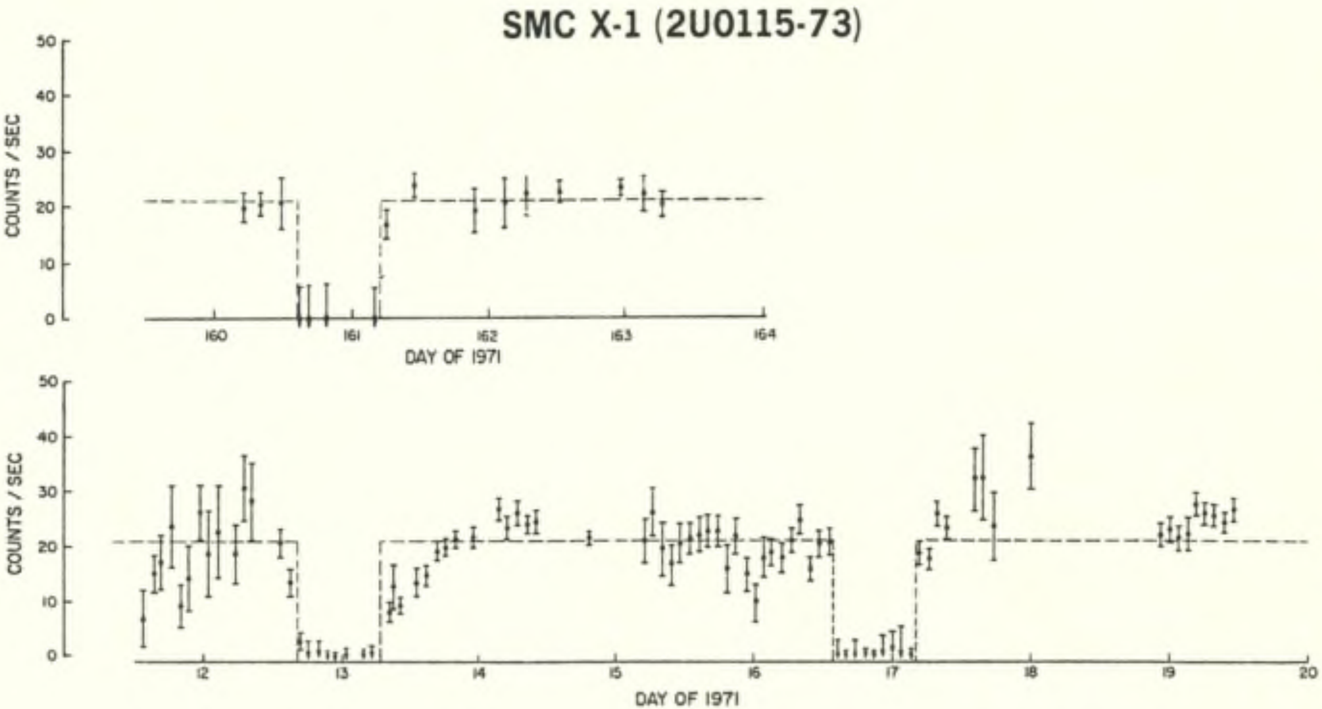

2U 0900-40
Data Folded With 8.96 Day Period
(May - June 1972)
RAW INTENSITY
160.00
120.00
80.00
40.00
0.00
-0.50
-0.25
0.00
0.25
PHASE

Fig. 30



CHARACTERISTICS OF X-RAY BINARIES

Source	Binary Period (Days)	Short Term Variability	Optical Candidate	Distance (kpc)	Peak Luminosity ($2 - 10$ keV ergs/sec)
Cyg X-1 (2U 1956+35)	5.600 ± 0.003 from optical. Not observed in X-ray.	Quasi-periodic pulsations as short as 50 millisec.	HDE 226868	2	1×10^{37} before transition. 3×10^{36} after transition.
Cen X-3 (2U 1119-60)	2.08712 ± 0.00004	4.842 sec pulsations	None	?	?
Her X-1 (2U 1702+35)	1.700167 ± 0.000006	1.23782 sec pulsations	HZ Her	5.8	1×10^{37}
2U 1700-37	3.412 ± 0.002	Non-periodic pulsations as short as 0.1 sec	HD 153919	1.7	3×10^{36}
2U 0900-40 (GX263+3)	8.96 ± 0.05	Non-periodic pulsations on times of secs	HD 77581	1.3	4×10^{36}
SMC X-1 (2U 115-73)	3.8927 ± 0.0010	Non-periodic pulsations on times of mins	Sanduleak 160	61	3×10^{38}

Fig. 31

optical data; and the last column gives the peak observed X-ray luminosity from 2-10 keV (the luminosities range from 10^{36} to 10^{38} ergs/sec), close to the Eddington limit at which radiation pressure should limit mass flow and thereby luminosity.

Typical counting spectra for these 6 binary sources are shown in Figure 32. Notice the deficit in counts for 5 of the 6 sources in the lowest energy channels. The only source without a substantial low energy cutoff is the only one which does not eclipse -- Cygnus X-1. We interpret this as being caused by the presence of much material in the orbital plane of the mass transfer binary systems. Systems which eclipse are observed at inclinations near the orbital plane, where much of the material of the system presumably lies, thus giving rise to the observed absorption. Cygnus X-1 is presumed to be observed from above the orbital plane and hence little obscuring material is along the line of sight. The fact that intensity variation appear to be a prevalent characteristic of the X-ray sources in binary systems makes it

BINARY SOURCE SPECTRA

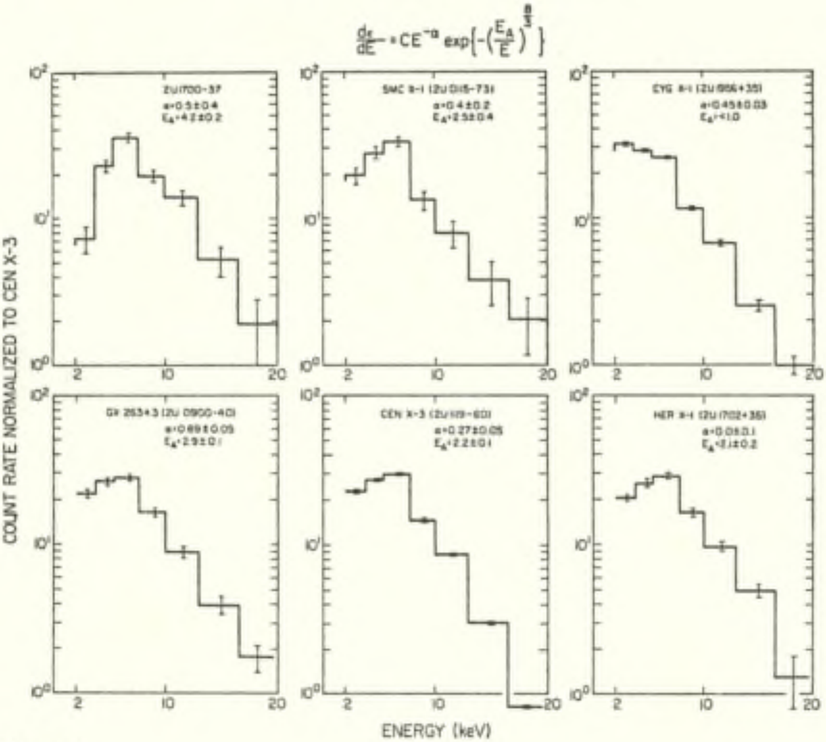
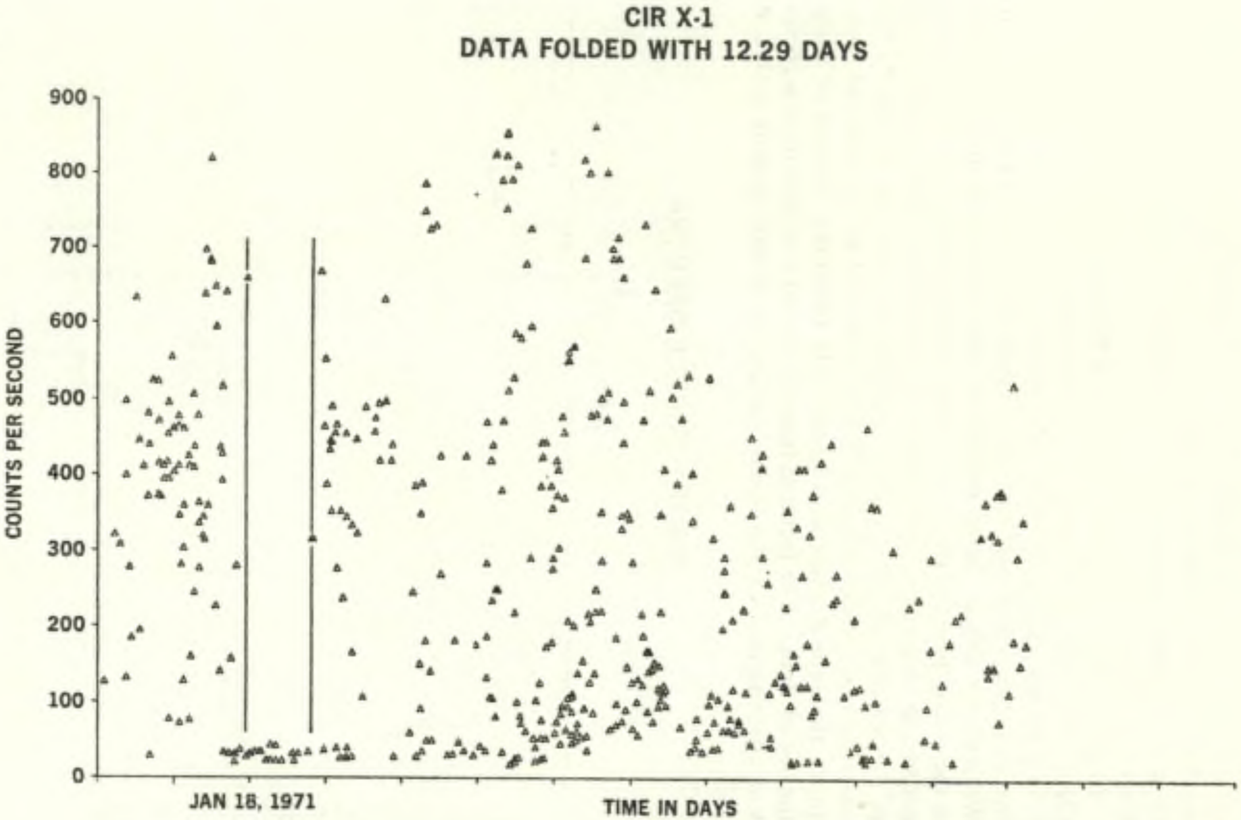


Fig. 32



CIR X-1
DATA FOLDED WITH 12.29 DAYS
COUNTS PER SECOND
900
800
700
600
500
400
300
200
100
0
JAN 18, 1971
TIME IN DAYS

Fig. 33

very difficult to establish the existence of eclipsing behavior. It is also clear that only a few of the sources even if they are in binary systems will have appropriate orbital inclinations to allow us to observe eclipses.

We have, therefore attempted to establish other criteria to aid us in determining how many of the strong galactic sources could be binaries.

The criteria of large intensity variations and substantial low energy cutoffs were used in selecting the 9 candidates for first study as possible new eclipsing binaries. While no new eclipsing binaries have been observed definitely, the data in Figure 33 indicate the situation with Circinus X-1 (3U1516-56). The data on this source were examined for possible periodicities less than 15 days and a period of 12.29 days was obtained. However, due to the erratic behavior of the source and the fact that it has not been possible to observe the source continuously for several periods, we regard this result as tentative. Figure 33 shows the data folded with a 12.3 day period and we see there is about 3/4 of a day with no high intensity sightings. As we earlier mentioned with 2U0900-40 there are many additional low points, as well as a large scatter, showing the extreme variability of this source. All of this contributes to the difficulty in obtaining a period and thereby confirmation as a binary. Given the very small location error box for this source, and its intrinsic interest, we hope that optical astronomers may succeed in finding an optical counterpart.

In view of the limited success of this approach, we have begun a systematic search for short and long term variability, adopting the point of view that compactness is a necessary, if not sufficient, criterion for establishing the binary nature of the sources. Although this work is just beginning, I would like to report some preliminary results that have been obtained by our group and primarily by Christine Forman Jones.

Long Term Variability

In the first figure (Figure 34) a compilation of the day-to-day variability of the strongest among the known 161 X-ray sources that comprise the 3U Catalog is given (Giacconi, et al, 1973). We have divided the sources into two intensity classes, the 44 sources greater than 35 counts/second and the 10 sources between 20 and 35 counts/second (sources fainter than 20 counts/second have insufficient counting statistics to be considered for this purpose). Before proceeding to examine this sample for variability, we eliminate known extragalactic sources (NGC 1275 and Virgo Cluster), known supernova remnants (the Crab and Cas A), and the known extended source at our Galactic Center. This leaves 40 and 9 sources in the two categories, respectively. Of the 40 sources brighter than 35 counts/second, 36 are found to vary in average intensity on different days (1 day to several months apart) in the 3U Catalog. Additional study has shown that 3U1636-53 and 3U1728-24 also vary in intensity, leaving only 3U1735-44 (for which we have parts of 4 days of data) and 3U1822-00 at 37 counts/second as not variable. Based on past experience, 3U1735-44 might be found to vary as more data become available, although another possibility is that this source could correspond to a supernova remnant with weak, extended, non-thermal radio emission yet undetected. As for 3U1822-00 a factor of two variability could easily have gone undetected given the source intensity of 37 counts/second, as is also suggested by the absence of detected variability in 6 of the 9 sources with intensity between 20 and 35 counts/sec. This is apparently the boundary at which the UHURU instrument becomes limited in sensitivity. Thus, our data are consistent with the view that all sources exhibit large variations of intensity, on the scale of days. These results have been obtained by comparing the average intensity of the sources from one sighting to the next.

DAY-TO-DAY VARIABILITY OF 3U SOURCES

161 SOURCES IN 3U CATALOG

	$I > 35$ counts/sec	$35 \geq I \geq 20$
Total sources	44	10
Known extragalactic	1 (NGC 1275)	1 (Virgo)
Known supernova	2 (Crab, Cas A)	
Known extended	1 (Gal. Center)	
Total remaining	40	9
Vary in 3U catalog	36	3
Vary from further study	2	
Not presently known to vary	2 (3U 1735-44 $\langle I \rangle = 210$ 3U 1822-00 $\langle I \rangle = 27$)	6
Known binaries	6*	1
Candidates under study as possible eclipsing binaries	9	

Fig. 34

* Includes Cyg X-1 which does not eclipse in X-rays and Cyg X-3.

Short Term Variability

We can now examine each individual sighting in order to study the short time scale variability (0.1 to 1 sec) of the more intense galactic sources. Figure 35 is a table summarizing the status of this survey.

SHORT* TIME SCALE VARIABILITY OF 3U SOURCES

PERIODIC VARIATIONS

1. Cen X-3 (3U 1118-60)
2. Her X-1 (3U 1653+35)

PREVIOUSLY REPORTED AS PULSATING

3. Cyg X-1 (3U 1956+35)
4. 3U 1700-37
5. Circ X-1 (3U 1516-56)
6. GX263+3 (3U 0900-40)

NEWLY FOUND TO PULSATE

7. 3U 1636-53
8. GX9+1 (3U 1758-20)
9. GX349+2 (3U 1702-36)
10. Serpens X-1 (3U 1837+04)
11. 3U 1820-30
12. GX3+1 (3U 1744-26)
13. GX5-1 (3U 1758-25)
14. GX340+0 (3U 1652-45)

PROBABLY PULSATE

15. Cygnus X-3 (3U 2030+40)
16. GX13+1 (3U 1811-17)
17. Cygnus X-2 (3U 2142+38)
18. 3U 1658-48

NO EVIDENCE FOR LARGE PULSATIONS

19. GX17+2 (3U 1813-14)
20. GX9+9 (3U 1728-16)
21. 3U 1630-47
22. 3U 1705-44
23. SCO X-1 (3U1617-15) - shows ~ 5 % variations in 1 SEC

STUDIED BUT TOO WEAK TO DRAW CONCLUSIONS

24. 3U 1702-42
25. 3U 1822-00
26. 3U 1727-33
27. 3U 0115+63
28. SMC X-1 (3U 0115-73)

Fig. 35

* Short — → typically less than 1 second and often on times of 0.1 second.

The table starts with the 2 periodically pulsing sources - - Cen X-3 and Her X-1, and 4 sources already reported by us in the literature as pulsating but with no evidence for regular, persistent periodicity. Then, we list 8 sources newly found to pulsate and 4 more which probably pulsate. The results are obtained by means of a χ^2 analysis of the intensity fit to individual passes of durations from 2 to 100 seconds and refer to non-periodic variations. Not all passes exhibit a statistically significant χ^2, but for the sources numbered 7 to 14 there are sufficient passes with large enough χ^2 to conclude the sources do pulsate (at least some of the time). For sources 15-18, not enough data have yet been analyzed, but preliminary indications strongly suggest the existence of pulsations. In the case of Cyg X-2, variations of at least 25 % in less than 1 second are observed on one occasion. For the other sources, 7 to 18 in the table, variability is often observed on times of 0.1 seconds - - the best time resolution available with UHURU and with amplitudes that are consistent with 50 % changes in intensity. What we have not yet done is determine quantitatively the characteristic time and amplitude of the variability - - if such categorization should prove significant - - or the fraction of the time that the source is active.

Sources 19-23 in the table were studied in a number of passes and have not yet demonstrated large amplitude, short time scale variations, although all have been observed to vary on times of days or less.

The last 5 objects in the table are sources whose intensity is too low to allow us to draw any conclusions from our analysis. Although this survey has not yet been completed and we cannot yet give a quantitative description of the characteristic time scales and amplitude of the variation, yet the above results allow us to conclude that essentially all of the strong galactic X-ray sources with the exception of supernova remnants are variable and many of them appear to vary on times less than 1 second, indicating source regions of 10^{10} cm or less.

It may well be that the variety of X-ray behavior we observe is caused by combinations of the various parameters which a binary system with a compact secondary may exhibit, starting with the nature of the secondary, its magnetic field, the mass transfer rate, and the inclination angle to the observer, to name a few. We hope that the survey for eclipsing X-ray sources, the study of the X-ray light curves of the 40 or so brightest sources, the search for short time scale variability, and the searches for optical counterparts presently underway, will contribute to our understanding of the physical processes taking place at the source and perhaps allow us to answer, in at least a statistical sense, the question of the binary nature of all of the galactic X-ray sources and to begin to understand the complicated variety of behavior exhibited. In the near future with the advent of orbiting X-ray telescopes, such as the one planned for the HEAO-B mission of NASA, it will become possible to extend the study of sources of 10^{36} - 10^{37} ergs/sec intrinsic luminosity to 30 Mpc. It is our hope that this will greatly expand the number of sources we will be able to investigate in detail, and make it possible to give a substantial contribution to our understanding of the properties of objects near the end point of stellar evolution.

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Discussion of the report of R. Giacconi

J. Bahcall: Could you say how the Japanese group found the changing periodicities in Cyg X-1 and whether you believe it?
R. Giacconi: The Japanese group at Tokyo including M. Oda and others have been investigating the existence of possible regularities in the changes of the period of the trains of pulses we observe in Cyg X-1. They appear to find that some specific frequencies (about a few Hertz) are observed more often then others. The certainty of the results is dependent on the interpretation of the Fourier analysis which is notoriously difficult.
Y. Ne'eman: Could you describe the source in the smaller Magellanic Cloud? It must have a very large X-ray luminosity?
R. Giacconi: The source SMC - X1 appears to have an intrinsic luminosity of $L_X = 10^{39}$ ergs/sec in the energy region from 1 to 100 keV.

OPTICAL PROPERTIES OF BINARY X-RAY SOURCES*

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and

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I. INTRODUCTION

We review the information that has been obtained from optical observations of the X-ray sources that are known to be in binary systems. We refrain as much as possible from discussing theoretical ideas and papers since these will be covered in other talks at this conference, and concentrate instead on summarizing the observations that have the greatest theoretical significance. The probable identification of the optical counterpart of Cyg X-1 occurred about two years ago, and the first certain identification of an optical counterpart (HZ Herculis) of a binary X-ray source occurred only about one year ago, so the subject is still in its infancy. At the time of the writing of this article (July 1973), there are six X-ray sources that are believed or known to have optically-identified counterparts in the same binary system. In addition there are many other galactic, variable X-ray sources (most of them recently discovered with the aid of the UHURU satellite) for which optical counterparts have not yet been identified; presumably many of these X-ray sources are also in binary systems. Thus the subject we are reviewing may very well exhibit further rapid growth in the future. We are grateful to many friends and colleagues for supplying us with preprints and preliminary accounts of their work; without their active collaboration this report could not have been written with any pretense of completeness.

Some of the principal characteristics of the optical counterparts are given in Table I. Detailed summaries of what is known regarding each

*Work supported in part by NSF contract 32394 with Princeton University and NSF contract with the Institute for Advanced Study.

system are given in sections IIA-F. In section III we make some remarks about the accuracy of mass determinations for X-ray sources. This problem is of crucial importance in helping to decide if black holes have been definitely observed (the most promising candidate at this writing is Cyg X-1) and in determining the mass of a neutron star. Our cautionary remarks are intended to remind both observers and theorists of the present uncertainties with the hope that the situation may be improved by further joint efforts*.

II. INDIVIDUAL SYSTEMS

IIA. HZ HERCULIS/HER X-1

A. X-Ray Properties

The pulsating X-ray source Hercules X-1 was shown by Tananbaum et. al. (1972) to be a member of an occulting binary system and has since been studied in detail by many workers [see Clark et. al. 1972; Ulmer et. al. 1972, 1973; Doxsey et. al. 1973; for the most complete description of X-ray properties see Giacconi et. al. 1973 and Tananbaum et. al. 1972]. There is now a wealth of detailed X-ray information on this system. We only recall the fact that there exist in the X-rays three distinct periods: (1). an X-ray pulsation period of 1.2^{sec}; (2). an orbital period of 1.7^{d}; and (3) a 35-day " on-off " cycle [cf. also Fabian et. al. 1973]. Most authors have interpreted the 1.2^{sec} periodicity as the rotational period of a neutron star (although Cameron, Brecher and Morrison, and Lamb and Van Horn have proposed rapidly rotating or pulsing white dwarfs). [There are no outstanding peaks in the X-ray power spectrum between 0.2 Hz and 0.8 Hz, see Avni et. al. 1974].

* It has become conventional in discussions of X-ray binaries to distinguish between regularly pulsing X-ray sources (Her X-1 and Cen X-3, commonly believed to be rotating neutron stars) and sources for which no regular pulsations have been observed (Vela X-1, 3U 1700-37, and Cyg X-1, all frequently proposed as candidates for black holes). This distinction is useful in discussing the observational situation, and we adopt it here, but the distinction does not *necessarily* refer to different kinds of objects. We mention as a possibility, consistent with the observations discussed in the following, that all or most of the X-ray sources in binary systems emit regularly pulsed X-rays which are largely confined to the plane of the binary orbit. This possibility, which may merit detailed investigation, requires that the beam angle of the pulsed X-rays be small (e.g., one half or one-third of the angle subtended by the optical star at the X-ray source, i.e., a pulsed beam angle $\lesssim 15^{\circ}$). The observed difference between pulsing and non-pulsing sources is attributed to different inclination angles, i, on this hypothesis.

B. Optical Identification

The optical identification of HZ Herculis was established by Bahcall and Bahcall (1972a, b), who showed that the period and phase of the optical star are the same as for Her X-1. Their work followed an improved X-ray position measurement by Clark et. al. (1972) and a suggestion by Liller (1972) [see also Forman et. al. 1972] that HZ Herculis was an interesting candidate for further optical study.

C. Optical Properties

The optical properties of HZ Herculis have been studied by many different groups [for two rather complete discussions see N. Bahcall (1973) and Bahcall, Joss, and Avni (1974)]. We shall only summarize the most important results.

The average light curve is well described [Bahcall and Bahcall 1972b; Petro and Hiltner 1973] by a simple cosine law: observed flux $= C_1 + C_2 \times \cos(2\pi \times \text{orbital phase})$, although there are statistically significant fluctuations of the order of $\pm\ 0.25^{m}$ that are most pronounced near maximum light [see, e.g., Bahcall and Bahcall 1972b; Petro and Hiltner 1973; Boynton et. al. 1973; Bahcall et. al. 1974]. The optical light curve is much narrower and sharper near minimum intensity than is the X-ray intensity curve; this fact has been explained using several different models, including light originating above the photosphere of HZ Herculis [Joss, Avni, and Bahcall 1973; Pringle 1973], rapid transfer of energy from the irradiated side of HZ Herculis to the side facing away from the X-ray source [Crampton and Hutchings 1973; Wilson 1973], several different sources of optical light [Boynton et. al. 1973; Crampton and Hutchings 1973; Strittmater et. al. 1973], and optical emission by a disc around the X-ray source [Basko and Sunyaev 1973]. Most authors agree that the basic features of the light curve are due to the fact that the side of HZ Herculis (photosphere + corona) that faces Her X-1 is hotter and radiates more strongly.

The X-ray and optical phases agree to within $\pm$ 15 minutes (Bahcall and Bahcall 1972a, b; Bahcall et. al. 1974). Using the agreement between X-ray and optical periods over a timespan of more than 50 years found by Jones et. al. (1973), one deduces that

$$< \frac{d}{dt} \ln P >_{50 \text{ years}} \lesssim 10^{-6}\ \text{yr}^{-1}.$$

This is a useful constraint on the mass loss from the system.

The amplitude of the modulation of the X-ray intensity ($\geq$ a factor of 10) is more than 50 times the *mean* modulation of the (B-band) optical light ($\leq 0.2^{m}$) in the 35^{d} cycle. This conclusion led Avni et. al. (1973) to suggest that the heating of HZ Her is due to a *steady* energy source that has not yet been observed. There may be a small but systematic optical variation with a 35-day periodicity [Kurochkin (1973); Petro and Hiltner (1973); Boynton et. al. (1973)]. Brecher (1972) suggested precession as an explanation of the 35^d cycle.

The time-averaged excess (above minimum light) optical flux $<\Phi>_{\text{excess optical}}$ (observed at earth at wavelengths $\geq$ 3200 Å) is 4×10^{-11} erg cm^{-2} sec^{-1} [Avni et. al. 1973; Bahcall et. al. 1974]. One thus has $<\Phi>_{\text{excess optical}}/\ \Phi_{\text{X-ray at } 35^d \text{ maximum}} \sim 0.01$ and $(\Phi_{\text{excess optical}}/\ \Phi_{\text{X-ray at } 35^d \text{ minimum}}) \gtrsim 0.1$.

Jones et. al. (1973) have shown that HZ Her undergoes extended inactive periods (lasting months or years) during which the luminosity remains nearly constant at its minimum value ($m_{pg} \sim 15^m$). However, small-amplitude variations persist with a period equal to half the orbital period of HZ Her and Her X-1 and a phase consistent with tidal distortion of HZ Her by Her X-1. The amplitude of these variations in photographic magnitudes are 0.09 ± 0.06^m [Jones et. al. (1973); Bahcall et. al. (1973)].

Spectroscopic observations of HZ Her have been published by Crampton and Hutchings (1972) and Bopp et. al. (1972) and extensive observations, which confirm the existence of weak emission lines, have just been reported [Crampton 1973; Bopp et. al. 1973]. Additional programs are underway. Unfortunately, the determination of the orbital velocity depends sensitively upon the model adopted for the non-uniform surface brightness of HZ Her and upon the assumed rotational velocity [see Bahcall et. al. 1974 who also show that the observed light curve does not uniquely determine the surface brightness].

An accurate optical position has been determined by P. Veron (see Table 2) from a 1972 plate taken by the Bahcalls at Wise Observatory. The position was also measured on the Sky Survey Print E1069 using a Mann x-y measuring engine. The two positions agree within the expected errors of $\pm$ 0.4″. The upper limit on the proper motion derived in this way is 0.04″/yr, which is consistent with the earlier result of Bahcall and Bahcall (1972b).

Optical pulsations from HZ Her are never a large fraction of the total light [$\lesssim 0.2$ % at all times; Groth and Nelson 1972; Davidsen et. al. 1972] and are sometimes less than 0.01 % [Cocke et. al. 1973]. However, optical pulses have definitely been detected on some occasions (Davidsen et. al. 1972; Middleditch and Nelson 1973; Groth et. al. 1973) although the velocity-shift of the pulses suggests they may be reflected by gas moving in the binary system rather than by HZ Her.

N. Bahcall (1973) has shown that the distance to HZ Her could be 2-3 kpc if HZ Her is a late A or early F main-sequence star with an apparent magnitude equal to that observed at minimum light but is more likely 5-6 kpc if HZ Her fills its Roche lobe.

D. Mass Estimates
The mass of Her X-1 is of special interest because it may be the first neutron star for which binary-star techniques can be applied to the mass determination. Four independent methods of estimating the masses of Her X-1 and HZ Her are available; they involve using (1): orbital velocities of HZ Her; (2). spectral classification of HZ Her; (3). a Roche-lobe model; and (4). a tidal lobe model. Bahcall et. al. (1974) have investigated the uncertainties in each method and have concluded that all four methods are subject to large errors (a factor of three is not extremely pessimistic) but that each of the four methods is consistent with $M_{Her\ X-1} \sim 1M_\odot$ and $M_{HZ\ Her} \sim 2M_\odot$ [for earlier references to specific determinations see Tananbaum et. al. 1972; van den Heuvel and Heise 1972; Forman et. al. 1972; Bahcall and Bahcall 1972b; Crampton and Hutchings 1972; Lyutiy et. al. 1973; Davidson and Ostriker 1973; Leach and Ruffini 1973; N. Bahcall 1973].

E. Exceptional Properties
HZ Herculis differs in several ways from all the other optical counterparts of X-rays sources in binary systems that have been identified so far [see Table 1]. HZ Herculis is so far unique in: (1). having a spectral type as late as A9 or F0 (all the others are of B or O spectral type); (2). having a smaller stellar luminosity than the observed X-ray luminosity; (3). showing very large optical variations (1.5^m instead of $\sim 0.1^m$); and (4). being within our galaxy but lying at a very high galactic latitude ($b = 38^o$) instead of the typical value of several degrees. Properties (1) through (3) are presumably related by the fact that X-ray heating is a smaller perturbation in the brighter stellar systems. Property (4) is a puzzle in the context of the usual theory of stellar evolution.

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IIB. CENTAURUS X-3

Centaurus X-3 was the second X-ray source (Crab pulsar was the first) for which periodic variations were observed [Giacconi et. al. (1971)]. This source was originally detected by Chodil et. al. (1967) and was later shown to be highly variable by Cooke and Pounds (1971). The existence of a regular 4.8^{sec} period was demonstrated by Giacconi et. al. (1971) using the UHURU satellite; these authors showed that most of the X-ray power (1-20 keV) exhibits this pulsation period. Schreier et. al. (1972) showed that Cen X-3 is a member of an eclipsing binary system with an orbital period of 2.087^{d}, an eclipse or low state of 0.49^{d}, and a transition time (from low to high states) $\simeq 0.04^{d}$. They showed that this interpretation is consistent with both the observed intensity variations and the sinusoidal variations in arrival time of the 4.8^{sec} pulses. The pulse period was found to decrease between

Jan. and May 1971 at a rate $\frac{d \ln T}{dt} = 10^{-4.5}$/yr. Schreier et. al. also determined the projected orbital radius $= 1.2 \times 10^{12}$ cm, the orbital eccentricity ≤ 0.05, the projected orbital velocity $= 4 \times 10^2$ km/sec, and the mass function

$$\frac{M_{opt}{}^3 \sin^3 i}{(M_x + M_{opt})^2} = 15.4\ M_\odot$$

For $M_x \sim 1M_\odot$, $M_{opt} \sim 20M_\odot$, i.e., the optical counterpart is expected to be an intrinsically bright star (perhaps a supergiant) with a relatively small projected orbital velocity $\sim$ 20 km/sec. Wilson (1972) has derived much more severe limits on M_x by assuming a Roche-lobe model.

Baity et. al. (1973) have shown that the spectral shape of Cen X-3 is variable and that in December 1971 and January 1972 it was rather steep ($\sim E^{-3}$). They also reported a non-periodic low intensity state lasting 12 days.

B. Optical Identification

A number of identifications have been suggested for the optical counterpart of Cen X-3 (see, e.g., Margon and Wray 1972; Brucato, Kristian, and Westphal 1972; Peterson 1972). These candidates have been largely eliminated by improved position determination with Copernicus [Parkinson et. al. 1973] and UHURU [Giacconi et. al. 1973].

A tentative optical identification has been suggested by Liller (1973) who proposes a star with $m_B \sim 14.9^m$ and an apparently cyclic component of brightness with a semi-amplitude $= 0.08 \pm 0.03^m$ and a period close to that of the X-ray binary. Support for this identification has been provided by the improved X-ray position of Parkinson et. al. (1973) and the report by Schreier et. al. (1973) of fractional changes in the orbital period $\sim 10^{-5}yr^{-1}$. Van Genderen (1973) finds a photoelectric period of Liller's star that disagrees with the X-ray period. Further photoelectric and spectroscopic studies are required.

[Note added in proof, May 1974. Krzeminski has identified Cen X-3 with a 13^m optical counterpart, IAU Circ. No 2612 and Ap. J. Letters to be published].

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IIC. HD 153919/3U 1700-37

A. X-Ray Properties

Jones et. al. (1973) have shown that 3U 1700-37 has an orbital period of 3.4^d with an unusually long eclipse, or low-state, of duration of 1.1^d. Evidence was also found for secondary minimum at a phase of 0.5 (the center of the occultation is at phase 0.0). Variability was observed on time scales of minutes to tenths of seconds but a 3-σ upper limit of 17 % was found for the fraction of pulsed energy with periods from 0.1 to 10 seconds. Jones et. al. (1973) suggested HD 153919 as the optical counterpart of 3U 1700-37 on the basis of it being the most conspicuous star in the X-ray error box [see also Gursky 1972; Liller 1972] and because, like some of the other optical candidates, it is an early-type supergiant. The star lies within the small 3U error box (0.009 sq. degrees) and the much smaller error box of Rappaport et. al. (1972) (0.002 sq. degrees). Jones et. al. (1973) suggested that the X-ray source might be a black hole.

B. Optical Properties

The optical identification of 3U 1700-37 with HD 153919 has been established by a number of workers [Thackery and Walker 1973; Walker 1973; Jones and Liller 1973; E.P.J. van den Heuvel 1973; Hutchings, Thackery, Webster, and Andrews 1973; Penny, Olowin, Penfold, and Warren 1973; Bessell, Peterson, Wickramasinghe, and Vidal 1973]. The optical object is an O7f star with a visual magnitude 6.6^m [Crampton 1971]. Hutchings et. al. have found the following parameters from a least squares fit to their spectral data (42 plates on which He II λ 4541 was measured): $K_a = 29 \pm 6$ km/sec and $V_0 = -67 \pm 4$ km/sec. They also showed that best-fit period was in good agreement with the X-ray period. Similar results have been obtained by Hensberge et al. (1973) who deduce $K_a = 22$ km/sec, and by Wolff and Morrison (1973). Strong emission lines are present [see especially Walker 1973; Hutchings et. al. 1973; and Wolff and Morrison 1973]. These include He II λ 4686; a blend of N III λλ 4641, 4642; N III λ 4634; and unusually strong C III λλ 4647, 4650, 4651. The emission and absorption lines show different radial velocities, suggesting an expanding atmosphere; the absorption lines have larger negative velocities than the emission lines as expected on this picture. The probable presence of large non-orbital velocities in the spectral lines complicates the determination of the mass of the X-ray source. There is evidence that the primary is not corotating (see Wolff and Morrison 1973).

UBV observations of HD 153919 have been reported by Jones and Liller (1973) and Penny et. al. (1973). They find a double-peaked light curve with an amplitude $\simeq 0.1^{\mathrm{m}}$ in B or V and a period of 3.4^{d}. One of the two minima coincides with the X-ray minimum.

Kemp and Wolstencraft (1973) have reported strong variable magnetic fields of up to 10^4 gauss inferred from observations at H-β. These results were not confirmed by Angel, McGraw, and Stockman (1973) who made measurements at H-α, which has a strong emission component, or Hensberge et. al. (1973) and Wolff and Morrison (1973) (see however, Kemp and Wolstencroft 1973). Further observations are required.

The source has been detected at infrared wavelengths (1.25 μm to 3.5 μm) by Frogel and Persson (1973) who conclude that the infrared emission is not markedly different from that of other stars in the same spectral range.

The best current estimate for the distance modulus is (Hutchings et. al. 1973; Hensberge et. al. 1973): $(m_0\text{-}M) = 11.2^{\mathrm{m}}$, corresponding to a bolometric magnitude of -8.5^{m} and 2.7^{m} of reddening [Hutchings et. al. 1973; Bessel et. al. 1973; Hensberge et. al. 1973]. The distance of the order of 1.7 kpc is consistent with the value derived from the interstellar K line and the diffuse interstellar band λ 4430 (Wolff et. al. 1973).

C. Mass Estimates

Two independent phenomenological estimates can be made for the mass of the X-ray source 3U 1700-37 (see especially Hutchings 1973). Both estimates make use of an estimate for the mass of the optical primary, HD 153919, based on its spectral type ($M_{\mathrm{HD\ 153919}} \sim 35M_{\odot}$) and an assumed radius ($R_{\mathrm{HD\ 153919}} \sim 30R_{\odot}$). These numbers are characteristic of isolated O stars and may be somewhat in error for stars in close binary systems where extensive mass exchange and evolution may be occurring. The first argument uses the obvious inequality

$$\frac{R_{\mathrm{HD\ 153919}}}{R_{\mathrm{orbit}}} \leq \frac{M_{\mathrm{HD\ 153919}}}{M_x} + 1.$$

The value of the orbital radius of HD 153919 is determined by the radial velocity measurements and period to be (for sin i = 1) $R_{orbit} \sim 2R_{\odot}$. Therefore $M_x << M_{HD\ 153919}$ and M_x can be calculated directly from the mass function determined by optical observations. A similar result is obtained if one assumes that the apparent X-ray eclipse duration is determined by the radius of the O star. One finds: $(R_{HD\ 153919}/d) = 0.7$, where d is the center-to-center separation of the X-ray source and the primary optical star. This argument also implies that $R_{orbit} << d$, i.e., $M_x << M_{HD\ 153919}$. With this result, the mass function determined from optical observations implies $M_x \sim 2M_{\odot}$.

This conclusion is subject to the uncertainty associated with the original assumption that $M_{HD\ 153919} = 35M_{\odot}$. The deduced value of M_x depends on $M^{2/3}_{HD\ 153919}$. Had we assumed that HD 153919 has lost much of its mass by evolutionary and exchange processes and consists now of a He core ($\sim 15M_{\odot}$) with a small atmospheric mass, we would have obtained $M_x < 2M_{\odot}$. Hensberge et. al. (1973) obtain a lower limit to the mass M_x of between 0.7 and $1.5M_{\odot}$, depending on the assumptions made.

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IID. HD 77581/Vela XR-1 (= 3U 0900-40)

A. X-Ray Properties

The X-ray source Vela XR-1 was first detected by Chodil et. al. 1967 and has since been observed by many groups. Ulmer et. al. 1972 showed that the X-ray object was an eclipsing binary with an extended low intensity or eclipse state. Forman et. al. 1973 confirmed the results of Ulmer et. al. 1972 and reported a period of 9.95 ± 0.02^d with a duration of the occultation equal to 1.9^d. Variability has been observed on time scales of tenths of seconds to hours, but a 3σ upper limit of 10 % has been established on the fraction of radiation (2-6 keV) that is pulsed with periods between 0.1 and 10 seconds.

B. Optical Properties and Inferences

Based upon the first UHURU observations, Kellogg and Murray (1971) and Bradt and Kunkel (1971) [see also Liller 1972] suggested HD 77581, a B0.5I star (Morgan, Code, and Whitford 1955) with $m_v = 6.9^m$ (Hogg 1958) as an optical candidate for the X-ray source. Brucato and Kristian (1972) interpreted previously known (Feast et. al. 1957) variable radial velocities of HD 77581 as caused by a binary period of less than 10 days and noted that there were no ultraviolet-bright objects like Sco X-1 on a Palomar Schmidt two color plate.

The optical identification has been definitely established by Hiltner, Werner and Osmer (1972), Hiltner (1972) and Hutchings (1972) [see also Jones and Liller (1973), Vidal et. al. 1973 and Wickramasinghe et. al. 1973] by both spectroscopic and photoelectric observations that show period and phase agreement between the X-ray source and the optical primary. Hiltner et. al. deduce an absolute visual magnitude $M_v = -6.0$, a distance modulus $(m-M)_0 = 10.5$ (or a distance = 1.3 kpc) and remark that the spectrum is similar to ε Ori (a B0Ia standard) on blue plates. There are no very obvious spectral peculiarities on blue plates. Photoelectric variations of 0.08^m in B-magnitudes (Hiltner 1972) and 0.09^m in V-magnitude (Hutchings 1972, 1973) have been reported; [see also Vidal et. al. (1973), and Wickramasinghe et. al. (1973) and Jones and Liller (1973)]; these variations are periodic with a total period of the familiar double sinusoidal variation of 8.9^d.

Kemp and Wolstencroft (1973) have reported evidence for magnetic fields of as large as 10^4 gauss, measured by the circular polarization in the wings of H-β. These results have not been confirmed by Angel, McGraw, and Stockman (1973). The star has been observed at near infrared wavelengths by Frogel and Persson (1973) and appears normal for its spectral class.

Hiltner et. al. (1973) estimate a primary mass $\sim 15 M_\odot$ [Wickramasinghe et. al. argue for a much larger primary mass $\sim 50 M_\odot$ based on model calculations and distance determinations] and Hutchings (1973) estimates a mass for Vela XR-1 $\sim 1.7 \pm 0.2$ M$_\odot$. This value is uncertain but is plausible based upon the presently available optical spectroscopy and X-ray parameters.

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IIE. Sk 160/SMC X-1

A. X-Ray Properties

The source associated with SMC X-1 ($\equiv$ 3U 0115-73) is of special importance because the distance is known accurately. Schreier et. al. (1972) showed that the X-ray source is an eclipsing binary with a period of 3.9^d and a low or eclipse state lasting 0.6^d [for earlier X-ray work see Leong et. al. 1971 and Price et. al. 1971]. The fraction of radiation in the 2-6 keV range that is pulsed has been shown to be less than 10% for periods in the range of a few tenths of a second to 10 seconds (Schreier et. al. 1972). Extended lows may occur [Schreier et. al. 1972; Ulmer et. al. 1973]. The known distance to the SMC [Sandage and Tammann 1971; Osmer 1973] implies a large X-ray luminosity: $\sim 5 \times 10^{38}$ ergs/sec [Schreier et. al. 1972; Ulmer et. al. 1973].

B. Optical Properties and Inferences

The star Sanduleak 160 was first proposed as the optical counterpart by Webster et. al. 1972 on the basis of color and spectra showing that Sk 160 was a B supergiant with a radial velocity appropriate to the SMC. Schreier et. al. 1972 had earlier argued on statistical grounds that SMC X-1 was indeed in the Small Magellanic Cloud. The upper limit to the radial-velocity variations obtained by Webster et. al. 1972 imply that the X-ray source has a rather small mass, $M_x \lesssim 1.5\ M_\odot$.

The identification of Sk 160 with SMC X-1 was conclusively established by Liller (1973) who showed that Sk 160 exhibits a small periodic intensity variation (full amplitude $= 0.10^m \pm 0.03^m$) which approximates a double sine curve with a periodic equal to that of SMC X-1. The X-ray and optical minima agree to within $\pm\ 0.3^d$. Liller's results were obtained by examining old plates from the Harvard collection taken between 1896 and 1952.

Mass estimates of $M_{opt} \sim 20 M_\odot$ and $M_x \sim 1.5 M_\odot$ have been made by Hutchings (1973) using the limited data of Liller (1973).

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Sk 160/SMC X-1

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IIF. HDE 226868/Cyg X-1

A. X-ray Properties

The X-ray source Cygnus X-1 has been of especially great interest since the discovery by Oda et. al. (1971) that the X-ray intensity exhibits variations on a time scale of less than seconds (originally believed to be periodic with a period $\sim$ 73 ms). Rappaport, Doxsey, and Zaumen (1971), with millisecond time resolution, found no regular pulsations in the range 0.01 to 1 second with a pulsed power $\geq 5\ \%$ of the X-ray emission in the 2-10 keV band, although they did observe significant variations (or flaring) on a time-scales as short as 50 ms. Schreier et. al. (1971) [see also Holt et. al. 1971; Shulman et. al. 1971] made an extensive analysis of 6 months of UHURU data and concluded, in agreement with Rappaport et. al. (1971), that no single period is consistently present. Schreier et. al. (1971) suggested that there are pulse trains with periods between 0.3 seconds and 10 seconds, lasting several tens of seconds, and containing 10-25 percent of the X-ray power (2-6 keV). On the other hand, Terrell (1972) showed that he could reproduce the appearance of the observed intensity versus time data and the calculated power spectra by assuming that the X-ray intensity has the form of shot noise. In Terrell's picture, there is also no consistent periodicity but instead of pulse trains he describes the data in terms of randomly occurring and overlapping (equal strength) bursts of X-rays.

There is as yet no proof of uniqueness in the various forms of description of the X-ray data. However, all workers agree that there are significant variations on time scales of less than a tenth of a second and that there is some kind of irregularity (or randomness) in the observed intensities.

B. Optical Identification

The widely accepted identification of HDE 226868 as the optical counterpart of Cyg X-1 is based mainly on three facts: (1) HDE 226868 coincides in position (accuracy better than 0.5 arc seconds) with a weak radio source [Braes and Miley 1971, 1972; Hjellming and Wade 1971; Hjellming et. al. 1971; Wade and Hjellming 1972]. (2) There was a change in the X-ray intensity of Cyg X-1 and in the radio-intensity of the radio source associated with HDE 226868 in late March 1972 [Tananbaum et. al. 1972; Hjellming 1973] which suggests correlated behavior; and (3). HDE 226868 lies within the very small ($\lesssim 30''$) X-ray error box [Rappaport, Zaumen, and Doxsey 1971]. The combination of these facts does strongly suggest that HDE 226868 is associated with Cyg X-1, but the identification is not quite as certain as for the other X-ray binaries where an identity of X-ray and optical periods has been established. In order to make the identification more certain, one would like to observe further correlations between X-ray and radio intensities [especially since the radio source does exhibit fairly frequent variations, (Hjellming 1973) and the precise time of the apparent X-ray " event " is slightly ambiguous in the data of Tananbaum et. al. 1972].

C. Optical Properties

The optical star HDE 226868 is classified approximately as a B0Ib supergiant (it may be as early as O9.5 at some phases) in the system of Morgan et. al. (1955) [see Murdin and Webster 1971; Bolton 1971, 1972a, b; Smith et. al. 1973; Walborn 1973; Hutchings et. al. (1973)].

The star exhibits a 5.6^d spectroscopic period [Webster and Murdin 1972; Bolton 1972a, b; Brucato and Kristian 1973], although no corresponding periodicity has been established in the X-rays [Tananbaum et. al. 1972]. The distance to HDE 226868 can be estimated [Murdin and Webster 1971; Bolton 1972a, b; Kristian et. al. 1971] from the measured apparent magnitude, and an estimate of the reddening, plus the assumption that spectral classification determines the luminosity; the distance of about 2 kpc determined in this way is consistent with estimates made from the intensities of the interstellar K-lines and the 4430 Å diffuse band. However all methods for distance determination used so far are rather uncertain [see especially Smith et. al. 1973 and reference to Kraft quoted therein for a warning against using luminosity classification, which is mainly an indication of surface gravity, as an indicator of distances and for mass estimates].

Small intensity variations have been reported by Walker (1972), Lyutiy et. al. (1972), and Lester, Nolt, and Radostitz (1973).

D. Mass Estimates

The mass function for the binary system containing HDE 226868 has been determined by spectroscopic observations to be [Murdin and Webster 1971; Webster and Murdin 1972; Bolton 1973; Brucato and Kristian 1973]:

$$M_x^3 \sin^3 i/(M_x + M_{HD})^2 = 0.23 M_\odot$$

where we have denoted by M_x the mass of the unobserved companion of HDE 226868. In order for the M_x to be less than $2M_\odot$ (a likely upper limit for a neutron-star mass), the mass of HDE 226868 must be $\leq 4M_\odot$ *and* $\sin i = 1$. (If one requires only that $M_x \leq 3M_\odot$, then $M_{HDE226868} \leq 8M_\odot$ *and* $\sin i = 1$). These results indicate one of the main reasons that the system containing HDE 226868 has attracted so much observational and theoretical attention; *it is a natural candidate for a black hole in a binary system.*

Reasonable estimates for *normal* stars of the same spectral type as HDE 226868 are $M_{HD} \sim$ 15-35$M_\odot$, but we cannot be sure that HDE 226868 has not lost much of its mass during the course of its evolution (a precise estimate would involve stellar model calculations which are at least somewhat uncertain). However, geometrical models of the binary system made assuming that the X-rays from Cyg X-1 are produced by mass transfer from HDE 226868, and which also take account of the fact that no X-ray eclipse is observed, yield values of $i \lesssim 45^\circ$ (see references quoted above and Hutchings 1973). If one requires both that $i \leq 45^\circ$ and $M_x \leq 2M_\odot$, one is led to the rather unreasonable result that $M_{HD} \leq 1.5M_\odot$.

The conclusion that Cyg X-1 is a black hole is very plausible if one assumes: (1). that the identification of HDE 226868 as the optical counterpart is correct and (2). the X-rays are produced in roughly the manner described by conventional models of mass transfer onto a collapsed object (white dwarf, neutron star, or black hole). The consequences of other theoretical models (see Bahcall, Kulsrud, and Rosenbluth 1973) must be explored to test the uniqueness of the explanation of X-ray sources as arising from accretion onto a collapsed object. Attempts to optically detect large magnetic fields in this system would be of great interest, although the mechanism of Bahcall et. al. (1973) could work if the strongest fields were on the optically unobserved secondary star. *At present Cyg X-1 is the best candidate for a black hole among the known binary X-ray sources.*

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IIG. CYG X-3

The X-ray source discovered by Giacconi et. al. (1967) has a $4^h.8$ period with an approximately sinusoidal light curve at low X-ray energies (2-6 keV) [see Parignault et. al. 1972; Sanford and Hawkins 1972; Canizares et. al. 1973]; there is no convincing evidence of a $4^h.8$ modulation at higher X-ray energies [see Baity et. al. 1973]. The X-ray period is an order of magnitude shorter than for the known X-ray binaries and the X-ray intensity does not go to zero at minimum (it decreases by only a factor of 2), suggesting that the X-ray modulation is not due to a simple eclipse. There is evidence for strong and variable X-ray absorption (Parsignault et. al. 1972). There has not been any reported evidence for either very short time scale variations or regular pulsations. although the average intensity is not constant and variarions over scales of thens of minutes have been observed [Canizares et. al. 1973]. The shape of the light curve (especially the ratio of minimum to maximum intensity) may also be variable.

In addition to a fascinating X-ray source, the present X-ray error box contains the radio source (Braes and Miley 1972) that exhibited spectacular increases in intensity in September of 1972 [Gregory et. al. 1972a, b; Hjellming and Balick 1972). Most of the attempts to find an optical or infrared counterpart of the X-ray source have assumed that the radio and X-ray sources are coincident in position [see, however, the work of Bahcall and Bahcall 1973 described below]. This identification is not yet absolutely certain (although it is strongly suggested by the positional agreement within the known error) because there was no evidence of a change in the behavior of the X-ray source following the enormous radio outburst [Parsignault et. al. 1972]. Note added in proof, May 1974. Becklin et. al. *Nature* **245,** 302, (1973) have established the identification with certainty]. Laque, Lequeux, and Nguyen-Quang-Rieu (1972) have determined the distance of the radio source to be between 8 and 11 kpc from 21 cm absorption measurements.

Becklin et. al. (1972) detected an infrared source on October 2-3, 1972 within 2 arc sec (estimated error also 2") of the radio source. They measured a flux density $\sim 10^{-28}$ Wm^{-2} $HZ^{-1} = 10^{-2}$ f.u. The ratio of the flux at 2.2μm to that found at 1.6μm was $F(2.2\mu)/F(1.6\mu) \simeq 2$. The infrared source is apparently discrete (diameter $\leq 7''$), as was shown by scans in right ascension and declination. No optical object brighter than $m_v = 17^m$ could be seen at the position of the infrared object. The infrared source was constant in intensity during 1.5 hours of observation (the radio source was also constant during this period). Becklin et. al. (1972) estimate that the probability of finding a source by accident with at least the observed strength is only $\sim 10^{-2}$.

Westphal et. al. (1972) found no visible object on 6 nights in September

1972 to a limit of $\sim 10^{-5}$ f.u. ($V \sim 21^m$). They used a new two-dimensional integrating silicon vidicon photometer at the prime focus of the 200″ telescope and also several direct photographs taken with the Palomar Schmidt camera. Gaustad et. al. (1972) used the Lick 120″ telescope to establish a limit (at 0.9μm) of $I > 16.3$ and derive an absorption $A_V \geq 17.1$ by comparison with the results of Becklin et. al. (1972).

Bahcall and Bahcall (1973) have searched an area of 10′ radius around the position of the radio source using an ITT image-tube with an S-25 photocathode behind a W-88A filter (effective wavelength for reddened objects $\simeq$ 0.9т, which is similar to I). No object was osberved at the position of the radio source and no strongly variable star (variations $\gtrsim 0.4^m$) was observed anywhere in the field. Two-color plates failed to reveal any very blue objects in the field. Their results indicate that $I > 18^m$ and they suggest, on the basis of models, that $m_v = 26^m \pm 2.5^m$. [See Becklin et. al. 1973, *Nature* **245,** 302, 1973 for important IR observations.]

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Cyg X-3

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III. MASS DETERMINATION: GENERAL REMARKS

A. Sources with No Regular Pulsations

For the X-ray sources in which no regular pulsation has yet been found, the primary quantity upon which mass estimates depend is the optically-determined mass function:

$$\frac{M_x{}^3 \sin^3 i}{(M_x + M_{opt})^2} \equiv f\, M_\odot. \tag{1}$$

The parameter f is determined by period and radial-velocity measurements of the optical primary: $f \equiv \tau_{orbit}\, v^3_{opt,\ proj.}/2\pi G M_\odot$. In order to calculate f accurately from the spectroscopic measurements of Doppler velocities, it is necessary to estimate how much of the observed velocity-variation is to be attributed to orbital motion and how much to atmospheric winds and gas exchanges within the binary system. This problem is complicated and has not yet been completely solved for early-type supergiants that are losing mass (see, e.g., the discussion of HD 153919/3U 1700-37 in section IIC and references quoted therein).

Perhaps the most important uncertainty in the present-day state-of-the-art is the mass of the optical primary. It follows from equation (1) that

$$M_x \geq f^{1/3}\, M_{opt}^{2/3}\, M^{1/3}{}_\odot. \tag{2}$$

The equal sign in equation (2) applies if $\sin i \equiv 1$ and $M_x << M_{opt}$. For classification purposes (deciding whether a presumed collapsed object can be a white dwarf, neutron star, or black hole), one is interested in lower bounds for M_x and equation (2) represents the most conservative such bound.

Under the most favorable conditions, estimates for the masses of early-type supergiants can differ by a factor of three or more (as evidenced by the spread in quoted values in the literature) causing an uncertainty of at least a factor of two in the inferred masses of the X-ray sources. Van den Heuvel and Ostriker (1973) have recently stressed the fact that the luminosity and effective temperature of a supergiant within the core helium-burning stage are practically independent of the envelope mass. Thus a supergiant that has undergone mass loss in a binary system can have, for example, any mass between its core mass ($\sim 8M_\odot$) and the total mass ($\sim 25M_\odot$) of a normal supergiant without mass loss [see the model of Kippenhahn (1969) of a $25M_\odot$ star that has lost nearly all of its envelope; also Giannone, Kohl, and Weigert (1968)].

It is therefore easy to see how a factor of three or more uncertainty in M_{opt} can exist even in the best circumstances. In addition, one must remember that there may be considerable additional uncertainty involved in using stellar models for calibration purposes, although this uncertainty is more difficult to quantify. Among the X-ray objects discussed in this review, only Cyg X-1 seems to be clearly too heavy to be a neutron star or white dwarf.

B. Sources with Regular Pulsation

For sources in which regular X-ray pulsations have been discovered, one might hope that the situation would be much more certain. After all, the X-ray observations provide one mass function (and other ratios) and the optical observations provide a second mass function (and other quantities). The observational situation is somewhat analogous to the classical problem of double-lined spectroscopic binaries. Unfortunately, there are serious complications in the case of X-ray binaries. A detailed investigation of the present uncertainties in the mass determination of Her X-1/HZ Her by four independent methods has been carried out by Bahcall, Joss, and Avni (1974) with the net result that the mass of Her X-1 is uncertain by at least a factor of three. The determination of the masses from spectroscopic observations depends sensitively upon the assumed rotational period of HZ Her; the non-uniform surface brightness caused by X-ray heating causes the rotational velocity to contribute significantly to the observed spectroscopic shifts. In addition, gas moving between the stars can complicate the observed velocity curves. Geometrical models for the binary system are subject to additional uncertainties [see Bahcall et. al. (1974)]. Even if one makes the unproved assumption of corotation, the geometrical models are rather uncertain; if the radius of HZ Her differs from the mean radius of the Roche-lobe by only 10 % the mass estimate can be erroneous by a factor ~ 3.

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TABLE I

Optical Properties of Binary X-Ray Sources

Object	Orbital Period	m_v	Distance (kpc)	$\lvert b_{II} \rvert$	Spectral Type	Optical Variations	M_{opt} ($M_\odot$)	M_x ($M_\odot$)	$\mathscr{L}_{stellar}/\mathscr{L}_x$ (2 to 10 keV)	X-Ray Variations Observed
HZ Her/ Her X-1	1.70^d	13.2-14.7	≥ 2	38°	late A	1.5^m	~ 2	~ 1	$\sim 10^{-1.5}$	1.2^{sec} period; 35^d period
Cen X-3	2.09^d	see	text	IIB						4.8^{sec} period; extended lows
HD 153919/ 3U 1700-37	3.4^d	6.6	~ 1.7	2°	O7f	0.1^m	$\sim 10^{1.5}$	~ 2	$\sim 10^2$	variations on scales of tenths of seconds to minutes and longer
HD 77581/ Vela X R-1	8.95^d	6.9	~ 1.3	4°	B0.51b	0.1^m	$\sim 10^{1.5}$	~ 1.7	$\sim 10^2$	variations on tenths of seconds to hours
Sk 160/ SMC X-1	3.9^d	13.3	$7 \times 10^{+1}$	44°	B0I	0.1^m	$\sim 10^{1.5}$	~ 1.5	~ 1	extended lows
HDE 226868/ Cyg X-1	5.6^d	8.9	~ 2	3°	B0Ib or O9.71ab	0.1^m	$10^{1.5}$	$\gtrsim 3$	$\sim 10^2$	variations on time scales as short as 100 ms; also long term variations

TABLE II

Positions

Object	α(1950)	δ(1950)
HZ Herculis	$16^h56^m01^s.7$	$+35°25'04''.5$
Cen X-3	$11^h18^m55^s$	$-60°19'5''$
HD 153919	$17^h00^m32^s.7$	$-37°46'27''$
HD 77581	$09^h00^m13^s.2$	$-40°21'25''.2$
Sk 160	$01^h15^m44^s.3$	$-73°42'53''.6$
HDE 226868	$19^h56^m28^s.8$	$+35°03'54''.5$

Discussion of the report of J. and N. Bahcall

E. P. J. van den Heuvel: I would like to comment on the large outflow velocities observed in the spectrum of 2U 1700-37. These velocities of around 4000 km/sec which were first found by Walker (1973) and confirmed by our observations probably arise at several stellar radii away from the surface. On the other hand, the higher Balmer lines indicates outflow velocities of the order of 100 to 200 km/sec near the orbit of the X-ray source. Therefore, the ratio of the outflow velocity to the orbital velocity is probably less than a factor of 10. This is still quite bad and possibly will distort the radial velocity curve.

As to the mass of the O9.7 Iab star HD 226868 (optical candidate of Cygnus X-1) I would say that the lower limit to its mass certainly is not below 8-10 solar masses. Now that we know that its distance is over 2kpc, we know a lower limit to its luminosity. Even if the star would be a practically pure helium star (having lost its hydrogen envelope) it would still need 8 solar masses to support such a luminosity. I do not see any model that could get away with a still lower mass.

J. Bahcall: A recent paper by J.N. Bahcall, F.J. Dyson, J.I. Katz, and B. Paczynski *(Ap. J. Letters*, **189**, L17, 1974) has suggested that the Cygnus X-1 system may consist of three stars. The secondary could then be a main sequence star (not observable because of its relative faintness) and the X-ray source a light third object. (see also Fabian et. al. Nature 1974).

M. A. Ruderman: What can one tell about the age of X-ray " stars " from the absence of reported supernova remnants about them. Presumably these collapsed stars where formed in supernova explosions.

E. P. J. van den Heuvel: The collapsed objects in the X-ray binaries are expected to be old. From the pulse period of Cen X-3 (4.8 sec) we infer that it is an old neutron star. This is just what one expects from close binary evolution, as was pointed out by Heise and myself (1972)*. In an evolving massive close binary the more massive primary is the first one that fills its Roche-lobe and transfers its hydrogen-rich envelope (over two-third of its mass) to the secondary. This secondary then becomes the more massive component and is still close to the zero-age main sequence. The remaining helium nucleus of the primary lives for about 10^6 years and then explodes as a supernova, leaving behind a neutron star or black hole. The explosion does not disrupt the system, as it is the less massive component which explodes. The secondary remains in the hydrogen burning stage for several more

* van den Heuvel, E. P. J. and Heise, J., 1972, Nature, 239, 67.

million years. During this time it does not fill its Roche lobe and no X-rays are produced. When the secondary finally leaves the main sequence and begins to transfer matter to the collapsed object, the system becomes an X-ray source. As at that time the collapsed object is already several million years old, any supernova remnants around it will since long have disappeared. So one does not expect supernova remnants around such systems.

THE PHYSICS OF BINARY X-RAY SOURCES

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1. GENERAL CONSIDERATIONS

In this review, I shall discuss some of the theoretical implications of the remarkable new results which Giacconi has described in his paper (these proceedings, p. 000). Before doing so, however, it is perhaps worth recalling that the idea of X-ray sources being associated with close binary systems dates back to the earliest days of X-ray astronomy. It was Hayakawa and Matsouko (1964) and Zeldovich and Guseynov (1965) who first made the suggestion that binary stars might be X-ray sources. After Sco X1 had been identified with an object reminiscent of an old nova, many other theorists proposed that X-ray sources involved transfer of matter from one star onto a compact companion (see Burbidge (1972) for an account of these developments). It is still unclear whether this is actually happening in Sco X1; but there now seems little doubt that it *is* the case for a major class of X-ray sources in the Galaxy.

What, then, can be said concerning the general nature of objects like Her X1, Cen X3, Cyg X1 and Cyg X3? The first general point is that the rapid variability suggests, though of course it does not prove, that a very small object—probably even smaller than a white dwarf—is involved. The gravitational potential well associated with such an object is very deep indeed, and accretion therefore provides an efficient energy source. If, as seems to be the case in the observed X-ray binaries, the compact object is in a close orbit—almost a grazing orbit—around another star, then a copious supply of material is available from the companion. Because the efficiencies are so high, the accretion rates need only be in the range $10^{16} - 10^{18}$ gm sec^{-1} ($10^{-10} - 10^{-8}$ $M_\odot$ year^{-1}) in order to produce the observed luminosities. These are modest compared to the inferred transfer rates in other binary systems, and could be supplied by a stellar wind even if the companion star does not overflow its Roche surface.

A second general point is that most of the gravitational energy is liberated deep in the potential well—at or near the surface of the compact object if it is a neutron star; within a few Schwarzschild radii if it is a black hole. Thus the effective dimensions of the source (assuming that the compact object is in the stellar mass range) are only $\sim 10^6$ cm. If $10^{36} - 10^{38}$ erg sec^{-1} are radiated thermally from such a small region, a temperature high enough that the energy emerges predominantly in the X-ray band is therefore guaranteed.

In this report, I shall confine attention exclusively to what might be called the ' standard model ' in which the X-ray source is regarded as being associated with either a neutron star or a black hole. Although, as Giacconi has argued, the evidence favouring this model seems fairly compelling, the case is certainly not completely watertight; and one should bear in mind that some quite different interpretations for various aspects of these phenomena still remain tenable. These include models involving pulsating or rapidly rotating white dwarfs, or strong magnetic fields in a binary system which does not contain a compact component at all. I shall first outline the main features of the ' standard model ' and then comment on how it may apply to some particular sources.

My reason for concentrating on this one model is that it seems more plausible than any specific alternative so far proposed. Also this model has formed the basis for most of the detailed theoretical work carried out so far. Already so much work has been done that it will only be possible for me to sketch most of it; and some interesting aspects of the phenomena will be left out entirely.

The X-ray binaries obviously involve all the problems connected with ordinary close binary systems—problems which are still ill-understood despite having been with us for many years—together with a whole range of new ones connected with the compact object. For convenience of exposition, it is convenient to split the subject into three parts: the mass transfer (relevant length scales $\sim 10^{11}$ cm); the accretion disc (dimensions $\sim 10^{10}$ cm); and the compact object itself, which is also the region where the X-rays are presumed to originate ($10^6 - 10^8$ cm). Unfortunately these three areas cannot be regarded as entirely disjoint, despite the very different length scales involved. For example, the X-ray intensity and spectrum is probably determined by processes occurring close to the compact object, but it may nevertheless have an important influence on the flow of matter from the companion because of heating and radiation pressure effects.

The mass transfer ($\sim 10^{11}$ cm)

Much theoretical work has been based on the hypothesis that the companion star fills its Roche lobe, and that material flows across the Lagrangian point. It is important, however, to remember that these analyses are only strictly valid if the star corotates with the orbital period. This is probably quite a good assumption in these close systems, unless they were perturbed so recently that tidal effects have not yet re-established synchronous rotation. Some calculations—for example, estimated limits on the masses of the X-ray sources—depend rather heavily on this postulate. It is also possible that the star does *not* fill its Roche lobe, but has a strong stellar wind. Gas streams may cause the optical emission lines observed in these systems (and also, incidentally, confuse attempts at radial velocity determinations). I shall say nothing about this topic, which is similar to that which arises in all close binary systems.

The accretion disc ($\lesssim 10^{10}$ cm)
By whatever process material is captured from the companion star, it is likely to have so much angular momentum that it cannot fall directly onto the compact object. The matter will instead dissipate its motions perpendicular to the plane of symmetry and form a differentially rotating disc, the rotational velocity at each point being approximately Keplerian, and then gradually spiral inwards as viscosity transports its angular momentum outwards. If the companion star is overflowing its Roche lobe, it is conventionally assumed that the matter joins the disc at the radius where its angular momentum relative to the compact object equals that of a Keplerian orbit. This argument suggests that the so-called 'hot spot' appears at a radius which is $\sim 20\%$ that of the Roche lobe around the compact star. The structure of the outer part of the disc is not well understood. The disc must extend further out than the hot spot, because *some* of the material transferred from the companion star has to carry away the angular momentum—it cannot all be accreted by the compact object. A further complication is that the gravitational field of the companion star probably cannot be ignored in the outermost part of the disc, so the gas will not circulate in simple Keplerian orbits.

If the accreted matter is captured from a strong stellar wind, it will tend to have less net angular momentum; but the disc would still extend out to a radius $\gtrsim 10^9$ cm in general.

The structure of accretion discs has been discussed by many authors —for example Prendergast and Burbidge, 1968 (who considered a disc surrounding a white dwarf), Lynden-Bell, 1969; Pringle and Rees, 1972, Shakura and Sunyaev, 1973, Novikov and Thorne, 1973—and the details will not be repeated here.

An obvious prerequisite for the existence of a disc (whose thickness must, by definition, be only a small fraction of its radius) is that radiative cooling should be efficient enough to remove most of the energy liberated by viscous friction, so that the internal energy is small compared with the gravitational binding energy.

$$\text{i.e.}\quad kT\left(1+\frac{p_r}{p_g}\right) << \frac{GMm_p}{r} \tag{1}$$

where p_r/p_g is the ratio of radiation pressure to gas pressure, and m_p is the proton mass. For accretion flows with the parameters appropriate to X-ray sources the densities are high enough, and the timescales long enough, to ensure that (1) is almost certainly fulfilled. Also, the mass in the disc is gravitationally negligible compared to that of the central object.

If a steady state has been set up, the structure of the disc is governed by the following system of equations. First, the same mass flux $\dot{M}$ must flow across any radius r, so that

$$\dot{M} = 2\pi r \int \rho\,(r,z)\, v_r\,(z)\, dz \tag{2}$$

for all r, when z is the coordinate perpendicular to the disc measured from the plane of symmetry.

A second, and somewhat less trivial, requirement is that in a steady state the *flux of angular momentum* should be the same at all r. Angular momentum is transported inward by the accreted matter, but transported outward by the viscous stresses. The difference between these quantities represents the rate at which the central compact object is gaining angular momentum. Following Novikov and Thorne (1973) we assume that angular momentum is being accreted at a rate $\beta\,\dot{M}(GMr_1)^{1/2}$, where r_1 is the radius of the inner boundary of the disc. Since the specific angular momentum deposited on the compact object cannot exceed the Keplerian value at r_1, we have $\beta \leq 1$. One then finds that the heat dissipated per unit surface area of the disc at a radius $r > r_1$ is

$$p(r) = \frac{3\dot{M}}{4\pi r^2}\, G\,\frac{M}{r}\left(1-\beta\left(\frac{r_1}{r}\right)^{1/2}\right) \tag{3}$$

It is important to note that β is a second parameter which is not completely determined by $\dot{M}$—one can imagine situations with the same $\dot{M}$ but different torques in the disc, and therefore different values of p(r). When $r >> r_1$, however, one finds, independently of β, that the energy radiated at radii $\geq$ r is *3 times larger* than the energy lost by the accreted material while spiralling inward to that radius. The extra contribution arises because the viscous stresses transport *energy* outward as well as momentum. One might at first sight worry about the energy budget for the disc as a whole. However, when $\beta = 1$ one finds that the total energy radiated, integrating over all $r \geq r_1$, is precisely equal to $\dot{M}$ multiplied by the binding energy of Keplerian orbit of radius r_1; when $\beta = 0$, the factor of 3 enhancement applies right in to $r = r_1$, but in this case the extra energy comes from viscous torques which apply a drag to the compact object*. (i.e. twice as much energy in this case is supplied by the central spinning object as comes from the infalling material itself).

These deductions do not depend on the magnitude of the viscosity—if this is low, then the radial velocity v_r is small, so the equilibrium value of ρ needed in order to give a given $\dot{M}$ must be high; and conversely. But to analyse the structure of the disc in any further detail one *must* know something about the viscosity, and this is the stumbling-block to further progress. Possible causes of viscosity include turbulence induced by the differential rotation, convective motions, or sheared magnetic fields. Pringle and Rees (1972) and Shakura and Sunyaev (1973) made specific simplifying assumptions about the viscosity, which enabled them to discuss the vertical structure of the disc (i.e. the balance between the pressure gradient perpendicular to the disc and the component of gravity in that direction), and the spectrum of the emergent radiation. However one has little confidence that one knows even the appropriate order of magnitude for the viscosity, and it therefore seems premature to discuss the spectrum of the disc in great detail. The dominant emission mechanism is probably thermal bremsstrahlung, though the spectrum may be appreciably distorted as a result of scatterings by the hot thermal electrons. All that can be said is that the effective temperature must be at least as high as the black body temperature needed to radiate a power p(r).

The compact object and the x-ray emission (10^6 — 10^8 cm)

When the compact object is a black hole, the disc extends inward to the innermost stable circular orbit, the emission being concentrated within a few Schwarzschild radii. The efficiency is $\sim 6\ \%$ if the black hole is described by a Schwarzschild metric, and efficiences of up to 42 % are possible for accretion discs around Kerr black holes, the precise upper limit depending on how much of the emitted radiation is captured by the hole. (Note that the efficiency would be very much lower if the accreted matter had so little angular momentum that it could fall almost radially inward. This situation, which is relevant to isolated black holes accreting interstellar matter, has been discussed by Schwartz-

* Note that the above analysis is strictly Newtonian. When one considers an accretion disc surrounding a black hole, then one finds that the total energy radiated by the disc *equals* the energy lost by infalling matter when the black hole accretes a specific angular momentum appropriate to the circular orbits at the inner edge of the disc (see Novikov and Thorne (1973) for the details of the relativistic case). The appropriate inner boundary condition in this case is that the viscous stresses should be zero at $r = r_1$. If the viscosity is high enough, one may still get radiation even from $r < r_1$, and this raises the efficiency.

man (1971) and Shapiro (1973a, b). In general, only a small fraction of the mass-energy is radiated away before the infalling matter is swallowed by the hole).

Attempts to determine the expected radiation spectrum from accretion discs are impeded by our ignorance about the viscosity, which introduces far larger uncertainties than those corresponding to the difference between a Schwarzschild and extreme Kerr black hole. In general, the temperature decreases outwards and, even though the emission is thermal, the integrated spectrum may resemble a power law. Radiation from the outer parts of the disc would not be energetically significant unless, as discussed by Shakura and Sunyaev (1973) the disc were so thick in relation to its radius that X-rays from the inner regions were intercepted by the disc and reradiated at softer energies. Some further aspects of this model, as it may apply specifically to Cyg X1, are discussed later.

When the central object is a spinning, magnetised neutron star, a far more complex situation ensues, which has been discussed extensively by Pringle and Rees (1972), Davidson and Ostriker (1973) and Lamb, Pethick and Pines (1973). If the neutron star were unmagnetised, then the disc would extend inwards until the accreted material grazed the star's surface. If, however, the neutron star has a surface magnetic field of the same strength as is inferred for pulsars ($\sim 10^{12}$G) then the magnetic stresses will influence the dynamics out far beyond the surface of the star. We define the ' Alfvèn radius ' to be that distance at which the magnetic stresses are comparable with the viscous stresses in the disc.

$$\text{i.e.} \quad \frac{(H(r_A))^2}{4\pi} \simeq \rho\,(r_A)\, v_r\,(r_A)\, v_\theta\,(r_A)$$

The Alfvèn radius depends on $\dot{M}$, but somewhat insensitively because H^2 depends on r at least as steeply as r^{-6}, and for typical parameters r_A is 10 — 100 times larger than r_*. r_A, defined as above, is fortunately independent of the viscosity except insofar as this affects the scale height. The disc would not be expected to extend inward to radii much less than r_A, which means that the radiation from the disc itself is relatively unimportant. Once matter penetrates within r_A the high field strengths and conductivity ensure that it is constrained to follow the field lines. If the star has an oblique dipole field, the infalling plasma will impact on the surface in the vicinity of the magnetic polar caps. The physics at $r \simeq r_A$ is so complicated that one cannot really estimate which of the magnetic field lines can capture matter. These field lines will certainly, however, be a subset of those which would have reached out o radii $\gtrsim r_A$ in the absence of infalling plasma. This guarantees that, when $r_A >> r_*$, the material will be channelled onto only a small fraction of the stellar surface.

The dominant radiation mechanisms would be bremsstrahlung or cyclotron radiation (including emission at the first few harmonics of the basic cyclotron frequency). Lamb *et al.* (1973) and Gnedin and Sunyaev (1973) have discussed the likely beam shape of the emergent radiation. If the dominant opacity were ordinary Thomson scattering, the radiation would tend to leak out of the sides of the accretion column, yielding a fan beam. If the magnetic field is so strong that the cyclotron frequency exceeds the radiation frequency under consideration, then electron scattering is inhibited for radiation propagating along the field direction, and also for radiation travelling across to the field which is polarised such that the electric wave vector is at right angles to the magnetic field. Realistic models can yield either pencil beams or fan beams, depending on the strength of the magnetic field and the polarization of the radiation. Modulation of this beam pattern each time the neutron star spins generates the X-ray pulse shape. The radiation would generally be expected to display a high degree of both linear and circular polarization. Detection of such polarization from variable X-ray sources would lend strong support to the accreting neutron star hypothesis.

Some other aspects of this scheme are discussed later in connection with particular sources.

2. CAUTIONARY REMARKS ON THE " EDDINGTON LIMIT "

An important role in these models is played by the so-called ' critical luminosity ' or ' Eddington limit ' at which radiation pressure balances gravity. If Thomson scattering provides the main opacity and the relevant material is fully ionized, then this luminosity is

$$L_{edd} = \frac{4\pi G M m_p c}{\sigma} \simeq 10^{38} \left(\frac{M}{M_\odot}\right) \text{ ergs sec}^{-1}, \qquad (4)$$

σ_T being the Thomson cross-section.

One might therefore expect that the accretion rate $\dot{M}$ could approach, but in no circumstances exceed, the value needed to yield this luminosity. Recently, Margon and Ostriker (1973) have in fact analysed the data on X-ray sources, and find that there does indeed seem to be a luminosity cut-off at around the expected value of L_{edd} for $M \simeq M_\odot$, and that there is a class of sources whose luminosities cluster close to this value. Because this issue is an important one, it is perhaps worth pointing out that the ' Eddington limit ' is physically significant only under relatively restrictive circumstances—circumstances which are *not* generally met by the kinds of X-ray source models usually considered.

As emphasized by Buff and McCray (1974) the luminosity of a source powered by accretion cannot even approach L_{edd} if the effective cross section per electron is larger than σ_T. This is quite likely to be the case for a source emitting soft X-rays, because the relevant opacity (unless all the ions are completely stripped) is then primarily due to photo-ionization, for which $\sigma >> \sigma_T$. If the value of $\dot{M}$ in binary X-ray sources is controlled by processes occurring near the surface of the companion star or the critical Roche surface, as in the ' self-excited wind ' hypothesis (Basko and Sunyaev, 1973; Arons 1973) then one might expect the luminosity to stabilise at a value *well below* L_{edd}.

There are, however, several types of situation where luminosities $>> L_{edd}$ are possible, especially under the extreme conditions prevailing near compact objects. Some of these are mentioned below:

(i) The effective opacity may be much *less* than that provided by Thomson scattering. In the context of X-ray sources this may, for instance, happen in the accretion column above the magnetic polar caps of neutron stars, where the scattering cross section is $<< \sigma_T$ for photons below the cyclotron frequency travelling along the magnetic field direction.

(ii) The Eddington limit can also be violated in any non-spherically-symmetric configuration. Consider again, for example, the accretion column near a magnetised neutron star. If the magnetic field does *not* modify the opacity and make the scattering highly anisotropic, then radiation will tend to escape from the *sides* of the column. This means that the radiation flux along the column, and therefore the pressure opposing gravity, is then less than it would be in an isotropic situation. (An analogous argument may also apply to accretion discs).

(iii) As has been pointed out by Lamb *et. al.* (1973) there are conceivable circumstances when the luminosity may exceed L_{edd} even when the appropriate cross section *is* σ_T *and* the accretion is isotropic. This is because $L > L_{edd}$ is merely the condition that infalling matter should be *decelerated*. But unless the total optical depth is sufficiently large, this does not guarantee that radiation pressure can halt the accretion. The infalling matter carries momentum across a sphere of radius r at a rate $\dot{M}v(r)$, where v(r) is of the order of the free fall speed. If its kinetic energy is converted into radiation at a radius r_{min} the outward momentum flux, ignoring relativistic corrections, is $\sim \dfrac{\dot{M}}{2c}\left(v\,(r_{min})\right)^2$ (and less, of course, if the conversion efficiency is low). This means that the average photon must undergo more than $\frac{2c}{v(r_{min})}$ scatterings if radiation pressure is to stem the accretion flow (unless the main contribution to the opacity comes from radii $r >> r_{min}$).

(iv) The Eddington limit is of course irrelevant in an *unsteady* or *explosive* situation: it is, for instance, violated by factors $\sim 10^5$ in supernovae.

A luminosity exceeding L_{edd} obviously entails a correspondingly higher accretion rate. If, however, the accretion has the high efficiency expected in X-ray sources, the observations seem to rule out a value of $\dot{M}$ of $\gtrsim 10^{-7}$ year^{-1} in all cases. If the companion star loses mass at a higher rate than this (as is likely at certain stages of stellar evolution, and as would seem required if the changes of *orbital* period in Cen X3 are attributable to mass loss from the companion star) then most of the material would presumably escape from the system.*

3. PHENOMENA OBSERVED IN PARTICULAR BINARY X-RAY SOURCES

Her X1 and Cen X3 are clear candidates for systems when the X-ray source is a neutron star (and it is gratifying that the mass of Her X1 seems to be within the allowable range 0.3 — 1.6 $M_\odot$ for neutron stars). There are several specific observations which can be tentatively explained on the basis of this model. These systems resemble pulsars in that rotation provides the "clock". However the X-ray power radiated derives *not* from rotational kinetic energy—which could maintain the observed X-ray luminosity of Her X1 for $\lesssim 10$ years—but from accretion. (This, as Schwartsman has pointed out, suggests at least part of the reason why pulsars are not found in binary systems. An isolated spinning neutron star, surrounded only by diffuse interstellar gas, generates the electromagnetically driven relativistic wind which is believed to be a precondition for the coherent pulsed radio emission. When such an object is embedded in a denser environment, the pressure of the relativistic outflow cannot hold the external matter at bay, and we instead get accretion, manifesting itself in the emission of thermal X-rays. One can estimate that Her X1 would have displayed pulsar-like behaviour only if its period were $\lesssim 0.1$ sec).

* The possibility has been raised (Zeldovich *et al.*, 1972, Ruffini and Wilson, 1973) that a neutron star may be able to accrete at a rate $\sim 10^{-3}$ $M_\odot$ year^{-1} and get so hot that the energy escapes mainly as neutrinos, but the photon luminosity remains below L_{edd}. There seem severe doubts, however, about whether this situation could actually be set up by gradually increasing $\dot{M}$, and whether it would be stable.

Changes in the pulse period

Since an accreting neutron star is not drawing on its rotational energy as its main power supply, it is not obvious whether its spin rate should slow down or speed up. An element of gas accreted by the star carries angular momentum corresponding to corotation at the Alfven radius. This suggests that the spin rate would speed up on a timescale

$$\frac{M}{\dot{M}}\left(\frac{r}{r_A}\right)^2 \tag{5}$$

Even though $\frac{M}{\dot{M}} \simeq 10^8$ years, this " lever-arm " effect certainly allows a speedup as rapid as that observed in Cen X3. There is, however, a possible opposing effect tending to *brake* the rotation: this is the viscous torque exerted by the accretion disc outside r_A. These two effects can be of the same order of magnitude if

$$\left(\frac{GM}{r_A}\right)^{1/2} \simeq \Omega\, r_A$$

(and of course if $\left(\frac{2GM}{r_A}\right)^{1/2} < \Omega\, r_A$ it would be energetically possible for material at the inner edge of the disc to be flung out of the system by magnetic forces, leading to a further braking effect). Davidson and Ostriker (1973) suggest that Ω tends asymptotically to a value such that the net torque on the neutron star is zero. This value of Ω depends on r_A, which is itself a function of $\dot{M}$. Therefore, if there were fluctuations in the accretion rate, then Ω would tend to increase (decrease) s $\dot{M}$ increases (decreases). If Her X1 were close to this equilibrium state, and the fluctuations in $\dot{M}$ were small in amplitude, one could perhaps understand why Ω has been observed both to increase and to decrease, and why the timescale for these changes is slower than is the case for Cen X3.

The effects mentioned above are the dominant ones for causing changes in period. Other effects—for example, the spin-up due to the contraction of the star as it accretes mass—occur on the much slower timescale of $M/\dot{M}$.

Her X1: the long-term variability

Giacconi has summarized the X-ray data on the ' 35 day cycle ' and I shall here briefly mention some of the numerous suggestions already made to explain this puzzling behaviour. In this connection, it is important to recall that optical observations impose an important constraint on such suggestions. It appears that the 1.7 day period light variations persist throughout the 35 day cycle with more or less the same amplitude (even though a 35 day periodicity may be discernable in some of the fine details of the light curve (Kurochkin, 1973, Boynton *et. al.*, 1973)). Since the thermal inertia of the relevant layers of the companion star is small, this implies that the heating mechanism operates throughout the $\sim$ 23 days out of $\sim$ 35 when UHURU detects no X-rays from Her X1.

(a) *Modulations in mass transfer rate*

One class of theory for the 35 day cycle involves supposing that the *mass transfer* is modulated with this period. It seems unlikely that this could be due to some pulsation of the companion star because the expected pulsation periods would be $<<$ 35 days. Another possibility (Pringle, 1973, Henriksen, Reinhardt and Aschenbach, 1973) is that the spin period of the companion differs by $\sim$ 5 % from the orbital period. If the star displayed some departures from axisymmetry—a ' magnetic spot ' associated with an especially vigorous wind for instance—then the transfer rate could vary with a synodic period of 35 days.

Conceivably some kind of feedback process may be operating. McCray (1973) has developed an ingenious model which utilises the fact that the X-ray luminosity is a significant fraction (perhaps $\sim$ 10 %) of L_{edd}. When the X-rays are ' on ', the X-ray source behaves with respect to the surrounding gas as though it had a somewhat lower mass. The ' effective ' Roche lobe around the companion star might then expand so that material no longer overflowed it. Mass transfer would then cease, and no material would be added to the disc. The disc would then drain away, and the X-ray emission would stop. Mass transfer would then begin again, the disc would be replenished, and so on. McCray speculates that some kind of limit cycle is set up. The time-scale of this cycle would be determined by the length of time taken for a typical element of gas to spiral inward to the central object. A period of the general order of 35 days would certainly not be unreasonable, but one cannot claim to " predict " it, because of the wide uncertainty about the efficiency of viscosity in the disc.

A fully developed theory along these lines must also take account of a competing process which might cause *positive* feedback. This arises because the X-rays, by heating the surface layers of the companion star, tend to *raise* the mass transfer rate by increasing the scale height in the atmosphere and/or by stimulating an enhanced stellar wind (Arons, 1973, Basko and Sunyaev, 1973, Alme, 1973). It has in fact been proposed (Lin, 1973) that the 35 day cycle could result from this type of positive feedback if the X-rays stimulate a mass transfer rate which ' overshoots ' to such an extent that opacity effects around the compact object quench the X-rays. Plainly a proper theory of the 35 day cycle along the above lines must await a fuller understanding of how the X-rays interact with the companion star, and also of the factors that determine the residence time of material in the accretion disc.

(b) *Processes occurring in the accretion disc*

Katz (1973) has suggested that the rim of the accretion disc may not lie in the orbital plane of the system. This might happen if the companion star possessed a component of spin angular momentum which was not aligned with the orbital angular momentum. In this situation, the rim of the disc would precess, and could obscure the X-rays for some fraction of each precession period. To obtain a precession period of 35 days, Katz has to assume that the disc extends outwards to a larger radius than is customarily supposed.

It is also conceivable that the disc might be subject to convective or other instabilities which might cause it to dump material periodically onto the central object.

(c) *Modulation of inflow from Alfven radius*

Pines *et al.* (1973) have developed a model according to which the neutron star undergoes free precession in such a way that the angle between the magnetic axis and the plane of the accretion disc varies periodically. When this angle is small, accretion along the ' magnetic funnel ' can proceed; but when the magnetic axis points too far out of the plane of the disc accretion is suppressed, and material transferred from the companion accumulates in the disc outside the Alfvèn radius. It is not clear how large the precession amplitude would have to be in order for such an ' accretion gate ' to operate. However Pines *et al.* list some other reasons why the accretion flow near the Alfvèn surface could be sensitive to the orientation of the neutron star's rotation axis, so it is conceivable that a wobble through only a few degrees could suffice. On the basis of this model, Pines *et al.* have attempted to explain the other features of the 35 day cycle. The asymmetry between the sharp rise and the gradual fall in X-ray intensity during the 12 day ' on ' period is readily explained. Matter accumulating during the ' off ' period will be opaque to the X-rays until it has been photoionized. The fact that the switch-off occurs near orbital phases 0.25 or 0.75 is attributed to the higher density of obscuring matter along the line joining the two stars, which makes it more likely that the first X-rays to be seen will escape perpendicular to this line. The hypothetical ' hot spot ' where the gas stream merges with the disc may be thick enough to obscure the X-rays at the phase of the orbit when it lies along our line of sight. The outer radius of the disc would decrease during the ' on ' period, and the location of the hot spot would change (it is claimed) in such a way that the dip ' marches ' in phase in the matter observed. The apparent tendency of the small amplitude 1.24 sec *optical* pulsations (which probably come from gas with cooling time $\lesssim$ 1.24 sec which is being heated by the X-rays) to occur at particular orbital phases can also be explained.

(d) *Precession of pencil beam*
Another idea involving precession of the neutron star (Brecher, 1972, Strittmatter *et al.*, 1973) is that the X-rays remain ' on ' for the whole 35 day cycle, but that they emerge in a pencil beam which sweeps through our line of sight only for 12 days out of 35. There are some geometrical difficulties associated with this idea. In particular, the broad and relatively smooth observed X-ray pulse profile tells us something about the shape of the beam, and it is hard to reconcile this with the sharp onset of the high state or with the apparent lack of any marked systematic changes in the pulse shape during the ' on ' state. A very large wobble amplitude ($\gtrsim$ 45 degrees) would certainly seem required by this model.

At least in models (a) and (c), the continuous heating of the companion star can only be explained by invoking a steady heat source. One possibility (Avni *et al.*, 1973) is that the neutron star emits a steady non-pulsed flux of soft X-rays, powered by the $\sim$ 8 MeV per nucleon resulting from nuclear fusion of the accreted matter. This energy is liberated well below the neutron star surface, and emerges isotropically. This steady isotropic emission could also be powered by *inward* conduction from the hot magnetic polar caps. But a serious problem arises with any model in which soft ($\lesssim$ 0.5 keV) X-rays play the dominant role in the heating, because these photons (unlike harder X-rays) are absorbed predominantly *above* the photosphere. The associated energy input would then distort the temperature stratification, resulting in the formation of strong emission lines and suppression of the ordinary stellar absorption spectrum (Basko and Sunyaev, 1973; Strittmatter, 1974). It seems more likely that the star HZ Her is heated mainly by *hard* ($\gtrsim$ 10 keV) X-rays, though the problem then is the inefficiency resulting from the high albedo (unless one considers photons of $\gtrsim$ 0.5 MeV). Heating by fast particles is another possibility. In models (b) and (d), one may suppose that X-rays always hit the companion star even when they cannot propagate along our line of sight (though this requirement places further constraints on the geometry). A more attractive variant of (d) might be to postulate that the star is heated by hard X-rays which are not so strongly beamed as those detected by UHURU. This is theoretically plausible because the circumstance which might most naturally cause a pencil beam—the reduced scattering cross section for photons travelling along the magnetic field direction—would not be so effective at high photon energies.

35 days is much too short a free precession period for a neutron star with a liquid core. However a neutron star with a *solid* core and the 1.24 sec spin period appropriate to Her X1 could plausibly sustain a sufficient deviation from axisymmetry to yield a 35 day precession, and would then automatically be rigid enough to be able to wobble through a large angle. (Mechanisms for exciting this kind of wobble and for sustaining it against damping processes are discussed by Pines in his report).

In assessing the various models for the 35 day cycle it is of course crucial to know just how regular a phenomenon it really is. It is also relevant that Cen X3 displays extended lows which are apparently *not* strictly periodic. Finally, some explanation is also required for the *very* long ($\sim$ 10 year) time-scale variability in Her X1 (Jones *et al.*, 1973). If one were optimistic one might therefore hope that *two* of the possibilities mentioned above might actually be relevant!

Orbital period of Cen X3

The changes in the orbital period of Cen X3 have been attributed to a mass loss or mass transfer rate far higher than the minimum needed to provide the X-ray power. However this large amount of gas, even if it were escaping from the system, would cause so much opacity that the X-ray source could not be observed. A more plausible possibility (Pringle, private communication) is that angular momentum is still being exchanged between the orbital motion and the spin of the companion star. This suggests that the event which formed the neutron star happened within the last $\sim 10^4$ years, and that the timescale of the observed orbital period change corresponds to that required for tidal effects to establish synchronous rotation.

Cyg X1

This is the prime candidate for being an X-ray source involving a black hole. One would expect the accretion disc around a black hole to be subject to various instabilities: thermal instabilities, magnetic instabilities (perhaps analogous to those which Parker has discussed in the context of the interstellar gas in our Galaxy), or perhaps instabilities resulting from irregularities in the mass transfer rate. These could give rise to irregular flickering on all time scales down to the orbital period associated with the most tightly bound stable circular orbits, but no regular period would be expected. Even if one had no evidence on its mass, one would therefore suspect Cyg X1 of being a disc around a black hole, and it is therefore gratifying that the evidence on its mass strongly supports this interpretation. Sunyaev (1973) proposed that attempts should be made to search for pulse trains due to regions of enhanced emissivity orbiting the hole. The typical orbital periods would be $\sim 0.5(M/M_\odot)$ milliseconds for a Schwarzschild black hole, but ~ 8 times faster if the black hole has a maximal Kerr metric whose angular momentum was aligned with the disc, but with the same mass. (Further interesting complications can occur if the black hole is obliquely oriented relative to the disc).

Further information would be derived if an X-ray spectral feature originating in the disc could be discovered and its profile measured, but this seems unlikely to be feasible before 1980. It is important to remember that black holes are a consequence of almost all " viable " theories of gravity, and much further work is needed before one can diagnose whether the properties of a given black hole agree better with those expected on the basis of general relativity than with the predictions of a rival theory.

Sco X1

It is still unclear whether Sco X1 belongs to the same family as the other X-ray sources. Basko and Sunyaev have argued that Sco X1 could have a normal star as a binary companion with $L \lesssim L_\odot$. However, because the X-ray output is $\sim 10^4\ L_\odot$, the geometry and orientation of the system must be carefully specified in order to reduce the X-ray heating of the hypothetical companion to an undetectably low level.

The general spectrum of Sco X1 (in the infrared and optical bands as well as in X-rays) is very well fitted by an accretion disc model (Pringle and Rees, 1972). Maybe Sco X1 could be a compact object surrounded by a *massive* disc, the disc being a remnant of a companion star destroyed by tidal forces (c. f. Faulkner, 1971).

Cyg X3

The gradual and incomplete character of the X-ray eclipses suggests that in this system the eclipse is caused not by the surface of the companion star, but by scattering and absorption in a strong wind. The observed 2.2μ infrared variations imply that the relevant layer of the heated side of the companion star has a temperature $\lesssim 10^6$ °K. This however, is quite possible if one is seeing emission from the wind, which is heated to this temperature (Pringle, 1974).

Although Cyg X3 has a shorter period than the other X-ray binaries, the period is longer than that of systems such as DQ Herc. It may differ from such systems merely in having a neutron star (or black hole) as the compact component, instead of this being a white dwarf.

4. CONCLUDING REMARKS

Many important and interesting aspects of binary X-ray sources have not even been touched on in the foregoing review.

There is a whole complex of problems associated with the *optical light curve* of these systems (see the report by Bahcall and Bahcall for a review of the data). Heating of the companion star causes the side facing the compact object to be hotter and brighter than the eclipsed side. A full understanding of this effect involves detailed computations (along the lines of those already done by Arons (1973) and Basko and Sunyaev (1973)) of the structure of a stellar atmosphere irradiated by X-rays. A second quite different effect which leads to optical variations with *half* the orbital period arises from the distortion of the companion star by the compact object's gravitational field. Interpretation of actual light curves is complicated by further effects (emission by gas streams, radiation and absorption by the accretion disc itself, etc.) and one suspects that detailed model-building may prove somewhat fruitless unless some very clear-cut correlations between X-ray and optical variability are found. In Her X1, X-ray heating (or heating by some other radiation flux emanating from the compact object) is the dominant effect, and the effects of gravitational distortion are relatively minor; in Cyg X1, where the companion star is much more luminous relative to the X-ray source than is the case for Her X1, the heating augments the stellar luminosity by only $\sim 2\%$. The occurrence of X-ray heating, however, sets a *lower limit* to the apparent brightness of the optical counterpart for any eclipsing X-ray source. If there were no interstellar absorption, any X-ray source with an intensity of C UHURU counts which is observed to eclipse for a fraction f of every period should have an optical apparent magnitude

$$m \lesssim 15 - 2.5 \left(\log\left(\frac{C}{10}\right) + 2\log(4f)\right)$$

Thus any eclipsing source in the UHURU catalogue would be optically identifiable were it not for the often severe effects of interstellar extinction.

The radio properties of the X-ray binaries are completely unexplained. However the fact that—even in the extreme case of Cyg X3 at the peak of its radio flare—the radio luminosity is a tiny fraction of the X-ray output, suggests that to concern ourselves with the details of the radio variability may be as premature as it would be to worry about solar flares before understanding the basic elements of stellar structure. Moreover, some binary systems containing two relatively normal stars have similar radio properties and this suggests that the radio behaviour, fascinating and puzzling though it may be, is unlikely to be intimately connected to the compact object itself.

The existence of these close binary systems with compact components raises many astrophysical questions. How do they fit into the general scheme of binary star evolution? How do they evolve to their present state (and, in particular, how did they avoid disrupting during the catastrophe which formed the collapsed component)? Why, nevertheless are there only $\sim$ 30 such systems in the Galaxy? What will be their eventual fate?—for example, what happens if a neutron star accretes so much material that it comes to exceed the limiting mass; or what happens when, later in its evolution, the companion star swells up and engulfs the compact object? Others at this meeting are far better qualified than I to discuss these issues. Far more detailed information can be expected from the next generation of X-ray detectors and from more refined optical observations. The discovery of binary X-ray sources ranks as one of the major astronomical advances of this decade, and there seems little doubt that they will remain at the focus of theoretical attention for several years to come, eventually allowing us to check the theory of black holes (and neutron stars) against observations in many key respects.

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Discussion of the report of M.J. Rees

R. Hofstadter: Do you know any way in which gamma rays could be produced in the compact binaries?

M. Rees: I think we would expect the emission to be predominantly thermal. The radiative processes are, moreover, efficient enough to ensure that the temperature need not be far above the black body temperature of a few kev. This means that there is no reason to expect strong γ-ray emission (though there may be production of electron-positron pairs and consequent γ-ray emission).

On the other hand, an *isolated* black hole sitting in the interstellar medium would accrete matter in a spherically symmetrical fashion and the infalling matter may then get hot enough to emit predominantly in the γ-ray band. The luminosity of these objects would however be much lower, when their masses are much larger than typical stellar masses.

F. Pacini: A fairly large flux of γ-rays is possible from Sco X-1 in the model of Davidson, Salpeter and myself where the X-ray emitting gas is heated by a central pulsar emitting fast particles.

M. Rees: Yes. This also raises the interesting question of how Sco X-I is related to the other compact X-ray sources.

B. Carter: I have the impression that no one has seriously considered the way in which the excess angular momentum is carried off from the surface of the accretion disc. Can you say something about this problem?

M. Rees: It is certainly impossible for *all* the matter transferred from the companion star to fall onto the compact object. The flow-pattern at the outer edge of the disc is very complicated, and I do not know of any detailed quantitative discussions of this point. All that can definitely be said is that a certain fraction (maybe $\gtrsim 50\,\%$) of the matter *must* either return to the companion star or else escape from the system altogether.

J. Bahcall: Could you describe the physical processes that are responsible, on the black-hole models, for the 100 ms observed fluctuations in the X-rays and how reliable are the calculations of these processes?

M. Rees: If the magnetic field is important in the disc, one might expect the gas to fragment into clouds, and loops of magnetic flux to rise out of the plane. (This situation is analogous to that discussed by Parker in the context of the interstellar gas in the Galaxy). These clouds—as they spiral inward—could give rise to pulse trains with the frequencies characteristic of orbits close to the black hole. For a black hole of 10 $M_\odot$, this allows timescales down to $\sim$ 5 ms for a Schwarzschild black hole, and down to $\sim$ 0.6 ms for a " maximal Kerr " metric.

There is also the possibility of thermal instability in the disc. The situation is plainly far too complex for detailed calculations to be worthwhile at this stage. All that can be said is that one can very easily think of mechanisms that would give rise to rapid variability. None of these processes would, however, generate strictly periodic variations. The discovery of any variability of this latter kind would of course seem to be stong evidence against an interpretation in terms of an accretion disc around a black hole.

G. Burbidge: If I understood you correctly you suggested that the case for a black hole in Cyg X-1 is stronger than the case for a neutron star in H Z Her. Can you say something about the difficulties associated with having a neutron star in H Z Her, and what are the plausible alternatives?

M. Rees: The only difficulty which has been raised at this conference concerns the problem of forming a neutron star without disrupting the binary system. I am surprised, however, that any theorist working on these complex phenomena should have sufficient confidence in his conclusions to regard this argument as a strong one! It would, on the other hand, be pleasant if one could find some argument which decisively supports the spinning neutron star hypothesis. An X-ray source with a period much less than one second would obviously strengthen the case for neutron star. One specific prediction of the neutron star model is that the pulsed X-rays should be circularly polarised. It is probably not feasible to search for this effect observationally. However, I think this is a phenomena which would be almost impossible to explain on the basis of the white dwarf hypothesis. It might, however, be feasible to search for high linear polarisation.

D. Pines: I quite agree with Prof. Rees that the principal, and perhaps the only argument against Her X-1 being a rotating neutron star is the problem of forming a neutron star in a binary system. I would like to inquire whether one does not encounter similar difficulties in making a black hole as part of a binary system; if that were the case, then from the identification of Cyg X-1 as a black hole, one might argue that it is not all that difficult to make a neutron star as part of a binary.

M. Rees: There are many other people herew ho are more competent to comment on this than I am myself.

A. Cameron: While it is possible that mass may be ejected during the process of forming a black hole, we know of no absolute need for this to occur, since the details of the formation process are still unknown.

The ejection of mass in a supernova explosion can easily disrupt a binary system, or at least place the neutron star remnant in a somewhat distant and quite elliptical orbit. The ejection of mass in the formation of a white dwarf star in a binary system, can take place rather gently over a long period of time, and this mass can be transferred to the companion or lost from the system, but there is no indication that the system is likely to be disrupted in this case.

In the formation of a black hole the core must undergo hydrodynamic collapse, and it is quite possible that all of the envelope will participate in his hydrodynamic collapse and that none of it will be ejected. The only mass loss would be the mass equivalent of the energy of the ejected neutrinos. Under these conditions the orbital parameters of the system would suffer only minor perturbations.

R. Giacconi: I should like to point out that a strong argument against a white dwarf interpretation for binary X-ray sources stems from the fact that old novae which we know to be binaries containing white dwarfs have not been observed to be X-ray emitters to about 10^{-11} erg/cm^2 sec from the Uhuru survey (in the 1 — to 10 Kev. region). Also lower energy surveys to .25 Kev have found *no* evidence for such emission.

M. Rees: This seems to me an extremely relevant point. Indeed one could argue that the only difference between X-ray binaries and other close binaries with compact components is that in the former cases one has a neutron star or black hole, whereas in the latter one has a white dwarf.

E. P. J. Van Den Heuvel: I would like to point out that the disruption of a binary does not occur (or is rather unlikely) if it is the less massive component which explodes as a supernova. And this is just what happened in the history of the close massive X-ray binaries; due to mass exchange, the most evolved component had become the less massive one before it exploded. Therefore, the systems remained bound. Only if the initial binary period was less than about 2 days impact of the supernova shell onto the other component may have disrupted the system.

M. Rees: Yes. But I think the main problem concerns the momentum transferred to the companion during the explosion. (A shock wave propagates into the companion. Material heated by the shock is ejected at much more than the escape velocity; and this yields a " rocket effect " which amplifies the momentum transfer by an order of magnitude. It nevertheless seems to me that the uncertainties are still too great for this problem yet to be regarded as serious).

GENERAL DISCUSSION.

THE EVOLUTIONARY ORIGIN OF MASSIVE X-RAY BINARIES AND THE TOTAL NUMBER OF MASSIVE STARS WITH A COLLAPSED CLOSE COMPANION IN THE GALAXY

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INTRODUCTION

Five out of the seven binary X-ray sources known to date seem to belong to the same class. They all consist of a massive supergiant primary together with a low-mass secondary which is the source of the X-ray emission. Table 1 lists the basic physical parameters of these five systems. In the four systems for which optical identifications are available, the primary is an O- or B-type supergiant and therefore—if it has not undergone extensive mass loss—is likely to have a mass in excess of 15 $M_\odot$ (cf. Bahcall and Bahcall 1974; Underhill, 1966; van den Heuvel and Ostriker 1973). Also in the Cen X-3 system the mass function indicates that the primary certainly has $M > 15\ M_\odot$, while the secondary is unlikely to have a mass in excess of 1 $M_\odot$ (Wilson 1972, van den Heuvel and Heise 1972; Davidson and Ostriker 1973). Although in Cyg X-1 and possibly also in SMC X-1 the secondaries may have masses $\geq 3\ M_\odot$, the secondaries in 2U 1700-37 and 2U 0900-40 are likely to be low mass objects (cf. Bahcall and Bahcall 1974). In view of the large observed X-ray fluxes, these secondaries are likely to be neutron stars or black holes. White dwarfs are unlikely here, since even nearby (< 70 pc distance) close binary systems which contain an accreting white dwarf, such as the cataclysmic variables (cf. Kraft, 1962) show neither hard X-ray emission nor stable pulsations. (Nevertheless, accretion rates $\geq 2.5 \times 10^{-9}\ M_\odot$/yr are observed in these systems, Robinson 1973). Unlike the pulsations of Cen X-3 and Her X-1, the observed optical pulsations of accreting white dwarfs in cataclysmic variables are extremely unstable, showing large jumps in frequency on time scales of a few minutes (Warner and Robinson 1972). The formation of a neutron star or black hole requires a supernove (SN) explosion. The fact that in the case of the X-ray binaries the

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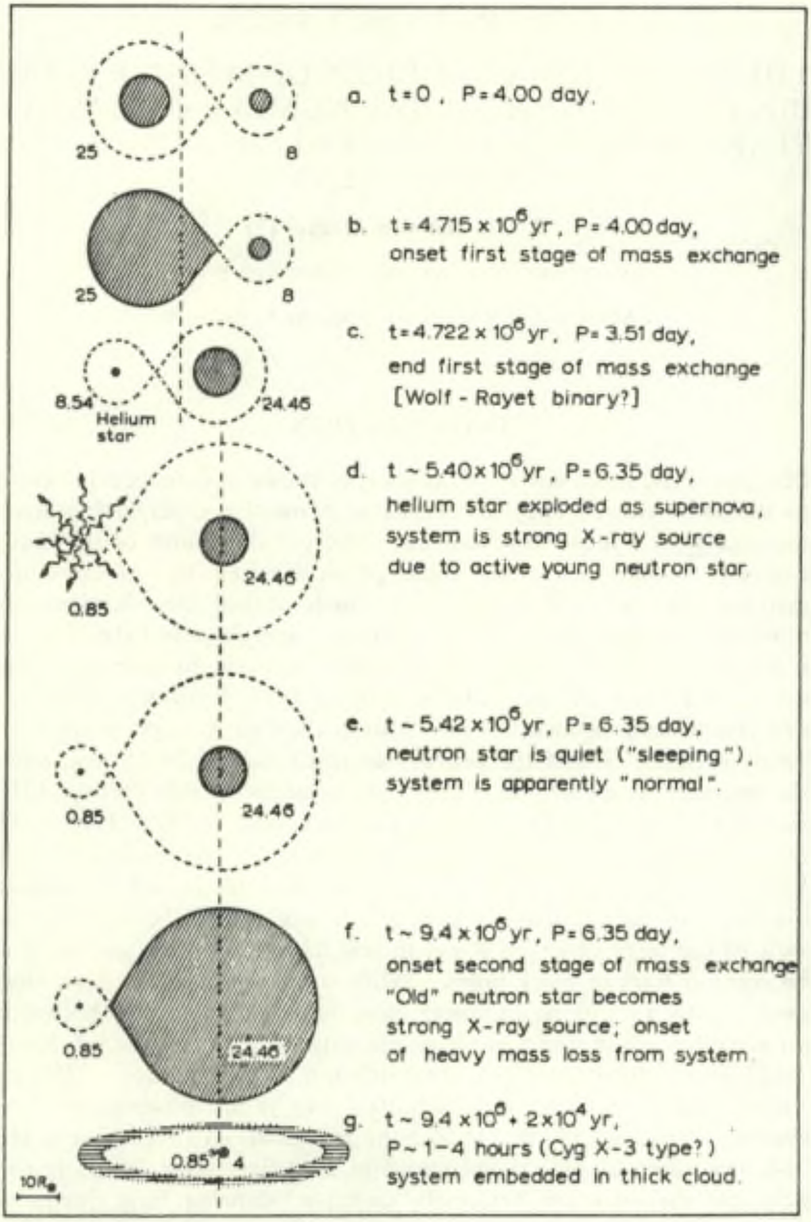

Fig. 1. Subsequent stages in the evolution of a massive close binary with components of 25 M⊙ and 8 M⊙.
Notice that the system becomes an X-ray binary a long time after the supernova explosion of the most evolved component. (It is assumed here that the supernova explosion leaves behind a 0.85 M⊙ neutron star. The remnant might, however, also be a black hole).

explosion did not disrupt the close binary system is a logical consequence of the mass exchange which caused the most evolved component of the system to have less than half of the total mass of the system before it exploded (van den Heuvel and Heise 1972). (The effects of the impact of the SN shell onto the unexploded component—as put forward by Colgate (1970)— are in most cases insufficient for disrupting the system (Sutantyo, 1974a, b)).

I would like to outline here the, in our opinion, most likely evolutionary history of these systems and, from this evolutionary picture, make an estimate of the number of massive close binaries with one collapsed component (neutron star or black hole) that is expected to exist in the galaxy.

THE EVOLUTION OF CLOSE BINARIES

I restrict myself to the most common type of close binary evolution, designated by Kippenhahn and Weigert (1967) as case B. This case—which takes place for over 80 per cent of the OB-type spectroscopic binaries (van den Heuvel 1969) occurs if the separation of the components in the unevolved system is such that the primary star overflows its Roche lobe after it has terminated core hydrogen burning, but before it has started core helium burning, i.e. when it is crossing the Hertzsprung gap in the Hertzsprung-Russell diagram (shell hydrogen burning stage).

In case B, two types of evolution of the primary are possible, depending on the initial mass of this star (Paczynski 1971a), viz.:

(1) *Low mass case B* (primary mass $< 3\ M_\odot$. This case is not of interest here, as the primary finishes its life as a (helium) white dwarf.

(2) *Massive case B* (primary mass $\geq 3\ M_\odot$. In this case, after the end of core hydrogen burning, the helium core of the primary star has a mass $\gtrsim 0.4\ M_\odot$ and contracts until it starts helium burning. During the stage of core contraction it is burning hydrogen in a shell and transfers practically the entire hydrogen-rich envelope ($\sim$ 70 to 80 per cent of its mass) to the secondary. What remains is a practically pure helium star in the stage of core helium burning (Kippenhahn and Weigert 1967; Paczynski 1967). The tiny remaining amount of hydrogen (a few tenths of a solar mass) in the outer layers may strongly increase the outer radius but it does not affect the luminosity, structure or evolution of the helium core (cf. Giannone, Kohl and Weigert 1968). For this reason, for the study of the further evolution of the primary star, one may simply consider it as a pure helium star. Evolutionary tracks of pure helium stars were computed by Paczynski (1971b) and by Arnett and Schramm (1973). These show that (i) helium stars ignite carbon non-explosively, (ii) during helium and carbon burning, helium stars

TABLE 1

Massive X-ray Binaries

X-ray source			Optical candidate				
Name	P_x(day)	$t_{eclipse}$ (day)	Name	Spectral type	m_v (mag)	Binary period (day)	Estimated primary mass ($M_\odot$)
Cyg X-1	—	—	HD 226868	B 0 Iab	8.9	5.6	≥ 15
2U 0900–40	8.95	1.7	HD 77581	B 0.5 Iab	6.9	8.96	≥ 15
SMC X-1	3.8927	0.6	Sanduleak Nr. 160	B 0 I	13.6	3.89	≥ 15
2U 1700–37	3.412	1.1	HD 153919	07 f	6.6	3.412	≥ 23
Cen X-3	2.087	0.5	—	—	—	—	≥ 15

TABLE 2

Double-lined and Eclipsing WR Binaries with Known Orbits

	Star	Spectral type	P(day)	$M_w \sin^3 i$	$M_{OB} \sin^3 i$	$\frac{M_W}{M_{OB}}$	P_F (day) $M_F = M_\odot$	$M_F = 4M_\odot$
1	HD 152270	WC7+O8	8.82	1.85	6.85	0.28	15.8	11.4
2	HD 186943	WN5+OB	9.55	5.8	21.0	0.28	14.4	11.6
3	HD 193576 (V 444 Cyg)	WN6+B1	4.21	9.5	24.1	0.39	7.4	6.2
4	HD 211853	WN6+B0	6.68	7.6	19.1	0.39	11.8	9.3
5	HD 214419 (CQ Cep)	WN5+B1?	1.64	(f(m)=4.38)		—	2.5	1.99
6	HD 228766	WN7+OB	10.6	4.6	22.3	0.21	14.3	11.2
7	HD 68273 (γ^2 Vel)	WC7+O7.5	78.5	13.0	46.3	0.28	148	130
8	HD 168206	WC7+O	29.67	8.2	24.8	0.33	51	42
9	HD 190918	WN5+B0	85	0.21	0.78	0.27	145	119

with $M \leq 3$ to $4\ M_\odot$ evolve to a second red giant phase, whereas helium stars with $M \geq 4\ M_\odot$ never expand to radii larger than a few times the solar radius. Hence, in close binary systems, helium stars with $M \leq 3$ to $4\ M_\odot$ may undergo a second stage of mass loss and finish life with a mass below the Chandrasekhar limit, i.e. as white dwarfs. On the other hand, in massive close binaries with $P \geq 0.6$ days, helium stars with $M \geq 4\ M_\odot$ will probably never fill their Roche lobes before the onset of oxygen burning.

Due to neutrino emission, the lifetime after the onset of oxygen burning is reduced to less than the thermal time scale of the stellar envelope (cf. Rakavy et al. 1967; Arnett 1973), which is the timescale needed for expansion to the Roche lobe. Consequently it seems unlikely that helium stars with $M \geq 4\ M_\odot$ in close binaries with $P \geq 0.6$ days will ever be able to expand to their Roche lobes before the final collapse of the core. Therefore, such stars are likely to finish life as a supernova.

Computations by Arnett (1973) do indeed show that helium stars with $M \gtrsim 5\ M_\odot$ will leave a dense, collapsed remnant (neutron star of black hole). Whether the exact limit is at $4\ M_\odot$ or $5\ M_\odot$ is still uncertain. (Mass loss from the helium star due to a stellar wind—i.e. in the Wolf-Rayet stage (see below)—may somewhat increase the lower limit). As a working hypothesis we will assume that the lower mass limit for a helium star supernova is at $4\ M_\odot$. In order for the helium core to have a mass $\geq 4\ M_\odot$, the primary star of a case B binary system should originally have had $M \geq 16\ M_\odot$. Hence, for a component of a close binary, the lower mass limit for finishing life as a SN is probably $\geq 16\ M_\odot$, whereas for single stars it is only around $4\ M_\odot$ (Gunn and Ostriker 1970). To illustrate the evolution of a massive case B system, we consider the evolution of a close binary with components of $25\ M_\odot$ and $8\ M_\odot$ and $P = 4$ days, as presented in figure 1. Since in case B the transferred amount of matter depends practically uniquely on the mass of the primary star, we adapted the first part of the evolutionary history of the system from the evolution of a binary with components of $25\ M_\odot$ and $15\ M_\odot$ as computed by Kippenhahn (1969). The initial composition is X (hydrogen) $= 0.604$, Y (helium) $= 0.352$, Z (heavier elements) $= 0.044$. After 4.71×10^6 yr the $25\ M_\odot$ star has finished core hydrogen burning and reaches its Roche lobe. At this moment it has a helium core with a mass of $8\ M_\odot$. After reaching the Roche lobe it transfers in 7000 yr $16.46\ M_\odot$ of its outer layers (which still have the initial composition) to its companion. At the onset of the mass transfer the helium content of the core of the $8\ M_\odot$ secondary has—due to the much slower evolution of this star—only increased from 0.35 to 0.40. After the reestablishment of thermal equilibrium (~ 7000 yr), the mass of the hydrogen fusing convective core of the secondary will—due to the accretion of the $16.46\ M_\odot$—have increased from $1.5\ M_\odot$

to $\sim$ 8 M$_\odot$. Due to mixing in of hydrogenrich matter, the hydrogen content of this core is, at that moment, only 1 percent below its value at age zero. Hence, upon termination of the mass exchange, the system consists of a 8.54 M$_\odot$ almost pure helium star—which we will from here on simply call " the helium star "—and a 24.46 M$_\odot$ main-sequence star which is nearly at zero age. Assuming that—after rearrangement of angular momentum—the orbital angular momentum of the system is conserved, the binary period after the exchange is (see Plavec 1968; Paczynski 1971a) reduced to 3.51^d. Main-sequence stars with M $\sim$ 25 M$_\odot$ are expected to have spectral types around O7-O8 (Underhill 1966) As was pointed out by Paczynski (1967), binary systems resulting from massive case B evolution—viz., consisting of a helium star together with an OB-type main-sequence star—very closely resemble Wolf-Rayet binaries. Indeed, the characteristics of Wolf-Rayet stars and helium stars are strikingly similar (see also Smith (1968a) and Kippenhahn (1969)) e.g.: (i) both types of stars are very overluminous for their masses (i.e. have luminosities comparable to those of 3 to 4 times more massive main-sequence stars); (ii) in Wolf-Rayet binaries the Wolf-Rayet star is always the less massive component, being 3 to 5 times less massive than its OB-type companion (which always has a spectral type earlier than B2, i.e. has M $\gtrsim$ 15 M$_\odot$); nevertheless, the luminosity of the Wolf-Rayet star is in most cases comparable to that of its companion; (iii) helium stars as well as Wolf-Rayet stars have $T_e \gtrsim 50000$ K and radii of only a few times the solar radius.

As an illustration of the above mentioned characteristics table 2 lists the system parameters of a number of well-known Wolf-Rayet binaries (cf. Smith 1968a). The table shows the similarity between these systems and evolved case B binaries, just after the first stage of mass exchange (stage c in figure 1).

EVOLUTION AFTER THE FIRST STAGE OF MASS EXCHANGE

The following evolutionary stages are expected to occur, as depicted in figures 1c to 1g:

c-d. Evolution up to the SN stage.

The 8.54 M$_\odot$ helium stars finishes helium and carbon burning in $\sim 7 \times 10^5$ yr. Due to neutrino emission the lifetime of the subsequent stages will be less than a few thousand years (Rakavy et al. 1967). Consequently, about 7×10^5 yr after the first stage of mass exchange the helium star will explode as a SN (figure 1d). As the hydrogen burning stage of the 24.46 M$_\odot$ star lasts for about 4.8×10^6 yr, this star is still almost unevolved when its companion explodes.

We have tentatively assumed that the SN explosion leaves behind a 0.85 $M_\odot$ neutron star remnant and that the remaining 7.69 $M_\odot$ is ejected instantaneously as a spherically symmetric SN shell. Due to this sudden mass ejection the orbital period increases to 6.35 days and the eccentricity becomes $e \sim 0.25$ (cf. van den Heuvel 1968). The center of gravity of the system has been accelerated to about 100 km/sec, due to the explosion. The impact of the SN shell onto the unexploded component has been neglected here, and will slightly change these values (cf. Sutantyo 1974 a,b). The situation just after the explosion is depicted in figure 1d.

We could also have assumed that the remnant is a black hole with a mass of say, 4 $M_\odot$. In that case the system parameters after the explosion would be $P \sim 4.9$ days, $e \sim 0.13$, $V_{\mathrm{run\ away}} \sim 50$ km/sec.

Similarly, in columns 7 and 8 of table 2 we have listed for a number of well known Wolf-Rayet binaries the final binary periods P_F, which would result after a spherically symmetric SN explosion of the Wolf-Rayet components of these systems, for remnant masses $M_F = M_\odot$ and 4 $M_\odot$ respectively.

One notices the similarity between these final system parameters (masses, periods) and those of the massive X-ray binaries listed in table 1. However, just after the explosion the X-ray binary stage is still far off as the OB-type components of the systems are still practically unevolved and, consequently, will not overflow their Roche lobes within the first few million years.

d-e. Young neutron star or black hole.

If the remnant is a neutron star it will, for a short time after the explosion rotate very rapidly. Similar to the Crab pulsar it will during a few thousand years be a source of relativistic particles, X rays and γ rays. One certainly expects that this will have influence on the appearance of the OB-type component. From observed pulsar deceleration rates (cf. Ruderman 1972) one infers that after some 10^4 to 2×10^4 yr the rotation of the neutron star will be slowed down so far that its total energy emission will have dropped below 10^{35} ergs/sec. As this value is only between 0.01 and 0.001 times the energy emission of the OB-type companion we expect that the presence of the neutron star will become difficult to detect after this time.

Due to the enormous flux of energetic particles and photons which bombarded the OB star during this stage, one expects extensive nuclear processing to have taken place in the surface layers of this star. In this way considerable abundance anomalies may have been produced. Possibly the peculiar helium weak early B-type stars such as *a* Cen (Norris and Baschek 1972, Mihalas 1973), 3 Cen A (Hardorp 1966) and ι Ori B (Conti and Loonen 1970; Dworetsky 1973) have resulted from this type of evolution. In case that the remnant is a black hole, this short lasting active stage will be absent.

e-f. Quiet collapsar binary.

During this stage the presence of the collapsed companion is undetectable. The OB-type component is on the main sequence (hydrogen burning stage) and does not overflow its Roche lobe or emit a strong stellar wind unless it is very massive. One expects that the duration of this stage ($> 4 \times 10^6$ yr) is in most cases sufficiently long for tidal forces to restore the circularity of the orbit (Sutantyo 1974c).

f-g. Massive X-ray binary.

Some 4×10^6 yr after the explosion of the helium star, the OB star leaves the main sequence and, when it becomes a supergiant a strong stellar wind emission will develop (cf. Lucy and Solomon 1970). This wind will activate the *old* neutron star (or *old* black hole) which then becomes a strong X-ray source. (Recently some early type main-sequence stars have also been observed to emit stellar winds; however, the fact that all but one of the massive X-ray binaries contain supergiants indicates that apparently only in the supergiant stage the stellar wind density is sufficiently large for producing a strong X-ray source).

The duration of the strong stellar wind stage is rather short: from evolutionary tracks computed by Simpson (1971) one infers that in a binary system with $P < 10^d$ the time elapsed between the moments at which the star leaves the main sequence and at which it reaches the Roche lobe is 2×10^4 yr and 5×10^4 yr, respectively, for stars with masses of 30 $M_\odot$ and 15 $M_\odot$, respectively. As soon as the stellar envelope overflows the Roche lobe (or the tidal lobe, in the case of absence of corotation) the rate of mass loss will become of the order of 10^{-3} $M_\odot$/yr (van den Heuvel and De Loore 1973) and the X-ray source will be completely extinguished (cf. Pringle 1973).

The further evolution of the system is uncertain, as it depends on the way in which the neutron star or black hole accretes or expells the large amounts of matter dumped onto it. Several simple modes of mass loss and mass exchange in the system during this stage, involving various assumptions about the amounts of angular momentum transported by the expelled or exchanged matter, were considered by van den Heuvel and De Loore (1973). One possible final configuration—resembling the Cygnus X-3 system—is depicted in figure 1g. Obviously, this is not the only possible final configuration.

E.P.J. VAN DEN HEUVEL

THE NUMBER OF MASSIVE X-RAY BINARIES AND QUIET COLLAPSAR BINARIES EXPECTED IN THE GALAXY.

From the foregoing paragraph it is clear that:

(i) the duration of the stage in which the system consists of a quiet collapsed component together with a main-sequence OB star covers some 82 per cent of the life time after the first stage of mass exchange;

(ii) the duration of the Wolf-Rayet binary stage and the X-ray binary stage take only some 17 per cent and 0.5 to 1 per cent of this lifetime, respectively. (These percentages do not change much if one assumes another initial mass ratio for the system).

From these lifetimes and the observed number of unevolved massive close binaries in the galaxy, one can estimate the total galactic numbers of OB stars with collapsed " quiet " and collapsed X-ray emitting companions, respectively.

We will assume here that the lower mass limit for becoming a supernova is 16 $M_\odot$ for a component of a close binary. This mass corresponds approximately to that of a B1 V star (Underhill 1966). We estimate the total number of stars more massive than 16 $M_\odot$ in the galaxy as follows:

(1) We assume that the stellar mass distribution Φ (M) as derived by Limber (1960) from the van Rhijn luminosity function represents the correct *relative* mass distribution of stars in the galaxy;

(2) to find the *absolute* distribution of stellar masses we use accurate estimates of the total number of OB stars in the galaxy.

Roberts (1957) estimated the total number of stars earlier than B8 in the galaxy as 2×10^6. Adopting masses of 3.5 $M_\odot$ and 20 $M_\odot$ for B8 and B0 stars, respectively, one finds a total number of 6000 O-stars in the galaxy, in good agreement with Parenago's (1948) estimate of 6500 O-stars. Shnirelman (1952) gives a number of 1.8×10^6 B0 — B5 stars, which corresponds to 3×10^6 stars earlier than B8. As a reasonable mean we therefore adopt the real number of stars earlier than B8 to be 2.5×10^6, which corresponds to some 20000 stars with $M \geq 16\ M_\odot$ in the galaxy. The percentage of spectroscopic binaries in young OB associations is at least 20 % (van Albada 1968; Blaauw and van Albada 1974) most of which will be unevolved. So, the total number of unevolved close binaries with primary mass $\geq$ 16 $M_\odot$ in the galaxy will be around 4000. As the lifetime of the system after the first stage of mass exchange nearly equals the lifetime before this stage (see figure 1), one expects that the total number of systems after the first stage of mass exchange is also around 4000. From the above given fractional ifetimes of the Wolf-Rayet stage (17 %), the " quiet " collapsar stage (82 %) and the massive X-ray binary stage (0.5 to 1 %) one then expects the respective numbers of systems in these three stages to be about 680, 3280 and 20 to 40. A more nearly exact estimate, taking into account the actual distribution of mass ratios of unevolved close binaries, etc., gives similar results (van den Heuvel, 1973). We compare these numbers with the observed numbers of Wolf-Rayet binaries and massive X-ray binaries in the galaxy.

Wolf-Rayet binaries. Smith (1958b) finds 28 Wolf-Rayet stars within 3 Kpc distance. Due to the intrinsic brightness of Wolf-Rayet stars ($<M_V> = -4^m.5$) the discovery of Wolf-Rayet stars within this distance is expected to be nearly complete (cf. also Roberts 1962). Hence, adopting a radius of the galactic disk of 13.5 Kpc and a uniform distribution of Wolf-Rayet stars in the galactic plane, one estimates the total number of these stars in the galaxy to be around 600, in good agreement with the above mentioned theoretically expected value.

Massive X-ray binaries. Here the observed numbers are very small. Nevertheless, the three optically identified galactic sources of this type are all within 3.5 Kpc distance. (From interstellar features one derives that the distance of Cygnus X-1 is probably ≥ 2.5 Kpc. (Bregman et al. 1973; Margon et al. 1973), the distance of 2U 1700-37 is around 2.5 Kpc (Hensberge et al. 1974) and the distance of 2U 0900-40 is around 2 Kpc (Hiltner et al. 1972)). In view of their brightness it is unlikely that there would be undiscovered sources of this type within this distance. Assuming a uniform distribu ion of massive X-ray binaries in the galactic plane one expects the total number of these systems in the galaxy to be around 50. Due to the very small numbers involved, the statistical uncertainty in this number will be $\pm$ 30. The resulting number of between 20 and 80 systems is of the same order as the above mentioned theoretically expected number of 20 to 40 systems. We therefore conclude that the observed numbers of Wolf-Rayet binaries and massive X-ray binaries in the galaxy are in satisfactory agreement with the numbers expected on the basis of the evolutionary picture outlined in figure 1.

Therefore, one cannot escape from the conclusion that there must also be in the galaxy some 3300 massive close binaries consisting of a main-sequence OB star and a " quiet " collapsed companion (neutron star or black hole). This means that there must be some 20 of these objects within 1 Kpc distance and some 80 within 2 Kpc distance. Adopting $M_V < -4^m$ for a star with $M > 15\ M_\odot$, one expects that some 20 of these binaries are present among the stars brighter than $m_v = 6^m$.

The main uncertainty in the number of systems with a quiet collapsed component is due to the uncertainty in the total number of O — B2 stars in the galaxy. Ostriker et al. (1974) estimate the total number of stars more massive than 15 $M_\odot$ in the galaxy to be $\sim$ 10000. Even with only 10000 stars more massive than 15 $M_\odot$, the number of quiet collapsar binaries is still expected to be around 1600 and about 10 of such systems are expected to exist among the stars brighter than $m_v = 6^m$. The absence of radio pulsars in binary systems might imply that most of the collapsed objects in these quiet binaries are black holes.

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Discussion of the communication of E.P.J. van den Heuvel

A. G. W. Cameron: 1. In regard to the behavior of a binary system in which a supernova explosion takes place, I would like to call attention to some work done by Colgate. Colgate found that the companion star, not undergoing the supernova explosion can receive a greatly enhanced momentum transfer when the ejected gases from the supernova impinge upon its surface. The shock progresses inwards below the surface, weakening as it goes, and eventually is reflected. This causes a great deal of mass in the forward facing hemisphere of the star to be violently ejected, thereby giving a large kick to the remainder of the star, and tending to disrupt the system. Colgate found that an approximately equal increment to the momentum obtained by the stellar remnant comes from each scale height of the mass which is ejected in the facing hemisphere. In this way the momentum transferred to the stellar remnant can exceed that brought in by the impinging supernova gases by an order of magnitude or more. From these calculations he found it difficult for a binary system to survive intact with a period of less than about one month.

I would also like to remind you that the typical pulsar seems to have a rather large proper motion, which is indicative of the disruption of a binary system.

2. At the present time there is a great deal of discussion concerning the manner in which carbon thermonuclear reactions ignite in a degenerate core composed of carbon. Supernova theories are in trouble because it appears that if the ignition of carbon thermonuclear reactions occurs explosively, then the entire star is likely to be disrupted, leaving no neutron star remnant. My guess is that there is a good chance that carbon thermonuclear reactions may ignite non-explosively. In such a case there is a good chance that the carbon thermonuclear reaction can burn out in the center of the star, to be followed by a carbon burning shell, with a continuation of stellar mass loss or mass transfer in a binary system. In such a case there is an excellent chance that a white dwarf remnant will be formed in the binary system, and hence that supernova explosions in binary systems may take place only in the most massive examples.

E. P. J. van den Heuvel: As to your first comment I wish to point out that there is a very large difference between the binary systems considered by Colgate and the ones which I consider here. In the systems which he considered it was the *more* massive component which exploded; the hydrodynamic effects to which he called attention made the disruption of these systems—which is already very easy when the more massive component explodes—even easier. However, in the picture which I outlined here, the exploding star is the *less* massive component, due to the preceding mass transfer. Even if one takes the kick due to the enhanced momentum transfer into account by using Colgate's equations, one finds that the probability for disrupting such systems is very small. This was shown by Sutantyo (to be published in Nature and Astron. Astrophys., 1974). Therefore, in my opinion, close binaries which underwent extensive mass exchange have no problem in surviving a supernova explosion of the more evolved component. As to your second remark: Paczynski's (1971b) calculations show that helium stars ignite carbon non-explosively. He also found that throughout helium and carbon burning helium stars more massive than 4 solar masses never expand to radii in excess of a few solar radii (whereas helium stars with smaller masses make excursions to the giant region). Therefore, and since the subsequent evolutionary stages are of negligible duration, such stars in binaries are unlikely to undergo further mass transfer. Consequently, as they are too massive to finish as white dwarfs, they are likely to blow up as supernovae. Since a 4 solar mass helium star in a binary was the mass transfer remnant of an approximately 16 solar mass main-sequence star this will indeed happen only in fairly massive binary systems; I stressed this also in my talk. In close binaries with primaries less massive than 16 solar masses there is indeed a fair possibility of finishing with a white dwarf. Before drawing definitive conclusions about the evolution of these lower mass systems, detailed calculations should be made.

G. Baym: On the basis of the picture you present, one would expect the neutron star to exhibit pulsar behavior in the "sleeping" stage, before the onset of the second stage of mass accretion. How can one then account for the fact that such pulsars are not observed? Of the 105 pulsars so far, none appear to be in binaries.

E. P. J. van den Heuvel: This is indeed a puzzling question to which I do not know the answer at this moment. My guess is that possibly some gas is present in these binaries which in some way absorbes or smears out the pulsar's radio emission or renders it otherwise unobservable. But I have not made any calculations to support this hypothesis. It is also possible that the majority of these quiet systems contains a black hole instead of a neutron star.

D. Pines: During the discussion following a presentation of the work of the Illinois group at the recent Texas Symposium, (Pines, Pethick, and Lamb, Proceedings of VIth Texas Symposium, to be published), Prof. Colgate raised just the objections cited by Prof. Cameron to the existence of neutron stars in close binary systems. I would give now the response I gave to Prof. Colgate then: the scenario which ties together the formation of a neutron star with the supernova explosion has not yet been rendered plausible by theoretical calculation. In other words, if we were to rely upon the theorists who have been attempting to calculate supernova dynamics (and such calculations are truly difficult), we would not believe in a supernova-pulsar association. It would therefore seem that we should treat with some reserve the suggestion that neutron stars cannot be made in close binaries, since if supernova theory has not yet reached the stage where it predicts a neutron star, it may well not be sufficiently reliable to tell us anything about neutron stars in binary systems.

F. G. Smith: We can be sure that none of the known 105 pulsars is a member of a close binary system. The only possibility that binary pulsars remain undetected lies at the short period end of the period distribution, say below 100 ms. Here the search techniques are restricted in sensitivity; there is also a pulse blurring effect on distant pulsars which may smooth out a rapid pulsation. We would also have some difficulty in finding a period modified by Doppler shift in a binary with orbital period of only an hour or two. However, the range of periods found for the X-ray sources would present no problems, and we are faced with a real question: why are there no binary pulsars?

E. P. J. van den Heuvel: Could it be that dispersion of the pulsar signal by gas in the binary system renders this signal unobservable?

F. G. Smith: The search techniques do not seem to be limited by the dispersion from interstellar electrons. There would, for example, be no difficulty in detecting a pulsar behind a typical large H II region.

ACCRETION ONTO MAGNETIC NEUTRON STARS: COMMENTS ON THE POSITION OF THE ALFVÉN SURFACE*

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I. INTRODUCTION

Professor Rees (1974) has described a " standard " model of compact X-ray sources in which the X-rays are produced by accretion onto either a neutron star or a black hole. If the central condensed object is a rotating magnetic neutron star, the Alfvén radius, r_A, at which the magnetic field of the neutron star first dominates the behavior of matter flowing towards the star, plays an important role in determining many properties of the model. Among these properties, we may mention the location of the inner edge of any accretion disk, the dimensions of the accretion columns above the stellar magnetic poles, and whether there is speeding up or slowing down of the X-ray pulsation frequency. The interplay between the position of the Alfvén surface and changes in the X-ray pulsation frequency, Ω (which is identified with the stellar angular velocity in the standard model), holds particular interest since Ω is a quantity immediately accessible to observation. If we can understand this interplay, the behavior of Ω may provide valuable information about the flow pattern of accreting matter around the neutron star and the size of magnetic and viscous torques acting near r_A.

A complete discussion of the flow of matter around the star, its effect on the magnetic field, and the flow of angular momentum, is a formidable task. No accurate description of the magnetic field structure and the flow of angular momentum is yet available, so that our present understanding relies on rough estimates. The possible situations which may arise divide naturally into two categories according to whether (i) the neutron star rotates slowly and accretes matter which has little

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angular momentum, or (ii) the neutron star rotates rapidly or accretes matter from a Keplerian disk which extends all the way down to r_A. These categories will be made more precise below. There seems general agreement on the correct criterion for determining r_A in the first situation. For situations which fall into the second category, on the other hand, various authors have proposed a number of different criteria for determining r_A. For a given set of X-ray source parameters (luminosity, neutron star mass, surface magnetic field strength, etc.) the different criteria give estimates of r_A which in certain cases may range over more than an order of magnitude. An uncertainty of this size in r_A is not important for some properties of the accreting neutron star model, but is in considering the effect of accretion on the angular velocity of the star, and the resulting X-ray pulsation frequency.

In the following sections, we first discuss the various criteria which have been proposed to determine r_A and then comment briefly on the interplay between the position of the Alfvén surface and changes in the X-ray pulsation frequency.

II. DETERMINATION OF THE ALFVÉN RADIUS

In the case of radial inflow of matter towards a non-rotating neutron star, the Alfvén surface occurs where the pressure in the magnetosphere balances the external pressure of the infalling matter. This criterion is

$$\frac{(B\,(r_A))^2}{8\pi} + P_m\,(r_A) \sim \rho(r_A)\,(v_r\,(r_A))^2 + P_a\,(r_A), \tag{1}$$

where B is to be identified with the stellar magnetic field strength, P_m is the particle pressure in the magnetosphere, $\rho v_r{}^2$ is the ram pressure of the infalling matter, and P_a is the (isotropic) thermal pressure in the infalling matter. Provided r_A is large compared with the stellar radius, we may estimate $B(r_A)$ by using the dipole expression.

In the case of a neutron star which is rotating or accreting matter with finite angular momentum, we expect the magnetosphere to have two different zones. There should be a zone corotating with the star, threaded by field lines which close within the corotation radius, r_{co}, and a second zone where accreting matter flows towards the stellar surface along field lines which extend beyond r_{co} to the neighborhood of r_A. It is the matter and magnetic field stresses in this second zone which speed up or slow down the star's rotation. The radii r_{co} and r_A are unlikely to differ from one another by as much as an order of magnitude, however, since the energy density of the dipolar component of the stellar magnetic field falls off very rapidly with radius, whereas in almost any picture the properties of the inflowing plasma change much more slowly with radius. This conclusion is not altered by the presence of screening currents flowing in the neighborhood of the magnetospheric boundary layer, since they lead only to corrections in the magnetic field strength, B, of the order of the dipole value there. Thus, although the character of the Alfvén surface of such a star will be more complex than in the previous case, as long as the angular velocity, Ω, of the star satisfies

$$\Omega << \Omega_K (r_A), \tag{2}$$

and the angular momentum of the matter approaching r_A is small enough so that

$$\Omega_a (r_A) << \Omega_K (r_A), \tag{3}$$

where Ω_a is the angular velocity of the accreting matter and Ω_K is the angular velocity corresponding to a Keplerian circular orbit at radius r_A, the value of r_A should not differ much from the estimate given by (1). This type of intermediate situation could arise if the neutron star is slowly rotating, in the sense of (2), and either (i) the accreting plasma has insufficient angular momentum to form a fully-developed thin Keplerian disk by the time it approaches r_A or (ii) plasma which forms a Keplerian disk at larger r acquires a radial velocity, v_r, comparable to the free-fall velocity, v_{ff}, as it approaches r_A.

If the neutron star is rotating rapidly ($\Omega \gtrsim \Omega_K (r_A)$) or an accretion disk extends all the way in to the Alfvén surface (so that $\Omega_a (r_A) \sim \Omega_K (r_A)$), then the situation is less clear-cut. The simple pressure-balance criterion (1) does not take into account a number of effects which are possibly quite important in this case. These include (i) the " viscous " drag on the stellar magnetic field which results from mixing of the accreting plasma and the magnetospheric plasma within the boundary layer, (ii) the interaction which (except in the aligned rotator case) exists between the magnetospheric boundary and the plasma just outside, due to the non-circular shape of the boundary, and (iii) the tendency of infalling plasma within the magnetosphere to shear the stellar magnetic field. We can obtain one criterion for r_A in this case by arguing that the stellar magnetic field will be able to control the accreting plasma only if the energy density of the stellar magnetic field exceeds the energy density of the inflowing plasma. This gives (Lamb, Pethick, and Pines, 1973)

$$\frac{(B (r_A))^2}{8\pi} \sim \frac{1}{2} \rho (r_A) (v(r_A))^2, \tag{4}$$

where again B is to be identified with the stellar field. [Note that the total velocity appears here, not just its radial component, as in (1)]. The same result can be obtained by arguing that non-uniformities in the motion of the plasma from one point to another cannot be removed by the magnetic field in a time shorter than that taken by a hydromagnetic wave to travel from one point to the other. Criterion (4) can be derived from hydromagnetic solutions for the flow inside the magnetospheres of aligned rotators (Lamb and Pethick, 1974), and we expect a similar result for oblique rotators.

A slightly different criterion for the Alfvén radius can be derived from angular momentum conservation. If one assumes (a) that viscous stresses are at most of the same order of magnitude as the magnetic and matter stresses at r_A and (b) that the net angular momentum flux across the Alfvén surface is at most of the same order of magnitude as the angular momentum flux transported by the magnetic and matter stresses separately, then the contribution to the momentum flux from the stresses in the distorted stellar magnetic field must be comparable to that from the matter stresses. This gives

$$\frac{B_r(r_A)\, B_\phi(r_A)}{4\pi} \sim \rho(r_A)\, v_r(r_A)\, v_\phi(r_A). \tag{5}$$

A difficulty in using (5) is that, in the absence of the solution for the situation at hand, one does not know the appropriate values for the parameters which appear in it (if the solution is known, then this type of order-of-magnitude estimate is, of course, unnecessary). We note that if $B_r(r_A)/B_\phi(r_A) \sim v_r(r_A)/v_\phi(r_A)$, then (5) becomes equivalent to (4). In general, the value of r_A given by (5) depends crucially on what assumptions one makes about the ratios B_r/B_ϕ and v_r/v_ϕ at r_A.

At this point, it may be useful to compare these criteria with some of the others which have been suggested in the literature. Davidson and Ostriker (1973) used the condition

$$\frac{(B(r_A))^2}{8\pi} \sim \rho(r_A)\,(v_r(r_A))^2, \tag{DO}$$

which is equivalent to (1) with $P_m = P_a = 0$, or (4) if $v_r \sim v$. They chose $v_r = v_{ff}$ at r_A and a value of ρ appropriate to accretion inflow over a solid angle (as seen from the neutron star) of $d\Omega/4\pi = 1/\sqrt{2}$. This is appropriate for an accretion inflow of the intermediate type, which does not form a fully-developed Keplerian disk. Using a surface magnetic field of 10^{12} gauss and their estimates of the other parameters together with a binary separation of 5×10^{11} cm (of the order indicated in HZ Her/Her X-1, for example) one does find $\Omega_a(r_A) < \Omega_K(r_A)$ for their model.

Pringle and Rees (1972) used the criterion

$$\frac{(B(r_A))^2}{8\pi} \sim P_a(r_A) \tag{PR}$$

to determine r_A in the case of a fully-developed thin Keplerian disk extending all the way in to r_A. This is equivalent to (4) if $\frac{1}{2}\rho v^2$ is replaced by P_g. However, since the condition $P_g << \frac{1}{2}\rho v^2$ is necessary for the existence of a thin disk, (PR) leads to a much larger value of r_A than does (4) if the other parameters of the model are equal.

Rees (1974) has given the criterion

$$\frac{(B(r_A))^2}{4\pi} \sim \rho(r_A)\, v_r(r_A)\, v_\phi(r_A). \tag{R}$$

If $B_r(r_A) \sim B_\phi(r_A)$, this condition is equivalent to (5); however, if the stellar field is strongly distorted by the plasma, so that either $B_\phi(r_A) >> B_r(r_A)$ or vice-versa, then the Rees condition will lead to a much larger value for r_A than will (5) if the other parameters of the X-ray source model are equal. The estimate of r_A given by (R) depends crucially on the assumed value of v_r/v_ϕ at r_A.

III. SPINUP OR SPINDOWN?

We have seen that the value obtained for r_A depends rather sensitively on what criterion one uses to determine r_A and on the ratio of B_r to B_ϕ at r_A. How does the position of the Alfvén surface affect whether the star is speeding up or slowing down in its rotation? A detailed account of the effect of accreting matter in causing spinup or spindown of the neutron star has been given elsewhere (Lamb, Pethick, and Pines, 1973); here we wish to make a few additional remarks about the effect of viscous and magnetic torques acting in the neighborhood of r_A and the role of the critical stellar angular velocity, Ω_c (at which centrifugal and gravitational forces are comparable for matter corotating with the star near r_A). Both the effect of torques and the value of Ω_c depend strongly on the value of r_A.

Consider first the torque on the neutron star due to the interaction of the stellar field with accreting matter near the Alfvén surface. If $\Omega < \Omega_a(r_A)$, the interaction will lead to spinup of the star in the direction of the circulation of the inflowing matter (here and below we adopt the convention that the angular velocity of the accreting matter is positive). In the case where Ω satisfies

$$\Omega_a(r_A) < \Omega < (r_A/R_g)^2\, \Omega_A(r_A), \tag{6}$$

where R_g is the radius of gyration of the neutron star, the angular velocity of the star exceeds that of the accreting matter at r_A, but the specific angular momentum of the accreting matter is greater than the average specific angular momentum of matter in the star. The net effect of viscous and magnetic torques associated with the infalling matter will therefore be to slow down the star's rotation only if most of the angular momentum of the matter at r_A is transported outwards from the Alfvén surface. This can happen if matter which is spun up by the star is flung out of the system or if it communicates most of its angular momentum to matter which is not accreted to the surface of the star [this communication may, for example, occur through an accretion disk (Lamb and Pethick, 1974)]. Otherwise, when matter at r_A to which angular momentum was earlier transferred later accretes to the stellar surface, it will simply give back that angular momentum to the star. In order for the net viscous and magnetic torques *associated with matter accreting to the stellar surface* to slow down the rotation rate of the star, the specific angular momentum of the accreting matter at r_A must be less than the average specific angular momentum of matter in the star, that is,

$$\Omega_a (r_A)\, r_A{}^2 < \Omega R_g{}^2. \tag{7}$$

This requirement is very restrictive: for $\Omega_a (r_A) \sim \Omega_K (r_A)$, as would be the case for matter in a Keplerian disk at r_A, Ω must, in order to satisfy (7), exceed the Keplerian angular velocity at the radius, R, of the surface of the neutron star by the factor $(R/R_g)^2\, (r_A/R)^{1/2}$.

Whether one can reasonably expect the rotation of the star to eject inflowing matter from the system depends on whether Ω is greater than or of the same order as the critical angular velocity of the neutron star, given by

$$\Omega_c \equiv \Omega_K (r_A) = (GM/r_A{}^3)^{1/2} \tag{8}$$

where M is the mass of the star, and therefore is rather sensitive to the value of r_A. In discussing the possibility of a self-regulating mechanism for the spin rate of an accreting neutron star, Davidson and Ostriker (1973) assumed that the angular velocity of the star could exceed Ω_c. The stellar surface magnetic field needed to achieve such a situation depends rather strongly on the character of the flow near r_A and on which criterion one uses to determine r_A.

IV. CONCLUDING REMARKS

It is clear from the preceding discussion, which has been entirely based on a macroscopic hydromagnetic or fluid point of view, that a number of important questions concerning the precise character of the Alfvén surface and the correct criterion for locating it are at present still unanswered when the neutron star is rapidly rotating or the accreting matter forms a thin Keplerian disk extending all the way down to the Alfvén surface. Further, we have seen that these questions cannot be separated from the question of what overall flow patterns of accreting plasma are possible (Can a thin Keplerian disk actually extend all the way down to the Alfvén surface? Is it possible for significant quantities of matter to be ejected from the system after reaching the neighborhood of r_A?). These unanswered questions are in turn reflected in uncertainties as to whether, in the case of a rapidly rotating neutron star surrounded by a disk, the neutron star model predicts a secular speeding up or slowing down of the X-ray pulsation frequency, or self-regulation at a constant frequency.

We have only so far hinted at a number of other questions at this same macroscopic level which are also likely important: (i) Is the matter flow pattern relatively steady (at least in some average sense) or, on the contrary, does it undergo large fluctuations? (ii) What are the effects of possible merging of swept-up magnetic fields in the accreting plasma with the distorted stellar field in the boundary layer, and to what extent is angular momentum transported by stresses in the swept-up field near r_A? (iii) To what extent does the difference between the stellar field configuration near the magnetic poles and that near the magnetic equator play a role in determining the flow pattern? (This should be reflected to some extent in differences between aligned and oblique rotators). (iv) Does the stellar magnetosphere ever develop an extended (thickness $\sim r_A$) turbulent boundary layer?

At a more detailed microscopic level, one can ask many more questions, such as the role of trapped particles in the boundary layer, and the detailed structure of the layer. If answers to questions at this level of detail are required in order to gain a basic understanding of the character of the Alfvén surface then, to judge from the number of such questions which remain unresolved in the case of Earth's magnetosphere, where they can be decided by observation much more easily than in the compact X-ray sources, there is little hope of gaining such an understanding in the near future. On the other hand, the success of the macroscopic point of view in predicting the gross features of the geomagnetosphere lends hope that answers at such a detailed level may not be necessary, and that calculations presently underway will be able to decide the flow pattern of accreting plasma near r_A and the correct macroscopic criterion for determining r_A.

Many of these ideas originated in discussions at Nordita, Copenhagen, and we are grateful to Professor B. Strömgren for the hospitality extended to us there. One of us (F.K.L.) wishes to acknowledge helpful discussions with Mr. R. Elsner and Drs. S. Hinata and D. Roberts, and to thank Professors G. Baym, D. Lamb, and D. Pines for critical readings of the manuscript.

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COMMUNICATION OF A.G.W. CAMERON

There is a very great difficulty in forming a neutron star in a binary system which remains in a close circular orbit about the companion star. On the other hand, it is very easy to form a white dwarf star in such a situation. Therefore it is very desirable to see if one can get pulsating x-rays from such a close white dwarf in a binary system.

Several years ago one of my students, Michael Mock, investigated the type of x-ray emission which might result from white dwarfs or neutron stars with vibrating atmospheres. He found that when he placed a spherical piston at the base of a white dwarf atmosphere and wiggled it with a period of two seconds and an amplitude of one per cent, he generated progressive waves which steepened into shocks in the atmosphere. When these shocks broke out of the atmosphere at an optical depth of about 1, a burst of x-rays came out. Since no pulsating x-ray sources were known at that time, these results were never published.

This idea was later picked up by Blumenthal, Cavaliere, Rose, and Tucker, who suggested just such a vibrating white dwarf atmosphere as the basis for a possible model of the x-ray pulsations of Centaurus X3. They noted that the pulsations could be driven by a hydrogen burning shell near the surface of the star, with infalling matter providing the hydrogen needed to supply the hydrogen thermonuclear reactions. It seems plausible that one should extend the same concept to the x-ray pulsations arising from Herculis X1.

Mock's calculations produced an average x-ray luminosity in the range of 10^{35} to 10^{36} ergs per second, two orders of magnitude below what is needed to explain the above two x-ray sources. A straightforward extrapolation of these results indicates that a luminosity of 10^{37} ergs per second would require a pulsation amplitude of 10 per cent. This would lead to a rapid rate of mass loss from the white dwarf atmosphere. This might be a very nice feature in the case of Herculis X1, because it provides a way of shutting down the observable x-ray emission during part of the 35 day cycle. If enough mass is lost so that the amplitude of the pulsation decreases, then the x-ray luminosity becomes too small to be detected, and accretion of additional material back on to the white dwarf envelope can take place. The x-ray luminosity might then intensify, and if the star has not stopped vibrating completely, then the new phase of the amplitude will be consistent with the old one. This is a kind of model which is relatively simple and has practically no adjustable parameter. Therefore theoretical calculations of the process can be more easily tested against the x-ray observations than is the case for accretion onto a rotating neutron star, where much of the physics remains unknown.

Dr. J. Littleton and I have started to reconsider the vibrating white dwarf model for these pulsating x-ray sources. We are starting to calculate the vibrational x-ray emission from white dwarfs which will have the observed vibrational periods of the x-rays. This establishes a relatively small number of white dwarf masses and radii, and hence surface gravities, to use as an input to the calculations. For example, if Centaurus X3, with a 4.84 second period, is vibrating in its fundamental mode, then it has a mass of 1.14 solar masses and a radius of 4.6×10^8 cm On the other hand, if it is vibrating in its first overtone mode, then its mass is 0.56 solar masses, and its radius is 8.3×10^8 cm. There have been a number of suggestions that the mass of the x-ray source in this system is closer to the latter value than to the former, although John Bahcall tells me that the data are not sufficiently good to establish this question unambiguously. In the case of Herculis X1, with a 1.24 second period, no fundamental mode is possible, but the first overtone mode would correspond to a mass of 1.2 solar masses, with a radius of 4×10^8 cm. However, it would be necessary to consider the possibility of vibration in the second overtone mode, for which the mass would be 1.1 solar masses and the radius 5×10^8 cm. There may be a tendency to have an excitation of higher vibrational modes, since the nuclear burning source lies close to the surface of the star, and the higher vibrational modes have a considerably higher amplitude in this region, with much less vibration amplitude near the central core.

The actual calculations will have to be carried out for white dwarfs having hot non-degenerate atmospheres, which are therefore distended to somewhat larger radii than given by the above figures. We will have to recalculate vibrational periods for such models, and there is a possibility that the actual vibrational period may change somewhat with time, both in the positive and the negative directions, if the temperature structure in this hot atmosphere should change. Similar changes could occur if there is a variation in the hydrogen to helium ratio in the non-degenerate atmosphere, with this ratio both increasing and decreasing as the matter is gained and lost from the white dwarf atmosphere as a result of accretion and vibrational ejection.

The test of these calculations will be whether the x-ray observations can be reproduced in a reasonable manner.

Discussion of the communication of A.G.W. Cameron

J. Bahcall: How can a pulsating model explain the fact that the X-rays turn-on at specific *orbital* phases?

A. G. W. Cameron: I do not know how any kind of model can explain the fact that x-rays turn-on at specific orbital phases. I would have to guess that the flow of mass from the primary star varies as a function of the orbital phase, and therefore that the mechanism of accretion, whatever it is, plays some role in determining the onset of such x-ray emission in a vibrating white dwarf model.

L. Woltjer: Di Gregoria and I have tried to understand the periods and period changes of Her X-1 on the basis of a model of an accreting (pulsating) white dwarf. For a white dwarf composed of 1.2 solar masses of pure helium the period of the first overtone is 1.2 sec. Accreting helium at such a rate that the accretion energy is equal to the X-ray luminosity we find a period decrease of 9 sec/year due to the mass increase of the white dwarf, slightly larger than the average observed value. If hydrogen is accreted a more extended envelope would presumably result and the period could increase. In intermediate situations some hydrogen burning could energize the pulsations; it is perhaps not inconceivable that the burning rate would not be constant and that this might lead to some period fluctuations. The maximum period decrease for Cen X-3 is about half the present Uhuru value. Di Gregoria also investigated the hydrodynamics of the accretion flow and the effects of the pulsation there on. An amplitude of about 10 % of the white dwarf radius appeared to be required in agreement with the results described by Dr. Cameron.

A. G. W. Cameron: In the case of a hot pulsating white dwarf star the period may vary due to changes in the structure of the atmosphere, since the eigenfrequency is fairly sensitive to the precise surface boundary conditions. For example, one would have to take into account also possible secular and perhaps irregular variations in the mean molecular weight of the material in the outer envelope of the vibrating star.

D. Pines: What about the change in $\dot{P}$ which has been observed by Giacconi et al to occur in Her X-1?

L. Woltjer: The period variability certainly remains hard to understand.

P. Ledoux: I wonder if, in the circumstances considered, non radial pressure modes and particularly the first p-mode corresponding to a spherical harmonic of degree 2 would not be more appropriate than purely radial modes as one would expect that, in a binary system, there would be a natural source of excitation for such a mode. As far as the periods are concerned, they would be of the same order as those of the first radial mode so that in that respect the agreement with observation would not be affected. On the other hand, amplification and maintenance of this oscillation by nuclear reactions in the external layers, although probably a little weaker than for a purely radial mode, would still be very efficient.

A. G. W. Cameron: This is an interesting suggestion and would also have the advantage of accounting for a double pulse in the case of the Hercules x-ray source.

B. Carter: Returning to the conventional accreting neutron star interpretation of the Her-X1 source, I would like to defend the idea that the 35 day period is controlled by the rate of matter flux out of the optical primary. It seems that the earlier talks have rather brushed this possibility aside in favour of alternative mechanisms in which the accretion rate is modulated by some kind of gate in the neighbourhood of the neutron star itself. As was emphasised earlier by the Bahcalls, the optical primary is of comparatively low luminosity so that the effect of heating by the secondary has a major effect on its luminosity. One of the difficulties in most of the models which have been proposed is to account for the relatively small variation in this heating effect during the 35 day cycle, despite the fact that the observed hard X-ray flux changes by orders of magnitude. (The surface area of the neutron star is much too small for soft thermal X-rays powered by thermonuclear energy release from the accreted matter to be able to provide sufficient heating).

In the original theory (due to McCray) in which the 35 day cycle was attributed to modulation of the outflow from the primary the same difficulty arose, but it has been overcome in a very satisfactory manner in the newer version of this theory due to Lin. The theory of Lin differs from that of McCray in that it postulates that the hard X-ray emission is cut off in consequence of an increase (rather than a decrease) in the matter outflow from the primary. One expects that there will be a critical maximum rate of accretion onto the neutron star corresponding to a maximum rate of energy generation which will probably be comparable in order of magnitude with the Eddington limit as was explained by Martin Rees this morning. Lin suggests that when the flux from the primary exceeds this critical rate, the excess matter will be ejected to form an extended cloud surrounding the neutron star. This cloud could absorb the hard X-rays emitted near the neutron star so that the accretion energy would ultimately emerge in the form of much softer X-rays whose frequency would be too low for them to have been observed, but which would be none the less effective in heating the primary. The existence of a positive feedback mechanism controlling the primary outflow as required by Lin's theory would appear to be at least as plausible as the negative feedback mechanism postulated by McCray.

OBSERVING NEUTRON STARS: INFORMATION ON STELLAR STRUCTURE FROM PULSARS AND COMPACT X-RAY SOURCES*

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INTRODUCTORY REMARKS

In this talk, I wish to discuss some of the ways in which observations on neutron stars can provide a guide to theoretical calculation, so that by combining theory with observation one might be able to distinguish between different classes of neutron stars (if indeed distinct classes exist) and go on to assign a mass, moment of inertia, oblateness, internal structure, magnetic field, etc. to a particular star. As we shall see, in this and the succeeding talks, the study of neutron stars is an ideal topic for a Solvay Congress, combining as it does elements of solid-state and low temperature physics, nuclear physics, many-body theory, plasma physics, high energy physics, geophysics, and general relativity; developing a consistent interpretation of existing observations may involve an equally broad range of disciplines. While nearly forty years have passed since Baade and Zwicky (1934) made their remarkable prediction of the existence of neutron stars, and their formation in a supernova explosion, it is only during the past five years or so, following the identification of pulsars as rotating neutron stars, that physicists from many of these different disciplines have joined together in developing a detailed physical picture of the structure of a magnetic neutron star and its co-rotating magnetosphere.

One year ago at an IAU Symposium, Ruderman, Shaham, and I endeavored to summarize our understanding of neutron star structure as derived from pulsar observations (Pines, Shaham and Ruderman, 1973); we concluded that the principal observational evidence for stellar structure came from the Crab and Vela pulsars, which have been observed to speed up suddenly on one or more occasions, and which display as well a generally restless behavior (Richards *et al.*,

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1969; Papaliolios *et al.*, 1970, 1971; Lohsen, 1972; Boynton *et al.*, 1972; Radhakrishnan and Manchester, 1969; Reichley and Downs, 1969, 1971). The theoretical interpretation of observations of the behavior of the pulse frequency in the period following a large speedup, or macroglitch, enables one to conclude that both superfluid neutrons and protons exist in the core of these stars, and provides as well an estimate of the stellar abundance of the neutron superfluid (Baym *et al.*, 1969). Consideration of the origin of both macroglitches and microglitches (the smaller frequency jumps which characterize the restless pulsar behavior) led us to the tentative conclusion that the Crab and Vela pulsars represent two distinct classes of neutron stars, in that they possess a quite different internal structure; one with a liquid core (the Crab pulsar), one with a solid core (the Vela pulsar).

During the past year, the theoretical arguments that the pulsating compact X-ray sources, Her X-1 and Cen X-3, are rotating neutron stars which are accreting matter from more massive binary companions, have grown increasingly persuasive (Pringle and Rees, 1972; Davidson and Ostriker, 1972; Lamb *et al.*, 1973; for a summary of such arguments see Pines *et al.*, (1973)). The identification of these X-ray sources as neutron stars (and the likelihood that many others will likewise turn out to be neutron stars) increases substantially the number of neutron stars for which observations might yield useful information on stellar class and interior structure; in particular, it offers the possibility of direct determination of stellar mass and free precessional motion. Her X-1 is an especially promising star in these respects, and indeed detection of precessional motion, if confirmed, already establishes that Her X-1 possesses a solid core (Pines *et al.*, 1973).

I shall consider three neutron stars, Crab pulsar, the Vela pulsar, and Her X-1 in turn; for each, I shall discuss some pertinent observations and the theoretical interpretation thereof. I shall then discuss briefly free precession in neutron stars, with particular attention to physical processes which may act to excite or damp such a mode. In discussing the origin of the Crab and Vela macroglitches, I shall confine my attention to starquakes. A number of alternative explanations (planets, magnetospheric instabilities, accretion, superfluid hydrodynamic instabilities) have been advanced for these speedups, and a critical review of such theories was presented at last year's IAU Symposium; rather than repeating the arguments, pro and con, I have included a brief summary as an Appendix to this paper.

Given the fact that a typical neutron star (cf. Figure 1) bears at least superficial resemblance to the earth [with a solid outer crust, a mantle (the neutron drip region), a liquid interior, and, perhaps in some cases a solid inner core], and that we shall be concerned with physical processes (such as starquakes and free precession) which have a well-defined terrestrial counterpart, it seems appropriate to examine such processes in the light of present day geophysical information, and we shall endeavor to do that wherever possible.

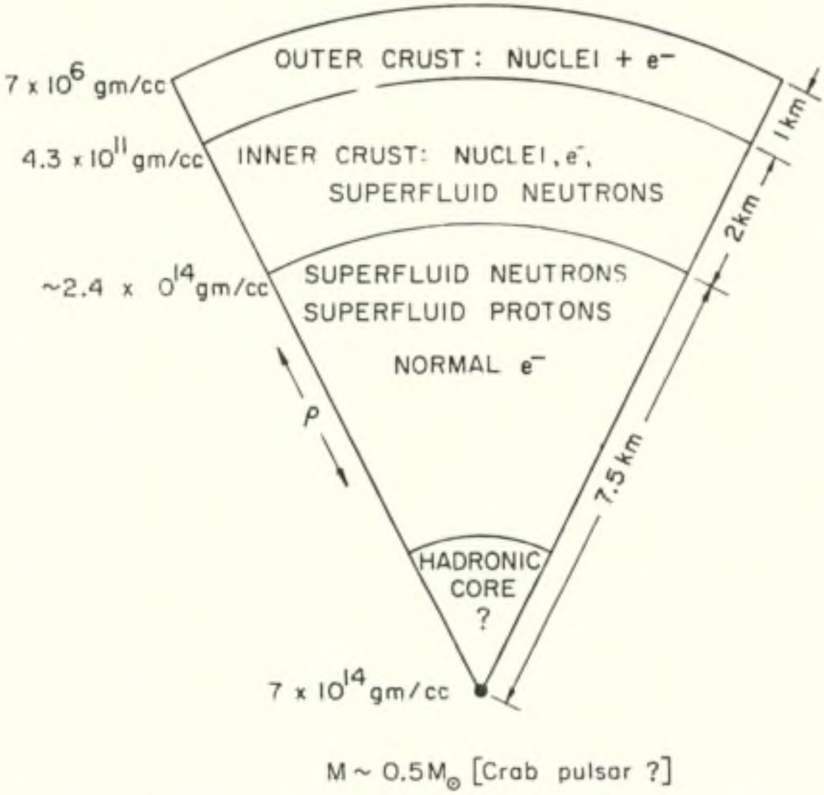

Fig. 1. Cross-section of a neutron star of mass ~ 0.5 M⊙ (Crab pulsar?)

THE CRAB PULSAR

The following observations, together with associated theoretical arguments, provide possible handles on the mass and internal structure of the Crab pulsar:

(1) The luminosity of the Crab nebula. The assumption that the rotational energy of the Crab pulsar is the sole source of power for the Crab nebula, together with estimates of the energy required to keep the nebula ablaze, in principle provides a determination of the pulsar moment of inertia (and hence its mass) (Gold, 1969, Ruderman, 1972).

(2) Short time-variations in the pulse frequency. These include the speedup of ~ Sept. 29, 1969, the post-glitch behavior, and a generally restless behavior of the pulsar, which has been shown to correspond to shot noise in the pulse frequency. (Boynton *et al.*, 1972; Groth, 1973). According to the two-component theory of postglitch behavior (Baym *et al.*, 1969a), the spin-up of the crust acts as an external probe of the interior neutron superfluid: study of the superfluid response provides strong evidence for the presence of both superfluid neutrons and protons in the stellar interior; it provides as well a measure of strength of the crust-superfluid coupling and the stellar abundance of the neutron superfluid. If one argues that the origin of the speedup is a starquake (Ruderman, 1969), one can place a lower limit on the pulsar mass (Baym and Pines, 1971), from the fact that no comparable speedup has yet been observed. If the restless behavior is attributed to starquakes resulting from a misalignment of the rotation axis and the elastic reference axis, one can estimate both the magnitude and direction of the magnetic torque responsible for such microquakes, and estimate the crustal critical strain angle as well (Pines and Shaham, 1972a).

(3) Long time variation (~ months to years) in the pulse signal. Let us consider these observational handles in more detail.

NEBULAR LUMINOSITY AND ENERGY BALANCE

The rate at which the Crab pulsar loses rotational energy is known with considerable precision; it is

$$\frac{dE_{rot}}{dt} = I\Omega\dot{\Omega} = \frac{I\Omega^2}{T}$$

which, upon substituting the observed values for Ω and the slowing-down time, T ($\equiv \Omega/\dot{\Omega}$), becomes

$$\left(\frac{dE_{rot}}{dt}\right)_{37} \simeq 4.6\ I_{44}$$

where the power supplied by the Crab pulsar is measured in units of 10^{37} ergs/sec, while the stellar moment of inertia is measured in units of 10^{44} gm-cm^2. The problem is that one does not know what fraction of this energy is required to keep the Crab nebula ablaze, since

(1) One cannot be certain of the absolute luminosity of the Crab nebula because its distance is not known to within a factor of two or so.

(2) One is not sure what fraction of the nebular luminosity must be supplied by the pulsar—is it only the energy radiated by the highest

energy (and shortest lived) electrons (5×10^{37} ergs/sec according to current distance estimates) or does one have to supply as well nebular radiation and the kinetic energy of expansion (4×10^{38} ergs/sec over-all, again using current distance estimates).

If we argue that a lower limit on the rotational energy loss of the Crab pulsar is 5×10^{37} ergs/sec, while an upper limit is $\sim 4 \times 10^{38}$ ergs/sec, we find

$$I_{44} \simeq 4.9 \pm 3.9 \text{ gm-cm}^2$$

and

$$M \simeq 1.1 \pm 0.8 \, M_{\odot}$$

on using the stellar structure calculations of Baym *et al.* (1971a). The range of Crab pulsar masses so deduced is essentially that permitted by the equation of state of Baym *et al.* (1971b). We can, however, derive some comfort from the fact that the mass range is that of expected neutron star masses, rather than lying outside it!

One can go farther, as Carter and Quintana (1973) have recently done, to argue that the existence of a lower limit on the stellar moment of inertia already serves to exclude some of the very soft equations of state which have been proposed for neutron matter (Leung and Wang, 1971), since the latter give rise to stellar moments of inertia which are $\lesssim 10^{44}$ gm cm^2.

SPEEDUP AND RESTLESS PULSAR BEHAVIOR: MACROGLITCHES AND MICROGLITCHES

Ideally, in analyzing Crab pulsar timing data, one would like to obtain the following information:

(1) the magnitude and time of successive macroglitches

$$\left[\left(\frac{\Delta\Omega}{\Omega}\right) \gtrsim 10^{-9}\right]$$

(2) the post macroglitch behavior

(3) the nature of any residual " restless " behavior.

As Boynton *et al.* (1972) have shown, all these aspects of the data are interrelated, so that it is non-trivial to carry out an analysis which distinguishes unambiguously among them. The most recent and thorough analysis of the data is due to Groth (1973) who concludes that:

(1) In the four years of optical timing observations (1969-1973) at Princeton and elsewhere, there has been only one macroglitch—the speedup of late September, 1969.

(2) Much of the remaining restless behavior which has been observed by both the Princeton group and other observers (e.g. Nelson *et al.*, 1970) can be regarded as a manifestation of frequency shot noise,

corresponding to microglitches which occur quite frequently (at intervals of hours to days, say) and which may represent spindowns as well as spin-ups.

(3) The speedup of initial magnitude, $(\Delta\Omega/\Omega)_o \sim 10^{-9}$, reported by Lohsen (1972) to have taken place in October 1971, is of quite different character than the Sept., 1969 speedup; no postglitch relaxation occurred, and indeed this speedup might be regarded as a particularly striking example of the shot noise observed in the pulse frequency throughout the span of radio and optical observations of the Crab pulsar.

According to Groth, the speedup of Sept., 1969 was of magnitude $[(\Delta\Omega_o/\Omega] = 8.8 \pm 3.8 \times 10^{-9}$ and took place on $14^h \pm 22^h$ Sept. 29, 1969. The postglitch behavior could be fit by the expression derived by Baym *et al.* (1969a)

$$\Omega(t) = \Omega_o(t) + (\Delta\Omega)_o\,[Qe^{-t/\tau} + (1 - Q)] \tag{1}$$

where $\Omega_o(t)$ is the extrapolated frequency in the absence of the glitch and

$$Q = 0.916 \pm 0.070 \tag{2}$$

$$\tau = 4.8 \pm 2.0 \text{ days} \tag{3}$$

As we have remarked, the appearance of a macroscopic relaxation time strongly suggests not only the presence of superfluid neutrons in the core of the Crab pulsar, but also the superfluidity of those protons which interpenetrate the neutrons (Baym *et al.*, 1969a, 1969b). Under these circumstances the coupling between the crust and the superfluid neutrons arises from the scattering of the core electrons (which are tied to the charged particles in the crust by the substantial stellar magnetic field) against that tiny fraction of " normal " neutrons located inside the cores (of radius $\sim 10^{-12}$ cm) of the vortex lines found in the rotating neutron superfluid. Feibelman (1971) has calculated this relaxation time, and finds it to be of the order of days for an energy gap in the neutron superfluid ~ 1.7 Mev. If the protons which interpenetrate the neutron superfluid were normal, this time would be reduced to a fraction of a second, while if the neutrons formed a " normal " degenerate quantum liquid, the electron-neutron interaction would spin up the neutron liquid in a time $\sim 10^{-14}$ sec. To the extent then that the macroscopic relaxation time observed following the Sept., 1969 macroglitch is correctly interpreted as the time required for the crust and interior neutrons to come into equilibrium with each other, one may conclude that one has both superfluid neutrons and protons in the stellar interior.

Q both furnishes a direct measure of the extent to which the glitch reduces the total stellar moment of inertia (cf. Eq. 11) and provides an indication of the stellar abundance of neutron superfluid, since

$$Q = (I_n/I)\left[1 - \frac{(\Delta\Omega)_n}{(\Delta\Omega)_o}\right] \simeq (I_n/I) \tag{4}$$

where I_n is the moment of inertia of the superfluid neutrons, and $(\Delta\Omega)_n$ is the initial jump in their angular velocity. The comparatively large value of Q observed for the Crab pulsar suggests that its interior consists mainly in superfluid neutrons; such stars are, however, not necessarily especially massive, since one finds $(I_n/I) \sim 0.9$ for a star as light as $\sim 0.5\ M_\odot$ (Table I).

TABLE I

Stellar Parameters for Neutron Stars with Liquid Interiors*

ε_{Crab} is the fluid oblateness and P_W(Crab) is the free precessional period of a star with the angular frequency of the Crab pulsar.

$M/M_\odot$	$I/10^{44}$ gm cm^2	I_n/I	$A/10^{52}$ (ergs)	$B/10^{48}$ (ergs)	$\varepsilon_{Crab}/10^{-4}$	ω_q^2 (s^{-2})	P_W(Crab)
0.15	0.60	0.16	0.11	23.1	5	1.7×10^9	.58 h
0.20	0.70	0.41	0.19	16.4	3	6.3×10^9	2.1 h
0.25	0.84	0.56	0.32	10.1	2	2.4×10^{10}	7.9 h
0.30	1.1	0.72	0.45	7.9	2	4.7×10^{10}	16 h
0.46	1.5	0.86	7.7	3.1	7	5.2×10^{11}	7.3 d
0.80	3.1	0.97	3.2	0.94	0.9	7.0×10^{12}	98 d
1.08	4.7	0.98	5.8	0.40	0.7	3.7×10^{13}	500 d
1.41	5.9	1	11.4	0.11	0.5	4.0×10^{14}	5500 d

* from Pines and Shaham, 1972a.

Starquake theory suggests a somewhat lower mass for the Crab pulsar. We consider first a one-parameter description of elastic energy storage and release (Baym and Pines, 1971) in which quadrupolar stellar deformations are described by a single time-dependent distortion parameter, the crustal oblateness, ε, defined according to:

$$I = I_o (1 + \varepsilon) \tag{5}$$

The time-varying portion of the mechanical energy is

$$E = -(L^2/2I_o)\,\varepsilon + A\varepsilon^2 + B(\varepsilon_o - \varepsilon)^2 \tag{6}$$

where A and B measure the gravitational energy and elastic energy stored in the star, and ε_o is the *reference* oblateness, which changes only as a result of plastic flow or starquakes. For a self-gravitating incompressible sphere one has

$$A = (3/25)\,(GM^2/R)$$
$$B = (57/50)\,\mu V_{cr}$$

for a star of radius R, with crustal material of volume V_{cr} and shear modulus μ. The equilibrium value of the oblateness is obtained by minimizing the energy, (6), with respect to ε, and is

$$\varepsilon = \frac{I_o\Omega^2}{4\,(A + B)} + \frac{B}{A + B}\,\varepsilon_o \tag{9}$$

$$\cong I_o\Omega^2/4A \qquad \text{(liquid core star)} \tag{10}$$

since $B << A$ for all stable neutron stars with liquid cores.

If one attributes the speed-up of September, 1969 to a starquake which took place in the solid outer crust of the pulsar, then it represented a change in the stellar oblateness, ε and moment of inertia I of

$$\Delta\varepsilon = (\Delta I/I) = -(\Delta\Omega)_\infty/\Omega = -(1 - Q)\,[(\Delta\Omega_o/\Omega)]$$
$$= -(9 \pm 4) \times 10^{-10} \tag{11}$$

where $(\Delta\Omega)_\infty$ is the glitch-induced long-term increase in the pulsar crustal rotation frequency and we have taken $Q \sim 0.9$. Such a change corresponds to shrinking the entire crust of the pulsar by some 4 microns; that we can observe directly such small changes in the radius of a pulsar located several light years away is a measure both of the accuracy of the pulsar timing measurements and the extraordinarily high density of the neutron stars.

According to Table 1, the oblateness of a neutron star rotating at the angular velocity of the Crab pulsar is in the range $1 - 2 \times 10^{-4}$ for masses which range from $M_\odot$ to $0.3\ M_\odot$; the quake-induced change in the oblateness corresponds, then, to a change of several parts in a million. The corresponding fractional energy release in the starquake is much larger, and sensitive to the assumed critical strain angle $[\sigma_c/\mu = (\varepsilon_o - \varepsilon)]$ and stellar mass. For a model star of $\sim 0.3\ M_\odot$ with crustal material characterized by a strain angle of 5×10^{-4}, the total reservoir of elastic energy is some 2×10^{42} ergs, while the energy release for a quake which would give rise to the Sept., 1969 Crab speedup would be

$$\Delta E = 2B(\varepsilon_o - \varepsilon)\,\Delta\varepsilon_o = 2B(\sigma_c/\mu)\,\Delta\varepsilon_o = 2A(\sigma_c/\mu)\,\Delta\varepsilon$$
$$\cong 4 \times 10^{39} \text{ ergs} \tag{12}$$

on making use of (9) and Table 1. The question naturally arises as to whether it is reasonable to expect quakes of this size to occur with some frequency (i.e., every few years) in the crust of a neutron star.

This question is posed still more starkly if one judges such starquakes by terrestial standards (Pines and Shaham, 1972b). Compared to a neutron star with a liquid interior, the earth has a much more substantial cœfficient of elastic energy vis-a-vis gravitational energy ($B \sim 1.3 \times 10^{39}$ ergs $\cong 4A$), while the elastic energy released in the biggest (Richter magnitude, 8.7) earthquakes, $\sim 5 \times 10^{24}$ ergs, is but a tiny fraction of the earth's elastic energy reservoir, $\sim 10^{32}$ ergs; moreover, such a quake gives rise to a change in the ε (and of the length of the day) which is at most a few parts in 10^{10}. Compared to the earth, then, a starquake big enough to explain the Sept., 1969 Crab pulsar spin-up, is a very big quake indeed (corresponding to a Richter magnitude of ~ 19); it would further seem necessary that a considerable portion of the strain energy released in one quake be replaced before the next, if the starquake era is to persist for any reasonable length of time ($\sim$ thousands of years, say...).

Pulsar spin-down offers a natural way of building up strain energy in the stellar crust since, according to (10), the stellar oblateness is proportional to Ω^2. The time, $t_q(\Delta\varepsilon)$, to replenish the strain ($\Delta\varepsilon_o = (A/B)\,\Delta\varepsilon$) released in a quake of magnitude $\Delta\varepsilon$ is

$$t_q(\Delta\varepsilon) = T\,(\omega_q^2/\Omega^2)\,|\Delta\varepsilon| \qquad (13)$$

where T, the pulsar slowing down time, $\sim$ 2500 y for the Crab pulsar and

$$\omega_q^2 = 2A^2/BI_o \qquad (14)$$

is a rapidly varying function of stellar mass (Table 1). For a star of $0.3M_\odot$, the strain, $\Delta\varepsilon_o \cong 5 \times 10^{-7}$, released in the Sept., 1969 event would be built up in $\sim$ 2.9 years. Assuming that once its strain energy is replenished, a subsequent large Crab quake would take place, we conclude that $M_{crab} \gtrsim 0.3\ M_\odot$; on the other hand, assuming that such a speedup is not an altogether unique event, one can put an upper limit on the pulsar mass by arguing that a further substantial speedup ($\Delta\varepsilon \sim 10^{-9}$) is to be expected in the next decade; this gives a Crab pulsar mass $\lesssim 0.37\ M_\odot$.

The above arguments suggest only a comparatively narrow range of Crab pulsar masses; however, that range is broadened once one takes into account the possibility that

(1) Not all the energy released in a starquake necessarily goes into heat; physical processes associated with an appreciable fraction of that energy release may act to build up stresses in another part of the stellar crust.

(2) A decrease in ε is not the only way to build up strain energy in the crust; Dyson (1970) has suggested that as a consequence of vulcanism mountains may form on the stellar surface to such a height that one gets a starquake, Goldreich (1970) has considered the buildup of strain energy as a result of a misalignment of the rotation axis and the magnetic field axis, while Shaham and I have examined a comparable effect associated with the misalignment of the rotation axis and the elastic reference axis (Pines and Shaham, 1972a).

All of these effects can act to decrease the time between crustquakes and hence would suggest the possibility of larger Crab pulsar masses than those arrived by considering only rotationally-induced strain buildup.

On the basis of his recent analysis of the Crab timing data, Groth (1973) concludes that the average value of the rate of frequency jumps times their magnitude squared is

$$R_J < \Delta\Omega^2 < \sim 3.4 \times 10^{-21}\ s^{-3} \qquad (15)$$

a value in good agreement with that earlier deduced by Boynton *et al.* (1972). Microquakes induced by the above-mentioned angular strains in the stellar crust represent a possible explanation for this frequency noise. As discussed in Pines and Shaham (1972a), since a crustquake induced by purely angular strains (a "Θ" quake) involves a much smaller area of the stellar surface than one associated with oblateness strains, the magnitude of the quakes will be correspondingly smaller; such a "Θ" quake will give rise to a spindown, involving as it does a crustal motion which tends to orient the elastic reference axis toward the instantaneous axis of rotation. When one takes into account both oblateness and angular strains, one finds that such microquakes can give rise to either spin-up or spin-down, depending on the quake geometry and the relative importance of the two kinds of strains. To the extent that there exist substantial magnetic torques which are nearly perpendicular to the rotation axis, the angular strain energy may be expected to build up much more rapidly than the oblateness strain energy; as a result microquakes of magnitude and frequency sufficient to explain the observed frequency noise are required in order to keep the over-all strain energy within reasonable bounds. Such microquakes, it should be noted, are still quite substantial compared to even the largest earthquakes; for a star of mass 0.3 $M_\odot$ in a microquake of magnitude $\Delta\varepsilon \sim 10^{-12}$ as much as 4×10^{36} ergs of elastic energy may be released. From (15) we see that quakes of this size could, in principle, occur at hourly intervals; from a seismic point of view, the crust of the Crab pulsar would then be a very active place indeed!

SYSTEMATIC LONG TERM VARIATIONS IN PULSE SIGNAL: FREE PRECESSION?

If one examines the residual phase as a function of time over a comparatively long time interval ($\sim$ months, say) it would seem that there exist systematic sinusoidal variations in the pulse arrival time; indeed such observations by Richards *et al.* (1969) led Ruderman (1970) to suggest that these might be attributed to a free precession of the Crab pulsar, analogous to the Chandler wobble of the earth. [Such precession (and ways of exciting it) had been earlier considered by Goldreich (1970)]. If, however, one then considers a longer time span of the data, the periodicity in question is no longer so evident and may, in fact, be replaced by one involving a still longer characteristic period. As Groth (1973) has emphasized, this is just the behavior one would expect for random frequency noise, and indeed with such a model one can explain all long term quasi-periodicities in the pulsar frequency thus far observed.

This is just as well, for observation of free precession of the Crab pulsar with a period of even a few months would be difficult to reconcile with the mass of $\lesssim 0.8\ M_\odot$ which has been inferred from postglitch behavior and starquake theory. The free precessional frequency is (Pines and Shaham, 1972b)

$$\Omega_W = \frac{3}{2}\frac{B}{A+B}\varepsilon_o\Omega; \tag{16}$$

for a star with a liquid interior, the reference oblateness, ε_o, is close to its fluid value, $I_o\Omega^2/4A$, so that one has

$$\Omega_W = \frac{3}{4}\Omega^3/\omega_q{}^2 \tag{17}$$

From the values of $\omega_q{}^2$ cited in Table 1, one derives the wobble periods likewise cited there: because the Crab is rotating comparatively rapidly we see that in order to get a wobble period as long as a year one must go to a neutron star of mass $\sim 1\ M_\odot$. Such a star has, however, such a small crustal volume that the time between rotationally-induced speedups would seem far too long to be compatible with observation.

A similar problem arises with the recent suggestion of Cocke *et al.* (1973) that precession may be responsible for possible systematic secular variations in the optical polarization of the Crab pulsar. Although no secular variations in the light curve have been detected, Cocke *et al.* suggest that free precession could affect the angular parameters specifying the orientation of rotating vectors which describe the optical polarization, without influencing the light curve directly. This interpretation requires a free precessional period of ~ 2 years; according to Table 1, this would imply a pulsar mass of $> 1\ M_\odot$, a result again in conflict with the mass inferred from postglitch behavior and starquake theory. Since one has only some three years of observations of the optical polarization it may be that the secular variation observed is not systematic, but rather a further consequence of the physical processes (microquakes?) responsible for the observed frequency noise in this pulsar.

THE VELA PULSAR

The Vela pulsar was first observed to speed up between February 26 and March 3, 1969 (Radhakrishnan and Manchester, 1969; Reichley and Downs, 1969). Equation (17) of the two-component theory provides a good fit to the observed postglitch behavior (indeed it was developed in an attempt to explain that behavior), with the following glitch parameters

$$\frac{(\Delta\Omega)_o}{\Omega} = 2.3 \times 10^{-6} \tag{18}$$

$$\tau \simeq 1.2 \text{ yrs} \tag{19}$$

$$Q \simeq 0.145 \tag{20}$$

In August, 1971 the pulsar was observed to speed up again (Reichley and Downs, 1971). The magnitude of the glitch was somewhat smaller

$((\Delta\Omega)_0/\Omega \simeq 2 \times 10^{-6})$, with postglitch behavior which resembled qualitatively that observed for the earlier speedup; as of this writing the analysis of the data has not yet been carried out with sufficient accuracy to permit a quantitative test of the two-component theory.

The value of Q which is observed suggests that the Vela pulsar has a structure (and mass) quite different from the Crab pulsar; if we assume that the pulsar has a liquid core, and that $Q \sim I_n/I$, then the solid outer crust extends far more deeply into the stellar interior; according to Table I, the pulsar mass would be $\sim 0.15\ M_\odot$, corresponding to an outer crustal radius of 17 km and an inner crustal radius 4.3 km. Such a star is at the edge of the stability limit; as Ruderman (1972) has pointed out, masses less than $0.15\ M_\odot$ are bound less tightly than white dwarfs with the same number of baryons, and are unbound, as well, relative to the same amount of matter dispersed as helium gas. It would therefore not be surprising if τ, which is quite sensitive to the effective energy gap of the neutron superfluid, were substantially longer than that observed for the Crab pulsar.

On the other hand, the magnitude of the two observed speedups poses a problem. For a $0.15\ M_\odot$ neutron star rotating at the angular velocity of the Vela pulsar the calculated oblateness is $\sim 7 \times 10^{-5}$; if we interpret the two speedups as due to crustquakes, the fractional charge in ε in each quake is $\sim 3.4\ \%$, an alarmingly large value. (For a star of mass $0.3\ M_\odot$, it approaches 10 %). Matters get only worse if we attempt to attribute the origin of the quakes to rotationally-induced strain, and assume that the strain energy released in one quake is replenished before the next. The strain released in a Vela quake is much larger than in the Crab, while both the slowing down rate, T, and the rate at which strain is replenished $(\alpha\Omega^{-2})$ are an order of magnitude smaller; as a result the time to replenish the strain released in the March, 1969 event is of the order of millions of years, rather than the two years which passed before a further large speedup.

The suggestion that the heavier neutron stars might possess a solid inner neutron core (Anderson and Palmer, 1971, Canuto and Chitre, 1973) prompted Ruderman, Shaham and me to suggest that if the Vela pulsar possessed a solid core, one might be able to understand its frequent large speedups as resulting not from crustquakes but from corequakes which represent the sudden release of elastic energy stored in the *inner* solid neutron lattice (Pines, Shaham, and Ruderman, 1972). Let us first note that the presence of a solid core leads to a significant reduction in the fraction of superfluid neutrons, Q, measured in postglitch behavior, since the solid neutron core corotates rigidly with the crust and interior charged particles within a microscopic time. If, for example, neutron solidification begins at densities $\sim 1.5 \times 10^{15}$ gm/cc, then it follows that one gets $(I_n/I) \gtrsim Q = 0.15$ for a neutron star of mass $\sim 0.7\ M_\odot$.

Next we note that the size of the Vela glitches no longer presents a great problem; because the shear modulus of the solid neutron lattice is very large ($\mu \sim 10^{35}$ dyne cm^{-2}), the elastic energy cœfficient, B, can easily be an order of magnitude larger than the gravitational energy cœfficient, A. As a result, the stellar oblateness $\varepsilon \simeq \varepsilon_0$, the "rigid-body" reference oblateness, which changes only in starquakes. In the time since the formation of the Vela pulsar ($\sim 10^4$ years), an initial core oblateness of order $\sim 10^{-2}$ would thus have been reduced by only an order of magnitude or less by starquakes of magnitude $\Delta\varepsilon \sim 10^{-6}$ which take place every few years. In each such starquake the fractional changes in the oblateness now assumes a more reasonable value ($\Delta\varepsilon/\varepsilon \sim 10^{-4}$) so that such corequakes may be regarded as involving only a small part of the equatorial bulge, rather than the several percent required on the crustquake interpretation.

One can invert the order of the above arguments, to infer the presence of a solid inner core for the Vela pulsar solely from the observation of such large speedups. If we accept the interpretation of the speedups as produced by events which take place inside the neutron star [and, as discussed in Appendix A, at the present stage of our understanding such an interpretation is to be preferred over the alternative—that they are produced by events which occur in the corotating magnetosphere], then the frequency of such large oblateness changes strongly suggests that the current oblateness of the Vela pulsar must be $\sim 10^{-2}$ to 10^{-3}. Inspection of the expression (9) for the stellar oblateness shows that such large oblateness is only possible if the rigid body contribution to ε is large compared to the fluid contribution, that is, if

$$\frac{B\varepsilon_0}{A + B} >> \frac{I_0\Omega^2}{4(A + B)} \lesssim 7 \times 10^{-5}$$

Since $\varepsilon \sim \varepsilon_0$, this in turn requires that

$$B \sim A \simeq 10^{52} \text{ ergs,} \tag{21}$$

or what is equivalent,

$$\mu_{\text{solid}} \sim 10^{35} \text{ dyne cm}^2. \tag{22}$$

It is then straightforward to show that such a high rigidity is only found in solid matter in which the characteristic interaction energy is ~ 100 MeV/baryon, that is, in a lattice made up of strongly interacting particles at densities equal to or greater than that found in nuclear matter—hence a solid stellar core. Let us emphasize that the solid inner core need not be made up of neutrons; a solid hyperon core would provide an equally satisfactory basis for explaining the Vela speedups.

According to the above interpretation, the Vela pulsar would resemble the earth. Its seismic behavior would likewise resemble that of the earth (but on a suitably astronomical scale); elastic energy is stored primarily as a consequence of cataclysmic events which take place at an early stage in its geological history, (perhaps upon solidification of the inner core) and the corequakes which release elastic energy may be triggered by a wide variety of physical processes, from crustquakes to magnetic torques associated with pulsar activity. Thus a given corequake, in addition to releasing strains present in a particular region of the equatorial bulge, may produce weak spots in the adjacent regions; a slight additional stress, perhaps produced by pulsar spindowns, would then suffice to set off a corequake in one of these regions, and so on. One would thus have a continuing chain of corequakes, in which the buildup of strain energy required to trigger a quake is far less than the energy released in the preceding quake.

If we assume that $(\sigma_c/\mu\varepsilon_o)$ has a value not dissimilar to that found on earth ($\sim 1/7$), we find that the fractional elastic energy release in the recent corequakes is of the order of 0.1 %, a very large fraction indeed. More specifically, assuming $\varepsilon_o \sim 10^{-2}$, $\sigma_c/\mu \sim 2 \times 10^{-3}$, we find a current elastic energy reservoir of some 4×10^{47} ergs, while the energy released in the recent corequakes would be 8×10^{44} ergs. If, however, we allow for the possibility that physical processes resulting from corequakes in one region act to build up elastic energy in another, and that additional elastic energy build up may take place as a result of pulsar torques, we conclude that the corequake era may well continue for another 10^4 years or so. Thus corequakes, like crustquakes, are to be expected only for comparatively young pulsars.

HERCULIS X-1

Assuming Her X-1 to be a neutron star accreting matter from its more massive companion, HZ Her, there are two possible observational handles on its mass and internal structure:

(1) From the observed period (1.24^s) and Doppler shift of the X-ray pulsations, Tananbaum *et al.* (1972) have obtained the following mass function for the binary system:

$$\frac{M_{HZ}^3 \sin^3 i}{(M_1 + M_{HZ})^2} \simeq 0.85 \, M_\odot \tag{23}$$

here M_{HZ} is the mass of HZ Her and M_1 is the mass of Her X-1. To the extent, then, that one can determine from observation both M_{HZ} and the orbital inclination, i, one can then determine the mass of Her X-1.

(2) If one attributes the 35^d period of the high-low X-ray cycle to a free precession of Her X-1, then from the magnitude of that period alone one can conclude that Her X-1 possesses a solid inner core (of hyperons or neutrons) and an oblateness $\sim 2.6 \times 10^{-7}$. [Pines, Pethick, and Lamb, 1973].

The possibility of a direct mass determination through (23), while at first sight appealing, wanes upon closer examination, since neither sin i nor M_{HZ} are known with sufficient accuracy to obtain a mass assignment for Her X-1 which is much more meaningful than, say

$$M_1 \simeq 1.1 \pm 1\ M_\odot \tag{24}$$

Thus while from the duration of the X-ray eclipse one can conclude that $\sin^3 i \gtrsim 0.5$ (and one may well have $\sin i \simeq 1$), there is such a large scatter in the measured optical absorption-line velocities (Crampton and Hutchings, 1972) that this method when combined with the above estimate of $\sin^3 i$ has not thus far led to a better than order of magnitude estimate of M_1. In similar vein one doesn't know to what extent the spectral type of HZ Her at minimum light (late A or early F) is an accurate indicator of the nature of that star, since there is an excellent possibility that even under these conditions its spectral behavior is influenced by Her X-1; hence this method of determining M_{HZ} is not promising. Finally, since we do not know to what extent HZ fills its Roche lobe, use of a mass formula based on the assumption that it does so is likewise open to serious question. [For a recent discussion of these various methods of mass determination, see, for example, Avni *et al.*, 1973].

Although the explanation proposed by Brecher (1972) for the 35^d high-low X-ray cycle—that one is seeing the precession of an always-on X-ray beam in such a way that the beam strikes the earth for $\sim 12^d$ and misses it for $\sim 23^d$—does not hold up under close scrutiny (cf. Pines, Pethick, and Lamb, 1973), free precession of Her X-1 continues to be an appealing clock mechanism for the following reasons:

(1) There are a number of ways in which free precession, by changing the angle between the magnetic and rotation axes, could control the accretion of matter onto the stellar surface.

(2) As a result, it is quite possible that mass is transferred from HZ Her to the Roche lobe of Her X-1 at a comparatively steady rate, with a magnetic accretion gate, controlled by the free precession of Her X-1, which permits accretion of matter onto the stellar surface only during the high part of the 35^d X-ray cycle.

(3) Theoretical interpretations of both the "marching" of the dip in the X-ray intensity (Giacconi *et al.*) and the 35^d variations in the optical appearance of the binary system (Boynton *et al.*, 1973) likewise favor a regulatory clock mechanism associated with Her X-1—and free precession is perhaps the only viable candidate for a 35^d neutron star clock.

If the period for the free precession of Her X-1 is, then, 35^d, one has

$$(\Omega_W/\Omega) \sim 4 \times 10^{-7}, \tag{25}$$

for the ratio of the precession frequency to the rotation frequency, while the theoretical value for this ratio is given by (16). Assuming Her X-1 to be a neutron star with a liquid interior, one has $B << A$ and a reference oblateness, ε_o, which is close to the "fluid" value, $I_o\Omega^2/4A$, so that one finds

$$(\Omega_W/\Omega)_{max} \sim 3 \times 10^{-9}, \tag{26}$$

this maximum occurring for a light star of mass $\sim 0.2\ M_\odot$. Hence free precession of a neutron star with a liquid interior can be ruled out as the mechanism for the 35^d clock. On the other hand, if Her X-1 has a solid neutron or hyperon core, one would have $B >> A$, and

$$\Omega_W/\Omega = 3/2\ \varepsilon_o, \tag{27}$$

its "rigid-body" value. The value of ε_o ($\simeq 2.6 \times 10^{-7}$) then required to explain the result, (25), is consistent with a number of possible stellar histories.

The existence of more than a single candidate for a "hard-core" neutron star has already served as a considerable stimulus for theoretical work on the possibility of the solidification of neutron matter at densities $\gtrsim 5 \times 10^{14}$ gm/cc. As we shall hear in some detail from Pandharipande (1973), some such stimulus may be required to encourage further theoretical work on this problem, in view of the fact that the formation (or lack of formation) of a neutron lattice depends on a number of quite subtle physical effects, so that the calculation of the liquid-solid energy difference is a delicate affair at best.

FREE PRECESSION OF NEUTRON STARS*

Because detection of free precession represents one of the most promising methods of determining both the mass and internal structure of a neutron star (and, as we have just remarked, such precession may indeed have already been seen for Her X-1) it seems worthwhile to examine it in further detail. I would like to summarize briefly some work in progress which Jacob Shaham and I have been carrying out on ways of exciting and maintaining such precession, as well as mechanisms which will act to damp it (Pines and Shaham, 1973b). This problem has been considered earlier, by Henriksen *et al.* (1972), whose starting assumptions and conclusions differ somewhat from ours; let me postpone until later a discussion of these differences.

To put the problem in a terrestrial perspective, let us note first that eighty years after the discovery of the free precession of the earth by S. C. Chandler in 1893, the mechanism which is responsible for exciting this Chandler wobble remains in doubt. It is now generally believed that the dissipation in shallow oceans of energy associated with polar

* Work carried out in collaboration with Jacob Shaham.

tidal motions is sufficiently great that left to its own devices the Chandler wobble would damp in less than 100 years (Munk and McDonald, 1960). Yet after almost 100 years of observation, the amplitude of this wobble [$\sim$ 6 meters (10^{-6} radians)] remains essentially unchanged; a mechanism to pump the Chandler wobble is clearly required.

Of the various suggested terrestrial pumping mechanisms, three are of particular interest to the astrophysicist interested in pumping the wobble of a neutron star:

(1) Random pumping by earthquakes [Cecchini (1928); Smylie and Mansinha (1968)]

(2) Random pumping by the core-mantle interaction [Runcorn, 1970]

(3) Resonant pumping by earthquakes as a result of in-phase release of angular elastic energy, triggered by the Chandler wobble itself. [Pines and Shaham, 1973a].

Of these, the first mechanism turns out to be too inefficient to maintain the Chandler wobble at its current amplitude; its analogue may, however, be operative for a comparatively young neutron star with a solid core (the Vela pulsar?). To the extent that our interpretation of the giant Vela spin ups as produced by corequakes is correct, such corequakes could excite a substantial stellar wobble through the discontinuous jumps in the center of nutation they bring about.

An analogue of the second mechanism—random angular momentum transfers between the core and crust of a neutron star—could well act to maintain the precession of a neutron star with a liquid interior (the Crab pulsar?). We lack, however, any physical picture for what might be causing such transfers (which would be reflected as frequency jumps analogous to the changes in the length of the day observed for the earth); moreover, because of its randomness, such a mechanism is comparatively inefficient, so that pumping a large amplitude stellar wobble in this fashion is difficult.

An analogue of the resonant pumping mechanism might be operative in a neutron star like Her X-1, for which we have suggested that the free precessional motion of the star acts to regulate the accretion of matter to the stellar surface. If corequakes in the solid core of Her X-1 are triggered by such accretion (and crustquakes could provide the catalyst needed to bring this about), we would then have a quite efficient self-regulating system: on the one hand, stellar wobble regulates the accretion of matter to the stellar surface (which we see on earth as a high X-ray state); on the other hand, corequakes triggered by that accretion act to maintain the core wobble. Although the direct detection of such corequakes would be difficult (even in the case of a frequency jump, $\Delta\Omega/\Omega$, as large as 10^{-6}), such quakes could be indirectly detected through the changes in amplitudes of the stellar wobble they produce and may indeed have already been detected. For a sizeable jump in the amplitude of the stellar wobble would alter the pattern of the high-low X-ray states, so that one would observe the high X-ray state for a greater (or smaller) number of days within the $\sim 35^d$ over-all cycle. Since such shifts in the high-low X-ray cycle have been frequently seen, it is tempting to attribute them to shifts in the amplitude of the stellar wobble; more work is, however, needed before such an interpretation can be taken seriously.

In a pulsar one has an additional mechanism which might be responsible for exciting stellar wobble—the radiation torque associated with the pulsar magnetic field. Goldreich (1970) has examined the change in nutation amplitude brought about by the electromagnetic torque associated with magnetic dipole radiation (Davis and Goldstein, 1970); he finds that free nutation grows or damps on the slowing-down time scale, ($T \sim 2\,500$ years for the Crab pulsar), depending on whether the angle, χ, between the magnetic axis and elastic reference axis exceeds or is less than $\sim 55^o$; the rate of growth is, however, proportional to the amplitude of nutation, so that it seems difficult to achieve an appreciable nutation amplitude in times which are not long compared to T. Shaham and I have examined the problem from a somewhat different point of view; in considering the influence of a torque which acts perpendicular to the reference axis, $\underset{\sim}{n}_o$, we found that the effect of such a torque depends on whether it is fixed in the star, or adjusts to the reference plane formed by the angular momentum $\underset{\sim}{L}$ and the reference axis, $\underset{\sim}{n}_o$ (Pines and Shaham, 1972a). In the former case, the maximum amplitude of the stellar wobble is $\sim 1/\Omega_W\tau_\perp$, where $(L/\tau_\perp)$ is the magnitude of the external torque; in the latter case, the wobble amplitude increases linearly with time, with a growth rate independent of its amplitude; the maximum amplitude is then determined by either the elastic properties of the stellar crust or by crust-core coupling which may act to damp the wobble motion. We found that in order to achieve nutation amplitudes sufficiently large to be observable (either directly or through microquakes induced by the corresponding growth in the angular part of the elastic energy stored in the crust), one requires a radiation torque fixed in the reference plane, $L/\tau_\perp$, which is large compared to the slowing down torque, L/T.

We have recently examined a possible candidate for such a torque—a component of the stellar field, $\underset{\sim}{B}_o$, which is sensitive to $\underset{\sim}{\Omega}$ and the actual figure axis; such a component, instead of being fixed in the star (with respect to the reference axis, $\underset{\sim}{n}_o$), will be fixed with respect to the reference plane (and can exert a sizeable torque perpendicular to it). We find in this fashion an effective torque

$$\underset{\sim}{N}_{eff} = -\varepsilon_B I_o (\underset{\sim}{\Omega} \times \hat{\underset{\sim}{B}}_o)(\underset{\sim}{\Omega} \cdot \hat{\underset{\sim}{B}}_o) \qquad (28)$$

where ε_B is the " magnetic " stellar oblateness,

$$\varepsilon_B \sim \frac{B_o^2 R^3}{4(A+B)}\ ; \tag{29}$$

The torque, (28), is independent of the wobble amplitude; it is of order of magnitude 10^{39} sin 2χ, and can, in principle, be several orders of magnitude larger than the slowing down torque; it would, however, be observable only through its influence on the wobble amplitude, which would, under these circumstances, be sufficiently large both to bring about microquakes and to be directly detectable. The question then reduces to the difficult one of whether currents in the core can produce and maintain a magnetic field of this character (tied to $\underset{\sim}{\Omega}$, and to the crust), a question for which we have no definitive answer at present.

Assuming that mechanisms for exciting a stellar wobble exist, the amplitude of that wobble will depend, of course, on the extent to which it is damped; here, the situation looks rather favorable for would-be wobble observers. First, in view of the likely homogeneity and sharpness of the stellar surface, the latter a consequence of the strong stellar magnetic field (Ruderman, 1971), it is reasonable to conclude that no shallow oceans exist on the surface of a neutron star, so that the mechanism responsible for damping the earth's wobble will be absent. Second, the crustal temperature of a neutron star is exceedingly low compared to its melting temperature ($T/T_m \lesssim 10^{-5}$), so that the only creep mechanism operation is the so-called logarithmic, or low-temperature, creep; the time scales for the resulting plastic flow are therefore long compared to the slowing-down time, T (Baym and Pines, 1971). Indeed, to the extent that pulsar spindown increases the elastic energy content of the crust, T provides an upper limit to the extent to which plastic flow in the crust can act to damp pulsar wobble.

Crust-liquid coupling is likely the major source of wobble damping, but this too is comparatively ineffective. From the postglitch behavior of the Crab and Vela pulsars one knows that the time, τ, required for the crust and superfluid neutron liquid to come into equilibrium is of the order of days to years; such coupling gives rise to a damping time for the pulsar wobble which is far longer, being

$$\tau_W = \frac{\Omega}{\Omega_W}\tau = \frac{2}{3}\left(\frac{A+B}{B}\right)\frac{\tau}{\varepsilon_o}. \tag{30}$$

For the Crab pulsar, since $\dfrac{\Omega_W}{\Omega} \lesssim 10^{-7}$, one has $\tau_W \gtrsim 3 \times 10^4$ years,

at least an order of magnitude greater than the slowing-down time;

for the Vela pulsar, the wobble damping time is shorter than T_{Vela}, being $\sim$ 500 years for an assumed ε_o of 3×10^{-3}.

Given such long wobble damping times, it is reasonable to expect large amplitude stellar wobble for those neutron stars for which the effective magnitude of the exciting torque, $L/\tau_{\perp}$, exceeds the slowing-down torque, L/T. In such cases, the amplitude of the stellar wobble is likely limited by the maximum storage of angular elastic energy ($\sim \theta_o^2$, where θ_o is the wobble amplitude) in the solid crust (or core) of the star. For the Crab pulsar, Shaham and I have estimated that starquakes will limit the maximum wobble amplitude for a star of mass 0.25 $M_\odot$ to some 5° if the angular contribution to the elastic energy is comparatively small, 15° if this contribution dominates the elastic energy (Pines and Shaham, 1972a). For a pulsar with a solid core, comparable wobble amplitudes may be expected; for example, if $\sigma_c/\mu\varepsilon_o \sim 1/7$, one gets a limit on the wobble amplitude which is $\sim$ 6°.

In their consideration of this problem Henricksen *et al.* (1972) have assumed that the Goldreich mechanism for exciting the wobble is operative, and have calculated the maximum wobble amplitude for the following two cases:

(1) Stabilization of the wobble amplitude either by internal friction in the crust or by core-crust coupling.

(2) Wobble at the elastic limit, followed by plastic flow.

The estimates we have given above suggest that Case (1) is likely not realized in practice; the very low temperature of the crust would seem to preclude an effective internal-friction mechanism, while for the Crab pulsar, at least, the damping time associated with core-crust coupling is long compared to the excitation time, T, assumed by Henricksen *et al.*, so that core-crust coupling could not act to stabilize the wobble amplitude. Case (2) likewise does not seem realizable in practice. If only the Goldreich excitation mechanism is operative, examination of the time-rate of change of the oblateness and angular mismatch terms in the elastic energy shows that the oblateness mismatch contribution will grow at a faster rate than that associated with the wobble amplitude; as a result the wobble amplitude could never be large enough to induce either plastic flow or a microquake. Finally, should it reach such a critical size, we would argue that because the crust is so cold, a series of microquakes are more likely to ensue than plastic flow.

To conclude this discussion let us emphasize that our proposed mass and structure assignments for the Crab and Vela pulsars put definite limits on the precession frequencies one would expect to observe. Thus for the Vela pulsar, the corequake interpretation of its macroglitches requires an $\varepsilon_o \gtrsim 10^{-3}$; taking $B \gtrsim A$, we are led to a wobble period,

$$P_W \lesssim 100 \text{ sec.} \qquad \text{(Vela pulsar)} \tag{31}$$

For the Crab pulsar, assuming it to have a liquid interior and a mass

$\gtrsim 0.3\ M_\odot$, the wobble period is

$$P_W \gtrsim 16 \text{ hours.} \qquad \text{(liquid core Crab pulsar)} \qquad (32)$$

On the other hand, assuming the stellar structure of the Crab pulsar were such that it had a solid core *and* $(I_n/I) \sim 0.9$, one would expect that $\varepsilon_o \gtrsim 10^{-3}$ and $B \gtrsim A/10$, in which case one finds

$$P_W \lesssim 5 \text{ minutes.} \qquad \text{(solid core Crab pulsar)} \qquad (33)$$

Observation of a Crab pulsar wobble could thus provide a decisive determination of its stellar structure.

CONCLUDING REMARKS

Table II represents a " state of the art " summary of our continuing attempt to deduce the properties of the Crab and Vela pulsars and the compact X-ray source, Her X-1, from observations on these three neutron stars. The table can only be regarded as tentative and incomplete;

TABLE II

Structure and Structure-Sensitive Parameters of Three Neutron Stars

Neutron star	Nature of inner core	Superfluid n and p	τ	$\frac{I_n}{I}$	$\frac{M}{M_\odot}$	ε_o	P_W
Crab pulsar	Liquid	Yes	$\sim$5 d	$\sim$0.9	$\lesssim$0.5	$\lesssim 2\times10^{-4}$	$\gtrsim$ 16 h
Vela pulsar	Solid	Yes	$\sim$1 y	$\sim$0.15	$\gtrsim$0.7	$\gtrsim 10^{-3}$	$\lesssim$100 s
Her X-1	Solid	?	?	?	$\gtrsim$0.7	2.7×10^{-7}	$\sim$35 d

perhaps the major reason for composing it is to stimulate both observers and theorists to consider ways in which one can pin down the uncertainties in the structure and mass assignments which have been made. I have not attempted to incorporate a " best guess " for the strength of the magnetic fields associated with these stars; present estimates of field strengths for all three have tended to cluster about 10^{12} gauss, but a determined theorist can make strong arguments (derived from the appropriate pulsar theory or accretion theory) in favor of fields which are an order of magnitude larger or smaller for any of these stars. One of the best prospects for direct observation of neutron star magnetic fields lies in the detection of the Alfvén waves which are likely excited in a speedup (Baym *et al.*, 1969). Perhaps the strongest argument which can be made in favor of the conclusion that both the Vela pulsar and Her X-1 have a solid inner core is that in this way one can explain simply observations which are otherwise quite difficult to interpret.

A more accurate mass assignment for these stars will presumably have to await a definitive calculation of the density at which either neutron or hyperon solidification occurs, if indeed, it occurs at all. As we have emphasized, observation of the free precession of the Crab and Vela pulsars would be decisive in sorting out their stellar structure, and there would seem reasonable theoretical grounds to encourage observers to continue their search for such a periodicity in pulsar signals.

In relating theory to observation, much work remains to be done. Starquake theory is still in its infancy; for example, one would like to know to what extent the elastic energy released in a localized quake is converted into heat, radiated as gravitational waves, or acts to build up elastic energy elsewhere. If a substantial fraction of the energy release is converted into heat, would there not be further observational consequences of the giant Vela core quakes? Will the entire star be heated, or might one get localized hot spots in the vicinity of the magnetic poles as a result of the highly anisotropic thermal conductivity associated with the strong internal magnetic fields? Does one power thereby a thermal X-ray source; a pulsating X-ray source, or perhaps X-ray flares? (Such possibilities have already been suggested by Henricksen *et al.* (1972) as observational consequences of frictional heating associated with pulsar wobble). Back-of-the-envelope calculations indicate that any, or all, of the above three possibilities might be realized; detailed microscopic calculations are both feasible and necessary in order to decide the matter. In similar vein, Greenstein (1973) has suggested that friction arising from crust-core coupling could act to heat the Crab and other pulsars; further work on this coupling is likewise in order.

A number of other physical problems deserve further exploration, and I shall mention but a few. Are starquakes the only viable explanation for the Crab and Vela macroglitches, or can one or more of the other proposed explanations be developed and refined to the point that it both meets the more obvious objections which have been raised to non-starquake theories, and provides as well a consistent account of such speedups? Again, as we shall hear from subsequent speakers, it has been suggested (Hoffberg *et. al.*, 1970) that the neutrons in the liquid core might form a p-state, rather than an s-state, superfluid. Does this have any observational consequences? [It is likely that this problem will receive increasing attention, now that there is a good prospect that the atoms of He^3 likewise form a p-state superfluid at sufficiently low temperatures]. Finally, if the pion condensation proposed by Migdal (1971), Sawyer (1973), *et al.* takes place in the interior of a neutron star, does it in turn have observational consequences, or will we have to rely solely upon the intricacies of the theoretical calculations for a resolution of the question of its formation?

Hopefully by the time of the next Solvay meeting devoted to astrophysics, we shall be a good deal farther along the way to deciding these and related questions. We can, in any event, anticipate much more in the way of observational data on these and other neutron stars (and Cen X-3 is especially promising in this regard), data which may help decide some of the above questions, but which is still more certain to raise new ones, the resolution of which will hopefully bring us that much closer to understanding the structure of the neutron stars we observe.

I should thank Professor E. P. Groth for providing me with the fruits of his analysis of the Crab timing observations in advance of publication, and Professors Ruderman, Shaham, and my colleagues in the University of Illinois astrophysics group for stimulating discussions on these and related topics. Part of the manuscript for this talk was prepared while I was resident at the Aspen Center for Physics, and it gives me pleasure to acknowledge their support and assistance.

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Appendix

Origin of Macroglitches in the Crab and Vela Pulsars

The origin of these macroglitches has been a subject of considerable controversy. As of August, 1972, when Ruderman, Shaham, and I (Pines, Shaham, and Ruderman, 1973) essayed a critical review of this topic, more than 50 articles dealing with various macroglitch mechanisms had appeared; during the past year, however, there have been few new developments on either the theoretical or observational side. For this reason, rather than repeating that discussion in detail once more, I have chosen simply to reproduce a summary of the glitch facts which any theory must explain, together with a Table from the

Table III. Present Status of Macroglitch Theories

□ ≡ Not inconsistent with observation

◪ ≡ May be inconsistent with observation

■ ≡ Inconsistent with observation

Observational Property / Theory		Magnitude	Rapid rise time	Sign	$(\frac{\Delta\Omega}{\Omega})$ Vela >> $(\frac{\Delta\Omega}{\Omega})$ Crab >> $(\frac{\Delta\Omega}{\Omega})$ other	macro-glitch function	Q values	Frequency	$t_a \neq \tau$	Persistance of pulse shape	Explain micro-glitches as well ?	Physically plausible for neutron stars
Planets :	Crab	□	■	◪	□	■	◪	◪	◪	□	■	□
	Vela	□	■	◪	□	◪	◪	□	◪	□	◪	□
Magneto-spheric instability		◪	◪	□	◪	◪	□	□	□	■	□	◪
Accretion		◪	■	◪	◪	◪	□	◪	□	□	◪	■
Crust-quakes :	Crab	□	□	□	□	□	□	□	□	□	□	□
	Vela	◪	□	□	◪	□	□	■	□	□	□	□
C. G. Hydro-dynamic instability		□	□	◪	■	■	■	□	■	□	□	□
Core-quakes :	Crab	□	□	□	□	□	■	◪	□	□	□	□
	Vela	□	□	□	□	□	□	□	□	□	□	□

aforementioned work, in which there is summarized our view of the extent to which each of the proposed theories is able to explain each "glitch fact", and, finally, a brief summary of some of the principal arguments against some of the suggested mechanisms.

If one defines a macroglitch as a frequency jump such that $(\Delta\Omega/\Omega) \geq 10^{-9}$, the observational facts which a macroglitch theory must explain include the following:

(1) $(\Delta\Omega/\Omega)_{Vela} >> (\Delta\Omega/\Omega)_{Crab} >>> (\Delta\Omega/\Omega)_{Other}$, or why have macroglitches been seen only for the Crab and Vela pulsars, with the size of the latter some two orders of magnitude larger than the former?

(2) Why do all macroglitches thus far observed have the same sign?

(3) The time, t_g, between these macroglitches is of the order of years.

(4) $t_g \neq \tau$, the healing time.

(5) Macroglitches have a rapid rise time; they take place in at most a few days and quite possibly occur on a far shorter time scale ($\lesssim$ hrs., if one identifies the Oct., 1971, event as a macroglitch).

(6) The immediate post-glitch behavior can be fit by the "glitch function", (1).

(7) $(Q)_{Vela} << 1 \neq (Q)_{Crab} \cong 1$.

(8) No appreciable change in pulse shape is seen following either Crab or Vela speedups.

Of the suggested macroglitch theories, *acccretion* (Börner and Cohen 1973) is perhaps the easiest to rule out, for the following reasons:

(1) There is no plausible source for the large amount of accreting matter required ($\sim 10^{-9}$ M$_\odot$/yr. for the Crab pulsar, $\sim 10^{-6}$ M$_\odot$/yr. for the Vela pulsar). Observations rule out the existence of a close binary companion, while any fallback from the supernova explosion which created the pulsar takes place within the first year of the supernova.

(2) Any large chunk of accreting matter will be ripped apart by tidal forces; the resulting bits will start moving independently of each other at a distance of $\sim 3 \times 10^{11}$ cm from the pulsar surface; moreover since any infalling chunk likely carries a large amount of angular momentum, the pieces will form a disk which falls in only gradually; hence, there is no way of getting a "sharp" glitch with a rapid rise time. Finally, if one were able to get such a rate of mass infall, one would get substantial X-radiation (which is not observed), and, in all likelihood, the neutron star would not function as a pulsar, since the "accretion" action would dominate the "pulsar" action (Shvartzman, 1971; Lamb *et al.*, 1973).

Planetary perturbations (Michel, 1970; Rees *et al.*, 1970) can now be ruled out for the Crab pulsar as being inconsistent with observation (Groth, 1973), while for the Vela pulsar, in view of the different magnitude of the two observed glitches, one will need at least two planets in orbit about the Vela pulsar in order to fit the data; moreover, one would have to account for the apparent rapid rise time of the Vela speedups, the failure to observe any apparent sudden large spindowns, the fit to the macroglitch function (1) etc. Given the existence of four years of Vela observations, it should now be possible for a planetary enthusiast to investigate in sufficient detail the consistency of this hypothesis with observation to rule it in or out as a possibility.

Among the principle questions which proponets of *magnetospheric instabilities* (Pacini and Scargle, 1971; Roberts and Sturrock, 1972) need to answer are the following:

(1) Why do only the Crab and Vela pulsars have unstable magnetospheres and why, in these cases, is the instability such that in the glitch an appreciable fraction ($\sim$ 100 % for Vela) of the pulsar magnetosphere is blown away?

(2) Why should it take years for an instability to develop when the characteristic time to fill the magnetosphere is of the order of seconds?

(3) Why, if a substantial fraction of the magnetosphere is involved in the glitch, is there no change in pulse shape after a macroglitch?

(4) Why are pulsar plasmas such that one gets a gigantic instability, instead of the fluctuations about a " minimal " instability one observes for most terrestrial plasmas?

The proposal that *hydrodynamic instabilities* associated with the neutron superfluid are responsible for macroglitches (Greenstein and Cameron, 1969) has not been followed up to any great extent; among the problems with such a macroglitch theory are the following:

(1) Why does one see glitches only for the Crab and Vela pulsars?

(2) The interval between glitches is not the relaxation time for the fluid to return to its preglitch state, as one could expect for a typical hydrodynamic instability.

(3) It has not been established that one can set up the hypothesized unstable rotational flow (in which the angular momentum per unit mass decreases with distance from the rotation axis) in a non-turbulent superfluid.

Discussion of the report of D. Pines

L. Rosenfeld: I should like to add a footnote to Pines' historical survey. I recall that when the news of the neutron's discovery reached Copenhagen, we had a lively discussion on the same evening about the prospects opened by this discovery. In the course of it Landau improvised the conception of neutron stars—" unheimliche Sterne ", weird stars, which would be invisible and unknown to us unless by colliding with visible stars they would originate explosions, which might be supernovae. Somewhat later, he published a paper with Ivanenko in which he again mentioned neutron stars as systems " to which quantum mechanics would not be applicable ".

This leads me to my question: how far is the picture of a solid neutron core to be taken seriously, or metaphorically? What is the order of magnitude of the lattice constant in such a solid core?

F. Pacini: Concerning the possibility of explaining the glitches in terms of magnetospheric instabilities, I would like to stresst he following points:

a) The frequency of such instabilities could depend on the amount of activity at the neutron star surface and it appears plausible that there is more activity on young pulsars.

b) We don't yet understand the origin of pulses. If the pulses arise in the open magnetosphere along selected field lines (close to the surface or in the proximity of the speed of light distance) it is not clear a priori whether the pulses should be affected by what happens in the closed magnetosphere.

I agree that the model has not been developed to an extent which would make it attractive from a theoretical point of view. On the other hand, I also feel that one should wait for the next large glitch in the Crab pulsar. We would expect that such a glitch should occur when the braking index η is fairly low (say $\eta \sim 2.2$ or less); that after the glitch η would go up to a value closer to 3; finally, we would expect that the wisps of the Crab should become excited after the event. If such expectations were verified, then a large theoretical effort in understanding magnetospheric instabilities would be justified and necessary.

J. Shaham: I would like to briefly comment on two matters related to the Vela corequakes; one concerns Professor Cameron's remark about the melting at the solid-liquid interface. Without specifying whether surface reshaping is actually in operation, it is safe to say that even then it is incapable of relieving the slowing-down-induced strains. The latter are present throughout the core and no surface melting will be of much importance.

The second concerns another consequence of corequakes. They cause a release of some 10^{44} ergs/yr in the Vela interior, a substantial fraction of which is not radiated away as gravitational waves. Thus, they can sustain a steady state thermal emission, possibly modulated by the dipolar magnetic field, from the neutron stellar surface, in the soft X-ray range. I am told by Professor Giacconi that such an X-ray source may indeed have now been identified with the Vela pulsar. Furthermore, even starting with a cold star, the storage of core elastic energy and its release in the stellar interior, provides a rather efficient way of converting rotational energy into thermal radiation. A neutron star must contain some $10^{47} - 10^{48}$ ergs to have that soft X-ray temperature, and, amusingly enough, the age of Vela, $\sim 10^4$ years, is just right for this energy to be stored in it by yearly $10^{43} - 10^{44}$ ergs corequakes.

Professor Pines, Professor Ruderman and myself are currently looking at this in some more detail.

M. J. Rees: You mentioned that the loose coupling between crust and core did not lead to rapid damping of the wobble. Does this result depend on assuming strong magnetic fields threading the core?

PHYSICS OF HIGH DENSITY AND NUCLEAR MATTER†

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1. INTRODUCTION

Within a few days after formation, the neutron stars are believed to cool down to temperatures of less than 10^{10} °K ($\simeq$ 1 MeV) by neutrino emission, while the degeneracy temperatures of the constituent nuclear fluids are $\simeq$ 50 MeV. Hence to a good approximation the star-matter can be treated as being at a temperature of absolute zero. Various investigations suggest that the highest density which is likely to occur in neutron stars is $< 10^{16}$ gm/cm^3.

In the present report we will mainly be concerned with cold matter in two density ranges.

1) Sub-Nuclear densities: $\rho < 2.5 \times 10^{14}$ g/cm^3.
2) High density: $2.5 \times 10^{14} < \rho < 10^{16}$ g/cm^3.

Much of our understanding of range 1 comes from the theory of nuclear-matter ($\rho \simeq 2.5 \times 10^{14}$ g/cm^3), and hence the discussion is restricted to it.

The mean interparticle distance $2r_o$, where r_o is the radius of sphere containing one particle,

$$\frac{4\pi}{3} \rho r_o^3 = 1, \tag{1.1}$$

is > 0.7 fm in range 2. Since this distance is larger than the effective core radius $r_c \simeq 0.5$ fm in nucleon-nucleon interaction it is hoped that a good starting point is to describe the system as an ensemble of particles interacting through two-body potentials. At least in atomic physics the properties of liquid and solid helium up to $r_o/r_c \simeq .8$ can be understood from 2-body potential.

A non-relativistic theory of dense-matter has been developed in the past few years. Calculations of corrections to it from relativistic kinematics, and the inadequacy of 2-body potentials are just being initiated. These corrections have been studied in nuclear matter, and are found to be small.

In order to give a hopefully adequate status of this field in a limited space the discussions in this report are very brief. The details may be found in the quoted references.

2. NUCLEAR POTENTIALS

At high densities the nucleons can come close to each other, and the potential at $r \sim r_o$ should be known. However scattering experiments up to 400 MeV involve de Broglie wave length > 0.5 fm, and hence do not give details of v ($r \simeq 0.5$ fm). Experiments at higher energy involve inelastic scattering, and it is difficult to deduce a potential from them.

It can be argued[1] that the nucleon-nucleon potential must contain a strong repulsive core, which can be explained as due to the exchange of the vector meson ω. Bethe[1] has suggested that the major interaction at small r may be estimated from the static limit of the neutral vector meson field[2]. This is the familiar Yukawa potential

$$Y(r) = g^2 \frac{e^{-\mu r}}{r}, \tag{2.1}$$

where g is the coupling constant, and μ the reciprocal compton wave length of the meson.

In addition to the repulsion, the low energy scattering experiments indicate an intermediate range attraction, attributed to simultaneous exchange of two pions[3,4]. The long range part is well described by the one-pion exchange.

The nuclear potentials have explicit dependence on the angular momentum and spin. The leading contributors to this dependence are the ρ and π exchanges which give central potentials proportional to $(\sigma_1 . \sigma_2)$ $(\tau_1 . \tau_2)$, and the intermediate range attraction[4].

The phenomenological soft core Reid[5] potential is the only available 'static' potential with Yukawa repulsive core, and it has been widely used in the study of dense matter. The central components of this potential can be written as superposition of Yukawas,

$$v(lSJ,r) = \sum_n a_n (lSJ) \frac{e^{-n\mu r}}{\mu r} \tag{2.2}$$

Here $\mu = 0.7\ \mathrm{fm}^{-1}$ is the inverse compton wave length of a pion. Some of the cœfficients a_n (lSJ) are given in Table 1. The potentials in $T = 1$ $^1S_0$, $(^3P_2 - {}^3F_2)$, and $^1D_2$ channels are shown in Fig. 1. Apart from a strong exchange term (i.e., a big difference in the potentials in even and odd l states) there is a significant difference in $v(l = 0)$ and $v(l = 2)$. Little is known about the potential in states with $l \geq 3$ from experiments, and these are treated approximately with either the phase-shifts, or assuming

$$\begin{aligned} v(l = \text{even} \neq 0, S = 0, r) &= v(^1D_2) \\ v(l = \text{odd}, S = 1, r) &= v_c(^3P_2 - {}^3F_2). \end{aligned} \tag{2.3}$$

The spin-averaged direct potential (Fig. 1)

$$\bar{v} = \frac{1}{4} \left\{ v(^1D_2) + 3v_c(^3P_2 - {}^3F_2) \right\} \tag{2.4}$$

has very little attraction in it. The l dependence of the interaction has a strong effect on the solidification of neutron matter.

The $T = 0$ triplet potential in $(^3S_1 - {}^3D_1)$ coupled channels is dominated by the tensor force (Fig. 2). The higher order contributions

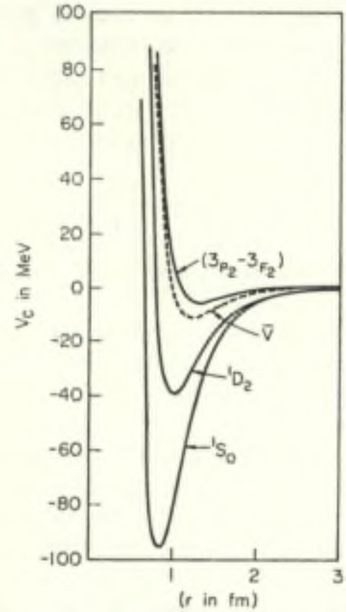

Fig. 1. Central potentials in T = 1 states.

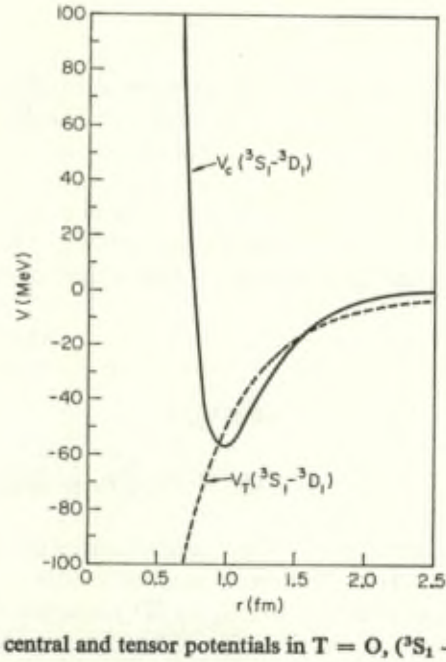

Fig. 2. The central and tensor potentials in T = O, ($^3S_1 - {}^3D_1$) state.

of this tensor force are very density dependent, and significantly contribute to the saturation of nuclear matter. The $({}^3S_1 - {}^3D_1)$ central force is much less attractive than the $({}^1S_0)$.

Little is known about the tensor force in $T = 1$ states from experiments. The tensor force in Reid ${}^3P_2 - {}^3F_2$ coupled channels is compared with that projected out from the ${}^3P$ states:

$$V_T\,({}^3P) = -\,[V\,({}^3P_0) - 1.5V\,({}^3P_1) + 0.5V\,({}^3P_2)]/7.2,$$

with

$$V\,({}^3P_2) = V_c\,({}^3P_2 - {}^3F_2) - 0.4\,V_T\,({}^3P_2 - {}^3F_2) + V_{LS}\,({}^3P_2 - {}^3F_2), \tag{2.5}$$

in Fig. 3. At short distances even the signs of $V_T\,({}^3P)$, and $V_T\,({}^3P_2 - {}^3F_2)$ are different. It is possible simultaneously to change the repulsion and attraction in a potential without seriously affecting the phase shifts. Hence the projected $V_T\,({}^3P)$ should not be too reliable, and the $V_T\,({}^3P_2 - {}^3F_2)$ is rather weak. There is of course a strong *l*.S force in the triplet states. It can be argued[6] that to prevent the collapse of nuclear matter there cannot be any attractive force which increases with velocity. This of course rules out the survival of *l*.S force in high *l* states. As a matter of fact approximations (2.3) tacitly assume that the *l* dependence (barring the exchange) is limited to $l < 2$.

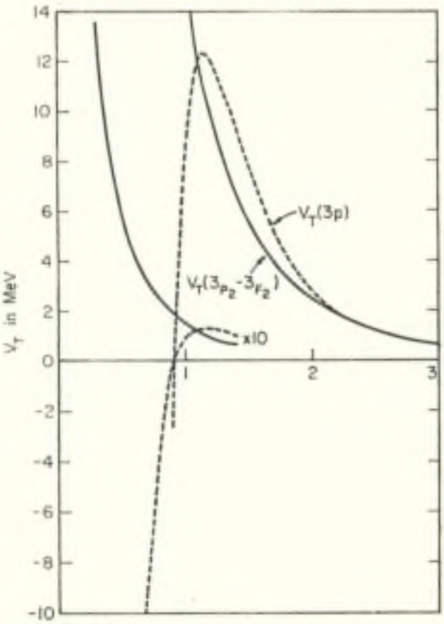

Fig. 3. The T = 1, S = 1 state tensor potential.

The scattering data allows many different potential shapes, and the Reid potential is by no means unique. A review of different nuclear-potentials is given recently by Bethe[6]. Many of these have the general features discussed above.

3. THE BRUECKNER-BETHE-GOLDSTONE THEORY

In this theory the total Hamiltonian,

$$H = \sum_i -\frac{\hbar^2}{2m_i}\nabla_i^2 + \frac{1}{2}\sum_{ij} v_{ij}, \tag{3.1}$$

is written as $H_0 + H_1$, where

$$H_0 = \sum_i \left(-\frac{\hbar^2}{2m_i}\nabla_i^2 + U_i\right), \tag{3.2}$$

and

$$H_1 = \frac{1}{2}\sum_{i<j} v_{ij} - \sum_i U_i. \tag{3.3}$$

The U_i is a single particle potential, and H_0 should be exactly solvable. The contributions of H_1 are given by the linked diagrams of H_1 by Goldstone's[7] theorem. The lowest order of Brueckner expansion[8] corresponds to a sum of two particle ladder diagrams shown in Fig. 4, and gives

$$E = T + W,$$

$$T = \sum_i \frac{\hbar^2}{2m_i} k_i^2,$$

$$W = \frac{1}{2}\sum_{ij} (\varphi_i\varphi_j - \varphi_j\varphi_i\,\delta_{ij},\, G\varphi_i\varphi_j). \tag{3.4}$$

Here T is the kinetic energy of the eigenstate of H_0 (which has Fermi gas wave function in case of fluids) $\delta_{ij} = 1$ if particles i and j are identical, and the reaction matrix G is given by the integral equation

$$\langle mn|G|ij\rangle = \langle mn|v|ij\rangle - \sum_{ab} \langle mn|v|ab\rangle \frac{Q\,(ab)}{e} \langle ab|G|ij\rangle, \tag{3.5}$$

where the Pauli operator $Q\,(ab) = 1$ if states a and b are both unoccupied, and zero otherwise, and

$$e = \frac{\hbar^2}{2m_i}(k_a^2 - k_i^2) + \frac{\hbar^2}{2m_j}(k_b^2 - k_j^2) + U_a + U_b - U_i - U_j. \tag{3.6}$$

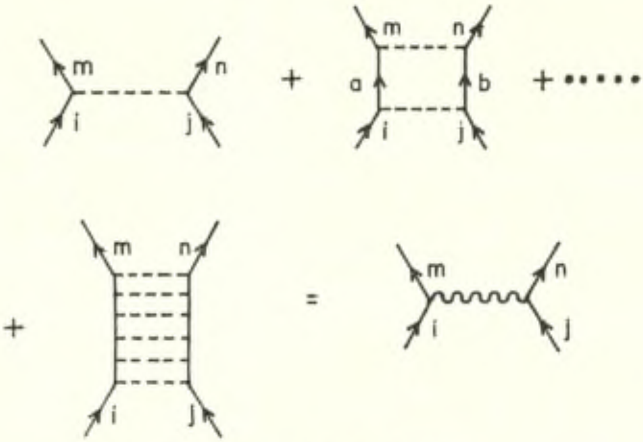

Fig. 4. The two-body diagrams.

The G can be easily calculated with the Bethe-Goldstone equation for the correlated relative wave function ψ,

$$\psi = \varphi - \frac{Q}{e} v\psi. \tag{3.7}$$

The φ is unperturbed relative wave function, and

$$G\varphi = v\psi. \tag{3.8}$$

The single particle potential U_i in occupied states can be self-consistently determined in Hartree-Fock fashion;

$$U_i = \sum_j < ij - ji\delta_{ij}|G|ij > \tag{3.9}$$

to cancel the " bubble interaction " shown in Fig. 5. However a similar choice is not advisable for the U_a in unoccupied or " particle " states.

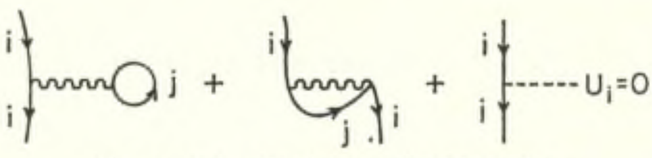

Fig. 5. Choice of hole state potential energies.

The Brueckner-Goldstone expansion in G can be obtained by summing all possible ladder sequences in v. Rajaraman[9] has shown that it does not converge in powers of G. Bethe[9] regrouped the diagrams into a hole line expansion. Some of the three hole lines diagrams are shown in Fig. 6a, and the first of these is the bubble interaction with particle line. Hence U_s is commonly taken to be zero in particle states, thus restricting the applicability to systems in which U_i is negative. When U_i are large negative the correlated wave function ψ rapidly goes over to φ as r increases. This process is popularly called " the rapid healing ".

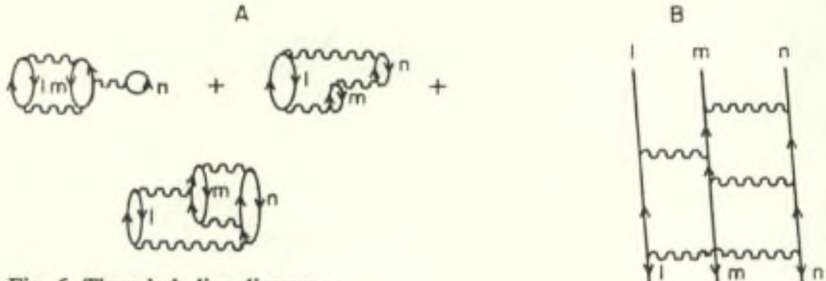

Fig. 6. Three hole line diagrams.

The diagrams in Fig. 6a can be drawn as 3-body ladders as shown in Fig. 6b. Bethe[9] was able to sum these using the method of Faddeev and Day[10] extended it further to sum the four hole line diagrams. It was noticed that the major part of 3- and 4-body correlated wave functions could be approximated by the Jastrow form in which correlations are represented by the product of 2-body correlation function between all pairs. Day[11] subsequently generalized this result to any number of particles.

Bethe[9] also showed that such a hole line expansion converges provided the wound integral κ

$$\kappa = \rho \int (\varphi - \psi)^2 d^3r \qquad (3.10)$$

is small. The κ gives the probability of the particle being outside the Fermi sea, and the n hole line term is proportional to κ^{n-2}.

4. THE VARIATIONAL METHOD

For practical reasons the trial wave function is restricted to the so called " Jastrow " form:

$$\Psi (1 \ldots . N) = \Pi f_{ij} \, \Phi (1 \ldots . N) \qquad (4.1)$$

where Φ is the model wave function, and f_{ij} is determined by minimizing the energy:

$$E = \frac{(\Psi, H\Psi)}{(\Psi,\Psi)} \qquad (4.2)$$

Most variational calculations have been carried out with a real spherically symmetric f described by a couple of parameters, and the energy is minimized with respect to these[12]. The $\nabla_J f_{Jm} \cdot \nabla_J \varphi_J$ terms in the energy expectation value are zero for these f, and the effective mass of an impurity always equals the bare mass. Experimentally of course it is known that ^{3}He impurity in liquid ^{4}He has an effective mass of more than twice its bare mass[13]. So these f are too simple.

Alternatively the f can be calculated analytically with suitable approximations. We[14] calculate them with the constraint $f = 1$ for $r > d$, and $\nabla f(d) = 0$, by minimizing the two-body term in the cluster expansion of (4.2). The healing distance d was subsequently allowed, by Pandharipande and Bethe[15] (PB is used to denote Ref. 15 in this section), to be so large that the effects of the constraint are negligible. PB have also shown that with these f the two-body term dominates. Its contribution from the correlation volume ($r < d$) is

$$\frac{1}{\Omega} \int_0^d \psi^* \left\{ v - \frac{\hbar^2}{m} (k^2 + \nabla^2) \right\} \psi d^3r \tag{4.3}$$

where k is the relative momentum, m the reduced mass, and formally

$$\psi = f\varphi = fe^{i\vec{k}.\vec{r}} \tag{4.4}$$

The ψ and φ are decomposed into partial waves,

$$\psi = \sum_{l=0}^{\infty} i^l (2l + 1)\, U_l (r)\, P_l (\cos \theta),$$

$$\varphi = \sum_{l=0}^{\infty} i^l (2l + 1)\, J_l (r)\, P_l (\cos \theta), \tag{4.5}$$

and the contribution of each partial wave is minimized separately (except in case where tensor or other forces couples the waves[16]) to obtain the " Schrödinger equation " for the radial wave function $u_l = rU_l$.

$$-\frac{\hbar^2}{m} \left\{ \frac{\delta^2 u_l}{\delta r^2} - \frac{l(l+1)}{r^2} u_l \right\} + v_l u_l = \left(\frac{\hbar^2}{m} k^2 + \lambda^l (k) \right) u_l \tag{4.6}$$

The λ^l (k) are determined from the boundary condition on f.

The f is complex, and its real and imaginary parts are given by

$$\mathrm{Re}f = \mathrm{Re}\psi \mathrm{Re}\varphi + \mathrm{Im}\psi \mathrm{Im}\varphi,$$
$$\mathrm{Im}f = \mathrm{Im}\psi \mathrm{Re}\varphi - \mathrm{Im}\varphi \mathrm{Re}\psi.$$

In the limit of small k the Bessel functions J_l can be expanded in powers of kr, and only $l = 0$, and 1 need be considered. The wave function (4.1) for a slowly moving impurity with momentum $\vec{k}_J$ in a Bose fluid can be shown to be[17]

$$\Psi(k_j) = e^{i\vec{k}_j\cdot\vec{r}_j} \prod_m \left\{ f_o(r_{jm}) + i\vec{k}\cdot\vec{r}_{jm} \left[f_1(r_{jm}) - f_o(r_{jm}) \right] \right\} \prod_{m<n} f_{mn} \quad (4.7)$$

with

$$f_l = U_l/J_l, \quad (4.8)$$

and it resembles the Feynman-Cohen[18] wave function. The Feynman-Cohen back flow is included in the dependence of f, and (4.7) can quite satisfactorily explain the impurity effective mass[17] (the calculated effective mass of ^{3}He in ^{4}He $\simeq$ 2.2 ^{3}He mass).

The lowest order approximation[19] is obtained by noting that the contribution of farther neighbors of a particle i to the instantaneous potential $\sum_j v_{ij}$ seen by i should mostly be included in the average field of which φ_i is an eigenfunction. Hence distant neighbors should not be strongly correlated, and the effect of their correlation on the energy should be small. If this effect is neglected, one could in principle work with correlation functions satisfying the condition

$$\text{if} \quad f_{ij} \neq 1 \qquad \text{all} \quad f_{ik} = 1, \quad (4.9)$$

when ij are nearest neighbors. When f satisfy (4.9) all direct contributions of many-body (meaning $n \geq 3$ by many) clusters are zero, and PB also found that their exchange contributions are small. However (4.9) couples all f_{ij} in a complicated way, and in practice (4.9) is satisfied only in average by choosing d such that on average there is only one particle within a distance d of an average particle. Thus correlations are at times allowed between second and more distant neighbors, while at times even the first neighbors are treated as uncorrelated. We hope that these effects cancel. In the lowest order constrained variational calculations PB could interpret λ^l as the contribution to the average field from v_l $(r < d)$, and to consider it as a constant independent of r is the simplest assumption.

PB found it more advantageous to write the energy expectation value (4.2) as

$$E = T + W + U + U_F$$

$$T = \sum_i \frac{\hbar^2}{2m_i} k_i^2,$$

$$W = \frac{1}{2\Omega} \sum_{ij} < \varphi_i\varphi_j - \delta_{ij}\, \varphi_j\varphi_i \mid [\sum_l f_l^2(k, r)\, V^l(k, r)\, P^l]\, h' \mid \varphi_i\varphi_j >,$$

$$U = -\frac{\hbar^2}{2m} \frac{1}{\Omega^2} \sum_{ijk} g_3(\vec{r}_{ij}, \vec{r}_{ik}) \frac{\nabla_i f_{ij} \cdot \nabla_i f_{ik}}{f_{ij} f_{ik}} d^3r_{ij}\, d^3r_{ik}, \quad (4.10)$$

and

$$U_F = \frac{\hbar^2}{m}\frac{1}{\Omega^2}\sum_{1,2,3} g_3(\vec{r}_{12},\vec{r}_{13})\, e^{i(\vec{k}_1-\vec{k}_3)\cdot\vec{r}_{13}}\left(i\vec{k}_1\cdot\frac{\nabla f_{12}}{f_{12}}\right) d^3r_{12}\, d^3r_{13}.$$

Here

$$V_l(k, r < d) = \lambda_l(k),$$
$$V_l(k, r > d) = v_l,$$

and P^l are l projection operators. The g_3 is the three particle distribution function, and is approximated by

$$g_3(\vec{r}_{12}, \vec{r}_{13}) = g(r_{12})\, g(r_{23})\, g(r_{31}). \tag{4.11}$$

The h' represents the effects of other particles on the pair distribution function g of particles in states $^k i$ and $^k j$. If the exchanges are neglected $h' = g/f^2$. It is close to unity, and is calculated with a single average f^2.

In lowest order calculations the U and U_F are neglected, and the $h' = 1$. It is interesting to evaluate the many-body cluster contributions when $d \simeq 2r_0$ (Eq. 1.1), and $r_0/r_c \simeq 1$. In this range the cluster expansion does not converge with respect to the number of particles in the cluster. Wu and Feenberg[20] however noted that the exchange contributions decrease rapidly with respect to the number of exchanges.

The main contributors to h' are thus the direct many-body clusters schematically shown in Fig. 7. All the diagrams E.2 to E.8 give comparable contributions to g_{mn}, and form the so called hypernetted chains (HNC). No four particles of a hypernetted chain need to have simultaneous correlations with each other as for example E.9 has. Since the probability of four particles to come within correlation range is much smaller, E.9 and the diagrams like E.10 which have E.9 as a subdiagram have much smaller contributions than the HNC diagrams. The HNC with and without the four particle cluster E.9 are together called HNC/4. The HNC and HNC/4 can be summed by successive approximations of integral equation derived by van Leeuwen, Groeneveld, and de Boer[21]. PB find the convergence in these successive approximations to be satisfactory.

PB have applied this formalism to liquid ^{3}He and ^{4}He with significant success. The use of state dependent correlation functions lowers the energy per atom of ^{3}He liquid to —2°K, whereas that with the best spherically symmetric f is —1 . 3°K.

5. REVIEW OF NUCLEAR MATTER RESULTS

Results of lowest order constrained variational (LOCV) and Bruckner-Bethe-Goldstone theory (LOBT) calculations are shown in Figs. 8 and 9. Curve B_1 of Fig. 8 is obtained by Haftel and Tabakin[22] with LOBT and the Reid potential in low l ($l \leq 2$) states. The potential in high l

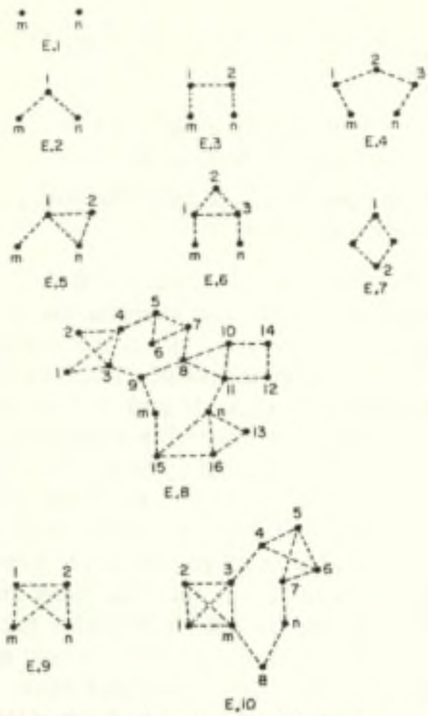

Fig. 7. Direct diagrams contributing to g_{mn}.

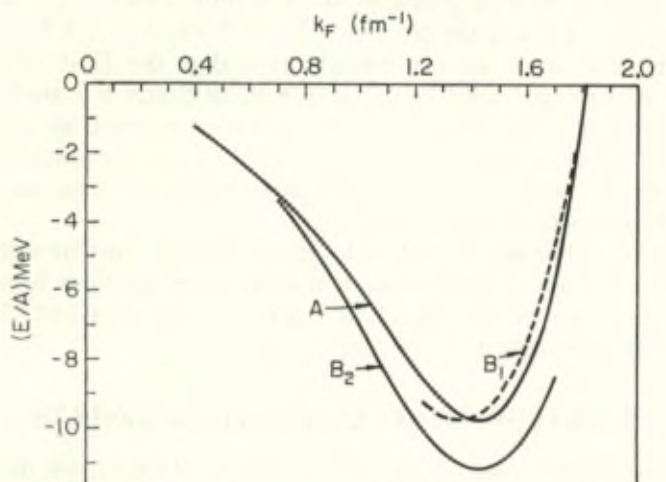

Fig. 8. The energy of nuclear-matter. Curves A, B_1, and B_2 respectively give the results of LOCV and LOBT calculations with OPEP, and phase shift approximation in $l \geqslant 3$ states.

states is assumed to be that of one pion exchange. Curve B_2 gives the results of Siemens,[23] and Banerjee and Sprung[24]. These also are with LOBT and the Reid potential, however the interaction in higher partial waves is estimated directly from the phase-shifts by a method due to Srivastava[25]. The difference in curves B_1 and B_2 reflects the effects of uncertainty in the potential in high l states. The LOCV results[16] are shown by curve A, and use the approximations (2.3) in higher partial waves. The LOCV and LOBT give very similar results.

The contributions of various partial waves are shown in Fig. 9. The curves and points respectively give the LOCV and LOBT results. The $^3S_1$ state interaction gives the strongest contribution to saturation, most of which comes from the decrease in $^3S_1 - {}^3D_1$ tensor correlation with density. Haftel and Tabakin[22] have most strikingly demonstrated the importance of tensor force in saturation by showing that equilibrium density can be made three times too large by choosing a very weak tensor force.

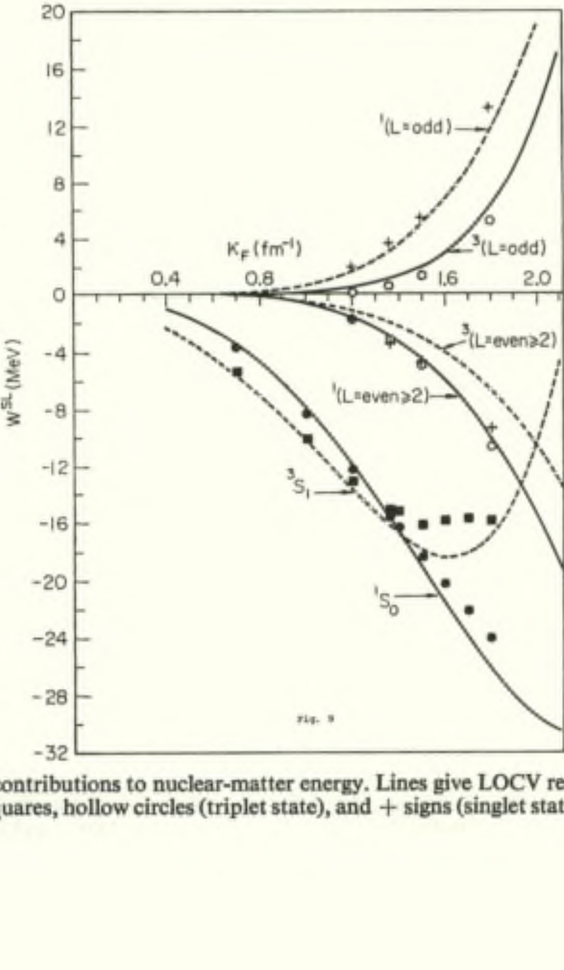

Fig. 9. The contributions to nuclear-matter energy. Lines give LOCV results, and the full circles, squares, hollow circles (triplet state), and + signs (singlet state) give LOBT results.

The other contributors to saturation are the exchange nature of the interaction which makes the odd state contributions increase rapidly with ρ. Second, the extra attraction in $l = 0$ states gets lower and lower weight at high ρ. The repulsive core by itself, is probably unable to saturate nuclear matter at reasonable density. Its effective radius is $\simeq$ 0.5 fm, and it becomes dominant only at few times nuclear matter density (r_0 in nuclear matter is $\simeq$ 1.15 fm). It should also be noted that the interactions which turn more repulsive at high ρ are isospin zero neutron-proton interactions in triplet-even, and singlet-odd states.

The 3- and 4-hole line contributions have been respectively calculated by Dahlblom[26] and Day,[10] and have been summarized by Bethe[6]. These are given in Table II, along with the convergence estimates of Section 3. The corrections are estimated at $k_F = 1.36$, the empirical nuclear matter equilibrium density; for which $\kappa \simeq 0.14$. Thus the nuclear matter energy with Reid potential and the phase shift approximation is estimated to be $\simeq -14.3$ MeV/N against the empirical volume energy of $\simeq -15.8$ MeV/N. The energy obtained with various phase shift equivalent potentials has of course a significant spread[6]. More correction to the nuclear matter energy will be discussed in subsequent sections.

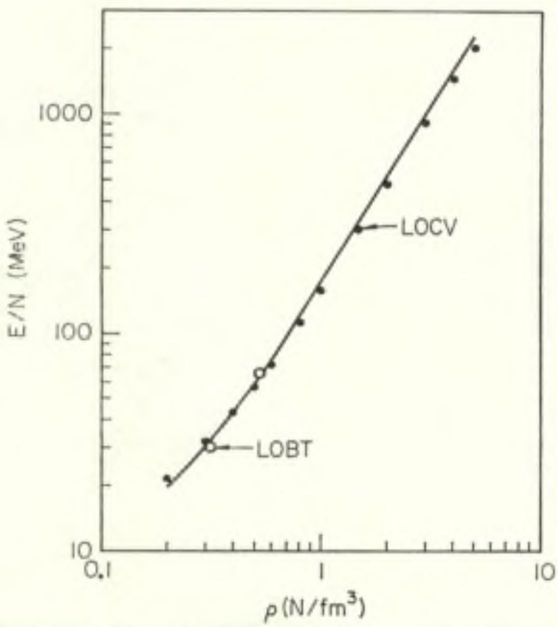

Fig. 10. The energy of neutron matter. Full line gives the results of variational calculations, while the full and hollow circular respectively give LOCV and LOBT results.

6. REVIEW OF DENSE NEUTRON MATTER RESULTS

Results of variational calculations of neutron matter[15] are summarized in Fig. 10. The healing distance d is $\simeq 1.2\, r_0$ in the lowest order constrained variational calculations. In full variational calculations the energy becomes insensitive to d around d $\simeq 2r_0$ as shown in Table III. At nuclear matter densities the LOCV overestimates the energy by $\sim 5\,\%$ while at gifgh densities it is underestimated by $\sim 5\,\%$.

The LOBT[27] results at $\rho < 0.6\ \mathrm{fm}^{-3}$ (Fig. 10) are also in close agreement with the variational results. The wound integral κ is very small (< 0.12) in neutron matter and hence the LOBT should be a fair approximation. However at $\rho > 0.6\ \mathrm{fm}^{-3}$ some of the hole state potential energies become positive, and this has limited the Brueckner-Bethe-Goldstone theory calculations to $\rho < 0.6$.

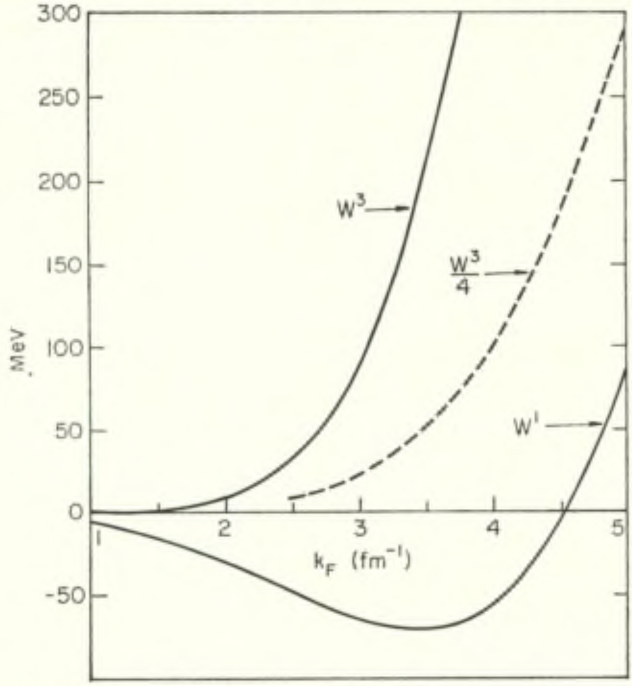

Fig. 11. Singlet, and triplet state contributions to the neutron matter energy (LOCV).

Figure 11 gives the contributions to the potential energy from singlet and triplet state interactions (Fig. 1). At low densities the attraction in singlet states gives the dominant negative contribution, while at high densities the singlet state attraction and repulsion cancel while the triplet states give a large positive contribution.

Earlier it had been suggested[28] that neutron matter may become ferromagnetic at high density. However the large repulsion in triplet states was neglected in this work. The magnetic susceptibility of neutron matter with the Reid potential is actually very small, and (χ_F/χ) (Fig. 12) increases with ρ.[29–30].

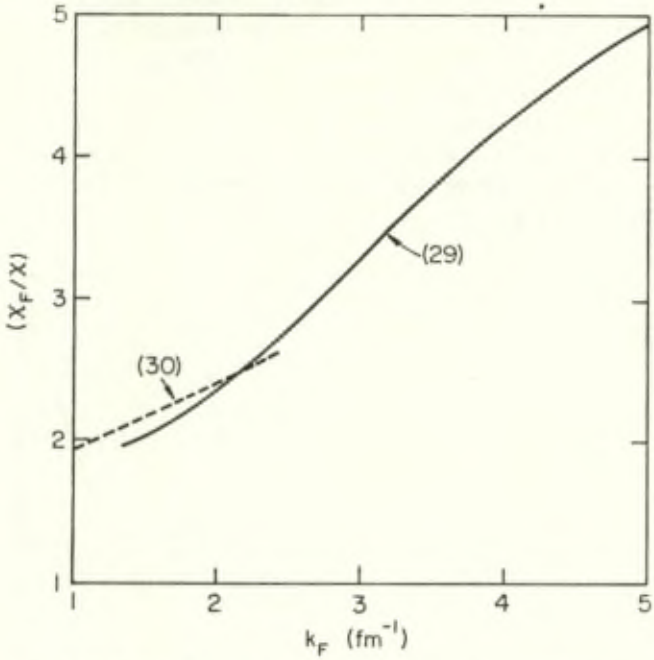

Fig. 12. The magnetic susceptibility of neutron matter.

The χ as estimated from Landau parameters[30] is quite close to that calculated from LOCV[29].

The equation of state P (ε),

$$P(\rho) = \rho^2 \frac{1}{N} \frac{\delta E(\rho)}{\delta \rho}, \tag{6.1}$$

and

$$\varepsilon(\rho) = \frac{1}{\Omega} (E(\rho) + Nmc^2) \tag{6.2}$$

is shown in Fig. 13. At high densities P approaches ε. This asymptotic behavior may be understood as follows. At very high densities the effects of correlations and exchange will become small. Hence as $\rho \to \infty$

$$E(\rho) \to \frac{1}{2} N \rho <\overline{V}> \tag{6.3}$$

where $<\overline{V}>$ is the volume integral of the spin average direct potential (2.4). This gives both

$$\varepsilon(\rho) \to P(\rho) \to \frac{1}{2} \rho^2 <\overline{V}>, \tag{6.4}$$

and hence

$$\varepsilon \to P. \tag{6.5}$$

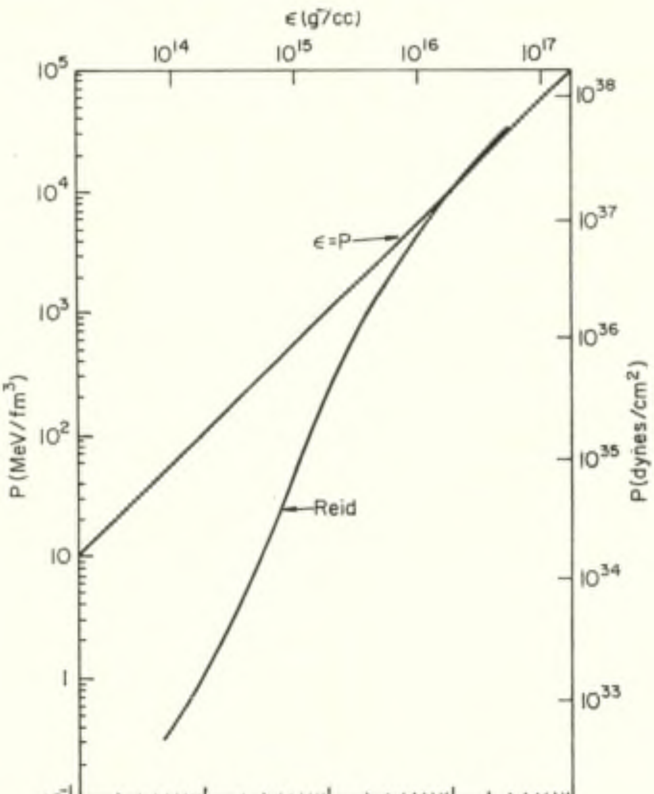

Fig. 13. The equation of state of neutron matter.

We have studied[14] neutron-matter equations of state with various phase shift equivalent potentials. At low density the potentials give similar equations of state on account of their phase shift equivalence, while at high density all tend to $\varepsilon \to P$. The spread between them is not too large. Causality would require that P approaches ε from below. However the non-relativistic calculations give $P \to \varepsilon$ from above. The maximum speed of sound in neutron star regime ($\varepsilon_{max} = 4 \times 10^{15}$ g cm^{-3}) is however 0.7 speed of light, and there is no causality violation in the high density range 2.

7. SOLIDIFICATION OF NEUTRON MATTER

Considerable interest in this subject has been stimulated by the suggestions of Pines, Ruderman and Shaham[31], and Pines, Pethick, and Lamb[32] that the Vela speedups, and Her X-1 35^d high-low X-ray cycle can be understood if these pulsars have solid cores. Theoretically it is a rather controversial subject at present. Earlier[33–34] estimates of rather low solidification densities (ρ_S) based on corresponding states arguments are not too reliable because of the complex, soft core nature of the nucleon-nucleon interaction.

Schiff[35] has carried out calculation of the neutron fluid, and solid phases with a state independent interaction

$$v(r) = (6484.2\, e^{-7\mu r} - 825.8\, e^{-4\mu r} - 5.261\, e^{-\mu r})/\mu r, \qquad (7.1)$$

with $\mu = 0.7$ fm^{-1}. This interaction is qualitatively similar to the direct spin average interaction $\overline{v}$ (Eq. 2.4). A perturbation method developed by Kalos, Levesque, and Verlet[36] is used. Briefly this method corresponds to i) perturbatively solving the problem of bosons interacting with interaction 7.1 from a numerical solution of 256 hard sphere boson Schrödinger equation, and ii) correcting the Bose fluid energy for exchanges to obtain the Fermi fluid energy by Wu-Feenberg[20] expansion. The solid energy is assumed to be independent of statistics. This method can be criticized on two grounds. [1] The perturbative method may be more applicable to hard core potentials, where it is verified, than to soft core potentials of type (7.1). [2] For reasons discussed in Section 4 the correlations in Fermi fluids are more complex, and lower energies can be obtained by solving the Fermi fluid problem directly rather than by simply correcting the Bose fluid for exchanges.

Schiff's results are shown in Fig. 14, and they indicate that the system solidifies at $\rho \simeq 1.75 \pm 0.3$ fm^{-3}, which is very close to the maximum density in neutron stars (a little above $\rho_{max} = 1.6$ fm^{-3} as per Baym, Pethick, Sutherland[37] calculations.) There is very small difference between the fluid and solid energies, and the difference in Fermi and Bose systems is also very small. Our results with LOCV and $\overline{v}$ are similar to these, however the (7.1), or $\overline{v}$ is too simple to represent the N-N interaction.

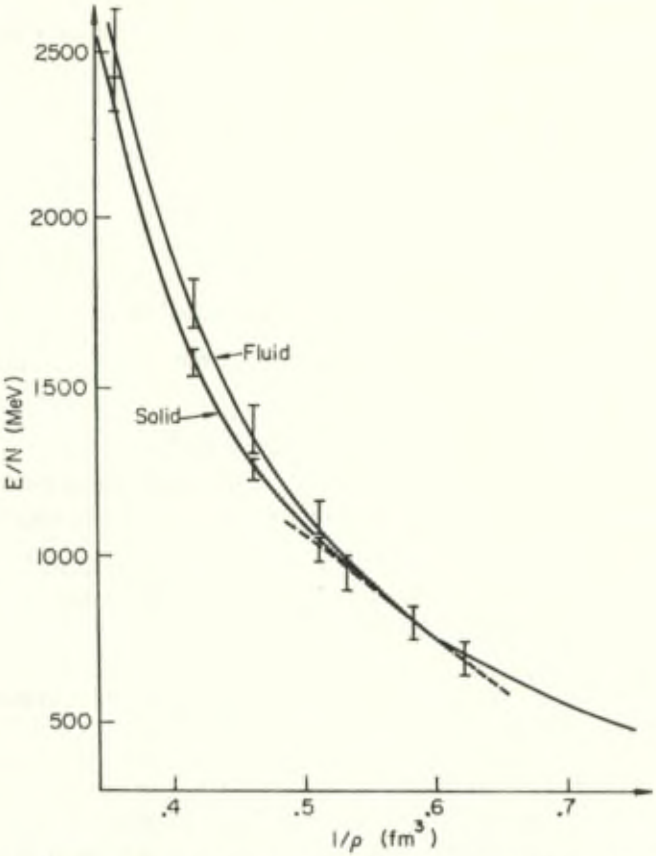

Fig. 14. The fluid and solid energies with a state independent potential.

Nosanow and Parish[38] (NP) have carried out Monte-Carlo calculations for fluid and solid phases assuming the $^1S_0$ state Reid potential in all even states, and the V_c ($^3P_2$ — $^3F_2$) in all odd states. It is known that the $^1D_2$ potential is much less attractive than the $^1S_0$ and hence this choice of interaction also may not be too realistic. NP find the solidification at very low density ($\simeq 0.3\ fm^{-3}$), and their solid has very large zero point motion. The exchange corrections are very large, and are evaluated in only the lowest 2-body approximation. We believe that the NP treatment of exchange (discussed subsequently in more detail) and hence their result, is not convincing.

The LOCV calculations[39] assume the variational wave function

$$\Psi_V = A \left\{ \prod_{m<n} f_{mn} (r_{ij}) \right\} \left\{ \prod_{m} \varphi_m (\vec{r}_i) \right\} \tag{7.2}$$

for the solid phase. The $\varphi_m (\vec{r}_i)$ are gaussians centered around lattice points $\vec{R}_m$

$$\varphi_m (\vec{r}_i) = \left\{ \frac{v}{\pi} \right\}^{3/4} e^{-\frac{v}{2} (\vec{r}_i - \vec{R}_m)^2}, \tag{7.3}$$

and the f_{mn} and v are to be determined variationally.

The long range periodic correlations in solids are included in $\varphi_m (\vec{r}_i)$, and f represents the short range correlations. Neighbors of a particle i form an approximately spherically symmetric shell of radius $2r_o$ around $\vec{R}_m$. The mean square displacement of i from the lattice point $\vec{R}_m$ implied by the $\varphi_m (\vec{r}_i)$ is

$$<(\vec{r}_i - \vec{R}_m)^2>^{1/2} = R_{rms} = (1.5/v)^{1/2} \tag{7.5}$$

If f has a range larger than $2r_o - R_{rms}$, the short range correlations with close neighbors would effectively confine i in a smaller volume around $\vec{R}_m$. Much of this effect could have been included by simply increasing v and thus decreasing R_{rms}. Hence a reasonable choice of the range of f appears to be:

$$d = 2r_o - R_{rms}. \tag{7.6}$$

This choice of d gives correct high density limit, E and v approaching ∞ as $2r_o \rightarrow C$, for hard spheres. It also explains the solidification densities of atomic helium solids within 15 %. For d given by 7.6 the direct 3-body cluster contributions are $\sim$ 5 % of the 2-body cluster contributions, and we simply neglect them hoping these repulsive contributions get cancelled by the gain on increasing d (as in Table III). The solid energy is then given by

$$E = \sum_m C_1 (m) + \frac{1}{2} \sum_{m,n} C_2 (mn),$$

$$C_1 (m) = \left(\varphi_m, \frac{\hbar^2}{2m} \nabla_i^2 \varphi_m \right) = \frac{3}{4} \hbar\omega,$$

$$C_2 (mn) = - \frac{\hbar^2}{m} \frac{I_1}{I_2} - \frac{3}{4} \hbar\omega + \frac{I_3}{I_2},$$

$$\begin{aligned} I_1 &= (\varphi_a f, f\nabla^2\varphi) \\ I_2 &= (\varphi_a f, f\varphi) \\ I_3 &= (\varphi_a f, Vf\varphi) \end{aligned} \tag{7.7}$$

Here

$$V = v - \frac{\hbar^2}{m}\left(\frac{\nabla^2 f}{f} + \frac{2\nabla f \cdot \nabla\varphi}{f\varphi}\right), \tag{7.8}$$

the model wave function for relative motion is

$$\varphi(\vec{r}) = \left(\frac{v}{2\Pi}\right)^{3/4} e^{-v/4\,(\vec{r}-\vec{R}_{mn})^2},$$

and (7.9)

$$R_{mn} = \vec{R}_m - \vec{R}_n.$$

The antisymmetrized wave function φ_a is simply

$$\varphi_a(\vec{r}) = \varphi(\vec{r}) + (-1)^S \varphi(-\vec{r}) \tag{7.10}$$

where S is the total spin of the pair.

The $\varphi(r)$ is localized in a narrow solid angle in the direction of $\vec{R}_{mn}$. The f on the other hand is dominated by the strong $v(r)$, and must be small at small r, and rise to unity by $r = d$. Only $f(r, \theta \simeq 0)$, θ being the angle between $\vec{r}$ and $\vec{R}_{mn}$, is relevant, and hence we opt to approximate $f(\vec{r})$ in solids by spherically symmetric function. It is then possible to obtain a differential equation for f in solids, (the f of course depends on v), and carry out the variational calculation of E (v).

At small values of v however the exchange terms involving $\int \varphi(-r)\,\varphi(r)$ become very large. This is because the φ_m in wave function (7.2) are not orthogonal to each other. When the φ_m's overlap strongly the lattice structure gets very diffused, and in a Boltzmann system the ψ_V goes smoothly over to the Boltzmann gas. We define here a localization parameter Λ as the ratio of gross available volume per particle $(= 4\pi r_0{}^3/3)$, and the actual volume occupied by the zero point motion of the particle. The latter is estimated from

$$R_{\text{equivalent}} = \left(\frac{5}{3}\right)^{1/2} R_{rms} = \left(\frac{2.5}{v}\right)^{1/2} \tag{7.11}$$

giving

$$\Lambda = (v r_o{}^2/2.5)^{3/2} \tag{7.12}$$

The diffusion of the lattice structure as seen grom the uncorrelated Boltzmann density

$$\rho(\vec{r}) = \sum_m \varphi_m{}^2(\vec{r}) \tag{7.13}$$

is shown in Fig. 15. Classical solids obeying Lindermann's law have $\Lambda \simeq 30$, and it is 3.5 in helium solids. We search for a minimum in E (Λ) up to $\Lambda = 1$, and disregard any minimum in E (Λ) at $\Lambda << 1$.

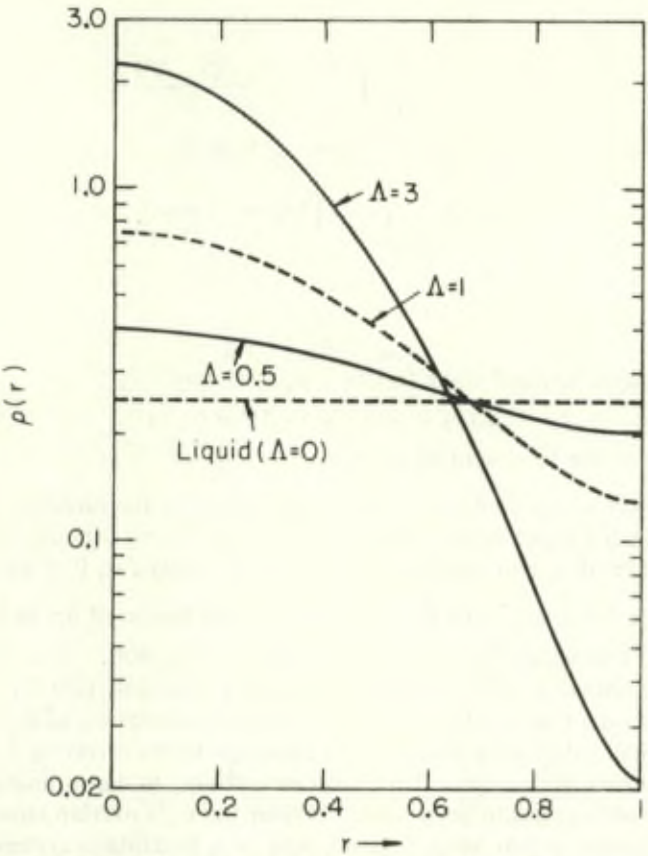

Fig. 15. The variation in ρ (r) from the body center lattice point to the center of cube face, at various values of Λ.

Because when $\Lambda << 1$ the many-body exchange diagrams may not be convergent even when the correlation range is arbitarily small. One indication of this is the direct kinetic energy 3 $\hbar\omega/4$ going below the Fermi gas energy $T_F = 0.3\ \hbar^2 k_F{}^2/m$. Nosanow and Parish have $\rho = 0.3\ \text{fm}^{-3}$, $\Lambda = 0.13$ at which $3h\omega/4 = 23$ MeV while $T_F = 52.9$ MeV.

The results are given in Tables IV, V, and VI. The Boltzmann system with interaction $\overline{v}$ (Table IV) solidifies at $\rho \simeq 2\ \text{fm}^{-3}$ in qualitative agreement with Schiff's result. The solid and fluid energies are very close. (This appears to be a reflection of the softness of the Yukawa repulsive core.) When one considers ther Fermi system with direct + exchange interaction, the solid energy is almost unaffected while that of the liquid goes down due to the attractive exchange interaction (Table V). The most spectacular effects is of the extra attraction ΔV_s in $l = 0$ states.

The relative $l = 0$ state component of ψ is given by

$$\psi_{l=0}(r) = \frac{1}{4\pi} \int \psi(\vec{r})\, d\Omega, \qquad (7.14)$$

and is directly proportional to $\Delta\Omega$, the solid angle in which the relative wave function is localized. The $\Delta\Omega$ is very small in solids, and decreases with Λ, where as the $l = 0$ state interaction is substantial in liquids. The result is i) we do not obtain a minimum in E (ρ, Λ) for $\rho < 2$ fm^{-3}, and ii) the liquid energy goes substantially below the solid energy. Thus it appears that with this choice of interaction the neutron matter does not solidify in the density range of interest. It should be noted here that following the discussion in Section 2, the only alternative to approximations (2.3) is to assume an interaction which goes on becoming more and more repulsive as l increases. Such an interaction will obviously favor the fluid phase rather than the solid.

Canuto and Chitre[41] have attempted to calculate the neutron solid energy with the t matrix theory, and the Reid soft core potential in $^1S_0$, $^3P_0$, $^3P_1$, $^3P_2$, and $^1D_2$ states. The potential in $l \geq 3$ odd states is taken to be V_c $(^3P_2 - {}^3F_2)$, while that in $l \geqslant 4$ even states is V $(^1D_2)$. Thus they include the non-central forces in their calculation. Secondly by doing a partial wave expansion they avoid the $\theta \simeq 0$ approximation mentioned earlier. By comparing their solid energy with LOCV fluid energy solidification is suggested at $\simeq 1.2$ fm^{-3}. The problems here are the following: 1) Since the fiuid energy is calculated with central forces only, the comparison may not be meaningful. 2) The extent to which their methods are valid for noncentral forces is not clear. The point is that in calculating the interaction of neutrons localized at lattice points $\vec{R}_m$, and $\vec{R}_n$ the relative wave function is expanded in partial waves with $\vec{Z}$ axis in the direction of $\vec{R}_{mn}$. Here the neutron spins are always assumed to be either parallel or antiparallel to $\vec{R}_{mn}$, a situation which is impossible in a 3-dimensional lattice. Canuto and Chitre are aware of this difficulty, and are currently devising an approach designed to deal with it. 3) The calculations which have been carried out are only lowest order ones, and the importance of higher order corrections is not known.

8. HYPERONIC MATTER

Pure neutron matter is of course an idealization of dense matter because the neutrons could decay into other particles. The ground state of matter can be thought as an equilibrium mixture of baryons and leptons. The equilibrium ocndition is given by the following relations between the chemical potentials μ_m,

$$\mu_m = \frac{\delta\varepsilon}{\delta\rho_m}, \tag{8.1}$$

for the species m. Here ρ_m is the partial density of baryons m. The

$$\mu_{\text{neutral baryons}} = \mu_0,$$
$$\mu_{\text{positive charged baryons}} = \mu_0 - \mu_e,$$
$$\mu_{\text{negative charged baryons}} = \mu_0 + \mu_e,$$

and

$$\mu_{\text{leptons}} = \mu_e \tag{8.2}$$

The two chemical potentials μ_e and μ_0 can be determined from charge neutrality, and total density.

The problem here is of course the ignorance of the interactions between various baryons. Various models have been tried, and the results of these can be summarized as follows.

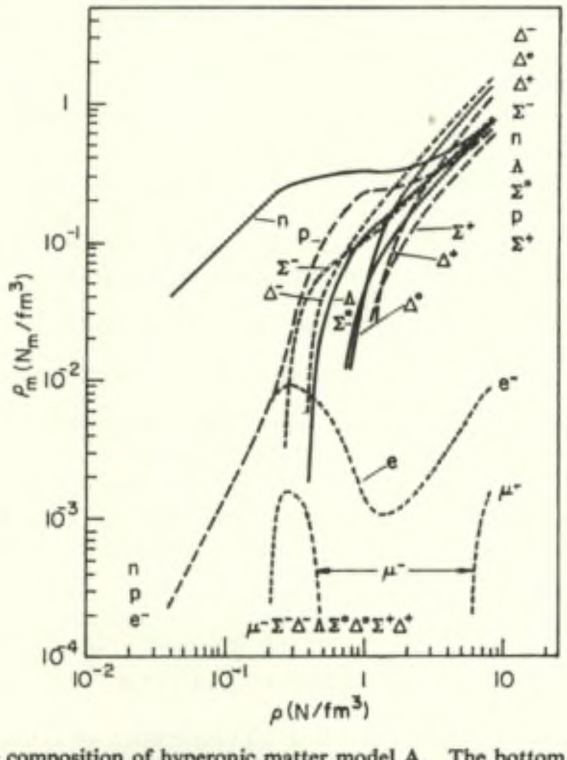

Fig. 16. The composition of hyperonic matter model A. The bottom row of symbols gives the sequence in which various hyperons appear as stable particles, whereas the right hand column indicates the order of partial densities at the highest total density.

Model A[14]: In this model all baryons interact with the Reid $T = 1$ state interaction. This choice of interaction is certainly too attractive for the neutron-proton pairs, as well as many other baryon-baryon pairs. The composition is shown in Fig. 16, and is very heterogeneous at high ρ.

Model C[14]: Same as model A, except the strength of intermediate range attraction is reduced by 10 % in all pairs except neutron-neutron, and proton-proton. The composition is similar to that in Fig. 16.

Model E[42]: Only neutrons and protons are seriously considered, and the Reid interaction is used. The striking feature is that the protons disappear around 1 fm^{-3} (Fig. 17) because the Reid $T = 0$ neutron-proton interaction becomes very repulsive at high ρ. (See Fig. 9 of Section 5). This of course shows that the composition is very sensitive to the interaction choice.

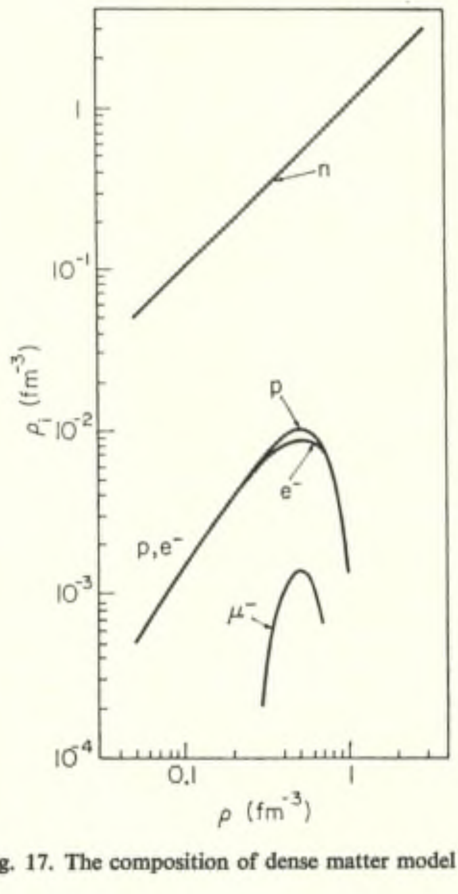

Fig. 17. The composition of dense matter model E.

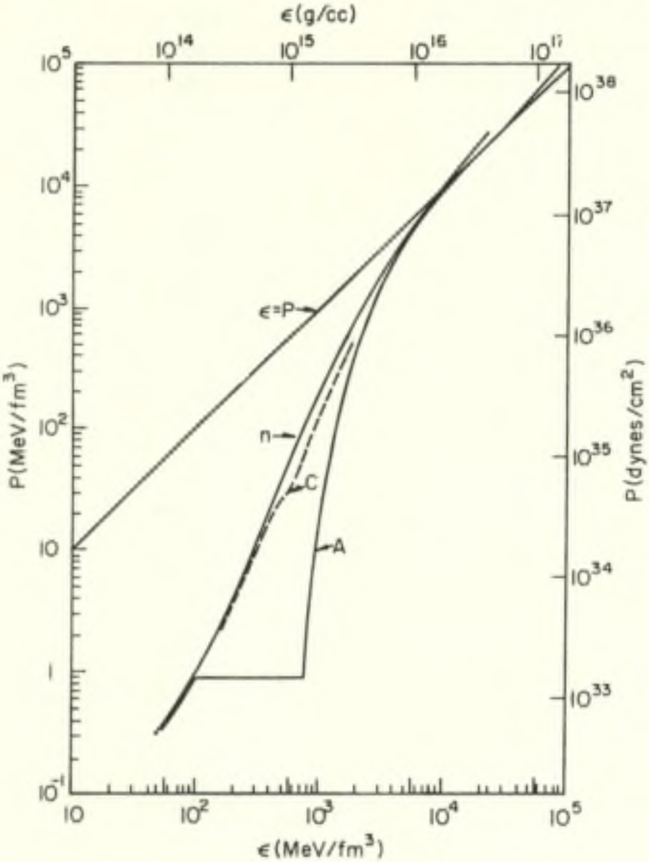

Fig. 18. Hyperonic matter equations of state.

Model of Bethe and Johnson[1]: Bethe and Johnson (BJ) note that the phase shifts leave the repulsive core strength undetermined, while the composition is very sensitive to the differences in the interactions of various Baryon pairs. Hence they argue that to obtain any meaningful results the core strengths in various pairs have to be theoretically prescribed. Since ω meson is isoscaler $T = 0$, BJ attempt to fit all N-N phase shifts assuming similar core in all states. The core is assumed to be $\simeq 6484 \exp(-4.9\ r)/0.7\ r$, and suitable averages over spin and isospin are carried out to generalize the N-N interaction to treat other baryon pairs. The composition obtained by BJ is similar to that in Fig. 15.

The equations of state for models A, C, and neutron matter are shown in Fig. 18. Those of modes E, and BJ are very similar to the neutron-matter equation os state. With model A the transition from neutron to hyperonic matter is discontinuous. All the P (ε) curves rapidly bend over to $\varepsilon = p$ around 5×10^{15} g/fcm^3, for reasons discussed in Section 6, and there is little spread between them in this density region. Fortunately the neutron-star mass limits are mostly determined by the equation of state in this region. Baym, Pethick, and Sutherland[37] obtain maximum masses of 1.27, 1.41 and 1.66 $M_\odot$ respectively with models A, C, and pure neutron matter. The BJ equation of state gives 1.6 $M_\odot$.

It is probably reasonable to rule out model A as too unrealistic, in which case the uncertainties in the equation of state of hyperonic matter become tolerable, however its composition still remains too sensitive to the choice of interaction.

9. MINIMAL RELATIVITY

The repulsive core in the N-N interaction excites particles to momenta up to $\simeq 6\ \mathrm{fm}^{-1}$, and hence the validity of non-relativistic calculations presented in preceding sections is not obvious. These non-relativistic calculations however are dictated by the fact that the Reid potential is obtained by fitting the experimental phase shifts using non-relativistic Schrödinger equation.

The relativistic kinematical corrections, popularly known as 'minimal relativity corrections' have been studied by Brown, Jackson, and Kuo[43] with Blankenbeckler-Sugar[44] equation for the scattering matrix $\mathcal{T}(k, k')$:

$$\mathcal{T}(k,k') = \mathcal{V}(k,k') - \frac{2\,m^2}{(2\,\pi)^3} \int d^3 q\ \frac{\mathcal{V}(k,q)\,\mathcal{T}(q,k')}{(q^2 - k'^2 + i\,\varepsilon)\sqrt{m^2+q^2}} \tag{9.1}$$

They note that the m/E factor can as well be associated with the potential, and scattering matrix, and obtain the Lippman-Schwinger equation

$$T(p,q) = V(p,q) - \frac{2\,m}{(2\,\pi)^3} \int d^3 k\ \frac{V(p,k)\,T(k,q)}{k^2 - q^2 + i\,\varepsilon} \tag{9.2}$$

with

$$V(p,q) = \left(\frac{m^2}{m^2+p^2}\right)^{1/4} \mathcal{V}(p,q) \left(\frac{m^2}{m^2+q^2}\right)^{1/4}, \tag{9.3}$$

and

$$T(p,q) = \left(\frac{m^2}{m^2+p^2}\right)^{1/4} \mathcal{T}(p,q) \left(\frac{m^2}{m^2+q^2}\right)^{1/4}. \tag{9.4}$$

The N-N phase shifts are fitted with Eq. (9.2) and a $\mathcal{V}$ (p, q) very similar to the Reid Potential. In $^1S_0$ state the two potential are:

$$V_{Reid}\,(^1S_0) = (-\,10.463\,e^{-x} - 1650.6\,e^{-4x} + 6484.2\,e^{-7x})/x$$
$$\mathcal{V}\,(^1S_0) = (-\,10.463\,e^{-x} - 1650.6\,e^{-4.0932x} + 6484.2\,e^{-7.3115x})/x. \quad (9.5)$$

Here $x = 0.7\,r$ (in fm). The Reid and (9.5) are equivalent on the energy shell; however they differ off energy shell due to the minimal relativity effects.

LOBT calcultations with (9.5)[43] give slightly lower energies than those with Reid potential, the difference is only — 0.5 MeV, and comes entirely from the neutron-proton $^3S_1$ interaction. The relativistic correction is almost zero for the $T = 1$ $^1S_0$ interaction at nuclear matter densities. The smallness of this effect may be because the high momentum components come from very short range correlations ($r \simeq 0.2$ fm), and are almost identical in free scattering and matter wave functions. The difference in these wave functions, and hence the correction, should increase with density.

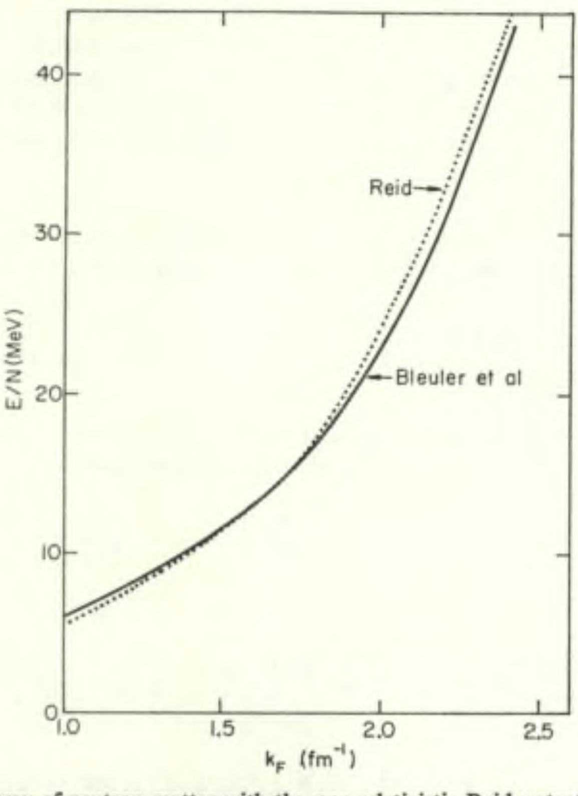

Fig. 19. Energy of neutron matter with the non-relativistic Reid potential, and the OBEP with « minimal relativity ».

Bleuler *et al*[45] take $\bar{V}$ (p, q) to be one-boson exchange potential derived from covariant perturbation theory. It thus includes the retardation of exchanged mesons, and is energy dependent. The meson masses, coupling constants etc. are adjusted so that V(p, q) gives the experimental phase shifts, and neutron-matter is studied in LOBT. Their results are compared with those with Reid potential and LOBT[27] in Fig. 19. The two calculations give very similar energies up to $\rho \simeq 0.5\ \mathrm{fm}^{-3}$.

The potential (9.5) is non-local, and no variational calculations have been carried out with it. Thus even the relativistic kinematical effects are not studied in the important density range of $\rho = 1 - 2\ \mathrm{fm}^{-3}$. The violation of causality discussed in Section 6 certainly indicates effects greater than $\simeq 10\ \%$ around $\rho \simeq 5\ \mathrm{fm}^{-3}$.

10. THE NN-NΔ COUPLED CHANNEL PROBLEM

The two-pion exchange[3-4] gives a large contribution to N-N interaction, and some of the diagrams that contribute to it are shown in Fig. 20. The diagram C is simply the virtual scattering of a nucleon-nucleon pair in to nucleon plus Δ (1236) resonance, and implies a new correlation in the wave function of the NN and virtual NΔ channels. This correlation, and hence a part of the two pion exchange potential, will be suppressed in matter due to the presence of other nucleons. The effect is similar to the reduction in Λ-potential energy in nuclear matter, obtained by attributing a part of NΛ attraction to virtual scattering into N Σ state[46].

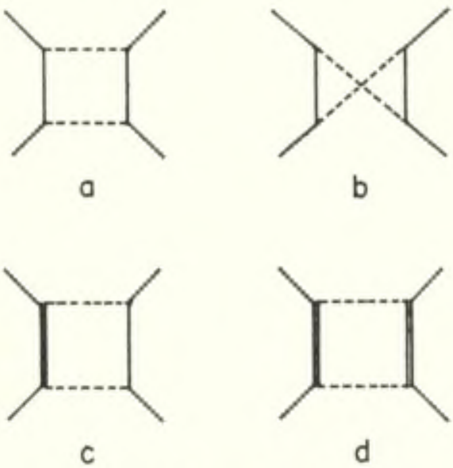

Fig. 20. Two pion exchange diagrams.

Green and Haapakoski[47] have demonstrated this effect in neutron matter by assuming that the entire two-pion exchange contribution comes from the virtual scattering.ë They assume the interactions in $T = 1$, $^1S_0$ (NN), and $S - 2$, $l = 2$, $J = 0$ ($N\Delta$) coupled channels to be :

$$<NN|v|NN> = -10.5\frac{e^{-0.7r}}{0.7r} - 43\frac{e^{-2.75r}}{2.75r} + 5000\frac{e^{-3.9r}}{3.9r}, \tag{10.1}$$

$$<NN|v|N\Delta> = 44\frac{e^{-0.7r}}{0.7r}\left[1 + \frac{3}{0.7r} + \frac{3}{(0.7r)^2}\right]\left[1 - e^{-1.8r^2}\right]^2. \tag{10.2}$$

The first two terms in (10.1) are due to one π and η exchange respectively, the last is due to ω and ρ exchange, and its strength is obtained by fitting $^1S_0$ phase shifts. The (10.2) is suggested by von Hippel and Sugawara[47a], and the last coefficient of it is meant to remove the $1/r^3$ singularity of one π exchange. The exponent $-1.8\ r^2$ is also obtained by fitting the scattering data.

Green and Haapakoski calculate the $^1S_0$ state contribution of potentials (10.1) and (10.2) in neutron matter, using the LOCV method[16]. They find it to much less (Fig. 21) than that with a simple two-body potential between neutrons.

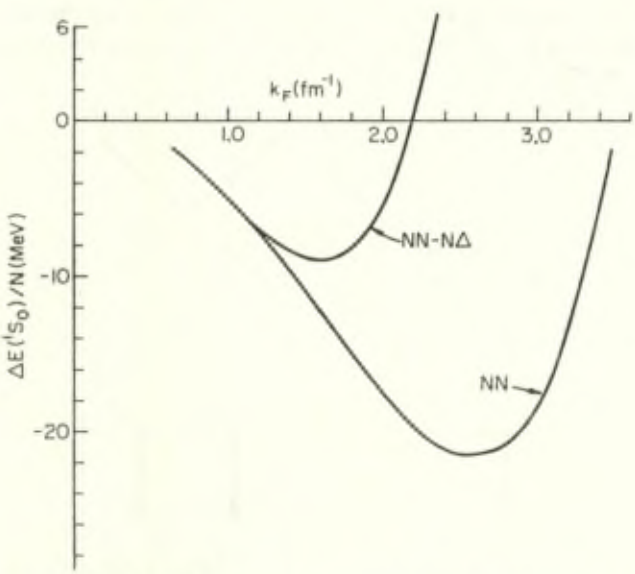

Fig. 21. The contribution of $^1S_0$ state to the energy of neutron matter as estimated with a static N-N potential, and the potentials (10.1) and (10.2) (from ref. 47).

The magnitude of the effect in Fig. 21 is certainly much too large because the choice of potentials (10.1 - 10.2) assumes that the entire intermediate state attraction is due to NN — NΔ virutal scattering, which is certainly not true. Addition of another attractive term in (10.1) for example will reduce the coupling (10.2) and hence the effect of the coupled channels on energy.

A qualitative effect of this type is welcome in nuclear-matter, because it will aid to obtain saturation. In dense neutron matter the contribution of $^1S_0$ state is not too important (Fig. 11), and the magnitude of the effect should be studied in ^3P and ^1D states with realistic coupling 10.2. The NN — NΔ coupling will make the equation of state (Fig. 13) harder at lower densities, and it may not have too large an effect at high ρ.

11. THREE-BODY FORCES

The contributions of two pion exchange 3-body force, shown in Fig. 22 have been estimated in nuclear matter. In lower-order processes (22a) the pion scattered by nucleon 2 has momentum $\simeq 0$. Brown *et al*[48] have shown that the isospin symmetric scattering amplitude of $q \simeq 0$, zero energy pions by nucleons should be very small, and hence 2-pion exchange 3-body force is small. Its contribution is estimated by Bhaduri *et al*[49] to be $\simeq 0.5$ MeV/nucleon at $k^F = 1.36$.

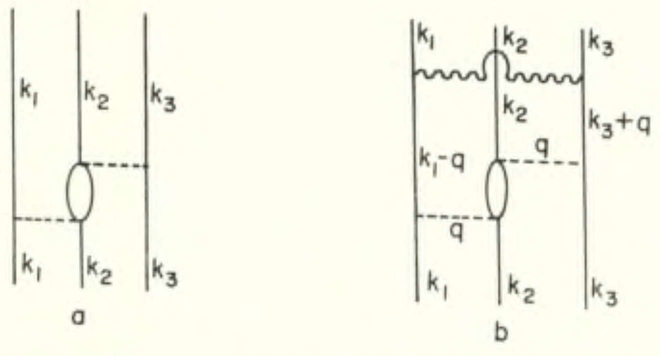

Fig. 22. The two pion exchange three body force contributions in lowest and higher orders.

In higher order processes (22 b) however the pion momentum can be large, and these give the dominant contribution. With reasonable pion form factors Bhaduriet *et al*[49] estimated it to be $\simeq -6$ to -7 MeV/nucleon.

McKellar and Rajaraman[50] have formally generalized Brueckener-Bethe theory to treat strongly interacting systems with 2 - and 3 - body forces. They argue that the 3-body force is strong when the 3 particles are close, but the repulsive core in two-body interaction cuts off the wave function when any of r_{12}, r_{23}, r_{31} is small. Hence the short range correlations should drastically reduce its contribution. Loiseau *et al*[51] using cut off at $\simeq$ 0.8 fm in r_{12}, r_{13}, r_{23} estimate the 3-body force contribution to be $\simeq -1.14$ MeV/nucleon at equilibrium density.

The effect of this force in neutron-matter has not been studied. However it may be mentioned that at nuclear matter densities the dominant contribution of (22 b) comes when 1 and 3 are neutron and proton interacting via the ($^3S_1 - {}^3D_1$) tensor force. This contribution will be absent in dense neutron-matter, but the other contributions may become non negligible due to high density.

12. PION CONDENSATION

Low momentum π^- can not possibly occur in the ground state of dense matter because i) the recent calculations that $\mu_n - \mu_p$ never exceeds the pion rest mass, and ii) the low energy $\pi^- - N$ interaction, as estimated from pion-nucleon scattering length, gives a repulsive contribution of $\simeq 219\ (\rho_n - \rho_p)$ MeV/pion (ρ_n, ρ_p in fm^{-3}).

Recently Sawyer and Scalapino[52], and Migdal[53] have pointed out that the attractive p-wage interaction would greatly reduce the energy of high momentum pions in matter. If μ_π becomes less than $\mu_n - \mu_p$ the pions, since they are bosons, will occupy a single microscopic mode (a condensate) $e^{i\vec{k}\cdot\vec{r}}$ with finite $\vec{k}$. The equilibrium conditions of such a condensed state have been discussed by Baym[54].

It is probably to premature to decide if such a condensate is possible. Sawyer and Scalapino[52], using a wave function having average neutron momentum zero, and average proton momentum equal and opposite to the π^- momentum, estimated that i) π^- may condense at $\rho \simeq 0.5 - 1.0$ baryons/fm^3, and ii) the condensate would decrease P (ρ) by amounts $\lesssim P_F(\rho)$, the ideal Fermi gas pressure. This amounts to a substantial change in the equation of state for neutron matter at $\rho \simeq 0.5 - 1.0$. However the core repulsion still dominates at high ρ, the ΔP being only 15 % at $\rho \simeq 2\ fm^{-3}$.

Bethe and Johnson[55] have argued that the Sawyer-Scalapino wave function essentially has a displaced proton Fermi sphere, and hence large relative momentum between protons and neutrons. The $N - N$ interaction, due to its l-dependence, becomes less and less attractive at large relative momenta. They estimate that this increase in the nuclear interaction energy could more than compensate the π^- condensation energy gain, thus preventing condensation.

Baym[54] however has shown that the Sawyer-Scalapino wave function is too simple. At equilibrium the spatial averages of spatial baryon, charge, and lepton currents must be zero. Baym and Flowers[56] find that i) in absence of nucleon-nucleon and $\pi - \pi$ interaction the n, p, π^- matter is unstable, (repulsive N — N interaction of course stabilizes it). ii) The threshold for π^- occurrence is very sensitive to assumed N — N interaction. They demonstrate this with a simple interaction of form.

$$H_{N-N} = \frac{1}{2} \zeta k^2 (N^+ \tau\sigma N)^2 \tag{12.1}$$

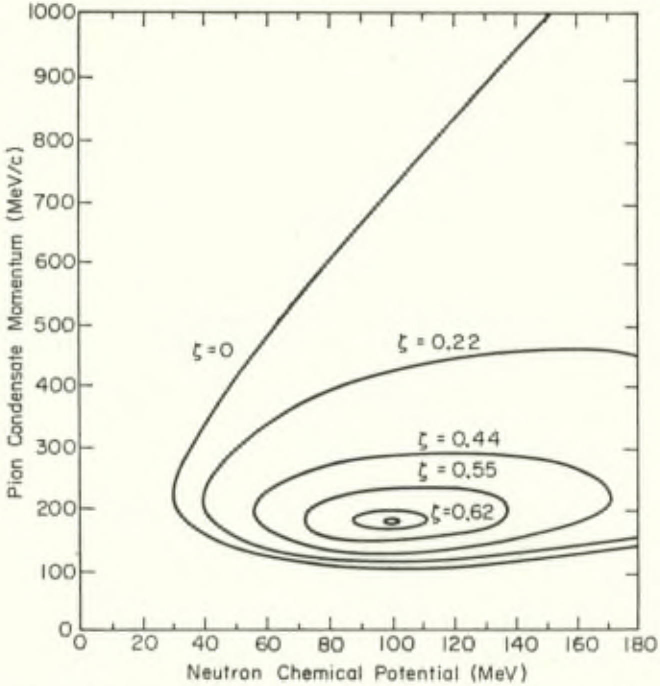

Fig. 23. Thresholds for appearance of π^- at momentum k in pure neutron matter (from ref. 56).

The k, and μ_n thresholds are shown in Fig. 23 for various values of ζ. Around $\mu_n \simeq 40 - 60$ MeV the ζ value estimated from the $(\sigma_1 . \sigma_2)$ $(\tau_1 . \tau_2)$ components of LOBT G matrix is $\simeq 0.45$. It is expected to increase with ρ_n as more and more of the $(\sigma_1 . \sigma_2)$ $(\tau_1 . \tau_2)$ repulsion due to ρ exchange is felt by the nucleons. Finally Barshay *et al*[57] have indicated that coupling of π^- to neutron hole — Δ^- state will greatly favor the condensate.

Migdal[58] and Barshay *et al*[57] have considered the possibility of π^0 condensation, and Migdal[57] has also considered $\pi^- - \pi^+$ pair creation unstability. Recent calculations[56, 57] indicate small negative condensate energies which might not affect the equation of state appreciably.

13. COLLAPSED MATTER

Bodmer[59] has considered the possibility that at very high density ($\rho_c \simeq 10$ baryons/fm^3) there may be a new, as yet unobserved phase of matter. If the matter in this phase is stable, i. e. its binding energy is greater than the nuclear binding energy, then the probability of transition from nuclear matter to this matter must be very small. The lifetime of nuclear matter τ_N must be $\gtrsim 10^{31}$ sec on observational grounds. Considering the collapse of matter to this phase as a barrier tunneling problem

$$\tau_N = \tau_0 P_N^{-1} > 10^{31} \text{ sec} \tag{13.1}$$

where τ_0 is the breathing mode period ($\simeq 10^{-22}$ sec), and

$$P_N \simeq \exp\left\{-2\left(\frac{2AM_N}{\hbar^2}\right)^{1/2} \int_{R_c}^{R_N} \Big(W(r) - W(R_N)\Big)^{1/2} dr\right\} < 10^{-53} \tag{13.2}$$

is the penetrability for A nucleans to collapse from a uniform spherical distribution of radius R_N to the collapsed radius R_c. The barrier is given by

$$W(r) = A\varepsilon(\rho)/\rho,$$

with

$$\frac{4\pi}{3} r^3 \rho = A \tag{13.3}$$

The penetrability decreases rapidly with A. With the neutron matter $\varepsilon(\rho)$ it can be concluded that the collapsed matter must not be stable for $A \gtrsim 10$ provided $\rho_c \simeq 10$ fm^{-3}.

Bethe[60] has pointed out that if the collapsed state is to be bound by an attractive Wigner potential of radius $\simeq 0.25$ fm between the nucleons, the fact that it can not bind less than 10 particles indicates that its volume integral is very small. The ratio

$$\frac{\text{Volume integral of attraction inside core}}{\text{Volume integral of the repulsive core}} < 10^{-2} \tag{13.4}$$

is very small, and hence such forces if present will have negligible effect on the equation of state of the normal phase.

In neutron star interiors however the R_N (penetrability equation [13.2]) is much smaller, and $W(R_n)$ is large. Naturally the probability of collapse of neutron star matter is much larger. It has been suggested by Gold[61] that a slow collapse of core could explain the Vela speedups.

14. MATTER AT SUB-NUCLEAR DENSITIES

At densities much less than nuclear-matter density the nucleons cluster to form nuclei which in turn form a Coulomb solid in degenerate electron gas. The problem is to calculate the most stable nuclear species. At low densities ^{56}Fe is the most stable nucleus, while as the density and the electron Fermi energy increases, more and more neutron rich nuclei are favored. Table VII lists the stable nuclei at various densities as calculated by Baym et al[37]. These calculations are primarily based on liquid-drop semi-emperical mass-laws.

Around 4.3×10^{11} gms/cc neutron drip begins, and the most stable configuration is a nuclear solid in a sea of neutron and lepton gas. Most recent calculations in this regime have been carried out by Negele and Vautherin[62] with density-dependent effective interaction based on LOBT G matrix in nuclear and neutron matter, with appropriate corrections (for many-body clusters etc.) determined by fitting the properties of stable nuclei. Their results are summarized in fig. 24.

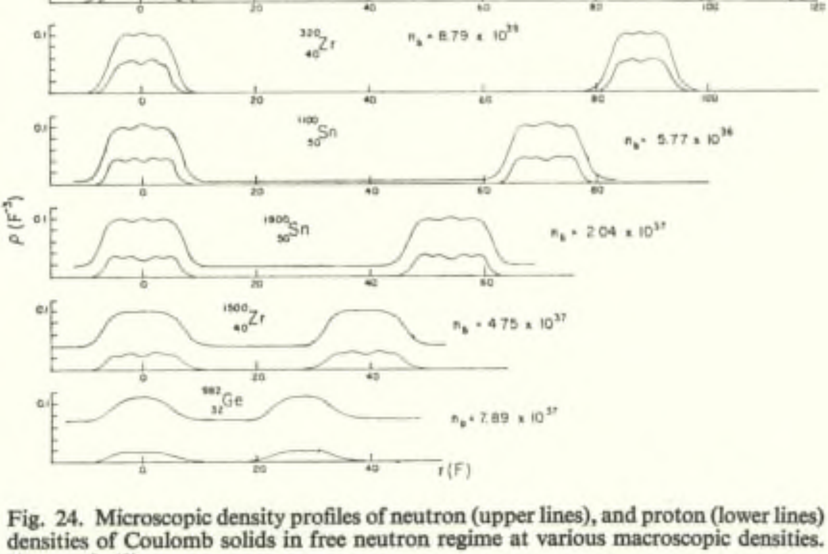

Fig. 24. Microscopic density profiles of neutron (upper lines), and proton (lower lines) densities of Coulomb solids in free neutron regime at various macroscopic densities. (from ref. 62).

At $\rho \sim 2.5 \times 10^{14}$ g/cc the neutron and proton density variations smooth out suddenly with a first order transition, and the matter becomes uniform fluid of neutrons, protons, electrons, and muons. The neutrons and protons in this density range, and the neutron fluid in the solid crust is believed to be superfluid[62]. The cause of neutron superfluidity may change from $^1S_0$ pairing to $(^3P_2 - {}^3F_2)$ state pairing as shown in Fig. 25.

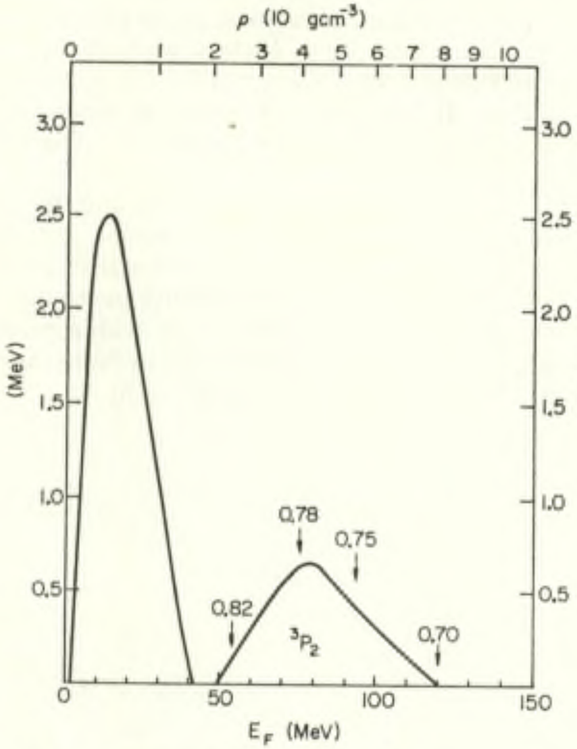

Fig. 25. The $^1S_0$ - and $(^3P_2 - {}^3F_2)$ energy gaps in neutron matter. The effective mass values (m^*/m) used in the calculation are shown by arrows at various densities. (from ref. 63).

15. COMPARISON WITH „ EXPERIMENTAL DATA "

With the density-dependent Hartree-Fock theory Negele[64] has shown that to obtain the experimentally observed binding energies, and radii of nuclei, the starting nuclear-matter E (ρ) should have its minimum at $k_F = 1.31$ fm^{-1} with $E_{eq} \simeq 16$ MeV. We will regard those as reasonable " experimental " numbers, rather than those obtained with semiempirical fits[6] ($k_F \simeq 1.45$ fm^{-1}). Adding the corrections due to minimal relativity, and three-body forces to the results in Section 5 the theoretical k_F is estimated at $\simeq 1.5$ fm^{-1} with $E_{eq} \simeq 16.3$ MeV. The density dependence of the corrections is not too well studied[6] and hence the estimates may not be very accurate. However they definitely suggest that the k_F, and E_{eq} are both a little too high. The NN-NΔ coupled channel correction however will help to decrease k_F significantly.

Some experimental information on the equation of state at high densities comes from the infered moment of inertia $I \gtrsim 1.5 \times 10^{44}$ g/cm^2 of Crab pulsar from its luminosity, and slowdown[65]. The neutron or hyperonic matter equations of state discussed here can give stable pulsars with $I \lesssim 6 \times 10^{44}$ g/cm^2,[37] and thus explain the Crab luminosity. It may be pointed out here that $I < 10^{44}$ g/cm^2 if the repulsive core interaction is neglected[66].

Fig. 26. The arrows show the direction of σ (and S) in the proposed cubic lattice. All particles in a plane perpendicular to the Z direction have parallel σ (and S) vectors. The numbers give neighbor positions, and all the 3rd neighbors are off the XZ plane.

If one accepts the crustquake mechanism[67–68] for pulsar speed up, and the two component-theory of post-glich healing[69], the behavior of Crab pulsar can be understood from a $\sim 0.5\ M_\odot$ model with the present equations of state, and only one solid-fluid phase transition at $\sim 2.5 \times 10^{14}$ g/cc. However the large Vela speed ups cannot be easily understood as crust quakes if only the above transition exists. Pines et al[31] have suggested that the Vela speed ups could probably be understood as core quakes provided a second fluid solid transition is assumed at $\sim 10^{15}$ g/cc. Such a transition, as discussed in Section 7, has not yet been demonstrated with a convincing theory. It us very unlikely in neutron matter because of the *l*-dependence of the interaction. It may however be mentioned here that other phase transitions of the type discussed in Section 13 could also possibly explain Vela quakes. Secondly a strong tensor force between particles can also give a solid on account of spin order. The $T = 1$ neutron-neutron tensor force (Fig. 3) is probably too weak to crystallize matter, however the NN-NΔ interaction is dominated by a strong tensor formed from the nucleon spin $\vec{\sigma}$, and a vector $\vec{S}$ related to the polarization vector of the Δ[47a]. We are currently studying a solid in which the particles are superpositions of neutron and Δ resonances. The simple cubic solid with spin structure as shown in Fig. 26 will probably be favored because it gives attractive tensor interaction from the first and second neighbors. The tensor interaction from the 3^{rd} and 4^{th} neighbors is zero, while that from the fifth is repulsive but rather weak due to the large separation in position space.

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TABLE I

	1	2	4	6	7
$v_c(^1S_0)$	-10.463		-1650.6		$+6484.2$
$v_c(^1D_2)$	-10.463	-12.322	-1112.6		$+6484.2$
$v_c(^3P_2-^3F_2)$	$+3.488$		-933.48	$+4152.1$	
$v_c(^3S_1-^3D_1)$	-10.463	$+102.012$	-2915	$+7800.0$	
$v_c(^1P_1)$	$+31.389$	-240.0		$+17000.0$	

Coefficients $a_n(lSJ)$ of the Reid potential. The very strong repulsion in $T = 0, S = 0$ $^1P_1$ state clearly indicates the presence of a $(\sigma_1 . \sigma_2)(\tau_1 . \tau_2)$ component in the core.

TABLE II

n	E_n	$E_2\kappa^{n-2}$
2	35	35
3	2	5
4	1.1	0.7

Contributions of n-hole line diagrams to the binding energy of nuclear matter at $k_F = 1.36$.

TABLE III

ρ (N/fm^3)	(E/N) MeV	1.2	1.6	2.0
1.0	Total	204.4	177.3	169.7
	L.0	156.3	123.7	108.0
	H.0.	38.1	53.6	61.7
3.0	Total	1174.0	1037.7	997.4
	L.0.	982.5	710.6	544.3
	H.0.	191.5	327.1	453.1
5.0	Total	2467.7	2266.0	2237.6
	L.0.	2117.5	1595.0	1223.0
	H.0.	350.2	671.0	1014.6

The lowest order and higher order contributions to the energy of neutronmatter at values of d/r_0.

TABLE IV

ρ \ Λ	2.36	1.85	1.48	1.20	0.98	0.80	Liquid
1.2	457.6	416.7	390.7	376.0	*370.9*	374.4	337.3
1.6	635.1	589.2	560.7	547.2	*545.7*	554.8	526.7
2.0	831.4	780.6	751.1	*738.8*	741.4	756.8	736.3
2.4	1043	989.0	958.4	*948.1*	954.8	976.8	959.6
2.8	1269	1212_x	1180	*1172*	1184	1212	1204
3.2	1508	1447	1415	*1410*	1426	1462	1456
3.6	1757	1693	1662	*1659*	1681	1723	1720

The E (ρ, Λ) in MeV of a Boltzmann solid interacting with spin averaged dierct potential $\bar{v}$. The liquid energies are given for comparison.

TABLE V

ρ \ Λ	.8 2.36	.77 1.85	.74 1.48	.71 1.20	.68 0.98	.65 0.80	Liquid
1.2	440.5	402.1	378.2	366.3	*364.1*	370.1	334.8
1.6	616.6	572.6	547.2	*536.8*	538.6	550.5	508.6
2.0	812.3	764.1	737.6	*729.0*	734.6	752.8	702.0
2.4	1025	973.0	946.0	*939.3*	949.6	974.3	909.7
2.8	1252	1197	1170	*1165*	1180	1212	1137
3.2	1493	1434	1407	*1405*	1425	1464	1373
3.6	1744	1683	*1656*	1657	1682	1727	1621

The E(ρ, Λ) in MeV of a Fermi solid interacting with $v(^1D_2)$ in all singlet states, and v_c ($^3P_2$ — $^3F_2$) in all triplet states.

TABLE VI

ρ \ Λ	2.36	1.85	1.48	1.20	0.98	0.80	Liquid
1.2	404.4	358.0	325.3	303.7	291.0	285.8	227.6
1.6	567.4	512.9	475.7	452.4	440.3	437.4	362.6
2.0	750.3	688.9	647.9	623.3	611.8	611.7	520.2
2.4	950.5	882.7	838.4	812.9	*802.9*	805.9	697.8
2.8	1165	1092	1045	1018	*1010*	1017	893.4
3.2	1394	1315	1265	1238	*1232*	1243	1105.2
3.6	1633	1550	1497	1471	1467	1481	1332.8

The E(ρ, Λ) in MeV for ' neutron ' solid.

TABLE VII

Nucleus	b (MeV)	Z/A	ρ_{max} (g cm^{-3})	μ_e (MeV)	$\Delta\rho/\rho$ (%)
^{56}Fe	8.7905	0.4643	8.1×10^6	0.95	2.9
^{62}Ni	8.7947	0.4516	2.7×10^8	2.6	3.1
^{64}Ni	8.7777	0.4375	1.2×10^9	4.2	7.9
^{84}Se	8.6797	0.4048	8.2×10^9	7.7	3.5
^{82}Ge	8.5964	0.3902	2.2×10^{10}	10.6	3.8
^{80}Zn	8.4675	0.3750	4.8×10^{10}	13.6	4.1
^{78}Ni	8.2873	0.3590	1.6×10^{11}	20.0	4.6
^{76}Fe	7.9967	0.3421	1.8×10^{11}	20.2	2.2
^{124}Mo	7.8577	0.3387	1.9×10^{11}	20.5	3.1
^{122}Zr	7.6705	0.3279	2.7×10^{11}	22.9	3.3
^{120}Sr	7.4522	0.3166	3.7×10^{11}	25.2	3.5
^{118}Kr	7.2002	0.3051	(4.3×10^{11})	26.2	...

b is the binding energy per nucleon; ρ_{max} is the maximum density at which the nuclide is present; μ_e is the electron chemical potential at that density, and $\Delta\rho/\rho$ is the fractional increase in the mass density in the transition to the next nuclide. The value of $\rho_{max} = 4.3 \times 10^{11}$ g cm^{-3} is the density at which neutron drip begins.

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Discussion of the Report of V.R. Pandharipande

J. SHAHAM: It seems that in the solid core subject, there is more of a problem in the two body potentials involved than in the many-body calculations. Could you briefly comment on the prospect of getting better experimental data on the two body potentials in the next few years?

V. PANDHARIPANDE: The limitation on the energy in scattering experiments comes from competing inelastic processes, and hence no new information can be expected from these. There have been suggestions (M. J. Moravesik and P. Ghosh, Phys. Rev. Lett. to be published) that better determination of the deuteron wave function at $r < 0.5$ fm is possible by measuring tensor polarization in elastic electron-deuteron scattering. A known deuteron wave function will of course put additional constrains on the potential core. I am not up-to-date on other measurements that could give valuable information on the NN potential, particularly off the energy shell.

J. SHAHAM: But are the two body potentials known with sufficient accuracy at the densities involved?

V. PANDHARIPANDE: Various potentials, with static Yukawa core that fit the scattering data, do not give widely different results for neutron matter equation of state. This could be because then the equation of state at low and high densities is uniquely determined. Latest studies on the solidification problem (S. Cochran and G. V. Chester, Cornell University, USA) indicate that this family of potentials will not give a solid neutron matter.

The situation however is much less clear for hyperonic matter, and also for other classes of potential models.

V. CANUTO: I would like to point out that historically the first proposal of a solid model for neutrons at high density appeared in a 1966 paper in Nuovo Cimento by Cazzola, Lucaroni and Scaringi. Secondly, it seems to me that as far as the NN potential is concerned we surely have a problem from the fundamental point of view, since at this high densities we are not quite sure if the concept of potential makes sense at all. However as a working hypothesis suppose we use the best NN potential known today. Suppose also that such a potential is for instance given by Reid. Then I do think that we should not make any more changes to it as you seem to have done, because this doesn't help an already difficult phase of our computation.

V. PANDHARIPANDE: Since it has not been proved that the Reid potential gives the real NN interaction I see no reason why we should not improve it where it appears to be inconsistent with out theoretical concepts.

R. HOFSTADTER: Will your equation of state of nuclear matter predict correctly the skin thickness of finite nuclei, which is a constant over all the nuclei?

V. PANDHARIPANDE: First, because of the rapidly varying density the problem of nuclear surface is much more difficult than that of infinite matter. Second, the present equation of state does not explain nuclear matter density satisfactorily. Hence detailed properties of surface have not been this extensively studied. However Negele[64] could reproduce the surface thickness of many nuclei using a local density approximation and a phenomenological correction to the equation of state for obtaining correct central density.

R. HOFSTADTER: I want to suggest that you look at two number, not one. Not only the average density but the magnitude of the surface thickness, would be an important test of your equation of state for the nuclear fluid.

V. PANDHARIPANDE: I certainly agree, but I am afraid that it is a much more difficult problem.

R. OMNES: Is it established that the corrections to self-energy and to forces which may be due to the exclusion principle are negligible?

V. PANDHARIPANDE: The self energy corrections are not negligible, however they are included in the two-body potential. The one pion exchange potential for example can be thought of as a self energy correction due to exclusion principle.

NEUTRON STARS: GENERAL REVIEW

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INTRODUCTION

Neutron stars have an interesting conceptual history. It was foreseen by Landau (1932) that such highly condensed stable objects might exist. The first models of neutron stars were constructed by Oppenheimer and Volkoff (1939), using a very simple equation of state in which no nuclear interactions were assumed between the neutrons. At about the same time there was speculation by Baade and Zwicky (1934) that the formation of neutron stars might be an important event associated with supernova explosions, which had then only recently been discovered.

The lack of any relevant observations then led to a decline in the theoretical interest in neutron stars, and further work on their theory did not begin again for another two decades. During this extended interval, only Zwicky appears to have been a consistent advocate of the probable role played by neutron stars in astrophysical processes.

Interest in the theoretical properties of neutron stars was revived by Harrison, Wakano, and Wheeler (1958), who became interested in the issue of the final state that would be reached in the collapse of a massive astrophysical body. They developed an equation of state for cold catalyzed matter extending from ordinary densities to the densities associated with nuclear matter, and constructed models of neutron stars using this more complicated equation of state. At about the same time Cameron (1959) became interested in neutron stars as a probable end product of supernova explosions, which might provide at high densities a suitable source of neutrons for the r-process production of heavy elements in stellar nucleosynthesis. He noted that a large variety of hyperons might be expected in the cores of neutron stars.

He also made an early attempt to include some of the effects of nuclear forces in the neutron star equation of state. The existence of hyperons in neutron star matter was also independently foreseen by Ambartsumyan and Saakyan (1960), who computed the relative abundances of the various hyperons in dense matter, but without including the effects of nuclear forces upon the relative number densities.

Relatively little further work was done until celestial x-ray sources were discovered in the early 1960's. At that time, it was thought that the x-rays being observed might result from thermal emission from the surface of a hot neutron star. If so, the temperature of the neutron star surface would have to be about 5×10^7 °K. This led to some theoretical work on the cooling of neutron stars (Morton 1964, Chiu 1964, Tsuruta and Cameron 1966*a*). Both neutrino and radiative cooling were considered at that time. It was concluded that an x-ray source such as the Crab nebula was unlikely to involve thermal emission from a neutron star surface, because the surface temperature should have decreased to at least an order of magnitude less than the observed temperature during the age of the Crab nebula. At that time, also, it was experimentally observed that the source diameter of the x-ray emission from the Crab nebula was too large to be consistent with the dimensions of a neutron star.

During the next few years study of the general relativistic stability and instability of neutron stars was continued by Wheeler and his colleagues (Harrison, Thorne, Wakano, and Wheeler 1965; Wheeler 1966; Thorne 1967). Following a hypothesis by Cameron (1965) that vibrations of a neutron star might lead to acceleration of cosmic rays and production of x-rays in the corresponding vibrating magnetosphere, a number of studies of the vibrational properties of neutron stars were carried out. These ultimately showed that a number of damping mechanisms should exist which would quickly damp out the vibrations of a neutron star, hence making such a hypothesis untenable.

Then came the discovery of the pulsars. After some initial months in which a variety of possible models of pulsar emission were suggested, the view of Gold (1968) that the only plausible model for a pulsar is a rotating neutron star, gradually prevailed. In early 1969, after the discovery that the Crab nebula pulsar was gradually spinning down, this view became well established, since only rotating neutron star models seemed to be capable of providing an energy output through loss of rotational energy at a rate corresponding both to the observed slowing down of the Crab nebula pulsar, and the observed energy output from the Crab nebula.

A rapid explosion of papers on the properties of neutron stars followed. The neutron star literature is now very extensive. In this review no attempt will be made to cite all of this literature, but rather an attempt will be made to describe those properties of neutron stars upon which there seems to be general agreement, and to indicate those areas which currently remain in doubt.

PHYSICS OF THE INTERIOR OF A NEUTRON STAR*

1. The physics we need.

As we have briefly noticed in the Introduction, after Landau's pioneering proposal and order of magnitude computation, few theorists had the bravery to speculate in great detail upon the structure of matter at super-high densities.

The work of Oppenheimer and Volkoff was not actually concerned with the structure of matter under these unusual conditions, but rather with the justification of Landau's results on a more rigorous basis. In their work, the structure of the star was not taken to be more sophisticated than in Landau's model, i.e., a gas of non-interacting degenerate neutrons. We may also notice that it would have been unwise to elaborate to any greater length about the structure of nuclear forces and their influence upon the equation of state at a time when the existing experimental data could not possibly be used to predict anything meaningful about the behaviour of V(r) at $\rho \simeq 10^{14}$ g.cm^{-3} or equivalently at distances less than $\sim$ 1 fm.

In the thirty years that have elapsed since the appearance of this work until the discovery of pulsars, very little was done to improve upon Oppenheimer's work. During the same period, however, the problem of nuclear forces was under massive study. As H. Bethe has remarked, the study of the nucleon-nucleon scattering has attracted more man-hours than any other problem in physics in this century. Not only was the two-nucleon problem thoroughly investigated but also the mathematical foundations were laid for the treatment of an assembly of nucleons, the so-called nuclear many-body problem, the solution of which has a fundamental bearing on the problem of the structure of neutron stars. Both the fields just described were characterized by periods of strong disagreement among the several working groups and an agreement was reached around or slightly before the time pulsars were discovered. From a nuclear physicist's point of view, the year 1968 could not have been chosen by astronomers more appropriately! However, had pulsars been discovered 20 years ago

* A comprehensive review article on the equation of state for neutron stars has been recently prepared by one of the authors (V. Canuto) for the *Annual Review of Astronomy and Astrophysics*, vol. 12.

(and no technological reasons forbade that from occurring) the history of that chapter of astronomy dealing with the end point of stellar evolution would have been dramatically altered.

Two other chapters of physics apparently disconnected with neutron stars but that will play a great role in what follows, i.e., solid state and low temperature physics, also received a great impulse in the years before 1968. The phenomenon of superfluidity, discovered more than half a century ago, was not understood on a microscopic basis until 1956 when the BCS theory appeared. It was later realized that non-magic nuclei and a gas of neutrons under appropriate conditions of density can also exhibit a superfluid behaviour. We shall see that such conditions can indeed be found in the interior of most neutron stars and this probably constitutes one of the few possibilities of relating observational data to the internal structure, a task particularly difficult but exceedingly important. Here again, the peak interest in the superfluid behaviour of nuclei and nuclear matter was around 1960-1966 and the major theoretical problems were cleared up just before or around 1968.

Another great help in understanding the structure of neutron stars is being borrowed these days from that chapter of nuclear physics that studies strange particles, or hyperons, as they are usually called. Here the situation is more delicate. The host of elementary particles presently known has not yet received a fully systematic justification from a fundamental point of view. However, for our purposes it suffices to say that besides neutrons and protons there exist heavier fermions whose lifetimes are exceedingly short, $\leq 10^{-13}$ sec, and thus particularly difficult to study. Among them the lightest is the Λ hyperon (1115 MeV), followed by the triplet $\Sigma^{\pm}_{0}$ (1190 MeV), the Δ (1236 MeV), and so on. These particles are so heavy that their appearance requires an amount of energy of the order of 1 GeV. If the system is dense enough, i.e., if we squeeze the original neutrons in to smaller and smaller volumes, a point will be reached at which the neutrons will have an energy of confinement ε such that $mc^2 + \varepsilon$ will exceed the mass of the lightest hyperon. Early in 1959 (Cameron 1959) it was proposed that when the interior of a neutron star becomes dense enough for this to occur, the original picture of a gas of pure neutrons should be changed into a more realistic one of neutrons and hyperons (of all types) with different concentrations. At the time this idea was put forward, the knowledge about hyperonic forces was at the same level of sophistication as the nuclear forces were at the time of Oppenheimer's work.

It is therefore only natural that in the first detailed computation the nuclear forces acting among them were totally neglected (Ambarstumyan and Saakyan 1960). Since 1959 to the present date, the knowledge about hyperonic forces has not proceeded at the same pace as for the nucleon-nucleon case, and today we are still far from having anything more than a few empirical formulae to work with. It is also unfortunate that, so far, the low energy data ($\simeq$ 50 MeV) on scattering involving hyperons have only clarified the lowest angular momentum ($l = 0$) component of V(r); almost nothing is known about the P, D and higher partial waves, which, however, enter in a determining way at high density (Argonne Report, 1969). Hopefully, the need for better and more reliable data on hyperonic forces requested by astrophysicists will be a reason for a stimulating collaboration between two differently oriented types of physicists.

We have so far introduced the question of the composition of the several different layers of a neutron star in the way they are supposed to appear as one goes from the surface down towards the center of the star, i.e., from low to high density regimes. Evidently, the higher the density, the more uncertain the situation becomes until a density regime will be reached in which we have to admit that the present knowledge is still too primitive and the few computations done too fragmentary, to allow for any general conclusion to be drawn at present. This situation of total despair can be more or less arbitrarily drawn at around 10^{17} g.cm^{-3}.

Nonetheless, at the rate at which the research is being conducted in this area, it should not come as a big surprise if, in a few years time, the gap is definitely bridged.

It is a rather debated subject of this Conference whether a fluid of neutrons exhibits a phase transition to a solid structure, i.e., if a quantum crystal made of neutrons might exist at densities around 10 times the ordinary nuclear density. Here, too, we face an example of how the apparently unrelated subject of the properties of solid ^{3}He at zero temperature, came to help the understanding of the structure of a neutron star. Helium is known to be the only substance still liquid at T = 0 and to solidify only under a pressure of about 25 atm. The helium atoms repel each other rather strongly at small distances and attract each other at large distances. The analogy with a liquid of neutrons is highly appealing and one might hope that the neutron liquid will also solidify under appropriate pressure. The essence of the question is to ascertain if the solidification pressure turns out to be within the range of pressures found in neutron stars. The many-body formulation of the ^{3}He case was outlined in 1966, when it was realized that the experimental data could not be reproduced without properly accounting for the short range repulsion. The difficult task of including such a feature in a consistent formulation was successfully accomplished and one can safely say that the experimental energy per particle of the

^{3}He and ^{4}He solid substances are rather satisfactorily reproduced at present time (Guyer 1969). Even though the move toward the understanding of the neutron solidification problem has not been an easy one, it is certainly true that it would have been almost an impossible one without the previous experience acquired in the ^{3}He case. The crystaline structure, if it exists, does not change the mass and radius of a stable configuration in any substantial way with respect to the case of a liquid core (Canuto and Chitre 1973). Its importance is more directly involved in explaining dynamical effects, as we shall see when discussing the occurrence of speed-up in two pulsars. Here again, as in the case of the superfluid behaviour, we seem to have the unique possibility of testing the truthfulness of our assumptions about the behaviour of dense matter through astronomical data. The hope is not only to explain whatever data we could possibly have, a task extremely difficult and praiseworthy if successful, but perhaps to use the astronomical data to infer some constructive suggestion about the way elementary particles behave when their distances approach their Compton wave lengths, a hope commonly shared with the elementary particle physicists.

As we have seen during these cursory remarks, the understanding of the interior of a $\sim$ 10 km object, whose density varies by $\sim$ 8 orders of magnitude from the surface (10^{7-8} g.cm^{-3}) down to the interior, has called for a collaboration of nuclear, solid state, low temperature, and high energy physicists. Each has contributed the knowledge he has acquired in his own field during the last 20 years. This collaboration, originating around 1968, allows us today to draw a rather definite picture of a neutron star.

2. The surface

After having very briefly outlined the chapters of physics that will enter in the study of neutron stars, we shall now proceed to expose the results: first well established, then less well established and finally totally discrepant. We shall proceed from the surface down toward the interior, a path which, besides growing in density, also grows in difficulties of all sorts.

The formation of a neutron star from a supernova explosion is treated in the second part of this review, and here we shall treat it as a $T = 0$ object and try to determine its composition. The actual temperature of about 10^7 °K constitutes too small a correction to the Fermi energy of an average nucleon ($\sim$ 50 MeV) to be of any relevance. Needless to say, the nature of neutron star constituents is strictly related to their past history, i.e., their relation with the supernova explosion where they presumably originated. We shall, however, treat the problem in a rather academic way: we shall seek to determine the lowest energy state and the composition of matter at $T = 0$ in the density range 10^6 - 10^{15} g.cm^{-3}.

For densities lower than 10^6 g.cm^{-3}, the total energy is minimized by having ordinary nuclei as the only component, among them ^{56}Fe being the most abundant. This result is obtained through the use of a Thomas-Fermi theory and it has been obtained by more than one group (Baym, Pathick, and Sutherland 1971). By increasing the density, the atoms become more compressed and the first to suffer from the closer packing are the electrons around the nuclei, for they are only loosely bound by atomic forces. The work done by the compression can be used to perform a cold ionization, i.e., the work pdV is great enough to detach electrons from their nuclei, a process that the Thomas-Fermi model predicts to be almost totally completed at densities of the order of $\rho \sim 6AZ$, i.e., 10^4 g.cm^{-3} for ^{56}Fe. By increasing the density beyond 10^6 g.cm^{-3}, the electrons not only are totally free but they become also completely relativistic.

We are therefore facing a new structure: a system of nuclei imbedded in a gas of freely moving relativistic electrons. It is a simple matter to show that, in this case, the nuclei will most certainly arrange themselves in a crystal structure rather than in a gas or liquid form. This statement has never been rigorously proven but the arguments that led to it are sound enough to make the physical picture trustworthy (Salpeter 1960). The essential action of the electrons, when they were still attached to the nuclei as in ordinary matter, is primarily that of screening the strong repulsion among the Z charges of each nucleus, action that they cease to exercise once they have been squeezed out by compression. Nuclei see each other as bare charges and the only way to minimize the energy of a system of strongly repulsive particles is to arrange them in locked positions, i.e., by having a crystalline structure.

By further increasing the density, we will encounter a rather unique process, the so-called neutronization. We know that, in vacuum, the neutron is an unstable particle, since it decays into p, e^-, and, $\bar{\nu}$ via ordinary beta decay. The neutron mass is slightly higher than that of the proton and that energy is being shared by the three final particles. In our case, however, the gas of electrons can reach a point in density where the energy of each electron is high enough to overcome the negative amount $m_p - m_n$ and therefore make the inverse reaction,

$$e^- + p \rightarrow n + \nu \tag{1}$$

proceed, i.e., free electrons are being captured by the protons bound in the nuclei and transmute them into neutrons. Such a process is totally ineffective unless we take care not to let the newly formed neutron decay back via ordinary beta decay. One ensures that by relying on the Pauli principle, by filling up the space with enough electrons to make any cell in phase space unavailable for the would-be new electron, in such a way that reaction (1) cannot be inverted. Very few electrons are needed to accomplish that and a simple computation says that at $\rho \gtrsim 6.10^7$ g.cm^{-3}, the chemical potential μ_e of the electrons becomes greater than the Q of the reaction, 1.19 MeV. The simplest way to describe the energy of the nuclei is by adopting the nuclear mass formula. Up to $\rho \simeq 10^{11}$ g.cm^{-3}, its application, in principle limited to those nuclei for which N does not exceed Z by a large amount, is still valid as indicated by the identical results obtained using different techniques.

In fact, the ratio $Z/(N + Z)$, indicating the percentage of protons with respect to the total number of nucleons in the nucleus, decreases from 0.46 for the most abundant nucleus, ^{56}Fe, encountered at $\rho \simeq 8.10^6$ g.cm^{-3}, down to 0.305 at $\rho \simeq 4.10^{11}$ g.cm^{-3}, where the most abundant nucleus is now ^{118}Kr, a strange nucleus with $N = 82$ and $Z = 36$. The situation as described above is rather schematic but probably not far from the truth. By going from the surface, say at $\rho = 8.10^6$ g.cm^{-3}, down towards the center, we pass through regions whose most abundant constituents are successively: ^{56}Fe, ^{62}Ni, ^{82}Ge, ^{80}Zn, ^{124}Nb, etc. The last four nuclei are unstable in the laboratory but, as explained before, the presence of a surrounding gas of electrons blocks the phase space and the predominant component of neutrons cannot undergo ordinary beta decay.

We must stress that up to 4.3 10^{11} g.cm^{-3}, all the computations performed are in agreement, even with the calculations performed back in 1960, when many fine details of the mass formula were still not completely understood. As well known, the neutronization process (1) occurs only among isobars, i.e., nuclei with the same A. This means that the transition from ^{56}Fe to ^{118}Kr described before is not due to such a process. As a matter of fact there is no known process at zero temperature by which one could possibly think of transforming Fe into Kr, because that would require at least a few billion degrees. We could think of photodisintegrating some iron nuclei into α particles and then rebuild heavier nuclei by capturing α-particles by those iron nuclei that were not photodisintegrated. In order to follow the " actual " building up of these unstable nuclei, one has to construct a complicated kinetic network of all the possible reactions ocurring, and to estimate the reaction cross sections for all links in the network by averaging over the predicted level densities of the nuclei involved; this has not been done for this problem. What one does in the work described above is an equilibrium computation after a long enough time has elapsed for those reactions to take place.

The last number we have been quoting, $4.3\ 10^{11}$ g.cm^{-3}, is the result of one of the more accurate computations done in this region (BPS, 1971). This point is commonly known as the Neutron Drip Point, since it so happens that by building up nuclei with an increasing number of neutrons we render them less and less able to stay together as an entity, i.e., as a finite configuration in space with a sharply changing density at the edge. The great preponderance of neutrons, and the fact that on the average n-n forces are less attractive than n-p forces, to the extent that a stable di-neutron does not exist, renders a nucleus with $N >> Z$ unable to keep all the neutrons in a well defined region of space. It is computed that at $4.3\ 10^{11}$ g.cm^{-3}, the neutrons begin to drip out of the nuclei. Again, we must stress that the drip point differs very little from one computation to another.

A much more serious problem arises in the description of the next region, i.e., for that part of the star for which ρ is greater than 4.10^{11} g.cm^{-3}. The system is made of three components: electrons, nuclei, and free neutrons.

A first problem is presented by the neutron gas whose energy has to be described by using a reliable many-body theory. The Brueckner theory or its version following the lines of Bethe and his school, was originally devised to handle nuclear matter, i.e., an hypothetical system of $N = Z$ particles, for which the ground state energy vs. density curve is known to have a minimum at nuclear density ρ_0 with $E(\rho_0) \simeq -16$ MeV Great skill and care has been exercised to account for all the possible effects and a satisfactory answer has recently been obtained (Bethe 1971). On these solid grounds the many-body technique was extended to handle a system of pure neutrons by essentially shutting off all the $T = 0$ states, and reliable results were so obtained (Siemens 1970).

A second and more serious problem is related to the nuclei, so far successfully described by a mass formula. We shall first discuss what one can expect from physical arguments and then the technical way to describe the detailed behaviour of the nuclear components.

Due to the manifest inability of the nuclei to keep the neutrons together, it is plausible to expect that the process of losing neutrons will continue until a point will be reached at which the nuclei will disappear. More exactly, we should say that the clusters of neutrons and protons ($N >> Z$) that we have been calling nuclei, will, as the density increases, have a density distribution in space, $\rho(r)$ vs. r, less and less localized around their centers, up to the point where $\rho(r)$ for nucleus 1 will greatly overlap with $\rho(r)$ of nucleus 2, instead of going to zero at some point between 1 and 2. This smooth merging of the nuclei into the surrounding medium is predicted to occur at around 10^{14} g.cm^{-3}. A graphical representation of the merging of nuclei into the surrounding

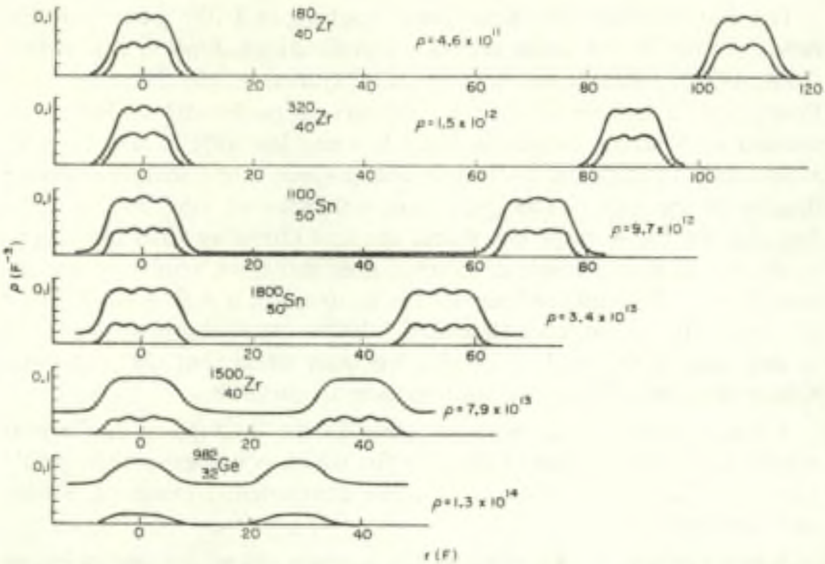

Fig. 1. Merging of the clusters of neutrons and protons ($N >> Z$) as the matter density increases .n_b is the baryonic density in particles per cm^3. (From Negele and Vautherin 1973).

medium, as the density increases, is given in Figure 1 (Negele and Vautherin 1973). How does one describe such a phenomenon?

As the density increases, the mass formula is bound to become almost totally unreliable if applied bodily to regions of the $N - Z$ plane too removed from the restricted strip $N \simeq Z$ originally used to fit its parameters. Many attempts have been made in the last ten years to obtain a version of the traditional mass formula that could be reliably extrapolated in the $N >> Z$ region, but the truth of the matter is that such efforts have not yet been very successful. Finer and finer adjustment of empirical mass formulae to experimental binding energies seem to indicate the existence of new terms which, being much smaller than the traditional volume, surface, asymmetry and Coulomb terms, are rather difficult to determine. Numerically speaking, they represent noise and very skillful methods have to be devised for their determination. Among the most debated ones we can quote the so-called surface asymmetry and curvature terms. The last one is still so uncertain that there seems to be no general agreement on whether it should increase or decrease the binding energy. We have brought up these questions because, historically speaking, the mass formula was the first tool used in the description of the system and, as we shall see, it gave rise to debatable results.

In a rather sophisticated computation that appeared in 1971 (Baym, Bethe, and Pethick 1971), the problem was handled by essentially employing a mass formula. One of the main results was that the curve $Z = Z(\rho)$ was an increasing function of ρ, with a value of $Z = 445$ at the point of disappearance of nuclei.

The mass formula used in this work contained volume, surface, asymmetry and Coulomb energies. One can easily show that, by minimizing the total energy of the system with respect to A and by making use of the mass formula with the terms just described, the value of Z that minimizes the energy is given as a function of A by the simple relation

$$Z = \sqrt{a_s/2a_c}\ \ A^{1/2} \tag{2}$$

i.e., it is an increasing function of A. Here a_s is the surface energy cœfficient and a_c is the Coulomb energy cœfficient. However, a second group of workers (Buchler and Barkat 1970; 1971; 1972) noticed that if one adds an extra term to the previous mass formula (the so-called surface-asymmetry term, with cœfficient a^*, needed in all the most recent best fits to the empirical binding energies) then (2) would change to

$$Z = \sqrt{a_s/2a_c}\ \ A^{1/2}\left\{1 - \frac{a^*}{a_s}(1 - 2\,Z/A)^2\right\}^{1/2}$$

i.e., the function $Z = Z(A)$ is now non-monotonic, has a maximum around $Z \simeq 40$, and then decreases again. On the basis of this criticism, the authors employed a formulation independent of the use of the mass formula, the Thomas-Fermi model previously developed by nuclear many-body theorists. The results were in agreement with the last relation for Z and there appeared to be a rather serious discrepancy between the two results. The solution to the problem came with the appearance of a full quantum mechanical computation, the results of which agreed with the ones obtained from the Thomas-Fermi model as shown in Figure 2 (Negele and Vautherin 1973). Another useful insight into the problem was offered in a paper where the authors still used a mass formula approach but performed a full quantum mechanical computation of one of the most troublesome terms, the surface term. The results again indicate that Z has a maximum around ≈ 40, and it does not overshoot to high values (Ravenhall, Bennet, and Pethick 1972).

We must, however, note that this discrepancy has little effect upon the $P = P(\rho)$ curves obtained by the several groups, since they came out to be rather similar. We can, therefore, assert that, this last point having been clarified, we do possess a reliable description of the composition of matter at $T = 0$ up to $\sim 10^{14}$ g.cm^{-3} and the corresponding equation of state.

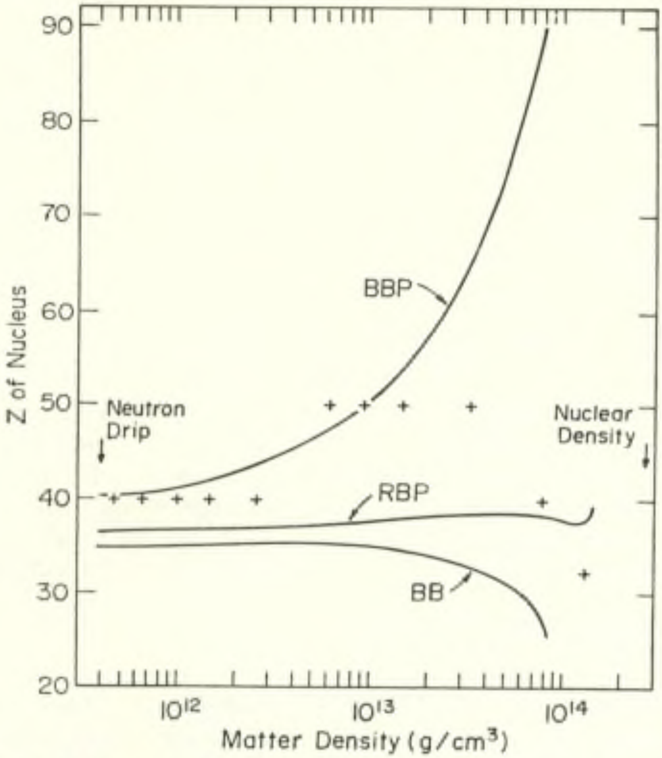

Fig. 2. The behaviour of the curve Z vs. ρ as a result of different computations. The crosses are the results of the work of Negele and Vautherin.

3. The central region

After the disappearance of the nuclei into the surrounding medium, we are left with a system of neutrons, protons and electrons of unknown concentration. Two complications arise in this region. First of all, the neutrons, being always the most conspicuous among the other species, can be in a superfluid state. We know from the application of the BCS theory of superconductors to ordinary nuclei (Bohr, Mottelson and Pines, 1958; Belyaev 1959) that the largest matrix element of that part of the two-body interaction that is left after the common well has been built up, is that corresponding to coupling the particles with the same j but with opposite projection of j_z. This is perfectly analogous to the BCS theory where the largest matrix elements are the ones corresponding to particles having opposite momentum and spin. Finite nuclei parameters like momenta of inertia, pairing energies, etc., were found to be rather well explained on the basis of this model. Furthermore, it was also found that not only a finite nucleus but also a gas of neutrons could exhibit a superfluid behaviour represented by the alteration of the single particle energy spectrum by an amount Δ, the so-called energy gap. The physical significance of the gap is that $2\,\Delta$ is the minimum energy required to break a pair of particles coupled in the way explained before. If one knows the nucleon-nucleon potential, the curve Δ vs. ρ can be computed (Kennedy, Wilets and Henley 1964). The best results seem to agree about a value of 2 MeV at around $\rho \simeq 2.9$ 10^{14} g.cm^{-3}. Recently, however, it has been pointed out that a new type of superfluidity can set in when the $^1S_0$ contributions vanish. This is due to the $^3P_2$ wave which exhibits a strong attraction, essentially due to tensor forces (Hoffberg *et. al.* 1970). The curve Δ vs. ρ containing the two contributions is represented in Figure 3, where one sees that for a quite relevant density interval just after the nuclei dissolve, the neutron component can be in a state of superfluidity. We must again be reminded that such an interesting possibility of having a layer of the star in a superfluid phase can have many interesting observational consequences, but very little effect on the equation of state. In fact, the superfluid state hardly affects the bulk of matter but only those few neutrons not completely surrounded by other particles (as it happens inside the Fermi sea). They, in fact, can arrange themselves in such a manner as to give rise to zero total momentum state as required by the superfluidity. This is especially true in finite nuclei where the pairing energy corrections to the total energy are minute. The existence of a gap in the $^3P_2$ wave has been only a theoretical exercise until now.

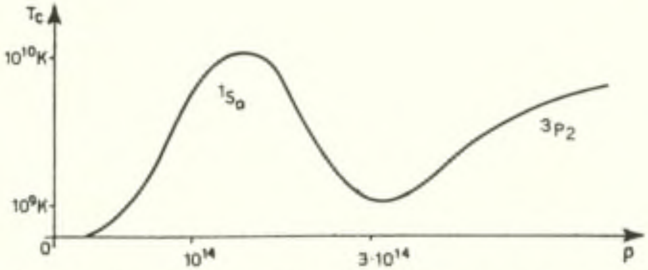

Fig. 3. Behaviour of the energy gap ($\Delta \simeq kT_c$) vs. density in g. cm^{-3} showing the contributions of two partial waves $^1S_0$ and $^3P_2$ (From Dyson 1971).

Quite unexpectedly a possible experimental proof for its existence seems to come from the behaviour of liquid ^{3}He at temperatures around a few millidegrees. The experimental data are accumulating (Osheroff, Gully, Richardson, and Lee 1972) and the predominant opinion is that a new phase has been discovered that would correspond to an anisotropic gap of the type generated by the interaction of two fermions in a P-state. This interesting possibility is now being studied intensively by low temperature physicists and new developments have to be expected in the near future.

The electrons are very unlikely to be superconducting, since the effect of the Coulomb interaction, compared to the kinetic energy, is of the order of the fine structure constant.

One might expect that if the protons are superconducting, the magnetic field would be expelled by the Meissner effect. However, this is not so, since, due to the enormous electrical conductivity, the time required for the expulsion is comparable with the age of the universe.

One knows from the theory of superconductivity that if the ratio of the proton coherence length to the penetration depth is greater than the $\sqrt{2}$ (Type I superconductors), the magnetic flux can penetrate the superconductor in such a way that there are alternative regions of normal material containing flux and superconducting material exhibiting the Meissner effect. If, on the other hand, the ratio is less than the $\sqrt{2}$ (Type II superconductors), there is a periodic array of vortices of supercurrent corotating with the protons. It turns out that for the conditions prevailing in a neutron star, protons should be Type II superconductors. (Bayn, Pethick, and Pines, 1969).

Since the flux in the superconducting regions is frozen, superconductivity has far less effect on magnetic properties than previously believed. Independently of whether protons are superconducting or not, they must corotate with the electrons, since otherwise an appreciable differential rotation would produce very high magnetic fields.

A second complication that can arise at densities greater than nuclear is the possible appearance of strange particles like hyperons.

We have already dealt with the subject of how and why hyperons are likely to appear at densities slightly above nuclear. The very first computation back in 1960 indicated that the Σ^- would be the first hyperon to appear at $\rho = 10^{15}$ g.cm^{-3}, Λ would then follow at 2.10^{15} g.cm^{-3} and then Σ^o, Σ^+, and Δ. More detailed computations have recently been performed, but they lack in two respects the degree of credibility that characterizes the nuclear matter studies. First of all, no published version has yet appeared that treats the problem of a gas of hyperons with a many-body technique of a comparable degree of sophistication to the one presented before. In particular, one should perform a many-body multi-component computation, an exercise of extreme complexity that has been defeating the ingenuity of most people in the field. Simplified versions of such an approach have appeared (Pandharipande and Garde 1972) and the presently most reliable way of approximating the scheme just described is the one employed by Moszkowski (1974), where several approximations are employed to test the sensitivity of the problem to the density dependence of each species.

A second serious trouble affecting all the published computations is the lack of a reliable hyperonic potential. We have already discussed this point at the beginning and again we feel that those computations in which the potential was tailored from the known nucleon-nucleon potential by just cancelling the one pion exchange contribution (for ΛN scattering) are highly suspicious and the results cannot be taken but with reservation (Pandharipande 1971). A new way of handling the nuclear force problem has been devised by Moszkowski, who has changed the Reid nucleon-nucleon potential by looking at the nature of the exchanged particles. We feel that this is a much better way of handling the problem. His results indicate that Σ^- and Δ^- (together) appear as early as 3.8 10^{14} g.cm^{-3}, a rather low density compared to the original value for the free particle case.

A more thorough approach to the hyperonic liquid has recently been worked out by Canuto and Datta (1974) who have, first of all, derived from first principles the hyperonic potential by exchanging heavy mesons and by determining the unknown coupling constants using the prescription of SU_3 and the experimental data on ΛN scattering cross section. The results indicate a potential rather different from the Reid nucleon-nucleon potential but not very different from the simplified form employed by Moszkowski. As a second improvement, a multicomponent many-body theory was set up and the previously determined potential used therein. Preliminary results indicate that the Σ^- meson is the first hyperon to appear at $\rho = 6.68\ 10^{14}$ g.cm^{-3}, whereas Λ makes a first appearance at 1.42 10^{15} g.cm^{-3} followed by Σ^0, Σ^+, and Δ. This way of handling the hyperonic problem is the best one can hope to do today. Much work remains to be done but we feel that the framework in which this last work has been performed is the only one that will allow us to incorporate the improvement that will derive from the future work on hypernuclear forces. Once more we have to stress the important fact that all the computations but the last (where such a problem has not yet been touched upon) indicate that the pressure vs. density relation is not sensibly altered with respect to the case of pure neutrons.

4. The solid core

The region just described has provided us with a good perspective of the difficulties one can expect when trying to describe, in a rigorous way, regions of densities and pressures far beyond the domain attained in terrestrial laboratories. The closer one gets to the center of a neutron star, the more intricate the situation becomes and less reliable are the theoretical approaches presently available.

We have already described why the treatments of the hyperonic matter presented before, though praiseworthy for the light they shed on the problem, are still highly unsatisfactory, because the way they deal with the problem is far from complete. What is at stake here is not a question of principle, since one can hardly foresee an overall change of the baryonic composition with respect to the one described before when the problems not yet solved today will be overcome. The physical principles, i.e., the appearance of hyperons, are well founded and universally accepted.

The same cannot be said about the region we are about the describe. Here the problem is still of a primary physical nature: does a system of strongly interacting baryons solidify at pressures and densities of the magnitude encountered in the interior of most neutron stars? Needless to say, the answer depends almost entirely upon the behaviour of the n-n potential near the origin and, since we lack an exact knowledge of such a region, we can only speculate and finally pursue the computations by adopting what we presently believe is the correct potential.

The problem of the possible solidification of neutron matter has been vigorously pursued in the last three years and all the computations, except one, so far presented provide a positive answer.

The solidification densities vary from 4.2 10^{14} g.cm^{-3} to (2.9 $\pm$ 0.5) 10^{15} g.cm^{-3} and we see there is still room for discrepancy. The earlier computations rely on the possible analogy between a neutron liquid and ^{3}He liquid, an analogy that a closer inspection reveals to be not completely trustworthy in that the stiffness of the Lennard-Jones potential, r^{-12}, has no analogue in the presently known nucleonic potentials, whose forms tend to be rather soft compared to many years ago when it was believed that a hard-core existed between two nucleons at distances shorter than say $\sim$ 0.5 fm. One can therefore say that the analogy with ^{3}He can be instructive but the solidification density obtained by applying the so-called law of corresponding states, i.e., $\rho_s = 5.10^{14}$ g.cm^{-3} cannot be taken too literally (Anderson and Palmer 1971). The main reason for this is that the Lennard-Jones potential is a spherically symmetric potential, whereas we know that the n-n potential has many important components that are due to spin and angular momentum, dependences that cannot be brought into a simple analytic form resembling the Lennard-Jones function. The positive result obtained by such a simple approach and the puzzling properties of the nuclear force structure prompted other computations to be done. An attempt using the Harthree-Fock method to handle the many-body system was tried by Coldwell (1972), who obtained a solidification density around 7.10^{14} g.cm^{-3}. Variational methods corroborated by Monte Carlo techniques to handle the multi-configuration integrals were also proposed with the idea that the problem of higher order diagrams, always present in any many-body theory, would in this way be properly handled. This is surely a most welcome feature but it forces the nuclear physics part (i.e., the angular momentum and spin dependence of nuclear forces) to be treated only approximately. The results obtained in this way correspond to a liquid-solid transition around 4.4 10^{14} g.cm^{-3}, just after the superfluid region (Nosanow and Parish 1973). Due to the good features of this approach, one should try to understand how to include the state dependence of the nuclear forces.

The same variational technique was employed by Pandharipande (1973) who, however, resorted to the Bethe-Goldstone (BG) equation to determine the two body wave function Ψ.

The B-G equation was treated in such a way as to erase two peculiar features that characterize it when applied to a crystal. In the form usually employed in the study of quantum crystals, the hamiltonian H describing the interaction among two nucleons is not invariant under the exchange of particle 1 with particle 2, i.e., H has no symmetry with respect to $\underset{\sim}{r} \rightarrow - \underset{\sim}{r}$. This implies that the wave-function itself does not possess any symmetry and this in turn implies that all the angular momentum components of Ψ are linked together. This makes the problem very hard to solve mainly because at high densities one has to include interactions up to $l \simeq 5, 6$; this in turn gives rise to ~ 20 to 24 coupled differential equations. The coupling term is of a rather sophisticated nature and the decoupling of all the waves performed by Pandharipande erases many features. In particular, such a simplified treatment of the B-G equation yielded correlation functions that behave rather differently near the origin. The N-N potential was consequently modified to account for such an effect. A close scrutiny of the problem indicates that one can arrive at such a conclusion only when all the features of the B-G equation are fully accounted for.

Such features are extremely difficult to embody in a fully satisfactory way and even the most complete treatment of them (Canuto and Chitre 1973) has not yet reached the degree of perfection necessary to allow one to think of the NN force as the only one responsible for any misbehaving results one could possibly derive. The result of this computation is that a system of neutrons would not solidify before $\sim 8.10^{15}$ g.cm^{-3}, a region where the presently available many-body theories are surely totally inadequate to represent the situation.

The most extensive way of handling the general features of the BG equation, the coupling of all waves and the most general

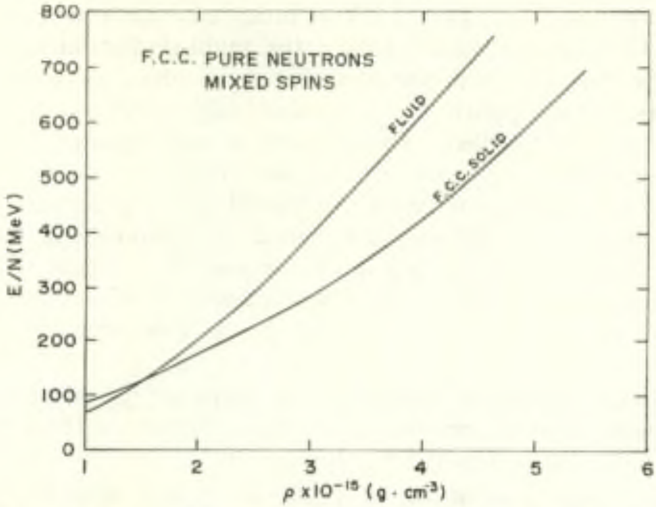

Fig. 4. The energy per particle vs. matter density showing the onset of a liquid-solid transition at $\rho \simeq 1.6\,10^{15}$ g. cm^{-3} (From Canuto and Chitre 1973).

form of the nucleon-nucleon problem has been worked out by Canuto and Chitre (1972; 1973). Once the BG equation is solved, the wave functions can be used in either the t-matrix or variational approach. By employing a simplified potential like the central part of $^1S_0$ suggested by Bethe, these authors also found that the two alternative many-body techniques yielded almost the same results. This was rather different from the results obtained by Pandharipande who employed a variational method. This example was important in proving that the several approximations performed on the BG equation in the way described before were the origin of the difficulties. The results obtained by employing the t-matrix approach substantiated with the BG equation in its integrity, gave a solidification density of 1.6 10^{15} g.cm^{-3} (Figure 4). It is important to notice at this stage that the problem still calls for further improvements even though the source of error that led to the negative result explained before seem to have been localized. Finally, a recent computation by Schiff (1973), using a previously tested many-body technique to handle a quantum solid on a variational basis, gave a solidification density of $(2.9 \pm .5)\ 10^{15}$ g.cm^{-3}. The present situation on the solification of a neutron liquid is graphically presented in Figure 5.

In conclusion, we may notice that more work has to be done to improve the treatment of the BG equation by trying to render the hamiltonian invariant under the $\underset{\sim}{r} \rightarrow - \underset{\sim}{r}$ transformation, in such a way that only waves with the appropriate parity will enter into the problem as they should. This is by no means an easy task. Here again, a rather different problem, the fission of nuclei, where similar problems arise, can be of some assistance in sorting out the right way of handling things. We may also notice that this method would also be highly beneficial to the study of solid He^3, for which the theoretical ground state energies are not yet given to within a better margin than $\sim 1\,^{\circ}K$ (Canuto, Chitre, and Lodenquai 1973).

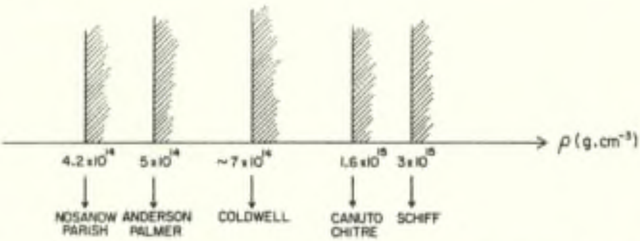

Fig. 5. The solidification pressures obtained by the various authors as discussed in the text.

5. The ultra-high density regime

If a neutron star has a central density higher than $\sim 10^{15}$ g.cm^{-3}, the description of its physical content in that region can become very debatable. We have already seen how difficult it is to have a universally accepted version of the composition of matter at lower densities and the main reasons why this is so. At densities higher than 10^{16} g.cm^{-3}, it is only fair to say that we do not know of any method of handling the problem that could not be strongly criticized on many grounds. First of all, no experimental data have yet been accumulated to give us a unique description of the behaviour of V(r) for $r < .3$ fm. Various forms have been employed during the last twenty years: from the original proposal that the potential is infinitely strong (hard core) (Jastrow 1950), to the suggestion in the middle sixties that a soft core (but still infinite at $r = 0$) potential is actually more suitable to explain experimental data and more in line with the theoretically derived form of V(r) (Reid 1968), to the Japanese school proposal of a gaussian shaped potential finite at $r = 0$ with a value $V(0) \simeq 2$ GeV. No simple theoretical model can be attached to this last potential, but it is surely a fact that on the basis of the phase-shift fitting criterion it can be made probably as good as the soft-core potential (Otsuki, Tamagaki, and Wada 1964). More recently it has even been proposed that the n-n potential near the origin is neither infinitely repulsive nor softly-repulsive but on the contrary, very close to $r = 0$ it turns down and becomes negative (Neeman 1968; 1972; Bodmer 1971). The reasoning behind that proposal is as follows: the best theoretically understood model for nuclear forces calls for the exchange of mesons of different nature between two nucleons, Many models have been constructed based on the exchange of π, σ, ω, and ρ, where we have written the particles in order of increasing mass or decreasing range $\lambda = \hbar/mc$ of action. The heavier bosons exchanged are the vectors ω and ρ, that are known to produce a repulsive central force. However, if instead of stopping at the vector mesons, one exchanges the next heavy meson, the spin-2 f^0-meson, one finds an attractive contribution that could overweight the repulsion at shorter distances since $m_f \cong 2$ GeV. No detailed model has yet been proposed, either on the effect of the f^0-meson on the analysis of phase-shifts, or much less on the astrophysical implications. It is, however, an important idea that has to be pursued with all its consequences. A sketch of the possible behaviour of V(r) near r = 0 is given in Figure 6.

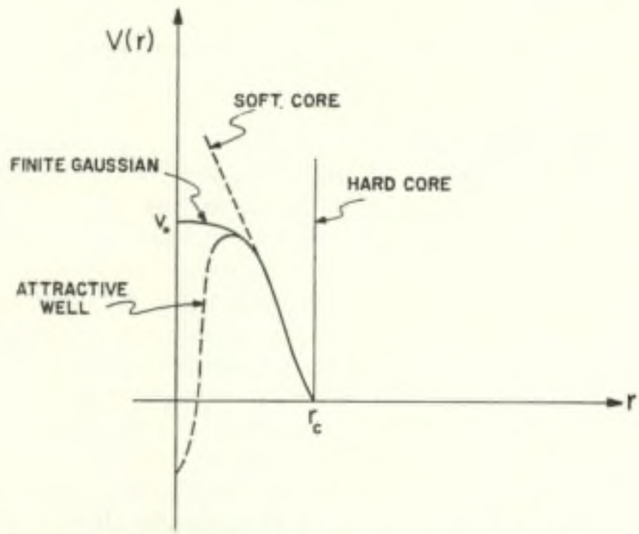

Fig. 6. Several possible forms for the NN potential near the origin.

An equally important comment has to be made regarding the many-body techniques. Even if we had a reliable V(r) all the way down to r = 0, we hardly would know what to do with it! By that we simply mean that a relativistic many-body theory has not yet been proposed to handle such high density regions. Both variational and t-matrix approaches are low density, non-relativistic treatments and there is no point in thinking that in the present form they could be of any use. For that reason, several other paths have been followed and at the present time it is difficult to see which of them, if any, is the more appropriate. In one model, the relativistic nucleons are treated as being properly described by Dyson's equation for electrodynamics (Bowers, Campbell, and Zimmermann 1973). One has a set of coupled matrix equations for the nucleon Green functions and the meson Green functions that satisfy Dirac and d'Alambert type of equations with a mass operator and a polarization operator in them. The mass operator is computed to the lowest order in the coupling constant, i.e., up to g^2, and in so-doing one obtains a definite result for the Green functions that in turn give almost immediately the desired pressure vs. ρ relation. First of all, it is not at all clear why a strongly interacting system of nucleons treated essentially in a perturbative manner should be trustworthy, once we recognize that for instance πN scattering would be off by orders of magnitude, if treated in the same approximation. Secondly, the only example worked out at the present by the authors is the one with a pion as the exchanged meson and this is surely not the dominant particle at high densities. Even taking the computation at face value, one cannot hope to continue any previous equation of state in this manner due to the unrealistic type of meson being used. The main problem remains as to whether one can in any way justify the g^2 approximation. If so, this method sounds very promising since temperature effects can be easily embodied, which would make the results interesting not only for cold neutron stars but also for the hot-big-bang theory where such an equation is needed.

Another method consists of trying to relate the equation of state $P = P(\rho)$ to some known experimental function measurable at high energy. One sure candidate is the two-body cross section for which we possess an ever increasing number of data and good analytic fits. One of the first of such attempts gave rise to a rather strange $P = P(\rho)$ relation in that there was a region between $1.7\ 10^{15} \leq \rho \leq 2.10^{16}$ g.cm^{-3}, where the pressure became negative (Libby and Thomas 1969). Many reasons can probably be found for that, but it is difficult to ascertain any one in particular, since this approach had no follow-up by other researchers and the fundamental hypotheses upon which the paper rests have never been fully tested. A superficial impression one immediately gets is that the many-body aspect of the problem is almost completely overlooked since only two-body free scattering data are used. It is hardly conceivable that two nucleons in a medium should have the same scattering matrix elements as when they are free without any influence of the dense surrounding medium. No other improvement on this work is known to the authors and the work can only be catalogued as an interesting idea that needs further checking and criticism. Finally, a third approach considers that at super-high densities one can hopefully describe the medium as being composed of all the elementary baryons presently known taken as being free. Leung and Wang (1973) set up a simple statistical mechanics set of equations for the thermodynamic quantities of interest in which the only parameter needed is the mass spectrum, N(m) dm, the number of particles with mass between m and m + dm. This is easily constructed by plotting the known number of baryons against mass and then parameterizing it with a polynomial equation. The simple idea, attractive as it might be, cannot be taken completely seriously unless one is able to include some kind of repulsion among the baryons. The argument that runs behind the taking a free particle model is that much of the interaction has already been used precisely to create those baryons that one is including in the problem. This might well be so, but with this heuristic argument one can only justify the neglecting of the attractive part of V(r), but surely not all of the repulsion.

We may conclude here by pointing out that these last three approaches, totally different in their fundamental line of attack, are still plagued with uncertainties and represent the best example of the state of affairs, nowadays, on this subject matter. The answer will come, and probably very soon, due to the extended interest among physicists and theoretical astrophysicists on the subject.

6. Equation of state and neutron star models.

By joining the equations of state obtained in each separate regions of density, one can obtain a reliable function $P = P(\rho)$ from 10^6 up 10^{15} $g.cm^{-3}$. The four best equations of state have been tabulated by Canuto (1973) and the interested reader is referred to that publication for specific details. The neutron drip point and the nuclei dissolution point differ slightly from one author to another but no appreciable effect can be traced in the resulting mass and radius. These four equations of state are good up to, say, 5.10^{14} $g.cm^{-3}$, where hyperons might be expected to change the situation. If one, however, continues the $P = P(\rho)$ as if the pressure was contributed almost entirely by the neutron gas, then the function $P = P(\rho)$ is good up to $\sim 10^{15}$ $g.cm^{-3}$. At that point, the possible existence of a solid core has some effect. In Figure 7 we present the results of the integration of the general relativistic equation of hydrostatic equilibrium using two different $P = P(\rho)$ functions from 10^{15} $g.cm^{-3}$ onward. The possible existence of a solid core decreases the maximum allowed mass from 1.66 $M_\odot$ to 1.5 $M_\odot$ and slightly shifts the position of the maximum. As we know, only the rising portions of the curve correspond to stable configurations, i.e., those for which $\Gamma > 4/3$. The first peak around 10^8 $g.cm^{-3}$ corresponds to the region of white dwarfs, whose maximum allowable mass is seen to be 1 $M_\odot$. In the original Oppenheimer and Volkoff paper, the maximum mass for a neutron star was only .7 $M_\odot$, a rather small value. However, most modern treatments seem to converge around 1.5 $M_\odot$, which is the value we shall assume.

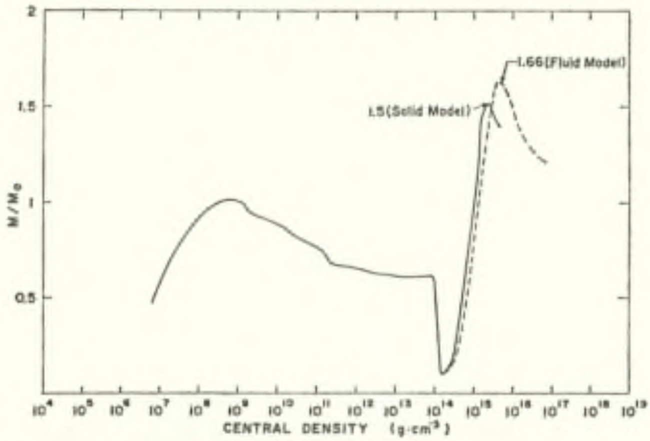

Fig. 7. Mass of the hydrostatically stable configurations of cold matter vs. central density, in the range $10^6 - 10^{16}$ g. cm^{-3}. The two maxima correspond to white dwarfs and neutron stars respectively. For neutron stars, the solid core model reduces slightly the maximum value of M.

Another interesting quantity that one can compute once the equation of state is known is the density profile, i.e., $\rho(r)$ vs. radius for several values of the mass $M/M_\odot$. The results are shown in Figure 8.

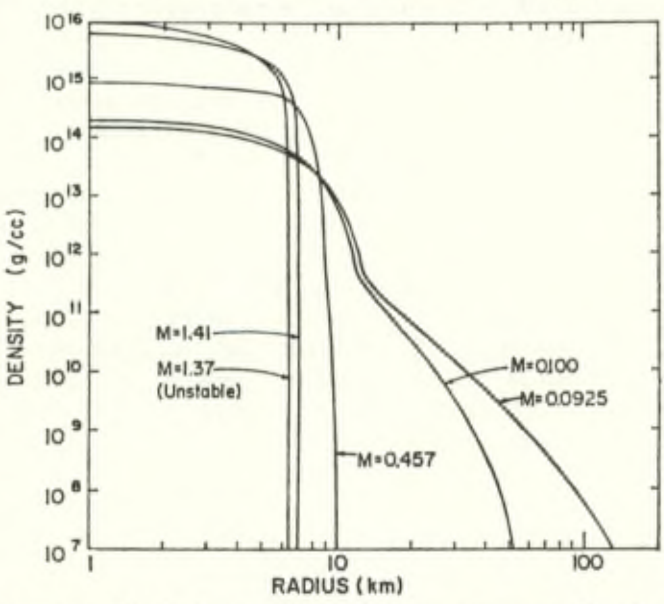

Fig. 8. Density profiles for five neutron stars. Masses are measured in solar masses. The star with M = .0925 is the lightest stable neutron star. That, with M = 1.41, is the heaviest stable star found by using the equation of state without a solid core. The two lightest stars are entirely solid. (From BPS 1971).

7. Magnetic Structure

We have so far described the structure of a neutron star without considering the very important fact that, most likely, within such a star is imbedded a magnetic field of the order of 10^{12} Gauss. We do not have a direct observational proof for such a number, but a series of sound arguments all lead us to believe that such a field is indeed present. If that is so, one easily suspects that the behaviour of matter will be substantially altered with presumably important observational consequences. We shall, therefore, discuss the changes brought in when such fields are present, leaving for the second half of this review the astronomical implications.

The presence of magnetic fields in nature is a rather common phenomenon. Our earth for instance possesses an approximately dipole field with a strength of $\sim .5$ Gauss, and according to fossil evidence the field has an ancient history. The sun, an average main sequence star, possesses a magnetic field of rather complicated structure and configuration. On the surface, the average value is ~ 1 Gauss, but such a value is derived from a very heterogeneous distribution of fields ranging from zero to several thousands of Gauss in the sunspots. Many stars are known to possess fields of the order of ~ 500 Gauss. Our galaxy also possesses a magnetic field of a few times 10^{-6} Gauss. Recently, white dwarfs were found to possess magnetic fields as high as 10^6 Gauss (Angel and Landstreet 1971). This last finding has lent support to the idea that a neutron star could probably possess as high a field as 10^{12} Gauss. In fact, if the flux conservation law holds true (i.e., if the quantity HR^2 is a constant during the evolution of a star) then by taking $H_1 = 10^6$ Gauss, $R_1 \simeq 10^{-2} R_\odot$ and $R_2 = 10^6$ cm, it turns out $H_2 \cong 10^{12}$ Gauss. The only assumption underlying the application of the flux conservation law is the existence of an infinitely great electrical conductivity σ, an hypothesis that has been lately verified, since computations show that σ is at least of the order of 10^6 times higher than the value for the best conductor in the earth (Canuto 1970). It, therefore, seems plausible that a huge field is indeed present. The behaviour of matter in such a huge field may be very different from what we can suspect on the basis of our experience in the laboratory where we can achieve only fields of the order of $\sim 10^6$ Gauss for very short times. The flux conservation law should not be used as a justification of the true mechanism generating such field. This is an entirely different problem that has not yet been fully solved. Several hypotheses exist and we shall review them once we have investigated the behaviour of matter in superstrong fields. We can distinguish two cases: 1) behaviour of a free gas of electrons imbedded in such a field, and 2) behaviour of atomic structure as the ones encountered at the surface of a neutron star.

As we have described before, the crust contains both an electron gas and atoms and, therefore, we shall treat them separately. Before going into that, it is important to sketch the results in their more fundamental aspects. The physical properties of an electron gas can be studied rather accurately since all the necessary ingredients, wavefunctions and eigenvalues, are known for a charged particle moving in a magnetic field of essentially arbitrary strength. Since the gas is supposed to be dense enough to render the Coulomb interaction relatively unimportant, the many-body problem reduces drastically and one can essentially claim to have solved the problem exactly. In this framework an exhaustive list of physical processes was investigated (Canuto and Chiu 1971) and a summary of the most important effects can, therefore, be made with a certain degree of confidence that no fundamental aspect of the problem has been overlooked.

Unfortunately, not quite the same conclusion can be reached when one studies the behaviour of atoms imbedded in a strong field. Perturbative methods are out of the question, both with respect to the magnetic field (too strong), and the Coulomb potential (no bound state). Even for the simple hydrogen atom, no exact solution can be obtained, in that the two fields have different symmetry (spherical and cylindrical), and no system of coordinates is known that allows a separation of variables. Worse than that, the systems of actual interest made of complicated atoms with many electrons are exceedingly difficult to handle. Much effort has gone into these problems and despite the difficulties, a qualitative picture can probably be put forward.

We shall review first the behaviour of an electron gas. If the magnetic field is considered on the atomic scale, its geometrically complicated structure can be simplified to that of a uniform field, constant in time, along the z-axis. By solving the Dirac equation with such a configuration, one finds that the motion parallel to the field is unaltered, whereas the perpendicular energy, $P_{\perp}^2/2\,m$, is changed into a discrete variable $2nH/H_q$, where $H_q = 4.10^{13}$ Gauss $= m^2c^3/e\hbar$. The quantum number n takes only discrete values and, in any given problem, the maximum value N^* of it is given by the following relation: $N^* = (E/mc^2)/(H/H_q)$. For a cosmic ray in the intergalactic medium, this number can easily be of the order of 10^{15}, indicating the totally classical nature of the problem. For a degenerate system, however, $E/mc^2 \rightarrow \rho_6{}^{2/3}$ and we see that quantum effects, $N \underset{\sim}{*} 0{,}1$, enter only at low densities and high fields. One can expect that at densities of the order of 10^{14} g.cm^{-3} inside neutron stars, the magnetic influence upon the existing electrons will be rather small in view of the overwhelmingly more important density effects. At the surface of the star, however, the conditions seem to be appropriate for the magnetic effects to be rather conspicuous. A general review of the major processes and their possible astrophysical implications has already been published (Canuto and Chiu 1971) and we shall only review the major features.

A) *Equation of state.* An exact relativistic equation of state for an electron gas under a magnetic field of any strength can be derived and the major feature is the anisotropic character of the pressure. In fact, $P_{zz} > P_{xx}$ up to 10^8 g.cm^{-3}, where the degree of anisotropy tends to disappear due to the high density effects (Canuto and Chiu 1968).

B) *Neutrino Processes.* The whole series of neutrino processes that are known to be important in the cooling of neutron stars have been recomputed in the presence of a magnetic field. The corrections do not seem to be overwhelmingly important compared to a totally new process that can exist solely due to the presence of an external field. That is the neutrino synchrotron process, $e^- \rightarrow e^- + \nu + \nu$. It is found that it dominates over the remaining processes (plasma neutrinos, photoneutrinos and pair neutrinos) at low temperatures $T \lesssim 10^8$ °K and high densities $\rho \gtrsim 10^6$ g.cm^{-3}. This new channel has to be considered an important sink of energy entirely caused by the magnetic field (Canuto, Chiu, Chou, and Fassio-Canuto 1970; Landstreet 1967). A general survey of neutrino processes in strong magnetic fields has been recently prepared (Canuto, Chiuderi, Chou, and Fassio-Canuto 1973).

C) *Neutron Beta Decay.* Due to the different and more abundant phase space available to an electron in a magnetic field, it is found that the neutron lifetime can decrease by as much as 10-20 per cent at magnetic field strength of the order of 10^{13} Gauss. Since the production of helium in the early history of the universe, in the frame work of the Big-Bang theory, depends sensibly upon the beta decay rate, it was found that the production of primeval helium is increased (Fassio-Canuto 1969; Greenstein 1969).

D) *Transport properties.* The decay time of a magnetic field due to ohmic dissipation follows an exponential law with a decay time given by $4\pi\sigma L^2/c^2$, where σ is the electrical conductivity and L a typical dimension characterizing the problem. The transport properties in a magnetic field are far from being a well understood subject and much work is necessary to clarify the methods to be applied. Supposing the validity of Boltzmann transport equation even at such high fields and in the presence of quantum effects, a computation of σ was carried out with the result $\sigma_{||} > \sigma_0 > \sigma_{\perp}$, where the subindex zero stands for zero magnetic field. The degree of anisotropy $\sigma_{|}/\sigma_{\perp}$, can be as high as $\sim 10^2$ at $\rho \simeq 5.10^6$ g.cm^{-3} and then it levels off rapidly due to density effects. The units of σ are 10^{21} sec^{-1}, to be compared to $\sim 10^{16}$ sec^{-1} for copper. Greater values, about $\sim 10^{26}$ sec^{-1}, can be achieved if the scatterers become degenerate at higher densities. With an anisotropic electrical conductivity and, consequently, an anisotropic thermal conductivity it is to be expected that the heat transfer process will also be affected giving a non-uniform temperature distribution on the polar caps and at the equator. We still do not know of any quantitative evaluation of these effects on the overall cooling mechanism of neutron stars (Canuto and Chiu 1969; Canuto and Chuideri 1970).

E) *Compton Scattering*. Among the many processes that can significantly alter the transfer of heat from the center to the surface, the scattering of radiation from electrons is one of the dominant ones. By analyzing the process imbedded in a magnetic field, it was found that radiation travelling along the field lines has a cross section σ diminished in the ratio of $(\omega/\omega_H)^2$ with respect to the $\sigma\,(H = 0)$ case. This is because the action of the electric field of the wave upon the electrons is negligibly small due to the high degree of localization of the perpendicular motion of the electrons in the field. In other words, the strong quantization effects make them move in tiny circular orbits around H and any action of an external field of frequency ω_H is decreased in the ratio $(\omega/\omega_H)^2 << 1$. It follows that the mean free path is larger and consequently the cooling is faster. In the perpendicular direction, only one type of polarization can profit from the quantization condition ($k \perp H$, $E \perp H$), whereas for the other polarization, the magnetic field has no effect at all (Canuto 1970a; Canuto, Lodenquai, and Ruderman 1971).

We can conclude, therefore, that the overall effects of the magnetic field is to accelerate the cooling of neutrons stars as explicit computations of the thermal history confirm (Tsuruta, Canuto, Lodenquai, and Ruderman 1972).

The next problem to deal with concerns the behaviour of atomic structures in magnetic fields. In a magnetic field, electrons move in circular, quantized orbits perpendicular to the field but are free to move parallel to the field. In the lowest energy state, there is an infinite number of degenerate orbits, but the radius of a given orbit decreases inversely as the square root of the magnetic field strength. Thus, in a uniform magnetic field, electrons in their ground states move on the surfaces of coaxial shells with axes along the field lines. For $B > 10^9$ Gauss, the radius of the smallest orbit is less than the Bohr radius of a normal hydrogen atom. The result is that magnetic confinement perpendicular to the field dominates the Coulombic binding attraction of an electron to a proton. For $B \gtrsim 10^{12}$ Gauss, typical of a neutron star surface, the electrons are tied to the field lines such that their response to the Coulomb attraction of a nucleus at the center of their orbits is essentially restricted to a onedimensional motion parallel to the field. An immediate effect is a great increase in binding energies of atoms since an electron is much more likely to be found near its binding nucleus. The usual shell structure of heavier atoms disappears. The minimum ionization energy of a neutral atom in a superstrong ($B >> 10^{12}$ Gauss) magnetic field does not fluctuate with atomic number Z but increases slowly and monotonically. For example, in a field $B \sim 2 \times 10^{12}$ Gauss, the ionization energy of a hydrogen atom is ~ 180 eV, more than 13 times its normal value. The atom is cylindrical in shape with a height to width ratio ~ 12. For multi-electron atoms, the binding energy increases in magnitude as $\ln Z$ when exchange effects are taken into account.

In a neutron star surface, where matter is abundant, these " cylindrical " atoms will coalesce to form more tightly bound solids for two reasons. They have large quadrupole moments, with symmetry axes parallel to the magnetic field; the quadrupolar interactions give a bound body-centered orthorhombic lattice in which nearest neighbours attract, next nearest repel. More important, however, a quantum mechanical binding which causes nuclei to arrange themselves in a chain parallel to the magnetic field surrounded by a cylindrical electron sheath. These chains will be packed closely together to form a solid with a density $\gtrsim 10^4$ g.cm^{-3}, having the properties of a one-dimensional solid and an energy per " atom " ~ 30 KeV for iron nuclei. The work function of this solid is ~ 1 KeV along the field. Because of rotation, a neutron star surface has electric fields $E \sim 10^{-8} (2\pi R_s P^{-1} B)$ volts/cm, where R_s and P are the radius and period of the star, respectively. Ions remain in the lattice despite such fields. The field-emission electron current generated by the component of E parallel to B is given by $J \sim 10^3 E^2 \exp(-30\ \varphi^{3/2}\ PB_{12}^{-1})$ A/cm^2, where φ is in KeV and B_{12} is the magnetic field strength in units of 10^{12} Gauss. The field-emission current may have been effectively cut off for rotation periods greater than a few seconds. This may be why long period pulsars are not observed. (Cohen, Lodenquai, and Ruderman 1970; Ruderman 1971; Canuto and Kelly 1972; Lodenquai 1972).

After having dealt with the effects brought by a huge magnetic field it seems only natural to deal with the problem of the possible origin of such a field.

As far as one knows, magnetic fields can be generated and maintained by one of the following processes:

1) Moving charges

2) Alignment of spin magnetic moments

3) Alignment of magnetic moment due to the orbital angular momentum of some type of atoms

4) LOFER states, Landau orbital ferromagnetism.

The presence of an electric current in a conductor can generate a magnetic field according to Maxwell's equation, rot H = j. Such a current is subject to ohmic dissipation that in conductors of ordinary size will dissipate it completely in matter of milliseconds. In the case of astronomical objects, however, the conductivity is so high and the size of the object so large, that the decay time ranges from millions to billions of years. To obtain a magnetic field through the alignment of spin magnetic moments, as in the case of ferromagnetism, one invariably needs a crystalline solid. The nature of the solid that we think exists in a neutron star is of a rather different nature and, besides, the temperature almost certainly exceeds the Curie temperature of the order of 10^3 °K, above which the phenomenon would not exist. Therefore, it seems that processes 2 and 3 have little chance of being of any help in understanding the magnetic field of interest to us.

A new possible source of strong magnetic fields was suggested in analogy with the usual ferromagnetism where one has an alignment of the spins (Canuto, Chiu, Chiuderi, and Lee 1969; Canuto, Kumar, and Lee 1972). In this case a type of ferromagnetism was found that is entirely due to the " orbital " motion of the electrons. If one calls H the external inducing field and B the physical field actually felt by the charges, the relation $B = H + 4\pi M(B)$, was found to possess solutions, even when the external field has been turned off (i.e., in the limit $H \rightarrow 0$). The magnetization is contributed by both spin and angular momentum parts of the hamiltonian, but the quantization effects are solely produced by the orbital motion. The function M(B) vs. B has a spiky appearance due to the jumps from one quantum level to the next. The straight line B intersects 4π M(B) at a number of points and then the two curves diverge. If one takes the maximum value of B and plots this against ρ, one finds a curve of the type $B_{max} \sim \rho^{2/3}$, as one would expect from the flux conservation law. Since these self-consistent solutions of the magnetic moment are entirely due to the quantization effects, the process was named Landau orbital ferromagnetism, or LOFER.

Thermodynamically, the LOFER states are not the minimum energy states and, hence, not the stable states in the thermodynamic sense. Still, a system that is not in a thermodynamically stable state, may require a long time to reach the most stable state. As an example, consider a bottle of hydrogen at room temperature. The thermodynamically more stable state is when all hydrogen nuclei are catalyzed into a piece of iron, but the lifetime against such a transition is over 10^{100} years, enough to make one consider the bottle of hydrogen as a truly stable state for all practical purposes. Therefore, it seems that, even though the state corresponding to the maximum value of B is only a metastable state, the barrier between any two states is high enough to trap the system in that state almost indefinitely.

ASTROPHYSICS OF NEUTRON STARS

1. Precatastrophic Stellar Evolution

In order to place neutron stars in their proper astrophysical context, we must first consider the conditions in which they may be formed. For this we must rely heavily upon calculations of stellar evolution.

A massive star can be expected to achieve progressively higher values of temperature and density in the center. The history of such a star consists of a continued contraction of its core, with halts during stages of nuclear energy generation. There is a succession of nuclear fuels which can be effective in producing such halts: these include hydrogen burning, helium burning, carbon burning, neon burning, oxygen burning, and silicon burning. The last of these stages of nuclear energy generation produces the nuclear statistical equilibrium abundance peak centered about ^{56}Fe. At this stage all of the available sources of nuclear energy have been extracted from the matter.

Only the most massive stars can be expected to go through this full sequence. A star of about one solar mass will first undergo hydrogen burning in the core, and then hydrogen burning in a shell around the exhausted core (now composed of helium), during which the star becomes a red giant. The core becomes highly degenerate, and helium burning commences with a " flash " which causes the core to expand and the envelope to contract, while the star then becomes a part of the " horizontal branch " of the Hertzsprung-Russell diagram. After helium has burned to completion in the core, the star then takes up a double shell source configuration, with hydrogen and helium burning shells surrounding the carbon and oxygen core, which becomes electron degenerate. As these shell sources burn toward the surface, the luminosity of the star progressively increases, since there is a decreasing total opacity impeding the escape of photons toward the surface. Eventually the luminosity becomes so high that the radiation pressure causes a rather quick loss of the outer hydrogen envelope of the star, producing a planetary nebula, and leaving a white dwarf star. Paczynski (1970) has found this to be the general character of the evolution for stellar masses up to about 3.5 or 4 solar masses. As the main sequence mass of the star increases, the mass of the remaining white dwarf remnant also steadily increases, until this remnant has achieved the limiting Chandrasekhar mass of 1.4 solar masses for a main sequence mass of about 4 solar masses.

Now consider the mass range 4 to 8 solar masses. The same general type of behavior can be expected in this mass range as described above for the lower mass range, except that the carbon and oxygen core will form a degenerate configuration close to the Chandrasekhar limiting mass before the stage of planetary nebula ejection is reached. At this point the core will undergo a rapid contraction, and the details of the subsequent evolution of the core become essentially independent of the amount of overlying mass (Barkat 1971). This is because the overlying mass forms an extended envelope in a region of relatively low gravitational potential, so that the pressure exerted by it on the degenerate core is rather minor.

The core contraction will continue until the ignition of the next nuclear fuel, carbon burning. There has been a great deal of discussion of the way in which carbon burning will be initiated; this discussion has been dominated by the expectation that stars in this mass range should form neutron stars. We defer an elaboration of this discussion to the next section.

Beyond 8 solar masses, the main modification in the evolutionary scheme is that carbon ignites non-explosively before a degenerate core can be formed following helium burning at the center of the star. It seems likely that there will be at least a small range of mass in which an oxygen-neon degenerate core will form, and explosive ignition of the fuel may then occur in this core under conditions in which collapse to form a neutron star will take place. The only advanced evolutionary calculations of more massive stars which have been carried close to the point of catastrophy are those of Arnett (reported by Arnett and Schramm 1973). It appears that from about 15 solar masses and upwards, nuclear processing of the core through the iron equilibrium peak will take place. In the very late stages of evolution, the iron core was undergoing a rapid contraction in Arnett's models, and seemed on the way toward forming either a neutron star or a black hole. In the meantime, the core is surrounded by a great deal of mass which has not undergone as much nuclear processing, and in which an explosive energy release can eject the outer layers of the star in a supernova explosion. This state of affairs seems to exist up to stars with masses of at least 70 solar masses. If any stars exist with masses substantially in excess of this amount (and observations have failed to find any), it appears that the character of the late stages of stellar evolution will again be modified, with the formation of electron-positron pairs in the outer envelope leading to collapse and explosive nuclear energy release, forming either a small silicon remnant or leading to the complete dissolution of the star.

The number of stars formed per unit time in our galaxy is a rapidly decreasing function of the mass. Gunn and Ostriker (1970) tried to determine the range of stellar masses responsible for the formation of pulsars in the following manner. They determined a pulsar formation rate by utilizing the apparent pulsar ages to determine a mean pulsar lifetime, and corrected for the non-observability of a majority of the pulsars on the assumption that the pulsar emission mechanism involves a fairly narrow beam sweeping out angles in the vicinity of the neutron star equator, to determine the rate of formation of pulsars per unit time. By assuming that stars of mass less than about 4 solar masses form planetary nebulae, they then concluded that the mass range giving rise to the pulsars was about 4 to 8 solar masses. Because this was coincident with the mass range in which carbon-oxygen degenerate stellar cores behave in very much the same way, contracting toward carbon ignition, it has generally been assumed that this is the stellar mass range responsible for the formation of the pulsars.

However, Arnett and Schramm point out that the rate of formation of stars in the mass range 8 to 70 solar masses is about the same as the rate of formation of stars in the mass range 4 to 8 solar masses. Since Arnett's calculations indicate that neutron stars have a good chance to become the remnants left following the supernova explosions in this mass range, this would become an alternate mass range which could be equated to the pulsar formation rate according to the calculations of Gunn and Ostriker. However, the pulsar formation statistics are surely uncertain by at least a factor of 2, and hence one could not exclude the conclusion that the mass range 4 to 70 solar masses could be that which produces the pulsars.

There is also considerable interest in the possibility that a neutron star might be formed in a close binary system, since a popular model of the pulsating x-ray binary sources consists of a neutron star onto which mass accretion occurs. Stellar evolution is drastically modified in close binary systems. The more massive of the two stars burns out its central hydrogen in much the usual fashion. However, following the formation of an exhausted helium core, the star starts to evolve toward the red giant configuration. At this stage a deep outer convective zone forms in the extended envelope, and this leads to a rapid transfer of mass from the primary star onto the secondary star as soon as the primary fills its Roche lobe (Lauterborn and Weigert 1972). R. Kippenhahn has noted (in remarks at the 1973 Tucson Workshop on The Physics of Compact X-Ray Sources) that the mass transfer may take place in two stages, the first following the exhaustion of hydrogen in the central core of the primary, and the second stage leading to still further mass transfer following the end of helium burning in the remnant left after the first mass transfer. The primary star will be reduced to a white dwarf for original main sequence masses up to about 16 solar masses during these two burning stages.

Much or most of the transferred mass may add onto the secondary star, but some of it may be lost from the system through an outer Lagrangian point. The secondary star will then evolve in its turn, and when it enters the red giant stage, a reverse mass transfer may take place, with a division of mass between that added onto the original primary and that which is lost from the system. The evolution in these later stages can become exceedingly complex, and it must be regarded as rather speculative. Nevertheless, it does seem possible that one of the stars will eventually undergo a supernova explosion which could leave a neutron star remnant.

However, it is very unlikely that a neutron star can remain a member of a close binary system, if it is formed in a supernova explosion in that system. The ejected matter in the supernova explosion will give a large impulse to the companion star when the matter impinges on the atmosphere of that star. Furthermore, according to calculations by Colgate (1970), this impulse is likely to be magnified by about an order of magnitude, since as the shock progresses downward through the envelope, it will eventually be reflected, leading to a blow-off of a substantial amount of mass in the facing hemisphere of the companion star, thus amplifying the momentum transfer initiated by the supernova explosion. Colgate determined that the velocity thus imparted to the companion would exceed the escape velocity unless there was a very substantial separation between the two components. This separation must be enormously greater than that exhibited by the components in the pulsating x-ray sources.

This illustrates the need for substantial caution before accepting neutron star models of pulsating x-ray sources. Alternative models, such as binary systems containing vibrating white dwarf stars (Blumenthal, Cavaliere, Rose, and Tucker 1972) should be examined as a way to escape from this difficulty.

2. The Carbon Detonation Dilemma

Let us now return our attention to the case of the contracting carbon-oxygen degenerate core in an advanced stage of evolution of the 4 to 8 solar mass region. The conditions attendant on the ignition of carbon on this core have received a great deal of attention in the last few years. A real dilemma has arisen as a result of these calculations.

The basic issue behind this dilemma is as follows. Suppose that an ignition of carbon thermonuclear reactions occurs at the center of the degenerate core. The temperature rises as the carbon starts burning, and soon the temperature rise at the center has been sufficient to start a convective region transporting energy outwards from the center of the core. The rise in temperature and the onset of a convective core makes very little difference to the structure of the core, since the equation of state is almost entirely dependent on the density and very little dependent upon the temperature. Thus the rise in temperature at the center makes very little difference to the structure of the core, and a thermonuclear runaway can begin. A similar thing happens in the case of onset of the helium flash in a degenerate core, but in that case the thermonuclear runaway lifts the electron degeneracy at the point where only a very small fraction of the helium has been exhausted by the thermonuclear reactions, and subsequent adiabatic expansion of the core can then damp out the thermonuclear runaway. In other words, the time scale for thermal expansion of the core then becomes smaller than the thermonuclear burning time. However, the opposite will occur in the case of the carbon thermonuclear runaway. At the time that electron degeneracy is lifted by the rise in temperature, the thermal expansion time of the core is longer than the thermonuclear burning time. Hence the thermonuclear reactions go to completion, dumping a great deal of energy into the core. It is expected that this will produce a detonation wave which will progress outward through the core, being strengthened by carbon burning as it propagates.

The numerical calculations indicate that the detonation wave raises the pressure in the core about 10 percent above the values needed to uphold the overlying layers in hydrostatic equilibrium, and hence the core starts expanding, driving ahead of it a shock which propagates into and ejects the envelope. This would ordinarily lead to the complete explosion of the star, leaving behind no remnant. The formation of a neutron star remnant only becomes possible if the pressure near the center of the core can be sufficiently reduced to cause an implosion at the center, with the propagation of a rarefaction wave through the expanding core which will cause additional mass to add onto the central imploded region. Such a reduction in pressure can in principle take place as a result of electron capture on the nuclei in the statistical equilibrium region near mass number 56; since most of the pressure is due to the electrons, and since a significant part of this pressure has now become thermal in origin, the absorption of electrons and the emission of some of the thermal energy in the resultant neutrinos can suffice to lower the pressure below that needed to cause outward expansion of the central layers. The electron capture rates are very strong functions of the density of the matter in which they take place; hence if carbon detonation takes place at a small central density, complete disruption of the star should ensue, whereas if the carbon detonation takes place at a sufficiently high density, implosion can be expected to occur.

The quantitative numerical calculations indicate that the critical central density required for implosion is somewhere in the vicinity of 2 or 3×10^{10} g.cm^{-3} (Bruenn 1972, Mazurek, Truran, and Cameron 1973). On the other hand, it appears impossible for carbon detonation to be postponed to densities higher than about 10^{10} g.cm^{-3}, even making allowance for the possible presence of an Urca convection zone (Paczyński 1972, Couch and Arnett 1973). Thus all recent calculations have tended to indicate that the carbon detonation should result in the complete disruption of the star.

This has seemed to contradict two types of observational evidence. One of these concerns pulsar statistics, where it has seemed necessary to obtain the pulsars from stars in this mass range. However, the preceding discussion indicates that this argument cannot be considered a very strong one. On the other hand, a considerably stronger argument comes from a consideration of the amount of iron peak elements which would be produced by the thermonuclear detonation, and dispersed into space by the complete disruption of stars in this mass range. According to calculations by Truran and Cameron (1971), the production of 1 to 1.4 solar masses of iron peak elements which would be produced by such explosions, would introduce far too great a mass of these elements into the interstellar medium to be in accord with solar system abundances and stellar abundances of newly formed stars in general. Therefore, it does seem likely that nature finds a way to prevent the catastrophic destruction and dispersion of these stars.

Iben (1973) and Buchler (1973) have attempted to see whether off-center detonation of the carbon might result in a compression of the core leading toward central collapse and formation of a neutron star. These investigations have turned out negatively; if it is assumed that an off-center ignition of the carbon leads to a detonation wave, then the whole star is still disrupted in a manner essentially indistinguishable from that resulting from central ignition and detonation. Thus it seems inescapable that if carbon ignition leads to the formation of a detonation wave, then independent of the site of the ignition, the star must completely explode.

Thus the solution to the dilemma appears to require that a detonation wave will not form. The physics of the detonation wave is fairly subtle and cannot be reproduced by the numerical methods presently used in supernova hydrodynamic calculations. All investigations carried out to date have assumed that a detonation wave can be initiated; once it is initiated, it appears that it will probably propagate successfully. Thus, the initiation of the detonation wave deserves further attention. Also, none of the investigations to date have treated the evolution of the core up to the carbon ignition point completely realistically; the pre-ignition evolution of the degenerate cores has generally been taken to be isothermal. In fact there will be temperature gradients in the outer layers of the core, and the temperatures near the center are likely to be somewhat less than those near the outer portions of the core, as a result of neutrino-antineutrino pair emission processes. It is possible that this may lead to a non-explosive ignition of carbon near the surface of the core with a non-detonation propagation of the burning wave inwards, accompanying a core expansion that allows the core to undergo a stage of carbon burning without being disrupted.

If non-explosive carbon burning can be initiated in the core, then it appears that there may be two possible outcomes of this process. On the one hand, carbon may burn out in the core, leading to the formation of a triple nuclear burning shell source, and the formation of a degenerate core composed primarily of neon and oxygen, with the oxygen undergoing thermonuclear ignition at a high enough density to cause reimplosion and formation of a neutron star. On the other hand, it is possible that the high luminosity associated with the triple shell source will result in the ejection of the outer layers of the star, forming a planetary nebula and leaving a white dwarf remnant. Extensive further calculations will be needed to settle these issues.

3. Cooling and Vibrational Damping

When a neutron star is formed in a supernova explosion, there is a mass cut which divides the outflowing material from that which will tend to fall back onto the collapsed core. Actually, not all of the material on the interior side of the mass cut is likely to succeed in falling back onto the neutron star, since some of it would take rather a long time to do so, and it is unlikely that the pulsar emission mechanism will be delayed in starting until the infall is complete. When the neutron star becomes a pulsar, there is likely to be a strong pressure exerted on this infalling material which will reaccelerate it outwards. In fact, there is even a possibility that the onset of the pulsar emission mechanism may play a significant role in accelerating the entire supernova envelope outwards, and contributing to the energy which is radiated in the supernova light curve.

When the neutron star is formed, much of the released gravitational potential energy may be temporarily stored in several different forms of internal energy. These include internal thermal energy, vibrational energy, rotational energy, and the energy in the contained magnetic field. A number of investigations in recent years have been concerned with possible astrophysical consequences of these forms of energy storage.

It appears that oscillations of the neutron star are unlikely to persist for astrophysically interesting lengths of time. Ordinarily, in the cold neutron star interior, the Fermi energies of the constituents adjust themselves in such a way that transformations between constituents will not take place. Vibrations upset this balance, allowing phase space to open for such transformations in each part of the vibrational cycle. Transformations between protons and neutrons, with emission of neutrinos and antineutrinos, were considered by Hansen (1966), and by Hansen and Tsuruta (1967) who found that the neutrinos would greatly damp the vibrational energy of the star in a time of some centuries. Transformations between nucleons and hyperons can probably take without emission of neutrinos and antineutrinos, but still via weak interactions, and it was found that these transformations would convert vibrational energy into internal heat on a very short time scale, of just seconds, by Langer and Cameron (1969), and by Jones (1970). Jones (1971) also pointed out that these transformations would quickly damp out any initial turbulent motions which might exist in the neutron star interior. Any nonradial vibrational modes will also be quickly damped through emission of gravitational radiation (Thorne 1969). In fact, the combination of rotation and vibration is likely to lead to a coupling between the radial and nonradial vibrational modes, so that radial vibrational energy is channelled into nonradial modes which are quickly damped by the gravitational radiation. These dissipative processes also tend to operate on a time scale of seconds. Dyson (1972) has suggested that a solid core in a neutron star would be a particularly effective gravitational wave radiator because of the near equality of sound and light speeds.

The internal thermal energy is radiated away by a combination of neutrino emission from the interior and photon radiation from the surface of the star. This problem has recently been reconsidered by Tsuruta, Canuto, Lodenquai, and Ruderman (1972), who considered the effects on cooling processes of superfluidity, crystallization, and the presence of the intense magnetic field. The neutrino emission processes included in the calculations were the plasmon process, the URCA process, the neutrino bremsstrahlung process, the photoneutrino process, the pair annihilation process, and the neutrino synchrotron process. The URCA process involves emission of neutrinos and antineutrinos in the inter-conversion of neutrons and protons, and this process would be dominant in the absence of magnetic fields and superfluidity. However, the superfluidity of the nucleons suppresses this process significantly and it appears that the dominant neutrino cooling mechanism is the plasmon process. The neutrino emission dominates the cooling during the first 10^2 or 10^3 years, by which time the internal temperature of the star has been reduced to about 10^9 °K, and the surface temperature has been reduced to about 10^7 °K.

The effect of a strong magnetic field is to reduce the opacity in the surface layers of the star, since processes contributing to the opacity, particularly Compton scattering of photons by electrons, tend to be suppressed by the presence of the magnetic field which allows the electrons to travel only along the direction of the field lines. Following the onset of photon cooling dominance, the surface temperature of the star tends to drop fairly rapidly on a logarithmic time scale, as shown in Fig. 9. The fall in temperature is accelerated due to the effect of crystallization in the neutron star interior, since the heat capacity of such a crystal becomes quite small at the lower temperatures. It may be seen from Fig. 9 that most pulsars probably do not have a surface temperature in excess of 10^4 °K, if one can neglect possible heating processes in their interiors, which will be discussed subsequently.

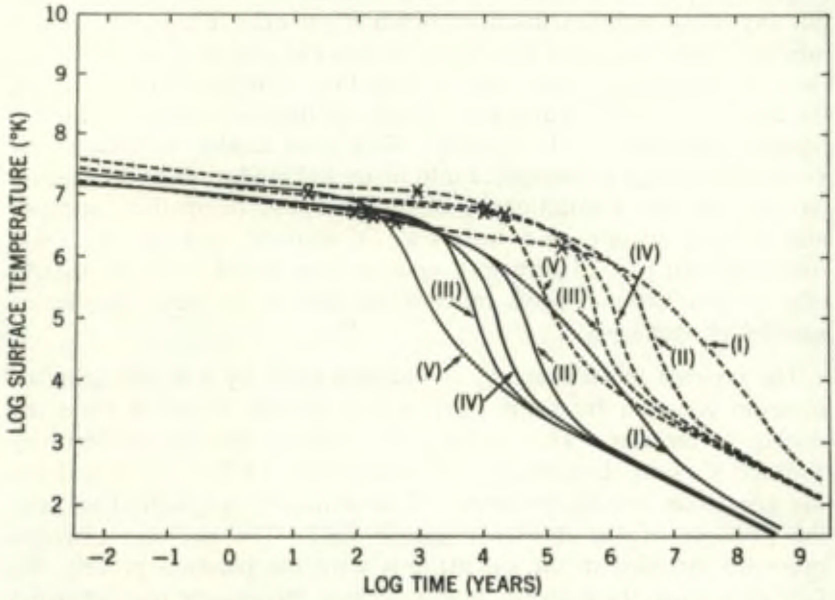

Fig. 9. Surface temperature vs. age of a neutron star. The dotted lines do not contain superfluid effects whereas the solid curves do. The models (I), (II), (IV) and (V) have $H/H_q = 0, 0.025, 0.1$ and 1 respectively. The model (III) has $H/H_q = .025$ in the region where radiative opacity dominates and $H/H_q = .25$ in the conductive opacity regions. (From Tsuruta *et al* 1972).

4. The Rotation of Neutron Stars

Nearly all supernova hydrodynamic calculations have been carried out with the assumption of spherical symmetry of the object, and hence with the neglect of internal angular momentum. Since, as we have seen, the issue of whether or not a neutron star is formed depends upon conditions in the presupernova involving central densities in the range characteristic of white dwarf stars, it is possible that the presence of substantial amounts of angular momentum is such a stellar core has relatively little influence upon the possibility of implosion of the core to form the neutron star. However, once such an implosion occurs, it is possible that the resulting stellar object will be rotating so fast to have achieved centrifugal instability in the equatorial plane, and hence it will flatten into a disk. However, it is also possible that various processes can extract sufficient angular momentum from the core during the course of stellar evolution so that one simply ends with a rapidly rotating neutron star having a substantial equatorial rotational bulge.

Since the equation of state of neutron star matter is very stiff at higher densities (see Fig. 8), the central portions of neutron star models tend to have a very flat density distribution. Tsuruta and Cameron (1966*b*) have pointed out that this may make a very rapidly rotating neutron star tend to deform into a Jacobi ellipsoid, essentially a rotating bar-like configuration. A rotating flattended disk at neutron star densities is likely to be subject to a similar type of deformation. Such a figure represents a time-varying mass quadrupole, and hence the shape is rapidly dissipated toward one of axial symmetry through the emission of gravitational radiation.

It is a well known characteristic of pulsars that they are slowing down. In fact, the combination of the known rate of slowing down of the Crab nebula, together with the total rate of energy emission observed from the Crab nebula, with the fact that the rate of loss of rotational energy is comparable to the observed rate of energy emission for objects having moments of inertia characteristic of neutron stars, constituents one of the strongest demonstrations that the pulsars are in fact neutron stars.

The pulsar emission mechanism remains a mystery, with many different theories having been put forward to explain it, but none of these having yet achieved general acceptance. However, it is worth noting that there seem to be two general types of phenomena involved, one, the main energy loss associated with the slowing down process, and the other the process or processes that lead to the emission of the observed radio pulses.

It is popular to suppose that the loss of rotational energy and angular momentum as the neutron star slows down may result from magnetic dipole radiation, which propagates electromagnetic waves of very large amplitude into the surrounding space, and may produce interesting particle acceleration effects. These effects depend upon the magnitude of the assumed surface magnetic field on the neutron star; if the observed slowing down rates are attributed to this process, then magnetic fields of the order of 10^{12} Gauss are inferred. Since magnetic fields of this order may reasonably be expected to be produced in neutron stars during the process of gravitational collapse from a configuration of much lower density, due to the freezing-in of the field lines, this picture has a certain appealing consistency with what is known about stellar magnetic fields. However, there have been no pulsar observational effects which can be unambiguously attributed to this mechanism.

On the other hand, the amount of observed coherent radio energy emitted in pulsar pulses is much less than one would expect for the associated loss of rotational energy from the same stars. Thus it seems likely that the physical processes responsible for the pulsar emission characteristics may not play a major role in the slowing down process itself, but may be principally responsible for giving the pulsars the distinctive signature which makes their detection possible.

Perhaps the phenomena which have given the greatest stimulus to the investigation of physical processes in neutron star interiors are the variations in the slowing down rates of the two shortest period pulsars, those associated with the Crab and Vela supernova remnants. Each of these supernova remnants has been observed to have two substantial discontinuities in the rotational period, in which there was an apparent speeding-up of the rotation taking place in a period of at most a few days. These period discontinuities are commonly called glitches. In addition to these major events, which were of much greater amplitude for the Vela pulsar than for that in the Crab, there are apparently rather ramdom departures of the pulsar timing phases from a smooth slowing down behavior. This general type of phenomenon is frequently called restless behavior.

This extraordinary behavior of these two pulsars has given rise to a large literature.

5. Theories of Glitches and Restless Behavior

There are three general classes of theory which attempt to explain pulsar glitches. The first class of theories involves phenomena external to the neutron star, such as dumping plasma out of the magnetosphere, or having external matter collide with the neutron star. We shall not discuss this type of theory here. The second class of theory involves redistribution of the internal angular momentum within the neutron star. The third class of theory involves starquakes, and these theories have received the greatest analysis in the literature. We shall discuss them first.

The starquake theory was first advanced by Ruderman (1969), and has subsequently been analyzed in great detail by Pines and various of his collaborators (Baym, Pethick, Pines, and Ruderman 1969; Pines 1971; Baym and Pines 1971). At a time when only one glitch had been observed in each of the two pulsars, the theory took a relatively simple form. The newly-formed neutron star was assumed to be rotating rapidly, so that there was a substantial equatorial rotational bulge, and this bulge would be frozen into the crust of the neutron star when the crustal material solidified in the form of the Coulomb crystal. As the neutron star slowed down, the equatorial bulge would extend beyond the new equilibrium figure of rotation, and the crust would be able to adjust to the new equilibrium figure only as a result of undergoing major fractures, called starquakes. As a result of such starquakes, there would be a sudden reduction in the moment of inertia of the star, and hence the conservation of angular momentum would require a sudden increase in the rate of rotation, giving the observed glitch. Since the observed magnitude of the glitch in the Vela pulsar was very much greater than that in the Crab pulsar, it was then predicted that the interval between glitches in the Vela pulsar should be much greater than those in the Crab pulsar.

An interesting aspect of the analysis accompanying this glitch theory concerned the behavior of the pulsar period following the glitch. At that time it was generally believed that superfluid neutrons and protons would constitute the majority of the matter inside the crystalline crust of the neutron star. There would be very little interaction between the superfluid neutron and proton components of the fluid. The existence of a magnetic field in the interior of the neutron star would keep the various charged components of the star coupled together in uniform rotation, but the neutrons could be coupled to the charged components only as a result of friction. Superfluid neutrons would exist throughout most of the mass in the crust, where their frictional interaction would be with the crustal nuclei, an interaction that has not received extensive physical analysis as yet. In the deeper portion of the star, the normal component of the neutrons would suffer friction through interaction with the magnetic moments of electrons and by scattering from the normal component of the superfluid protons. The normal component of the neutrons would exist in the cores of rotational vortices, and the normal component of the superfluid protons would exist both in the cores of the rotational vortices and in the magnetic flux tubes extending through the interior which would carry the magnetic field.

Following a glitch, the difference in angular velocities between the superfluid neutrons and the charged particle components would be reduced, and hence the friction between them would also be reduced. Thus the pulsar magnetospheric torque on the charged particle components, assumed constant through the glitch, would slow down the charged particle components at a more rapid rate than normal immediately following the glitch, in accordance with observation. The normal rate of slowing down would only be re-established after the difference in angular velocities between the components had been restored. This " healing time " of the slowing down process would depend upon several neutron star parameters, and the observed healing times thus suggested values for these parameters (the pulsars were concluded to be of relatively low mass: Sutherland, Baym, Pethick, and Pines 1970), and they led to the predictions of the interval between glitches.

It was obvious that there were a number of additional physical processes which might modify this picture. For example, Smoluchowski and Welch (1970) discussed the possibility that stresses in the neutron star crust might be relieved as a result of plastic flow rather than fracture. So far there has not been a quantitative analysis of the effect of a lessened pressure at the base of the crust near the equator and an increased pressure at the base of the crust near the pole as a result of the slowing down process. This should lead to a deposition of new crustal material in the equatorial region and the dissolution of crustal material at the base in the polar regions. However, the presence of a strong magnetic field would prevent the surface layers of the neutron star from flowing across parallels of latitude in order to establish the new equilibrium figure.

Pines and Shaham (1972) extended the theory of neutron starquake to include two types of quakes: major quakes and microquakes. The microquakes represent a continual small crumbling of the crust, and they can take place in such a way as either to increase or to decrease the rotational period. Pines and Shaham thus advanced the microquake theory to explain the restless behavior of the pulsars.

After the second glitches had been observed in each of the two pulsars, with a time separation of about two years in each case, it was evident that the starquake theory in its existing form had some serious deficiencies. The Vela quake had recurred much sooner than expected. Thus Pines, Shaham, and Ruderman (1972) suggested that the existing crustal quake theory might be applicable only to the Crab pulsar, and they suggested that the large frequent quakes in the Vela pulsar might represent a quake phenomenon taking place in an assumed solid core of the neutron star. This theory was advanced at a time when it was becoming plausible that neutron stars should form quantum crystals in their cores at greater than normal nuclear density.

Theories involving redistribution of internal angular momentum have not been extensively developed. Greenstein and Cameron (1969) suggested that a hydrodynamic instability in the star might produce glitches. This would require the existence of superfluid neutrons rotating very much faster near the center of the star than farther out, so that somewhere in the star there would be a negative gradient of angular momentum per unit mass. Under such conditions a hydrodynamic instability would lead to an interchange between different parts of the fluid, introducing superfluid neutrons with a greater angular velocity into the outer parts of the star, where a larger amount of friction could then transfer angular momentum suddenly to the charged particle components. At the present time there does not seem to be any reason to expect such a large change in the angular velocity of the neutrons to be established in the interior of the neutron star. However, it should be noted that this general type of process would produce a glitch if there were sudden variations in the friction between the neutrons and the charged particle components, leading to a rapid transfer of angular momentum from some part of the neutron superfluid to the charged particle system. It must be remembered that the greater part of the angular momentum of the neutron star resides in the superfluid neutrons, which are rotating at a considerably higher rate than the charged particle system.

Even if glitches cannot be produced in this way, there remains the possibility that variations in the friction between the two-component system may be responsible for the restless behavior. Such variations in friction could exist if the variation of the friction to which the normal component of the rotational vortices is subjected along the vortex lines running parallel to the rotation axis, should result in a deformation and resultant tangling of the vortex lines. Greenstein (1970, 1973) has suggested that such a turbulent scrambling of deformed vortex lines could be a common feature of the neutron star interior, but Ruderman (1973) has questioned this on the grounds that the vortex lines should act collectively to strengthen one another against such deformations.

Ruderman (1970) suggested another way in which angular momentum redistribution might take place within the neutron star interior. This would involve a large-scale oscillation, toward and away from the rotation axis, of the system of rotational vortices. Such an oscillation would lead to a sine-wave variation in the timing residuals of the neutron star period, and it was advanced at a time at which the restless behavior was thought to have such a regularity. At present there is no indication from the observations that such a regular oscillation in the period occurs which might be attributable to this process.

It will have been noted that friction between the two superfluid components in the neutron star interior plays a significant role in many of the different ideas discussed above. One of the consequences of this friction should be heating, as discussed by Greenstein (1971, 1972, 1973). Another possible source of heat might be dissipation of precessional wobble in the neutron star crust (Henriksen, Feldman, and Chau 1972). In either case such heating would offset the cooling processes in the neutron star interior, leading to a higher surface temperature than indicated in Fig. 9. For the Crab pulsar, Greenstein (1973) has estimated that the present surface temperature may lie between 10^5 °K and 5×10^6 °K. The higher range of these temperatures might result in observable x-ray emission by thermal processes from the surface of the neutron star, and an opportunity to make such observations will arise during the forthcoming lunar occultations of the Crab nebula during 1974 and 1975.

6. Neutron Stars and Cosmic Rays

Supernova explosions are thought to be prominant sites for cosmic ray acceleration, and in recent years there has been much discussion of the possibility that pulsars play a leading role in such acceleration. This role could occur via the strong electromagnetic waves radiated into the surrounding nebular gases during the slowing down process of the pulsar, or it might involve material accelerated from the neutron star surface through the operation of the pulsar emission process, whatever its nature should be. Some calculations have been carried out relevant to these questions.

When a neutron star is formed hot in a supernova explosion, the final infalling material is likely to have a composition consisting mainly of neuclei lighter than those in the iron equilibrium peak. Such nuclei then have a chance to diffuse inwards and be transformed in thermonuclear reactions during the early hot stages of the neutron star. The most extensive calculations concerning such diffusion and composition changes are those carried out by Rosen (1968, 1969). In the case of a neutron star surface without a strong magnetic field, he found that the inward diffusion would convert nearly all of the material to ^{56}Fe, with only a small amount of it remaining at the surface itself in the form of ^4He. In case a strong magnetic field is present, the opacity is considerably reduced in the surface layers, and the cooling time is shortened. However, the reduced surface opacity considerably decreases the temperature difference between surface and interior, and once again iron and helium are the principal constituents remaining after the star has cooled. However, in this case, the hotter atmosphere has a larger scale height at low densities, and this tends to protect much of the helium that is formed against thermonuclear transformation. Thus, in this case, a significantly larger amount of helium is left on top of the iron at the surface (Rosen and Cameron 1972).

Some questions have been raised by Ruderman (1971) about the possibility that the iron and helium nuclei can be lifted off the surface of the neutron star in order to be injected into the cosmic rays. If a magnetic crystal is formed in the surface layer as a result of the presence of a strong magnetic field, then a work function exists which tends to prevent the extraction of iron or helium nuclei from the surface layer. Ruderman therefore suggests that pulsar magnetospheres may lack heavy ions.

Normal cosmic ray composition contains heavier elements in very roughly the same proportions as in the solar system, although with a relative depletion of hydrogen and helium. However, the composition is consistent with some separate injection of iron nuclei from some of the cosmic ray sources, and the neutron stars are logical candidates for these sources.

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Discussion of the report of A.G.W. Cameron and V. Canuto

R. Ruffini: In the reports of Professors Cameron, Canuto and Pandharipande we have heard much of the detailed considerations of nuclear physics involved in the determination of an equation of state for neutron star material. From these presentations it becomes clear that although agreement is reached for an equation of state in the regimes of densities $\rho \lesssim 4 \times 10^{15}$ g/cm^3, very little is known on the equation of state at nuclear and supranuclear densities.

However, what seems to be of great interest is that only three quite well definite and different behaviours for the equation of state at $T = 0$ have been predicted in the asymptotic regimes of supranuclear densities (see Figure 1):

(1) It is well known how Zel'dovich[1] pointed out the possibility of reaching in the limit of very high densities an equation of state with velocity of sound equal to the velocity of light

$$p \to \rho.$$

In the presentation of Professor Pandharipande we have learned today that *if* the concept of a static potential is used *then* this limiting case is always reached *independently* of any form of the interactions.

(2) Landau[2] had explicitly pointed out, instead, that *independently* of any form of the interactions a limiting equation of state

$$p = \rho/3$$

should be expected to apply to matter in bulk at supranuclear density and $T = 0$. A detailed treatment leading to this limiting behaviour has been recently given by R. L. Bowers, J. A. Campbell and R. L. Zimmerman[3].

(3) Hagedorn[4] following a totally different statistical approach has shown how, again, *independently* of any form of the interaction a limiting equation with

$$p = \rho/\ln \rho$$

should be expected[5].

My reasons of focusing on this issue here is that this different behavior of the equation of state at supranuclear densities far from being only of academic interest does indeed influence in a drastic form the computations of the masses of the equilibrium configurations of neutron stars (see fig. 2). It is in fact important to realize[6] that the equation of state below 5×10^{14} gr/cm^3 does not influence drastically the value of the critical mass of the neutron star. The major contribution originates in fact from the asymptotic behaviour and the rate of reaching this asymptotic behaviour.

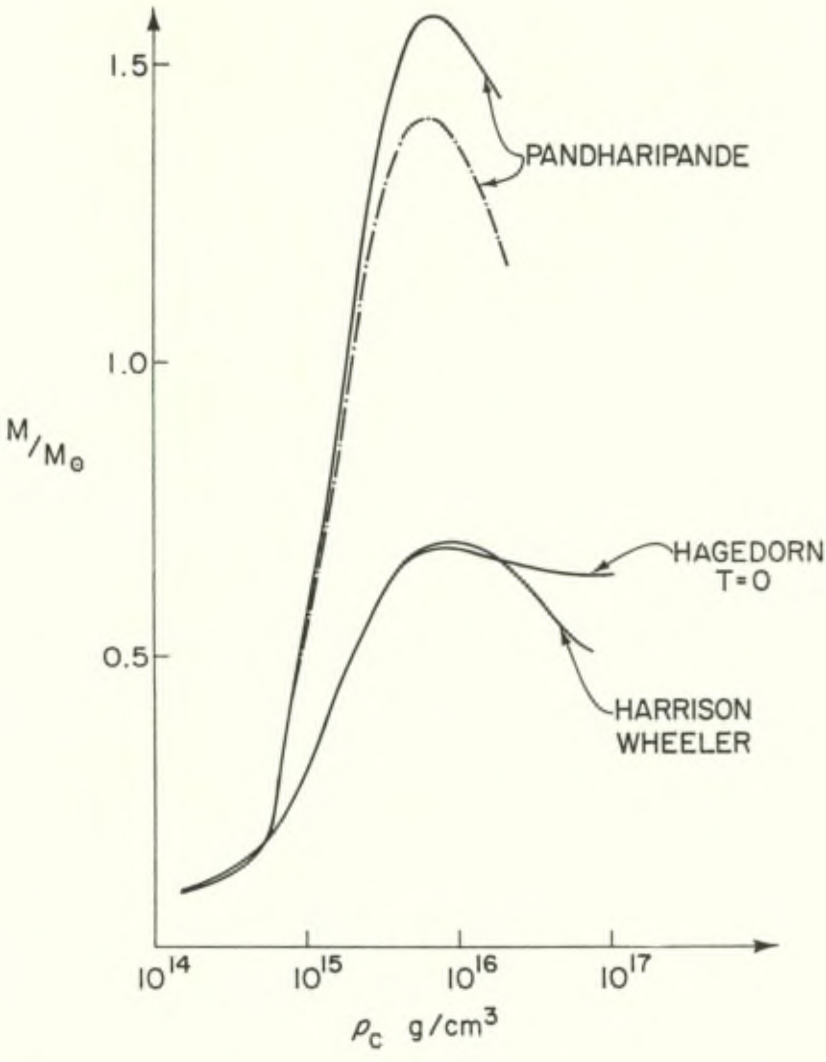

Fig. 1. Pressure versus density for selected equations of state descriving neutron star material. The free nucleons equation of state gives for densities smaller than 4.5×10^{15} g/cm^3 a value of the pressure systematically larger than the one given by an equation of state taking into account nuclear interactions and approaches in the high density limits an equation of state $p = \rho/3$. The Hagedorn equation of state takes into account the interactions indirectly by modifying the statistic. The equations of state taking into account nuclear interactions either they approximate asymptotically $p \to \rho$ or they give a velocity of sound greater than the light velocity! (Figure from R. Ruffini in « Black Holes » Gordon and Breach, 1973).

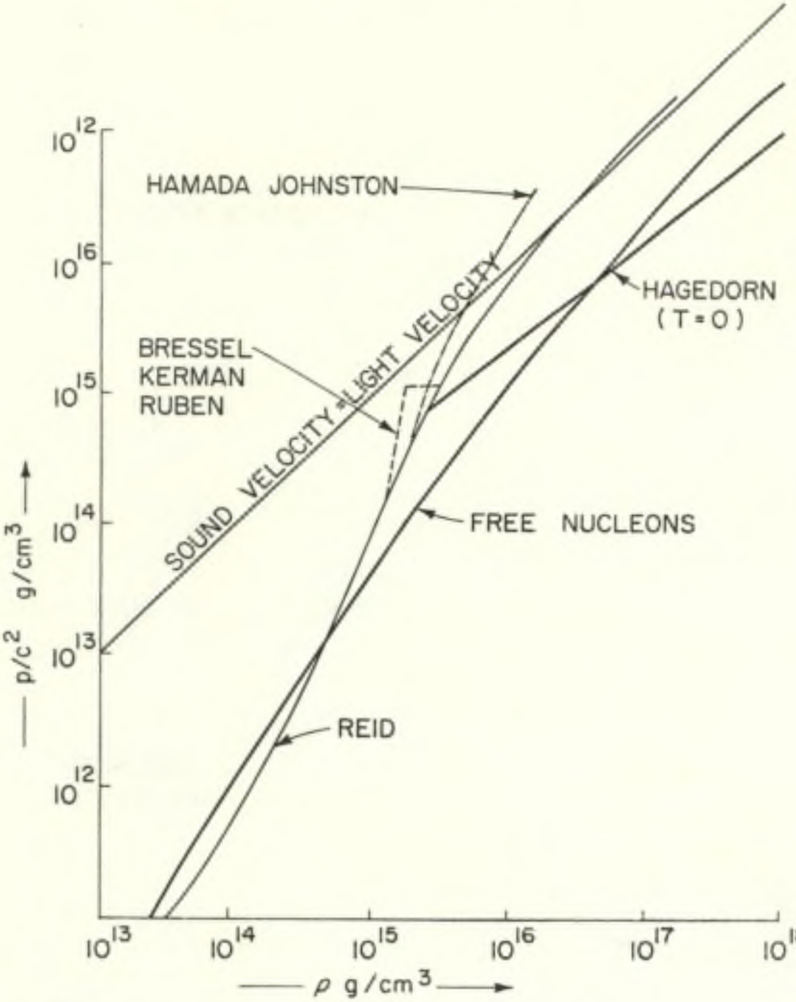

Fig. 2. Masses of the equilibrium configuration of neutron stars plotted as a function of the central density for selected equations of state. The Pandharipande equation of state goes $\lim_{\rho\to\infty} p = p$, the Harrison-Wheeler $\lim_{\rho\to\infty} p = \rho/3$, the Hagedorn (T = 0) $\lim_{\rho\to\infty} p = \rho/\ln \rho$.

On the other side, from an experimental point of view it has been clearly pointed out in the talk of Professor Giacconi[7] how the neutron star masses are the single most important parameter in the physics of binary x-ray sources. This for two good reasons:

(a) The determination of the mass of the neutron stars in pulsating x-ray sources will be made in the near future with an unprecedented accuracy and will allow to infer details on the equation of state of neutron star material[8].

(b) The difference in mass of collapsed objects together with their characteristic pulsating or bursting emission in the x-ray[9] is the most important tool we have today in differentiating between neutron stars and black holes.

My question therefore is the following: Could some of you, Professor Cameron or Professor Canuto or Professor Pandharipande comment or clarify this paradoxical situation. Three drastically different behaviour each one totally *independent* (so it seems!) from any detail of the interactions!

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A. G. W. Cameron: In a sense the three different asymptotic behaviors described by Professor Ruffini are not very relevant to the issue of the maximum mass of a neutron star, which is a potentially observable quantity in nature. The maximum mass of a neutron star is reached near a central density of 10^{16} g/cm^3, and while it is true that under these conditions the pressure has become an appreciable fraction of the proper energy density, it is still very far from the asymptotic limit. Therefore it is not clear that the determination of a maximum neutron star mass will cast light on this particular issue.

V. Canuto: As far as the first point is concerned, one has to be very careful about the physical meaning of Zeldovich $p \rightarrow \rho$ limit. Not withstanding the rather interesting rumour[1] that Landau never believed in such a relation (but unfortunately he never explained why) there is the fundamental consideration that at $\rho \rightarrow \infty$, more mesons will influence the equation of state and the vector meson cannot be the final and only one. For example $\rho \gtrsim 10^{17}$ gr/cm^3, the f^o, spin-2 meson will certainly be important. This gives *attraction* thus making the equation of state softer. This was first pointed out by Y. Ne'eman in 1968.

Zeldovich's argument $p \to \rho$ *should* in my view be considered as representing a mathematically soluble many body system. This is far from saying that at $\rho \to \infty$ matter is actually described by the relation $p = \rho$. As far as point 2 is concerned, let me first comment on the work of R. L. Bowers et al. you quoted. For the time being, they have solved a purely academic problem, i.e. Dyson's equation in the high density limit by exchanging a π-meson. The reason why they get $p = \rho/3$ is rather trivial. The effect of exchanging such a light meson is completely washed out in the high density limit. The system acts as being free and the relation $3\,p = \rho$ then naturally follows. Therefore, I most certainly would not take their work, as I know the authors themselves don't, as an indication of the correctness or otherwise of the relation $3\ p = \rho$.

I know that R. Zimmerman[2] and his collaborators are in the process of extending their computation by exchanging a set of mesons that most certainly dominate the behaviour of $p = p(\rho)$ at high densities. Therefore, until we know the result of their computations, no definite statement can be made.

Third, as far as Hagedorn is concerned, I do not agree that his equation of state is completely independent of any interaction. It is true that Hagedorn does not use any potential concept to derive his equation of state. However, some form of interaction is surely present was it not for the simple fact that a free system of particles gives rise to a density of state of the form

$$\rho \sim e^{\sqrt{M}}$$

whereas he gets

$$\rho \sim e^{M}$$

i.e. he has a higher entropy since he is dealing with Regge trajectories. Besides he uses resonances only and not Pomerons and this could mean that the attraction has been correctly incorporated but that a sizable part of the repulsion has been left out. I guess this remark was first made by Professor H. Bethe. If this is so, there is no mystery or difficulty in understanding the softness of $p = p(\rho)$ and therefore the low value of $M/M_\odot$ you just quoted. Hagedorn equation cannot be said to be independent of any type of interaction and before being used meaningfully, one has to devise some way to include repulsion. You will then get a $M/M_\odot$ vs. ρ_c relation very similar as we all get.

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1. I. M. Khalatnikov, 1973, private discussion.
2. R. Zimmerman, 1973, private communication.

V. R. Pandharipande: The pressure (P) approaches energy density (ε) as the number density $\rho \to \infty$ when a soft-core potential is used to describe the interactions between baryons. This statement is proved in my report. Bethe (ref. 1 in my report) has argued that potential description should be reasonable if the strong repulsion in nucleon-nucleon interaction is indeed due to vector meson exchange.

More recently Walecka (1973, September preprint) has proved with linearized quantum field theory that a vector field coupled to conserved baryon current will give $P \to \varepsilon$. The linearization of field equations should be reasonable at high densities.

Hence in the asymptotic limit P should approach ε as long as the ω exchange provides the strongest interaction among baryons.

V. Canuto: I agree that as long as ω provides the strongest interaction, the relation $p = \rho$ will follow. However, here again I must stress the point I have already made to Dr. Ruffini, that at higher and higher densities, ω is not the only meson that contributes; the f^o-spin 2 meson will enter and will decrease the pressure perhaps even changing the limit $p = \rho$. A very general and important point is the following. In the high energy ($\sim$ 30 GeV) p-p interaction, that is being studied very intensively in these days, the multiplicity of secondaries is known to be an increasing function of energy of the two protons, E, the general form being

$$N(E) \sim E^a \qquad a > 0$$

If one takes an equation of state of the form

$$p = \lambda\rho$$

it can be shown by simple thermodynamic-arguments and nothing else that

$$N(E) \sim E^{\frac{1-\lambda}{1+\lambda}}$$

If $p = \rho$, N (E) turns out to be independent of E, in disagreement with experiments. In the hydrodynamic treatment of multiparticle production, Landsau used $\lambda = 1/3$ thus getting

$$N(E) \sim E^{1/2}$$

in agreement with the data. More recent analysis[1] of the multiparticle production indicates that λ is either 1/3 or less (not greater) and surely not one. I must stress that this last result is based on the angular distribution analysis of the emitted secondary particle and not only on the multiplicity.

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E. Suhonen, J. Enkenberg, K. E. Lassila and S. Sohlo (*Phys. Rev. Letters* **31**, 1567, 1973).

G. Baym: (Comment to R. Ruffini). The problem of the asymptotic form of the equation of state of dense matter is certainly interesting, and as we see, unsolved. However, I wish to question whether the softer asymptotic models are really relevant in determining the structure of neutron stars. The point is that the maximum mass neutron star has a central density only in the range 10^{15} - 10^{16} gm/cm^3; the harder the equation of state, the lower is this maximum density. It is by no means clear that this density regime is anywhere near asymptotics. In particular, in this regime the repulsive forces between baryons play a decisive role in determining the equation of state, and they are certainly neglected in models based on non-interacting baryon resonances. An appeal to asymptotics can easily underestimate the important effects of the baryon repulsion.

V. Canuto: I most certainly agree with you.

Y. Ne'Eman: 1. I think I can offer a program for as reliable a study as one can ever hope to get from the microscopic level. Hagedorn's approach, which was inspired by thermodynamics, can now be seen to have included only a part of the strong interaction. Progress in Horn-Schmid duality and the Veneziano representation, plus two-component duality (suggested by Freund and Harari) tell us that the sequence of resonating states described by Hagedorn represents in effect the result of an interplay of attraction and repulsion due to the exchange of all ordinary Regge trajectories (i-e the same states). On the other hand one has to add the effects of *diffraction,* described by the exchange of the Pomeromchuk singularity, and producing non-resonating states. This will probably modify Hagedorn's result, perhaps in the direction of $\frac{\rho c^2}{3}$ ..... One can afterwards superimpose SU (3), and even better, *broken SU (3)*.

2. As to the point about high density mentionned by Prof. Canuto, I had in 1968 suggested that the recurrence of attractive forces (rthe f^0) at a shorter range than the hard core, and then another hard core etc... made it possible to expect that dense nuclear isomers might exist for some range of nuclei. Bodmer has estimated that it could occur beyond $A = 16$-20, and T. Gold has suggested that starquakes might be due to collapse into these states.

V. Canuto: Your point is very well taken and I hope that a realistic many body computation with the effect of the spin-2 meson will soon be undertaken.

L. Rosenfeld: It is perhaps not superfluous to recall the well-known fact that even in ordinary nuclear physics the concept of potential has a very limited validity. The attempt to study " nuclear matter " on the basis of nuclear potentials derived from scattering data yields too weak a binding for the densities corresponding to actual heavy nuclei; one should not forget, of course, that such nuclei have only about six nucleons across their diameter, and are thus very far from samples of extended nuclear matter.

To try to improve the analysis by introducing hyperon potentials does not seem very hopeful; in fact, nobody would dream of improving the equation of state of a crystal by treating the excited states of the individual atoms as so many components of the system. I am therefore inclined to a similar view as Ne'eman's, even though I do not share his optimism about the efficiency of present high energy techniques: I would nevertheless think that starting from what we know about the high energy behaviour and structure of the various " particles " might provide a more promising approach to the discussion of the equation of state in the asymptotic region of high densities. This would amount to studying an assembly of the components of neutrons, or rather hadrons on the basis of some appropriate assumptions about the couplings and properties of these components.

J. A. Wheeler: Perhaps we can summarize Professor Rosenfeld's remarks in these words: these is such a thing as a " soft " equation of state; there is such a thing as a " hard " equation of state: but there is also such a thing as a " too hard equation of state " - an equation of state that is too hard to calculate! Despite the ability of the workers who have investigated the properties of cold matter at densities of the order of 10^{15} g/cm^3 and greater, it may be that the question is just too difficult for the present state of physics. If so, observation itself must be our ultimate recourse. At some future time, when observations of slowly rotating neutron stars have given mass and radius for a sequence of objects, we can use the straightforward algorithm of U. Gerlach (Phys. Rev. 177, p. 1929 (1969)) to deduce the equation of state directly from the data.

Professor Cameron emphasizes a central question in neutron star physics, where does the compaction of mass come from that produces the neutron star. Also, in the I.A.U. meeting at Warsaw earlier this month Arnett stressed the difficulty of ever getting any star with white dwarf core more massive than 1.4 $M_\odot$ to serve as precursor for a neutron star. However, at Princeton last week Ostriker (who unfortunately could not be here) stressed the great influence of rotation, particularly differential rotation, in increasing the allowable mass for a white dwarf. His calculations show that a white dwarf of 3 solar masses, and also one of 4 solar masses, is stable against both fission and collapse. Ono is therefore tempted to consider such an object as the precursor for the typical neutron star we see.

A. G. W. Cameron: The principal difficulty with considering any model of a very massive rotating white dwarf star or stellar core has to do with the lifetime against alteration of the structure of differential rotation in such a star. If the matter composing the star should cool off, then crystallization would rapidly eliminate differential rotation and cause the core to collapse. On the other hand, if the configuration remains very hot, so that the thermal pressure gradient is not negligible, then one might expect circulation currents in the Eddington-Sweet sense to cause a redistribution of angular momentum, thereby eliminating differential rotation from at least the inner portions of the object, again causing collapse on a presumably small time scale. However, in either case one would be dealing with a collapsing core which is still spinning quite rapidly, so that the time evolution of the collapse is likely to be somewhat slower than that envisaged in the calculations which ignore the rotation, and this should probably make it still more difficult to form a neutron star as a result of a supernova explosion involving this core collapse.

R. Ruffini: I would like to say how appealing appears to me the program of research here outlined by Professor Ne'eman. It reminds very closely the approach presented in the different context of high energy two body collisions by E. Fermi as far as 1952. Of course the physical situation is here largely complicated by the presence of many body interactions and the fact that the system is in a degenerate state. I do believe however that the enthusiastic words used by Fermi in presenting this approach can still be applied to our present problem and in particular inquire on the possibility of " stiffening " the Hagedorn equation of state toward $P = \rho/3$.

Concerning the remark of Professor Baym I would like here to say that every one of us clearly realizes that the " action " in neutron star physics occurs in the limited range of densities.

$$5 \times 10^{14}\ \mathrm{g/cm^3} \leq \rho \leq 5 \times 10^{15}\ \mathrm{g/cm^3}.$$

The treatment of this region is largely unknown and only upper limits are possible with reliability (see e.g. C. Rhaades and R. Ruffini submitted to Phys. Rev. Lett.). The reason of emphasizing the asymptotic behaviour is simply given: only three asymptotic behaviours exist there or at least have been proposed. The rate of approaching this asymptotic behaviour starting from $\rho = 5 \times 10^{14}\ \mathrm{gr/cm^3}$ appears to me to be then the key issue.

V. R. Pandharipande: The situations in cold rare gas matter (atomic helium liquid for example) and nuclear or dense matter could be very different because the first excited state of Helium atom is $\sim 10^5\ {}^{\circ}\mathrm{K}$ while the liquid helium involves typical energies of $\sim 10\ {}^{\circ}\mathrm{K}$. Hence the internal degrees of freedom of the atom could be adequately absorbed in a potential between two atoms. However in nuclear or dense matter the first nucleon resonance is only at 300 MeV excitation, where as the energies of the order of 100 MeV are involved in the matter. Hence the resonance could play a more direct role in determining the properties of matter.

L. Rosenfeld: There is some misunderstanding about what I meant by the comparison with an ordinary crystal. I just wanted to say that if one starts considering other states of nuclear particles than the nucleons, there is no reason to restrict oneself to resonances.

D. Pines: It seems to me that our discussion has perhaps not been as helpful as it might have been to the observers. Let me try to summarize the situation with respect to the maximum allowable mass for a neutron star. Both Dr. Canuto and Dr. Pandharipande are in agreement that whether or not a neutron (or hyperonic) solid is formed at densities $\sim 10^{15}$ gm/cc, the effects of particle interaction are such that one will not get stable neutron stars of mass $> 1.7\ M_\odot$. I believe this conclusion is shared by everyone who has gotten his hands dirty with high density neutron star matter calculations. Thus to the extent that an observer can conclude that the mass of a compact star is definitely $\gtrsim 2\ M_\odot$ he can conclude with some confidence that it is a black hole.

There is, of course, the possibility that one cannot, in fact, make neutron stars anywhere near the maximum allowable mass. For example, some of the discussion remarks at this meeting lead one to expect that it may not prove easy to make neutron stars of mass $\gtrsim 0.5\ M_\odot$. I wonder if Professor Cameron would care to comment on this possibility.

A. G. W. Cameron: In fact it seems unlikely to me that a neutron star can be formed with a mass less than about 0.5 or 0.6 solar masses. The reason for this lies in the diagram of the mass of an object in hydrostatic (but not necessarily stable) equilibrium as a function of central density, going over the complete range of white dwarf masses and neutron star masses. After one passes the peak of the white dwarf maximum stable mass, the mass of the object in hydrostatic equilibrium decreases significantly, but not below about 0.6 solar masses, until almost the central density at which neutron stars can be formed in the stable range. Then there is a rapid drop to of the order of 0.1 solar masses, followed by the rise into the curve of stable neutron star masses. Therefore a collapsing core would have to have a mass of at least the 0.6 solar masses until central densities of the order of 10^{12} g/cm^3 were reached, whereupon large amounts of mass would have to be ejected if a significantly smaller stellar remnant were to be left. This seems dynamically somewhat implausible. On the other hand, if the preceding stellar evolution leading to the core collapse has in most cases formed a degenerate core which has a mass very close to the upper limit of the stable white dwarf star, then it could well be that the upper limit on the mass of the stable white dwarf star also becomes the practical upper limit in nature of the mass of any neutron star that is likely to be formed, rather than the maximum mass which most of the current equations in state would allow for the neutron star. Thus, the maximum realizable mass of a neutron star may be closer to 1.3 or 1.4 solar masses rather than the limit of 1.7 solar masses mentioned by Professor Pines.

R. Hofstadter: I want to ask Professor Cameron whether a possible braking mechanism for the friction he invoked could possibly be due to radiative phenomena in the interior of a neutron star. This could either be due to synchrotron radiation in the presence of a large magnetic field or bremsstrahlung.

A. G. W. Cameron: Whatever radiation exists in the interior of a neutron star will not travel very far from one part of the interior to another. While it is true that the effects of degeneracy and the effects of a magnetic field greatly reduce the sources of opacity, they are not eliminated, and the very high density of the neutron star thereby assures that photons will not travel very far in the interior before being absorbed. However, the absorption of the photons will inevitably involve phenomena in the charged particle components of the stellar interior, and whatever internal magnetic field exists is bound to require a co-rotation of all of the different charged particle components in the interior. Thus the presence of radiation is unlikely to influence the friction between the neutron superfluid and the charged particle components of the star.

R. Hofstadter: If the states of motion in the magnetic field are macroscopic quantum states then I would expect no radiation. If not then how can radiative phenomena not be included?

G. Baym: I just wish to point out that the major source of friction between the charged components—the electron fluid and proton superfluid—and the neutron superfluid, is believed to be the magnetic moment interaction between the electrons and the neutrons in the vortex cores of the rotating neutron superfluid. In assessing whether radiation can provide significant coupling one must bear in mind that the electron and proton fluids are co-rotating to a high degree of accuracy, else the magnetic field would be inordinately large. The plasma frequency of the degenerate electron fluid in the interior is $\sim$ 5-10 MeV, so that there would be no photons radiated below this frequency.

THE BLACK HOLE*†

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§ 1. Three levels, three handles, three signals

It is difficult to summarize black hole physics more briefly than in " three threes ": the three levels of gravitational collapse; the three handles required to specify the properties of a black hole; and the three signals from a black hole.

The three levels of gravitational collapse begin with the universe, and include not only the big bang at the start, but also the recontraction to ultimately singular conditions predicted by Einstein's 1915 and now standard general relativity at the end of time for any closed model universe that is topologically equivalent to a 3-sphere. The second level is the collapse of a single star to a black hole of stellar mass; and the collapse of a collection of stars and matter to a black hole of multistellar mass, as expected in sufficiently compact galactic nuclei. The third level is the quantum fluctuations taking place all the time and everywhere throughout space, in effect describing gravitational collapse all the time taking place and being undone, taking place and being undone, over and over, at the Planck scale of distances,

$$L^* = (\hbar G/c^3)^{1/2} = 1.6 \times 10^{-33} \text{ cm} \tag{1}$$

Three handles, apart from location and velocity, determine the properties of an isolated black hole: its mass, its electric charge, and its angular momentum: M, Q and J. A black hole has no other *classical* parameter, no other disposable characteristic, no " hair ", according to all available considerations of principle. It is far more massive than an elementary particle; it is not susceptible to formation in a single quantum state (see § 6 on the entropy of a black hole); but otherwise it rivals an elementary particle in its perfection (Figure 1).

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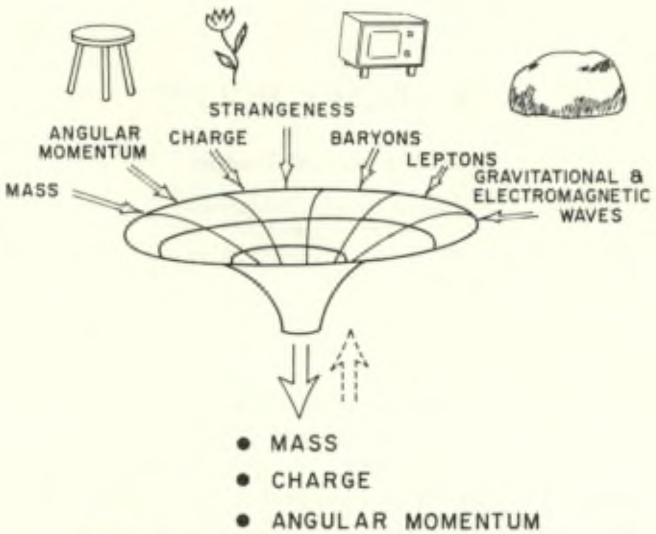

Fig. 1. The final black hole, regardless of what went in to make it, is parameterized by mass M, charge Q, and angular momentum J, and by nothing more, it is believed, so long as attention is limited to its macroscopic classical properties; *i.e.*, neglecting the Bekenstein-Hawking radiance (§ 6e). Two black holes of the most different provenance, one made for example from matter, the other from a mixture of antimatter and radiation, are indistinguishable by any *classical* measurement, according to all available arguments of principle, if only they have the same M, Q, and J.

In collisions and other macroscopic processes the so-called " irreducible mass " M_{ir} of the black hole (Christodoulou 1970) and the proper surface S spanned by its " horizon " (Hawking 1971, 1972)

$$S = 16 \pi M_{ir}^2, \tag{2}$$

may increase but can never decrease. The total mass, given by the formula

$$M^2 = \left(M_{ir} + \frac{Q^2}{4M_{ir}}\right)^2 + \frac{J^2}{4M_{ir}^2} \tag{3}$$

(Christodoulou 1970; Christodoulou and Ruffini 1971) may increase *or* decrease, according to the changes that are made in the three " handles ", M_{ir}, Q, and J.

In these formulas and in most formulas of black-hole physics all quantities are expressed in geometric units; thus,

$$M(\text{cm}) = (G/c^2)\, M_{\text{conv}}\,(\text{g}),$$

with

$$G/c^2 = 0.742 \text{ cm/g or } 1.47 \text{ km}/M_\odot;$$

similarly,

$$Q(\text{cm}) = (G^{1/2}/c^2)\, Q_{\text{conv}}\,(\text{e.s.u.})$$

and

$$J(\text{cm}^2) = (G/c^3)\, J_{\text{conv}}\, (\text{g cm}^2/\text{s}).$$

Three signals come from a black hole: a pulse of gravitational radiation at the moment of formation or the moment of mass augmentation; x-rays from gas accreting onto the black hole, heated to $\sim 10^{10}$ to 10^{11} deg K by gravitational compression before crossing the horizon into the black hole; and " activity ": energy imparted by a " live " or rotating black hole to matter or radiation that enters into a certain critical region, the " ergosphere ", that surrounds the horizon.

§ 2. Collapse to a black hole

No model provides a simpler picture of collapse than a uniform spherically symmetric cloud of dust, starting from rest, and falling inward under the mutual gravitational attraction of its parts. No model has been analyzed by a greater variety of mathematical methods [for references see for example Harrison *et al.* (1965) and Misner, Thorne and Wheeler (1973), (cited hereafter as MTW)]. None shows more quickly and more clearly three important features of collapse:

1) No elasticity that ultimately develops, no " equation of state ", will prevent the collapse to within the Schwarzschild radius (Table I) of a sufficiently dilute but sufficiently extended cloud of dust.

TABLE I

Mass of a cloud of dust sufficient to guarantee formation of a horizon and collapse to a black hole, free of any question about the equation of state of the matter composing the cloud. Relevant formulas [Harrison *et al.* (1965)]: $R = 2\,M$; $M = (4\pi/3)\rho\, R^3$, despite curvature of space.

Example	I	II	III
A momentarily static and spherically symmetric cloud of dust whose density is so low as..................	10^{-2} g/cm^3	10^{-12} g/cm^3	10^{-22} g/cm^3
has already surrounded itself with a Schwarzschild horizon if its total mass is..	2.70×10^{42} g	2.70×10^{47} g	2.70×10^{52} g
or (units of sun)	$1.36 \times 10^{9}\ M_\odot$	$1.36 \times 10^{14} M_\odot$	$1.36 \times 10^{19} M_\odot$
or geometric units).......	2.0×10^{14} cm	2.0×10^{19} cm	2.0×10^{24} cm
The Schwarzschild coordinate radius R of the cloud (proper circumference/2π) is $R = 2M$ or...........	4.01×10^{14} cm	4.01×10^{19} cm	4.01×10^{24} cm
or (light years)	4.24×10^{-4} lyr	42.4 lyr	4.24×10^{6} lyr

2) The observer will receive one or other of two dramatically different records of the collapse according as he stays safely outside or follows the cloud of matter inward in its collapse. In the one case the outer boundary of the cloud is at first seen to move in more and more rapidly, as in the Newtonian theory of free fall; but then its motion is recorded as slowing down. It approaches but never crosses the Schwarzschild value of the r - coordinate, $r = 2M$, and coasts down to that asymptotic limit according to the formula

$$r = 2M + \text{constant} \times e^{-t/2M} \tag{4}$$

The deviation decreases exponentially with a characteristic time that is given by the expression

$$T_{conv} = T/c = 0.98 \times 10^{-5} \text{ sec} \times (M/M_{\odot}). \tag{5}$$

There is no question of the collapse " turning around ", when even in an infinite stretch of the time coordinate t relevant to the far away observer the collapse has got no further in than $r = 2M$.

In contrast, when the observer falls freely with the outer boundary of the cloud, and he records what happens in terms of his own proper time, he finds that the collapse follows the Newtonian formula, as if τ were Newtonian time. There is no slowing down or any other anomaly in the motion at $r = 2M$. The collapse is complete ($r = 0$) in a perfectly finite amount of time.

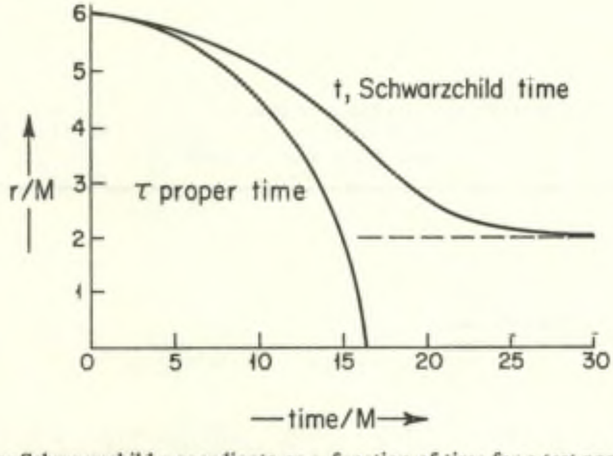

Fig. 2. The Schwarzschild r-coordinate as a function of time for a test particle falling radially towards a preexisting non-rotating black hole or, equally well, for the outer boundary of a uniform cloud of dust undergoing gravitational collapse. One curve gives r as a function of the time coordinate t appropriate to a far away observer. To him the collapse appears to get only as far as the Scharzschild radius even after infinite time, because of the difficulty or impossibility for signals to get out to tell him of the further stages of collapse. The other curve gives r as a function of the proper time τ recorded by an observer falling in alongside the test particle or alongside the boundary of the dust cloud. Diagram adapted from Ruffini and Wheeler (1971).

The two very different records for r as a function of time are compared and contrasted in Figure 2. The origin of the difference is clear. It arises primarily from the action of the gravitational field in delaying signals on their way to the far away observer or in its altogether preventing them from reaching him. He gets no information about the physics inside $r = 2M$, but his colleague, the infalling observer, does get such data.

The line of demarcation between the information accessible to the two observers is symbolized by a photon or light ray travelling radially outward at $r = 2M$. The pull of gravitation is strong enough to keep it from making any progress in its travel. A photon just a litte further out *is* able to make progress, at first slowly, then more and more rapidly. A photon just a little further in, though aimed out, is drawn in because overpowered by gravity, at first slowly, then more and more rapidly. There is nothing anomalous in the local tide-producing accelerations near $r = 2M$ nor in any other local feature of the geometry in that region. Only the global behavior of light rays reveals and defines the " horizon " at $r = 2M$. For more on the definition of horizon through the global properties of null geodesics, including situations deprived of spherical symmetry, as is the case for a rotating black hole, reference is made to Penrose (1968), also to the appropriate chapters in Hawking and Ellis (1973) or MTW (1973). Figure 3 shows how the horizon develops around a cloud of dust undergoing collapse.

The mass collapses first at its center. The mass collapses first farther out. Both apparently contradictory statements are true. Each shows on the appropriately programmed computer printout, not only for the idealized cloud of dust, but also (May and White, 1966, 1967) for matter endowed with a reasonable equation of state. The reason is simple. One has to distinguish between spacetime (the complete and invariant picture of space geometry evolving with time) and the " slices " through this spacetime (the sequence of configurations taken on by the 3-dimensional geometry of space, and by the physics in this 3-space). One slice is defined by " a particular reading on every wristwatch of an everywhere dense set of observers ". A later slice of the same family of slices is defined by a later reading of all these watches. But another army of observers, moving with respect to the first set, and equipped with quite other watches, define quite another " family of simultaneities ". The important concept is not the army of observers or the phalanx of watches but the individual " spacelike hypersurface " or " simultaneity " or slice through spacetime. It represents an instant of " many-fingered time ". One choice of spacelike hypersurface is on a footing as good as any other slice. Those who analyze the dynamics have full freedom to " push ahead the hypersurface in time "

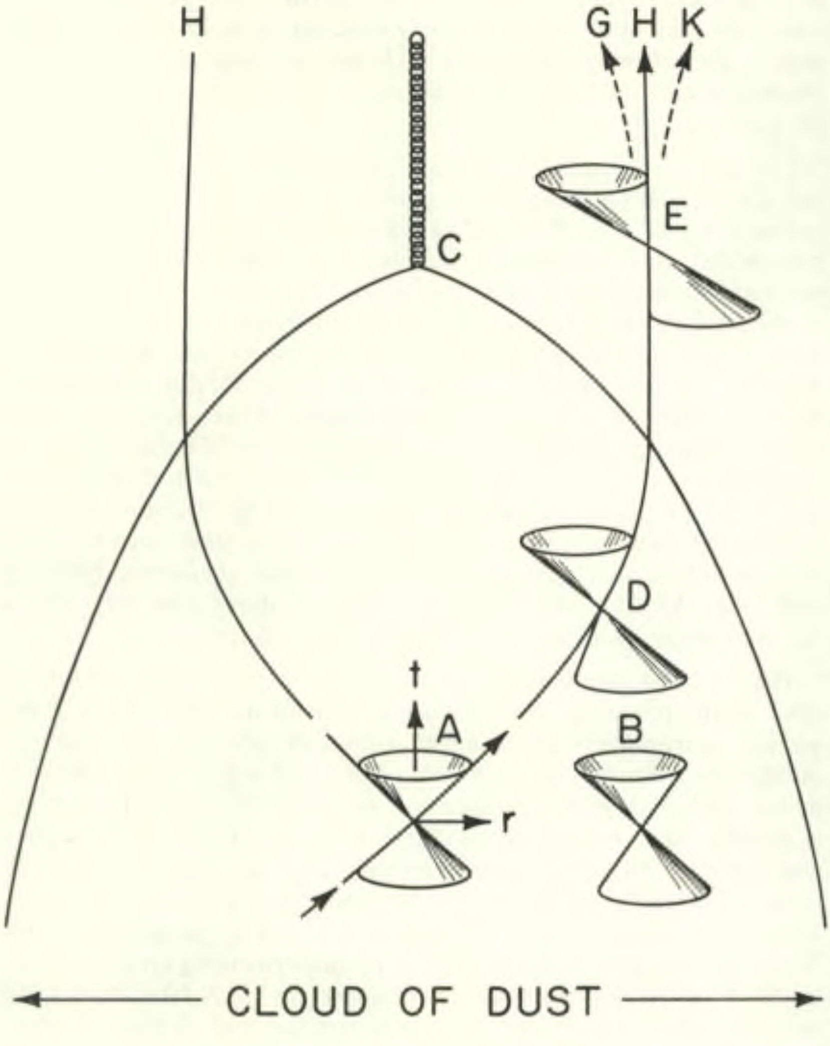

Fig. 3. Development of horizon H during collapse of cloud of dust. A cloud always at the large volume and low dilution indicated at the bottom of the diagram would let every photon escape (that did not strike a dust particle), as evident from the two lower light cones, A, B. However, the cloud falls in to complete collapse, as at C. The mass previously outside B falls to smaller r values. Light cone E therefore has a stronger inward tilt than light cone B. The null geodesic through A, D, E plus similar geodesics generates the horizon H. Of two only very slightly different null geodesics G and K, G falls to the singularity at $r = 0$, but K escapes.

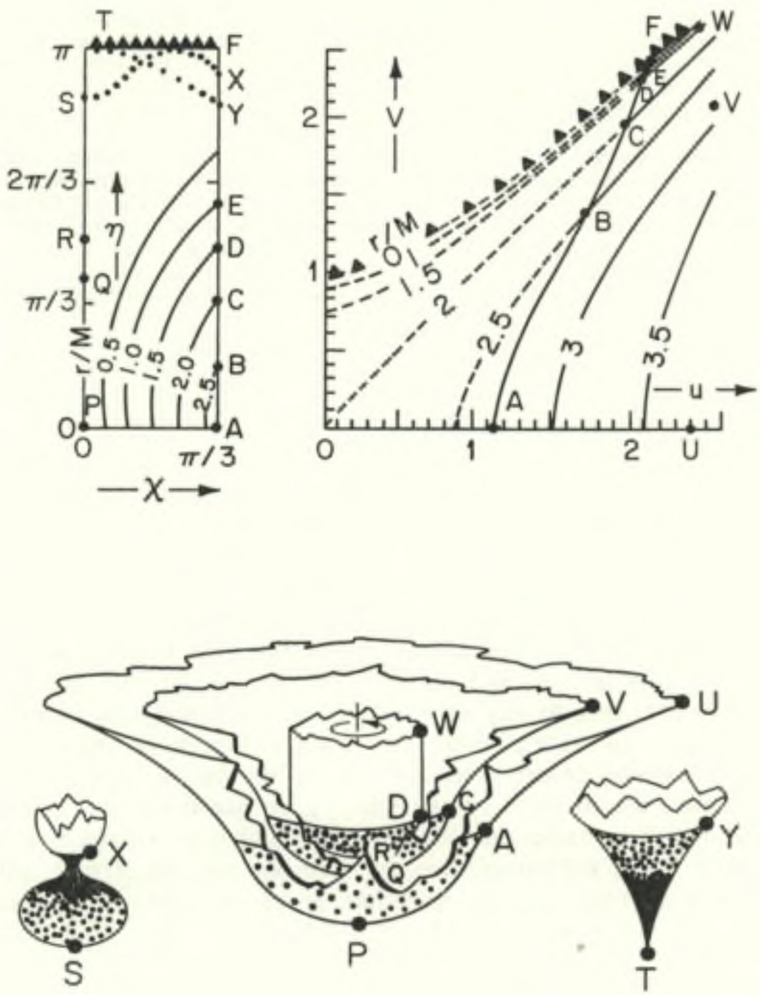

Fig. 4. Dynamics of collapse summarized in a single picture. Above: The spacetime associated with a collapsing cloud of dust. Below: Sample geometric configurations taken on by space in the course of this collapse. Example: The slice PAU through the spacetime above shows below as the 3-geometry PAU. This, like every other illustrative slice, actually endowed with three space dimensions and the symmetry of the rotation group in three dimensions, is depicted for simplicity as if a 2-geometry, with only one axis of rotational symmetry. The dotted region of each 3-geometry shows where the dust cloud is located and, qualitatively, its density. Complete gravitational collapse has first occurred at the center of the cloud on slice TY; first at some distance from the center of the cloud, on slice SX. Details: The spacetime could have been represented in a single diagram but is instead here to be sewed together in the mind's eye out of the relevant parts (above) of standard diagrams for Friedmann and Schwarzschild 4-geometries separately. The interface of smooth join between the two spacetimes is defined by the world line ABCDEF of a freely falling test particle at the periphery of the cloud of dust. Thus points A in the two top diagrams are to be identi-

fied; similarly points B, ..., and points F. A complete Friedmann geometry would extend from hyperspherical angle $\chi = 0$ (N-pole) to $\chi = \pi$ (S-pole); but the cloud of dust in the example reaches only from $\chi = 0^0$ to $\chi = 60^0$. The Schwarzschild r-coordinate (= proper circumference of a circle of equivalent particles/2π) of a particle in this cloud of dust is given as a function of the time parameter η by the formula $r = (a_0/2) \sin \chi (1 + \cos \eta)$. The Friedmann time itself is $t = (a_0/2) (\eta + \sin \eta)$. Here $a_0 = (16/3^{1/2}) M$ is the radius of the osculating Friedmann universe at the moment that the collapse starts. Light rays run at 45^0 slopes both in the Friedmann (χ, η) — diagram and in the Schwarzschild (u, v) — diagram (" Kruskal-Szekeres coordinates "). Light rays reach a far away observer from A or B; but not until after an infinite time, from C,; and never from D or E. Outward directed light rays from D and E are caught in the gravitational collapse of the geometry outside the dust cloud. The blacked-in zig zag marks complete gravitational collapse of the geometry whether inside or outside the cloud of dust.

faster here and slower there, or faster there and slower here. That is the meaning of many-fingered time. That is why in one scheme of calculation the cloud of dust first undergoes collapse at its center, and at some distance from the center in another (Figure 4).

Once a test particle or an observer has crossed the horizon, his subsequent history is short and ill-fated. No one in daily life can stop the advance of time by any effort of the will, no matter how firmly he plants his feet upon the ground. Inside the horizon decreasing r-values signify increasing proper time, for r there has the quality of a time coordinate. By no effort of the will, by no assistance of the most powerful rocket engine, can one maintain himself at a fixed r-value. As surely as the unforgiving minutes drag one forward in time, with that same relentlessness the spacetime geometry inside the horizon drags one down from $r = 1.5M$ to $r = M$, and from $r = M$ down to ever smaller r-values, ever larger tidal forces, ever increasing compaction, and final collapse.

All this is the consequence of crossing the horizon. Escape is not possible from this crossing for a star that is too compact and massive. Escape is possible for the single observer who does not venture too close. It is still possible even if he does come close to the horizon, provided that he has sufficient rocket propulsion at his command. But there is nothing anomalous in the physics that he encounters to give warning of that point of no return, or of having passed it, neither any anomaly in the local structure of spacetime nor any discontinuity in the ever rising strength of the local tide-producing acceleration.

The black hole presents the challenging choice, come in and see and suffer the final stages of collapse; or stay out, be safe, and not see them.

§ 3. Collapse of the Universe

The closed* universe of Einstein's standard geometric theory of gravity does not offer the choice between collapse and escape. For it a singularity is as inescapable at the end of time as at the beginning [Tolman (1934); Avez (1960); Geroch (1967); Hawking and Ellis (1968); Hawking and Penrose (1969); Hawking and Ellis (1973)]. There is no outside to which to escape. Collapse makes no distinction between one atom and another in the universal fate it imposes on all. In this respect collapse at the level of the universe contrasts with collapse at the level of the black hole.

In another respect the black hole of the here and now provides a kind of " experimental model " for the gravitational collapse predicted for the universe itself at a later era. A look at Figure 4 for the spacetime associated with a collapsing cloud of dust, and at spacelike slice SX through that spacetime, reveals a region centered on S that is hardly to be distinguished from a small closed universe undergoing collapse. At the point in that closed universe most remote from S there is a black hole. In the time subsequenty remaining to that small universe, neither do signals come into it through this collapsed region from the outside, nor can this inner universe act through the collapsed connection on that outer world. In all that concerns the little universe, the outside asymptotically flat space plays no part and effectively does not exist.

To make an advance test of the predicted collapse of our large universe, no observer is likely to go so far as to immolate himself in the little universe interior to a collapsing cloud of matter. Even if he did, his sacrifice would never be seen by, and his findings would never reach, the observers enjoying the comparative safety of the space far outside the horizon. Those observers would be able to check part of black-hole physics, and an important part, but not this most striking part.

* " Closure " is no less a part of what Einstein meant by general relativity for his having stated it in a word rather than in an equation. In this connection see his 25 June 1913 letter to Ernst Mach, reproduced photographically on pp. 544-545 of MTW (1973); also Einstein (1934, p. 52) " In my opinion the general theory of relativity can only solve this problem [of inertia] satisfactorily if it regards the world as spatially self-enclosed "; also Einstein (1950, pp. 107-108), " Thus we may present the following arguments against the conception of a space-infinite, and for the conception of a space-bounded, universe: (1) From the standpoint of the theory of relatibity, the condition for a closed surface is very much simpler than the corresponding boundary condition at infinity of the quasi-Euclidean structure of the universe. (2) The idea that Mach expressed, that inertia depends upon the mutual action of bodies, is contained, to a first approximation, in the equations of the theory of relativity;.... But this idea of Mach's corresponds only to a finite universe, bounded in space, and not to a quasi-Euclidean, infinite universe ". For the important rôle of closure in giving a unique solution to the initial-value equations of relativity (the modern formulation of Mach's principle), see for example MTW (1973), especially pp. 536, 543, 544, 549, 704, 1181.

A black hole is not a universe, and a universe is not a black hole. This is all the more reason why our look at how a small universe is contained in a black hole is appropriately followed by a look at how a black hole is contained in a large universe. Figure 5 shows how the region of complete collapse cocooned by the horizon of the black hole is really a part of completely collapsed state of the universe itself.

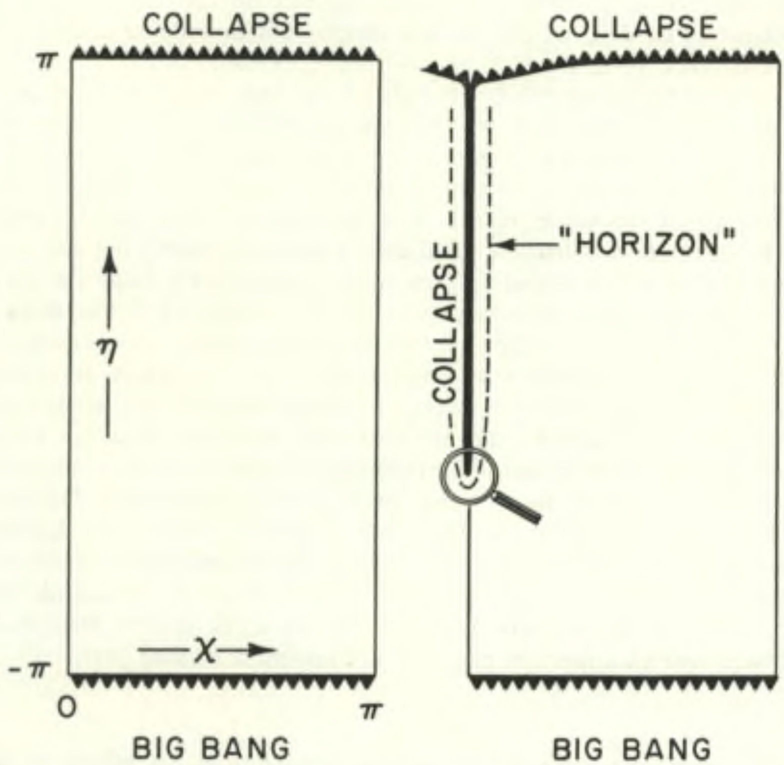

Fig. 5. Black hole compared to an icicle hanging from the roof of a cave. Left, dynamics of ideal Friedmann universe depicted in terms of the arc-effective time parameter η ($d\eta$ = dt/momentary radius of 3-sphere) and the hyperspherical angle χ ($\chi = 0$ at N-pole, $\chi = \pi$ at S-pole of 3-sphere; light rays of interest run at $\pm 45°$ in the diagram). Right, a nearly Friedmann universe in which gradual accretion of matter, or the collapse of a star, has led to the formation (see Figure 3 for magnified diagram) of a black hole at the N-pole. The region of complete collapse interior to the horizon is integral with and in principle indistinguishable from the domain of complete collapse of the universe itself. (Diagram inspired by oral report given by Roger Penrose at the meeting of working commission No. 64 of the International Astronomical Union on *Gravitational Radiation and Gravitational Collapse* at Warsaw, Poland on the second day of the 5-7 September 1973 sessions, scheduled for publication by Reidel, Dordrecht, Holland). The horizon loses definition just before collapse of the universe itself because there is *no* light ray that can escape the singularity.

Because collapse of the universe and collapse to a black hole are so closely related, it throws light on black hole physics to see the evidence on the universe. The predictions of Einstein's theory about the universe have faced three cycles of doubt and test. The first cycle began with Einstein's doubt that the predicted expansion was right at all, his introduction of a so-called " cosmological constant " explicity to obviate this dynamics, Hubble's 1929 observation that expansion is actually going on, and Einstein's conclusion that the cosmological term was " the biggest blunder of my life ". The second cycle began with apparent evidence that the expansion has been speeding up, contrary to what gravitation and Einstein's theory of gravitation lead one to expect; ran through an era of proposed " steady state expansion ", coupled with " continuous creation of matter " ; and ended when drastic corrections to the astrophysical scale of distances showed that the expansion has actually been slowing down, as expected. Nothing shows this slowing down more cleary or more directly than the two basic numbers themselves (first two entries in Table II): (1) the actual time, $\sim 10 \times 10^9$ years, back to the start of the expansion, as deduced from the astrophysics of star evolution and element formation; and (2) the " Hubble time " or time linearly extrapolated back to the start of the expansion, $\sim 20 \times 10^9$ years; that is, the time that would have been taken by galaxies to get to their present distances if they had always been receding with their present slowed down recession velocities.

TABLE II

Major features of the universe according to Einstein's theory, as deduced from the two key astrophysical data listed in the first two lines, and in the text, and each believed uncertain by an amount of the order of 20 percent. The numbers in subsequent rows [taken from MTW (1973)] like those in the first two rows, for the sake of consistency and illustrative value, are given with more significant figures than would otherwise be justifiable.

Time from start to now	10×10^9 yr
Hubble time now	20×10^9 yr
Hubble expansion rate now	$49.0 \dfrac{\text{km/sec}}{\text{Megabarsec}}$
Rate of increase of radius now	0.66 lyr/yr
Radius now	13.19×10^9 lyr
Radius at maximum	18.94×10^9 lyr
Time, start to end	59.52×10^9 yr
Amount of matter	5.68×10^{56} g
Density now	14.8×10^{-30} g/cm^3

The third cycle of doubt and test got into full swing when Oort (1958) [see also the analysis of this work by Peebles (1971)] found that the effective averaged-out density of matter present in the form of galaxies is only of the order of 1/30 of that required to curve the universe up into closure (last line of Table II). The search for the " missing matter " that then began is a saga in itself. The year 1973 has seen what appears to be an important forward step. Ostriker and Peebles (1973) conclude that the typical galaxy contains 3 to 20 times as much matter as previously recognized, most of it in the form of dark matter, outside the distance where the galaxy had previously been thought to end.

What general relativity has to say about the large scale dynamics of the universe, including big bang, expansion, and slowing down of expansion, on the way towards the predicted recontraction and collapse, has thus provided and continues to provide unique guidance to astrophysics. This is all the more reason to take seriously the predictions of Einstein's geometrodynamics about black hole physics in particular and gravitational collapse in general.

§ 4. Quantum Fluctuations in Geometry and Connectivity

Of all the remarkable developments in physics since World War II, none is more impressive than the prediction and verification of the effects of the quantum fluctuations of the electromagnetic field on the motion of the electron in the hydrogen atom. That achievment makes it impossible to overlook similar fluctuations taking place all the time and everywhere in the geometry of spacetime itself. They give one reason to consider seriously the picture of something close to gravitational collapse taking place and being undone, taking place and being undone, at the Planck scale of distances, throughout all space.

For the electromagnetic field in its ground state the probability amplitude for any divergence-free configuration $B\,(\mathrm{x, y, z})$ of the magnetic field is given by the Gaussian formula

$$\psi\,(B\,(\mathrm{x, y, z})) = \mathrm{N}\exp\left(-\iint \frac{B\,(\mathrm{x}_1).B\,(\mathrm{x}_2)}{16\,\pi^3\hbar \mathrm{c}\, \mathrm{r}_{12}{}^2}\, \mathrm{d}^3\mathrm{x}_1\, \mathrm{d}^3\mathrm{x}_2\right), \quad (6)$$

where N is a normalization factor. From this formula or otherwise one sees that, in a region of extension L, values of the magnetic field up to

$$\Delta \mathrm{B} \sim (\hbar \mathrm{c})^{1/2}/\mathrm{L}^2 \quad (7)$$

occur with practically the same probability as zero magnetic field.

Similarly [see for example MTW (1973), chap. 43], in a region of dimension L, where in a local Lorentz frame the normal values of the metric cœfficients will be — 1, 1, 1, 1, there will occur fluctuations in these cœfficients of the order

$$\Delta \mathrm{g} \sim \mathrm{L}^*/\mathrm{L} \quad (8)$$

where L^* is the Planck length of equation (1). These fluctuations are negligible in magnitude and effect at atomic, nuclear, and elementary particle dimensions. However, the predicted magnitude becomes larger at still smaller distances of examination, as the sea appears rougher to the aviator who approaches closer to its surface. Ultimately, as for the aviator who finds himself among the waves themselves, the calculated fluctuations become so large ($L \sim L^*$; $\delta g \sim 1$) that one finds it reasonable to think of the possibility that the connectivity of the geometry itself will change (" handles " on, or " wormholes " in, the space geometry). Electric lines of force of fluctuation origin will thread through the typical wormhole, carrying a flux of order

$$\int E.\mathrm{d}S \sim [(\hbar c)^{1/2}/L^2].L^2 \sim (\hbar c)^{1/2}. \tag{9}$$

Space on this picture has everywhere at small distances a kind of fluctuating foam-like structure, with everywhere positive and negative electric charges of order

$$q \sim (\hbar c)^{1/2} \sim 10e \tag{10}$$

continually being created and annihilated. These charges are not a property of elementary particles. The relevant scale of distances is 20 orders of magnitude smaller than elementary particle dimensions. The charges are not quantized in magnitude. The charges occur everywhere, not only where there is a particle. The charges are exclusively electric in character. The primordial quantity in the quantum-electrodynamic analysis is the potential A, and the equation

$$B = \nabla \times A \tag{11}$$

automatically excludes the trapping of magnetic lines of force in any wormhole.

Aside from the unsatisfactory picture of charge as a mystic magic electric jelly, or the equally unsatisfactory picture of charge as associated with a place where Maxwell's equations break down, one has never had any picture of electricity other than Hermann Weyl's concept of electric lines of force trapped in the topology of space. Nothing gives one more reason to take seriously the prediction of something like gravitational collapse going on all the time and everywhere at small distances, and continually being undone, than the existence of electric charge in nature.

§ 5. The Crisis Created by Collapse

Predicting ahead from step to step and from moment to moment the deterministic evolution of space and space curvature, classical geometrodynamics leads inevitably for the black hole and the closed universe to a singularity and an impasse. Einstein's equation says " this is the end " but true physics, as true physics has always been conceived, says " there is no end ". In this contradiction physics faces a crisis, a crisis even deeper going (see Table III) than the upset that brought quantum mechanics. At issue is the fate, not of matter alone, but of the universe itself. The stakes in the crisis of collapse are hard to match: the dynamics of the largest object, space, and the smallest object, an elementary particle, and how both began. Black hole physics offers a reasonable domain to study some of the pertinent issues without requiring one to confront all of the associated problems at once.

TABLE III

Black hole collapse, and collapse of the universe, as predicted by classical geometrodynamics, compared and contrasted with classically predicted collapse of the atom.

System	Atom (1911)	Universe (1970's)
Dynamic entity	System of electrons	Geometry of space
Nature of classically predicted collapse	Electron headed toward point-center of attraction is driven in a finite time to infinite energy	Not only matter but space itself arrives in a finite proper time at a condition of infinite compaction
One rejected "way out"	Give up Coulomb law of force	Give up Einstein's field equation
Another proposal for a "cheap way out" that has to be rejected	"Accelerated charge need not radiate"	"Matter cannot be compressed beyond a certain density by any pressure, however high"
How this proposal violates principle of causality	Coulomb field of point-charge cannot readjust itself with infinite speed out to indefinitely great distances to sudden changes in velocity of charge	Speed of sound cannot exceed speed of light: pressure cannot exceed density of mass-energy.
A major new consideration introduced by recognizing quantum principle as overarching organizing principle of physics	Uncertainty principle: binding too close to center of attraction makes zero-point kinetic energy outbalance potential energy; consequent existence of a lowest quantum state; can't radiate because no lower state available to drop to	"Participator" replaces the "observer" of classical physics. It is impossible in principle to separate what happens to any system, even the universe, from what this participator does. This principle of Bohr's of the "wholeness" of nature may be expected to come to the fore in a new and far deeper form.

§ 6. The Three Handles

How a collapsing object, with all its specialities, goes into a standard black hole, characterized at the classical level so far as one can see by M, Q and J, and nothing more, has been studied in considerable detail, especially as regards

a) uniqueness of the final configuration,

b) rate of approach to that configuration,

c) transcendence of baryon number and lepton number in black-hole physics,

d) motion of a test particle or a photon, or propagation of a wave, in the field of force of a black hole, and interchange of energy with the black hole,

e) surface area, irreducible mass, and the basic laws of black-hole physics,

f) the temperature and entropy and emissivity of a black hole, and

g) black holes and the direction of time. Some of the main considerations or key papers in each domain will be mentioned here, with no attempt at comprehensive coverage. For more detailed accounts, see De Witt and De Witt (1973), Hawking and Ellis (1973), and MTW (1973).

a. *Uniqueness*

A stationary black hole (one that has settled down into its final state) must have a horizon with spherical topology; and it must be either static (zero angular momentum) or axially symmetric or both, according to Hawking (1971, 1972). Combining this result with results of Israel (1967, 1968) and Carter (1970) or Wald (1971, 1972), one concludes that: All stationary black holes are axially symmetric. All static (nonrotating) black holes are endowed with the Reissner-Nordstrøm generalization of Schwarzschild geometry, uniquely specified by mass M and charge Q. All uncharged, rotating black holes fall into a single family, or into distinct and disjoint families, with each black hole in a given family characterized uniquely by mass M and angular momentum J. Turning from what has been proved to what has been surmised, one believes that " a black hole has no hair "; that is, that there is no particularity or adjustable parameter left in a black hole other than M, Q and J, and these parameters uniquely fix the properties of the black hole (Kerr-Newman geometry) at the *classical* level.

The Kerr-Newman black hole has an infinitude of effective mass moments, quadrupole, octupole, ..., as well as a magnetic moment and higher electromagnetic moments; but all of these properties are determined uniquely by M, Q and J. For example, the magnetic moment has the " Dirac electron value ",

$$\mu = (Q/M)\,J. \tag{12}$$

b. *Rate of Approach to the Final Configuration*

How rapidly do asymmetries disappear in the final stages of collapse? The answer depends on the nature of the asymmetry. No situation is simpler to consider than an already existing Schwarzschild black hole, towards the N-pole of which one drops a small additional mass δM. Nothing done with this much mass could easily produce a greater asymmetry. How the resulting " hair " on the black hole fades away is directly revealed by Figure 2 and equation 4: exponential fadeaway, with a time constant of the order of 10^{-5} sec for a black hole of solar mass.

Price (1972 a, b) has given a general analysis good for small departures from symmetry of any multipole order giving formulas for the rate of fadeout of the " hair " both for scalar and gravitational perturbations, and for all integer-spin zero-rest-mass fields.

Hanni and Ruffini (1973 and earlier work) and Cohen and Wald (1971) analyze what happens to the lines of force of a point charge as it is slowly lowered towards a Schwarzschild black hole. The lines of force, originally diverging away from the charge, come more and more to appear to radiate out from the center of the black hole itself, and this regardless of what point on the horizon sphere $r = 2M$ the particle approaches. Figure 6 shows the calculated pattern of lines of force in the intermediate case $r = 3M$.

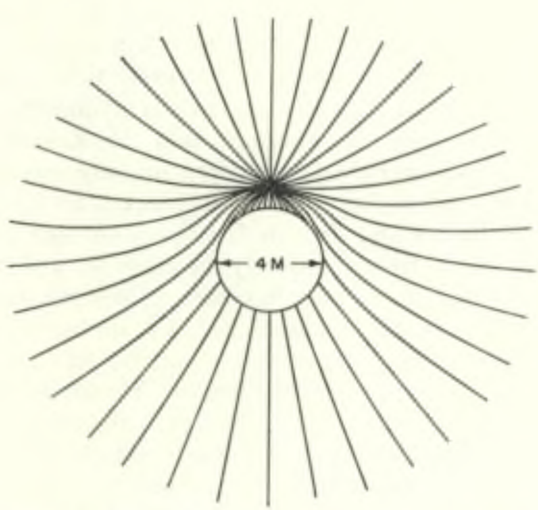

Fig. 6. Pattern of electric lines of force diverging from a point charge as modified by proximity ($r = 3$ M) to a Schwarzschild black hole [diagram from Hanni and Ruffini (1973)]. The closer the charge is to the horizon, the less the pattern reveals about where the charge is.

c. *Transcendence of baryon number and lepton number in black-hole physics*

If the lepton number " charge " carried by the collapsing matter were accompanied by a field, like the electric field, that obeys the Gauss flux theorem, there would be no more difficulty in defining the lepton number of a black hole than in defining its electric charge. However, not the slightest evidence of any such Coulomb-like field has ever been found. On the contrary, the theory of the weak interaction has led to the conclusion that between lepton and lepton there must exist a neutrino-intermediated $(1/r^n)$ coupling potential. In principle this interaction would seem to allow one to determine the lepton number of a compact object by the difference in the force exerted on a test lepton, and a test antilepton, in orbit around that object. However, all trace of this interaction is swallowed up, according to field-theoretic analyses of Hartle (1971, 1972) and Teitelboim (1972b, c), in the collapse of the compact object to a black hole. No measurement of the outside observer will be able to distinguish a black hole of given M, Q and J put together out of leptons from one of the same M, Q and J put together out of antileptons or one built largely out of pure radiation, according to these considerations.

The force between a baryon inside a collapsing object and a test baryon outside the horizon, as described by a meson-intermediated coupling, also fades out rapidly with time, according to Bekenstein (1972 a, b) and Teitelboim (1972 a, c).

It would go beyond the well-established to say that baryons and leptons are destroyed when matter falls into a black hole. One has rather to say that the baryon number and lepton number of a black hole are undefined and in principle undefinable by any *classical* scheme of experiment or principle whatsoever that anyone has ever been able to put forward. In this sense the long established laws of conservation of baryon number and lepton number are not violated; they are transcended (Wheeler, 1971 a).

The central idea of " transcendence " is seen most simply in the context of flat space. Let all the baryons in this idealized flat-space universe be located initially in some limited volume V, let their number be known, and let this number be redetermined from time to time. Suddenly a robber grabs N of these baryons and speeds away into space with them on a rocket that has a constant high acceleration a :

$$\begin{aligned} x &= a^{-1} (\cosh a\tau - 1) \\ t &= a^{-1} \sinh a\tau \\ x &\sim t - a^{-1} \text{ for large t.} \end{aligned} \tag{13}$$

No attempt at counting the baryons on a spacelike hypersurface that goes through the point of robbery more than a^{-1} time units after the robbery will ever yield evidence on the stolen baryons. No light ray then sent out to search for them will ever catch up with them. The robber has too much head start. Those baryons have not been destroyed, but they have been forever lost to count. That is the sense in which the law of conservation of baryon number has been transcended here.

So it is in the case of the black hole. A bit of faraway infinity has in effect been cut free and wrapped around the collapsing mass to make its horizon. A baryon speeding towards that horizon, that bit of " captured infinity ", is irretrievably beyond detection by a photon that starts in after it too late. It too is forever lost to count. That is why the laws of conservation of particle number are transcended in black hole physics.

That matter falling within " captured infinity " is compacted more and more rapidly to higher and higher densities, while baryons accelerated towards idealized faraway infinity are not, is an important distinction, which however has nothing directly to do with the central feature that makes for transcendence of particle number: the horizon.

The law of conservation of particle number is central in elementary particle physics, but is transcended in black hole physics. One therefore expects (Figure 7) a transition between these two extreme situations at some intermediate mass. For an electron moving in the field of force of a Schwarzschild black hole this transition is calculated to occur (Wheeler, 1971a) when the mass of the black hole is equal in order of magnitude to the mass m^*_e dual to the mass m_e of the electron:

$$\begin{aligned} M \sim m^*_e &\equiv (M^*)^2/m_e = (2.2 \times 10^{-5}\text{g})^2/0.91 \times 10^{-27}\text{g} \\ &= 5.2 \times 10^{17}\text{g}. \end{aligned} \tag{14}$$

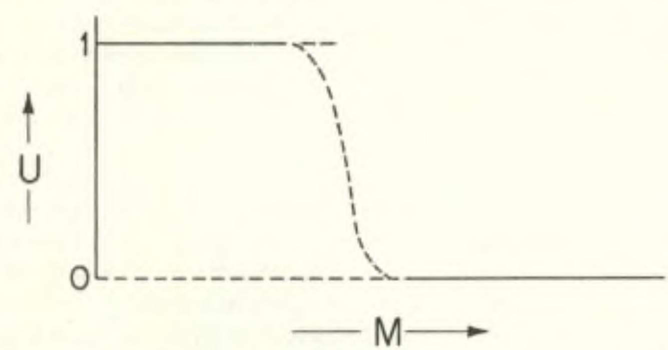

Fig. 7. " Usefulness ", U, of the law of conservation of leptons as dependent upon the mass, M, of the black hole in whose field of force the lepton moves. One hundred percent useful in the context of elementary particle physics ($U = 1$), this law is completely transcended ($U = 0$) when the electron impacts on a large black hole. U, defined for example by the probability of reflection of an electron dropped in from rest at infinity, changes rapidly from 0 to a value very close to 1 for a black hole mass of the order $m_{*e} = 5.2 \times 10^{17}$g.

This is the mass that pulls on an electron at several Schwarzschild radii or greater distances with the same force that would be exerted by a nucleus of charge

$$Ze = \hbar c/e \simeq 137e \tag{15}$$

on the charge of the electron. A black hole of greater mass is calculated to pull the electron in without inhibition. In contrast, a black hole of smaller mass has a Schwarzschild radius that is less than the Compton wavelength of the electron. The great difference in wavelength or the great " impedance mismatch " in this case between the de Broglie wave far away and near the horizon results in an almost complete reflection of the electron. The probability to go down the black hole goes exponentially fast to zero as the mass of the black hole is reduced below $\sim m_{*e}$, and particle number is soon conserved, according to the calculations, with extraordinary perfection, so long as one is entitled to treat the black hole as a classical object. Similar considerations apply to baryons and mu-mesons with appropriate changes in the dual mass of equation (14).

Actually to form a black hole of mass small compared to a solar mass is not only beyond the bounds of technology today, according to the analysis of Harrison *et al.* (1965), but also beyond the wildest dreams of anything one can hope to do in the future. Hawking (1971) and Carr and Hawking (1973) suggest however that primordial small black holes may have been formed by density fluctuations in the early universe.

In at least two ways the small black hole will depart from the idealized black hole. First, polarization of the vacuum by the strong curvature of space near a small black hole will cause the effective distribution of energy density and stress to take on non-zero values, and thereby alter the metric from its Schwarzschild value. Hawking (March 1974) argues that this effect will be slight except for a black hole as small in order of magnitude as the Planck mass,

$$M^* = (\hbar c/G)^{1/2} = 2.2 \times 10^{-5}\text{g}. \tag{16}$$

Second, the black hole will emit thermal radiation (section (f) below).

d) *Motion of Particle and Propagation of Wave in Black-Hole Geometry*

The three independent parameters, M, Q, J, of a black hole can be determined from the period and semi-major axis and difference of perihelion precession rates for a neutral test particle revolving in the same sense that the black hole spins, and one revolving in the opposite sense, and from the corresponding quantities for two test particles that bear some charge. Reaching far beyond this elementary motivation, investigators too numerous to cite here (see MTW, 1973, for a partial bibliography) have studied varied features of orbital motion in great detail. Among results obtained some of the most complicated illuminate non-equatorial orbits in the Kerr (M; Q = 0; J $\neq$ 0) geometry (Wilkins, 1972); and some of the simplest have to do with circular orbits in Schwarzschild geometry. Relativity effects not withstanding, such orbits fulfill exactly the Kepler 1-2-3 relation

$$M^1 = \omega^2 r^3 \tag{17}$$

provided that r is taken as the Schwarzschild coordinate (proper circumference/2π). There are stable circular orbits for all r values from $r = \infty$ down to r = 6M, and unstable circular orbits for all r values from $r = 6M$ down to $r = 3M$. In this last orbit the particle moves, or would move if it could attain this limit, with the speed of light. This is also the only circular orbit possible for a ray of light. The slightest departure from circularity grows exponentially, and according to its sign the particle or photon falls in to $r = 2M$ or escapes to $r = \infty$.

Waves of short wave length travel like particles, according to standard arguments of physical optics and the correspondence principle, whether they are de Broglie waves, electromagnetic waves, neutrinos, or gravitational waves. At wavelengths that are longer, and comparable to or greater than the dimensions of the black hole, characteristic diffraction and scattering phenomena occur that depend upon the spin of the field as well as its wavelength. It is not surprising to find *less* energy coming out than goes in, because one anticipates that some of the energy will go down the black hole. It was a surprise to discover (Misner, 1972) that a rotating black hole for a certain range of the angular momentum and frequency of the incident waves can give rise to the phenomenon of superradiance: *more* energy coming out than goes in. For the most favorable angular momentum and frequency, and for an extreme Kerr black hole ($J = J_{max} = M^2$) this effect gives a gain only of the order of 4.5 percent for electromagnetic radiation. but one of the order of 138 percent for gravitational radiation. For some of the relevant calculations, reference is made to Press and Teukolsky (1972), Starobinsky (1973), Starobinsky and Churilov (1973), Press (1973), and Teukolsky (1973b). Nothing did so much to open the door to these calculations as the discovery by Teukolsky (1973a) how to separate the wave equation for neutrino, electromagnetic and gravitational waves in Kerr geometry.

If only astrophysics provided in place of a rotating black hole, or in addition to it, a strongly charged black hole ($Q \sim Q_{max} \equiv M$), one would at once have, Ruffini (1973) has pointed out, a " gravitational wave converter ". Incident gravitational waves would cause the otherwise stationary lines of force of the black hole to wave back and forth and generate outgoing electromagnetic waves.

If energy can be extracted from a rotating black hole by electromagnetic and by gravitational radiation, it is not surprising that it can also be extracted by a particle process, a process conceived by Penrose (1969) and studied by him and Christodoulou (1970) before superradiance was known. The key to the energy extraction process is the " ergosphere ", a region outside the horizon of the black hole where a particle can have a positive energy in a local Lorentz frame and yet have negative energy with respect to the " common market standard " of energy as measured at infinity (Figure 8).

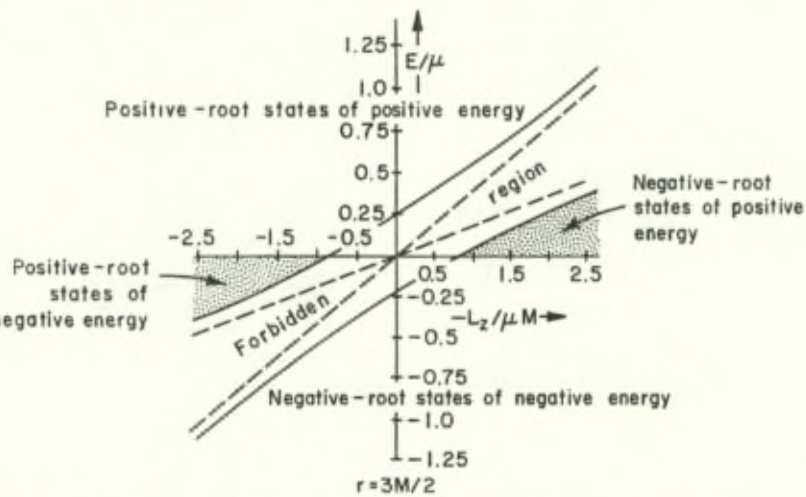

Fig. 8. Allowed values (" positive root states ") of the " energy-at-infinity " of a particle (in units of its rest mass, μ) as a function of its angular momentum L_z when it is moving in the equatorial plane of an extreme Kerr black hole ($J = M^2$) at a point with r-coordinate 3M/2. A point in the shaded region at the left symbolizes a particle which cannot escape from the ergosphere because it has negative energy-at-infinity but which in the local Lorentz frame has positive energy (4-momentum vector pointing into the future light cone). From MTW (1973) as adapted from Christodoulou Ph. D. thesis.

A particle with negative " energy at infinity " cannot get to infinity. It can get to the black hole. When it does, it delivers a negative increment of mass-energy to the black hole. The particle and the former black hole thereupon disappear from the bookkeeping, and the new and less massive black hole takes their place. This is how energy is extracted from the black hole. However, the account needs completion. In what way was the debt-laden particle created in the first place? By entry of a positive-energy particle of greater rest mass from infinity into the ergosphere, followed by explosion of this particle into two fragments, one of which is moving in such a way as to have negative " energy at infinity ". Elementary conservation laws tell one that the other particle, though endowed with less rest mass than the original particle, actually carries away to the outer world a greater total (rest-plus-kinetic) energy than that original particle brought in. That is how some of the mass of the rotating black hole is made available to the outside. The best that can be done in this process is to be found from Equation 3: keep fixed the irreducible mass M_{ir} of Christodoulou, or the surface area S of Hawking, and reduce the charge Q or angular momentum J or both. Such a process is called by Christodoulou a reversible transformation. In contrast, a less carefully executed process, an irreversible transformation, results in some increase in the irreducible mass, M_{ir}, and therefore less drop in, and utilization of, the mass-energy of the black hole (for the same changes in Q and J; Figure 9).

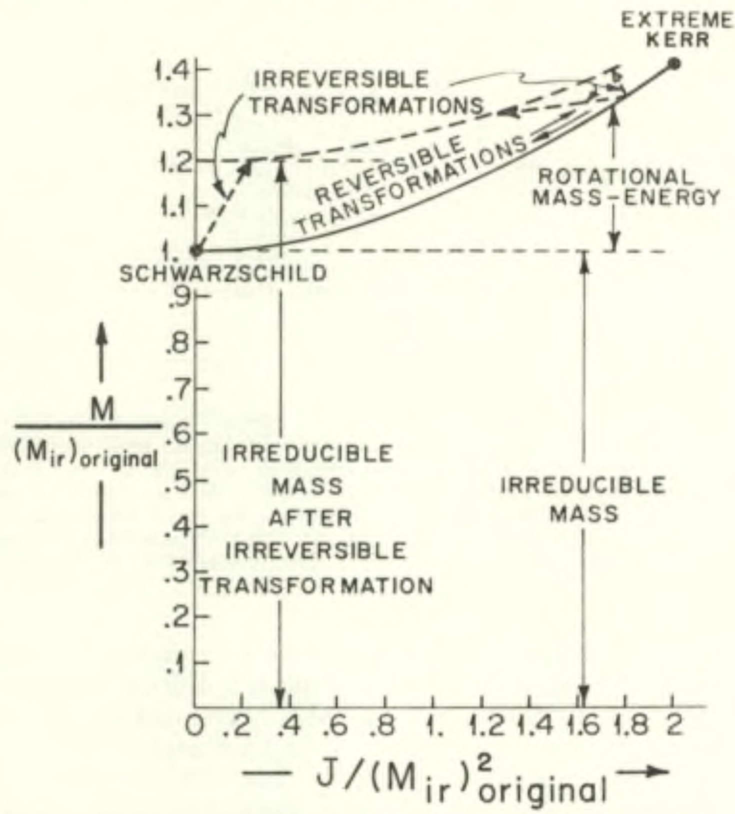

Fig. 9. Reversible *versus* irreversible transformation of a black hole. Starting with an extreme Kerr black hole in this example ($Q = 0$; $J = M^2 = 2M_{ir}^2$), one can in principle reduce its mass from $M = 1.414\ M_{ir}$ to $M = M_{ir}$ by a sequence of reversible transformations (Christodoulou, 1970) of the Penrose type, with the energy thereby set free being taken away by the outgoing particles. If one proceeds less efficiently, via irreversible transitions, one ends up with a dead black hole with $M = M_{ir,\ new}$ greater ($1.2M_{ir}$ in the example) than the original irreducible mass (diagram adapted from Christodoulou).

A " live " or rotating black hole, as contrasted to a " dead " or static black hole, not only can transfer energy to its surroundings (to a field, via " superradiance "; to a particle, via the Penrose process) but also cannot avoid transferring angular momentum to its surroundings, when those surroundings in any respect lack axial symmetry with respect to the axis of rotation of the black hole. The rate of transfer of angular momentum has been analyzed by Hawking and Hartle (1972).

e. *Blackbody Radiation from a Black Hole*

" Superradiance " means that the object sends out more radiation than is incident upon it, and raises the question whether a black hole may already without external stimulation spontaneously radiate. Hawking (January 1974, March 1974; these very important additions to black-hole physics, not available at the time of the Solvay congress itself, are added here thanks to the kindness of Dr. Stephen Hawking in communicating these two preprints) shows " that quantum mechanical effects cause black holes to [emit quanta and] create and emit particles as if they were hot bodies with temperature

$$T = kT_{conv} = \hbar g/2\pi, \tag{18}$$

or

$$T_{conv} \simeq 10^{-6} (M_\odot/M) \text{ degrees Kelvin}, \tag{19}$$

where g [$g\,(cm^{-1}) = g_{conv}\,(cm/sec^2)/c^2$, familiar notation, substituted here for Hawking's Greek κ, as also T_{conv} (" conventional units ") is substituted for his T] is the surface gravity of the black hole [$g = (1/4M)$ for a Schwarzschild black hole]. This thermal emission leads to a slow decrease in the mass of the black hole and to its eventual disappearance; any primordial black hole of mass less than about 10^{15}g would have evaporated by now " ..... " Near the end of its life the rate of emission would be very high and about 10^{30} ergs would be released in the last tenth of a second. This is a fairly small explosion by astronomical standards but it is equivalent to about one million one megaton hydrogen bombs " ...... " Although these quantum effects violate the classical law that the area of the event horizon of a black hole cannot decrease, there remains a Generalized Second Law; [the quantity]

$$S + A/4\hbar \tag{20}$$

[here we have inserted the factor $\hbar$ in the denominator of the second term as required for consistency] never decreases, where S is the entropy of matter outside black holes and A is the sum of the surface areas of the event horizons. This shows that gravitational collapse converts the baryons and leptons in the collapsing body into entropy ".

Bekenstein (1972a, b) had already earlier given reasons for thinking of the surface area of a black hole (expressed in units of the Planck area $\hbar G/c^3$; or here, expressed in units of $\hbar$ itself because we are using geometrical units such that $c = 1$ and $G = 1$) as not only analogous to entropy, but actually identical with entropy, apart from some then undetermined dimensionless constant of order unity; and $\hbar g/4\pi$ likewise not only analogous to temperature, but identical with a temperature, up to the reciprocal of the same undetermined dimensionless constant. By showing that a black hole radiates, and calculating its rate of radiation, Hawking established compellingly the point of Bekenstein, and evaluated the numerical factor as 1/4.

Hawking points out that the new radiation comes about through quantummechanical tunnelling through the " barrier " between the region inside the black hole where the Killing vector for t-translations is spacelike and the region outside where it is timelike. He adds that the surface gravity g measures the gradient of the magnitude of this Killing vector and therefore governs the probability for this tunnelling.

It provides insight into the Hawking radiance to consider a hot black sphere of radius R in the context of flat space. The number of photons and the amount of electromagnetic energy given off per unit time are given by the product of the following factors:

The average outward component of velocity,	$c/4$
The surface area,	$4\pi R^2$
The number of modes per unit volume in the interval of frequency $d\nu$ or of circular frequency $d\omega$ or of wave number dk,	$8\pi k^2 dk/(2\pi)^3$
The average number of photons, or the average energy, for this mode,	$\dfrac{1 \text{ or } \hbar\omega}{e^{\hbar\omega/T} - 1}$ (21)

A little consideration of solid geometry allows one to divide these photons into groups, to a semi-classical level of accuracy, such that those in a given group (j, m, π) have total angular momentum about the center of the sphere between $(j - \frac{1}{2})\hbar$ and $(j + \frac{1}{2})\hbar$; about the z-axis, between $(m - \frac{1}{2})\hbar$ and $(m + \frac{1}{2})\hbar$; and polarization π (either of two possible values):

$$\left\{\begin{array}{l}\text{number of photons or amount}\\ \text{of energy emitted per second}\\ \text{by the entire sphere in the}\\ \text{wave-number interval } dk\end{array}\right\} = \frac{c\,dk}{2\pi}\,\frac{1 \text{ or } \hbar\omega}{e^{\hbar\omega/T} - 1}\sum_{j,\,m,\,\pi}\Gamma_{\omega,\,j,\,m,\,\pi} \quad (22)$$

Here $\Gamma_{\omega,\,j,\,m,\,\pi}$ is an " access factor " that to the level of approximation just described has the value 1 for photons with semiclassical impact parameter $b = j/k$ less than R and the value 0 for j greater than kR. Thus the total photon current and photon power output are proportional to T^3 and T^4, respectively, multiplied by R^2, as expected.

Hawking shows that the very general formula (22) applies to black hole radiance, not only for photons, but for all kinds of radiation and particles, and not only for a Schwarzschild black hole, but for a black hole given by any allowed setting of the three handles, M, Q, J, with the following amendments and provisos:

1) The — 1 in the denominator of the Planck factor in (22) is to be replaced by a + 1 when one is calculating the rate of emission of fermions (neutrinos, electrons, protons, etc.), also c by v.

2) The $\hbar\omega$ in the numerator and in the exponential function is to be understood as including rest mass along with kinetic energy.

3) The Boltzmann factor $\hbar\omega/T$ in the exponential function is to be replaced by

$$[\hbar\,(\omega - m\Omega) - e\,\Phi]/T \tag{23}$$

where

$$\Omega = \delta M/\delta J \tag{24}$$

[see equation (3) for M] is the angular velocity of the horizon, e is the charge, if any, of the emitted particle, and Φ is the electrostatic potential of the horizon.

4) The index j, no longer a measure of total angular momentum because total angular momentum is not conserved in the field of force of a rotating black hole, now serves to distinguish one Teukolsky spheroidal harmonic from another.

5) The " access factor ", $\Gamma_{\omega, m, j, \pi}$, can be found by considering an incoming wave characterized by the indicated parameters (ω, j, m, π), as affected by the curved-space geometry around the black hole, and calculating what fraction of it, $\Gamma_{\omega, m, j, \pi}$, enters the horizon, or what fraction of it, $(1 - \Gamma_{\omega, m, j, \pi})$, is scattered back out. This latter fraction exceeds unity, that is, $\Gamma_{\omega, m, j, \pi}$ is negative, and one has " superradiance ", whenever one is dealing with bosons, provided only that (23) is negative [negative denominator in (22)] and provided that $\Gamma_{\omega, m, j, \pi}$ does not happen to be exactly zero.

One can uphold the laws of conservation of lepton number and baryon number in black-hole physics if one says that Hawking's results are valid for bosons but require for fermions the replacement of (23) by

$$[\hbar\,(\omega - m\Omega) - e\Phi - \mu_i]/T, \tag{24}$$

where μ_i is the *chemical potential* of the relevant family of particles (i = 1 for baryons; i = 2 for electrons and electron-neutrinos; i = 3 for muons and muon-neutrinos; etc.). To take up this point in more detail will require opening a new chapter in black-hole physics and will not be attempted here. However, in the next section appropriate changes will be introduced, and signalled as such, in the laws of black-hole physics as those laws have been previously accepted, on the new assumption that a black hole is indeed characterized by chemical potentials. On this view the properties of a black hole fall into two very different classes: the macroscopic properties and classical geometry, completely determined by M, Q and J, and by these parameters alone; and the microscopic properties, dependent not only upon M, Q and J, but also upon the chemical potentials $\mu_1, \mu_2, \ldots\ldots$; or, equivalently, upon the conjugate particle numbers, $N_1, N_2, \ldots\ldots$ No way is evident to determine these microscopic properties except from the radiance of the black hole which, in practical terms, would seem out of the question except for a mass very small in comparison with the mass of the sun. In this sense the laws of conservation of particle number is still transcended for a black hole in almost every astrophysical context.

f. *The Laws of Black-Hole Physics*

The " first law " of black-hole physics [Israel (1971); Bardeen, Carter, and Hawking (1973); Carter (1973); see also Smarr (1973)] states that the alteration in the mass of a black hole brought about by an alteration in the adjustable parameters of the black hole is the sum of products of two quantities, of which the first has a local meaning and is constant with respect to position on the horizon, and the second is global; or more specifically, with the addition of the chemical potentials as proposed in this report,

$$dM = (g/8\pi)\, dA + \Omega dJ + \Phi dQ + \sum_i \mu_i\, dN_i. \qquad (25)$$

Here the units continue to be geometric [M(cm); g(cm^{-1}); A(cm^2); Ω(cm^{-1}); J(cm^2); Φ(cm^0); Q(cm); μ_i(cm); N_i(cm^0)].

The " second law " as expounded and applied in earlier papers by Hawking, and generalized by Bekenstein (1973b) and Hawking (March 1974), states that when any particles or radiations are caught by a black hole, or when several black holes collide and coalesce or collide and scatter, or whenever any other process takes place that may or may not involve black holes, the generalized entropy

$$S + (A/4\hbar) \qquad (26)$$

will increase or at best remain constant. Some of the many applications of these two laws are spelled out in Carter (1973) and MTW (1973). Two additional laws are proposed and discussed by Bardeen, Carter, and Hawking (1973).

g. *Black Holes and the Direction of Time*

It is difficult to name any process in all of physics where friction or one-sidedness in time or irreversibility dominate mechanics on a vaster scale than they do in the formation and growth of a black hole. What assurance does one have that one is right in his assumptions on irreversibility in this new context? And what conditions can be imagined that might change one's conclusions?

When a fast charged particle passes sufficiently close to a nucleus to undergo an appreciable deflection, it sends a pulse of electromagnetic radiation out to " infinity ", and itself loses energy. Maxwell's equations allow not only this solution but also an alternative solution, in which a pulse of electromagnetic radiation comes in from " infinity ", converges on the particle in perfect timing with its sudden deflection, and thereby augments the energy of the particle. The argument that the first solution is the right one and the second solution is the wrong one, like the argument that heat flows from hot to cold rather than the reverse, is founded, one is unhappy but honest enough to say, on nothing deeper than experience: compelling experience, but only on experience. Moreover, one has never found himself to go wrong when he generalized this experience to new contexts. Therefore one has little doubt what will happen when the charged particle undergoes its sudden deflection in the immediate vicinity of the horizon of a black hole. It will radiate electromagnetic energy down the black hole; it will not receive electromagnetic energy coming up out of the black hole. What we say of the " infinity " of far away space we also say of the " captured infinity " that is a horizon: it receives, but it does not give. That is the generalization we make from everyday experience, and that is the source of our assurance that black holes will behave as expected.

On this reasoning we have a solid basis for disbelieving in the existence of any such object as a " white hole ", an object of mass large compared to $m_{*e} = 5 \times 10^{17}$g (equation 14), and thus dominated by classical effects rather than by Hawking's quantum radiance, and distinguished from a black hole in operating according to the opposite sense of time, suddenly and catastrophically vomiting forth all its mass as particles and radiation. There is a considerable literature on this idea, which it would be out of place even to cite here, not least because Zel'dovich (1973) has so effectively emphasized the impossibility of delaying the explosion of such an object more than a fraction of a second after the original big bang.

If only experience tells us that now, during the phase of expansion of the universe, all physical processes run in the same direction of time, from the flow of heat to the force of friction, and from biochemistry to radiation, then there is no experience to say what will happen to these processes when that recontraction of the universe sets in that is predicted by Einstein's theory. It is difficult to name any question of physics on which more distinguished physicists hold more divided opinions. It is also difficult to name any open issue of principle with more dramatic consequences for the fate of black holes in the era of recontraction. Do they live out their lives quietly until one by one or in great groups they amalgamate in the final catastrophe of collapse? Or have they long since turned into white holes and blown up? (formation of a black hole as seen by an observer living with the opposite sense of time!). It is possible to believe that the divergence of views on this point will only then be narrowed, when by measurements of the rate of radioactive decay or other appropriate process of long time scale, today as compared to long ago, one can clearly say yes or no to the idea of a zero rate for all irreversible processes at the phase of maximum expansion.

§ 7. The Three Signals

Nothing is of so great a help in the ongoing search for black holes as the circumstance that of the order of half of all stars are in the married state, members of binary or multiple star systems. Association with another star is not only essential for any unambiguous determination of the mass of the black hole from period and magnitude of the Doppler shift of the optical component, and from an astrophysical evaluation of the mass of that component. It is also important for the correct assignment of any of the three signals that a black hole may send out: gravitational radiation at the time of formation, x-rays from accretion after formation, and "activity" produced by release of rotational energy. In addition association with another star is important for determining the mass of a neutron star [see the summary of Giacconi (1973)]. In this way one has arrived at mass values of ~ 0.2 to $0.3\ M_\odot$ for Cen X-3 and $< 1.3\ M_\odot$ for Her X-1, and can hope in time to have a sufficient collection of mass values to identify the approximate point of cut-off; i.e., the critical mass for collapse of a neutron star to a black hole, a quantity of great interest.

a. *Gravitation Radiation at the Time of Formation*

Figure 10 illustrates one conceivable scenario for the formation of a black hole. A pulse of gravitational radiation is given out at each of several catastrophic events, seconds to hours apart, and in addition continuous gravitational radiation, of continuously changing frequency, is given off throughout this "settling down time".

The details of the scenario of Figure 10 will differ completely from one evolving star to another, depending on two parameters, the mass and the angular momentum of the white-dwarf core of this star. Even for the values of these parameters chosen in the construction of Figure 10, that picture can at best possess only qualitative validity. Detailed computer analysis would seem essential for any firm prediction about the course of any selected scenario.

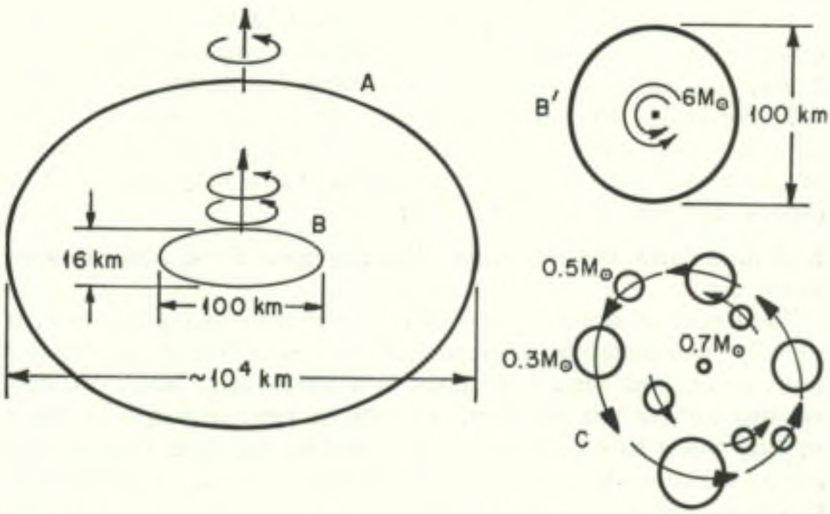

Fig. 10. "Collapse, pursuit, and plunge scenario" [schematic; from Ruffini and Wheeler (1971)]. A star with a whitedwarf core (A), slowly rotating, evolves by straight forward astrophysics, arrives at a condition of gravitational instability, collapses, and transforms for a brief period into a rapidly spinning neutron-star pancake (B, B′). It then fragments (C) because it has too much angular momentum to collapse into a single stable object. If the substance of this pancake were an incompressible fluid, the fragmentation would have a close tie to well-known and often observed phenomena ("drop formation"). However, the more massive a neutron star is, the smaller it is, so one's insight into this and subsequent stages of the scenario are of necessity subject to correction or amendment. One cannot today guarantee that fragmentation takes place at all; nevertheless fragmentation will be assumed in what follows. The fragments dissipate energy and angular momentum via gravitational radiation. One by one as they revolve they coalesce ("pursuit and plunge scenario"). In each such plunge a pulse of gravitational radiation emerges. Debris falls on the coalesed objects (neutron stars or black holes or both) changing their angular momenta. Eventually a single collapsed object is formed with a final pulse of gravitational radiation [caption adapted from MTW (1973)].

Estimates of the energy carried off in the gravitational radiation range from 0.01 to 0.1 or more of the rest-mass-energy of the final object, with the pulses cutting off for circular frequencies greater in order of magnitude than $1/M$. For one solar mass ($M = 1.47$ km) this means an $\omega_{\text{cutoff}} \sim 2 \times 10^5$/sec or less [MTW (1973)].

With some improvements, a Weber-bar detector should be able to pick up signals of this energy from a source located anywhere in this galaxy. If one says that the average interval between supernova events in a galaxy of this size is of the order of 50 years, that also might be the time interval to be expected between observable events.

The new generation of gravitational-wave detectors now under construction at Stanford, California, Baton Rouge, Louisiana, and Rome, Italy, with improvements reasonably foreseeable for the late 1970's or early 1980's, should be able to detect such events out to the distance of the Virgo cluster of galaxies, implying an expected frequency of responses of the order of one a month, uncertain by a factor of 10 [details summarized in MTW (1973)].

b. *X-Rays Given Out by Matter Accreting on a Black Hole After its Formation*

The theory of accretion, and of x-ray emission during accretion, is so fully summarized in the report of Rees elsewhere in this volume [Rees (1973)] that there is no point in a review here. Only can only say that without this radiation, and without x-ray telescopes to detect it, there would have been little hope of finding the object Cyg X-1 and getting evidence why it is plausibly identified as a black hole [Giacconi (1973) and report in this volume].

c. *Activity Produced by Release of the Rotational Energy of a " Live " Black Hole*

Despite arguments of theory that the Crab Nebula has to be powered by a source at its center, and that the source of this energy is most reasonably understood to be kinetic energy of rotation of a neutron star [Wheeler (1966)], no clear picture of a mechanism of transfer of energy from neutron star to surrounding plasma could be proposed, and it took the discovery of pulsars by Hewish and collaborators in 1968 to confirm the importance of rotational energy in the natural economy of the Crab Nebula. There is far more energy on a rotating black hole (up to 0.4142/1.4142 or 29.3 percent of its rest mass) than there is in a rotating neutron star, but there is also more ignorance about how this energy can be conveyed to the surroundings. Happily one already knows at least the basic principles of two mechanisms of energy transfer, one the Penrose mechanism, the other superradiance. A mechanism has also been put forward by which a rotating black hole can support a much smaller mass in a " floating orbit " against losses of energy by gravitational radiation and other causes [Press and Teukolsky (1972)]. One scenario has been suggested by which a star falling into a very large black hole (10^6 to $10^9 M_\odot$) will by reason of tidal disruption (Table IV) automatically undergo the Penrose process and lead to the ejection of a slug of mass [J. A. Wheeler (1971 b) and further work by B. Mashoon in progress].

Much more remains to be done to explore these and other possible forms of black hole " activity ", not least to discover a process with a signature so characteristic that it will bear unmistakable witness to the presence of a black hole.

TABLE IV

"Critical distance" for tidal disruption of selected stars near a Schwarzschild black hole. Approximate, based on the Newtonian theory of gravitation (reasonable for r substantially larger than 3M; L. Fishbone has developed foundations for a more nearly accurate relativistic analysis) and on the idealization that the victim star has uniform density (gives order-of-magnitude estimate of safe distance even for an object of very inhomogeneous density). The critical distance is evaluated from the simple Roche formula,

$$(\text{critical distance}) = 2.4554 \left(\frac{\text{mass of predator}}{\text{mass of victim}}\right)^{1/3} (\text{radius of victim}).$$

Data for the table were kindly calculated by Nathalie Deruelle and Philip Marcus. Boxes give the respective critical distances.

	Predator black hole (mass; Schwarzschild radius)			
Victim ↓	$10\ M_\odot$ 2.96×10^{6} cm	$10^{5}\ M_\odot$ 2.96×10^{10} cm	$10^{7}\ M_\odot$ 2.96×10^{12} cm	$10^{9}\ M_\odot$ 2.96×10^{14} cm
Sun $m = M_\odot$; $R = 6.96 \times 10^{10}$ cm ρ (central) $= 140$ g/cm^3 ρ (average) $= 1.42$ g/cm^3	Critical distance: 3.68×10^{11} cm	7.93×10^{12} cm	3.68×10^{13} cm	* * * *
Low-mass white dwarf $m = 0.26\ M_\odot$; $R = 1.24 \times 10^{9}$ cm ρ (central) $= 3.23 \times 10^{5}$ g/cm^3 ρ (average) $= 6.45 \times 10^{4}$ g/cm^3	1.03×10^{10} cm	2.22×10^{11} cm	* * * *	* * * *
Near-critical white dwarf $m = 1.18\ M_\odot$; $R = 3.53 \times 10^{8}$ cm ρ (central) $= 3.21 \times 10^{8}$ g/cm^3 ρ (average) $= 1.28 \times 10^{7}$ g/cm^3	1.76×10^{9} cm	3.81×10^{10} cm	* * NO TIDAL	* * * *
Low-mass neutron star $m = 0.18\ M_\odot$; $R = 3.23 \times 10^{7}$ cm ρ (central) $= 2.6 \times 10^{13}$ g/cm^3 ρ (average) $= 2.5 \times 10^{9}$ g/cm^3	3.03×10^{8} cm	* * * *	* * DISRUPTION OF VICTIM BEFORE CAPTURE	* *
Near-critical neutron star $m = 0.66\ M_\odot$; $R = 1.00 \times 10^{6}$ cm ρ (central) $= 3.2 \times 10^{15}$ g/cm^3 ρ (average) $= 3.15 \times 10^{14}$ g/cm^3	6.06×10^{6} cm	* * * *	* * INTO PREDATOR BLACK HOLE	* *
Black hole $m = 10\ M_\odot$; $R = 2.96 \times 10^{6}$ cm ρ (nominal) $= 1.85 \times 10^{14}$ g/cm^3	– – – – – – – – AMALGA-	* * * *	* * * *	* * * *
Black hole $m = 2\ M_\odot$; $R = 5.92 \times 10^{5}$ cm ρ (nominal) $= 4.61 \times 10^{15}$ g/cm^3	MATION – – – – – – – –	* * * *	* * * *	* * * *

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Discussion of the report of J.A. Wheeler

J. Ehlers: The assertion that stars with sufficiently large masses will give rise to black holes, as well as the special role given to Kerr spacetimes, depends on the cosmic censorship hypothesis. Recent work by Seyfert et. al at Hamburg shows that naked singularities will develop in some spherically symmetric collapse models. The model of matter used in those calculations is admittedly unrealistic, but on the other hand we also do not have theorems proving the cosmic censorship hypothesis under realistic assumptions about the equation of state. What is your opinion concerning the status of the censorship hypothesis?

J. A. Wheeler: You touch here on a central point to which we would all like to know the answer. At first sight it appears simple to exclude a singularity. For example, one operating formally can assign a negative value to the mass M in the Schwarzschild geometry. However, we rule out the negative mass, because the corresponding 4-geometry, having no horizon, presents an unacceptable singularity that is directly accessible to outside observation. About such a singularity and all singularities we recall Einstein's remarks, to this effect: If one permits a singularity at one place he should permit singularities at other places, and therefore anywhere and everywhere; but if one permits singularities everywhere, he has already abandoned the field equations. So we exclude singularities insofar as we can. We accept the big-bang singularity at the beginning of time and the corresponding singularity at the end of time for a closed model universe first because of what astrophysics tells us about early cosmology, second because well known singularity theorems leave no escape, and third because the initial singularity is spacelike (similarly for the final singularity). On a slightly later spacelike hypersurface the geometry is free of singularity and the dynamic evolution of the geometry proceeds deterministically with time. This is what we would like to believe to be the generic situation. But to our puzzlement we have examples such as those discovered by Seyfert *et al.* and the long-known Reissner-Nordstrøm solution (Schwarzschild solution generalized by the addition of electric charge) where a perfectly regular spacelike or " initial-value " 3-geometry has for its deterministic consequence a spacetime or 4-geometry that later on manifests a timelike or " naked " singularity [see for example the Penrose diagram, Figure 34.4, on p. 921 of MTW (1973)]. From the moment that such a singularity comes in, causality goes out. The course of the developing singularity is subject to modification from moment to moment according to the whim of the one who calculates the 4-geometry (unless he insists on analytic continuation, a demand that goes beyond dynamics and produces no warrant to justify itself).

The timelike singularity is an open doorway that not only admits " intervention from outside ", but even continually demands it. We feel we should say no to such " naked singularities ". But that requires legislation beyond the usual, legislation on initial conditions predicated not merely on the regularity of those initial conditions in and by themselves, but on the freedom of past and future from timelike singularities. The desirability of such legislation is clear; but from what foundation it is to be derived is not clear.

Y. Ne'eman: Shouldn't we divide black holes into two " social " classes:

a) " *Born-black* ", i. e. due to the initial conditions of the Universe. These can have any mass and very little can be said or known about their baryon number, etc...

b) " *Blackening holes* ", i. e., collapsed configurations. These have to have $M > 2\text{-}3\ M_{\odot}$ They will never *really* become ideal black holes, in the coordinates of a distant observer. Their quantum numbers therefore preserve some meaning, at least in the context of a " gedanken " experiment.

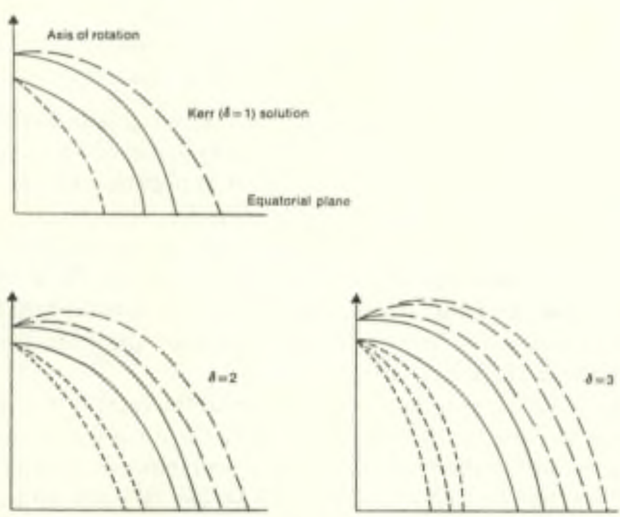
Note that we can always distinguish between even and odd total fermion number by counting the number of angular momentum $\frac{1}{2}\,\hbar$ quanta.

A. Trautman: The occurrence of singularities in gravitational collapse and cosmology indicates that the laws of physics used to build models of black holes and of the Universe are not applicable at large densities and high temperatures. According to Wheeler, the critical density is of the order of $C^5/G^2\,\hbar \approx 10^{94}$ g/cm^3, corresponding to a radius of curvature of the order of the Planck length, $(G\hbar/c^3)^{1/2} \approx 10^{-33}$ cm. At such densities and curvatures, quantum gravitational phenomena are presumed to play a dominant role, but elementary particles, as we know them, have little chance to survive.

There does not seem to be any doubt about the form of the relativistic equations describing the classical behaviour of a gravitational field in empty space. However, the Einstein equations corresponding to phenomena inside matter may be easily modified. This can be done without giving rise to any difficulties with respect to observations or the internal consistency of the theory. One of the simplest and most natural modifications of the Einstein theory of gravitation, which is due to E. Cartan (Ann. Ec. Norm. Sup., **40**, (1923), 325), consists in taking into account a direct influence of spin on the geometry of space-time. Within the framework of the Einstein-Cartan theory it is possible to build non-singular cosmological models (W. KOPCZYNSKI, Physics Lett. **A43** (1973), (63). According to one of the models, the density of matter in the Universe at the moment of maximum contraction is of the order of $m^2_{\text{nucl.}}\, c^4/G\hbar^2 \approx 10^{55}$ g/cm^3; i. e., is much smaller than the one implied by quantum gravitational fluctuations (A. Trautman, Nature Physical Sci., **242**

(1973) n° 114). In other words, it is likely that the basic laws of physics, as they are now known, require modifications which will play a role well before the quantum gravitational phenomena become important.

H. Sato: I comment about the metric solution with naked singularity. Recently, we found a series of metric solutions, which are axisymmetric, asymptotically flat and stationary solutions; that is, they have a rotational property. In this series of solutions, which are classified by an integer δ, the Kerr solution is the simplest one. The figure shows the structure of space-time. The dotted line is the surface of stationarity; the black dot is the singularity; and the solid line is the surface of the " event-horizon " -like surface. The multiplicity of the ring singularity is δ. One of the rings lies always outside the " event-horizon " -like surface except the $\delta = 1$ Kerr solution. Since the cosmic censorship hypothesis is tentative only, I think the space-time of a collapsed object *may* have a naked singularity.



L. Halpern: Consider a system in gravitational collapse; for a simple example, a spherically symmetric distribution of dust. Yet there may be more things between heaven and the black hole than dreamed of in this philosophy.

Assume that a finite (arbitrarily small) density of electromagnetic background radiation exists while the collapse goes on. This hardly alters matters up to the domain where present X-ray observations come into play — but far beyond this stage the process according to our preliminary considerations is strongly modified before the definite stage of collapse beyond the Schwarzschild radius occurs. We are at this moment not yet fully sure whether collapse then can at all occur.

B. Carter: I wish to reply to Professor Ne'eman's comment to the effect that we ought to think in terms of " blackening " holes rather than black holes. This comment is only fair in so far as it applies to black holes whose angular momentum J is very close to the maximum possible value determined in terms of the mass M by $J^2 = M^4$ (in units where $c = G = 1$).

Detailed analyses to which many people have contributed confirm the general principle that externally observable deviations from the final stationary black hole state of a collapsing body will normally fade away exponentially with decay timescales of the order of the time required for light to cross a distance equal to the Schwarzschild radius 2 M. For the kind of black hole which one would expect to be formed as the final state in the evolution of an ordinary star, with a mass not too large compared with that of the sun, the relevant timescales will be of the order of milliseconds at the very most. These times are so short that for most astrophysical purposes it is entirely reasonable to think in terms of the final asymptotic black hole equilibrium state.

However, it must be admitted that this would not apply to the formation of very rapidly rotating black holes in which the angular momentum is close to the critical value which would have arrested the collapse. The decay constant κ, whose reciprocal determines the half life of the deviations from the final state, is given by the formula

$$\kappa^{-1} = 2\,M \left\{ 1 + \left(1 - \frac{J^2}{M^4}\right)^{-1/2} \right\}$$

When J^2 approaches the critical value, M^4, κ approaches zero and the decay timescale becomes arbitrarily large compared with the Schwarzschild time 2 M. Bardeen, Hawking, and I have suggested that this condition that it should take an arbitrarily long time to approach the equilibrium state with the critical angular momentum, should be referred to as the " third law " of black hole mechanics since it is in many ways analogous to the statement of the third law of thermodynamics, according to which it takes an arbitrarily long time to approach a state of zero temperature. It is an immediate consequence of this law that in the special case of the collapse of a body with J^2 very close to M^4, Professor Ne'eman's suggestion that one should think in terms of " blackening " holes would indeed be appropriate.

R. Ruffini: Professor Ehlers' remark on the possibility of forming a naked singularity during spherically symmetric collapse appears to be strongly influenced by a model of matter which is unrealistic. I do believe that the only case in which the formation of naked singularities should be taken seriously is if they could be formed *without* the violation of any physical law. The formation of a naked singularity, otherwise, instead of showing the possibility of finding them in nature, would only testify, once more, to the extreme self-consistency of general relativity; the existence of naked singularities in the spacetime geometry would only show then the violation of some basic physical law. The existence of black holes is, instead, at the moment, the object of much work in comparing theoretical predictions with experimental data.

Only if the existing data (from Cygnus XI or other sources) will not be explained in the framework of black holes physics or more conventional objects should we adress our attention to these speculative possibilities. However there is no evidence of such a need at the moment.

J. Ehlers: The purpose of my remark was only to point out that, as far as I know, the problem of the formation of naked singulari.ies is open, and that it seems presently unjustified to base astrophysical considerations too firmly on this conjecture.

SEARCH FOR OBSERVATIONAL EVIDENCE FOR BLACK HOLES

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The phenomenon of a " black hole " is described in detail in prof. Wheeler's report. This object is undoubtedly one of the most striking and important of those predicted by theorists. And perhaps the most amazing fact is the prediction that such objects not only might occur in the Universe in principle but should necessarily emerge in the course of evolution of some celestial bodies. The problem of "black holes" in the Universe has developed through a number of historical stages:

1. The theoretical prediction** by Oppenheimer and Snyder in 1938 of the possibility of existence of black holes.

2. The gradual realization by astrophysicists (in the middle sixties) a powerful role of black holes as final stages of evolution of the massive stars and stellar systems ; the intensive development of the theory of " black holes " and investigation of their astrophysical properties, attempts to find the mout in the Universe by various astrophysical methods.

3. The discovery of binary X-ray sources and the growing confidence of a great number of specialists (in 1972 and 1973) that some of these sources are most likely " black holes ".

One may say that a new branch of astrophysics has appeared: " the astrophysics of " black holes ".

It was Zeldovich and his team who created the astrophysics of black holes. An important impact was given by the detection of QSS in 1963, the immediate answer was " Relativistic Astrophysics " of Fowler Hoyle and the 2 Burbidges, but the final issue with QSS is still unclear.

The present report deals with the observational properties of " black holes " that are of importance for attempts to find them in the Universe. It becomes more evident that the most promising places for the

* Report presented at the conference by R. Ruffini.

** As is well-known, the prediction of the «invisibility» of stars that are sufficiently massive and compact was made already in 1776 by Laplace on the basis of Newtonian gravity theory. But this brilliant prediction dates to the prehistory of the problem of « black holes ».

detection and investigation of " black holes " are close binary systems, one component of which is the normal star and the other a " black hole ". This is just the question being discussed through the major part of the report.

Before passing to the next part of the report some reviews will be listed, in which the problems under consideration are discussed in detail.

The astrophysics of black holes, with all needed theoretical background and results up to 1971, we tried to collect in Zeldovich and Novikov " Stars and relativity ". Further astrophysical aspects are given in Peebles (1972) Novikov and Thorne (1973) and numerous papers.

For theoretical development see Misner Thorne and Wheeler (1973), the book edited by De Witt, De Witt (1973); about the " black holes in binary systems see Shakura, Sunyaev (1972), Pringle, Rees (1972), Shakura, Sunyaev (1973), Novikov, Thorne (1973). The same reviews contain the detailed lists of references.

Where in the Universe Should One Search for " Black Holes " ?
According to the contemporary ideas some of the well-known astronomical objects must in the course of their evolution turn into " black holes ". The time of evolution of these objects up to the state of a " black hole " is considerably less than the age of the Galaxy. In the first place the massive stars are such objects (with a mass exceeding by a few times the solar mass). Just such " black holes ", the existence of which is almost beyond all question, should be looked for, first of all.

The central regions of some globular star clusters may be attributed to this class of objects too, but with somewhat less confidence.

The suspicion of " black holes " being present in other objects may proceed from the hypotheses on their structure and evolution. The nuclei of galaxies and quasars are just such objects. It should be mentioned that there is a great variety of hypotheses concerning the nature of the nuclei of galaxies and quasars, which is due to the great variety and complexity of the phenomena observed there. Just for this reason, it seems to be a difficult task to prove the phenomena in the nuclei of galaxies and quasars to be a reliable evidence of the existence of " black holes " inside them.

The hypotheses were put forward of " black holes " arising from hypothetical, unknown to us objects; from supermassive stars ($M > 10^5 M_\odot$) for instance, which might appear in the period when the galaxies began to form, and even from the large- scale inhomogeneities at the very beginning of expansion of the Universe. Besides that the hypotheses concerning the possibility of existence of " white holes " — the phenomena contrary to " black holes " — have been suggested (Novikov 1964, Neeman 1965). The discovery of the above mentioned phenomena would be of course a matter of great importance. However, one should not forget the very speculative nature of hypotheses concerning such phenomena, and we have no grounds up to now to establish a firm connection between these speculative hypotheses and any phenomena observed in the Universe.

In our further discussion we shall dwell upon the problem of the search for and study of " black holes " originating from stars. At the end the short notations concerning the " holes " of the other nature will be given.

The Origin of the Stellar Black Holes.

A stellar black hole is formed when at the very end of the nuclear evolution of a star, after possible mass losses in the course of calm evolution and after violent processes towards the end of it, the stellar mass exceeds the upper mass limit for the cold neutron star M_{max}. According to the calculations of Cohen and Cameron (1971), this quantity is $M_{max} \approx 2.4\ M_\odot$.

It should be emphasized that the binding energy of a neutron star with a mass close to M_{max} is large, and consequently such a star has a tremendous mass defect. For a neutron star with $M_{ns} \approx M_{max}$ the mass defect exceeds 30 % $\Delta M/M_{ns} \approx 0.3$. Thus, the mass of the initial star before it's collapse to the state of the neutron star with $M \approx M_{max}$ should be $M \approx 1.3\ M_{max} \approx 3.1\ M_\odot$. The possible mass losses during the collapse are certainly neglected in this calculations. The excess energy $\Delta Mc^1 \approx 1.6 \times 10^{54}$ erg must be realeased either during the formation of the neutron star or by it's cooling when all violent processes are damped.

Thus, if massive stars do not lose a considerable amount of mass in the course of their evolution and collapse, then all stars with $M > 3\ M_\odot$ should ultimately become black holes. The evolutionary timescale of such stars is less than 10^9 years. The stellar mass spectrum in our Galaxy and the evolution times being roughly known from the observational data, one can estimate that on the average one star in the Galaxy with $M > 3M_\odot$ terminates it's evolution every ten years. Hence, the rate of black holes formation in our Galaxy may approximate at present the value 0.1 $year^{-1}$. The real value of this rate is most likely much lower. It is due to a substantial mass loss at the final stages of the equilibrium evolution and especially by the supernova outburst, which is believed to be the end of the evolution of the massive star. Towards the end of the stellar evolution shells of nuclear burning appear, the star becomes essentially inhomogeneous, it's atmosphere swells and a stationary hydrodynamical mass outflow becomes possible. According to the preliminary estimates by Bisnovatyi-Kogan and Nadezhin (1972) a star with $M = 30\ M_\odot$, for instance, loses up to $0.5\ M_\odot\ \text{year}^{-1}$ at the stage of hydrogen shell burning, and it may throw off up to the half of it's initial mass at this stage. The mass losses of this kind should be taken into account in estimates of the rate of black holes formation*

The mass loss by a supernova outburst is even more important. The numerical calculations of the supernova outbursts give a contradictory results as to the possible mass lost. In principle, the flare may disrupt the whole star. The calculations are very sensitive to the initial state of the star before the flare, especially to it's chemical composition and inhomogeneity. Because of the technical computational difficulties nobody has yet managed to perform calculations of the slow evolution all the way to the flare and to determine the necessary stellar structure. For this reason a simplified stellar model is usually taken to be the initial state. The computations performed for such initial conditions show that the ejected mass fraction has sometimes a very large value, especially for stars with a low initial mass. We may tentatively make the supposition that even stars with a mass up to $10\ M_\odot$ (or even more) do lose by the outburst such a great amount of mass that the formation of a black hole becomes impossible (Nobikov, Thorne, 1973). It should be emphasized that this is an extremely rough guess adn the subsequent calculations may change it significantly.

Unfortunately, the observations don't give very much information about the mass fraction ejected by the explosion either. If it is true that the stars terminating their evolution with a mass $M > 10\ M_\odot$ collapse then to black holes, then the rate of black hole formation in our Galaxy is $\frac{dN}{dt} \approx 0.01\ \text{year}^{-1}$. Because of the large uncertainty in the lower mass limit M_{min} for the stars collapsing to black holes we give the following expression for the rate of black holes formation in our Galaxy

$$\frac{dN}{dt} \approx 0.1 \left(\frac{M_{min}}{3\ M_\odot}\right)^{-1.4} \text{year}^{-1}$$

Thus, the stellar black holes should be present in the Galaxy in a considerable number. What are their observational appearances?

Black Holes as Invisible Masses

Black holes reveal themselves only through their external gravitational field (the Kerr metric), they don't emit any radiation and have no

* About mass losses by components of binary systems see below.

magnetic field* (Wheeler: " a black hole has no hair ") Ginzburg (1964), Novikov, Zeldovih (1966).

Thus, the problem of discovery of a black hole reduces to the problem of detection of the gravitational field of the unshining and extremely compact mass concentrated in the region with dimensions $r_g = \frac{2GM}{c^2}$. Furthermore, this mass must exceed 3 $M_\odot$, for just in this case we may be sure that the compact object is neither a cold white dwarf (it's maximum mass 1.2 $M_\odot$) nor an old neutron star (maximum mass 2.4 $M_\odot$) but inevitably a black hole.

Guseynov and Zeldovich (1965, 1966), Trimble and Thorne (1969) proposed to search for black holes as invisible components of the spectral binaries. Earlier yet the question of black holes in binary systems was formulated by Hoyle, Fowler Burbidge and Burbidge (1964).

Very often, by the spectral observations of binary stars, one can detect lines of but one component—the brightest one. The lines of the secondary are invisible. It is usually not because the secondary does not emit any light, but simply because it is dim. In this case the Doppler shift of spectral lines allows to obtain the mass function

$$f = \frac{M_{invis} \sin^3 i}{[1 + (M_{vis}/M_{invis})]^2}$$

where i is the angle between the normal to the orbital plane and the line of sight. If it is possible to obtain the mass of the visible component M_{vis} from it's spectral type, then assigning sini = 1 we readily evaluate the minimum value of M_{invis} from the above expression for f. Now if $M_{invis} > 3\ M_\odot$ and we can see no light from M_{invis}, and if the presence of the bright companion does not account for this invisibility, then M_{invis} is supposed to be a candidate for a black hole. In the papers by Guseynov and Zeldovich (1965, 1966) and by Trimble and Thorne (1969) the concrete stars are listed which are suspected to contain black holes.

The invisibility as an argument sounds a bit funny just similar to an old joke about the thesis: " The absence of telegraph poles and wire in archaeological excavations is evidence that the ancients had wireless ". However, it is not very easy for the body of a large mass to remain invisible. The luminosity of the main sequence star is known, in the course of the further evolution the luminosity but increases and only at the very end of evolution, when the star becomes a white dwarf

* Under conditions encountered in astrophysics black holes also have no electric charge.

or a neutron star, it's luminosity decreases. But the condition $M > 3\, M_\odot$ is chosen just to avoid the trivial solution.

A new candidate was pointed out by Cameron (1971). He had put forward the supposition that the eclipsing binary ε Aur contains a black hole surrounded by a disk of solid particles, the size of which is of the order of a few millimetres. Some other candidates were proposed too*.

However, it should be mentioned that none of these candidates is convincing enough. In every particular case the cause of the invisibility may be explained in a more trivial way than the hypothesis of the black hole. The attempts to find out black holes in such a way encounter one more difficulty which is worth mentioning. This difficulty arises from the theory of stellar evolution in binary systems. The evolution in close binaries is accompanied by a mass transfer from one component to the other. In close systems with an orbital period less than a few days the mass transfer may already occur on the evolutionary stage of the main sequence. At the latest stages of evolution, when the shell energy sources appear and the stellar atmosphere swells, the mass transfer may occur in a more wide pair with an orbital period of years. The mass transfer is going on in the following manner. A more massive component[1] evolves faster than it's counterpart[2]. In the course of evolution, the star[1] swells, the gas from it's atmosphere flows to the less massive component[2]. When the star[1] terminates it's evolution and becomes (depending on the ultimate mass and the history) a white dwarf or a neutron star or a black hole, it's mass turns out to be less than the mass of the secondary star[2]. The example of Sirius is an impressive argument in favour of the evolutionary pattern given above. The mass of the white dwarf (Sirius B) $1\, M_\odot$ is less than the mass of the normal star (Sirius A) $2.3\, M_\odot$. Without mass transfer in the process of evolution it would be very difficult to understand how the less massive component has managed to become a white dwarf having outrun the more massive one. The above presented speculations on the mass transfer in binary systems imply that a black hole in a binary system should have a smaller mass than the normal component. In this case it is difficult to use the invisibility of one component as an argument in favour of it being a black hole, for were it a normal star, it's invisibility could be naturally explained by the fact that it is less massive and, consequently, has less luminosity, and it's radiation simply is " lost " in the energy flux from the companion. The attempts to find black holes in binary systems by their invisibility were not of a particular success.

* For the list of candidates, see for instance in (Zeldovich, Novikov, 1971).

The Luminosity of Single Black Holes Due to Accretion of Interstellar Gas.

Zeldovich (1964) and Salpeter (1964) pointed out the importance of the accretion of interstellar gas on the black holes.

The gas falling in the gravitational field of the black hole is heated up, the magnetic fields " frozen " into the gas greatly increase, and all this results in a significant energy emission by the infalling gas. It radiates a large amount of energy before being " swallowed " by the hole and becoming invisible to the external observer. Thus, the single black hole in the interstellar medium becomes visible.

To analyse the observational appearances of a black hole surrounded by interstellar gas, let us consider a hydrodynamical accretion pattern upon the single black hole being at rest with respect to the gas (Zeldovich, Novikov 1967).

In the stationary accretion pattern the amount of matter falling upon the black hole per unit time $\dot{M}$ and other basic parameters are determined by the gas properties and the gravitational field at large distances greatly exceeding the gravitational radius r_g where the gravitational field may be treated as a Newtonian one.

The general picture of accretion is as follows. At large distances from the black hole the influence of it's gravitation is negligible. The critical value of the distance is $r_{cr} \approx \dfrac{GM}{a_\infty^2}$, where a_∞ is the sound speed of the gas far from the hole. At $r > r_{cr}$ the gas is practically at rest, at $r > r_{cr}$ it is in the state of a free fall (Fig. 1). The accretion rate is approximately equal to

$$\dot{M} \approx 4\pi\, r_{cr}{}^2\, a_\infty\, \rho_\infty \tag{1}$$

where ρ_∞ is the density of the gas far from the hole. The numerical constansts being substituted, the equation (1) may be rewritten in the form

$$\dot{M} = (10^{11}\ \mathrm{g\ sec^{-1}}) \left(\frac{M}{M_\odot}\right)^2 \left(\frac{\rho_\infty}{10^{-24}\ \mathrm{cm^{-3}}}\right) \left(\frac{T_\infty}{10^4\ \mathrm{K}}\right)^{-3/2} \tag{2}$$

where T_∞ is the gas temperature far from the hole.

The magnetic field in the interstellar medium should strongly influence the accretion pattern (Shwartzman, 1971). It rapidly increases in the falling matter. It's energy becomes comparable to the kinetic energy of the gas and after that the magnetic field affects the motion and heating of the gas in itself. No calculations of the stationary flow pattern with magnetic field are available up to now. The estimates are based upon the assumption of the equidistribution between the magnetic and kinetic energy of the infalling matter, the energy of the turbulent motions and the thermal energy. The heating of the gas is due mainly to the transformation of the energy of the magnetic field into the thermal energy of the gas (Bisnovatyi-Kogan, Rusmaikin 1973). The radiation of the gas is due to the synchrotron emission of electrons in magnetic field. All the other radiative mechanisms turn out to be considerably less efficient. The optical depth of the infalling gas is negligible and the radiation is freely transmitted outward.

Near the horizon, where $r \sim r_g$ the relativistic effects cannot be neglected. In the proper reference frame of the infalling gas the temperature, the density of the gas and the emission power continue to increase. But the part of the radiation coming out into the external space decreases. This part is approximately equal to $\approx \frac{27}{64}(1 - r_g/r)$. Thus, the emissivity of the gas rapidly decreases near r_g. The main part of the total radiative flux is emitted from the region at a few* r_g.

The total luminosity of the single black hole with a mass about ten solar masses may amount to

$$L = (0.01 \div 0.1)\, \dot{M}c^2. \tag{3}$$

where $\dot{M}$ is given by the eq. (2). This energy flux is the radiation of the relativistic electrons of the gas in a magnetic field, emitted from the distances of a few r_g. The spectrum in this case has a very broad maximum in the range $\nu_{max} \approx 10^{15}$ Hz. In the optical range the spectrum is practically flat. At $\nu >> \nu_{max}$ it drops exponentially. In the X-range the spectrum depends on bremstrahlung radiation.

Note that the total outward energy flux from the black hole has a considerably smaller value than L_{cr}, at which the gravitational attraction is balanced by the light pressure (the Eddington limit) $L_{cr} \approx 1.3 \times 10^{38}$ erg $\sec^{-1} \left(\frac{M}{M_\odot}\right)$. For this reason the light pressure does not affect the gas flow.

The inhomogeneity and the instability of the gas flow may result in significant brightness fluctuations. The characteristic time-scale of these fluctuations equals to the time for which the gas descends from approximately 10 r_g to 2 r_g i.e. equals to $\Delta t \sim (10^{-3} \div 10^{-4}\ \sec)\ (M/M_\odot)$.

* In the case of rotating black hole (the Kerr metric) the gas is inevitably drawn into the circular motion near and inside the ergosphere.

If there are inhomegeneities in the form of bright spots in the orbiting gas, then this circular motion will result in the modulation of the light detected by the remote observer due to the Doppler shift and the curvature of light rays. The frequency of modulation for the hole with an extreme rotation is $\nu = 10^5 \sec^{-1} (M/M_\odot)^{-1}$.

What will be the accretion pattern when a black hole moves through the gas (Salpeter, 1964)?

In this case a shock wave is formed behind the black hole. The gas crossing the shock wave loses part of it's kinetic energy and is partially captured by the black hole. The rate of the mass capture is given by the eq. (1), where instead of a_∞ one should insert the velocity u of the black hole with respect to the gas. The gas emits mainly after passing through the shock front, when it falls towards the black hole, attains high temperature near the hole and the magnetic fields considerably increase. The detailed pattern of this infall is practically unknown because it is rather difficult to take into account the magnetic field and various types of instabilities. One might suppose that the averaged over time observable properties will be similar to those in the case of spherical accretion.

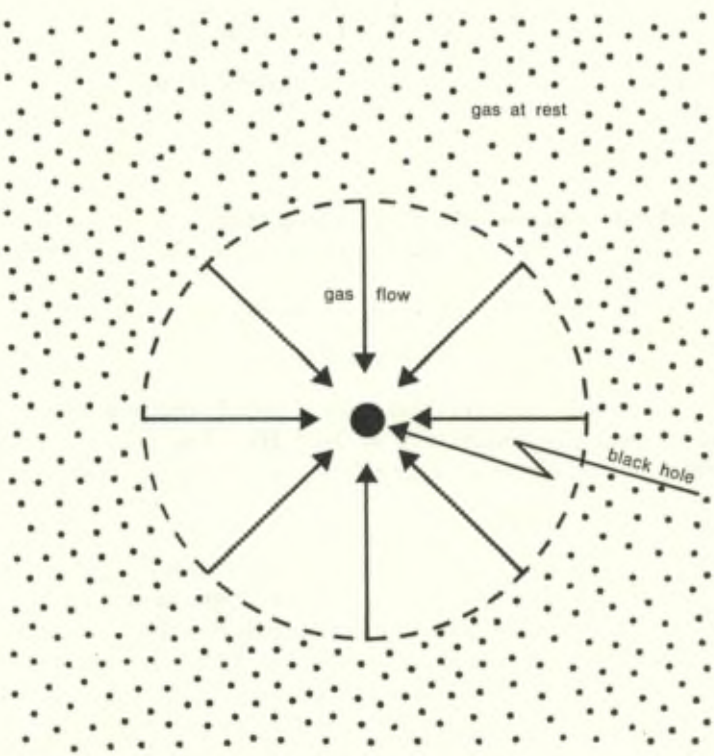

Fig. 1. The spherically symmetric accretion of the gas in the gravitational field of the black hole: $r_{cr} \approx \dfrac{CM}{\alpha_\infty^2}$, where $\alpha\infty$ is the sound speed far from the hole.

And finally one more aspect of this problem is worth noting. The gas captured by the moving black hole may have an angular momentum with respect to the direction of the motion of the black hole. If the specific angular momentum of the captured gas l exceeds the order of $r_g c$, then the centrifugal forces become of importance before the gas is " swallowed " by the hole. The gas is being drawn to the circular motion and cannot fall onto the hole until it loses the excess of the momentum due to the viscosity. In this case the radiation pattern may significantly change (see the next section). If the black hole passes from the region where the gas has the angular momentum of a certain sign to the region with the opposite sign of the angular momentum of the gas, than the collisions between the jets may result in violent emission flares (Salpeter, 1964, Shwarzman 1971). The parameters of the turbulent motion of the gas in the Galaxy being known, one can estimate the value of the specific angular momentum

$$\frac{l}{r_g c} \approx \left(\frac{M}{M_\odot}\right)^{3/4} \left(\frac{u}{10 \text{ km sec}^{-1}}\right)^{-7/2}$$

where u is the velocity of the black hole moving through the gas. Thus, in some cases the angular momentum of the captured gas may be of importance.

Observational Properties of a Single Black Hole.

Let us summarize the observational properties oft he single black holes in the interstellar gas.

They should have the luminosity of the order

$$L \sim 10^{30} \div 10^{34} \text{ erg sec}^{-1}$$

due to the synchrotron emission with a broad spectrum which has a maximum near the frequency $\nu \sim 10^{15}$ Hz. The similar properties are characteristic of the so called " DC white dwarfs " (without lines in the spectrum) (Shwartzman, 1971). Perhaps some of these stars may be candidates for single black holes.

The luminosity for single black hole may fluctuate with a characteristic time-scale $10^{-2} \div 10^{-4}$ sec. The bright flares are possible when the black hole passes from one turbulent cell to another. The time interval between the flares should be of the order of ten years.

From the above discussion one can see that it is very difficult to find out a single black hole. But it is even more difficult to prove that any object is a black hole indeed and not an old neutron star for instance. It is impossible to obtain a reliable value of the mass of a single black hole which could enable us to distinguish it from a neutron star. Perhaps, only the effects arising by the impact of the accreting matter with the surface of the neutron star may be of help to make a clear distinction between them.

Black Holes in Close Binary Systems.

Black holes in close binary systems with normal stars as companions are the best objects for the purpose of finding out and investigate black holes in the Universe. It is for the following reasons:

1) The gas accretion onto the black hole in a binary system should result in powerful X-ray emission (the accretion process is described below). By this emission combined with the optical methods it is much easier to detect a binary black hole than a single one.

2) The orbital motion in the binary system makes it possible to obtain the mass value of the compact object and thus to make a clear distiction between the black hole and the neutron star or the white dwarf.

Hayakawa and Matsuoka (1964) were the first to point out the possibility of X-ray emission by the binary systems with normal stars being the both components. Novikov and Zeldovich (1966) and Shklovsky (1967) pointed out that X-rays will be emitted in the binary system by the accretion onto a neutron star or a black hole and proposed to look for neutron stars and black holes in this way. Particular models of accretion in binary systems were constructed by Prendergast, Barbidge (1968), Shakura (1972), Pringle and Rees (1972), Shakura and Sunyaev (1972). About the calculations of relativistic effects see the review by Thorne and Novikov (1973). The survey of the application of theoretical results to particular systems is given by Shakura and Sunyaev (1973).

How does the gas accretes onto a black hole in the binary system? Two points are of a particular importance.

1) The accretion rate here depends entirely upon the rate of mass transfer from the envelope of the normal star to the black hole. From the observational data for the ordinary binary systems of β-Lyrae type we know that this mass transfer may be very intensive (up to 10^{-5} M⊙ year^{-1} in the systems of β-Lyrae type) and exceed by many orders of magnitude the mass flux by accretion onto the isolated black hole.

2) Due to the orbital motion of the system the gas overflowing to the hole acquires a considerable specific angular momentum $l >> r_g c$. The gas cannot fall on the hole along the radius, it spirals forming a disk*. Gradually, the specific angular momentum is transported outward due to the viscosity and finally gets removed from the system with the ejected fraction of the gas (see Fig. 2) The viscosity also heats the gas, causing it to radiate. It radiates mainly in X-rays.

* The similar pattern of an overflow is sometimes observed in the ordinary binary systems.

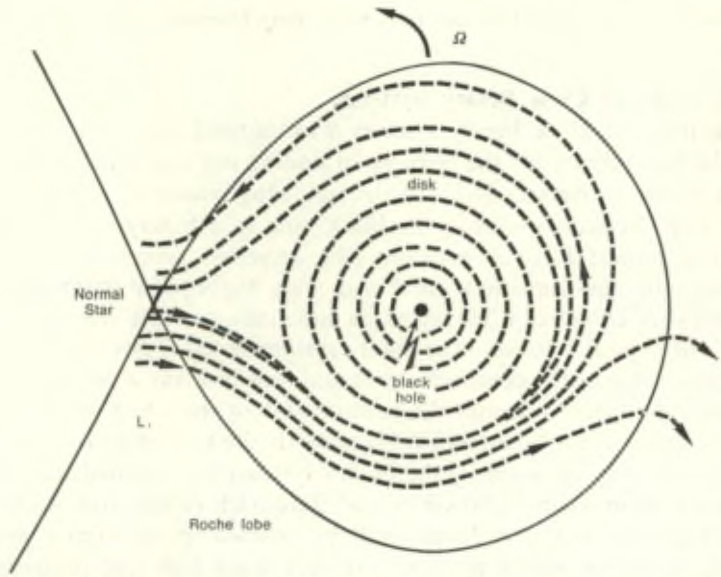

Fig. 2. The disk accretion onto the black hole in a binary system, The normal component fills it's Roche lobe.

The accretion rate in a binary system will attain high values in the case when the normal component fills it's critical Roche lobe (see Fig. 2) (A quasistationary star cannot exceed the size of it's Roche lobe). As will be seen later, the observations verify this fact. In this case the gas flows through the inner Lagrangian point to the black hole. The rate of the mass outflow depends on the structure of the star, it's slow evolution*.

To begin with, consider the accretion pattern for the case when the mass flux towards the black hole is comparatively small, so that the radiative flux arising in the heated gas is not powerful enough to balance the force of gravitational attraction, i.e. the produced radiative luminosity is below the Eddington limit $L_{cr} = 1.3 \times 10^{38} \dfrac{M}{M_\odot}$ erg sec^{-1} (For this the accretion rate $\dot{M}$ should not exceed $10^{-8} \div 10^{-9}$ $M_\odot$ year^{-1}, the exact figures are given below). The matter from the envelope of the normal star flows through the inner Lagrangian point to the black hole. This matter has a considerable angular momentum pre-

* As was shown by Basko and Sunyaev (1973) the rate of the outflow is sometimes also strongly influenced by the X-ray heating of the atmosphere of the normal star, the X-rays being emitted from the neighbourhood of the black hole.

venting it from the free fall onto the hole. At some distance from the black hole the centrifugal forces balance the attraction and the gas moves along almost the circular orbits, the disk is formed. The gas elements may gradually approach the gravitational radius, if only the effective mechanisms of momentum removal are available. The angular momentum losses are accompanied by the energy release. The angular momentum loss and the energy release occur through the viscous friction between adjacent layers of the disk, orbiting the black hole along almost the circular Keplerian orbits. The viscosity is due to the random magnetic field in the gas and to the turbulent motions.

In order to calculate the stationary disk accretion, one should take into account the laws of energy and angular momentum conservation and the emissivity characteristics of the gas.

Owing to the friction the gas elements approach the black hole along the gently sloping spirals. The major fraction of the gravitational energy is released in the inner part of the disk. Through the friction (mechanical strains) it is transported to the more outer parts of the disk, transformed into heat and is radiated by the disk surface. The thickness of the disk depends upon the balance between the gas-plus-radiative pressure and the vertical component of the gravitational attraction by the black hole. For the adopted above values of $\dot{M}$ the disk is rather thin (no more than 1/100 of the radius) Outwards it broadens to some extent (see Fig. 3). Inwards it stretches up to the closest stable circular orbit. For a non-rotating black hole the latter is at $3r_g$, for a black hole with an extremely possible rotation it is at $1/2\ r_g$. After leaving the last stable orbit the gas falls into the black hole radiating practically nothing (or very little) to the distant observer. The total amount of energy released in the gas element arriving to the inner disk boundary is equal to 6 % mc^2 for the nonrotating hole and 42 % mc^2 for the hole with the extreme rotation. A part of the radiation emitted by the inner parts of the disk is gravitationally captured by the black hole. Important details see Thorne (1974).

The total energy release and the spectral distribution of the outcoming radiation depends first of all on the accretion rate, i.e. the rate of the mass injection into the disk $\dot{M}$.

For the stationary accretion pattern, in the major part of the disk except for the closest to the black hole layers where the relativistic effects are to be taken into account, the energy release is proportional to r^{-1}. The amount of energy E transformed into heat and radiated by the disk layer between r_1 and r_2 per unit time is

$$\dot{E} = \frac{3}{2} \dot{M} \left(\frac{GM}{r_1} - \frac{GM}{r_2} \right) \tag{4}$$

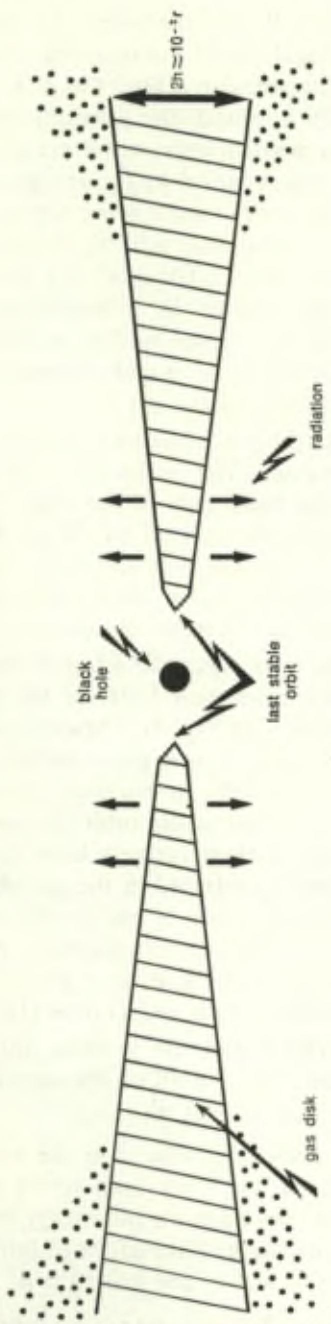

Fig. 3. The vertical section of the disk. X-ray emitted by the inner part of the disk heat the outer layers and evaporate the gas (dots).

One third of this quantity is the rate of the transformation of gravitational energy into heat, two thirds are the energy deposition from the inner parts of the disk by the viscous stresses (Thorne, see Novikov and Thorne, 1973). All the energy $\dot{E}$ is radiated by the disk surface (the radial energy flux is negligible).

In accord with the thermal energy release ([4]), the emissivity of the gas governs the temperature distribution over the disk. The optically thick outer parts of the disk radiate according to the Stephan-Boltzmann law $Q = \sigma T^4$. The more close to the hole, the higher is the temperature due both to the increasing energy release and to the increasing role of the scattering on electrons as an opacity mechanism. For the mass flux $\dot{M} = 10^{-9} \div 10^{-8}\ M_\odot\ \text{year}^{-1}$ the temperature of the inner parts of the disk emitting the major fraction of the energy flux is of the order $10^7 \div 10^8$ K. The disk radiates mainly in X-rays. The temperature distribution over the disk radius is shown in Fig. 4 according to Shakura and Sunyaev (1972). The total disk luminosity is

$$L \approx 0.1\, \dot{M}\, C^2 \approx (10^{37}\ \text{erg sec}^{-1}) \left(\frac{\dot{M}}{10^{-9}\ M_\odot\ \text{year}^{-1}}\right) \tag{5}$$

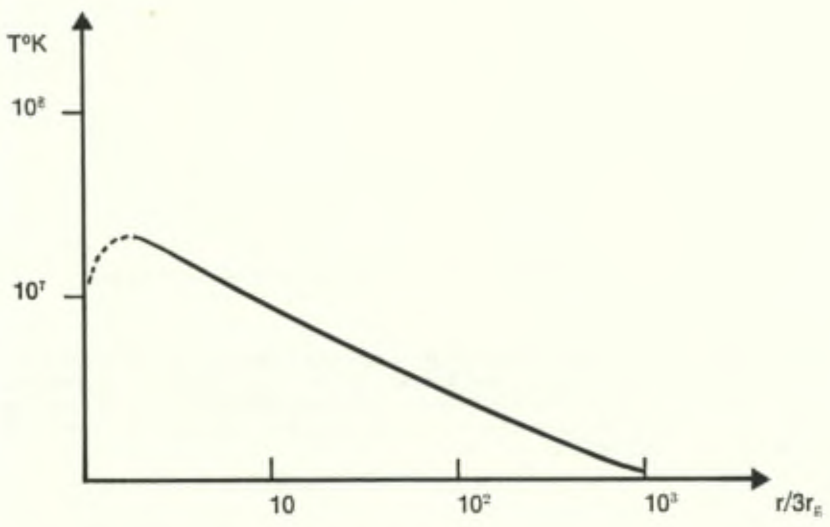

Fig. 4. The temperature distribution over the disk radius for the typical case of accretion in the binary system. The accretion rate is $\dot{M}/\dot{M}_{cr} = 0{,}1$ (Shakura, Sunyaev, 1972).

For the above given values of the rate of mass transfer from the normal star to the black hole $\dot{M} = 10^{-9} \div 10^{-8}\ M_\odot\ \text{year}^{-1}$ the black hole appears to be a powerful X-ray source with the mulinosity $10^{37} \div 10^{38}$ erg $\sec^{-1}$. Near the inner edge of the disk the relativistic effects result in the decrease of the frictional stresses, causing the gravitational energy to be transported to the outer disk layers instead of immediate transformation into heat. Thus, due to the relativistic effects the radiative emission decreases towards the inner disk boundary (see Novikov, Thorne, 1973). The integral spectrum of the disk as a whole is presented in Fig. 5 according to Shakura and Sunyaev (1972). In this figure we can see the weak power-law dependence of the intensity upon the frequency at $h\nu << \kappa T_{max,}\ F_\nu \sim \nu^\alpha$ where $\alpha = (1/3 \div 1)$. As a result appreciable also is the optical luminosity of the disk. At high frequencies the "tail" of radiation of inner parts of the disk or of nonthermal radiation is possible. The estimates show that even for the black hole $M = 10\ M_\odot$ with an accretion rate $\dot{M} = 10^{-9}\ M_\odot\ \text{year}^{-1}$ one may expect the optical luminosity to approximate that of the Sun.

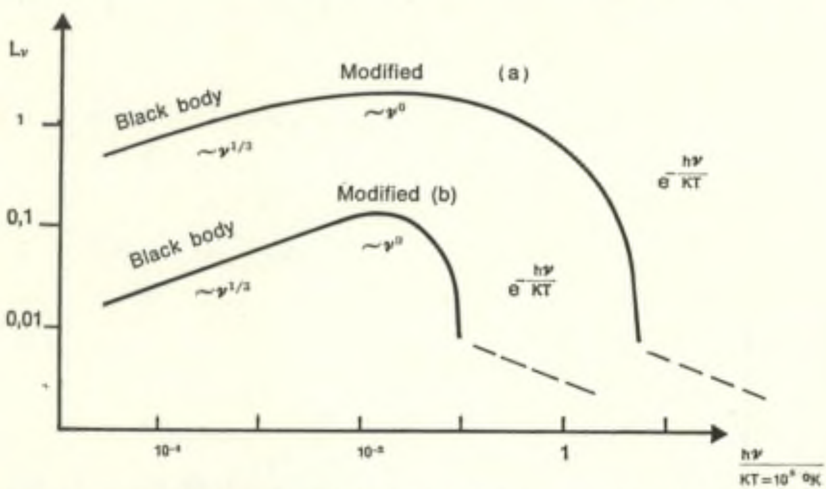

Fig. 5. The spectral distribution of the total radiative power of the disk around the black hole (according to Shakura and Sunyaev, 1972). The units of the spectral flux are arbitrary. The black hole is assumed to be a nonrotating one. Both cases (a) and (b) correspond to the same mass value of the black hole (arbitrary, of the order of a few $M_\odot$) and different accretion rates:

a) $\dot{M}/\dot{M}_{cr} \approx 1$; b) $\dot{M}/\dot{M}_{cr} \approx 0.01$.

Shakura and Sunyaev (1973) have shown that the optical luminosity of the disk may be even larger for the following reason. The disk slightly broadens outwards. The outer disk layers intercept the part of the X-ray flux, get heated and expand. As a result the fraction of the X-ray energy flux of about a few per cent is absorbed and reemitted in the optical lines as well as in the ultraviolet, due to the free-free and free-bound emission.

The light from the accreting black hole should reveal an aperiodicity and fluctuations when the bright spots appear on the disk surface. These bright spots may originate due to the intersection of lines of the magnetic field and to the turbulent motions. It is worth noting that the presence of such spots might enable us to distinguish the rotating black hole (the Kerr metric) from the nonrotating one (the Schwarzshild metric). This may be done in the following way (Sunyaev 1972). The bright spot near r_g may " live " long enough to orbit the black hole a few times. The Doppler shift and the curvature of light rays result in the luminosity of this spot be modulated with the orbital period. The orbital period at the last stable circular orbit in the Schwarzshild metric measured by the distant observer is

$$T_{min} = 12\,\pi \sqrt{\frac{3}{2}}\ \frac{2\,GM}{C^3} = 0{,}5 \text{ m sec } \left(\frac{M}{M_\odot}\right)$$

For the Kerr metric with an extreme angular momentum possible the mimum period is given by

$$T_{min} = 2\,\pi\ \frac{2\,GM}{C^3} = 0{,}06 \text{ m sec } \left(\frac{M}{M_\odot}\right)$$

Thus, the quantity T_{min} in the case of nonrotating black hole exceeds by $\sim$ 8 times that of the rotating one.

Assume the mass of the black hole M to be known from the analysis of the orbital motion in the binary system. The observations supply the information about the quasiperiodical brightness fluctuations resulting from the orbital motion of hot spots. In the case of nonrotating hole the minimum quasiperiod is $T_{min} \approx 0.5$ m sec $\frac{M}{M_\odot}$ and in the case of rotating one — $T_{min} \approx 0.06$ m sec $\frac{M}{M_\odot}$.

Such a difference makes it possible to distinguish the rotating black holes from the nonrotating ones.

We emphasize that accretion instabilities can arise and change the smooth picture.

All the above discussed accretion picture is valid for the case where the disk luminosity is beyond the Eddington limit $L_{cr} \approx 1.3 \times 10^{38} \frac{M}{M_\odot}$

erg sec^{-1}. From eq. (5) one can see that this case corresponds to the mass flux

$$M < M_{crit} \approx 10^{-8} \left(\frac{M}{M_\odot}\right) M_\odot \text{ year}^{-1}$$

For greater accretion rates the accretion pattern differs from that depicted above. The light pressure pushes the matter outwards along the radius of the disk where it exceeds the gravitational attraction. The black hole swallows only the critical mass flux. The rest of the mass flux is ejected forming an optically thick gaseous envelope. This envelope absorbs X-rays and reemits them in the ultraviolet and optical range. The total luminosity of the black hole in this case reaches the critical value $L_{cr} = 10^{38} \dfrac{M}{M_\odot}$ erg sec^{-1}.

Before passing to the analysis of particular binary systems in order to prove the existence of black noles, some notations are necessary concerning the effects of the black hole presence on it's normal companion.

In the first place, the tidal gravitational distortion causes the normal star to be an ellipsoid, and it may, in particular, fill it's Roche lobe (this is often the case in the ordinary binary systems too).

Secondly, the irradiation by X-rays heats the area of the normal star facing the X-ray source, it's optical brightness being considerably increased. This effect is of great importance when the incident flux of X-rays at the surface of the normal component exceeds the intrinsic stellar energy flux. It resembles qualitatively the reflection effect in the ordinary binaries, but the transformation of the X-ray energy flux into the optical radiation is of a specific character. This process has been calculated by Basko and Sunyaev (1973).

In combination with the orbital motion both these effects result in the optical variability of the normal component when observed from the Earth. They are thus of importance for the observational appearances of binary black holes.

The Observational Evidence of Black Holes in Binary Systems.

The X-ray observations from the satellite " UHURU " have revealed six binaries with one component a normal star and the other — a compact X-ray source (Tananbaum et al 1972, Tananbaum 1972, Hursky 1972). The survey of the observational properties of all these sources is given by Shakura and Sunyaev (1973).

Two of these sources are the X-ray pulsars (Her X1 and Cen X3). They are for sure not black holes, since it is impossible for a black hole to radiate with such an accurate periodicity (by contrast to the neutron star, the rapidly rotating active zone on the surface of which may account for the phenomenon of the pulsar). The quasiperiodic brightness fluctuations are only possible in the case of a black hole, similar for instance to those described in the above section.

According to the generally accepted interpretation, Her X1 and Cen X3 are magnetic neutron stars with matter accreting onto their surface.

We shall dwell upon the rest four sources suspected to be black holes.

The most interesting of them and best investigated of all is Cyg X1. The detailed account of it's properties is given in the report by prof. Rees, so we just recall them briefly. The distance of the source is about 2 kpc. The X-ray luminosity $L_x \approx 10^{37}$ erg $\sec^{-1}$. This source is not occulted by contrast tot he other five. Sometimes quasiperiodical brightness fluctuations are observed with the period $0.1 \div 10$ sec, sometimes they are aperiodical. Cyg X1 is identified with a weak radio source and, which is of a particular importance, with the optical star BD + 34° 3815 of B 01 b spectral type, $m \approx 9^{m}.7$ and slightly variable. The spectral observations of BD + 34° 3815 have revealed the orbital motion with the period $5^{d}.6$. The mass function is

$$f = \frac{M_x \sin^3 i}{\left[1 + \left(\frac{M_v}{M_x}\right)\right]^2} \approx 0.22.5\ M_\odot \tag{6}$$

The brightness fluctuations with $\Delta\tau \sim 0.1$ sec evidence the compactness of the X-ray source : $r_x < c\,\Delta\tau = 3 \times 10^9$cm. Hence, it is for sure not the usual star. The white dwarf, neutron star and black hole only may satisfy the restriction $r_x < 3 \times 10^9$ cm for the body with a stellar mass. To prove Cyg X1 to be an accreting black hole, one must obtain the mass value. Bolton (1972a, b) and Murdin and Webster (1972) evaluated by the spectrum the mass of the visible star $M_v = 20\ M_\odot$ and from the mass function (6) found M_x greatly exceed $3\ M_\odot$ and, consequently, to be a black hole.

Brucato and Kristian (1972) has measured the Doppler shift of the emission line He II λ 4686 Å. This line possibly arises in the vicinity of the X-ray source (or in the gas flowing out of the normal star to the X-ray source). The measurements allowed to obtain $M_x > 8\ M_\odot$ and thus confirmed the previous conclusion. It should be mentioned that the absence of X-ray eclipses is an evidence of the inclination angle i between the normal to the orbital plane and the line of sight be small and M_x even exceed the given above value.

And finally the independent estimate for the lower mass limit was obtained by Lyutiy et. al. (1973) on the basis of the observations of the optical variability.

It is shown in this paper that the period of the optical variability $2.^d8$ is two times less than the orbital period. The ellipticity of the visible counterpart naturally accounts for this fact*. The visible star should fill it's Roche lobe. The weak random optical fluctuations and the size of the supergiant B01 b approximating that of the Roche lobe justify the latter suggestion. Thus, the shape of the visible component coincides with the spape of it's critical Roche lobe. The parameters of the ellipticity of Roche lobe are insensitive to the mass ratio. If a, b, c are the half-axes of the ellipsoid, then for the reasonable mass ratios $b/a = 0.84 \div 0.86$, $c/a = 0.79 \div 0.80$. The amplitude of the optical variability of the rotating ellipsoid depends upon the angle between the rotational axis and the line of sight**. The ellipticity parameters and the amplitude of the optical variability $\Delta m \approx 0.1$ being known, one can easily estimate the inclination of the orbit i. It turns out to be equal to $i \approx 30^o$. Now the mass function gives

$$M_x = \frac{0.2 M_\odot}{\sin^3 i}\left(1 + \frac{M_v}{M_x}\right)^2 \approx 1.4 M_\odot \left(1 + \frac{M_v}{M_x}\right)^2 \qquad (7)$$

As was mentioned above the theory of stellar evolution in binary systems implies $M_v/M_x > 1$. Then from eq. (7) we find

$$M_x > 5.6 M_\odot$$

All the estimates given above tell us convincingly enough that M_x in Cyg X1 is a black hole. For the final sentence of course further observations are necessary.

The Source SMC X1

The source is located in the Small Magellanic Cloud and is identified with the optical star Sanduleak N_o 160 of the spectral type B01 (Liller, 1973) The optical luminosity of N_o 160 is of the same order as that of the visible component of Cyg X1. The luminosity of the X-ray source greatly exceeds that of Cyg X1 and equals to 10^{39} erg sec^{-1} For this reason the optical variability is due to the effect of illumination (the heating of the optical star by X-rays) as well as to the ellipticity. The period of the orbital motion in the system is determined from the X-ray eclipses: $T = 3^d.89$.

From the previous section we know that the luminosity of the X-ray

* The effect of irradiation by X-rays is negligible here because of the huge luminosity of the visible component: it's intrinsic energy flux greatly exceeds that of the X-rays.

** It goes without saying that one should take into account the effects of limb darkening and gravitational darkening the well-know effects in the theory of variable binary stars.

source cannot exceed the critical value $L_{cr} = 10^{38} \left(\frac{M}{M_{\odot}}\right)$ erg sec^{-1}

$$L_x \approx 10^{39} \text{ erg sec}^{-1} < L_{cr} \approx 10^{38} \frac{M_x}{M_{\odot}} \text{ erg sec}^{-1}$$

This unambiguously implies $M > 10\ M_{\odot}$ and a suggestion immediately arises that SMC X1 is a black hole (Sunyaev 1972).

There is much less information about the other two *sources* 2*U* 1700-37 and 2*U* 0900-40, and what is of importance the only ground to suppose the presence of black holes in them is the similarity of the X-ray emission and spectrum to those of Cyg X1. No estimates of components mass are available up to now. Both sources are identified with the optical stars and undergo eclipses in X-rays. The brightness fluctuations with $\Delta \tau > 0.1$ sec are observed for 2 U1700-37. Hiltner (1973) has discovered the effect of ellipticity in optical variability of the star ($m \approx 6^{m}\!.9$, Sp; B05Ib) identified with the source 2U0900-40.

The X-ray luminosity of both sources is of the order $L_x \approx 3 \times 10^{36}$ erg sec^{-1}.

The Search for Black Holes in Globular Star Clusters

As was shown by Zeldovich (1964), a compact stellar system in the course of evolution should finally form a black hole. Peebles (1972) having performed the analysis of the evolution of some particular globular clusters drew a conclusion that either the central parts of some of them should contain a great number of stellar black holes or these isolated black holes collapse merging into one massive black hole at the centre of the globular cluster.

Such an invisible massive body at the centre of the cluster must influence through it's gravitation the distribution of stars around the centre (Cameron and Truran, see Peebles 1972). If M^{bh} is the case of the black hole then it's influence will be appreciable at the distance

$$r_h = 2\ G\ M^{bh} / V^2$$

where V is the square mean velocity of stars in the cluster far from the hole. In this domain the number density of stars must be extraordinary high and rapidly increase towards the centre. A sharp peak of brightness near the centre should be observed from the Eearth. Basing upon the King's observations, Peebles draws a conclusion that nothing of the kind is observed and that no compact mass exceeding 0.008 M can be present at the centres of globular clusters

$$M^{bh}/M \leq 0.008$$

where M is the total mass of the cluster. Peebles emphasizes that if only all the stars with the mass exceeding 3 $M_{\odot}$ would have become black holes and concentrated to the centre then the ratio M^{bh}/M would be equal to a few tenthes. It badly contradicts the estimate given above. Perhaps it means that a small numbers of massive stars were formed in the globular clusters or that the explosions at the end of evolution prevented massive stars from collapsing into black holes.

Supermassive Black Holes.

The nuclei of the galaxies, where a cnosiderable mass is confined to a small volume, seem to be natural objects in which the supermassive stars and/or the supermassive black holes should emerge. Hypotheses of this kind were put forward more than once. The suggestions were also made that, if not in quasars themselves, then after termination of their active evolution at the places of their location supermassive black holes may exist. At XI and XIII Solvay Conferences Ambartsumyan (1958, 1965) suggested that the nuclei of the glaaxies must contain some unusual objects revealing themselves in active processes observed in the nuclei. Ambartsumyan believes that those are the bodies of an unknown nature, probably " white holes ".

What might be the test convincing us that a supermassive black hole is observed indeed but not any other object. Unfortunately, there is no reliable test up to now. One might attempt to explain the same observational data for the nuclei of the galaxies in any other way.

About the efforts to find the black holes in the nuclei of galaxies see Peebles (1972). In view of all said above we restrict ourselves to present some results concerning the possible observational appearances of a supermassive black hole according to the model built by Lynden-Bell (1969) and by Lynden-Bell and Rees (1971).

The mass of the supermassive black hole in the case of a quasar may be of the order of magnitude $M^{bh} \approx 10^{10}\ M_\odot$ and in the case of a galactic nucleus $M^{bh} \approx (10^7 \div 10^8)\ M_\odot$.

The gravitational attraction of the black hole at the centre of the galaxy will result in the accretion of surrounding gas. The gas in the galaxy has an angular momentum. Therefore, the gas will be drawn to the circular motion around the hole and form the disk. The whole pattern is similar to the accretion in close binary system.

The accretion rate $\dot{M}$ depends on the amount of gas flowing down to the galactic nucleus. It is difficult to determine it apriori.

If the total luminosity of the nucleus is known, one can estimate $\dot{M}$ according to the eq. (3). For the nucleus of our Galaxy $L \approx 10^{42}$ erg sec^{-1} and we have $\dot{M} = 10^{-4}\ M_\odot$ year^{-1}.

The frequency at which the major part of radiation is emitted, may be in ultraviolet or in soft X-rays depending on the particular conditions. The following expression may be of help for rough estimates

$$\nu_{max} \approx 10^{15} \left(\frac{\dot{M}}{10^{-3}\, M_\odot \;\; \text{year}^{-1}}\right)^{1/4} \left(\frac{M}{10^{8}\, M_\odot}\right)^{-1/2}$$

It is obtained under the assumtion that the disk around the black hole radiates as a black body. Aa was mentioned by Lynden-bell and Rees (1971), the incident gas may contain dust. This dust will be weeped out of the disk by the light pressure and form an opaque cloud. The disk radiation is absorbed by this dust cloud and reemitted in infrared.

The accretion instabilities of various types may give rise to the activity of the nuclei. In particular, the hot spots on the disc, where the lines of magnetic field reconnect, may emit in the radio band the synchrotron radiation of protons (the free path of electrons turns out to be too short and they cannot get accelerated, Lynden-Bell, 1969, Ekers and Lynden-Bell, 1970).

Perhaps, there are black holes in the nucleus of our Galaxy and in the nuclei of other galaxies ?.

Conclusions.

We do believe that the discovery of compact X-ray sources with mass $M > 3\, M_\odot$ in binary systems and their further thorough investigation will decisively prove in the nearest future the existence of black holes in the Universe.

The most important are 1) the ascertaining of the variability character with the best time resolution possible — to evaluate the size of the object and the minimum $\Delta\tau$ for quasiperiodic phenomena 2) the reliable mass determination from the orbital motion.

It looks like the witty prediction by Wheeler of the black hole discovery in 1972 come true.

The convincing evidence of the black hole existence would mean not only the discovery of a new interesting object but would be proof of quite extraordinary topological structure of the real spacetime — for the black hole is not only the figurative name (felicitously given by Wheeler)but a peculier kind of a hole in the spacetime in fact.

The discovery of the black hole would be together with the cosmological redshift one of the greatest discoveries of the astrophysics of 20-th century.

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Discussion of the report of I.D. Novikov

G.R. Burbidge: I shall confine myself to a few general arguments.

As far as normal stars are concerned, we know that stars on the main sequence have masses in the range $\sim 0.1\ M_\odot - \sim 100\ M_\odot$. Now any gravitational theory will lead to a critical mass M_c. The best estimate that we have at present is that $M_c \simeq 2\ M_\odot$. It is therefore clear that unless all stars with initial masses greater than M_c eject enough mass to end their evolution with $M < M_c$, they will collapse, and it is generally concluded that black holes will be formed. Thus ,unless we invoke a hitherto undiscovered law, that of stellar precognition, we must conclude that the existence of normal massive main sequence stars normally implies that black holes must also be present in large numbers.

Let us now turn to evidence for the possible existence of massive black holes. There is no evidence that massive superstars ever form in low-density regions in galaxies or in intergalactic space. However, in higher-density regions, particularly in the nuclei of galaxies, we know that quite high star densities ($>$ 1000 stars pc^{-3}) are present. The inevitable sequence of events is then:

Steady contraction of the central star cluster with evaporation of some stars, until the density is so high that the stars interact strongly, colliding inelastically to form more massive stars, some of which may evolve to give supernovae. Eventually, the composite stars will coalesce giving a single massive object. While rotation may stabilize it for some time, it will eventually collapse to form a massive black hole. The timescale for the whole evolutionary process to take place is in some doubt. In all conventional theories which attempt to explain explosive events in galactic nuclei and QSOs it is supposed that the large-scale energy release takes place in stellar collisions, or supernova explosions, or in the rotation phase, or in gravitational collapse, or because matter is accreted by the black hole. However, the occurrence of explosive events in galactic nuclei and QSOs is not by itself direct evidence that black holes must be formed, since less conventional theories invoking the presence of white holes, or the creation of matter in galactic nuclei are also able to explain the occurrence of violent events.

Can we directly detect massive black holes in galaxies? The mass-to-light ratios in solar units in elliptical galaxies are generally in the range 30-50. Part of this mass may be in the form of collapsed objects, but this conclusion is not necessarily demanded by the observations, since models can be made in which this mass-to-light ratio can be obtained if large numbers of low-luminosity main sequence stars, white dwarfs and neutron stars are invoked.

If massive enough black holes were present in the centers of galaxies, they might be discovered from studies of the light distribution at the centers and the form of the velocity dispersion of the stars in the nuclear regions. Wolfe and I carried out a detailed study of this problem and concluded that no very large masses were present. However, we could not rule out black hole masses $< 10^9 - 10^{10}\ M_\odot$ at the centers of elliptical galaxies. Nor could we rule out the possibility that a large amount of mass in the form of lower mass black holes is present and follows the stellar distribution.

Finally, we cannot exclude the possibility that much mass in clusters of galaxies is in the form of massive black holes.

M. Rees: I should like to add a few comments concerning the detectability of very massive ($\gtrsim 10^{11}\ M_\odot$) black holes in intergalactic space. There are two ways of detecting such objects within cluster of galaxies which seem by no means unfeasible. The first involves searching for galaxies tidally distorted by an invisible mass. Van den Berg has in fact argued that the Virgo Cluster could not be gravitationally stabilized by galactic-mass collapsed objects on the grounds that, in the majority of cases where a galaxy appears tidally distorted, a neighbouring galaxy responsible for the distortion is clearly identifiable. (This argument also sets a limit to the total mass in the form of dark objects *more* massive than galaxies; but it sets no significant constraints on the number of low-mass black holes in clusters). The second method of discovering black holes in clusters involves searching for the radiation arising from accretion. Because the accretion rate depends on M^2 and also because the accretion radius is so large that the angular momentum of infalling matter becomes important, the luminosities can be very high if the intra-cluster gas has the temperatures and densities inferred from other arguments.

Neither of these techniques would provide a sensitive test for the presence of black holes *between* clusters of galaxies, but there is still then the possibility of observing an instance of the gravitational lens effect. Many people have pointed out that scatter would be introduced into the magnitude-redshift relation for distant objects if the matter in the universe were inhomogeneously distributed. If there were enough black holes in the universe to contribute the " closure density ", then this scatter would amount to ~ 1 magnitude for objects with redshifts ~ 1 (provided that the angular size of the objects is sufficiently small). Press and Gunn have recently made the further point that the dominant effect in this situation is produced by the particular black hole which happens to lie closest to the line of sight, and that there is then a reasonable chance that the source image would have a characteristic " double crescent " shape. Detection of this phenomenon would perhaps be feasible by VLBI techniques if the black hole masses were $\gtrsim 10^4$ M$_\odot$ (characteristic angular scales $\gtrsim 10^{-3}$ sec) and by optical observations if the holes have masses $\gtrsim 10^{10}$ M$_\odot$ (implying image sizes $\gtrsim 1''$). An apparent binary caused by the gravitational lens effect could be distinguished from a real binary if the two components were observed to vary almost in unison. Masses $>> 10^{11}$ M$_\odot$ could be detected by this technique. (But they would produce the effect even if they had galactic dimensions, so some further argument would be needed to prove that the " lenses " really were black holes).

It is also perhaps worth mentioning the possibility that very small black holes (M $<<$ M$_\odot$) might form in the early stages of the " big bang ". Those " mini-holes " could have any mass down to $\sim 10^{-5}$ gms (the " Planck mass "). Their accretion rate would be negligible, unless they were in the middle of a dense star; and they would not be readily detectable unless they possessed an electric charge, in which case they could simulate the behaviour of very heavy cosmic ray particles,

E. P. J. van den Heuvel: I would like to comment on the expected number of black holes as companions to normal massive stars in the galaxy. As you heard from Dr. Bahcall, we have only one likely candidate for a black hole binary system, namely Cygnus X-1. Two more binaries in our galaxy (2U1700-37 = HD 153919 and 2U0900-40 = HD 77581) share many of the properties of Cygnus X-1, namely:

1) they contain an early-type supergiant;

2) their X-ray emission shows very rapid fluctuations on a timescale of fractions of seconds;

3) their X-ray fluxes are very large ($\gtrsim 10^{37}$ erg/sec).

(Also the extragalactic source SMC X-1 seems to be of this type). Although the mass functions of these systems are too small for indicating conclusively that we are dealing with black holes, it seems very tempting to consider these systems as black hole binaries because of their similarity to Cygnus X-1.

Only one massive X-ray binary—Centaurus X-3—shows regular pulsations. If we accept the evolutionary picture outlined in my talk of Tuesday, one observes that a binary system that contains a neutron star or black hole can only have been formed from the evolution of a massive close binary if the initial mass of the primary exceeded about 16 to 18 solar masses.

From Dr. Cameron's review of this morning—and especially also from the work of Gunn and Ostriker (1970) on the statistics of pulsars—it seems likely that single stars finish life as neutron stars or black holes if they initially has masses in excess of about 4 M$_\odot$. Specifically, pulsars are expected to originate from stars in the mass range between about 4 M$_\odot$ and 10 M$_\odot$. The much higher mass limit required for a supernova explosion of a component of a close binary is due to the mass exchange which, prior to the final stages of evolution, has removed over two third of the stellar mass.

On the basis of the above picture—what is the ratio of the formation rate of single neutron stars or black holes to the formation rate of neutron stars and black holes in close binaries?

The stellar birth rate function is proportional to the Salpeter-initial mass function, which, for $M \geq 2\,M_{\odot}$ can be approximated by (cf. Limber 1960): $\Psi(M) \sim M^{-2.5}$. Hence, the total formation rate of single main-sequence stars more massive than a certain value M_1, will be proportional to $M_1^{-1.5}$. Consequently, per unit time, eight times more stars will be formed in the range $M \geq 4\,M_{\odot}$ than in the range $M \geq 16\,M_{\odot}$. From the work of Blaauw and van Albada (1974) we know that some 30 per cent of the stars in young OB associations are spectroscopic binaries. Taking into account that some of these systems may already be evolved, the assumption that some 25 per cent of all OB stars are formed in close binaries seems a reasonable one. Under this assumption the formation rate of single stars with $M \geq 4\,M_{\odot}$ will be 32 times the formation rate of close binaries with primary mass larger than $16\,M_{\odot}$.

In a steady state (which holds for $M \geq 2\,M_{\odot}$ -cf. Schmidt, 1959) the death rate of stars equals the birth rate. Consequently, the production rate of single collapsed stars is expected to be 32 times the production rate of close binaries with a collapsed component.

A single radio pulsar lives probably for over 10^7 yrs (as ages between 10^7 and 10^8 yrs have been observed for a number of pulsars (Hunt, 1971)). In a close binary, the lifetime of the collapsed object is restricted by the time during which the companion remains on the main sequence. This time is of the order of 5×10^6 yrs for OB stars. So, if one assumes that all stars in the above mentioned mass ranges finish life as neutron stars, one expects that roughly one out of every 64 radio pulsars occurs in a close binary system. (If the lower mass limit for becoming a supernova in a close binary is $18\,M_{\odot}$ instead of $16\,M_{\odot}$, this fraction is one out of 80). From the remarks of Dr. Smith on Monday we know that there are 110 pulsars known to date, none of them in a binary. Under the above assumptions we would have expected one or two pulsars to be members of close binaries. The fact that there are none might perhaps be explained in two different ways, viz.:

(a) the radio emission of a pulsar in a close binary may be masked or modulated by the presence of the companion. Arons and Bahcall (private communication) have shown that if some plasma is around in the binary system, the pulsar signal may be smeared out in such a way that—although the plasma emits radio waves—the pulsed character of this emission has disappeared; or

(b) the majority of the stars more massive than 16 $M_\odot$ in close binaries leave black hole remnants instead of neutron star remnants.
The latter possibility seems attractive to me since, as I mentioned above, only one out of the four massive galactic X-ray binaries known so far is an X-ray pulsar. If we adopt the viewpoint that Cygnus X-1, 2U1700-37 and 2U0900-40 are indeed black holes, one might, on the basis of these rather poor statistics—possibly infer that only one out of every four stars more massive than 16 $M_\odot$ in a close binary finishes its life as a neutron star, and three out of four as black holes.

In that case, one would—if one corrects for the fact that single stars more massive than 10 $M_\odot$ might finish life as black holes too (Gunn and Ostriker 1970)—expect that only one out of every 192 pulsars is formed in close binary, instead of one out of 64.

Hence, we would have to wait until another 82 pulsars have been discovered before we have a fair chance for finding a pulsar in a close binary. From the statistics which I presented Tuesday one expects in the galaxy some 3000 to 4000 massive close binaries with a " quiet " collapsed companion (i.e. a companion onto which no accretion takes place).

On the basis of the above reasoning it seems not unlikely that some 75 per cent of these systems contain a black hole. This means that several thousands of normal main-sequence OB stars in the galaxy might have a close black hole companion.

A lower limit to the number of quiet black hole binaries follows from the fact that the " quiet " stage of a massive close binary with a collapsed component lasts for some 5×10^6 yrs (the nuclear timescale of an OB star), whereas the X-ray stage only lasts for some 2×10^4 yrs (a few times the Kelvin time scale). Hence, in a steady state of star formation, the number of massive binaries with a quiet collapsed component will be some 250 times larger than the number of massive X-ray binaries. From the fact that we observe three Cygnus X-1 type systems in the galaxy, one then infers that there must be at least some 750 massive binaries with a quiet black hole component.

From the existence of one Centaurus X-3 type system one similarly infers that at least some 250 massive close binaries with a quiet neutron star component must exist in the galaxy. Evidently, the real numbers will be several times larger than these lower limits, since—due to interstellar obscuration—we have, so far only optically observed the massive X-ray binaries within 3 Kpc distance.

Adopting the estimate of 3000 - 4000 massive close binaries with a quiet collapsed component in our galaxy, one finds that there must be some 20 of these systems within 1 Kpc distance and some 80 within 2 Kpc (a radius of 14 Kpc for the galactic disk was adopted).

The companions are expected to be main-sequence OB stars more massive than about 16 $M_\odot$; consequently they will be more luminous than $M_v = -4^m$. Let us adopt some 0.5^m interstellar extinction within 1 Kpc and 1.5^m within 2 Kpc distance. One then expects that some 20 OB stars brighter than $m_v = 6.5^m$ and some 80 brighter than $m_v = 9.0^m$ have a quiet collapsed companion.

Sutantyo (1974) has pointed out that—from our experience with the massive X-ray binaries, and also from the fact that mass exchange must have occurred prior to the supernova explosion—we expect close binaries with a collapsed component to *have very small mass functions* (instead of the large ones suggested by Trimble and Thorne (1969)). Therefore, normal single-line OB-type spectroscopic binaries with very small mass functions are expected to be the most likely candidates for the black hole binaries which must exist in considerable numbers in the galactic system.

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E. M. Burbidge: I would like to mention an observation which has always puzzled me, and which might have a bearing on the possibility or impossibility of observing either a single central massive black hole in an elliptical galaxy or a distribution of black holes, both of which G. Burbidge and A. Wolfe considered. This is the fact that certain elliptical galaxies which are strong radio sources have a different distribution of stars in the center. The best case is the double elliptical galaxy NGC4782/3, which apparently contains only stars with no gas, and is a 3C radio source. I believe it is not known which galaxy the radio source is centered on (they are about 40 arc sec apart), but the two galaxies, which are about the same outer dimensions and luminosities, and therefore, probably have similar masses, have very different light-distribution in the center. One has a central peak with isophotal countours very similar to other galaxies in a loose cluster centered on the double, while the other has a much flatter central peak and wider isophotes. This must represent different stellar distributions and should therefore have a bearing on whether a massive conglomeration of stars had formed in the center of one galaxy and not in the center of the other.

F. Pacini: I wonder if somebody could comment on the very minimum possible periodicities which can arise from very compact neutron stars. This is obviously relevant in discussing the possibility of " black hole signature ".

A. G. W. Cameron: The vibration of neutron stars has been calculated using some of the older equations of state. The minimum period occurs at a mass slightly below the maximum stable mass and is a fraction of a millisecond. Of course, overtones will have still smaller vibration periods, but it perhaps becomes somewhat implausible that high overtones should be excited with appreciable amplitudes.

For some years I was interested in problems associated with the vibrations of neutron stars, because it seemed to me at one time that there was a possibility that such vibrations might be responsible for a great deal of cosmic ray acceleration. This idea proved not to be viable because it turned out that neutron star vibrations can be quickly damped. The damping times were of the order of seconds, and two wholly different types of process can be responsible. One process results from a phase lag during the vibration period in the interconversion between neutrons, protons, and hyperons. It seems likely that such interconversion would take place without emission of neutrinos or antineutrinos, and hence this becomes a rapid way of converting vibrational energy into internal heat. K. Thorne and others pointed out a second mechanism: if a vibrating neutron star is also rotating, then the coupling between the vibration and the rotation will lead to an excitation of quadrupole deformations, with energy being channeled from the nonradial modes into the radial modes in a very short time. Thus neutron star vibrations are not expected to be important in nature unless one can find specific mechanisms for continually exciting them, and such mechanisms seem to require a great deal of power input.

GENERAL DISCUSSION

NEUTRON STARS, BLACK HOLES AND BINARY X-RAY SOURCES

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Invited talk delivered at the 1973 Solvay Conference on Relativistic Astrophysics, as updated on the basis of a colloquium delivered at Harvard University, February 1974 and an invited talk delivered at the A.A.A.S. annual meeting in S. Francisco, March 1974.

1. INTRODUCTION

For a long time it has been recognized that Einstein's theory of general relativity is very likely the most elegant theoretical framework in modern physics. However, all conceivable effects predicted by this theory and observable inside our own solar system are largely negligible and could be taken into account by a simple " tabulation " of the correction factors from the traditional Newtonian physics. If these observable effects were found to be so negligible in the entire Universe, the relevance of general relativity, despite its mathematical elegance, would certainly have been very limited. However, it has become more and more clear since the pioneering work of Landau[1], Chandrasekhar[2], Baade and Zwicky[3] that to properly describe the processes occurring at the late stages of evolution of a star after all the sources of its thermonuclear energy have been exhausted, a fully relativistic theory of gravity is needed and very large deviations from a Newtonian approach are to be expected. Then the process of gravitational collapse appears to be the natural testing ground where one may probe some of the most novel and unique predictions of Einstein's theory. From an astrophysical point of view, this process is also of the greatest relevance since it represents energetically, by far the most important part of the life of a star. (See § 7).

We have today credible evidence for the existence, and in some cases direct observation, of a large number of collapsed objects (neutron stars or black holes) inside our own galaxy. Their number is most likely larger than 10^8. (See § 7).

Nothing did more for the development of this entire field of research in relativistic astrophysics than the clear identification in 1968 of a pulsar (NP 0532) at the center of the Crab Nebula[4]. That the Crab Nebula, still expanding today at a velocity of $\sim$ 800 km/sec, was the rem-

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nant of the supernova explosion recorded with great scientific accuracy by the Chinese and Japanese astronomers in 1054 had been known for a long time[5]. But it was not until very recently that a large number of astrophysicists took seriously the idea presented by Baade and Zwicky[3] that a neutron star should be expected to be found at the center of this expanding envelope remnant of the supernova explosion. The unquestionable evidence for the identification of the star of 16.6 th, magnitude at the center of the Crab Nebula with the pulsar NP 0532 and its further identification with a neutron star came from the simultaneous observations in the radio[4] and in the optical wavelengths[6] of a sharply-defined pulsational period in the electromagnetic radiation emitted by this star (see Fig. 1). The arguments forcing the identification with a neutron star[7] are most convincing, thanks to their simplicity:

(1) The total electromagnetic energy emitted by the pulsar NP 0532 over the entire electromagnetic spectrum from radio waves to X-rays, is of the order of 5×10^{35} erg/sec ($\sim 10^2\ L_\odot$)[8]. This object has therefore to be very massive ($\sim M_\odot$) in order to account for this very large steady emission of radiation.

(2) The radiation is emited in sharp pulses with a period $P \sim 33$ m-sec. The object has therefore to be very compact in order to modulate that very large amount of energy over such a short period of time.

(3) The period of pulsation increases monotonically with time[9] with

$$\frac{dP}{dt} \simeq 13.5\ \mu\ \text{sec/yr.}$$

These three points and the entire energetics of the source and the remnant were explained at once by the assumption that the pulsar was indeed a neutron star. The period of pulsation and its monotonic increase could then be interpreted as given by the rotational period of the neutron star and by a loss of rotational energy due to braking processes occurring in its magnetosphere[10]. In turn if a value of the moment of inertia predicted by the theoretical computations of neutron star equilibrium configuration was adopted for the pulsar, then this loss of rotational energy came out to be of the same order of magnitude as the observed radiation flux from the pulsar and the entire nebula[11].

$$\left(\frac{dE}{dt}\right)_{\text{rot}} = I_{\text{theor}}\,\omega_{\text{obs}}\left(\frac{d\omega}{dt}\right)_{\text{obs}} \sim \left(\frac{dE}{dt}\right)^{\text{emitted}}_{\text{obs}}$$

This explanation of the nature of the pulsar NP 0532 gave rise to a deeper understanding of the Crab Nebula; but, even more important, it promoted a renewed and profound interest in the theoretical and experimental analysis of the two possible outcomes of gravitational collapse; neutron stars and black holes. Astrophysicists with a new spirit dictated by the experimental evidence of the existence in our own galaxy of these most extreme regimes of pressure, density and gravitational fields, returned with much attention to a detailed examination of the fully relativistic analyses presented in the works of Oppenheimer and his students[12, 13, 14] with the aim of observing and verifying some of the most novel and revolutionary predictions of general relativity.

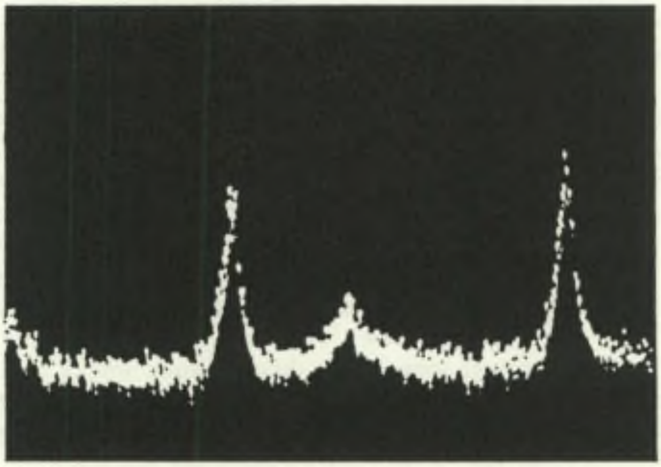


Fig. 1. Pulse shape of the pulsar NP0532 in the Crab Nebula. The two successive main peaks are ~ 33 msec apart. An interpulse follows 12 msec after the main pulse. (Picture taken with the 36 inch optical telescope at Princeton University and reproduced by courtesy of R. J. Groth III).

2. NEUTRON STARS

Oppenheimer and Volkoff[13] in 1939 were the first to give a detailed treatment of the equilibrium configuration of neutron stars. Their assumptions were very clear:

(1) Describe the microphysical structure of a neutron star by an equation of state obtained from the quantum mechanical treatment of a degenerate ($T = 0$) relativistic gas of neutrons fulfilling Fermi statistics.

(2) Describe the macroscopic structure of a neutron star (mass, radius, density distribution) by the use of Einstein equations as applied to a perfect fluid distribution of matter.

The two major conclusions of the article were of comparable clarity:

(1) Stable equilibrium configurations of neutron stars can only exist in a finite range of masses and densities

$$0.1\,M_\odot \leqslant M \leqslant 0.7\,M_\odot$$
$$1.0 \times 10^{14} \leq \rho \leq 3.6 \times 10^{15}\ \mathrm{g/cm^3}$$

(2) There is a critical value of the mass of a neutron star over which no equilibrium configuration can possibly exist. If the initial mass of a

star is large enough, unless fission due to rotation, or ejection of mass reduces the star to a mass smaller than this critical value, then, after the exhaustion of thermonuclear sources of energy, the star will gravitationally collapse contract indefinitely, never reaching true equilibrium.

Since the initial work of Oppenheimer and Volkoff, much work has been done in this field and the major assumptions adopted in their work have been critically reanalized. The major criticisms have ranged from the validity of any use of the concept of an equation of state for matter in these most extreme gravitational fields[15], to the use of the Einstein theory of gravitation for the computation of the equilibrium configuration[15], to the use of a degenerate non-interacting gas of Fermions (free neutrons) for the description of the neutron star material[16]. This last point particularly has lately generated much theoretical research[17]. In one direction there have been attempts to generalize to neutron star matter the two-body potentials obtained from laboratory experiments on the collision of two nucleons[18]. In a different direction, attempts have been made to generalize to neutron star matter the statistical treatment[19] first introduced by Fermi in the analysis of high energy collisions[20]. A summary of some of the different equations of state which have been purported is given in Fig. 2 where a direct comparison is made with the free nucleon equations of state used by Oppenheimer and Volkoff.

It is remarkable that after more than thirty years of theoretical research, not many of the conclusions reached by Oppenheimer and Volkoff have been changed. Recent analyses have shown that the effects of nuclear interactions in computing the masses of the equilibrium configurations of neutron stars are indeed far from negligible (see Fig. 3). However, neutron stars can still reach stable equilibrium configurations only in a finite range of masses:

$$0.1\ M_{\odot} < M \lesssim 1.45\ M_{\odot}$$

Apart from this mere change in the numerical value of the critical mass in no way has it been possible to overcome the necessity for the existence of the process of gravitational collapse.

Much work has also been done in the analysis of the composition of the crust of a neutron star. The following regimes are expected to be encountered as one goes down in depth from the " atmosphere " of a neutron star: magnetic fields of the order of 10^{12} Gauss are expected to exist on the surface. In the outermost layers of matter, as suggested by Ruderman[21] matter might exist in the form of very dense (average density of the order of 10^4 g/cm^3) one dimensional " hairs " parallel to the lines of force of the magnetic field. The next shell of material in the density range between 10^4 g/cm$^3 \lesssim \rho \lesssim 10^7$ g/cm^3 is expected to behave as a lattice of nuclei embedded in a degenerate gas of relativistic elec-

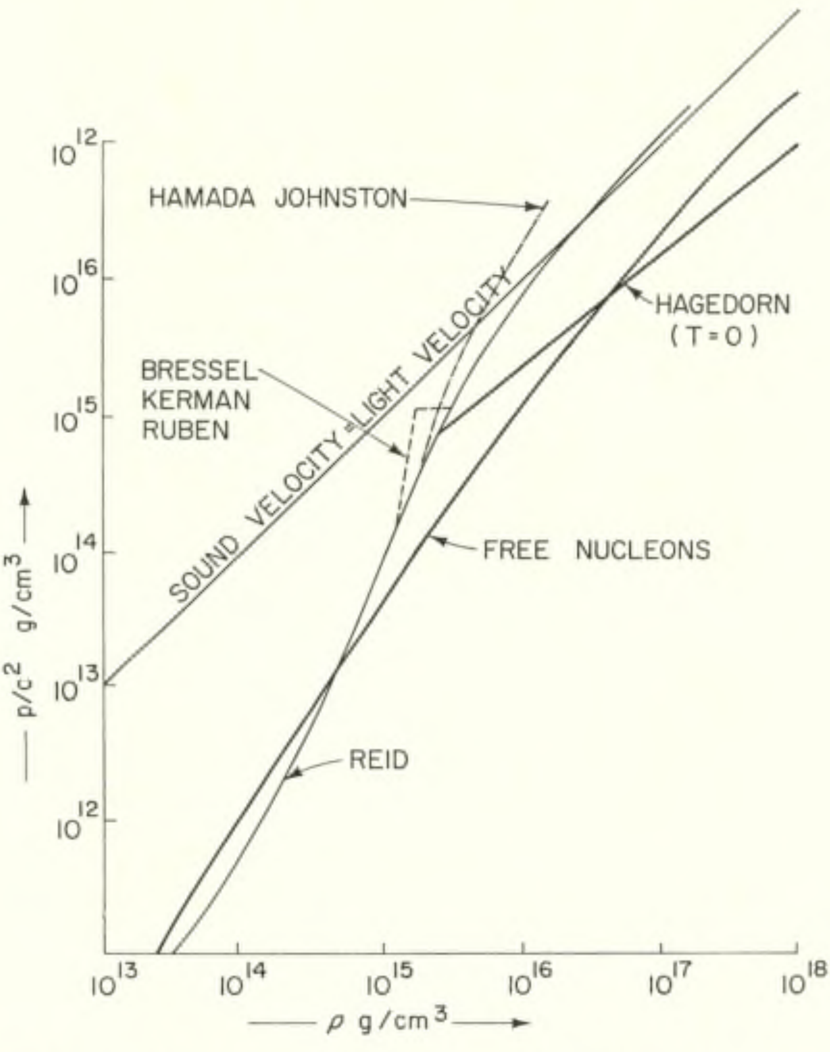


Fig. 2. Pressure versus density for selected equations of state describing neutron star material. Both the equations of state with a Reid and an Hamada-Johnston two body nuclear potential violate causality at supra-nuclear densities. The free-nucleons approximation for densities smaller (larger) than 4.6×10^{15} g/cm³ gives values of the pressure systematically larger (smaller) than the one given by an equation of state taking into account nuclear interactions. The masses of neutron stars with central densities $\rho \lesssim 4.6 \times 10^{15}$ g/cm³ ($\rho \gtrsim 4.6 \times 10^{15}$ g/cm³) computed with the free neutrons equation of state will always be larger (smaller) than the one computed out of a realistic equation of state. (Details in reference 17, see also Fig. 3).

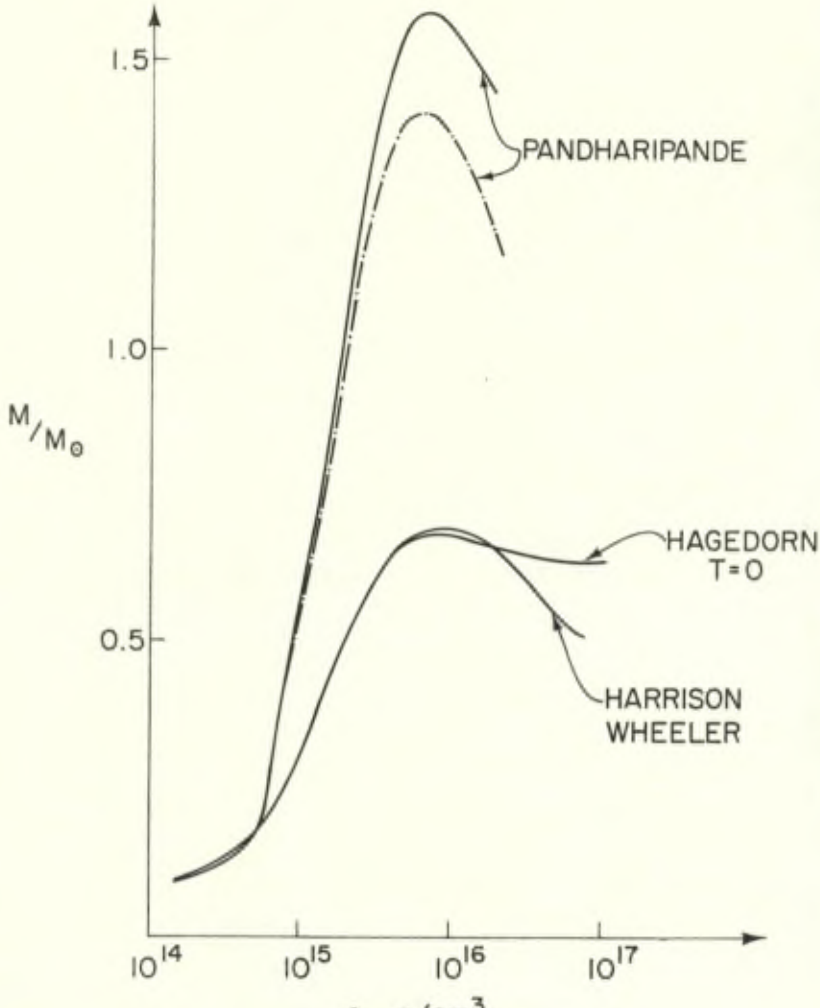


Fig. 3. Masses of the equilibrium configuration of neutron stars plotted as a function of the central density for selected equations of state. The Pandharipande equation of state takes into account the strong interactions between nucleons. The Harrison-Wheeler equation of state neglects all the nuclear interactions and uses substantially a "free particle" approximation. The Hagedorn equation of state is based on a thermodynamic approach derived from the theoretical analysis of high energy collisions between elementary particles and applies only asymptotically for $\rho \sim 5 \times 10^{15}$ g/cm^3. No matter what the different assumptions in the equation of state, the value of the critical mass is contained in a finite range $0.69 \lesssim m_{crit} \lesssim 1.45\ M_\odot$ (Details in reference 17).

trons (white dwarf material), the reason being that the Fermi energy of the electrons is very much higher than the ionization energy of the atoms. Since the material is expected to be at the complete endpoint of nuclear evolution, the nuclei are thought to be mainly iron nuclei. However, as pointed out by F. Dyson[22], in this material some incompleteness of combustion could occur (as, for example, H — burned to Helium, but not burned to iron) and the formation of some compounds could still be possible. The details of the thermonuclear reactions taking place in the collapse of a core of a white dwarf material are not sufficiently well known to state what nuclear material and in what amount it should be expected in the upper layers of a neutron star. The important point is just to give a conceivable example of the incompleteness of combustion implying the presence in any given layer of more than one nuclear species. The simpler example considered by Dyson clearly shows that a particularly stable configuration is given by a lattice with Na Cl structure and with Fe-He composition (Iron-Helide).

At densities between 10^7 g/cm^3 $\lesssim \rho \lesssim 10^{11}$ g/cm^3 relativistic electrons transform bound protons into neutrons. Under normal circumstances, in fact, the total packing of a nucleus, under the two conflicting effects of nuclear and electrostatic forces is minimized for a value of $Z = 28$ and $A = 56$. A relativistic electron transmutes a nucleus of charge Z and atomic number A by inverse beta decay.

$$e + (Z, A) \rightarrow (Z - 1, A) + \nu$$

The nuclei become neutron rich compared to nuclei unpressured by electrons. For these neutron rich nuclei the mass number $A = 56$ no longer represents the point of maximum stability. Stability shifts to higher A values. The details of this shifting process are far from being well understood. For any electron pressure there corresponds a nucleus with a fixed value of Z and A which is in beta equilibrium with the electrons and has the most favorable packing fraction. At still higher densities in the range 10^{11} g/cm^3 $\leqslant \rho \leqslant 5 \times 10^{12}$ g/cm^3 nuclei become so heavy ($A \sim 122$) and so neutron rich ($N/Z \sim 83/39$) that neutron " drip " occurs. An " atmosphere " of unbound free neutrons is formed. With a further increase in density, the Fermi energy of the electrons increases and the nuclei become even more neutron rich. The number of free electrons increases further. Three different components characterize this range of densities: (1) an ultrarelativistic degenerate gas of electrons, (b) a system of heavy nuclei, (c) a degenerate neutron gas.

The contribution of the nuclei to the pressure is always negligible while the contribution of neutrons becomes more and more important with the increase of density. At densities already of the order of $\sim 5 \times 10^{12}$ g/cm^3 the pressure of the degenerate gas of neutrons, extremely large by comparison to the nuclei pressure, is comparable to the pressure of the ultrarelativistic degenerate electron gas. To a further small increase in the density, there corresponds the disappearance of nuclei as such. The material of the star is uniquely formed of electrons, neutrons and protons in equilibrium against beta decay.

The properties of the material of the crust of a neutron star have been analyzed in depth in recent years largely using notions of solid state physics. Major contributions in this analysis have been made by D. Pines[23] and collaborators, M. Ruderman[24], as well as F. Dyson[22], R. Smoluchowsky[25] and C. Rhoades[26]. The major directions of research have been toward the determination of the composition, strength and conductivity of the material contained in the crust. The strength of this material is so small when compared with gravitational forces existing at the surface of the neutron star, that only " mountains " of a few centimeters or less could be supported on the surface. This entire analysis could, indeed, prove to be of importance for the explanation of the tiny " spin up " observed in the period of Pulsars[27, 28] ($\Delta P/P \sim 2 \times 10^{-6}$ in the Vela Pulsar and $\Delta P/P \sim 10^{-9}$ in the case of the Crab Nebula Pulsar) as well as in the understanding of the electrodynamic processes taking place near the surface of a neutron star. The detailed treatment of the crust of a neutron star is also of relevance for the determination of the value of the moment of inertia of a low mass neutron star. It can be, however, totally neglected in the computation of the value of the critical mass against gravitational collapse.

The reason is simply stated: The crust of a neutron star which extends a few tenths of kilometers in the case of a configuration of equilibrium corresponding to a central density $\rho_c \sim 10^{14}$ g/cm^3 becomes extremely thin for configurations of equilibrium with larger values of the central density. For a neutron star with a central density $\rho_c \sim 5 \times 10^{15}$ g/cm^3 the entire configuration of equilibrium has shrunk to a radius of $\sim$ 10 km, the crust is only a few hundred meters thick and only a few percent of the total mass of the star is contained at a density $\rho \leqslant 10^{13}$ g/cm^3.[17]

If we focus, therefore, on the fundamental issue of the unavoidability of a neutron star reaching a critical mass against gravitational collapse, our attention is mainly directed to the physical processes occuring at nuclear and supranuclear densities. However, in no way from our knowledge of laboratory nuclear physics can we hope to infer a realistic equation of state for these regimes of densities and for a system of 10^{57} nucleons.

Despite these complications, recently, on the ground of a completely general variational principle, it has been shown by Rhoades and Ruffini[29] that quite independently from any detail of the equation of state at nuclear and supranuclear densities, an absolute maximum mass to the neutron star equilibrium configuration can be established.

This variational principle applies in complete generality to any distribution of a perfect fluid in general relativity and simply establishes that the maximum mass of an equilibrium configuration for a fixed central density is obtained for an equation of state which maximizes at every density the velocity of sound of the material. Therefore, an absolute upper limit to the neutron star mass can be immediately obtained under the following conditions:

(1) At densities lower than 4.6×10^{14} gr/cm^3 we choose the equation of state of a degenerate ($T = 0$) non interacting neutron gas since this equation maximizes the speed of sound of the neutron star material by comparison to any realistic equation of state taking into account nuclear interactions substantially attractive in this range of densities.

(2) At densities larger than 4.6×10^{14} gr/cm^3 nothing is known for certain on the equation of state of neutron star material, we then choose that most extreme equation of state uniquely consistent with the conservation of causality with a velocity of sound equal to the speed of light.

On the ground of these two assumptions it is then possible to establish an absolute upper limit to a neutron star mass: $M < 3.2\ M_{\odot}$. It is hopeless to try to establish the effective value of the critical mass or, for that matter, of the radius, density distribution, or moment of inertia of a neutron star by direct theoretical arguments. Progress at this moment can be made only through collection of experimental data on neutron stars and through a direct comparison with existing theoretical predictions.

It is also clear that data cannot be collected from detailed analysis of pulsars for at least two different reasons:

(1) Out of the 120 pulsars observed[30] none is in a binary system and in no way can we then obtain a direct measurement of the mass of the neutron star.

(2) Apart from the glitches and microglitches[27, 28] the emission from pulsar is extremely steady and not giving any information on the internal structure of the neutron star.

In sharp contrast, all the binary x-ray sources observed (see § 5 and § 6) are in binary systems and the processes of accretion occurring in the collapsed object gives us a very large amount of information both on their electromagnetic structure and internal composition.

For the sake of an example it is interesting to stress how the knowledge of macroscopic parameters of a neutron star like the moment of inertia as a function of its mass can be used as a probe into the equation of state of neutron star material (see Fig. 4).

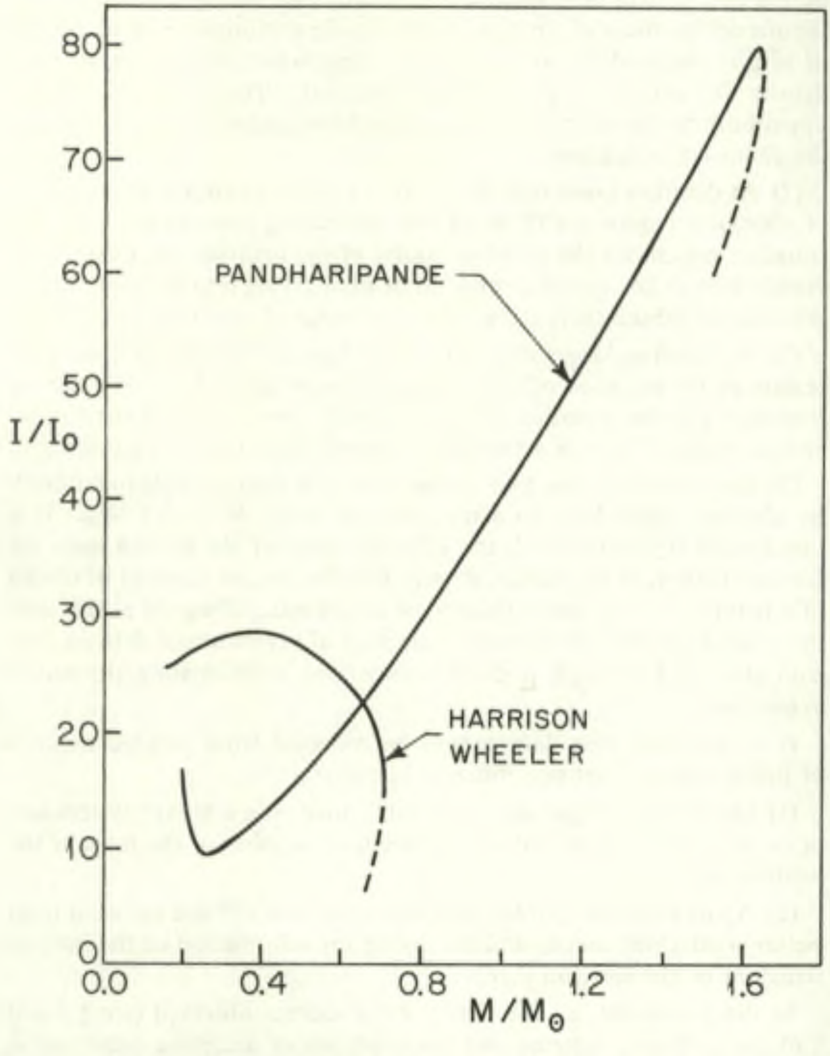


Fig. 4. Moment of inertia plotted as a function of the neutron star mass for selected equations of state. The dotted lines correspond to unstable equilibrium configurations. The major difference between the Pandharipande and the Harrison-Wheeler equations of state comes from the treatment of nuclear forces between nucleons at nuclear and supranuclear densities (details in ref. 17). It is important to realize that in a process of accretion, the mass of a neutron star increases and possible effects due to the change of the moment of inertia could be detected through the change of rotational period of the neutron star (see § 5). I_o is here given by $M_\odot$ km^2.

3. BLACK HOLES

If we turn now to the analysis of black holes, once again the fundamental work in this field goes back to Oppenheimer. In 1938, Oppenheimer and Snyder[14], in one of the most beautiful papers ever written in general relativity were able to describe with a few essential formulae all the major features of a star undergoing gravitational collapse: " The radius of the star approaches asymptotically its gravitational radius, light from the surface of the star is progressively reddened, and can escape over a progressively narrower range of angles ". " The total time of collapse for an observer co-moving with the stellar matter is finite... an external observer sees the star asymptotically shrinking to its gravitational radius ".[14] Much has been learned since 1938 about the physics of these totally collapsed objects[31], but, again, none of the conclusions reached by Oppenheimer has been modified or disproved.

We understand today that quite apart from the Schwarzschild black holes originally investigated by Oppenheimer and Snyder and uniquely characterized by their mass there exists an entire class of collapsed objects characterized by three different parameters m, mass, e, charge and L, angular momentum (see Fig. 5).

One major direction of research has been aimed toward a deeper understanding of analogies and differences between these different collapsed objects. One of the most powerful tool to advance in this analysis has been the study of gedanken processes of capture of test particles by the black hole and how transitions can occur from one kind of black hole to another by accretion of selected particles and by gain or depletion of charge mass and angular momentum. Examples of these processes are shown in Fig. 6 and Fig. 7.

The most striking result in the analysis of these transformations has been the possibility of differentiating in the accretion process between two radically different kind of transformations: reversible and irreversible[32, 33].

By capture of charged test particles endowed with angular momentum, we can always modify the mass (m), charge (e) and angular momentum (L) of a black hole:

$$\begin{aligned} m' &= m + \delta m \\ e' &= e + \delta e \\ L' &= L + \delta L \end{aligned}$$

By further capture of a test charge of opposite sign and opposite angular momentum, a black hole can reacquire its initial value of charge and angular momentum:

$$\begin{aligned} m'' &= m' + \delta m' \\ e'' &= e' - \delta e = e \\ L'' &= L' - \delta L = L \end{aligned}$$

Usually $m'' > m$ since two particles have been captured by the black hole. However, between all the possible transformations there exists a subset of transformations, the reversible ones, for which $m'' = m$.

As a direct consequence of these transformations, it has been possible to give a very simple formula governing the energetics of black hole physics. Christodoulou and Ruffini[30] have shown that the total mass energy of a black hole can be simply split into three contributions: the rest energy (the irreducible mass), the Coulomb energy and the rotational energy.

$$m^2 = (m_{ir} + e^2/4\, m_{ir})^2 + L^2/4\, m^2_{ir}$$

Here we have used geometrical units with ($G = c = 1$). To obtain the conventional units we have.

$$m = Gm_{conv}/c^2 \qquad e = G^{1/2}e_{conv}/c^2 \qquad L = G\, L_{conv}/c^3$$

The area of the black hole is given simply by[33]

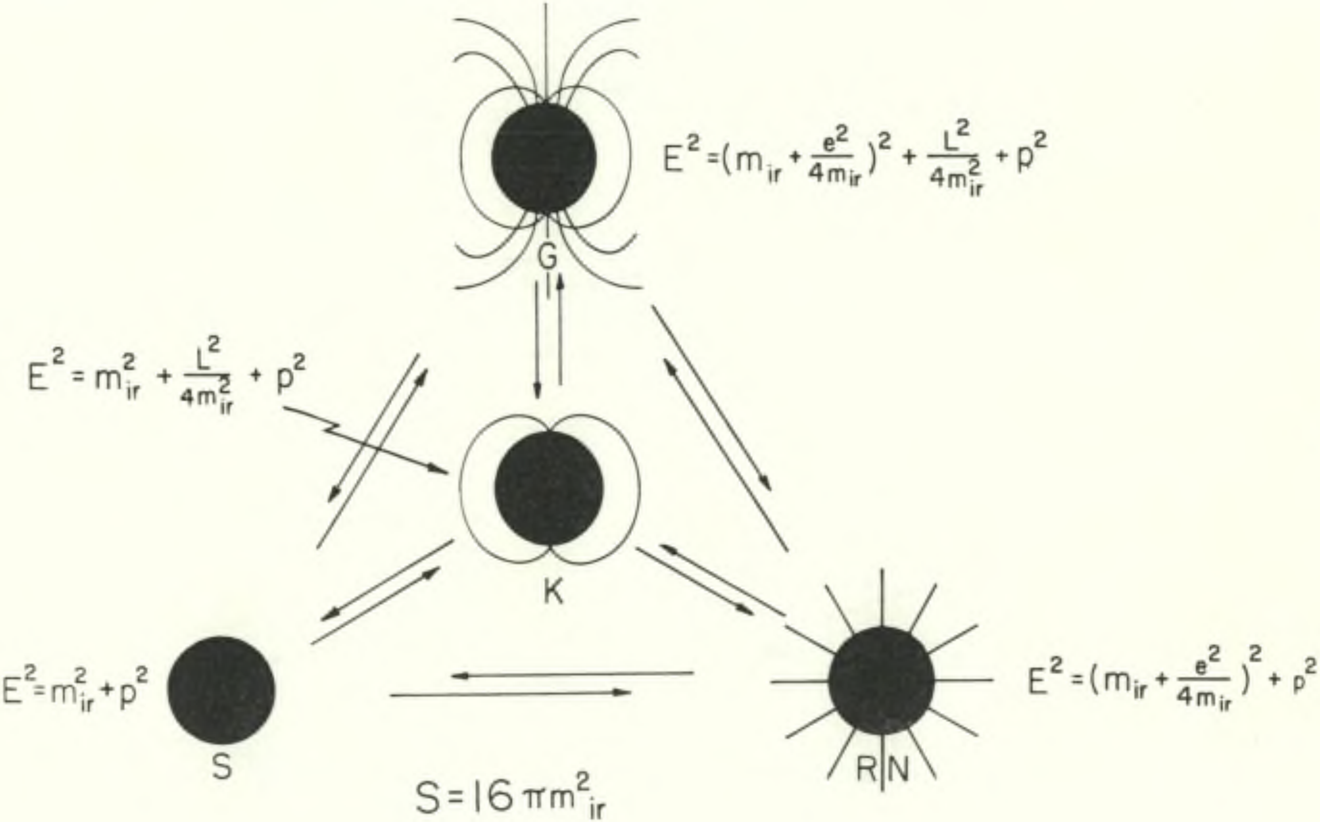
$$S = 16\, \pi\, m^2_{ir}$$

The irreductible mass is left constant in all reversible transformations and is monotonically increased by irreversible transformations[32, 33]. In contrast, the Coulomb and the rotational energy can be added and subtracted at will from the black hole under the following limitation:[34]

$$L^2/m^2 + e^2 = a^2 + e^2 \leq m^2$$

The processes of extraction of energy from a black hole are mediated in the effective ergosphere of a black hole[33, 35, 36, 37]. It is interesting to notice that for an extreme black hole ($a^2 + e^2 = m^2$) up to 50 % of its mass-energy can be stored in rotational and electromagnetic energy and is therefore extractable.

Fig. 5. The entire set of regular black holes is here summarized with the general formulae governing their mass energy as a function of their characteristic parameters[33]. Mass m, charge e, and angular momentum L are expressed in geometrical units. Reversible and irreversible transformations can lead from one kind of black hole to another through loss or augmentation of charge and angular momentum. The only black hole deprived of an ergosphere (region around a black hole from which it is possible to extract a finite amount of the total mass energy of the collapsed object) is the Schwarzschild black hole. The Reissner-Nordstrøm (R.N.) black hole is endowed with charge (e) and mass (m), the Kerr (k) with mass and angular momentum (1), the Kerr-Newmann (G for general) with mass, charge and angular momentum. Extraction of energy is possible if the black hole is endowed with angular momentum[38 34], charge [35] or both these parameters[36]. The effective ergosphere extends from the horizon r_+ to r_{erg}.[35 36] $m + (m^2 - a^2 - e^2)^{1/2} = r_+ \leq r \leq r_{erg} = m + [m^2 - e^2\,(1 - q^2/\mu^2)]^{1/2}$ where q/μ is the charge to mass ratio of the test particle which reduces the mass energy of the black hole. Up to 29 % (50 %) of the total mass energy can be extracted in the transition from a Kerr (Reissner-Nordstrøm) to a Schwarzschild black hole. In all these cases, the surface area of a black hole is most simply expressed by $S = 16\, \pi\, m^2_{ir}$. The result that the irreductible mass of a black hole can never decrease was independently obtained through a totally different derivation by S. Hawking[40] (details of this entire diagram in ref. 17).



$E^2 = (m_{ir} + \frac{e^2}{4m_{ir}})^2 + \frac{L^2}{4m_{ir}^2} + p^2$
G
$E^2 = m_{ir}^2 + \frac{L^2}{4m_{ir}^2} + p^2$
K
$E^2 = m_{ir}^2 + p^2$
S
$S = 16\pi m_{ir}^2$
RN
$E^2 = (m_{ir} + \frac{e^2}{4m_{ir}})^2 + p^2$

Fig. 5

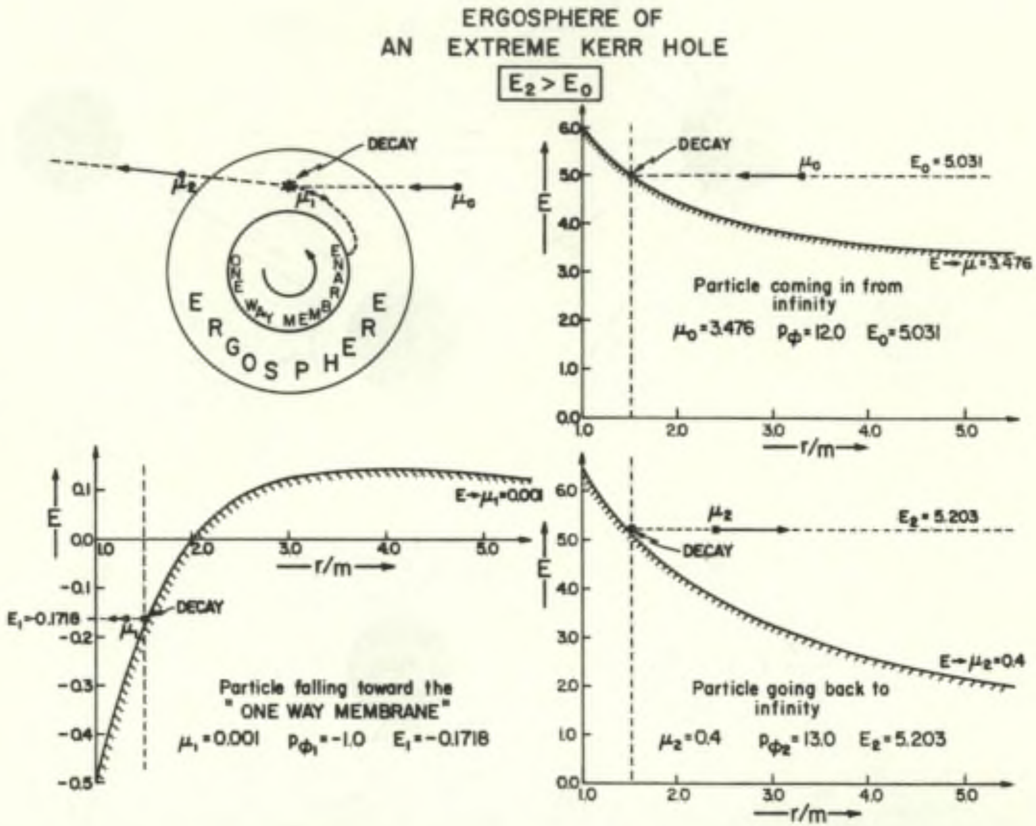

ERGOSPHERE OF
AN EXTREME KERR HOLE
$E_2 > E_0$
DECAY
μ_2
μ_1
μ_0
ONE WAY MEMBRANE
ERGOSPHERE
$E_0 = 5.031$
$E \to \mu = 3.476$
Particle coming in from infinity
$\mu_0 = 3.476$ $p_\phi = 12.0$ $E_0 = 5.031$
$E \to \mu_1 = 0.001$
$E_1 = -0.1718$
Particle falling toward the "ONE WAY MEMBRANE"
$\mu_1 = 0.001$ $p_{\phi_1} = -1.0$ $E_1 = -0.1718$
$E_2 = 5.203$
$E \to \mu_2 = 0.4$
Particle going back to infinity
$\mu_2 = 0.4$ $p_{\phi_2} = 13.0$ $E_2 = 5.203$
E
r/m

Fig. 6

Fig. 6. An example of how to slow down a black hole *and* reduce its total mass energy by the extraction of rotational energy. A particle of mass μ_0 coming from infinity with total energy E_0 and a positive value of the angular momentum p_φ, can penetrate the ergosphere of an extreme Kerr hole and there decay into two particles[38 39] One particle of mass μ_1, negative value of the angular momentum p_φ and a negative value of the total energy E_1, falls towards and penetrates the horizon. The second particle of mass μ_2, positive value of the angular momentum $p_{\varphi 2}$ and a positive value of total energy E_2, goes back to infinity. The remarkable feature in this process is that the energy E_2 of the particle coming back to infinity is larger that the energy E_0 of the particle coming in. In the detailed computations of the process here presented[39] we have assumed beside the conservation of the energy also the conservation of the total momentum of the system during the process of fragmentation of the particle μ_0 into the two particles μ_1 and μ_2. As is clear from the example here presented this energy extraction process can be made at the expenses of a very large reduction in the rest mass of the system. On the upper left side a qualitative diagram shows the main feature of the decay process in the equatorial plane of the ergosphere of a Kerr hole. In the upper right side is the effective potential (energy required to reach r as a turning point) for the incoming particle. The effective potential is plotted at the lower left and the lower right side for the particle falling toward the horizon and for the particle going back to infinity. This " energy gain process " critically depends on the existence and on the size of the ergosphere which in turn depends upon the value a/m of the hole. In the case of Kerr black hole considered here the ergosphere extends between the horizon and the infinite red-shift surface

$$m + (m^2 - a^2)^{1/2} \leq r \leq 2m$$

and when $a/m = 0$, (Schwarzschild black hole) the horizon expands and coincides with the infinite red shift surface, wiping out the ergosphere. The particle falling towards the one way membrane will in general alter and reduce the ratio a/m of the black hole. Details in references (38) (39) and (17). The corresponding processes for extraction of electromagnetic energy has been given in references (35) and (36). In the case of the existence of an electromagnetic field in the field of the black hole. The ergosphere will again start at the horizon of the black hole and will extend out to a surface $r_{erg} = m + [m^2 - e^2.(1 - q^2/\mu^2)]^{1/2}$ which is usually a function of the charge to mass ratio q/μ of the test particle falling into the black hole and reducing the total energy of the black hole. Christodoulou and Ruffini[37], to stress the difference between this more general case and the case of a Kerr solution, have called the region in which energy can be extracted the " effective ergosphere ".

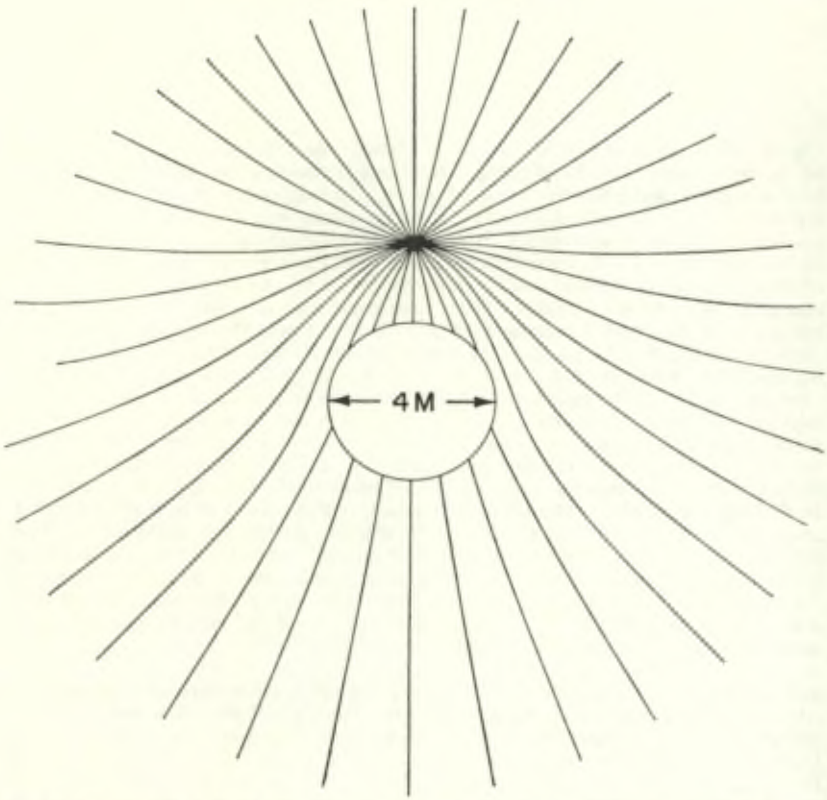


Fig. 7. Capture of a particle by a Schwarzschild black hole and transition to a Reissner-Nordstrøm black hole. Since the early days of black hole physics, it is well known that it takes an *infinite time* (as seen by a far-away observer) for a test particle to reach the horizon of a black hole. It is, therefore, natural to ask how the transition can occur from a Schwarzschild to a Reissner-Nordstrøm solution, since in this last case, the electric field should appear to a far away observer completely radial *as if* the charge was concentrated at the center of the black hole although the particle can never even cross the horizon as seen by that observer. This paradox was solved by the work presented in reference 41. The test particles " induces " charge on the surface of the black hole. The closer it approaches the Schwarzschild horizon, the larger the magnitude of the induced charge is, the " transfer " of the charge to the black hole occurs, therefore as a polarization effect. The electromagnetic field appears very distorted to an observer near the Schwarzschild radius and radially directed to a far away observer as if the test charge was indeed at the center of the black hole.

A second important line of research has been directed toward a detailed analysis of the radiation processes occuring in the field of black holes. The most powerful tool to carry out this program has certainly been the perturbation analysis introduced in 1957 by Regge and Wheeler[42] and further developed since 1971 by Zerilli[43]. The main idea of this approach can be easily summarized: to analyze the processes occuring in the background field of a collapsed object we can analyze small perturbations away from a given background metric $g^{(0)}_{\mu\nu}$

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu} \tag{2}$$

the Einstein equations for the metric $g_{\mu\nu}$ then give the following set of equations for the perturbation field $h_{\mu\nu}$

$$\begin{aligned} &h_{\mu\nu;\alpha}{}^{;\alpha} - (f_{\mu;\nu} + f_{\nu;\mu}) + 2\, R^{\rho\ \alpha}_{\ \mu\ \nu}\, h_{\rho\alpha} + h^{\alpha}{}_{\alpha;\mu;\nu} \\ &\quad + g_{\mu\nu}\,(f_{\alpha}{}^{;\alpha} - h^{\alpha}{}_{\alpha;\lambda}{}^{;\lambda}) = -16\,\pi\,\Delta\, T_{\mu\nu} \end{aligned} \tag{3}$$

where $f_{\mu} = h_{\mu}{}^{\alpha}{}_{;\alpha}$ and $R^{\alpha}{}_{\beta}{}^{\gamma}{}_{\delta}$ is as usual the Riemann tensor of the background metric and $\Delta T_{\mu\nu}$ is the physical perturbation driving the field $h_{\mu\nu}$. In this set of equations all the non-linear terms, quadratic in $h_{\mu\nu}$ have been neglected. When the background metric is given by the Schwarzschild metric, the set of equations (3) can be greatly simplified expanding the tensor perturbation $h_{\mu\nu}$ into a complete set of tensor spherical harmonics. Regge, Wheeler[42] and Zerilli[43] have then shown how to reduce the system of ten partial differential equations (3) to a system of two Schrödinger-like equations governing the most general perturbations of a background Schwarzschild.

This mathematical formalism has allowed for the first time to attack a fully relativistic analysis of gravitational and electromagnetic radiation processes in the field of collapsed objects. The spectrum (see Fig. 8), the directional properties (see Fig. 9), the shape of the burst (see Fig. 10), and the energetics (see Fig. 11) of a burst of gravitational radiation emitted by a particle falling or projected into a Schwarzschild black hole have been analyzed. The analysis has as well been carried out for the " eternal " problem of the emission of electromagnetic radiation from a charge infalling on a Schwarzschild or a Reissner-Nordstrøm black hole[53, 54].

An entire program has also been directed toward the exploration of possible beaming effects similar to the well known synchrotron radiation of classical electrodynamics, into the field of a collapsed object (see Fig. 12).

It has been pointed out by Partridge and Ruffini[55] how the enormous difference of cross section between electromagnetic and gravitational detectors make most promising an experimental search for coupled bursts of gravitational and electromagnetic radiation. The theoretical analysis of this coupling has generated a third major field of research. The first fully relativistic analysis of this coupling processe is based on the generalization[56] of the Regge-Wheeler[42] — Zerilli[43] formalism to the case in which the background field is endowed with an electromagnetic structure. Two major new effects have been pointed out as occuring in this regime: The gravitationally induced electromagnetic radiation[57] and the electromagnetically induced gravitational radiation[58, 54]. Both these analyses focus on the possibility of converting electromagnetic into gravitational radiation and vice-versa in the field of a collapsed object (see Fig. 14). In a specific example[57] it has been pointed out how this conversion can reach 100 % efficiency. The very large amount of gravitational radiation expected to be emitted during the process of gravitational collapse together with the clear possibility that the collapsed object be endowed with an electromagnetic field[59] make this analysis most interesting and attractive. An experimental search for bursts of electromagnetic radiation at radio and infrared wavelength is also currently being made[60].

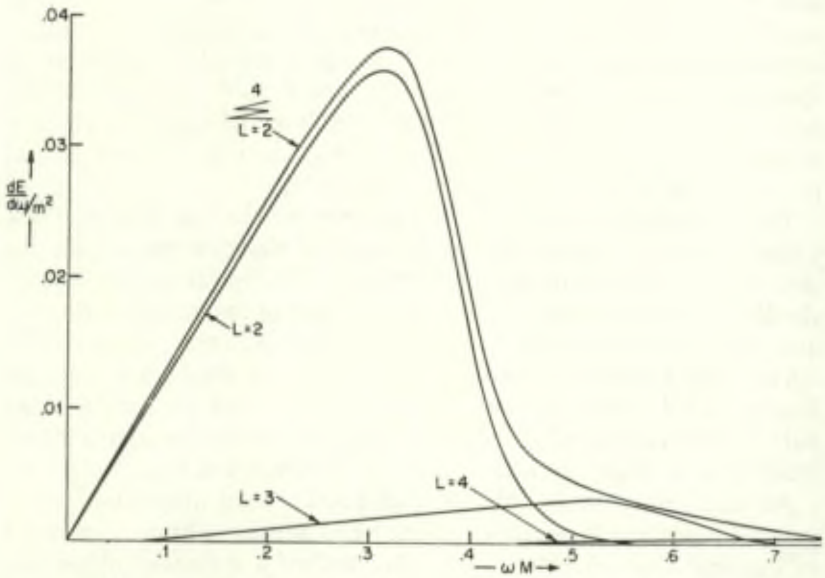


Fig. 8. Spectrum of gravitational radiation emitted by a test particle of mass m falling radially into a black hole of mass M (geometrical units $G = c = 1$). This work based on the Regge-Wheeler-Zerilli[42 43] formalism is the fully relativistic generalization of the semi-relativistic treatment presented by Ruffini and Wheeler[39] In both treatments, the spectrum is broad with a sharp decrease both in the low and high frequency limits. The major amount of radiation is emitted in the quadrupole ($l = 2$) mode and the peak of the radiation occurs at the $\omega \sim 0.4\, c^3/GM$. The total amount of radiation emitted, integrated over frequency and multipole distribution is:

$$\Delta E \sim 0.0104\, m\, c^2\, (m/M)$$

It is immediately clear that the highest efficiency can be reached in the limit $m \sim M$, however in that approximation, the validity of the approximation used in reference[42] and [43] ceases to be valid. In turn, it is also clear that the radiation of a one solar mass " test particle " into a very large black hole of $10^8 M_\odot$ or larger is negligible. (Details in ref. (44) and (45).).

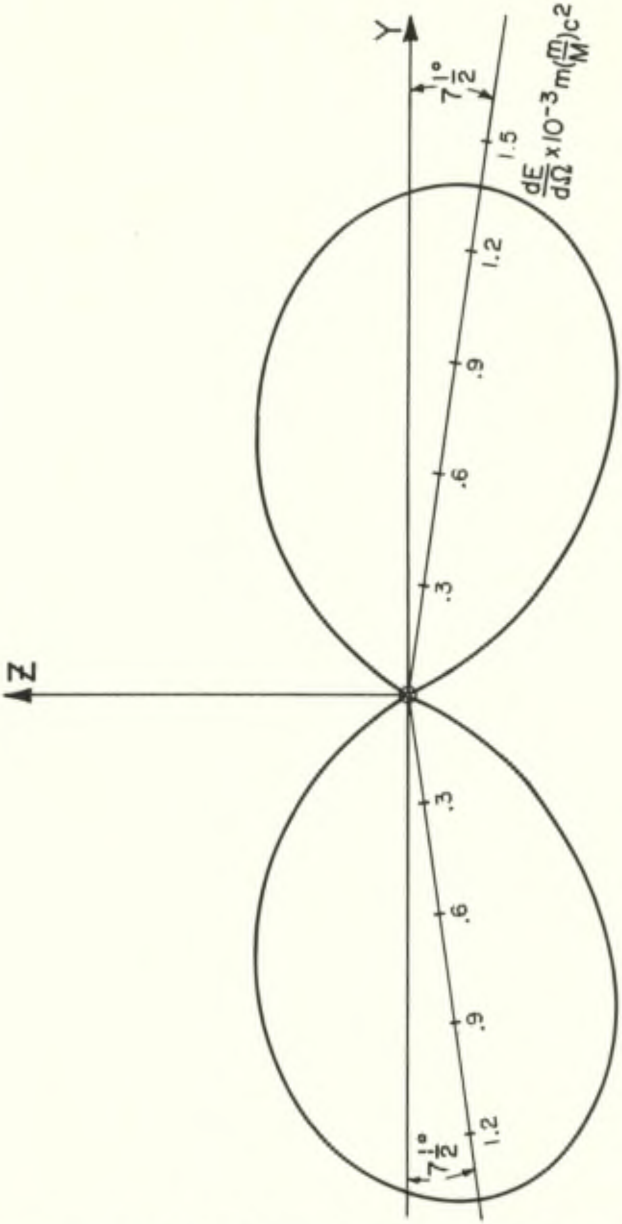
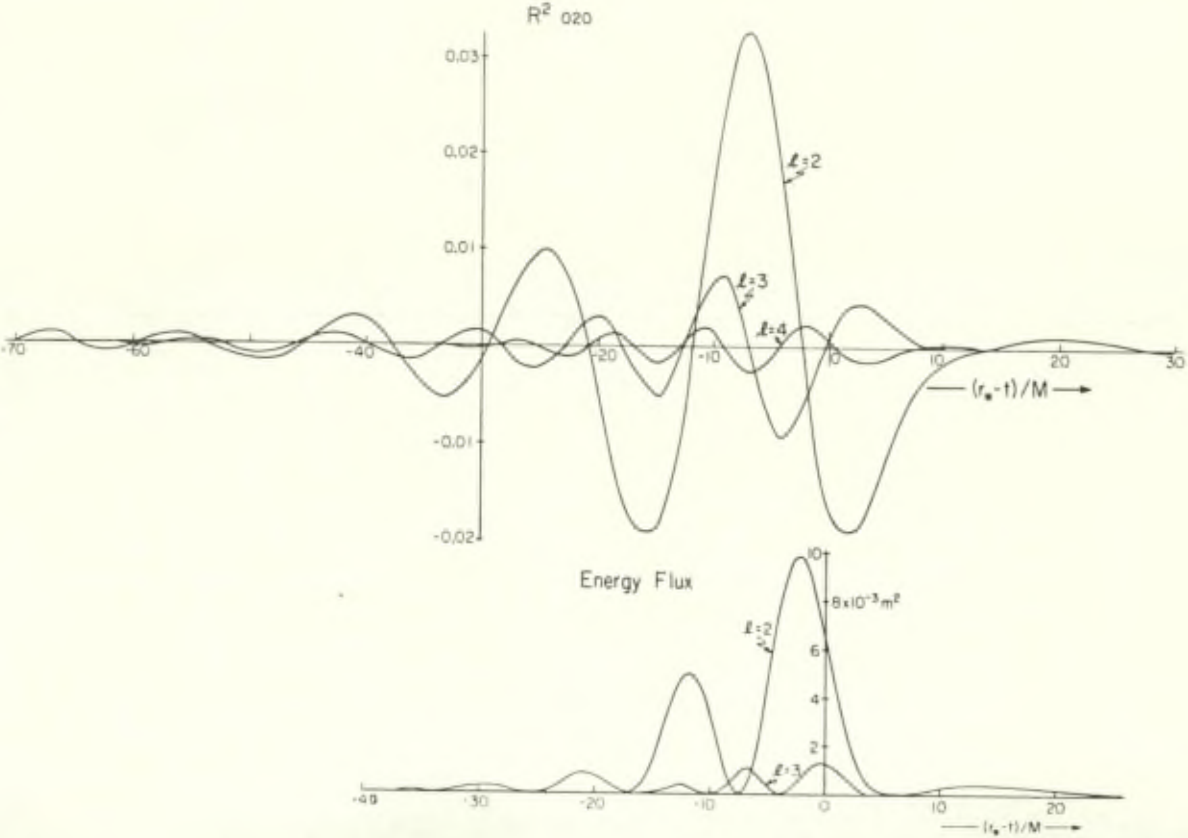


Fig. 9. Angular pattern of the radiation emitted by a test particle falling radially (Z axis) into a Schwarzschild black hole, assumed at the origin of the coordinate system. The forward beaming of the lobes is due to the relativistic velocity acquired by the particle in its final approach to the Schwarzschild horizon. Details on the intensity of the radiation ingoing into the black hole and of the polarization of the radiation as well as the details of this analysis are given in reference 46.



R^2 020
0.03
0.02
0.01
-0.01
-0.02
ℓ=2
ℓ=3
ℓ=4
-70
-60
-40
-20
-10
0
10
20
30
— $(r_*-t)/M$ →
Energy Flux
10
8×10⁻³m²
6
4
2
ℓ=2
ℓ=3
-40
-30
-20
-10
0
10
20
— $(r_*-t)/M$ →

Fig. 10

Fig. 10. A burst of gravitational radiation emitted by a particle of mass m falling into a black hole of mass M. The component of the Riemann tensor relevant for the detection of the burst of gravitational radiation (see also Fig. 13) as well as the intensity of the radiation are here plotted as a function of the retarded-time coordinate. A detector of gravitational radiation located far away from the source would first receive a weak signal emitted by the particle approaching the black hole (the "precursor" $5 \lesssim (r^* - t)/M \lesssim 30$) and then receive a very sharp pulse emitted during the last approach of the particle to the throat of the collapsed object ($-2 \lesssim (r^* - t)/M \lesssim 5$) where the particle reaches extreme relativistic velocities. Finally a sequence of impulses of decaying intensity is emitted during the amalgamation of the falling particle to the black hole. A new larger black hole has been formed by the capture of the test particle m (details in reference 46).

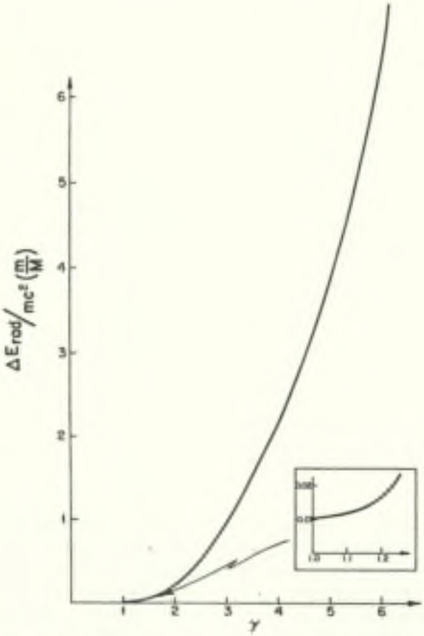


Fig. 11. Two major limitations govern the spectrum and the total energy emitted in the fall of a particle into a Schwarzschild black hole:

(a) As shown in Fig. 8, the total amount of energy is

$$\Delta E \sim 0.0104 \, m c^2 \, (m/M)$$

(b) The spectrum is peaked at a frequency

$$\omega \sim 0.4 \, c^3/GM$$

An extensive analysis has been made to see if under suitable conditions both constraints (a) and (b) can be relaxed, namely if we could have both a larger energy emission and at higher frequencies. The fulfillment of one or both these conditions would greatly facilitate any detection of gravitational wave bursts (see also Fig. 31). Of all the conceivable methods, one has proved to be very effective in largely enhancing the energy emission but only slightly shifting the peak of the radiation toward higher frequency: the projection of a particle into a black hole. Here the total energy emitted by a test particle of mass m projected into a Schwarzschild black hole of mass M is plotted as a function of the value $\gamma = 1/(1 - v^2/c^2)^{1/2}$, v being the velocity of the particle at infinity before starting its implosion. Interesting as it is for a better understanding of the radiation processes this process cannot be considered of astrophysical interest. (Details in reference 44).

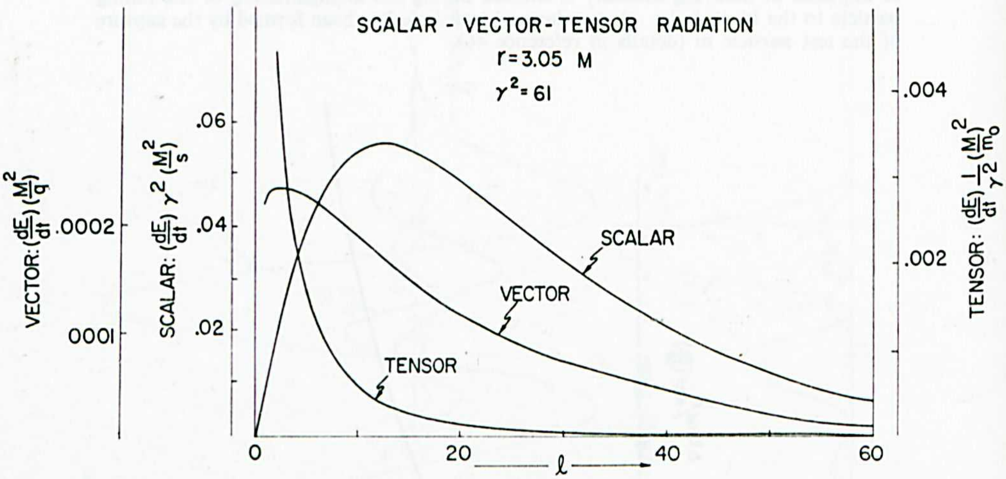


Fig. 12. Following a suggestion and a specific example given by Misner[48] and Misner et. al.[49] that beaming effects could be found in the emission processes of gravitational radiation emitted by particles in relativistic orbit around a black hole, a systematic search and analysis of the spectral distribution of both electromagnetic and gravitational radiation emitted by test particles in relativistic circular orbits around black holes was initiated and completed[50 51 52]. The interest in the possibility of beaming effects of gravitational radiation can be simply summarized: If it happens by symmetry reasons that the earth and our detection system is in the sharp beam of a source of gravitational rotation, then the required energy emitted by the source of the radiation will be smaller the stronger the beaming processes are. The fully relativistic analysis of electromagnetic and gravitational radiation has shown that any beaming process strongly depend upon the spin of the field under consideration: the higher the spin, the smaller are the beaming processes. Better than any further comment, this result is most clearly expressed in the present figure where the spectrum of scalar electromagnetic and gravitational radiation are compared and contrasted for the motion of a test particle in a relativistic orbit around a Schwarzschild black hole. The major conclusion is that in no realistic astrophysical conditions can we possibly find relativistic beaming processes of any relevance. Moreover in *no* way can a spin two field have beaming processes of the kind observed in electromagnetic radiation in flat space. The physical reasons behind these very interesting defocusing effects which forbid the beaming processes have been given in reference 51 and 52. In this figure, m is the mass of the particle, q the electric charge, s the scalar charge of the particle and l the multipole of the radiation under consideration (l = 1 dipole, l = 2 quadrupole, etc.)

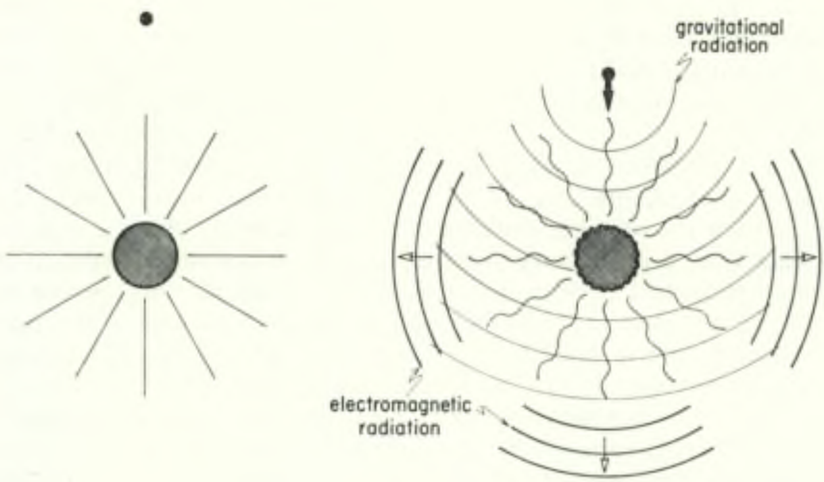


Fig. 13. Schematic representation of the gravitationally induced electromagnetic radiation. For the sake of example, we have considered the problem of electromagnetic and gravitational radiation emitted in the infall of an *uncharged* test particle of mass m in the field of a charged black hole (Reissner-Nordstrøm) of mass M and charge Q. Similar effects should be expected to occur in a more general situation in which the collapsed object is endowed with a more complex electromagnetic field structure (generalize Kerr solution with magnetosphere[17]). The infalling particle perturbs the background electromagnetic field both through the effect of the gravitational field of the test particle and the scattering of gravitational waves. The key and most important result can be simply summarized:

$$(\Delta E)_{elec} \sim (Q/M)\,(\Delta E)_{grav}.$$

The conversion of gravitational into electromagnetic radiation can reach a 100 % efficiency in the limit $Q \to M$. Details in ref. 57. The inverse phenomena, the electromagnetic induced gravitational radiation, has also been considered and details given in ref. 58.

All these researches on the ultrarelativistic radiation processes in the field of collapsed objects have also naturally lead to an in depth analysis of the properties of gravitational waves detectors (see § 4) in order to check the feasibility of observing some of these theoretical predictions.

If we turn now to perhaps the most interesting problem connected with the direct detectability of a black hole, namely the radiation emitted by an accreting plasma, much work still has to be done in order to prepare the ground for a detailed analysis. The detailed structure of the electromagnetic fields of a black hole *in vacuum* has been given by Christodoulou and Ruffini[61]. The trajectories of clouds of charged and uncharged test particles in the most general background geometry have been analyzed[62, 63] (see Fig. 14 and 15). Finally, the effect of a black hole on a uniform magnetic field has also been studied[64] (see Fig. 16). It is, however, now clear that these were only preliminary works. To approach the complete problem of plasma accreting about a rotating black hole, it is necessary to solve the entire self-consistent solution of the background electromagnetic field and of the accreting plasma in the magnetosphere. Preliminary results clearly show that in this process very large currents and electromagnetic fields should be expected to exist in the magnetosphere of such a black hole[65, 66].

Much work has therefore been done in recent years ranging from the analysis of the structure of black holes, to trajectories of particles in a given background geometry of a black hole, to the analysis of energy extraction from black holes, to the processes of accretion around collapsed objects to finally the electrodynamics processes near the surface of black holes. Thanks to this work it has become clear that each kind of black hole appears to have its own characteristic signature which could in principle be detected and observed. This entire program can be carried out only with a very strong set of new results in the experimental field in order to have feedback in the theoretical field not just for verification of the theoretical predictions but for new information in this drastically new field of physics.

Fig. 14. Motion of an uncharged cloud of test particles of mass μ, each endowed with an angular momentum $p\varphi/\mu m = 2$, corotating around an extreme spining black hole of mass m. The vertical lines are isochronous points as seen from infinity. The typical " Klein bottle " behavior of this cloud of particles is a direct consequence of the " Wilkins effect "[62, 63]. In the field of a Kerr solution a self gravitating orbiting cloud of particles acquires an additional periodicity in the θ direction and is confined to an equatorial region, It is very difficult for a particle to spiral toward the poles of the rotation axis. (see Ref. 62 and 63).

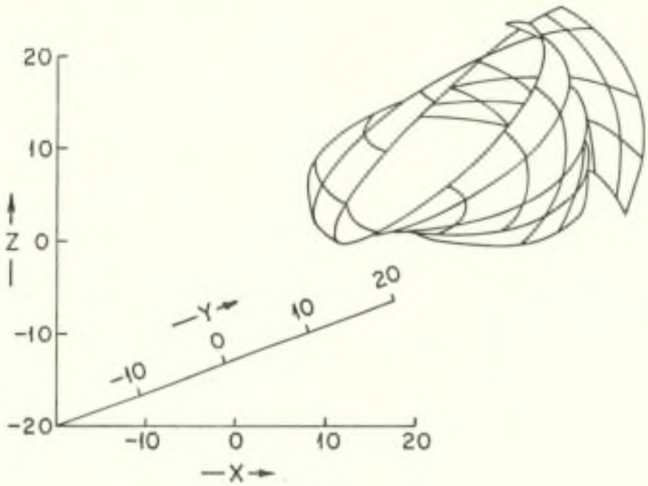


Fig. 14

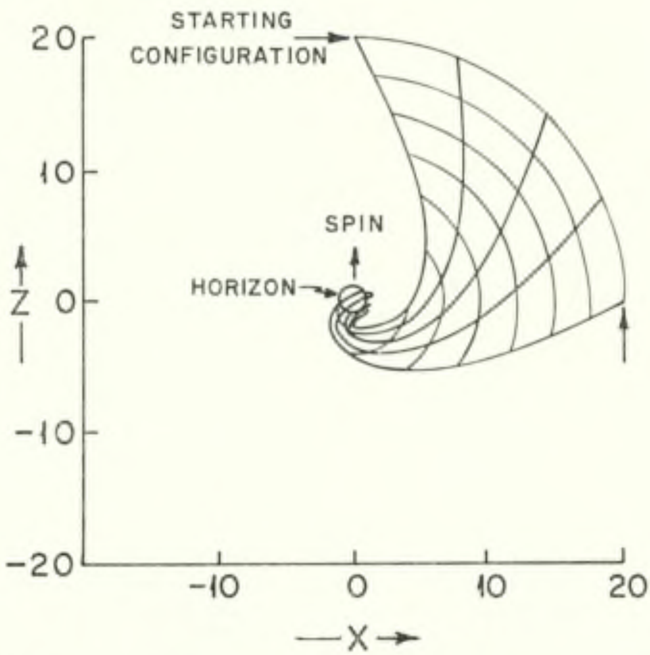


Fig. 15. This figure beautifully exemplifies the concept of angular velocity of a black hole introduced, in reference (61). A cloud of particles with initial zero angular momentum along the φ direction ($p_\varphi/\mu m = 0$) acquires an angular velocity along the φ direction which is simply given by:

(1) $$\omega = a/(r_+^2 + a^2)$$

as the cloud approaches the horizon $r_+ = m + (m^2 - a^2)^{1/2}$. Here, as usual, a is the angular momentum per unit mass of the black hole $a = L/m$. After this angular velocity of the black hole was introduced, J. A. Wheeler suggested that it should have been possible to obtain this same result from the formula governing the energetic of black holes[33]:

$$m^2 = \left(m_{ir} + \frac{e^2}{4\,m_{ir}}\right)^2 + \frac{L^2}{4\,m_{ir}^2} \tag{2}$$

Indeed we have

$$\omega = \frac{\partial m}{\partial L} \tag{3}$$

This result greatly contributes to a deeper understanding of the meaning and significance of the formula (2).

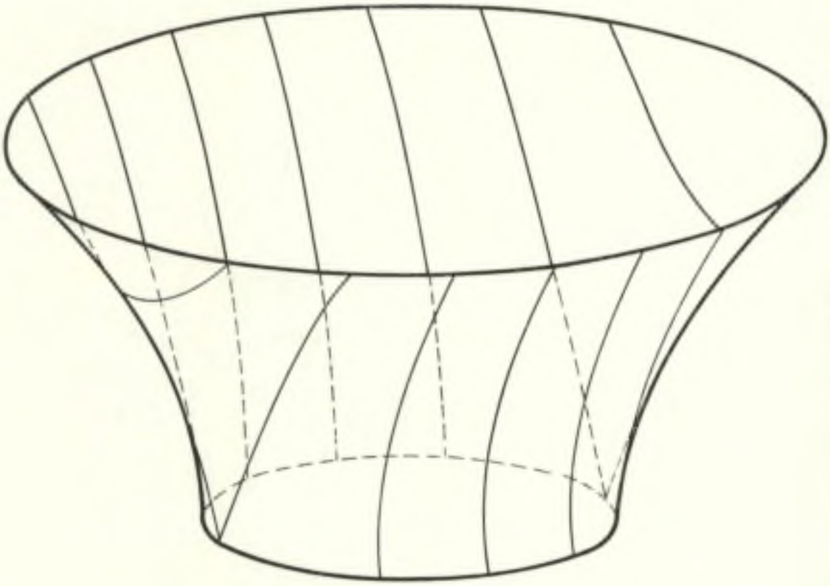


Fig. 16. The effects of a Schwarzschild black hole on a uniform magnetic field are here displayed in an embedding diagram. To solve this problem, the fully covariant Maxwell equations have been solved in a Schwarzschild background with the boundary conditions of a uniform field at infinity. We have adopted for the lines of force the definition given in references (61) and (17). This analysis is of the greatest relevance for the study of the accretion of plasma into a black hole since we expect from the usual condition of infinite conductivity, that the plasma will flow toward the black hole along the magnetic field lines and the final configuration of the magnetic field near the horizon will largely resemble the one here reproduced if, of course, the black hole is not rotating. (Details in ref. 64).

4. GRAVITATIONAL RADIATION'S DETECTORS

The detailed analysis of the processes of emission of gravitational radiation in extreme relativistic regimes has naturally lead to the study of the cross section, directionality, and polarization response of detectors of gravitational radiation[39, 67, 68]. The cross section of a gravitational wave antenna characterized by a resonant frequency ω_o is given for frequencies near resonance by a generalization of the Breit-Wigner formula[70, 69]

$$\sigma(\omega) = \frac{\pi \lambda^2}{2} \frac{A_{grav} A_{diss}}{(\omega - \omega_o)^2 + (A_{diss}/2)^2} \text{ cm}^2/\text{sec H}_z$$

with the value for the integrated cross section

$$\int_{res} \sigma(\nu) \, d\nu = (\pi/2) \lambda^2 A_{grav}$$

where $A_{grav} = -(dE/dt)_{grav}/E$ is the rate of damping of the detectors caused by radiation of gravitational waves and $A_{diss} = -(dE/dt)_{diss}/E$ is the rate of damping caused by all form of dissipation other than gravitational radiation. Under any realistic circumstance we have of course $A_{diss} >> A_{grav}$. If we then indicate by $I(\nu)$ (erg/cm^2 H$_z$) the spectrum of the gravitational radiation pulse we obtain for the energy E absorbed by the detector

$$E = \int I(\nu) \, \sigma(\nu) \, d\nu \text{ erg/sec.}$$

Any estimate of the power required for a source of gravitational radiation to deposit a fixed amount of energy E in the detector will strongly depend on the spectrum of the radiation. In the case of a broad spectrum we have $I(\nu_o) = E / \int_{res} \sigma(\nu) \, d\nu$. If the spectrum is flat from $\nu = o$ up to $\nu = \nu_o$ we have for the total energy in the pulse $\int I(\nu) \, d\nu = I(\nu_o) \, \nu_o$. If the pulse has a spectrum of the form $I(\nu) = I(\nu_o) (\Delta\omega/2)^2 / [(\omega - \omega_o)^2 + (\Delta\omega/2)^2]$ we obtain $\int I(\nu) \, d\nu = \Delta\omega \, I(\nu_o)/4$. If $\Delta\omega >> A_{diss}$; $\int I(\nu) \, d\nu = A_{diss} \, I(\nu_o)/2$ if $\Delta\omega = A_{diss}$ and $\int I(\nu) \, d\nu = A_{diss} \, I(\nu_o)/4$ if $\Delta\omega << A_{diss}$.

To evaluate explicitly the value of the integrated cross section for a given detector we have first to evaluate the A_{grav} for the mode at which the gravitational wave detector operates. Considering e.g. the detectors used by J. Weber[71], an aluminum cylinder of 153 cm in length and 66 cm in diameter, and limiting our considerations to the modes of longitudinal vibration of the form[39, 64]

$$\xi = \xi_o \sin(n \pi x/L) \sin(\omega t)$$

we have

$$A_{grav} = - < (dE/dt) >_{av/E} = \frac{64}{15} \frac{G}{c^5} \frac{M v^4}{L^2}$$

and for the integrated cross section over randomly polarized radiation[67]

$$\int_{\text{res, random}} \sigma(\nu)\, d\nu = \left(\frac{\pi}{2}\right) \lambda^2 A_{grav} = \frac{32}{15\,\pi}\,\frac{G}{c}\,\frac{v^2}{c^2}\,\frac{M}{n^2}$$

here $n = 1, 3, 5 \ldots$ correspond to the even vibrational modes, v is the speed of sound, and M the mass of the cylinder. The cross section for different modes of vibration has been also considered[72]. The directional properties of a detector of the kind used by Weber[71] as well as its response factor to gravitational radiation for selected polarization have been studied by idealizing the detector to a system of two masses m coupled by a spring of length L and resonance frequency ω_0 see Fig. 17 and Fig. 18.

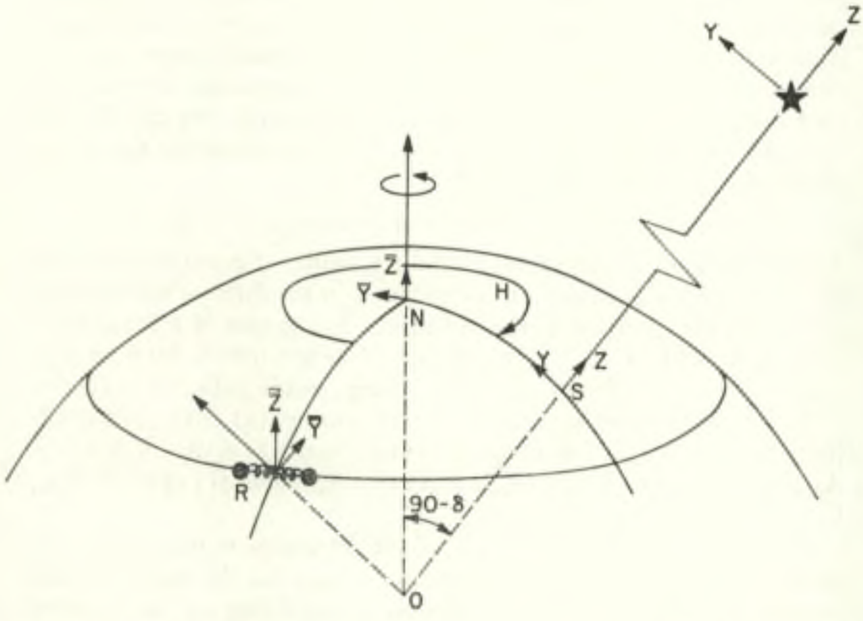


Fig. 17. Idealized detector of gravitational waves, R, on the surface of the earth is driven by a source on a far away star. The coupling between the gravitational radiation emitted at the source and the receptor on the Earth surface have been analyzed in r ference 39 by transforming tensorial components from the laboratory frame (double barred coordinates) to a frame at the north pole (barred coordinates) and then to a frame at S. If the source of gravitational radiation has random polarization and is located at declination δ and hour angle H the response factor of the detector is given by

$$W(H, \delta) = (\cos^2 H - \sin^2 \delta \sin^2 H)^2 + (\sin \delta \sin 2H)^2$$

see also fig. 18. (Figure reproduced from ref. 39 with permission of the authors).

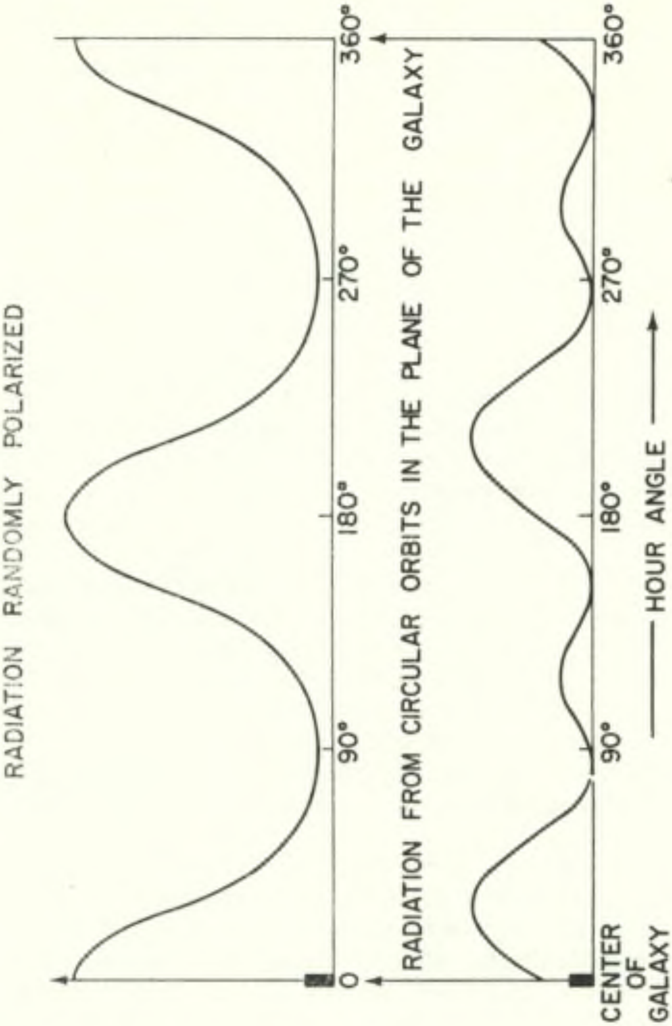


Fig. 18. The response factor of a gravitational wave detector depends drastically from the polarization of the gravitational radiation signal. In this figure we compare and contrast the response factor W (H, δ) of a detector of gravitational radiation aligned East-West (see Fig. 17) to a source of gravitational radiation located at hour angle H and declination $\delta = -28^{o}.9$ (center of the galaxy). Compared and contrasted are the two examples in which the radiation is randomly polarized (upper part of the figure) and the radiation is 100 % polarized and originates from circular orbits in the plane of the galaxy (details in Ref. 51). The intensity as well as the peak of the response function are clearly markedly different in the two cases. The response function of a detector directed East-West to a 100 % polarized source of radiation is given by

$$W(H, \delta) = [(\cos^2 H - \sin^2 H \sin^2 \delta) \cos 2\alpha + \sin 2\alpha \sin \delta \sin 2H]^2$$

the angle α is the angle between the plane of the polarization of the source and the plane y-z in Fig. 17 (details in Ref. 51).

5. OBSERVATIONS AND CRITERIA TO DIFFERENTIATE BETWEEN NEUTRON STARS AND BLACK HOLES

A direct comparison of the physical size and some of the parameters characterizing a neutron star and a black hole clearly summarize the similarities and the difficulties in distinguishing between these two possible outcomes of gravitational collapse (see Fig. 19). The value of the angular velocity as well as the magnitude of the magnetic fields and the radius of these two different kinds of collapsed objects can be extremely similar (see Fig. 19). One fundamental difference, however is the value of their masses, neutron stars can exist only for masses smaller than the critical mass against gravitational collapse ($< 3.2\ M_{\odot}$) and black holes can exist only for values of the mass larger than this critical value. To form a black hole from a star with mass smaller than the critical mass, enough kinetic energy should be given to the collapsing material in order to tunnel through the barrier of the neutron star equilibrium configurations (see Fig. 3). Another fundamental difference between these two families of collapsed objects follows from the structure of their electromagnetic fields. In a neutron star the magnetic field can have any inclination with respect to the rotation axis, and explicit solutions have been given by Deutsch for such configurations[10]. In particular, the existence of an off-axis magnetic field in a neutron star can explain most directly the very regular pulsation and the lengthening in pulsational period observed in pulsars. In a black hole the magnetic field always has to be aligned with the rotation axis in order to have a stationary metric at infinity[17].

Therefore, although we can expect very short time structure in the signal emitted by material falling into black holes with the characteristic time constant given by the revolution period of material orbiting down to the last stable circular orbit, this signal will last at most, a few revolution periods[17]. In no way, therefore, can we expect that a regular signal of the kind observed in a pulsar can be emitted from a black hole.

To further allow progress in this entire field of research, a large amount of data was needed to infer not only the mass and the angular velocity of the collapsed objects, but also the structure of their magnetospheres as well as, in the case of neutron stars, the details of their internal constitution. This was impossible to do on the basis of the data acquired from pulsars for two reasons: if we exclude a few glitches and microglitches which affect very slightly their pulsational period, pulsars are extremely steady in their emission processes and no variation occurs to give a hint on their internal structure. Moreover, no possibility exists of a direct measure of the mass of the pulsar since, of the 120 pulsars observed, none has been found to be a member of a multiple system[27].

The detection of an isolated black hole in space would have been even more hopeless: " No light comes directly from it. It cannot be seen by its lens action or other effect on a more distant star. It is difficult enough to see Venus, 12000 KM in diameter, swimming across the disc of the sun; looking for a 15 KM object moving across a far-off stellar light source would be unimaginably difficult! "[73]. Following the work of Zel'dovich and Guseynov[74], I. S. Shklovsky[75], Zel'dovich and Novikov[76] and Schwartzman[77], the emphasis was directed in 1971 not to isolated systems, but binary systems: " The possibility of capitalizing on double star system is most favorable when the black hole is so near to a normal star that it draws in matter from its companion. Such a flow from one star to another is well known in close binary systems, but no unusual radiation emerges. When one of the components is a neutron star or a black hole, a strong emission in the X-ray region is expected "[73] (see Fig. 20).

The discovery made by the team lead by Riccardo Giacconi[78] through the observations from the Uhuru satellite and the joint observations made from the ground in the optical and radio wavelength have given irrefutable evidence for the discovery and direct observation inside our own galaxy of a very large number of short period binary systems ($P_0 \lesssim 5$ days) with a normal star and a collapsed object as components (see Fig. 21). It is hard to overemphasize the relevance of this experimental discovery for the entire field of the astrophysics and for the physics of collapsed objects. For the first time, we are now in the position not only of measuring the masses of collapsed objects with great accuracy, but of also obtaining, from the observation of the detailed features and short time structure of the radiation emitted by material accreting into the collapsed object under a variety of conditions and regimes, an accurate description of the magnetosphere both of neutron stars and black holes. More important even the analysis of very short time variability allows having information from regions more and more near the surface of a collapsed object.

As in the physics of elementary particles where we can never " see " an elementary particle but we can " infer " its structure and form factor through an analysis of scattering experiments, similarly here we can never see the surface or the internal structure of a black hole, but we certainly can infer its " form factor ", through the large-scale scattering experiment originated by the accretion of matter in the field of the collapsed object. Finally, the reason that the collapsed object is continuously accreting material from the normal star implies that at least in principle we should be able to observe dynamical changes and increase in the mass of the collapsed object and consequently a direct " neutronization " of matter in a neutron star or an expansion of the horizon of the black hole.

One of the most impressive features of the binary X-ray sources consists in a sharp differentiation in the kind of X-ray spectrum they emit: In one family of sources the X-rays are emitted in pulses of great regularity recalling many features of pulsars, in the other, although variability down to time scale of a few milliseconds is observed, no regular pulsations are present in the spectrum with the possible exception of train of pulses of radiation. The difference between these two families of sources is exemplified better by a direct look at Fig. 22 than by any further word.

In 1972[17 79] a classification was proposed to identify all the pulsating binary x-ray sources with neutron stars, and the bursting sources, with short time variability but no regular pulsation, with black holes.

Today this classification appears to be greatly confirmed and supported by much experimental evidence (see § 6 and § 7). The crucial point of this classification consists in the clear possibility of determining from direct observations the value of the critical mass of a neutron star against gravitational collapse: The pulsating sources are expected to have masses *up* to this critical value, and *all* the bursting sources masses larger than this critical value.

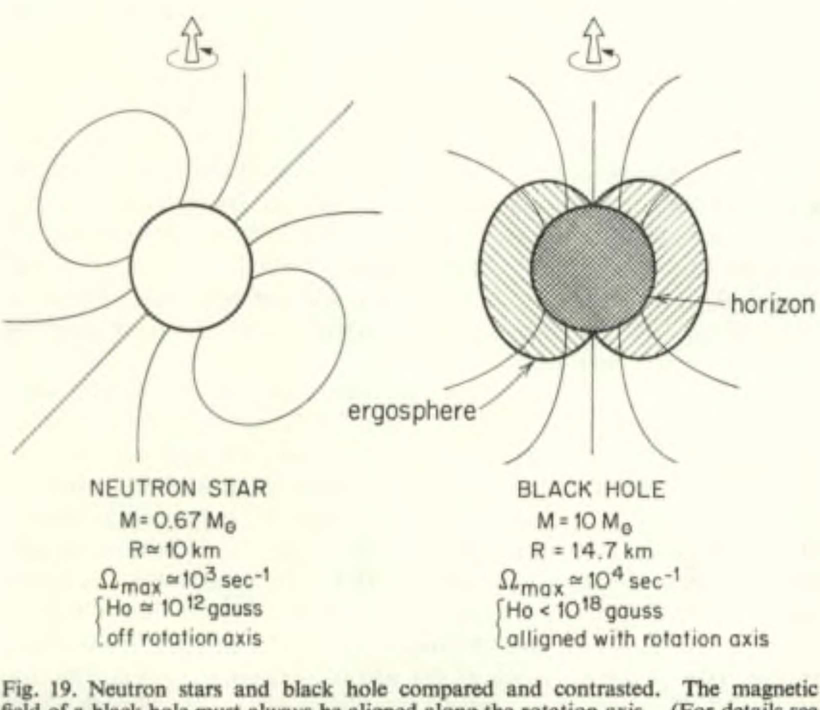


Fig. 19. Neutron stars and black hole compared and contrasted. The magnetic field of a black hole must always be aligned along the rotation axis. (For details see reference 17).

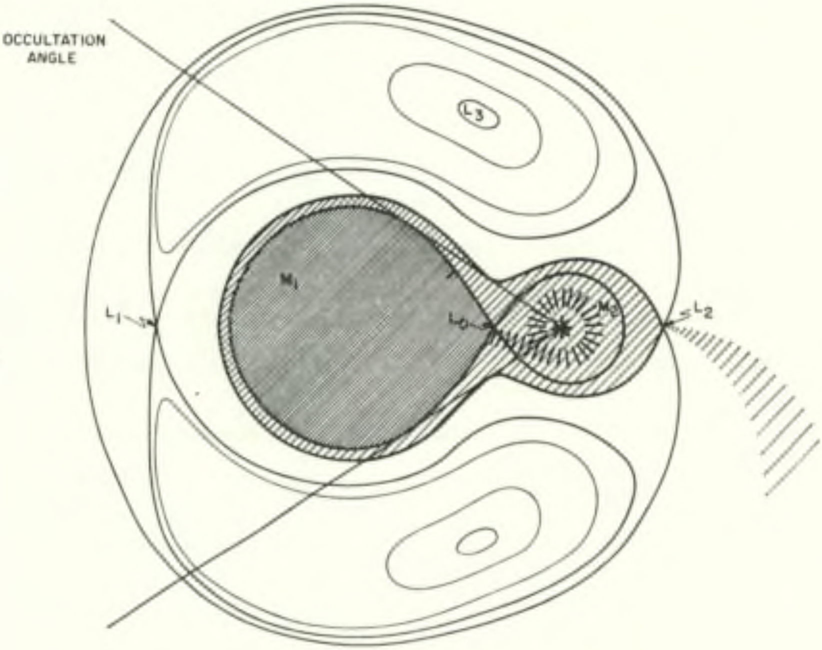


Fig. 20. Roche lobes and equipotential surfaces of a binary system formed by a normal star M_1, and a compact collapsed companion M_2. L_0, L_1, and L_2 are the Lagrange points of the system. X-rays are emitted by the matter from the main star being accreted into the very strong gravitational field of the collapsed companion star and, if the inclination of the orbit is high enough (see Tab. I), occultation will occur with the characteristic period of the binary. The equipotential surfaces here traced have been computed under the following simplifying assumptions of the Roche model: (a) The gravitational field generated by the two stars are computed as if the masses of the stars were concentrated in two points (b) the orbit of the two stars are assumed to be circular orbits around the center of mass (c) the axis of rotation of the two stars is perpendicular to the orbital plane and (d) the period of intrinsic rotation of the star M_1 is assumed to be the same as the one of orbital revolution (corotation). Information on the masses of the system can be immediately acquired by an application of the Kepler law. We have $G (M_1 + M_2) = (2 \pi/T)^2 a^3$, where a is the separation between the center of mass of the two stars and T is the period of the

binary system. If we know the projected velocity of both stars $(v \sin i)_1$ and $(v \sin i)_2$ and the orbital period T, or $\omega = 2\pi/T$, we can derive

$$M_2 \sin^3 i = [(v \sin i)_2 + (v \sin i)_1]^2 (v \sin i)_1/\omega G$$

$$M_1 \sin^3 i = [(v \sin i)_2 + (v \sin i)_1]^2 (v \sin i)_2/\omega G$$

By assuming that the main star fills its Roche lobe and material outflow through the Lagrangian point L_0, from the value of the binary period and the occultation angle $\Phi = (T_{occ}/T_{orb}) \times 180^\circ$, it is possible to obtain for selected values of the inclination angle i the value of the ratio $q = M_2/M_1$ see Tab. I. Profoundly different from Pulsars in which the energy source is due to the rotational energy of the neutron star, the energetics of these binary x-ray sources is totally determined by the gravitational binding energy of the infalling material. The luminosity of the x-rays is controlled by the Eddington limit[81] $L_E = 4\pi G M c/\sigma_c$ where as usual σ_c is the Compton cross section. Ruffini and Wilson[80] have pointed out that accretion processes can exist in which

$$\left(\frac{dE}{dt}\right)_{accr} = \frac{GM_2}{r_0}\frac{dM}{dt} >> \frac{4\pi G M_2 c}{\sigma_c} = L_E \,(\sim 10^{38} \text{ erg/sec for } M_2 \sim M_\odot)$$

due to the fact that a large fraction of the energy of the accreting material $\left(\frac{dE}{dt}\right)_{accr}$ could be emitted as neutrinos through to the reaction $\gamma \rightleftarrows e^- + e^+ \rightarrow \nu_e + \nu_e$. The accretion rate into the neutron star, far from being then constrained by the Eddington luminosity to the value $\left(\frac{dM}{dt}\right) < 10^{-9}\ M_\odot$/year, could reach much higher values e. g. $\frac{dM}{dt} \sim 10^{-6}$ M /year. This accretion rate would give rise to neutrino fluxes at $L_\nu \sim 8.5 \times 10^{39}$ erg/sec or to neutrino and antineutrino fluxes at the surface of the earth $\Phi_\nu = 22.2/\text{cm}^2$ sec assuming the source 1000 parsec away. During these processes (as pointed out in Ref. 80) the x-ray luminosity of the x-ray source is still equal to the Eddington luminosity. Details on the structure of the accretion disk can be found in references 82 — 88.

TABLE I

Occultation angle $\varphi = (T_{occ}/T_{orb}) \times 180°$ for selected values of the inclination of the orbit and selected value of the ratio $q = M_1/M_2$. The case $i = 90°$ corresponds to the line of sight in the orbital plane of the binary system. It has been assumed in these computations that the main star fills its Roche lobe and that the x-ray source can be considered point-like. For every value of q there exists a critical value of the inclination angle i_{crit} for which no occultation is possible if $i < i_{crit}$.

$q = \frac{m}{M}$	Ω_0	$\varphi_{i=90°}$	$\varphi_{i=80°}$	$\varphi_{i=70°}$	$\varphi_{i=60°}$	$\varphi_{i=50°}$	$\varphi_{i=40°}$	$\varphi_{i=30°}$	$\varphi_{i=20°}$
1,0	3.7500	22.00	19.81	10.46	—	—			
0.8	3.41697	23.30	21.29	13.25	—	—			
0.6	3.06344	25.03	23.22	16.40	—	—		no occultation possible	
0.4	2.67810	27.56	26.00	20.41	—	—			
0.3	2.46622	29.42	28.00	23.09	9.58	—			
0.2	2.23273	32.09	30.85	26.69	17.15	—	—	—	—
0.15	2.10309	34.00	32.88	29.16	21.16	—	—	—	—
0.1	1.95910	36.72	35.73	32.52	26.04	10.77	—	—	—
0.05	1.78886	41.32	40.52	37.97	33.15	24.50	—	—	—
0.02	1.65702	47.16	46.54	44.60	41.13	33.58	26.70	—	—
0.01	1.59911	51.29	50.77	49.17	46.37	42.10	35.92	27.01	12.59
0.005	1.56256	55.13	54.69	53.35	51.04	47.63	42.98	36.97	29.83
0.001	1.52148	62.87	62.55	61.63	60.06	57.87	55.07	51.83	48.54

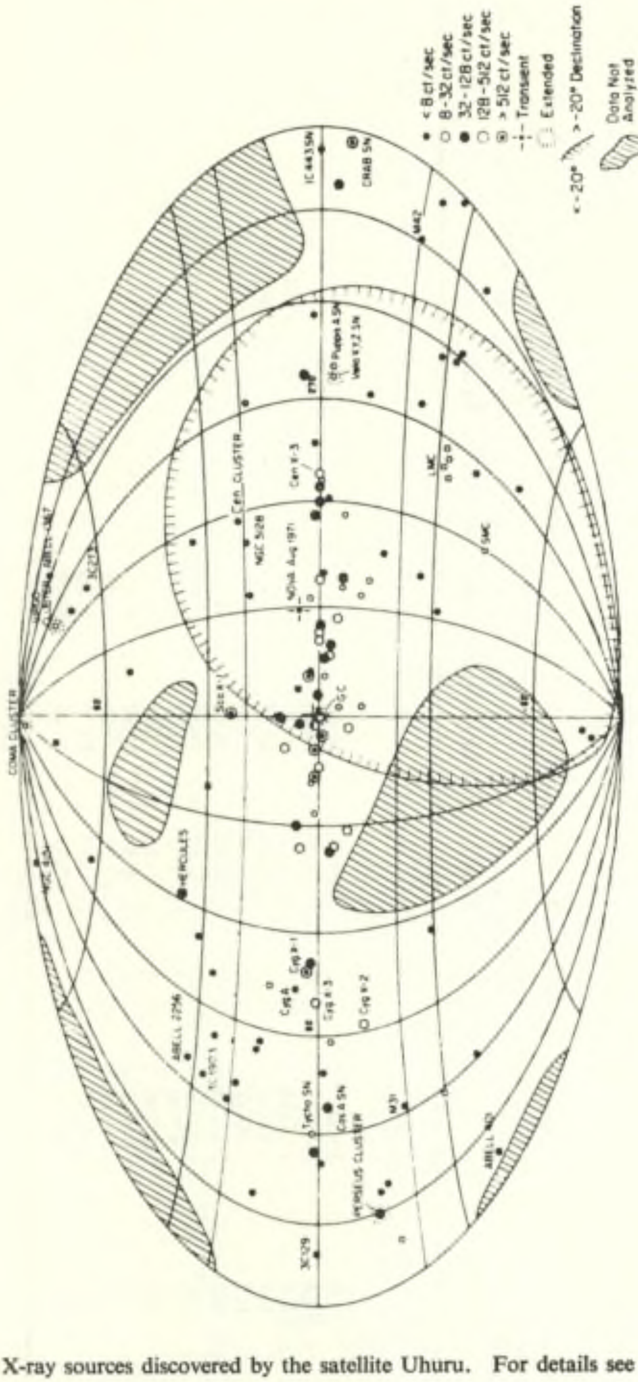


Fig. 21. X-ray sources discovered by the satellite Uhuru. For details see ref. 78.

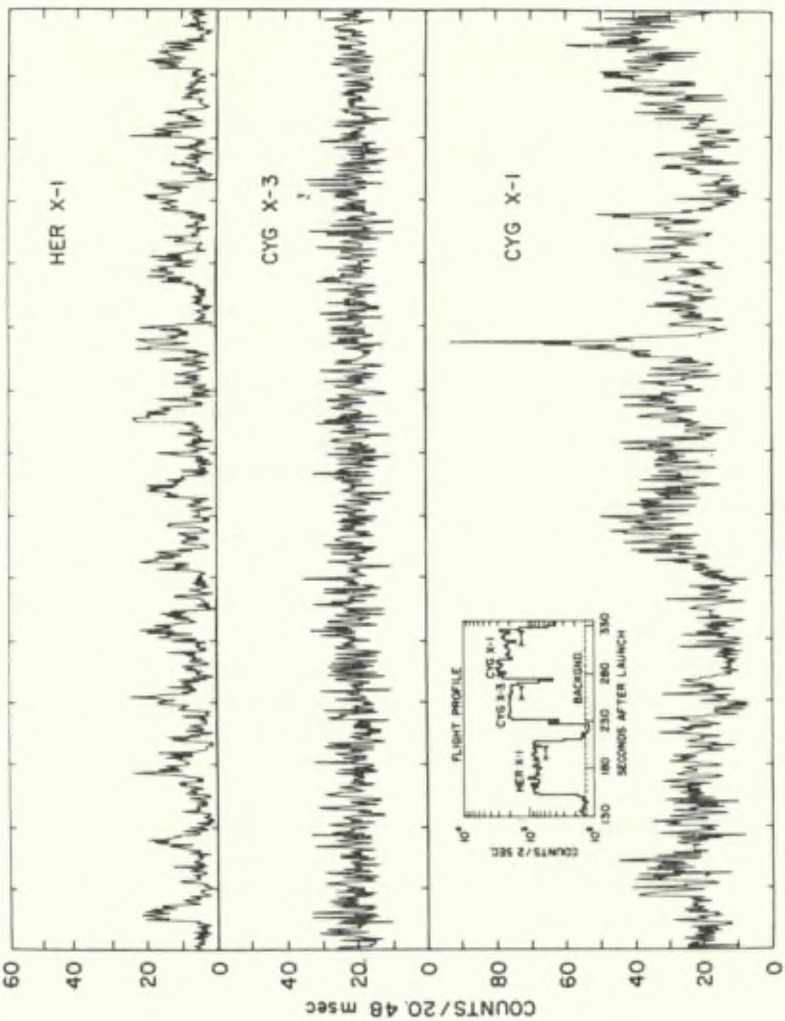


Fig. 22. X-rays data of the rocket flight from Goddard Space Center flown by Rothschild, Bolt, Holt, and Serlemitsos[125]. The flight lasted 330 sec with $\sim$ 70 sec observations of the pulsating X-ray sources Hercules X1, $\sim$ 40 sec on Cygnus X3 and $\sim$ 60 seconds on Cygnus XI. A direct comparison between the data of Hercules XI, Cygnus X3 and Cygnus XI clearly shows the marked differences in the X-ray signals in the three sources; regularly pulsating with a period of $\sim$ 1.2 sec the ones in Hercules XI bursting with time scales down to a few milliseconds the ones in Cygnus XI (for details see § 5 and § 6, and references mentioned there, also see reference 17).

6. REGULARLY PULSATING BINARY X-RAY SOURCES

There are two binary x-ray sources which have a sharply defined pulsational period in their x-ray emission in the range 1 — 20 kev: Hercules X1[91] has a pulsational period $P_o \sim 1.23$ sec, Centaurus X3[92] $P_o \sim 4.84$ sec. Their binary nature is most clearly shown by their occultation in the x-ray emission and by the Doppler shift in their intrinsic pulsation period P_o due to the orbital motion of the x-ray source (see Fig. 23). In the case of Hercules X1 the companion star has been identified with the star H_z Hercules[93] while the optical identification of the normal companion star of Centaurus X3 is still tentative[94].

The main arguments leading to the identification of these sources with rotating neutron stars, members of binary systems, leads to the simple understanding of three main experimental facts:

(1) An amount of energy of the order of $\sim 10^{37}$ erg/sec is emitted by these sources in the x-rays band.

(2) This radiation is emitted in sharply defined pulses with a period $P \sim 1$ sec.

(3) The intrinsic pulsational period of the x-ray source decreases with time (see Fig. 24).

As in the case of pulsars, the identification of these pulsating x-ray sources with rotating neutron stars endowed with an off axis magnetic field allows a simple explanation for the pulsating nature of the x-ray signal. However these x-rays sources depart from pulsars in a very important respect, ($dP/dt > o$), and the energetics of the system are most easily explained by the loss of rotational energy of the neutron star. In our case, instead, the period of the x-ray sources is observed to decrease with time. This clearly implies that the x-ray source is, in fact, gaining rotational energy! (see Fig. 24).

Both the energetics of the system and the gain of rotational energy are most easily explained if we account the accretion of matter from the main star into the collapsed object. The infalling material then imparts angular momentum to the neutron star while the energetics can be explained by the conversion into x-rays of the gravitational binding energy of the infalling material at the surface of the neutron star (up to 10 % of its rest mass).

Many of the features of the x-ray emission both of Hercules X1 and Centaurus X3 still present outstanding difficulties for their detailed explanation. However, we can emphasize that for the first time we shall be able in the near future to obtain from these two systems an accurate direct measure of the mass of a neutron star. Assuming that the main star fills its Roche lobe, then from the velocity of the neutron star in its binary orbit, the length of the occultation, and the binary period, we can directly estimate the neutron star mass for selected values of the inclination as shown in Fig. 23. We can then conclude that in the case of Hercules X1, it is most likely that the neutron star has a mass larger than the value of the critical mass as computed by Oppenheimer and Volkoff[13].

If this result will be confirmed by an analysis of the Doppler shift of the main star (H_z Herc.) associated with the x-ray source then we will have the first clear experimental evidence that the contribution of strong interactions in the description of the neutron star material has to be taken into serious account and cannot be neglected.

A detailed monitoring of the intrinsic pulsational period and of the binary period of the x-ray source can give important informations both on the amount of material being transfered in the binary system and the one accreting onto the neutron star. We can write the following general formula[89]

$$I \frac{d\omega}{dt} + \omega \frac{dI}{dm} \frac{dm}{dt} = \left(\frac{dJ}{dt}\right)_{\rm diss.} + \left(\frac{dJ}{dt}\right)_{\rm accr.}$$

Here $\left(\frac{dJ}{dt}\right)_{\rm diss}$ is always negative and takes into account all the loss of angular momentum due to dissipative processes from electromagnetic or gravitational radiation emitted from magnetic fields or changing gravitational quadrupole moments of the neutron star[90]. $\left(\frac{dJ}{dt}\right)_{\rm accr}$ is the angular momentum transferred to the neutron star by the accreting matter. $\frac{dI}{dm}$ determines the change in the moment of inertia of the neutron star as a function of accreting mass which is drastically dependent upon the details of the equation of state of the neutron star material (see Fig. 5). We can then conclude that to determine with great accuracy the change of the moment of inertia of a neutron star as a function of its change in mass can lead to basic informations about the equation of state of matter at nuclear and supernuclear densities in neutron star material. If we assume[89] for $\frac{dI}{dm}$ the value computed from selected equations of state as given in Fig. 5 and $dm/dt \sim 10^{-8}\ M_\odot$/year, then in both the case of Hercules X1 and Centaurus X3 we have $I \frac{d\omega}{dt} >>$

$\omega \frac{dI}{dm} \frac{dm}{dt}$. It is however conceivable that in some other binary x-ray sources not yet detected, or during some phases of the accretion process, the quantity $(dJ/dt)_{accr}$ is so small as to make observable both the change of moment of inertia and the dissipative terms in the loss of angular momentum. It is interesting here to remark that the change of the intrinsic period of pulsation also allows obtaining information on the structure of the accreting disc of material around the neutron star. If we assume that the accretion on the neutron star occurs from a disc in which angular velocity is removed by viscous stresses,[88] then, the accreting material transfer to the neutron star the Keplerian angular momentum of the inner edge of the disc R

$$\frac{dm}{dt} (G\ M\ R)^{1/2} = I\ d\omega/dt \tag{2}$$

Here we indicate by M and I the mass and the moment of inertia of the neutron star and by $\frac{dm}{dt}$ and $d\omega/dt$ the rate of mass accretion on the neutron star and the change of angular velocity, respectively. If we substitute for $d\omega/dt$ the observed values we can then obtain an absolute upper limit from (2) to the rate of matter accretion: $dm/dt \sim 1.1\ 10^{-10}$ $M_\odot$/year for Hercules X1 . This value is much smaller than the absolute lower limit on the rate of accretion obtainable on purely energetic grounds: assuming that up to ten percent of the rest mass of the accreting material could be transformed into x-rays we would obtain for a source intensity of $\sim 10^{37}$ ergs/sec an absolute lower limit of $dm/dt \lesssim 1.5\ 10^{-9}$ $M_\odot$/year. We can then conclude that the disc structure, if existing at all, is drastically modified by the presence of the magnetic field of the rotating neutron star. Additional information on the dynamics of the binary system can be acquired if we notice that the binary period also changes with time. In the case of Centaurus X3 the binary period T has been observed to decrease of $\frac{\Delta T}{T} \sim 3.5 \times 10^{-5}$ over one year in 1971.

It has been shown how this variation can be explained[89] with a very large outflow of matter $\left(\frac{dM}{dt} \gtrsim 10^{-3}\ M_\odot/\text{year}\right)$ from the binary system. It is most likely, therefore, that some of the low states of Centaurus X3 (see Fig. 24) are indeed due to the absorption of the x-rays by this very large outflow of matter it is also very important to correlate changes either of the intrinsic pulsational period or of the binary period with changes in the intensity of the x-ray emission. This correlation appears to be most promising for the understanding of the accretion processes.

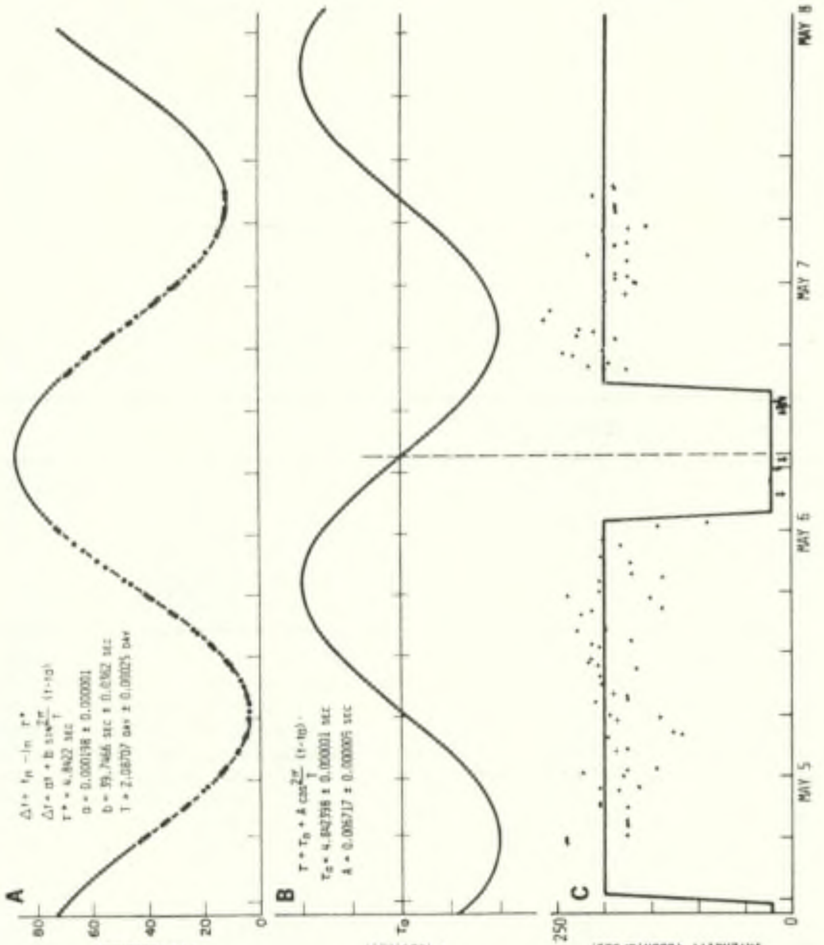


Fig. 23. X-rays data from Centaurus X3[92]. The clear occultation of the x-ray source by the normal star is shown in part (c). The binary period is T = 2.087 d and the eclipse or low state 0.55 d, the transition from the high to the low state takes place in $\sim$ 0.04 d (In the case of Hercules X1[91] the orbital period T = 1.7 d, the eclipse last 0.24 d, the transition between the high intensity state and the eclipse occurs in less than 12 minutes). The intrinsic period of the x-ray source is modulated by two different effects: A Doppler effect, part b, and an arrival time delay, part a. Due to its velocity in the orbital motion and to the Doppler effect the intrinsic period appears

to have a sine wave modulation, the amplitude being proportional to the projected velocity of the x-ray source along the line of sight. As a direct consequence the velocity of the x-ray source in the circular orbit can be estimated to be $v_x \sin i = 415.1$ km/sec, i being the inclination of the orbit. (The corresponding velocity for Hercules X1 is $v_x \sin i = 169$ km/sec). As a consequence of the fact that the x-ray source is moving in a circular orbit the arrival time of the pulse is delayed 39.7 sec at the center of the occultation and is 39.7 seconds early at the center of the high state. (The corresponding time delay of Hercules X1 gives 13.2 seconds). The delay time of 39.7 sec. gives a direct measurement in light seconds of the radius of the orbit of the x-ray source about the center of mass of the system as projected into the observing plane. The nearly sinusoidal feature of the curves in (a) and (b) allow to place a limit to the eccentricity of the orbit to $\varepsilon < 0.05$ ($\varepsilon < 0.05$ also in the case of Hercules X1). If we adopt the Roche model for the accretion of matter in the binary system[17 79] we can evaluate parameters of the binary system from the value of the period T, of the occultation time and the projected orbital velocity of the masses. For Centaurus X3 for selected value of the inclination we have:

i	$m_x/M_\odot$	$M/M_\odot$	$a/R_\odot$	$R/R_\odot$	v_x	v_M
90°	0.275	16.0	17.4	12.7	415.1	7.14
80°	0.250	16.6	17.7	13.3	421.5	6.32
60°	0.194	24.2	19.9	15.5	479.3	3.83

and in the case of Hercules X1

i	$m_x/M_\odot$	$M/M_\odot$	$a/R_\odot$	$R/R_\odot$	v_x	v_M
90°	1.20	2.1	8.9	3.8	169.0	96.3
80°	0.78	1.8	8.2	3.7	171.6	73.8
60°	0.17	1.6	7.2	4.1	195.2	20.5

Here we have indicated by $m_x/M_\odot$ and $M/M_\odot$ the mass of the x-ray source and of the main star, by $a/R_\odot$ and $R/R_\odot$ the separation between the center of masses of the two stars and the radius of the main star and by v_x and v_M the velocity of the x-ray source and of the main star. It is interesting to remark that both these estimates are very interesting: the neutron star in Cen. X3 appears to have a very small mass while the one in Hercules X1 has a mass which for a suitable inclination is larger than the critical value of neutron star as computed from an equation of state neglecting the nuclear interactions. Both results should be confirmed by a more model independent derivation, possibly, by the observation of the velocity of the companion star. (Figure reproduced by the kind permission of R. Giacconi[95]).

Fig. 24. The intrinsic period and intensity of the x-ray source of Cen. X3 ($P_u \sim 4.8$ sec) are here plotted as a function of time. In the case of Cen. X3 the intrinsic period has decreased of 1.1 msec from January to May 1971, 0.2 msec from Dec. 1971 to Sept. 1972 and of about 50 msec from September to October 1972. In the case of Hercules X1 the period decreased 5 msec from January to August 1972, increased of 3msec from September to October 1972 and decreased 3 msec from October 1972 to March 1973. Much can be learned from a detailed analysis of this intrinsic periods, see text, on the structure of the magnetic field of the neutron star and the rate of matter accretion. Also correlations between the changes in period and a changes of intensity of the x-ray emission can give both information on the rate of accretion and the composition of the neutron star, see text. (Figure reproduced with the kind permission of R. Giacconi)[95].

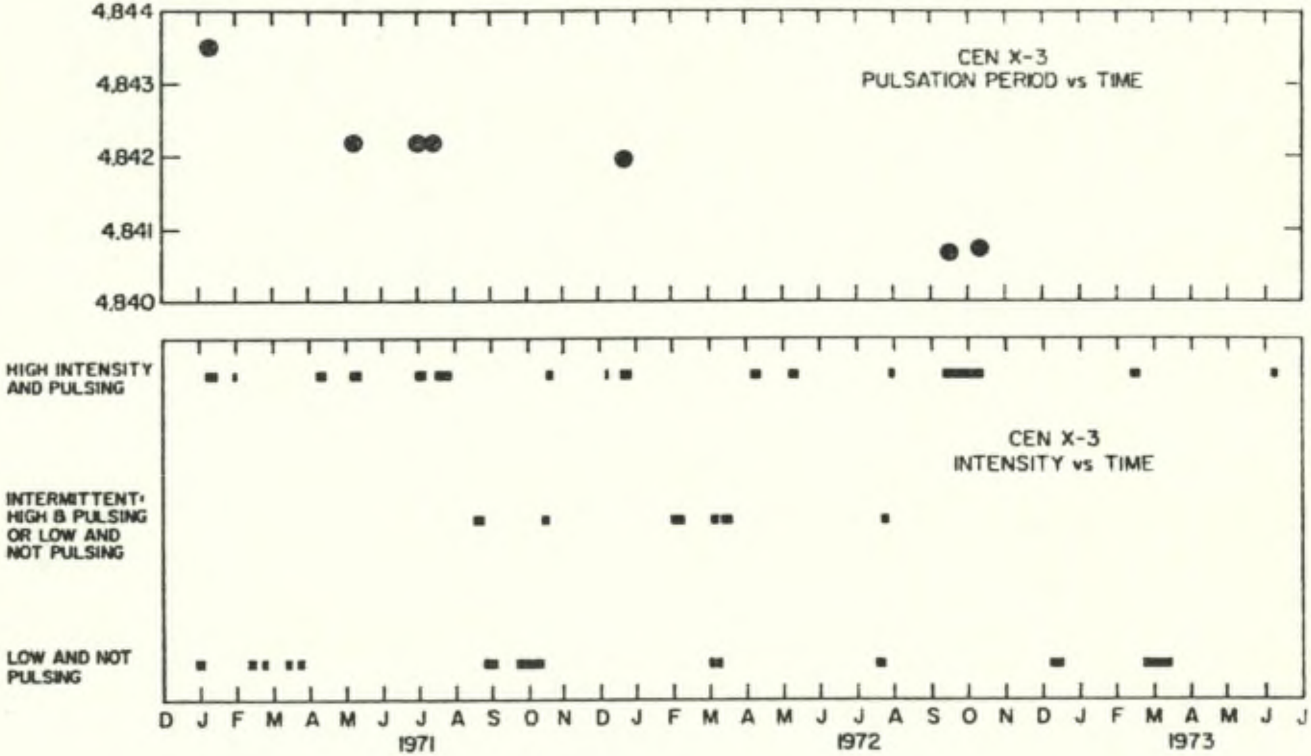

CEN X-3
PULSATION PERIOD vs TIME
4.844
4.843
4.842
4.841
4.840
CEN X-3
INTENSITY vs TIME
HIGH INTENSITY AND PULSING
INTERMITTENT: HIGH & PULSING OR LOW AND NOT PULSING
LOW AND NOT PULSING
D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J
1971
1972
1973

Fig. 24

7. BURSTING BINARY X-RAY SOURCES

The characteristics of these sources are very similar to those presented in the previous paragraph: they are members of binary systems and the energy they radiate in x-rays is $dE/dt \lesssim 10^{38}$ erg/sec. They drastically differ, however, from the ones presented there in one important respect: the x-rays are not emitted in regular pulses but they present only short intensity variations and flare like phenomena in the x-rays with intensity changes by a factor two or more on a time scale down to a few milliseconds. Since we are dealing again with close binary systems the most direct explanation for the strong x-ray emission of x-rays is, as in the previous case, accretion of matter from a normal star into a compact collapsed object. The main reasons for requiring that the object on which the accretion occurs be a collapsed object are made both on energetic grounds (we need a deep potential well in order to transform enough gravitational energy into electromagnetic energy) and on the grounds of the irregular variations in the x-ray intensity observed in some sources to extend down to a few milliseconds. This last experimental result clearly implies that the region of x-ray emission has to be very compact.

The absence of a regular pulsation in the x-rays can be ascribed to the fact that the collapsed object is either a black hole (see § 5) or a neutron star deprived of an off axis magnetic field of such an intensity as to modulate the x-ray emission of the accreting material[79]. The identification with a neutron star is clearly impossible if the collapsed object proves to have a mass larger than the absolute upper limit of the neutron star critical mass[29]. There are several binary x-ray sources which have these common features in their x-ray emission.

Cygnus X3 was originally discovered by Giacconi et al[96] as early as 1967. This source presents large intensity variations on time scale as short as 0.1 seconds (see Fig. 22) and a strong cutoff in the low energy x-rays due to considerable absorption[97]. The x-rays in the 2-6 Kev range have a nearly sinusoidal variation in intensity of about a factor two with a period of 4.8^h [97]. No occultation of the x-rays of the kind observed in Hercules X1 and Centaurus X3 due to the eclipse of the companion star is here present. However, as will be explained later, the absence of an occultation due to eclipse does not preclude in any way the possibility of the system being a binary. Similarly, the fact that the period of the variation of the x-ray intensity is much shorter than the binary period of the other sources can still easily be explained in the framework of a binary model (see particularly § 8).

No optical object down to 17 m has been associated with this source. Strong activities of Cygnus X3 both in the infrared and in the radio have been observed. Becklin et al[98] have observed from Cygnus X3 a very strong infrared emission (10^{-2} f.u. at 22 μm, 1 f.u. = 10^{-26} Wm^{-2} Hz^{-1}) synchronous with the 4.8^h periodicity in the x-ray source. In the radio Cygnus X3 also presents very strong activity. The radio emission (observed at 10.522, 6.630, 3.244 MHz) was observed to have an average value of 0.01 f. u. however on the 20th of September 1972 it was observed to rise[99 100] in less than two days to 22.5 f. u. decaying then to its usual value during the following week. A possible evolution of this system has been suggested (see § 8) by Ruffini and Treves.

2U 0115 — 73 or S M X1: This has been the first extragalactic x-ray source to be identified[101] Its binary nature is clearly shown by the periodic occultation due to the eclipse of the main star and lasting 0.60 $\pm$ 0.004 days with a binary period of 3.8927 days. The x-rays have again a large low energy cutoff due to absorption and are changing intensity considerably on a time scale as short as minutes (for a possible correlation between absorption and short time variability see the discussion later in this section on Cygnus X1). Since the distance to the source is accurately known, we can estimate with great accuracy the luminosity of this source to be $\sim 5 \times 10^{38}$ erg/sec, very close to the value of the critical Eddington luminosity (see Fig. 20). The optical counterpart of SMX1 has been identified with Sanduleak N° 160 by Liller[102] and Hiltner et al[103]. This star is a B01 and estimates of its mass give $M \sim 20\ M_\odot$.

From the binary period and the length of the occultation we can estimate (see also Fig. 20 and Tab. I) the mass function q = 0.4 for an inclination of 90° and q = 0.1 for an inclination of 60°. If we assume, as suggested in ref. 103 that the inclination of the orbit is i = 79° we then obtain for the x-ray source a mass $m_x \sim 5.6\ M_\odot$ well above the absolute upper limit of the value of the critical mass of a neutron star.

2U 0900 — 40 VelXR1: This source discovered by Chodil et al in 1967 [104] has been shown by Ulnmer et al[105] and more accurately by Forman et al[106] to have an occultation due to eclipse of the x-ray source of 1.9 $\pm$ 0.05 days with characteristic binary period of 8.95 $\pm$ 0.02 days. Again the spectrum of the x-rays presents low energy cutoff due to absorption and large variation in intensity on a time scale as short as a few seconds. The definite optical identification of the optical counterpart of Ve*l*XR1 has been given by Hiltner et al[107] and Hutchings[108] with the star HD77581. This star shows radial velocities caused by the binary motion which are in agreement with the 8.95 days period of the x-ray source. An estimate of the mass function of the system can, once again, be obtained from Tab. I. For a mass of the main star $M \sim 15\ M_\odot$ this gives $m_x \sim 1.5\ M_\odot$[79]. This system is certainly one of the most interesting borderline case to determine the experimental value of a minimum mass of a black hole or maximum mass of a neutron star.

2U 1700 — 37: This source, discovered in the x-rays by Jones et al[107] has a period of 3.412 days with an occultation due to the eclipse of the companion star of 1.10 ± 0.07 days. Assuming a distance of 1.7 kpc the energy of the x ray source is $\sim 3 \times 10^{36}$ erg/sec in the 2 — 10 Kev range. As in the previous case, the spectrum presents a strong cutoff at low energy due to absorption. Large changes of intensity have been observed with time constant down to a tenth of a second or less. The eclipse of this source is much to long to apply the results of a Roche Model as given in Tab. I and Fig. 29[79]. Jones et al[109] have suggested the star HD153919, as the optical counterpart of the x-ray source. This identification has been widely confirmed[110]. A detailed analysis of this source is however still needed both to ascertain value of the mass and explain the very long occultation.

3U 1516 — 56 or Circinus X1: This source presents a binary period consistent with 12.29 days and large time variability with time scales down to 100 msec (Uhuru limit). This source reveals, in its time variability, features very similar to Cygnus X1[111]. In this sense it is a very important source to analyze in much detail.

Finally *Cygnus X1*: In the following we shall mainly focus on the case of Cygnus X1 since we consider this by far the system with the greatest amount of data and the greatest physical interest. The first detailed observations of Cygnus X1 were obtained by Giacconi et al[112, 113] and Rappaport et al[114]. From the distance of the source[115, 116] and the observed flux and spectrum of the x-rays it was possible to infer that Cygnus X1 had to emit dE/dt $\sim 10^{37}$ erg/sec. The x-ray intensity was observed to have very large changes on a time scale of less than 50 milliseconds[114]. Very high energy flux and the short time variability in the x-ray intensity most naturally lead to the assumption that Cygnus X1 had to be an accreting collapsed object and a member of a binary system. However, the identification of this source with a binary system appeared very problematic from the beginning. The major " signature " characterizing an x-ray source member of a binary system (see Fig. 20) was missing in this case, namely, the x-ray source was not regularly occulted by the main star with the regular binary period of a few days. Moreover the absence of an intrinsic pulsational period, or for that matter of any regular long lasting structure in the intensity variations did not allow the use of the Doppler effect to infer the orbital motion of the x-ray source as in Centaurus X3 or Hercules X1.

The absence of regular occultation of the x-rays could still be made consistent with a binary system model if it was assumed that the angle between our line of sight and the orbital plane of the binary was larger than a critical amount ($i < 40^{o}$ see Tab. I), From the pure geometrical features of the Roche accretion model we should in fact expect that out of N_{obs} observed binary x-ray sources a number $N \sim 4\, N_{obs}/9$ should *not* present any occultation if indeed the orientation of their orbital plane is, as it should be, completely random. Clearly this figure should be taken with the due caution since is based on a direct application of the idealized case of a Roche model which at least in some sources (see e.g. 2U 1700 — 37) is proven not to fit the experimental results. In any case, since $N_{obs} \sim 8$, the fact that two sources, Cygnus X1 and Cygnus X3, do not have occultation should be considered in perfect agreement with the general expectation and explainable as an effect of the orientation of their orbital plane.

In sharp difference from the other x-ray sources, Cygnus X1 presents very little low energy cutoff due to absorption (see Fig. 25). This experimental fact gave the first confirmation that, most likely, we were looking at Cygnus X1 at a small inclination angle. The reason is simply explained: we expect in the accretion processes that the majority of the x-rays are emitted in a region very near the surface of the collapsed objects. From the geometry of the system we should then expect large absorption in the x-ray spectrum the more the line of sight will approach the orbital plane of the binary system.

In this sense Cygnus X1 will most likely be the most interesting system to examine: only in the case of small inclination angle will we be able to see the processes occuring near the surface of the collapsed object. Moreover the more the emission processes will occur near the surface of the collapsed object the shorter should expected to be their time variability. We can then conclude that we should notice a correlation in x-ray sources between the absorprion and their intensity changes: The smaller the absorption, the more structure we should find at shorter time scale.

To further analyze the nature of Cygnus X1 the region of the sky around this source was carefully anlyzed for the possible existence of radio emission. A detailed examination was made at 11 cm by Hjellming[117] who could not find any source down to the limit of 0.01 flux units. The motivation of this detailed analysis was dictated from a straightforward consideration: if Cygnus X1 was indeed a collapsed object and if its formation had occured through the usual supernova process then we should have expected a radio remnant around the collapsed object. The absence of a radio remnant together with the absence of the disruption of the binary system[118, 119] suggested the conclusion that the formation of Cygnus X1 had occurred through a different process than the usual supernova explosion (see § 9).

The key result which lead at once to the clear identification of Cygnus X1 as a member of a binary system and to the identification of the companion normal star of the system with the BO supergiant HDE

226868[120,121] has been due to the direct observation of an abrupt change in the x-ray spectrum occurred between 22 March and 28 April 1972. For still unexplained reasons correlated with this change of spectrum, a radio source appeared (0.02 f.u.) in the x-ray error box of Cygnus X1 see Fig. 26.

The location of the radio source allowed the determination with great accuracy of the position of Cygnus X1 and the immediate optical identification by Webster and Murdin[118] and Bolton[119] of the other member of the binary system with the BO supergiant HDE226868. See Fig. 27. The further evidence for the binary nature of the system was proved by Bolton through a detailed analysis of the Doppler shift of the line of the star due to its orbital motion (see Fig. 28).

Evidence for the accretion and infall of material from the main star into the x-ray source, and a direct estimate of the inclination of the orbital plane of the binary system with respect to the line of sight have been given by Hutching et al[122] ($i \sim 27^o$). By the analysis of the emission line He II λ 4686, first discovered by Bolton [121] and Brucato and Kristian[123], Hutching et al were able to show that these emissions line were originated in a stream of material from the main star to the x-ray source their velocity curve being 120^o out of phase from the lines of HDE226868 (see Fig. 29).

This entire set of observations by Bolton and Hutchings lead to the following clear conclusions:

(a) Cygnus X1 is indeed a member of a binary system,

(b) accretion is occurring between the main star and the x-ray source,

(c) the absence of an occultation of the x-ray source is a direct consequence of the high inclination of the orbit ($i \sim 26^o$).

The experimental data of Cygnus X1 have allowed a direct estimate of the masses of the system see Tab. II. It is important to remark here that in all cases the estimate of the mass of the x-ray source is much larger than the absolute upper limit to the neutron star mass of $3.2\,M_\odot$[29].

The three main arguments therefore[124]:

(1) Very large emission of energy $dE/dt \gtrsim 10^{37}$ erg/sec

(2) Intensity variations on time scale as short as 50 msec

(3) Mass larger than $3.2\ M_\odot$

all point to the clear evidence of a completely collapsed object or black hole in Cygnus X1 accreting material from the companion star. As pointed out in reference 17, the only alternative possibility is that a violation of a fundamental law of physics (either violation of general relativity or violation of causality) occurs in this system.

The discovery of a black hole in nature would prove to be completely sterile if we could not observe the processes occuring near its horizon.

The reason is simply explained: the most clear and important relativistic effects (gravitational redshift, dragging of inertial frames, and fully relativistic electrodynamic processes) give large and detectable effects only within a few kilometers from the surface of the black hole.

To probe the most novel predictions of this kind of physics a line of attack very similar to the one usually adopted in elementray particle physics is needed: the only way to infer the near field structure of a collapsed object is to proceed with scattering experiments. We define the entire set of the electromagnetic and gravitational multipole moments of a black hole as the " form factor " of the black hole. To determine this " form factor " we have therefore to consider emission processes which occurs more and more near the surface of the black hole or equivalently with shorter and shorter time scales[17 79]. It is already clear that as a direct consequence of the finite radius of the collapsed object we should also find a clear cutoff for signal with time constant shorter than the characteristic time of the last stable orbit of revolution of a particle around a black hole ($P \sim G\ M/c^3$).

Only in the case of Cygnus X1 in which the observations can be made at a small inclination angle can we expect to infer the properties of the processes occuring near the horizon of the collapsed object.

In this respect the most important results have been the observations made by Rothschild et al[125]. from a rocket flight flown on October 4th 1973. See Fig. 30 and Fig. 31. It is not our goal to go in a detailed explanation of the origin of the train of pulses observed from Cygnus X1 during this flight. Here, it is important only to stress the relevance of these results for black holes physics. If we look at the width of the last three pulses in the train ($\sim$ 1.28 msec, see Fig. 31) and examine their relative separation we find that the first two pulses are 4.48 msec and the last two 8.32 msec apart. Assuming for the black hole a mass $M \sim 10\ M_\odot$ we obtain for the last stable circular orbit of a particle around a Schwarzschild black hole $P = 12\ \pi\ (6)^{1/2}\ GM/c^3 \sim 4.5$ msec and in the case of an extreme Kerr black hole using the definition of angular velocity given by Christodoulou and Ruffini[61]

$\omega = a/(r^2_+ + a^2)$ with $r_+ = m + (m^2 - a^2)^{1/2}$ and $a = L/m = 1$ we then have $P = 4\pi\ GM/c^3 \sim 0.64$ msec

It is therefore clear that these observations are of the right time constant for a deeper understanding of the " form factor " of a black hole. We need more and continuous observations of this kind with extremely accurate statistical analysis and with as much intensity of x-rays as possible (larger collecting areas in telescopes!). Only through a direct analysis of similar data and a continuous feed back with the theoretical predictions we will be able to know if and how general relativity applies in these very strong ultrarelativistic regimes.

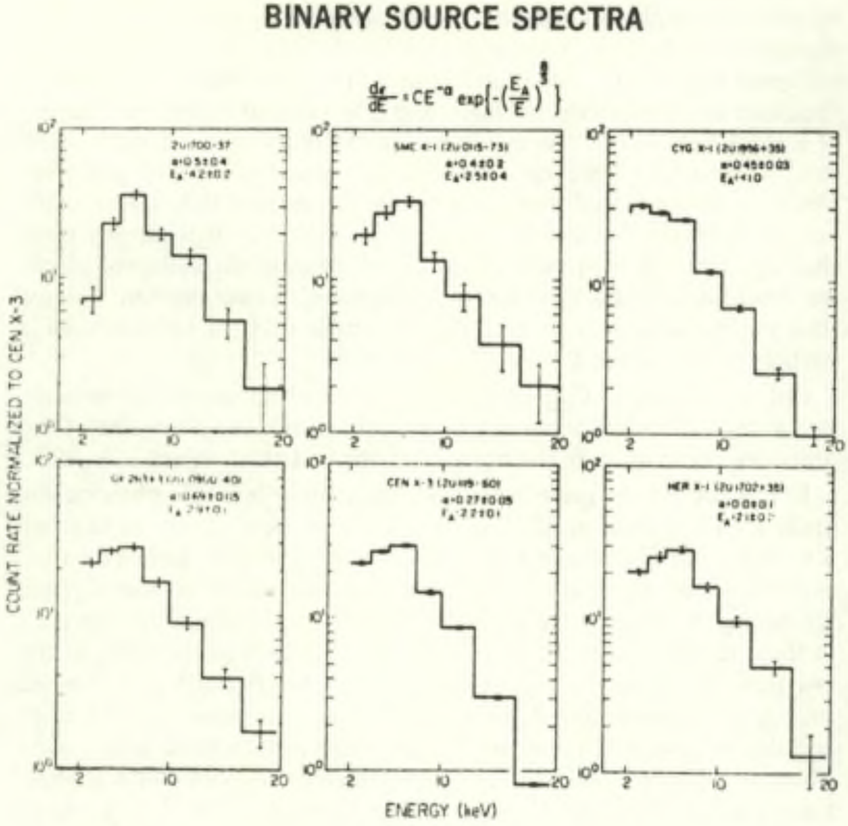


Fig. 25. Spectra of six binary x-ray sources in the range of energy 2 kev $\leq$ E $\leq$ 20 kev. In the first row from left to right are 2 U 1700-37, S M C X-1, C Y G X-1, in the second row again from left to right 2U 0900-40 Cen X3 and Her X1. All these sources present a drastic cutoff in the range of energies 2 kev $\leq$ E $\leq$ 6 kev due to absorption with the only clear exception of Cygnus X1. This result was extremely important in reaching the conclusion that we are observing the binary source in Cygnus X1 at a small inclination angle and therefore avoiding the absorption of the accreting matter. (Figure reproduced from ref 78 with kind permission of the author).

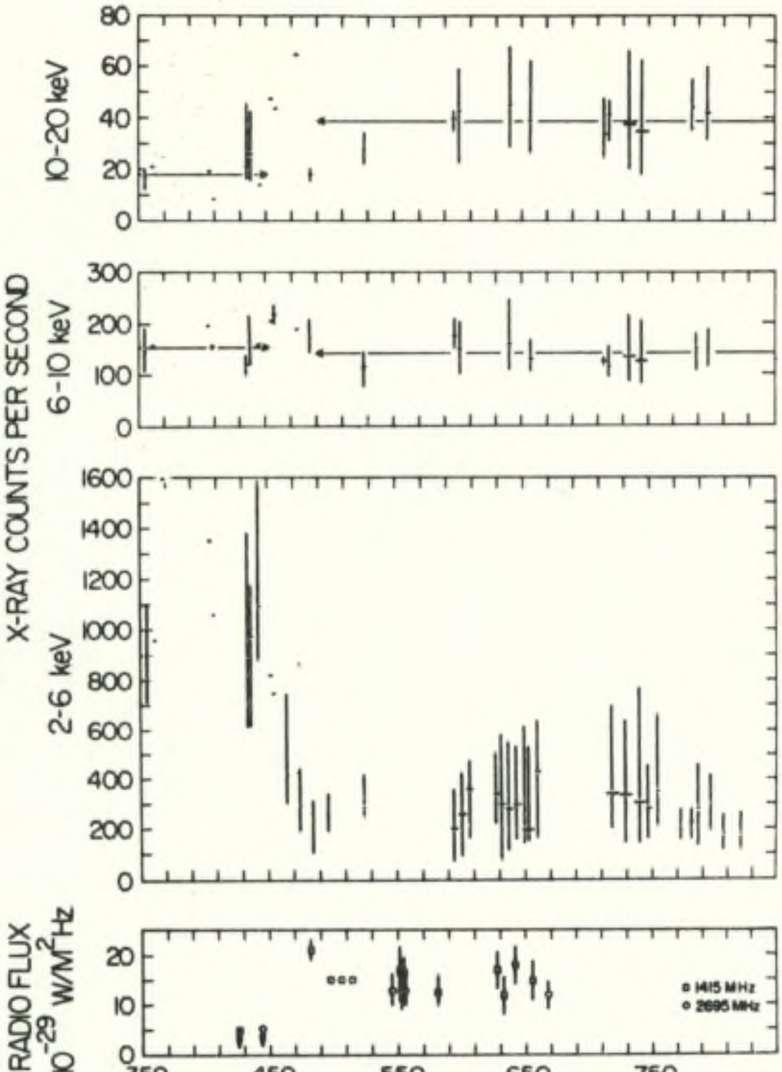


Fig. 26. Transition occurred in Cygnus X1 between 12 March and 28 April 1971. The spectrum drastically changed with an increase of intensity in the high energy (10-20 kev upper figure) of a factor 2 and a decrease of intensity in the low energy (2-6 kev) of a factor 4. Hjelming[117] has reported that sometime between March 22 and March 31 radio source first appeared and remained at a level of .02 f. u. in the x-ray error box (lower figure). No radio source was present in the x-ray error box before this transition down to a level of 0.01 f. u. This result allowed to give a much improved position for the x-ray source and led to the identification of the optical companion star HDE226868. (See Fig. 27). (Figure reproduced from ref. 78 with kind permission of the author).

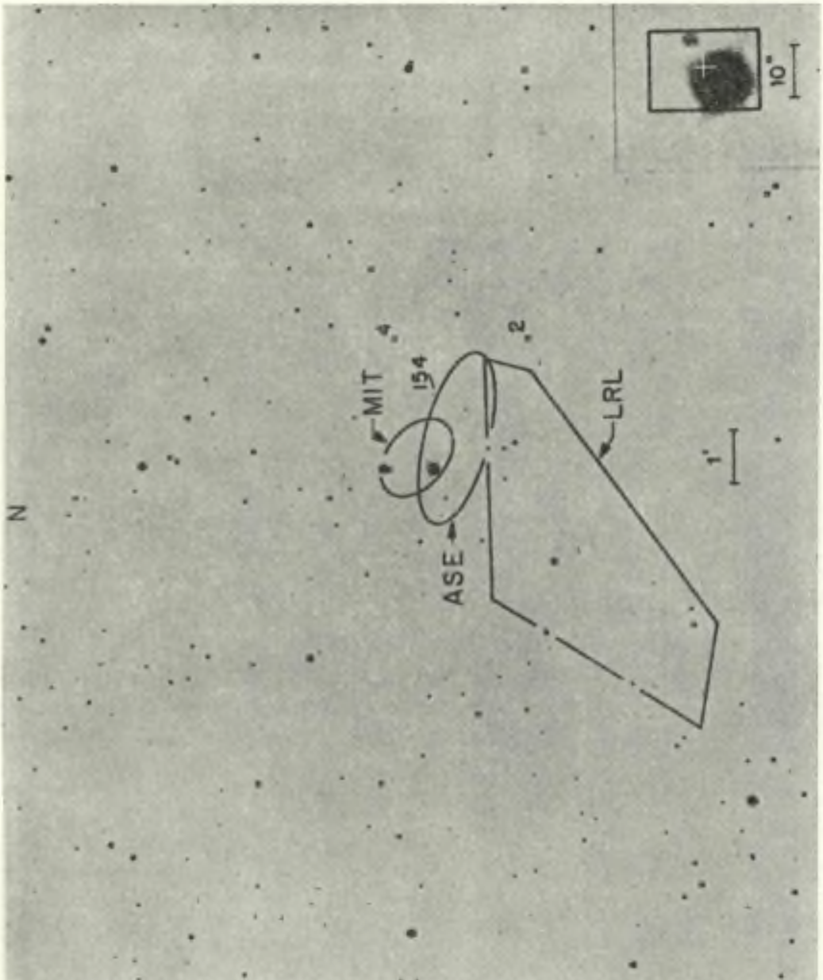


Fig. 27. Optical plate showing the star HDE226868. The error boxes indicate the x-ray position of the source as from rocket flights and Uhuru satellite data (ASE). In the corner is a magnification of the central region (10″) with the star HDE226868 and the radio error box (white cross). The analysis of the spectral lines of the star have clearly shown the sine wave shift typical of a star member of a binary system. (Figure reproduced from reference (95) with kind permission of the author).

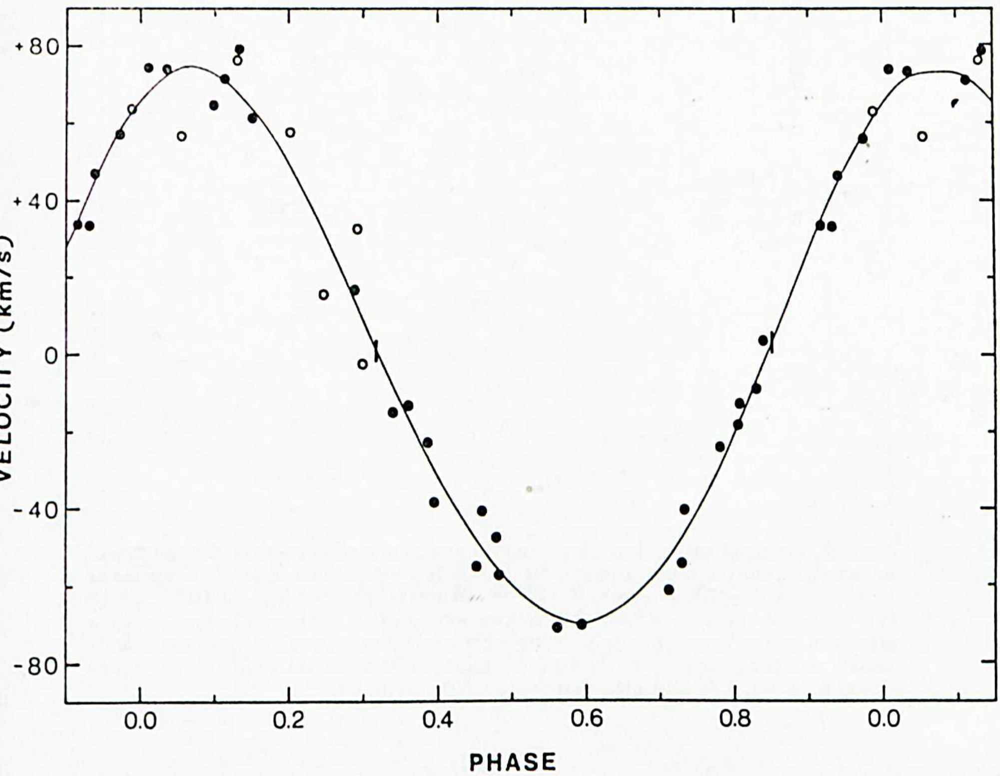


Fig. 28. Velocity curve of the absorption lines of HDE226868 obtained from 12 /mm spectrograms taken by T. Bolton[127] with the 74 inch telescope of the David Dunlap Observatory. The open circles are 1/2 points and the vertical stick marks on the velocity curve indicate times of conjunction. The orbital period deduced is P = 5.599823 ± 0.000037 days, the velocity of the center of mass of the system is v_o = — 1.7 ± 0.5 km/sec. The projected velocity of the star k = v sin i = 72.2 ± 0.8 km/sec the eccentricity of the orbit ε = 0.061 ± 0.11 the time of phase zero T = JD 244 1562.520 ± 0.305 the projected value of the semi axis a sin i = 5.549 ± 0.061 × 10^6 km and finally the mass function f (m) = 0.217 ± 0.007 M_0. T. Bolton quotes for the mass of the components of the system values which fall inside an " error box " whose corners in the M_1 (mass of HDE226868) M_2 (mass of x-ray source) plane are $(M_1, M_2)/M_\odot$ (12.7, 9) (21, 11.5) (35, 18.5), (19.5, 14.9). We thank T. Bolton for allowing to reproduce these data and for many enlightening discussions.

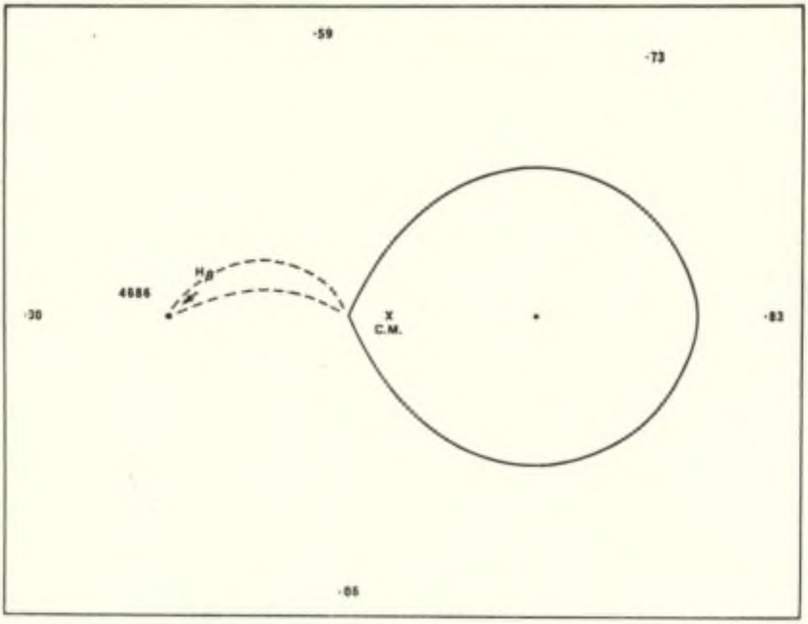


Fig. 29. Profile of matter leaving the main star and accreting on the collapsed object as derived from the observation of He II λ 4686 by Hutching et al[122]. The He II λ 4686 (v sin i ~ 100 km/sec) are 120° out of phase from the lines of HDE 226868 (v sin i ~ 80 km/sec). These observations were essential in strengthening both the argument of the binary nature of the system containing Cygnus X1 and the role of accretion in the generation of the x-ray emission. (Details in ref. 122. Figure reproduced from ref. 122 with kind permission of the authors).

TABLE II

Estimates of the masses of Cygnus X1 and HDE 226868 as given by Brucato-Kristian[123] Hutchings et al[122] Sunyaev et al[126] Bolton[121]. In all these estimates the mass of Cygnus X1 is well above the absolute upper limit of a neutron star[29].

	CYGNUS X1	HDE 226868
Brucato - Kristian	$M_2/M_\odot > 5.5$	$M_1/M_\odot \sim 22$
Hutchings et al	$10 \leq M_2/M_\odot \leq 18$	$16 \leq M_1/M_\odot \leq 23$
Sunyaev et al	$7.8 \leq M_2/M_\odot \leq 17$	$10 \leq M_1/M_\odot \leq 22$
Bolton	$10 \leq M_2/M_\odot \leq 20$	$25 \leq M_1/M_\odot \leq 35$

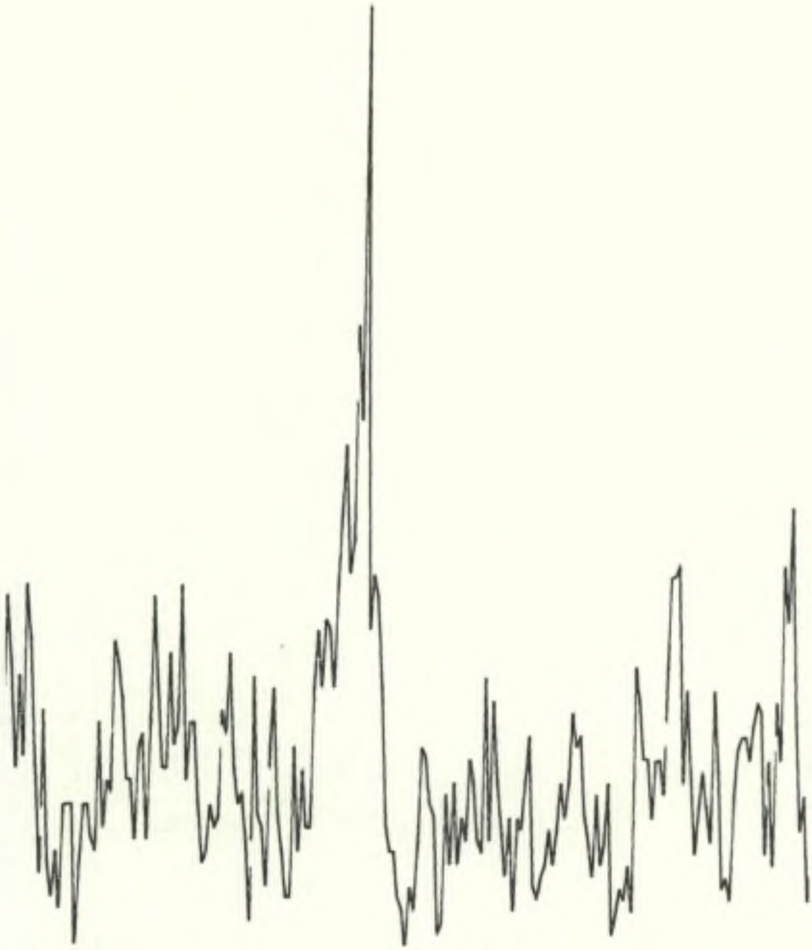


Fig. 30. Enhancement in the data recorded between 318 and 319 seconds after launch of the rocket flight by R.E. Rothschild et al[125]. During this time the counting rate increased from the overall mean value of 1274 counts sec^{-1} (see Fig. 22) to 2188 counts sec^{-1} averaged over 409.6 msec. The statistical significance of this enhancement is discussed in ref. 125. The interval of 409.6 msec was further divided into 320 bins each of 1.28 msec. Eight bursts were found in this set of data. (See also Fig. 31). (Figure reproduced from ref. 125 with kind permission of the authors).

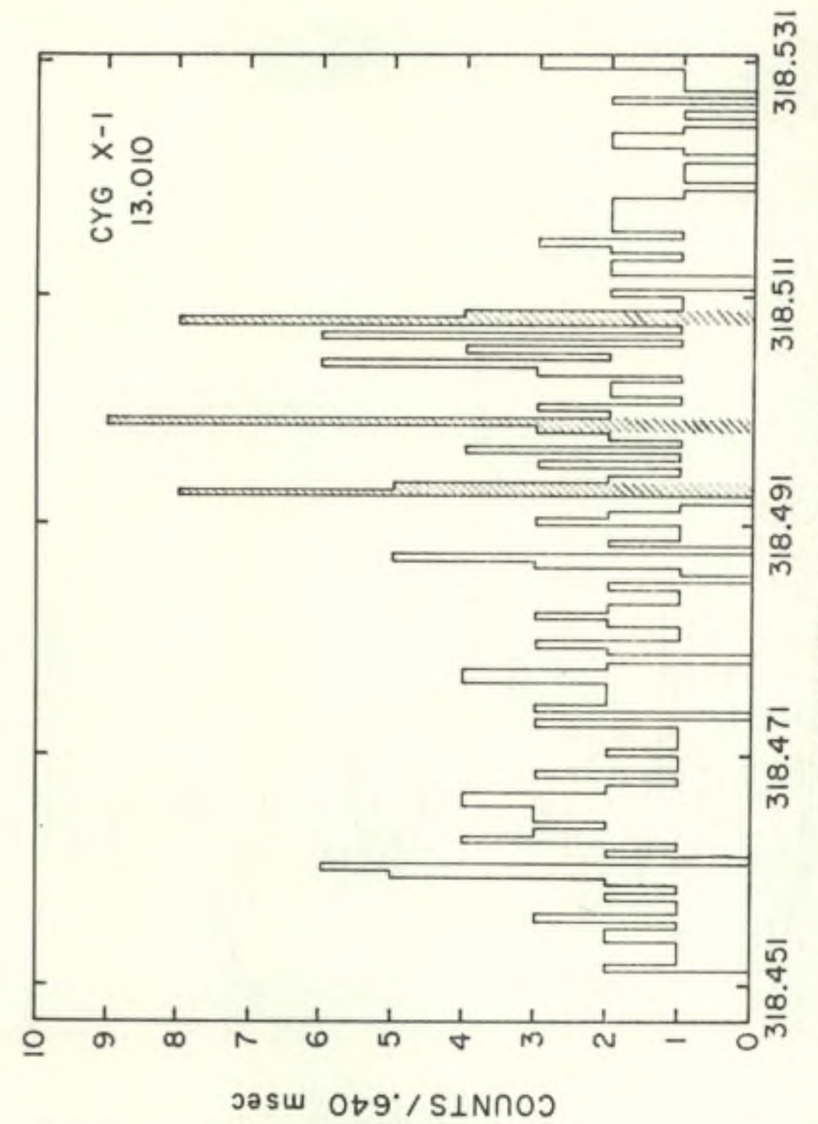


Fig. 31. Eighty milliseconds of exposure of Cygnus X1 containing the peak of the enhancement near 318 sec after launch. The count rates are binned every 0.64 msec. Bursts with $\geq$ 12 counts per 1.28 msec are shaded (see text and reference (125).) Figure reproduced from reference (125) with kind permission of the authors.

8. WHITE DWARFS IN CONTACT BINARY SYSTEMS

In the previous paragraphs we have emphasized how a direct observation of both families of binary x-ray sources and of their short time variability will most likely determine in the near future some of the major features characterizing the physics of collapsed objects. This task will be greatly simplified by the discovery of more sources inside our own galaxy, by a continuous monitoring of the existing sources and possibly by the detection of sources in nearby galaxies. As a by product of these observations we should also be able to determine the experimental value of the critical mass against gravitational collapse.

Thanks to the work of Paczinsky[128] and Arnett[129] it has become more and more clear in recent years the fundamental role that white dwarf stars play in the processes leading to the formation of collapsed objects. Paczinsky has given a detailed treatment of the evolution of population I stars with masses 0.8, 1.5, 3, 4, 10 and 15 $M_{\odot}$. In all these computations the evolutionary tracks of the center of stellar models were computed and some of the results are here reproduced in Fig. (32). The most striking aspect of these computations is the formation of a standard size degenerate core of material (white dwarf material) after the exhaustion of helium in the center of the star has occurred. The size of this core for initial configurations with $3 \lesssim M/M_{\odot} \lesssim 7$ is always the same: $M_{core} \sim 1.39\ M_{\odot}$. The remaining mass of the star, distributed in a large envelope, is expected to be expelled when the core becomes unstable and undergoes gravitational collapse if the star is single[129], or drive the entire dynamical aspects and transfer of material between the components, if the star is in a binary system[130].

If indeed these results will be confirmed, then a deeper understanding of the physics governing the white dwarf stars will not only be relevant in itself but will also be of basic relevance for the understanding of the physical conditions existing at the onset of gravitational collapse and of the evolution of binary star systems.

In this light, it is of the greatest importance to obtain a direct experimental verification not only of the main parameters of white dwarfs but also to try to infer as much information as possible about their internal constitution, and to particularly obtain an experimental value for their critical mass against gravitational collapse[131]. Masses of white dwarfs have indeed been measured with great accuracy in binary systems: the best known example being certainly the classical observations of Sirius a and Sirius b[132]. Measurements of this kind, though relevant for a mass measurement, are practically of no value to infer the physics of the internal constitution of white dwarfs.

The situation has been drastically changed by the detailed analysis done by Warner et al[133] of cataclysmic variable stars. These sources are in many respects very similar to the ones considered in the previous two paragraphs. They are very short period binaries ($P_o \lesssim 5$ hours) and they have as one of the component of the system not a neutron star or a black hole but a white dwarf. These binaries, however, are *not* strong x-ray sources. Also, in these systems matter is transferred from the main normal star into the white dwarf. The gravitational field is, however, not strong enough as to produce x-ray emission from the accreting plasma. Some of the best known systems are given with their typical parameters in Tab. III.

By far, the system most studied in this class is DQ Herculis[134]. This system has a binary period $P_o = 4^h39$ min which is observed to change at a rate $(dP_o/dt)/P_o = 3.484 \times 10^{-10}$ days^{-1}. The white dwarf luminosity is observed to pulsate with a period $P_1 = 71$ sec with an amplitude in 1971 of 0.016 mag[(133)]. Both the period and the amplitude of the pulsation of the white dwarf are observed to decrease with time ($dP_1/dt \sim -20$ μsec year^{-1}, amplitude of the pulsation 0.026 mag in 1961 and 0.016 mag in 1971). A very attractive and self consistent model has been advanced by Warner[133] in order to explain the major features of this system. The onset of the pulsation of the white dwarf in this model is triggered by the nova explosion which, occurred in 1934 in DQ Herculis. The change of the binary period is simply explained as due to the outflow of mass from the system which gives $(dM/dt)_{out} \sim 10^{-7}\ M_\odot$/ year. The observed decrease in the pulsational period is explained by Warner as due to an accretion rate of $(DM/dt)_{acc} \sim 10^{-7}$ $M_\odot$/year on the white dwarf: to an increase in mass corresponds a increase in density and therefore a decrease in period of pulsation since $\omega_{Puls} \sim (\pi G \rho)^{1/2}$. Finally a direct extrapolation of the observed binary period today to the 1934 value, compared with the binary period before the nova outburst shows a discontinuity which would imply an output of mass during the nova outburst of $\sim 10^{-3}\ M_\odot$. From this the conclusion that at the current accretion rate the system DQ Herculis would undergo a nova explosion once every 10^3 years. From this model[133] follows a considerable difference between systems like DQ Herculis and the binary pulsating x-ray sources. The intrinsic variation of luminosity (71 sec in DQ Herc, 4.8 sec in Cen X3, 1.2 sec in Herc X1) is due to rotation in the regularly pulsating binary x-ray sources and to non radial pulsation in the present case[135].

Nevertheless apart from the many differences (a) some of the major features of the accretion disk would be expected to be similar in all the binary systems here considered, and (b) it is certainly of the greatest interest to explore the possibility of nova-like phenomena around neutron stars and black holes particularly as a possible explanation of burst like phenomena as observed in Cyg X3, Cyg X1 (see § 7) or of the newly discovered γ-ray bursts[136].

A direct comparison of these three families of binary systems, with a direct measurement of the masses of the collapsed stars and of the white dwarfs, as well as a detailed interpretation of the short time structures of their intensity variations, will not only give a definite understanding of all the possible equilibrium configurations to be found at the endpoint of thermonuclear evolution of a star but also a definite knowledge of the physical processes governing these equilibrium configurations (see Fig. 33).

It is now clear that if the general results of the evolution of a star given by Paczinsky[128] and Arnett[129] will be confirmed by further analysis, then the abundance of neutron stars to black holes in nature is mainly determined by the ratio

$$(M_{crit})^{W.D.}/(M_{crit})^{N.S.}$$

If this ratio is larger than one, $(M_{crit})^{N.S.} < 1.39\ M_{\odot}$, than the majority of gravitational collapse processes will lead to formation of black holes. In this case neutron stars could be formed uniquely if enough matter will be expelled during the process of gravitational collapse. If instead this ratio is smaller than one then the formation of neutron stars will be largely enhanced. The formation of a black hole, since the critical mass of a neutron star must always be smaller than 3.2 $M_{\odot}$, will still be possible either by an implosion with enough kinetic energy to overcome the potential barrier of the equilibrium configuration of neutron stars or by a multiple step process through further accretion of material on the neutron star.

The fact that in all the known cataclysmic variable stars, the masses of the white dwarf is near the theoretical value of the critical mass, see Tab. III and the very high accretion rate expected in these systems ($M \gtrsim 10^{-7}\ M_{\odot}/_{year}$) has lead Ruffini and Treves to advance the hypothesis[137] that indeed we should be able to observe binary systems which have evolved from cataclysmic variable stars into binary x-ray sources. This model is particularly appealing for an explanation of Cygnus X3. Initially it is assumed that Cygnus X3 was a contact binary of short period one of the components having evolved to a white dwarf[138]. Due to accretion of material the white dwarf would have then collapsed to a black hole which is now orbiting inside the envelope of the main star. This could explain both the sine behavior in the intensity of the x-ray emission as well as the very large infrared luminosity measured by Becklin et al[98] (see § 7).

A direct comparison and intercorrelation of the sytems presented in § 6, § 7 and § 8 is strongly and urgently needed.

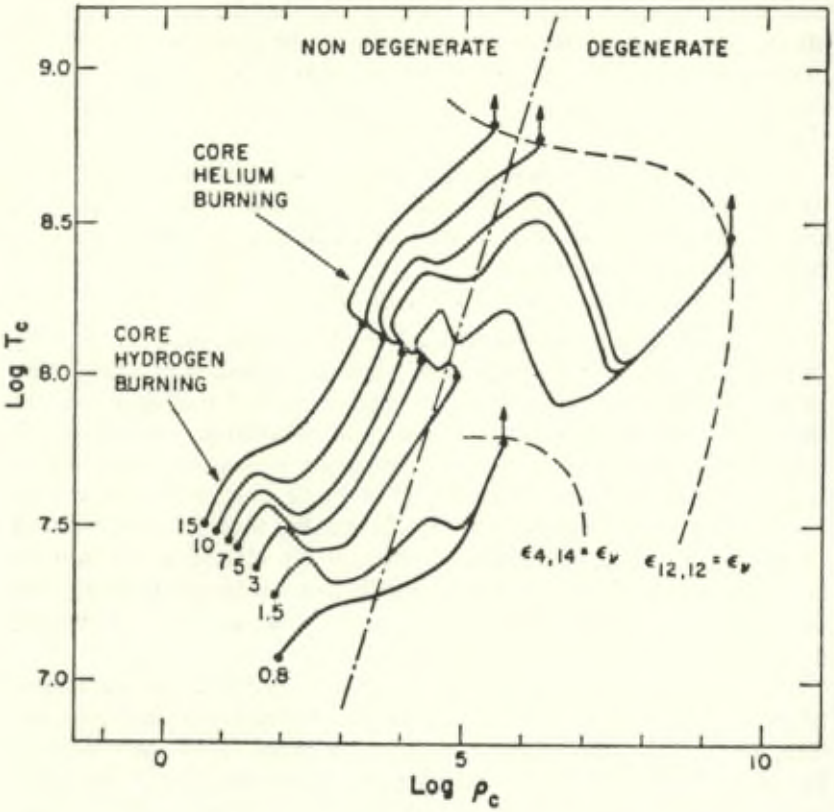


Fig. 32. Evolutionary track for stellar models proceeding toward the endpoint of thermonuclear evolution. The temperature at the center of the star is given as a function of the central density. At the beginning of each track the value of the mass of the star is given in units of the solar mass. The large dots indicate the position of the centers of the models on the main sequence and at the time helium and carbon ignition occur. The cores of the 3, 5 and 7 $M_\odot$ models are smaller and do not contract too rapidly after the helium exhaustion. Neutrino emission cools them down and carbon ignition takes place when the density at the center reaches 3×10^9 g/cm^3. Along the broken lines neutrino energy losses balance either nitrogen + helium burning or carbon burning. The ignition of carbon as suggested by Arnett[129] could be explosive. However, in these computations the effects of crystallization, electron capture, and general relativity were not taken into account. Some of the nuclear reactions governing carbon burning rate are also uncertain. The effects of these uncertainties could be so large as to make the system unable to cause a thermonuclear explosion and the core would collapse directly to neutron star density. Certainly the most striking feature in this model is the convergence of the evolutionary tracks for 3 $M_\odot \lesssim M \lesssim 7\ M_\odot$ into a common track after the exhaustion of helium in the core has taken place. (Figure and caption based on the paper in Ref. (139).).

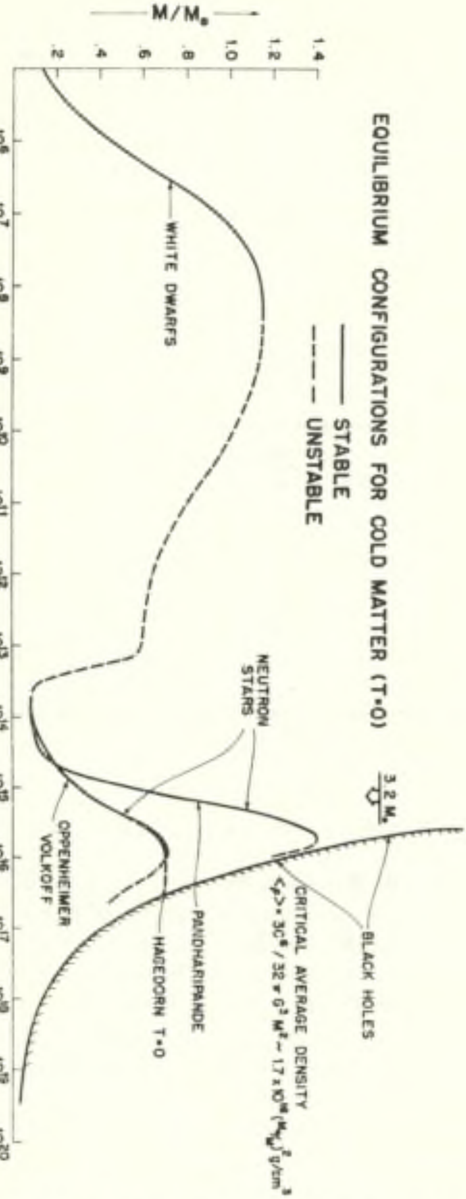


Fig. 33. Configurations of equilibrium of degenerate matter. The masses measured in units of solar mass are plotted here as a function of the central density of the configuration of equilibrium. In the case of black holes we have plotted an " effective " average density as given by $<\rho> = M/\frac{4}{3}\pi\,(2\,GM/c^2)^3$, strictly speaking the black hole never reaches an equilibrium but approaches asymptotically (as seen from us

far-away) its own horizon with a time constant $\tau \sim GM/c^3$ ($\tau = 4.9\ 10^{-3}$ sec for a 10 $M_{\odot}$ black hole!).[39] It is clear that the entire dynamics and outcome of gravitational collapse are totally governed by only two critical values: The critical mass of white dwarfs (Chandrasekhar limit), the critical mass of neutrons stars (largely unknown but certainly smaller than 3.2 $M_{\odot}$[29]). The experimental determination of the value of the white dwarf critical mass is extremely important as a consequence of the central role this mass appears to have as the " standard " starting point for gravitational collapse to occur. The value of the critical mass for neutron stars will drastically influence the ratio between neutron stars and black holes to be found in nature. (See text and caption to Fig. 32).

TABLE III

Parameters of cataclysmic binaries reproduced from references (133) M_1 is here the mass of the White Dwarf companion star, a_1 the semi major axis and i the orbital inclination (Details in references 133).

STAR	P (10^4 sec)	$\frac{2\pi a_1}{P} \sin i$ (km sec^{-1})	q	$M_2/M_{\odot}$	$M_1/M_{\odot}$
RV Peg	3.21	137	1.15	1.3	1.1
Z Cam	2.51	144	0.73	1.0	1.4
SS Cyg.	2.38	122	0.86	1.0	1.2
DQ Her.	1.67	150	0.72	0.70	1.0
ss Aur.	1.56	85	0.64	0.65	1.0
V603 Agl	1.20	37	0.46	0.50	1.1

9. THE MOMENT OF GRAVITATIONAL COLLAPSE

Since the first days of general relativity it has been clear that one of the major new predictions of Einsteins' theory of gravitation was the fact that gravitational interactions had to propagate with a finite velocity, equal to the velocity of light, and that waves carrying gravitational energy should exist in nature. It was, however, also very clear that the amount of energy radiated by any conceivable source inside our solar system was totally negligible and its effect well below any realistic limit of detectability. The main reason is simply given by a direct estimate of the gravitational waves energy radiated by a source as computed in the slow motion approximation[140]

$$\left(\frac{dE}{dt}\right)_{rad} = \frac{G}{45c^5} \dddot{Q}^{ij} \dddot{Q}_{ij}$$

with i, j = 1, 3, G the gravitational constant, c the speed of light, and Q^{ij} the third derivative of the quadrupole moment of the sytem. The only way to counterbalance the smallness of the factor G/c^5 is to consider very large systems ($M \sim M_{\oplus}$) and rapidly varying quadrupole moments (velocity of sound $\sim c$)[141].

It was suggested as far back as 1952 by Dyson[142] that similar regimes could be reached inside our own galaxy during supernovae explosions.

J. Weber[71] was the first to conceive the possibility of detecting gravitational waves from extraterrestrial sources and to build and operate the first detector to observe bursts of this radiation. The detector constructed by Weber consists of an aluminum-cylinder 153 cm in length, lm in diameter with a mass of 1.4×10^6 gr and a resonance frequency in its lowest mode of stretching vibration of $\tau = 6.03 \times 10^{-4}$ sec. The Q for this mode is estimated to be $Q \sim 10^6$ and the bandwidth of the detector $\Delta\omega = \omega/Q \sim 10^{-1}$ rad/sec. Following the considerations presented in Fig. 13 and reference (39) we can then evaluate the total cross section of this detector.

$$\int_{\text{resonance}} \sigma(\nu)d\nu = 1.0 \times 10^{-21} \text{ cm}^2 \text{ H}_z$$

To detect the stresses of the bar, Weber uses piezoelectric crystals which enable him to detect displacements as small as 10^{-14} cm, corresponding to vibration energy in the bar $\sim$ KT. To discriminate between locally produced noise and gravitational waves signal, Weber uses two identical detectors, one at Argonne National Laboratory (Chicago) and one at the University of Maryland and he looks for coincidences in the signals in the two widely separated detectors. What is now clear is that, on the basis of this experiment we can obtain the first observational upper limit ever for the intensity of gravitational radiation from outer space at a specified frequency[143]. If, in fact, we assume that no signals larger than KT have been seen by this experiment we can then very easily compute the value of this upper limit. If we assume that a signal of gravitational radiation produces a displacement $< \xi^2 > \sim 2KT/m\omega_o^2$, this represents an uptake of energy by the detector of 4×10^{-14} erg which divided by the cross section of the detector gives and upper limit to the energy $I(\nu_o)$ of an incoming pulse per cm^2 of intercepted area and per unit frequency at resonance of 4.0×10^7 erg/cm^2 Hz. Different limits can be placed on the total flux of incoming gravitational radiation as a function of the assumptions made about the original spectrum of the radiation (see § 3). If we assume that the pulse has a bandwidth peaked on the detector resonance frequency and equal to the detector bandwidth then we obtain a flux of 2×10^6 erg/cm^2. If we assume for the pulse a flat spectrum from $\nu = 0$ up to the frequency of observation $\nu = \nu_o$ we obtain a flux of 5.5×10^{10} erg/cm^2. If we then assume that the source is located at the center of our galaxy ($D = 8.2$ kpc) we have to multiply the above fluxes by $4\pi D^2$ (no possibility of beaming gravitational radiation! See Fig. (12)). We then obtain the *lower* limit for the output of gravitational waves from a source located at the galactic center in order to produce a signal $\sim$ KT in our detector to be respectively 1.6×10^{51} erg and 4.4×10^{55} erg in the two cases previously considered. These bursts would correspond respectively to an amount of mass m which would have to be annihilated in gravitational radiation (assuming 100 % efficiency of conversion!) of $m = 8 \times 10^{-4}\ M_{\odot}$ or $m = 22\ M_{\odot}$ respectively.

On the light of these experimental results it became clear that the emphasis of the theoretical research had to be directed toward processes of emission of gravitational radiation in which a sizable fraction of a solar mass could be radiated in form of gravitational waves in time scale of a few milliseconds.

Natural candidates of powerful sources of gravitational waves were then given by black holes. There are two different reasons for this: (a) large masses of material are indeed observed to accrete in these collapsed objects (see § 6 and § 7) and (b) the infalling matter reaches relativistic velocities during the accretion processes as a consequence of the very strong gravitational fields. Thanks to the detailed fully relativistic treatments presented in § 3. we can then estimate the energy emitted during these accretion processes.

The following general conclusions can be reached on the energy emitted by particle radially imploding into a Schwarzschild black hole:

(1) The peak of the radiation is emitted at a frequency $\omega_{\mathrm{peak}} \sim c^3/GM$ uniquely characterized by the mass of the black hole ($\omega_{\mathrm{peak}} \sim 2 \times 10^4$ rad sec^{-1} for a 10 $M_{\odot}$ black hole)

(2) the amount of gravitational radiation emitted at infinity during the infall of a mass m is given by (see Fig. 8 and Fig. 10)

$$\Delta E \sim 0.01 m\, c^2\, (m/M)$$

(3) the gravitational radiation is emitted in lobes of $\sim 70^{\circ}$ (see Fig. 9).

Although these considerations apply only to the idealized case of a particle infalling radially into a Schwarzschild metric we do expect their validity to apply also in the case of more general orbits of implosion as well as in the case of a Kerr metric. The major difference will occur only in the value of the numerical constants in front of the dimensional factors mc^2 (m/M) (for the energy radiated) and c^3/GM (for the peak radiation). Moreover we do not expect these numerical factors to vary drastically. It is therefore clear that the maximum efficiency in a radiation process is reached in the case $m \sim M$. On energetic grounds we can conclude also that these processes can turn out to be of physical interest only if $m \sim M_{\odot}$.

If we turn now to particles in circular orbits at a radius r around a black hole of mass M we can conclude that gravitational radiation will be emitted:

(1) at a frequency $\omega \sim (GM/r^3)^{1/2}$

(2) with an integrated intensity essentially proportional to the gravitational binding energy of the orbit. Stable circular orbits can exist only down to a minimum radius $r_{min} = 6\ GM/c^2$ in the case of a Schwarzschild black hole and $r_{min} = r_{horizon} = m + (m^2 - a^2)^{1/2}$ in the case of the Kerr metric. We can then conclude that the total amount of radiation emitted by a particle in circular orbit is simply proportional to the gravitational binding energy of the last stable circular orbit in the case of an extreme Kerr black hole $E_{bind} \sim 0.42\ mc^2$ and in the case of a Schwarzschild metric $E_{bind} \sim 0.051\ mc^2$.

We can then conclude that both in the case of radial infall and of circular orbits the black hole uniquely characterizes the frequency of the outgoing radiation. If we keep in mind the technological possibilities of gravitational wave detectors we must then also conclude that the only realistic situation in which a burst of gravitational waves can have enough intensity at the right frequency for detectability is reached in the special case where $m \approx M \approx M_\odot$!

It is also clear, however, that all the formulae and the treatment we have been speaking about applies only if the size of the infalling particle can be considered negligible by comparison to the size of the black hole. The constraints imposed by realistic astrophysical situations are then even stronger! If we look at the radius of a black hole of 10 $M_\odot$ we have $R \sim 29.4$ km (in the case of extreme Kerr solution $\frac{1}{2}$ as much) and the radius of a main sequence star of a solar mass is $R \sim 10^5$ km! If the pointlike assumption on the imploding mass cannot be adopted then tidal deformations have to be considered. A white dwarf imploding into a black hole will be disrupted by tidal forces at a distance.

$$R_{limit} = 2.45\ (\rho B.H./\rho W.D.)\ R_{B.H.}$$

from the black hole surface[39]. The disruption of the infalling star by tidal forces will totally spoil the effect of the fully relativistic asymmetric implosion. In the case of a normal star imploding into a black hole, since its radius is much larger than the radius of the black hole, the amount of gravitational radiation emitted will be totally negligible: the star will flow inside the black hole with a highly symmetric flow on a very long time scale essentially governed by the Eddington limit and, as a consequence, the third derivative of the quadrupole moment of the system will be negligible.

In conclusion as far as we can see today there exists only *one* regime in which, for certain, all the optimal conditions for the emission of a detectable amount of gravitational radiation can be reached at once: at the moment of gravitational collapse.

We have emphasized in the previous paragraph how a detailed knowledge is being gained on the two different families of collapsed objects inside our own galaxy thanks to the data acquired from binary x-ray sources. In no way can we escape the necessity that these objects have to be formed in a process of gravitational collapse (see Fig. 34). Then very general arguments lead inescapably to the result that during the process of gravitational collapse up to $4 - 5 \times 10^{-2}$ Mc^2 of the mass M of the collapsing star is emitted in a sharp burst of radiation, preceded by a low intensity " precursor " and a tail of exponential decreasing intensity[144]. If the collapse ends in a neutron star additional energy can still be extracted in the form of gravitational radiation either from the rotational energy or the vibrational energy[145] of the freshly formed neutron star (see Fig. 35) during the few seconds following the occurrence of the gravitational collapse itself.

One of the possible ways of estimating the number of gravitational collapses to be expected in the galaxy is to count the number of visible supernovae[5]. On this ground we would expect a gravitational collapse every thirty years. This number of supernovae would give a population of $\sim 10^9$ collapsed objects in our own galaxy, in substantial agreement with the number of pulsars observed if their lifetime and the geometrical effects of possible beaming are taken into proper account.

However, there are today reasons to believe that processes of gravitational collapse can occur under different regimes than the ones encountered in supernovae explosions.

In the previous paragraph we have seen the compelling evidence for the existence of collapsed objects in binary x-ray sources. Colgate[118] has pointed out that these collapsed objects have to be formed through a different process than usual supernovae, since the binary system would be disrupted by the relativistically expanding shell. It is therefore likely that the formation of a collapsed object in a binary system from a white dwarf core[130] will have a totally different behavior than in a supernova. Most likely the difference could be due to the gradual transfer of mass in the binary system from the initially more massive star to its compa nion[130]. Although not final understanding of these processes exists we stress the conclusion that today there is some evidence that we should expect a larger number of gravitational collapses inside our own galaxy than the one obtained from a direct count of supernovae.

To start a realistic program for the detection of gravitational radiation it is then necessary to drastically augment the sensitivity of existing gravitational wave detectors in order to observe all the gravitational wave bursts originating in gravitational collapses inside our own galaxy and possibly in nearby galaxies. With this aim in mind two different kinds of experiments have been conceived and are in an advanced stage of realization: one is a joint effort[146] by [147] the Universities of Louisiana (Baton Rouge), Rome (Italy), Stanford (California), the other is led by Dr. Braginsky at the Moscow State University[148] (U.S.S.R.).

The key idea beyond these two experiments is simply explained. The minimum energy detectable by a gravitational wave detector is limited by the information which can be extracted from its thermal noise. If Nyquist's theorem can be applied to analyze the noise of the detector then we find that the minimum energy flux detectable by the gravitational wave antenna is given by[149]

$$(E)_{grav} \times \int_{res} \sigma(\nu) d\nu = kT\, \tau/\tau_D$$

where $\int_{res} \sigma(\nu) d\nu$ is the cross section of the detector integrated over the resonance, T is the temperature of the detector, and $\tau_D = 2Q/\omega_o$, where ω_o is the resonance frequency of the detector and τ the time over which the signal is recorded. This formula applies uniquely to the case $\tau << \tau_D$ or in the limit that the time over which the observations are made is much shorter than the relaxation time of the antenna. There are only two ways of increasing the sensitivity of a gravitational wave antenna of a given cross section: (a) to lower its temperature and (b) to increase its Q.

The Louisiana-Rome-Stanford experiment is mainly directed at exploiting the gain coming from the reduction of the temperature of the gravitational wave detector. The antenna is expected to have a mass of $\sim 2 \times 10^6$ gr, to be magnetically suspended and thermally isolated and the cooling processes are made with the final aim of reaching a temperature of 3×10^{-3} °K[146, 147]. Drastic improvements in the electronics and in the detection system of the strains, clearly, are needed in order to gain a factor $\sim 10^6$ in sensitivity with respect to existing experiments[71]. (See Fig. 35)

The major effort of the Moscow State University experiment[148] is, instead, mainly directed at exploiting all possible gains obtainable from increasing the Q of the system neglecting, at least in a first stage, the possible gain from low temperature techniques. A sapphire monocrystal cylinder 152 cm long 45 mm in diameter with a mass of $\approx 2\ 10^3$ g and an eigen frequency of 34016 Hz was first used for detection. A Q = 4×10^7 was reached at room temperature, however it has been suggested that with a better polishing of the surface of the crystal and an improvement in the suspension, the Q could be increased of 2-3 orders of magnitude[148].

Both experiments are in an advance stage of realization and hopefully they will be operational in the next few years. If, indeed, since it is theoretically achievable, we will be able to gain the six orders of magnitude on existing detectors then we should be able to see for the first time with clear evidence a signal of gravitational radiation originating from gravitational collapse. It is not clear if this signal will be detected first as a clear signal over the thermal background of the detector by the Louisiana Rome-Stanford experiment or by the information extracted from the noise in the Moscow State University experiment. What is clear, however, is that either way we will have for the first time a record of one of the most important physical processes occurring during the moment of gravitational collapse.

Most likely this gravitational wave pulse will be emitted together with an electromagnetic signal (see § 3). The enormous difference of cross section between electromagnetic and gravitational wave detectors[150] then make the burst of electromagnetic radiation associated with gravitational collapse the ideal event to look for coincidences in signals of the gravitational wave detectors.

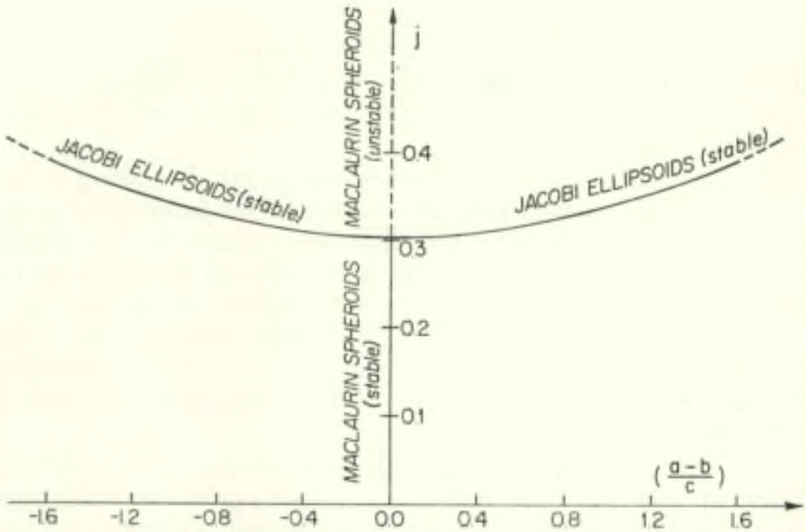


Fig. 34. Equilibrium configurations of an incompressible fluid turning at a uniform angular velocity. The solid lines indicate stable equilibrium configurations the dotted lines unstable. If the total angular momentum is larger than the critical value $L = 0.303751\ L_0$ with $L_0 = (G\ M^3)^{1/2}\ (abc)^{1/6}$, abc being the semiaxis of the ellipsoid of equilibrium, then the stable equilibrium configuration is given by Jacobi ellipsoids[152]. The horizontal scale gives the difference of semiaxis of the ellipsoid in the equatorial plane divided by the semiaxis along the axis of rotation. A neutron star with an equation of state with an effective politropic index $\gamma > 2.2$[151] will also, if endowed with large enough angular momentum, have equilibrium configurations given by a Jacobi ellipsoid. It will then radiate a very large amount of gravitational radiation with a characteristic signature. A neutron star of $M \sim 0.65\ M_{\odot}$ and $R \sim 15$ km rotating with a period of $P \sim 1.4$ msec will have an eccentricity in the equatorial plane $\varepsilon \sim 0.87$ and will radiate $dE/dt \sim 0.6 \times 10^{51}$ erg/sec with the period starting from $P \sim 1.4$ msec (extreme Hacobi configuration) decreasing in a time $\tau \sim 0.4$ sec to a value of 1.3 msec. Details of this process as well as gravitational radiation emitted by vibrating neutron stars are given in references (39) and (67).

Fig. 35. Gravitational wave antenna being assembled at the University of Louisiana (Baton Rouge). Identical detectors are being assembled at the University of Rome and at Stanford University. The final goal is to reach a sensitivity 10^6 times larger than existing detectors capitalizing, among other approaches, on reducing the temperature of the detectors (an aluminum bar of $\sim 2 \times 10^6$ gr) to 3×10^{-3} °k. The displacements at then end of the bar are being measured by a resonant diaphragm[149]. Displacements as small 4.9×10^{-18} cm are expected to be measurable at the final temperature of 3×10^{-3} °k. The magnetic suspension has already been tested at a temperature of 2° k both at Stanford (aluminium bar of 1.5 m and 30 cm in diameter covered with Nb Ti foil) and in Rome (aluminium bar 16 cm in diameter and 30 cm in length covered with Nb Ti foil). The first detectors are expected to be operational at 2° k at the end of 1974. The eigenmodes of the detectors are currently being studied theoretically and experimentally[153]. (Figure reproduced by courtesy of W. O. Hamilton).

CONCLUSIONS

For a long time the attention of astrophysicists has been directed towards the understanding of the nuclear processes occurring in the evolution and energy generation of stars.

Only recently it has become apparent that the late stages of evolution of a star is uniquely characterized from the energetics point of view by gravitational and rotational energy, and that the most violent and energetically relevant moments in the life of a star indeed take place after the exhaustion of the nuclear sources of energy has occurred.

Gravitational interaction either slowly generating the luminosity of white dwarfs through their continuous contraction, or producing up to $10^5\ L_\odot$ in the accretion processes in x-ray sources or, again, completely determining the physical processes during gravitational collapse or in the description of the totally collapsed objects ensuing from gravitational collapse, appear to be more and more the back-bone, the only fundamental field theory, of this drastically new domain of physics. In this sense the in depth analysis of a relativistic theory of gravity has become in the recent years not only much more easy due to the existence in nature of collapsed objects, but also much more relevant to our understanding of a very large number of physical processes.

During the next five or six years, if the resonance between so many different branches of astronomy (radio, x-ray, optical, infrared) will continue in close contact with the theoretical work we should be able to clearly determine all the details of the theoretical curves in Fig. 33 with direct experimental data.

If we turn to the long range program of research we have seen in the recent years the preparation of an entire new chapter of astrophysics what we could call " Burst Astronomy ". Detectors of gravitational, radiation, of neutrino bursts, and of possible associated electromagnetic radiation are all rapidly improving in sensitivity and sophistication while in the theoretical field basic advances are made in the analysis of fully relativistic short time phenomena. In the next few years we should be able to reach in all these different experimental fields the limits of detectability theoretically predicted.

The direct observation through different techniques of the moment of gravitational collapse appears to be of the greatest interest. More we learn of the physics characterizing the configurations of equilibrium of cold catalyzed matter (see Fig. 33), the more we see the need of processes of gravitational collapse to occur under a variety of regimes. It is also clear that it is unavoidable that all collapsed objects we are today observing either in pulsars or in binary x-ray sources had to be formed through this process.

The observation of the moment of gravitational collapse will disclose, among others, one of the most fundamental predictions of general relativity namely that gravity, as any other long range interaction has to propagate with a finite velocity equal to the speed of light and that gravitational energy can be carried by waves. The direct technological advance brought by these observations will most likely be very limited the influence however for our understanding of nature will certainly be enormous.

The moment of gravitational collapse appears since now to be the most energetic process occuring inside a galaxy second in the entire universe to the formation of the Universe itself.

This brings us to another domain of physics also dominated by a fully relativistic theory of gravity: Cosmology. The greatest success of our research on gravitational collapse will be reached if not only we will be able to describe using different techniques and detailed theoretical work the physical processes occurring in this very short phenomena ($\tau < 1$ sec) but if we will be able to apply to Cosmology the enormous amount of knowledge we are acquiring in this research

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Communication of M.J. Rees

The formation of a black hole would generally be accompanied by a burst of gravitational wave emission, with characteristic frequencies $\sim \left(\frac{GM}{c^3}\right)^{-1}$, or $(10^4 - 10^5)\,(M_\odot/M)$Hz. Formation of a neutron star would give rise to a similar (though less intense) burst at KHz frequencies. These are just the kinds of signal which Weber-type bar detectors are ideally suited to detect. The rate of supernova explosions in our galaxy is thought to be about one per 30 years. Some of these give rise to neutron stars (pulsars) and would probably yield much less than $\sim 0.1\, M_\odot c^2$ of gravitational waves; some supernovae however, may form black holes, yielding as much as $\sim M_\odot c^2$ of mass-energy in gravitational radiation. Even the latter type of event would only be detected by Weber's apparatus if it occured within 1 or 2 Kpc. This region encompasses only a few per cent of the stars in the Galaxy, and so presumably only a few percent of the supernovae. This suggests an event rate of 1 per $\sim$ 1000 years if supernovae are indeed the only significant sources. This is in one sense an optimistic prediction, because we have no firm evidence that many supernovae actually do form black holes. But, on the other hand, some stars could collapse to form black holes without the events manifesting themselves as anything so violent as a supernova. The upper limit to the rate of " star deaths " in our Galaxy would seem to be $\lesssim 1$ per year, and even if we assumed that then each caused a burst of $\sim M_\odot c^2$ of gravitational waves, the predicted event-rate for Weber's detectors would be $\sim$ 1 per century. Plainly one would need to invoke some extraordinary new phenomenon if these " events " really were gravitational in origin!

One may now ask: what are the chances of detecting a significant rate of gravitational wave events with more sensitive equipment? The most obvious candidates for detectability are supernova-type events in external galaxies. To make the search worthwhile, one would surely hope to count on several events per year. This means that the sensitivity must be adequate to register such an event at the distance of the Virgo Cluster ($\sim$ 15 Mpc.), necessitating an improvement by a factor $\sim 10^8$ over Weber's sensitivity! One might wonder whether apparatus with this level of sensitivity might detect other classes of event, but this (while of course possible) seems unlikely on the bases of 1973-vintage astrophysical ideas: to give just two examples, star-quakes on nearby neutron stars would probably not be detected; and any extra-galactic phenomena involving massive objects would tend (barring such possibilities as gravitational synchroton radiation) to give waves at frequencies far below the KHz range. If one drops the requirement that the expected event rate should be several per year, and is prepared to be patient, a sensitivity " only " 10^4 times Weber's might yield worthwhile results. Such a detector would record *any* star-deaths in our Galaxy (including those which give neutron stars rather than black holes). Even if *no* event had been recorded after (say) 30 years' operation this would still in a sense be interesting, because it would prelude the possibility that inconspicuous star-deaths might be much more frequent than ordinary supernovae.

Are there any likely sources of " monochromatic " gravitational waves rather than short bursts? Such sources must involve objects with radiative lifetimes $>> GM/c^3$, and therefore would generally be weaker than sources of sharp pulses. But different and more sensitive techniques exploiting resonances or long integration times can be used to detect a weak repetitive signal, especially if its period is already known. The least unpromising candidate for detection seems to be the Crab Pulsar, which could emit $\sim 10^{38}$ ergs/sec. as gravitational waves. (This would require a deviation from axisymmetry of about 1 part in 10^3, which could occur only if it has a solid core). Detection of gravitational waves from the Crab Pulsar, and measurement of their polarization, could also tell us how its rotation axis is oriented relative to the line of sight—an important parameter for pulsar models which does not seem discoverable in any other way. The gravitational wave emission from binary stars would be far harder to detect directly. As has been pointed out by many people, however, the loss of orbital angular momentum from binary systems as a result of gravitational wave emission may have an important effect on the rate at which the Roche Lobe encroaches into the primary star, and thus on the mass transfer rate to the secondary. Indeed, close binary stars are perhaps the only systems where one can already claim that gravitational radiation is having observable consequences.

Discussion of the communication of M.J. Rees

R. Giacconi: (1) I think it is important to emphasize that evidence for black hole existence in SMC-X-1, 2 U0900-40, 2 U1700-37 is very weak. (2) On the other hand evidence for Cyg X-1 being a massive ($> 3 M_\odot$) and compact object is conclusive. (3) It should be emphasized that on this basis it is important for the theoretical astrophysicists whether indeed there are tenable alternate interpretations to the black hole hypothesis.

Y. Ne'eman: Have the chances of observing extra-galactic gravitational waves been evaluated? Events such as the explosion in M 82, involving the projection of a mass of $5.10^6 M_\odot$ might be favored enough to overcome the R^{-2} weakening due to distance.

M. J. Rees: It is certainly possible that a collapsing supermassive object could emit a substantial fraction of its rest mass energy as a burst of gravitational waves. The characteristic period would however, tend to be of the order of days or longer, and so quite different techniques (for example, accurate measurements of the distance of an interplanetary probe) would have to be used. This type of event would not, I think, be detectable unless it happened in a nearby galaxy. It is nevertheless quite conceivable that a directed burst of gravitational waves from an active galactic nucleus may be responsible for the " jets " sometimes observed.

E. P. J. van den Heuvel: It is a pity that we have only one practically certain black hole candidate (Cygnus X-1). However, the situation may not be as bad as it seems. Dr. Gursky gave me a list of about 15 bright stars which fall within the error boxes of Uhuru X-ray sources. All of these stars are on the Southern Hemisphere. It seems to me that a systematic survey of the radial velocities and possible light variations of these stars might well reveal the existence of more X-ray binaries of the Cygnus X-1 type. None of these X-ray sources is eclipsing, but the same holds for Cygnus X-1. As nobody seems to be planning a systematic investigation of these stars—and as such an investigation involves a lot of work— a joined effort by several Southern Hemisphere observatories should be started. The prospects of finding at least one more good black hole candidate in this way seem not bad, in my opinion, as a number of these X-ray sources show flaring characteristics of a type similar to those of Cygnus X-1.

THEORIES OF QUASARS

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Quasistellar Objects, (QSO, quasars) were discovered more than a decade ago, but their nature is still a subject of strong controversy. We define this class of objects by two conditions: the optical images should be stellar, although some surrounding fuzz may be present, and the spectra should show strongly redshifted features. Defining a redshift parameter $z = (\lambda - \lambda_0)/\lambda_0$ with λ and λ_0, the observed and laboratory wavelengths respectively, we shall include in our discussion objects with $z > 0.1$.

In the published literature, redshifts are now available for 198 quasi-stellar radio sources (QSR) and for 61 QSO without noticeable radio emission, at least at the flux level of most current radio source surveys. Redshifts ($z = \Delta \lambda/\lambda_0$) range up to 3.53. For about a dozen QSO the spectra show only one emission line and so redshifts could not be determined. Many other redshifts are based on only two lines; however, in most cases — though not in all — where additional lines were later found in better spectra, such redshifts have been confirmed; there is little doubt that more than 90 % of the 259 redshift determinations are essentially correct.

Except at high redshifts, virtually all QSO are very blue (U-B large and negative). Some radio sources have been identified with blue stellar objects with featureless continuous optical spectra; these tend to be very strongly variable in the same way as many quasars. Five such " BL Lac type " objects are known (Strittmatter et al 1972) and it is possible that these are related to the QSO class. The optical object Weedman 2 may be of the same type (E. M. Burbidge et al 1972).

The number of redshifts decreases slowly up to $z = 2.2$. Only 8 QSO with $z > 2.2$ are known. This may to some extent be due to selection effects: at large z, the typical quasar color becomes less blue and the probability of recognizing a QSO decreases. It seems probable, however, that after allowing for this effect, we still are left with a rapid decline in the number of redshifts beyond $z = 2.2$. Various claims have been made concerning regularities or periodicities in the redshifts distribution, but the subject has remained controversial. (For a recent discussion see G. R. Burbidge and O'Dell 1972).

Several quasar spectra also show absorption lines. Redshifts have been determined in 28 objects. In 19 cases only one absorption redshift has been found, but in 5 objects, 2 redshift systems have been recognized and in 4 objects more - in 4C05.34 eight different redshift systems have been suggested, though some of these may still be spurious. The following statements about QSO with more than two absorption redshift systems all appear to be true: 1) the apparent brightness tends to be high, 2) the emission line redshift is near to or larger than 2.0 and 3) the absolute luminosity on a cosmological interpretation of the emission line redshifts tends to be high. Obviously 1) and possibly also part of 2) may be a selection effect while 3) may be simply a consequence of 1) and 2). Selection effects are particularly serious because the absorption lines are usually quite narrow and so they can be studied adequately only on spectra of fairly high resolution. Absorption lines are quite common at the larger z's: of the ten known QSO with $z > 1.4$ and a V-magnitude brighter than 17.0, two have one absorption system, one has two and three have several. Recently, Brown and Roberts (1973) have started a search for 21-cm absorption in QSO with sufficient resolution to detect narrow lines. To date, one very narrow absorption line has been found in 3C286 ($z_{em} = 0.846$) with a redshift of 0.69.

Most, if not all, QSO appear to be variable. In a study of 22 relatively bright QSO, Angione (1973) found evidence of optical variability in all. No clear periodicities have yet been found. Typical time scales for variability range from several hours up to many years. Radio variability at high frequencies (several GHz) appears to be common in QSO with flat radio spectra. Typical time scales appear to be of the order of weeks or months. Variations in BL Lac are even more rapid; optical variations occur on time scales of a few minutes and radio variations in a few hours. Variations in polaziration have also been observed in several QSO at both radio and optical wavelengths.

The radio variations may be understood on the basis of a simple model - first proposed by Shklovsky (1965) and further studied by Pauliny-Toth and Kellermann (1966) and van der Laan (1966). In this model, a cloud of relativistic electrons and magnetic fields is released " suddenly " (i. e. in a time less than a few weeks) in a small volume. Initially the cloud is opaque to its own synchrotron radiation at most radio wavelengths. The cloud expands under its own pressure and because the area of the radiating surface increases, the radio emission also increases. Subsequent expansion, however, makes the cloud more transparent—first at the higher frequencies—and as a result, the flux decreases there. On the basis of this model, an estimate of the energy involved in these events may be made; if the quasars are at distances corresponding to a cosmological interpretation of the redshifts, then the minimum energies are in the range 10^{54}-10^{55} ergs. In the more active sources, such an outburst may occur about once a year.

High resolution (intercontinental baseline) interferometry shows size variations associated with the radio bursts. These may frequently be interpreted on the basis of one of two very different models: 1) a double source with increasing component separation; for QSO at cosmological distances the velocities involved may be as much as 0.95 c, or 2) a triple or more complex source in which the components are at fixed positions, but of variable intensity.

Interpretation of the Redshifts

Much controversy has centered on the interpretation of the redshifts. Gravitational, kinematic and cosmological redshift origins have been considered, as well as the possibility that the redshifts are due to some new phenomenon—perhaps not explainable in terms of our present picture of physics. It is well to remember, however, that the assumption that local physics may be extrapolated throughout the universe has been incredibly successful. One certainly should not introduce novel physics until one is absolutely forced to do so. Of course there is no guarantee that the redshift is of the same origin in all quasars; in fact, recently, a number of theories with two types of quasars have been proposed. However, none of these appears to be particularly attractive. It would be rather surprising if two kinds of objects which would be physically entirely different would still look as similar as the quasars do. While the redshift issue is still far from settled, we believe that the gravitational and local kinematical interpretations meet insurmountable difficulties in the majority of the quasars. It would seem therefore that at the present time the only realistic alternatives are that either all QSO have redshifts of cosmological origin or that some or all have redshifts of a novel origin.

Gravitational redshifts—or any redshift mechanisms related to phenomena near the Schwarzschild radius—lead to difficulties with both the emission and the absorption line spectra. Beginning with Greenstein and Schmidt (1964) various authors have pointed out that in several quasars, forbidden lines are present in the emission spectrum, indicating a low density in the emitting region. Adopting a distance, we may obtain then a minimum value of the emitting volume on the basis of the observed emission line intensities. If the redshifts were gravitational the volume of the region were GM/rc^2 is of order unity should be larger than this and this immediately gives a lower limit to the mass M which generates the gravitational field. These masses are so large as to lead to the prediction of other effects which are not observed; if we assume small distances (10-100 kpc say) the masses of all QSO would exceed the mass of our galaxy by a large factor—something that hardly could have gone unnoticed; if we assume larger distances the upper limit to the mass density of the universe is substantially exceeded.

An equally strong argument may be made in the case of most quasars with absorption lines. The absorption lines with smaller redshifts than the emission lines must—on a gravitational model—originate outside the emission region in a place of smaller gravitational potential. In almost all cases in which there is a substantial difference between emission and absorption redshifts, there are no absorption lines arising from excited fine structure states (in the ground state) and as shown by Wolf and Bahcall (1968) this typically implies distances of more than 1 kpc between the continuum source and the absorption region if the redshift is cosmological. In case the redshifts are gravitational and associated with masses of $10^{12}M_{\odot}$, the condition that the average mass density contributed by quasars be less than the mass density of the universe allows distances smaller by a factor of a hundred. The absorption redshifts being of order unity this leads to an inconsistency because with a central mass of $10^{12}M_{\odot}$, the gravitational redshift at 10 pc is less than 10^{-2}. For smaller central masses the situation is even less favorable.

It could be objected that the conditions near the Schwarzschild radius are so extreme that ordinary emission and absorption theories are to be treated with some caution. However, an essentially geometrical argument may become possible in the case of the 21-cm absorption line in 3C286 referred to before. Radio interferometry shows that at 400 MHz, this radio source consists of three different components: two of these, which account for 90 % of the flux have smallest dimensions of $0''.03$ while the third one with 10 % of the flux is unresolved. At 1400 MHz the source appears to be about $0''.05$ by $< 0''.04$. According to Brown and Roberts (1973) the absorption line has a central depth of 12 % and this may well be an underestimate. It therefore seems quite likely—but this still should be confirmed by further data—that it is not only the unresolved component which is absorbed. If so, the absorption cloud should extend over a region of more than $0''.01$ across and this far exceeds the Schwarzschild radius of any conceivable object.

Both the amount of 21-cm absorption and the narrowness of the line fit well to the notion that the absorption is caused in the outer fringes of a galaxy or in a gas cloud along the line of sight.

Kinematical redshifts associated with the motion of fast objects in our neighborhood are no more promising. If such objects originate in many galaxies blue shifts would greatly predominate; if they all come from our own galaxy their total number is so large as to make the dynamics and energetics totally unbelievable. Chiu, Morrison and Sartori (1973) have suggested that a few QSO—in particular those involved in the close coincidences of Burbidge et al, could have kinematical redshifts and that the corresponding blueshifts might be found in the BL Lac type objects; the absence of lines in the spectra of these objects would then be due to the relative scarcity of strong lines in the red part of the spectrum. For several reasons, this appears unlikely. Even if the typical velocities are such as to give a maximum redshift of only $z = 0.5$, we would expect 4 times as many blueshifts as redshifts down to a given flux level. This would imply that five 3C sources with kinematic redshifts should be accompanied by 20 sources *in the 3C* with modest blue shifts. However, in the 3C, only two possible QSO without spectral features have been found (Schmidt 1968). In addition, it is not clear why Hα should be seen in the spectra of low redshift quasars and not in objects with modest blue shifts. If the velocities were much larger—corresponding to a maximum redshift in excess of $z = 1$, then Hα would, in fact, tend to become unobservable in the blueshifted objects, but their number would become excessively large.

Cosmological redshifts apparently do occur in several QSO. The positive evidence is as follows:

1) As shown by Setti and Woltjer (1973a) the subclass of QSR with steep radio spectra show evidence of a V-magnitude-redshift relation: all very bright QSO in this subclass have small redshifts.

2) As shown by Miley (1971) there is some correlation between redshift and angular radio diameter in the sense that the large angular diameters correspond to small redshifts.

3) As shown by Kristian (1973) several low redshift QSO are surrounded by some fuzz, with a size and brightness as would be expected if the QSO were the nucleus of a giant elliptical galaxy. Silk et al (1973) have shown that in the quasar B256 the B-V color of this fuzz is $0^m.8 \pm 0^m.1$, much redder than the QSO and exactly that of a typical giant elliptical. After separation of the luminosity contribution due to the two color components, the absolute magnitude of the reddish object comes out to be exactly that of a giant elliptical as well. This point had been made earlier in a more general way by Sandage (1971) who noted that the colors of the intrinsically faintest QSO frequently are rather reddish as would be expected if their light were contaminated by that of a galaxy.

4) The QSR 3C323.1 lies in a cluster of galaxies with exactly the same redshift (Oemler et al 1971). Some other similar cases may exist, but as discussed by Burbidge and O'Dell (1973) the statistical significance of those is rather doubtful.

Doubts about the possibility that the redshifts of all QSO could be cosmological have been expressed on many grounds, several of which—like the rather rapid light variations—are no longer relevant today. However the following points are potentially worrisome:

1) *Spatial associations* of objects of very different redshifts. In a sample of 40 QSR from the 3CR catalog, Burbidge et al (1971) found 4 located within 7 arc minutes from a bright galaxy; the *a priori* chance of this being the case in a random sample is small ($\sim 10^{-3}$). However, the *a priori* probability calculation is not applicable after the effect has been found in the sample itself. Bahcall et al (1972) investigated a larger sample of QSO and it appears that no significant effect is found in the additional sample. Similarly Hazard and Sanitt (1972) made up an equivalent 3CR catalog for southern hemisphere objects and again did not confirm the effect. Since the *a priori* statistics are only applicable to these additional samples, this greatly weakens the significance of the result of Burbidge et al. On the other hand, it was found, after the study of Burbidge et al, that the source 3C455 which had erroneously been identified with a bright galaxy, actually is a QSO, only 23 arc seconds from the galaxy. While the galaxy involved is part of a fainter sample than the one studied by Burbidge et al, the case is remarkable and, as noted by Arp et al (1972) more such misidentifications may exist. While we feel that, at this moment, it is premature to base far reaching conclusions on these few cases, a final verdict will have to await the results of further study of objects in well defined complete samples of QSO and galaxies.

A " double quasar " Ton 155-156 with a separation of 35 arc seconds, but very different redshifts, has been found by Stockton (1972), but as shown by Setti and Woltjer (1973b) from a study of the Tonanzintla catalog, the association is probably due to chance. A recently discovered, much fainter double, with one component associated with a 4C source and with a separation of 5″, will be described by Dr. Arp and Dr. E. M. Burbidge. The *a priori* chance of a random coincidence is non-negligible in this case—of the order of one per cent (Bahcall and Woltjer 1973). However, should two more doubles with a separation of less than 5″ and magnitudes brighter than 19 be found in the identification program of 4C sources, it would be difficult to consider these as random coincidences.

2) Possible difficulties also result from the observed expansion of radio sources. If the data are interpreted in terms of expanding doubles at cosmological distances, velocities close to the velocities of light have to occur. This should significantly affect the brightness distributions in the sources and sometimes, somewhat contrived models are needed to fit the observations. Also, there appear to be some sources which

vary surprisingly rapidly at 400 MHz (Hunstead 1972). These situations would be simplified if the sources were comparatively nearby so that the linear sizes would be smaller. However, many different models may be fitted to the presently available data and it seems premature to make too much of these difficulties.

We conclude that, at the present time, the hypothesis that the emission line redshifts of the QSO are of cosmological origin, has received substantial support. The possible difficulties that we have just discussed certainly deserve further study, but we feel that for the moment they do not justify such a drastic conclusion as that of the existence of redshifts unexplained in contemporary physics.

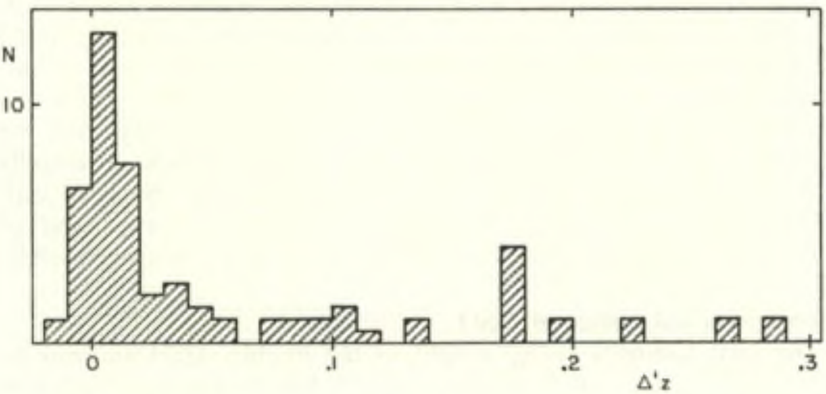

Fig. 1. The distribution of $\Delta'z = \frac{z_{em} - z_{abs}}{1 + z_{em}}$. Very close components which may be separated only on high dispersion spectra have been combined into one component

Absorption redshifts. In Figure 1, we display the frequency distribution of $\Delta'z$ defined as the difference $\Delta z = z_{em} - z_{abs}$ divided by $1 + z_{em}$. Note that in case the difference between emission and absorption components would be due to a velocity of the latter with respect to the former, the velocity would be equal to $c\Delta'z$. The distribution appears to consist of a narrow peak near $\Delta z = 0$ superimposed on a very broad distribution at positive values of $\Delta'z$. It has generally been assumed that the absorbing material near $\Delta z = 0$ is associated with the QSO. In 28 QSO there are 7 components with $\Delta'z$ between $-.011$ and $-.001$ and 22 components between .000 and $+.018$. The mean value of $\Delta'z = +.0056$ (1700 km/sec) and the RMS deviation from the mean $\pm .007$ (2100 km/sec). We suggest that the occurrence of negative $\Delta'z$ values and the magnitude of the velocities may be interpreted on the assumption that both the QSO and the absorbing matter are located in a cluster of galaxies through which both move. The mean difference of 1700 km/sec would indicate some interaction between the QSO and the cluster material; alternatively, we might consider the distribution made up of two components—one symmetrical around $\Delta z = 0$ composed of absorption in independent cluster matter and the remainder being associated with expanding matter around the QSO.

The nature of the absorption components at larger $\Delta' z$ is more uncertain. Some components may well represent absorption in in tervening gas clouds or galaxies. However, the distribution of redshifts suggests that most are connected with the objects themselves. The large Δz values are almost exclusively confined to objects with $z > 2$. But there are several objects with z between 1.9 and 2.0 which should show absorption between $z = 1.7$ and 1.9 if the components in this range in the $z > 2$ case were due to intervening galaxies. Although very serious selection effects make any conclusion provisional, we believe that it is therefore likely that most of the absorption redshifts are intrinsic to the objects. If so, they are probably associated with relativistically expanding clouds of matter. The fast expansion of the radio sources shows that such high speeds are not too implausible.

Energetics and Nature of QSO

The total luminous energy output of the brighter QSO amounts to 10^{47-48} ergs/sec. Lifetimes are uncertain, but the jet in 3C273 seems to indicate that values much less than 10^5 years are unlikely, corresponding to a total energy output of 10^{60} ergs at least if the luminosity is constant. In the case of quasars which are strong radio sources, the usual equipartition arguments show that at leat 10^{60} ergs in relativistic electrons and magnetic fields must be present. If the confinement of the radio source is due to its expansion into an intergalactic medium, the associated kinetic energy may be a factor of 10-100 larger still. Thus, it appears that the energy requirements are in the range of 10^{60-62} ergs, corresponding to $10^{6-8} M_0 c^2$. The radio bursts which we have discussed before inject 10^{54-55} ergs or more in the form of relativistic electrons and magnetic fields. In the more active sources, this happens once every few years and the resultant energy input is of the order of 10^{47} ergs/sec. It seems likely therefore that much of the energy input in quasars occurs in bursts rather than continously.

Most of the current proposals for sources of this energy involve either very massive single objects or very numerous supernovae or pulsars. Massive non-rotating stars are not very promising; because of their high internal temperature, they are almost fully radiation supported and, as a consequence, their effective γ ($= C_p/C_v$) is close to 4/3, their binding energy small, and relativistic instabilities important rather far from the Schwarzschild radius. Multiple supernovae also have difficulties: the typical energy yield per supernova appears to be around 10^{51} ergs, but much of this appears as kinetic energy of matter with non-relativistic velocities. The efficiency of the acceleration of relativistic electrons is likely to be low and an unreasonably large number of supernovae would be required.

The discovery of pulsars and the apparently very high efficiency with which the pulsar in the Crab Nebula generates relativistic electrons and magnetic fields, has led to various investigations of rotating objects in the present context. On one side is a very attractive model by Rees (1971) in which a large number of pulsars accelerate particles which then radiate in the low frequency wavefields of the pulsars. When the rotation axes are all parallel or when the pulsars are situated in a thin disk of matter, the waves propagate preferentially in two opposite directions—which could lead to an explanation of the characteristic double structure of radio sources. Unfortunately, there appear to be energetic difficulties. The Crab Nebula pulsar probably has emitted less than 10^{50} ergs from the beginning. While different pulsars differ, it seems unlikely in any case, that more than about 10^{52} ergs would be released in electromagnetic modes, even for rapid initial rotation, because in the very early phases, gravitational wave emission would be very efficient. This means that the relativistic efficiency (E/Mc^2) is no more than 10^{-2} -10^{-4}, and to account for the $10^{6-8}M_0c^2$ which are needed, $10^{10\pm2}$ pulsars would be required which seems very large.

An alternative possibility has been considered by Morrison (1969), Cavaliere, Pacini and Setti (1969), and by Woltjer (1971) (see also Ozernoy 1966), namely that a massive rotating object is responsible for the energy production in quasars. No proper theory has been worked out yet for such objects and all that has been done is to scale standard pulsar theory without worrying about the differences in boundary conditions at the surface of a neutron star and that of a rather low density gaseous body. If such a massive body is primarily rotationally supported, then loss of energy and angular momentum will lead to contraction and to a shortening of the rotation period. Because such objects—unlike the neutron star—do not have a cold equilibrium state, the contraction presumably will continue until relativistic conditions are met, unless instabilities intervene. It may easily be shown that if magnetic flux and mass are conserved and if solid body rotation prevails, the energy output varies with time as $(1 - t/\tau)^{-2}$ until relativistic terms become important. Here τ is a characteristic time defined by the initial parameters of the model. Ultimately, a Kerr type black hole would probably be formed and the energy output should greatly decrease. It thus would seem that a rather brief high luminosity phase would precede the ultimate extinction. This behavior appears to be consistent with the luminosity function of the QSO (Woltjer 1971, Morrison and Cavaliere 1971).

Since ultimately relativistic collapse ensues, very high relativistic efficiences ($> 10^{-1}$) would be expected and the energy requirements of the most energetic QSO could be met with a total mass of $10^{8\pm1}M_{\odot}$. A related model has been proposed by Lynden Bell (1969) who identifies the QSO with a black hole configuration which accretes matter. Efficiencies and total mass requirements are similar to those for massive rotators.

Instabilities in these rotating configurations may change the evolution significantly. Thin rotating disks with low internal random motions tend to be unstable. Probably the most worrisome instability is a large scale P_2 type mode which deforms the disk into a bar. If such an instability develops, efficient emission of gravitational waves becomes possible and the risk is that only a small part of the available energy enters the observable electromagnetic channel. In addition, fragmentation would also occur; the disk would break up into smaller self-gravitating rotating subdisks. Quirk and McKee (1971) evaluate the amount of random kinetic energy required to prevent these instabilities. Actually, the fragmentation instability is not at all serious. If the disk breaks up into smaller subunits, each of these will evolve in the same way as before. Each will have a brief epoch of high luminosity, but since the parameters of the different disks will be different, this phase will not occur at the same time for all. The total energetics remains the same, but the energy input becomes more discrete. Perhaps we may understand in this way why the energy input into the QSO occurs in bursts rather than continuously.

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Discussion of the report of L. Woltjer

R. Hofstadter: I want to ask a naive question. You assume isotropic emission of the radiation in the case of quasars. Is it not possible that there is beaming of energy to the observer?

L. Woltjer: In a few cases one can measure the quasar continuum close to the Lyman limit. From this an estimate may be made of the flux of ionizing photons; if isotropy is assumed this number agrees roughly with the number inferred from the emission lines produced in the plasma around the quasar, the latter of course being isotropic. While this type of argument does not prove that the radiation is isotropic to within a factor of two or so, it seems to exclude an extreme degree of anisotropy like that seen in pulsars.

R. Omnes: There is another class of models which has not been ruled out conclusively and which involves antimatter annihilation as the energetic mechanism of quasars. Without entering into details and staying at the level of numerology, it is worth noting that the most energetic quasars can live about 10^{10} years if the mass they can consume is a galactic mass.

F. Pacini: Concerning the possibility of matterantimatter annihilation, one can possibly explain the origin of mildly relativistic particles but one should not forget than a comparable amount of energy (possibly even more) should be generated as large scale magnetic fields. It is not clear to me how one can accomplish this simply by annihilating matter and antimatter.

GENERAL DISCUSSION ON REDSHIFTS

DISTANCES OF THE QUASARS AND EVIDENCE FOR NONVELOCITY REDSHIFTS

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In the following paper the observational evidence for anomalous, or non-velocity redshifts is briefly reviewed. The author concludes that *all* the present evidence favors the physical association of at least some quasars with nearby galaxies. In fact, it is discussed how all quasars might originate from galaxies of much lower redshift. In addition to the statistical evidence, it is shown that in the four cases where QSRs fall projected closest to bright galaxies, that in all four cases the galaxies show evidence of physical interaction. Evidence for high redshift, compact and peculiar companion galaxies is discussed. From the individual associations of high redshift QSRs and companions, an empirical continuity of observed characteristics is shown between compactness (youth) and excess redshift. Some extant theoretical explanations for intrinsic redshifts are mentioned.

1. INTRODUCTION

In these brief comments which were yesterday scheduled into today's final program I would like to present evidence for large redshifts in extragalactic objects which are not caused by recessional velocities in an expanding universe. One consequence of accepting this result is that, in these cases, the distances and the luminosities of the objects cannot be computed by the conventional application of the Hubble constant. An even more important consequence, however, is that if such non-velocity redshifts do exist, then there is no ready explanation for them in conventional physics and they would therefore present the greatest challenge to cosmological theories.

In the short time available to me I cannot cover all that I wish to say on this subject nor even mention very much of what F. Hoyle and others might want to say if they were here. Therefore, I will restrict myself to a very schematic summary of the evidence and one or two examples. But I would like to emphasize to you that the evidence I will now discuss represents only the tip of the iceberg of observational evidence which has accumulated in the last seven years on the existence of non-velocity redshifts.

2. ASSOCIATIONS OF QSR'S WITH NEARBY GALAXIES

Answering first the question of whether all quasars are at relatively nearby distances, it is possible to take the position that Rowan-Robinson and others have taken about QSRs, namely, that there are some at cosmological distances and others nearby which have spurious redshifts (for references see review by Arp 1973a, b). That situation would be much harder to conclusively prove or disprove than if they were all at cosmological or all at local (10-100 Mpc) distances. Therefore, in the interests of trying to answer a more preliminary question, I will investigate whether the proposition can be supported that " all QSRs are associated with nearby (< 100 Mpc distance) galaxies ".

(a) *Statistical evidence*

The initial investigations by Arp showed the QSRs to be associated with bright galaxies and peculiar galaxies, including those galaxies in our own Local Group. Later Burbidge, Burbidge, Strittmatter, and Solomon (1971) showed that four radio bright (3CR) QSRs fell very close (< 7 arc min) to moderately large spiral galaxies. The probability of this occurring by chance was extremely small. Some investigators subsequently argued that if these QSRs were physically associated with these galaxies, it should be possible to find additional cases by considering associations at greater distances, that is, fainter QSRs around fainter galaxies. So radio-fainter QSRs (Parkes 2700 MHz survey) were examined with respect to fainter galaxies (the Zwicky *Catalogue of Galaxies and Clusters of Galaxies* which reaches to $m_{pg} =$ 15.7 mag). No significant associations of fainter QSRs with fainter galaxies were found. Should we conclude that the association of QSRs and galaxies has not been supported by this result? Apparently not, because the most recent investigation by Browne and McEwan (1973) has turned up two new QSRs within 1.7 and 2.1 arc min of faint galaxies. The probability of chance association now becomes only 5 %. As it stands, this probability may be only marginally significant, but it can only be a lower limit to the real significance for the following reasons: The original Arp (1970) associations showed—

Associations	Galaxies (m_{pg})	QSRs (V)
Arp:	9 to 11 mag.	17 to 19 mag.
implies:	13 to 15 mag.	21 to 23 mag.

Since we do not optically identify many QSRs in the 21 to 23 mag range, we would not expect to find many QSRs around 13 to 15 mag galaxies.

Looked at another way, the original paper showed that QSRs with quite low redshifts as well as those with very high redshifts were distinctly less luminous than QSRs of intermediate redshift. The luminosity-redshift correlation must look something like the following Figure 1.

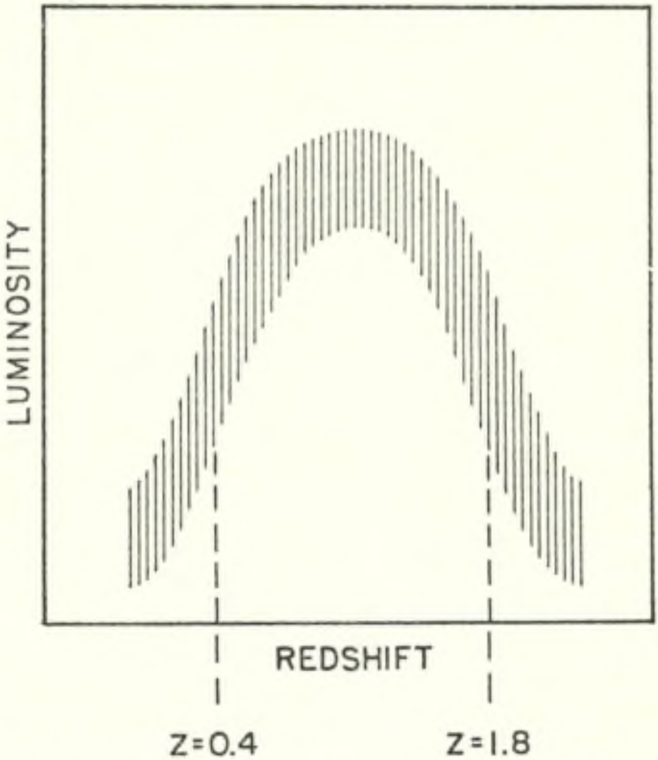

Fig. 1. Approximate derived luminosities of quasars as a function of their intrinsic redshift.

This would predict that the QSRs seen at the greatest distance, falling closest to galaxies and therefore singled out as associations, would be predominantly of intermediate redshift. Table 1 in the present paper shows all the QSRs presently believed to be most probably associated with galaxies. The first five cases are from B^2S^2 and the next seven are from my own compilation. A very significant result emerges when one examines the redshifts of these associated QSRs. All their redshifts fall between $z = 0.4$ and 1.8. If we take a normal distribution of QSR redshifts as in Barbierie, Battistini, and Nasi (1967), we see that the chance of accidentally selecting all the redshifts in Table 1 between $z = 0.4$ and 1.8 is less than 1 %.

TABLE 1

Close association between quasars and Galaxies

Object Pair	m	z	r (min)
3C 455 NGC 7413	19.7 15.2	.543 .033	0.4
3C 232 NGC 3067	15.8 12.7	.534 .005	1.9
3C 268.4 NGC 4138	18.4 12.1	1.400 .004	2.9
3C 275.1 NGC 4651	19.0 11.3	.557 .003	3.5
3C 309.1 NGC 5832	16.8 13.3	.904 .002	6.2
2020-370 Spiral galaxy	... ...	... 1,1	0.3
PHL 1226 IC 1746	... 14.5	.404 ...	0.9
3C 270.1 pec ring galaxy	18.6 (17)	1.519 ...	5.1
0159-11 IC 1767	16.4 (15)	.68 ...	39
Mark 132 NGC 3079	15 11.9	1.75 .041	45
3C 254 Mayall's pec object	18.0 (15)	.734 .035	126

The earlier paper showed that the very high and very low redshift QSRs were associated with very nearby galaxies, including the Local Group of galaxies which is dominated by M31. (Our own Galaxy falls somewhere near the edge of the M31 Local Group). These closest QSRs fall projected at large angular distances from their galaxies of origin. We can therefore answer the question: If all QSRs are associated with nearby galaxies why does not every QSR fall close to a galaxy? The answer would then be that a number of QSRs belong to very close-by galaxies and fall projected at considerable distance from them on the sky.

(b) *Individual evidence of QSRs with galaxies*
We list below the four closest cases of quasars adjacent to galaxies (excluding the two additional and as yet uninvestigated cases reported by Browne and McEwan):

(1) 2020-37 falls 21 arc sec from small spiral galaxy.
(2) 3C 455 falls 23 arc sec from NGC 7413.
(3) Mark 205 falls 42 arc sec from NGC 4319.
(4) PHL 1226 falls 55 arc sec from IC 1746.

In the first case the QSR falls very close to a very distorted spiral arm, which might be taken as evidence of interaction. In the second case the galaxy shows an unusual perturbation of its luminous isophotes in the general direction of the QSR. (See review by Arp 1973b for details of the previous and the following two cases). In the third case Markarian 205 shows a luminous connection back to NGC 4319. In addition, investigation of the interior regions of NGC 4319 shows perturbations of the inner isophotes, which appear to be associated with the ejection of a radio source in one direction and the ejection of Markarian 205 in the other direction from the nucleus of NGC 4319. This ejection is computed to have taken place at approximately 1000 km s^{-1} about 10^7 years ago. (It should be noted that an ejection velocity of about 3000 km s^{-1} is about what is needed to construct clusters of galaxies by fissioning and ejecting from central objects. This suggestion is along the lines of the kinds of conclusions initially presented by Ambarzumian in his classic report to the 1957 Solvay Conference). In the fourth case there is evidence for interaction between IC 1746 and the QSO-peculiar galaxy double that lies just off the southeast edge of the galaxy. There also appears to be a luminous filament connecting the QSO with the peculiar galaxy. Although the redshift of the galaxy is not known, it would be unprecedented for this fairly bright and diffuse object to have the redshift of the quasar, which is $z = 0.404$.

We can summarize this section on individual connections with the statement that, in the four cases where quasars lie closest to bright galaxies, in every case we see evidence for interaction between the quasar and the galaxy.

3. ASSOCIATION OF HIGH REDSHIFT GALAXIES WITH LOW

By now the most famous case of discordant redshift is Stephan's Quintet (see, for review, Arp 1973b). It has been proposed by Arp that the whole of Stephan's Quintet is associated with the bright, $cz =$ 800 km s^{-1} spiral NGC 7331, which is about 30 arc min northeast of the Quintet. That means that four members of the Quintet with redshifts between $cz = 5700$ km s^{-1} and 6700 km s^{-1} are really about

8 times closer in distance than their redshifts would indicate. In support of this picture a number of lines of evidence can now be cited:

(a) The H II regions are almost exactly the same size in both the low redshift member of the Quintet (NGC 7320) and the one high redshift which contains H II regions (NGC 7318).

(b) There is interaction between the high and low redshift members of the Quintet.

(c) There are excess numbers of radio sources in the region between NGC 7331 and the Quintet.

(d) There is a system of very faint, luminous optical filaments between NGC 7331 and the Quintet.

Now, independently, hydrogen measures by Heidemann with the Nancay radio telescope give a distance, independent of redshift to NGC 7319 which is much closer to the low redshift distance of NGC 7331 and NGC 7320 (about 10 Mpc) than the Hubble distance of NGC 7319 (which would be about 120 Mpc with $H = 55$ km s^{-1} Mpc^{-1}).

In addition, it has been recently shown by Arp 1973c that other systems like Stephan's Quintet (e.g., VV150 and the Burbidge chain northwest of NGC 247) *characteristically* occur near bright, relatively low redshift galaxies. Therefore, we see that Stephan's Quintet is not a unique isolated phenomenon, but this effect happens in other cases where multiple interacting galaxies appear to have strong components of nonvelocity redshift.

I would like to mention a photograph that illustrates one case of a bridge or luminous filament connecting a high redshift galaxy to a low redshift galaxy. Figure 2 in Arp (1971) shows a peculiar companion of $cz = 16{,}900$ km s^{-1} connected to a main galaxy (NGC 7003), which has a redshift of $cz = 8{,}800$ km s^{-1}. Aside from all the arguments originally made as to the physical association of this discordant pair of redshifts, we can now add the argument shown in shorter exposure photographs (see report of I.A.U. Symposium 63 Krakow Poland 1973). The photograph shown there is a lighter print than usual, and rather than emphasizing the bridge shows the extremely disturbed interior regions of the larger galaxy. The argument now becomes the simple but powerful one: " If the action of the high redshift companion has not disturbed the central galaxy, then what has? "

Finally, I would like to mention that all the cases of discordant redshifts, where individual high redshift objects have been associated with low redshift objects of known or estimated distance, can be combined in a diagram. Figure 2 shows the result of this compilation and illustrates how the excess redshift of an object is associated with its lower absolute magnitude. I believe that this diagram represents an evolutionary sequence in which a compact object or quasar is ejected

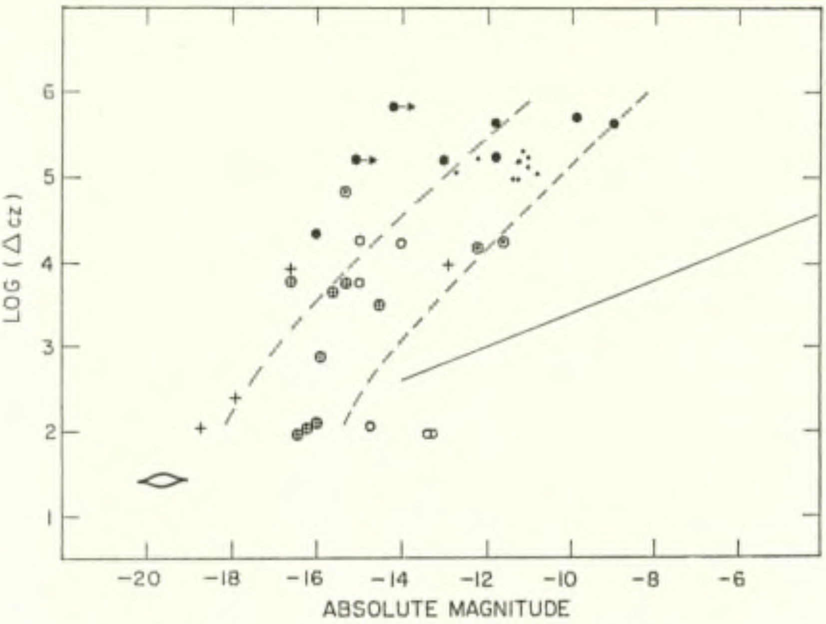

Fig. 2. Individual high redshift objects that are physically associated with low redshift objects. Ordinant is excess redshift and abscissa is absolute magnitude, both measured relative to bright central object shown schematically in diagram. Small filled cricles represent Local Group quasars; large filled circles represent quasars associated with individual galaxies. Remaining symbols represent individual compact galaxies, peculiar interacting systems like Stephan's Quintet, and companions to large nearby spirals like companion to NGC 7603, M82, M32, etc.

from a large galaxy. It has initially high intrinsic redshift, but as the compact object evolves into a compact galaxy, then into a disturbed young spiral, then spiral, and finally into a relaxed type II population system. I believe the intrinsic redshift decays along this evolutionary sequence leaving, eventually, only the true Doppler redshift due to the space motion of the object.

Among the astronomers and physicists who accept the reality of nonvelocity redshifts, about four kinds of explanations are being worked on. In reverse order of my judgement of their likelihood:

(1) gravitational redshifts;

(2) difference between proper and coordinate time at large space-time distances;

(3) photon-photon scattering; and

(4) creation of low-mass matter.

I will not go further into any of these explanations, but instead refer the reader to my review (Arp 1973b).

One final comment. I would like to describe the very exciting discovery by Joseph Wampler, communicated to me by telephone recently. The discovery consists of two quasars, 5 arc sec apart, which have redshifts of $z = 1.90$ and $z = 0.43$. There is a luminous, diffuse object on the other side of the bright quasar, directly across from the faint quasar, and a radio source somewhere on the same side as the nebulous object. This triple configuration, or pairing across a bright object, is a common finding in the discordant redshift and ejection phenomenon. Aside from the improbability of accidentally finding two quasars this close together on the sky, I would like to stress the extreme improbability of also finding this triple configuration.

In conclusion, I would like to point out that, of course, as is true of the earlier examples discussed in this paper, we need only one established case of a discordant redshift to force a crucial confrontation between observation and the current physics on which cosmology is based.

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RECENT OBSERVATIONS OF QUASI-STELLAR OBJECTS

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I shall discuss some observational results on QSOs which have a bearing on the problem of their redshifts. First, I shall review the situation concerning the absorption lines which appear mainly in QSOs of large redshift. In some objects there are many absorption-line systems. Second, I shall describe observations of recently-discovered objects with very large redshifts and also give a brief account of the observations by Wampler *et al.* of a double QSO separated by only 5 arc sec. Finally, I shall add something to the discussion of the statistics of the distribution of QSOs.

I. ABSORPTION-LINE SPECTRA: GENERAL DESCRIPTION

The early observations of QSOs, which were mostly 3C sources, revealed only broad emission lines. The history of the discovery and analysis of absorption lines has been reviewed elsewhere (Lynds 1970; Burbidge and Lynds 1970). Many fairly narrow absorptions were found in 3C 191 in addition to the usual broad emission lines (Burbidge, Lynds, and Burbidge 1966; Stockton and Lynds 1966). They were clearly of a type well known in spectra of hot stars with extended atmospheres—the P Cygni stars—in which a shell or ring of gas is driven outward, and that part of it lying between the star and the observer produces absorptions that are Doppler-shifted to lower wavelengths with respect to the emission lines. 3C 191 has $z_{em} = 1.955$ and resonance lines of ions of carbon, nitrogen, silicon, as well as the Lyα hydrogen line indicated an outward flow of gas at about 600 km/sec. The absorbing gas was clearly associated with the QSO. However, at about this time, Bahcall and Salpeter (1966) and Shklovsky (1965) had aroused interest in searching for absorptions produced by intervening galaxies or intergalactic clouds in the spectra of QSOs with large z.

When optical studies of QSOs were extended to the spectroscopic study of radio-quiet QSOs and QSOs of lower radio flux and flatter radio spectra than those of the 3C objects, several objects with absorption lines in their spectra were discovered. The most interesting were

PHL 938, Ton 1530, and PKS 0237-23 (Burbidge, Lynds, and Stockton 1968; Bahcall, Greenstein, and Sargent 1968; Chan and Burbidge 1971). These are all fairly bright, with magnitudes 16½-17, and have z_{em} around 2. When their rich absorption-line spectra were at least partly identified, they revealed many absorption-line systems at *different* z.

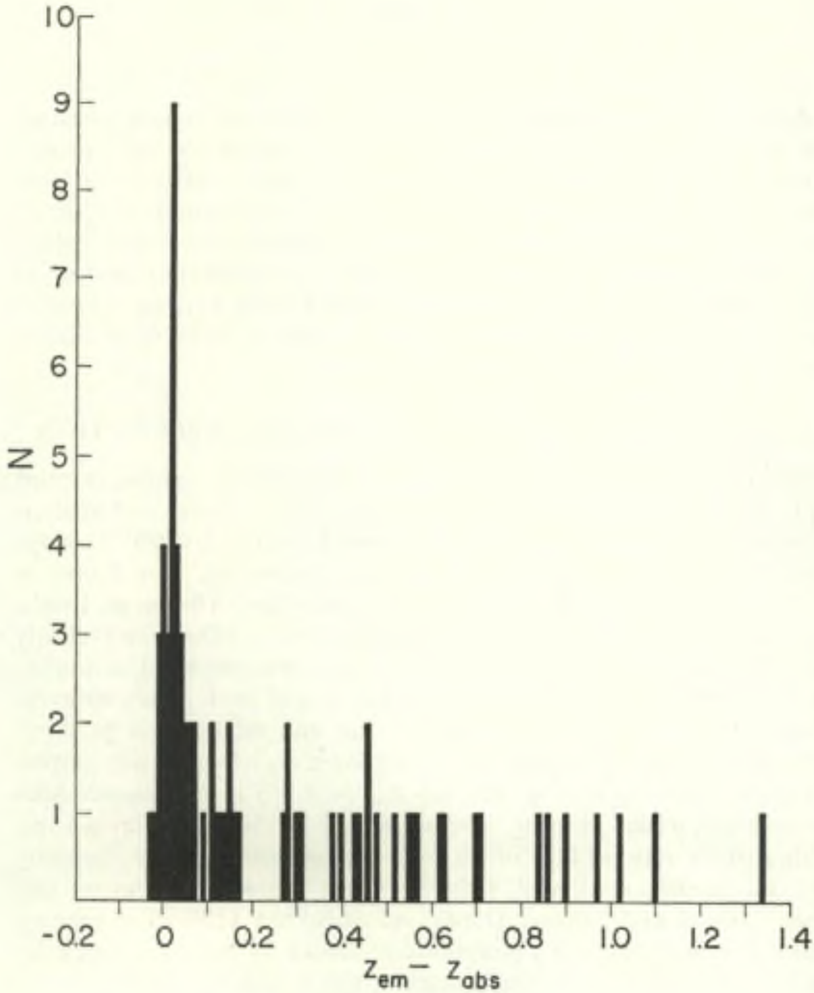

Fig. 1. Histogram showing the distribution of the difference between the emission-line and absorption-line redshifts in multiple-redschift QSOs. Ordinate gives numbers at particular values of $z_{em} - z_{abs}$. Data are compiled from all available sources. There are 32 QSOs with a total of 73 absorption redshifts.

The most extreme object, PKS 0237-23, with $z_{em} = 2.223$, has absorption-line systems ranging from $z = 1.36$ to 2.20.

The evidence that all these absorptions are intrinsic to the object, and do not arise in intervening clouds lying much nearer to the observer is very convincing, on the basis first of the distribution of the redshifts, and second on spectroscopic comparison of the systems. Let us consider the distribution of redshifts first. It is a feature of the absorption-line QSOs that among the high-z objects one tends to find *single objects* containing a *large number* of absorption systems, e.g. as many as 8 in PHL 957, and some 5-7 in PKS 0237-23, while there are still large-z QSOs that do not have any absorption lines. This is not what one would expect from any normal distribution of intervening galaxies or intergalactic clouds; calculations show that high-z QSOs should, if their redshifts are cosmological, show an average of 1-2 absorptions per high-z object (Roeder and Verreault 1969). Fig. 1 gives a histogram of (z_{em} - z_{abs}) for all QSO absorption systems so far identified. The break into two plots made by Dr. Woltjer for objects with z greater than and less than 2 does not give a physically meaningful separation, because the rest wavelengths of the resonance lines which must be used to study absorption-line systems are plentiful right through the wavelength range brought into the optical window of the spectrum observable from ground by redshifts centered about $z = 2$. In other words, there are many available resonance lines from the Lyman limit at 912 Å up to the strong resonance doublet of C IV at 1548-50 Å. Then there is a large gap in available lines until one reaches the weaker Fe II lines starting at about 2300 Å or, better, the strong Mg II doublet at 2796-2803 Å. Thus, a meaningful break in a separation of the $z_{em} - z_{abs}$ plot into two parts would be at about $z = 1.4$, below which C IV becomes unobservable. At the very least, one should make a separation no higher than $z = 1.6$, because the most characteristic absorption line, Lyα, is visible down to that redshift.

In any case, the following deductions may be drawn from Fig. 1:

(1) The peak at $z_{em} - z_{abs} \approx 0$ shows that most absorption systems have z_{abs} close to z_{em} so that the gas producing them must be associated with the QSO;

(2) gas may be infalling ($z_{em} - z_{abs} < 0$) as well as outflowing (if $z_{em} - z_{abs}$ has a Doppler interpretation);

(3) there is a continuous distribution out to $z_{em} - z_{abs}$ corresponding to $\Delta z \approx 1/3$ to $1/2$ c.

I do not have time to enter into a detailed discussion of the spectroscopy of the absorption lines; briefly, one finds that systems at $z \approx z_{em}$ are spectroscopically similar to systems at large $z_{em} - z_{abs}$, except for details in one case that are explicable in terms of the gas lying at different distances from the energy source. High-ionization lines like C IV appear at $z \approx z_{em}$ and z much less than z_{em}; this is not expected if the latter arises in relatively cold gas having nothing to do with the QSO. Finally, mention should be made of evidence from lines produced by ions whose ground states have fine-structure levels at a few hundredths of an eV. Bahcall and Wolf (1968) showed that absence of absorptions arising out of excited fine structure states, indicating zero population of these states, set maximum limits on electron density in the absorbing gas and minimum limits on the distance of the gas from the energy source, i.e. the central object. Some objects, e.g. PHL 938, must obey these limits because only zero-volt Fe II lines are seen in the absorption lines at $z_{abs} = 0.613$, and this object provides the best case for an intervening galaxy. However, recently some objects, discussed later, have been found in which the fine structure states show populations at or near thermal equilibrium values in absorption systems with sizable values of $z_{em} - z_{abs}$.

II. SOME INDIVIDUAL ABSORPTION-LINE OBJECTS

PHL 957 (Lowrance *et al.* 1972) is an especially interesting bright radio-quiet QSO discovered by Maarten Schmidt, with $z_{em} = 2.69$. It has an extremely broad Lyα absorption whose sharp-sided profile may be partly contributed by nearly coincident lines of other ions at other redshifts (Beaver *et al.* 1972; Grewing and Strittmatter 1973; Bahcall and Joss 1973), as well as sharp lines at this redshift and some seven sets, varying well determined, of sharp absorption lines at other redshifts.

4C 5.34, with $z_{em} = 2.88$ (Lynds and Wills 1970; Lynds 1971; Bahcall and Goldsmith 1971), has a very large number of absorptions, still mostly unidentified, shortward of the Lyα emission, as well as some eight separate absorption-line systems. The possibility exists that most of the unidentified absorptions are Lyα unaccompanied by other lines, except perhaps by Lyβ, at a very large number of discrete redshifts.

1331 + 170 — This Molonglo radio source, with $z_{em} = 2.082$ (Baldwin *et al.* 1973) has many absorption lines (Strittmatter *et al.* 1973) yielding two certain and one possible discrete systems which contain a number of lines from excited fine-structure states in Si II and Fe II. There are interesting coincidences between absorption lines in the two main absorption systems, which may be explained by the following hypothesis.

A Hypothesis for Production of Absorption Lines at Many Different Redshifts. If the absorption lines arise in gas associated with the QSOs, and if the differences between z_{em} and the various z_{abs} are velocity shifts, some explanation must be sought for the large outflow velocities required to explain the observations, and, since the lines are very narrow, for the very small velocity dispersion in each outflowing cloud or filament of gas. Furthermore, there is a well-marked tendency for certain coincidences in wavelength ratios to occur between the several z_{abs} or between the principal z_{abs} and z_{em}. The strongest absorption lines in the principal system often lie in the violet wing of a prominent emission line. These effects can be explained if the outflowing gas reaches its high outward velocities as a result of acceleration by radiation pressure, and if the outward force is nearly balanced by inward gravitational attraction by the central massive object. In the presence of such a near balance, in cases where radiation pressure is a strong function of wavelength as, for example, in spectral regions where there are emission lines, emission edges, or absorption by underlying gas, the following situation will arise. Gas being accelerated outward by the absorption of resonance lines will tend to accumulate at velocities such that radiation pressure just balances gravity, e.g. the absorption doublet of C IV λ1548-51 may fall in the wing of Si IV λ1400 emission with the C IV blueshifted relative to the Si IV, i.e. at a lower redshift as seen by the observer. Alternatively, groups of absorption redshifts may be found such that the shortward member of resonance doublets of e.g. Mg II and C IV fall on the wing of the longward member of the same doublet at a slightly lesser redshift. This is the case in 1331 + 170, and also in PKS 0237-23. The phenomenon has been described by Mushotzky *et al.* (1972) and has been named " line-locking ". In principle it can account for the large outward velocities, although details will depend on a real physical model for QSOs. Of course, the driving links may occur in unseen regions of the spectrum, and it is suspected that He II resonance lines and the He II continuum edge may play a prominent part.

3C 286 — A very interesting radio observation of an absorption line in 3C 286 has been reported by Brown and Roberts (1973). The optically-determined redshift of 3C 286 is $z_{em} = 0.849$, and a sharp absorption line, less than ~ 10 km/sec wide, has been detected which, if it is the 21-cm absorption line, would give a redshift of 0.692. The authors suggest this absorption arises in an intervening galaxy, since the absorbing gas must be well seperated from the source of the emission. Confirmatory observations at Arecibo have shown that the line is even narrower; a limit of 4 km/sec has been set. Such a small velocity spread would be hard to explain if the absorption does indeed arise in gas in an intervening galaxy, where, judging by known nearby galaxies, the velocity spread in a cut across the galaxy should be considerably larger than this. If the absorbing gas lies near the QSO, it must be cold and relatively dense.

III. QSOs WITH $z > 3$

Two radio sources singled out from the Ohio radio catalogues as having unusually flat or peaked radio spectra (Fitch *et al.* 1969) have been found to have redshifts exceeding $z = 3$. The first one is $0642 + 44$ (OH 471) (Carswell and Strittmatter 1973); it has $z_{em} = 3.42$ and absorption lines; in addition, at $z_{abs} = 3.346$ as well as other absorptions, suggesting further redshift systems. The second is $1442 + 101$ (OQ 172) (Wampler *et al.* 1973); it has $z_{em} = 3.53$ and a very rich and complex absorption-line spectrum most of whose lines lie shortward of Lyα emission. One might have hoped that, if and when QSOs having such large redshifts were located, they might be found to have some common property which had tended to prevent their discovery earlier. Indeed, $0642 + 44$ was found to have virtually a cutoff in the continuum near 4000 Å where the redshifted Lyman series limit occurs. Thus, the object did not appear noticeably blue on the Palomar Atlas, and would have been missed in a search for stellar objects with UV excess. However, $1442 + 101$ was found to have no detectable break at the Ly continuum edge; thus the first two objects with $z > 3$ are found to differ markedly in this important respect. The object $1442 + 101$ has many discrete absorption redshifts of which only a few have so far been identified; it may have several systems showing only the Lyman lines and is reminiscent of 4C 5.34 in this respect. The most prominent system is at $z_{abs} = 2.564$, and its strongest feature is the C IV doublet which falls in the shortward wing of the very strong Lyα emission. A second strong absorption system has $z_{abs} \simeq 3.09$, for which $(1 + z_{em})/(1 + z_{abs})$ has a ratio already found in several of the previously-known absorption-line QSOs. These coincidences lend support for the line-locking hypothesis.

Finally, it should be mentioned that the preliminary study by Mushotzky *et al.* (1972) indicated that, for reasonable values of the physical parameters, the central compact object would have a mass and radius such that the gravitational redshift would be non-negligible, although an order of magnitude smaller than the observed redshift. This fact has, however, rekindled our interest in the possibility of constructing a model in which large gravitational redshifts can occur.

IV. A DOUBLE QSO

Wampler *et al.* (1973) have recently discovered a close pair of QSOs with a separation of only 5″; the pair had been suggested as possible identifications for a 4C radio source. It was found that in fact both objects were QSOs of magnitudes 17 and 19 but that they had very different redshifts, 0.436 and 1.901, respectively. The radio source is double, one component coinciding with the brighter QSO and the other lying on the opposite side of it to the fainter QSO. Near this second source is a faint extended object. The close grouping suggests a physical association between the optical objects and the radio sources. Further, there are some interesting near-coincidences in wavelength; an absorption line in the object of larger redshift (b) occurs at nearly the same wavelength as strong Mg II emission in the object of smaller redshift (a). If the absorption were indeed Mg II, it might be explicable on the cosmological hypothesis if (a) were surrounded by an extensive cloud of gas which envelopes a background object (b). However, the preliminary spectroscopic measures suggest that the absorption is not Mg II but is the C IV doublet at a redshift somewhat less than the emission-line redshift of (b). Further, (a) has an emission line at the same wavelength as the strong C IV emission in (b). The conclusion drawn from these observations was that they add support to the non-cosmological redshift hypothesis; the probability of finding by chance a 19th magnitude QSO lying only 5″ from one of some 250 possible QSO radio sources is only about one in a hundred, taking the usually adopted background density of QSOs down to magnitude 19.4 of about 5 per square degree. The argument has been raised that such a probability depends *a posteriori* on the knowledge of the separation as 5″ and that one should, to be statistically rigorous, use a separation of 35″ since a double QSO at this separation, and having very different redshifts, was earlier found by Stockton (1972). To obtain the estimated probability of 1 in 100 one should have previously published this estimate before observing the spectra of the objects. However, in practice, the calculation of the estimate is so trivial that (i) it would not be possible to get it published and (ii) all observers intending to look at double blue stellar objects which are suggested as radio source identifications carry such figures in their heads and know very well that the probabilities are very low for separations $< 10''$.

V. ASSOCIATIONS BETWEEN QSOs AND GALAXIES

Considerable effort has been put into the search for associations between QSOs and galaxies, in order to test the cosmological hypothesis. The earlier investigation by Burbidge *et al.* (1971) into the distribution of the 47 QSOs of the 3C catalogue in relation to the fairly bright galaxies in the Revised Shapley-Ames Catalogue yielded 4 close associations between QSOs at fairly high redshifts and galaxies at low redshifts, for which the chance probability was found to be 5×10^{-3}. A fifth close association was subsequently found; for all 5 objects, the chance probability is 10^{-5} using the NGC galaxies as the sample (Burbidge, O'Dell and Strittmatter 1972). It has been argued that the adoption of the 3C and the Shapley-Ames catalogues involves an *a posteriori* choice; this, however, is not so as it was the close association of one 3C source and one Shapley-Ames galaxy which led the authors to undertake the study. Strictly speaking, one should thus use 3 out of the 4 coincidences in the original study; the fifth coincidence which was obviously *a priori* then brings the number back to 4 and the original estimate is valid. Recently, Kippenhahn and Vries have redone the random distribution calculations by another method but have obtained essentially the same result, a probability for chance occurrence of 4×10^{-3}.

Another attack on the problem concerns the search for associations between QSOs and faint galaxies. Here the results of Gunn (1971), Robinson and Wampler (1972), Oemler, Gunn and Oke (1972) and Miller *et al.* (1973) have yielded groups of galaxies having essentially the same redshift as nearby QSOs, suggesting that the redshifts of these QSOs are cosmological. However, the search for such associations was clearly biased toward finding this agreement. The search was conducted around low-redshift QSOs, in general having $z \leq 0.36$ in order that galaxies of this redshift could be detected. At this redshift, deep photographic plates show up myriads of galaxies, and recent calculations by Burbidge and O'Dell (1973) show that the probability of random coincidence between groups of galaxies and QSOs is not small. The statistical game cuts both ways in the argument on the nature of redshifts of QSOs!

Another attack on this problem is underway at present at Lick and Palomar Observatories. Hazard has selected a number of blue stellar objects which are good candidates for the identifications of radio sources having accurate positions determined by Murdoch and others at Molonglo. The selected objects have small groups of faint galaxies nearby, and form groupings which would certainly have been selected as associations had the stellar objects been already identified as low-redshift QSOs. But the important point is that one does not know beforehand whether the stellar objects are indeed QSOs, nor, if so, what their redshifts are. As a result of the first work on a few of these objects, the stellar objects have been confirmed as QSOs and their redshifts are *large,* considerably in excess of the redshifts that the faint galaxies could possibly have, based on their apparent magnitudes. This line of research is the only way to tackle the problem in an unbiased fashion, but it will require a great deal of telescope time.

VI. CONCLUSIONS

The QSOs are still a mystery as regards their physical nature, their origin, and their evolution. If their distances could be unequivocally determined, there would at least be a firm basis on which to attempt to construct physical models. That strong gravitational fields are present is the one undoubted fact, and one knows nothing experimentally concerning the interaction of radiation and matter, probably with strong magnetic fields present, in a very strong gravitational field. Theoretically the problem is very complex. Possibly we do not have sufficient knowledge yet to either construct or rule out theories of gravitational redshifts. That some component of the redshifts of some of the QSOs are non-cosmological seems to this writer to be very probable, on the basis of the evidence discussed above. The strongest arguments concern the existence of multiple redshifts in single objects, in which the evidence favors the view that all the redshifts are associated with the object, and the statistics of the distribution of QSOs relative to galaxies. However, it may well be that the QSOs do not form a homogeneous sample. Some of them have double radio sources lying on either side of the object, and have steep radio spectra, both of which properties ally them with radio galaxies. There may be a real separation between such objects and the compact sources in which one radio component (often the only one) coincides with the object, and in which the radio spectrum is usually flat or peaked. These objects include especially the high-redshift QSOs with absorption lines in their spectra. If all QSOs have cosmological redshifts, then the cosmology must definitely be more complicated than the simple models usually discussed. If only the galaxy-like QSOs have cosmological redshifts, then one may be able to live with one of the simple cosmological models.

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Discussion of the communications of H. Arp and E.M. Burbidge

G. R. Burbidge: Some remarks have been made about the double QSOs recently investigated by WAMPLER and his colleagues. There are two points of interest. The first is that there are two objects less that 5 arc seconds apart with very different redshifts. Statistical arguments, admittedly made a posteriori, suggest that this is very unlikely unless they are physically associated*. The other point of great interest is that there are two *coincidences* in wavelength in the line spectra of the QSOs. One is a coincidence between an absorption feature in the larger redshift QSO and an emission line in the other QSO, and the other is a situation in which two emission lines, one in each object, appear at the same wavelength. It is for this reason, as much as any other that the authors have argued that their discovery is a point in favor of the non-cosmological redshift hypothesis.

M. Schmidt: If the case against the cosmological redshift is mainly made up of the two double quasars and the four 3CR quasars near galaxies, then it seems to me that the case is very weak indeed. Has the evidence of redshift concentrations at 1.955, 1.95, 0.075, 0.061, 0.032 been put to sleep?

H. Arp: I would like to remind Dr Bahcall (*) that the hypothesis of double quasars is not a posteriori. That conclusion was reached in my 1970 paper. Since then evidence has accumulated that compact galaxies (Bertola et al) and Markarian galaxies (Heidemann + Kalloghlian) have a strong tendency to be double. Since all these compact objects have been previously demonstrated to be physically double, it is reasonable to test this result with a probability calculation on the new, Wampler double quasar.

Also, I would like to remind Dr. Bahcall that my major argument in the physical association of the double quasar was based on the *aggregation* of unusual objects in this very small region as well as the *alignment* of these objects, two important pieces of information which he has ignored.

Thirdly, in discussing the statistics of association of quasars and galaxies, he has ignored the new results of Browne and McEwan, which when coupled with the small number of quasars predicted to be associated with these fainter galaxies, presents results which appear to be a quite significant confirmation of the associations.

* For statistical discussions see BAHCALL and WOLTJER. *Nature* **247**, 22. 1974, and BURBIDGE-BURBIDGE and O'DELL. *Nature* 1974 in press.

(*) The comments made by J. Bahcall during the discussion have been incorporated in: J. N. Bahcall and L. Woltjer. " Close Pairs of Q.S.O's " *Nature* **247**, 22 (1974).

Finally I would like to at least mention the important question of anisotropy in distribution of radio sources on the sky. Results increasingly point to an anisotropy of radio sources, particular in flat spectral index sources. These kinds of radio sources are characteristically associated with quasars and substantial non-homogeneities of distribution on the sky would imply a small distance for them, comparable with the distribution of near by, anisotropically distributed, galaxies.

I hope there is time for further discussion of this important point.

E. M. Burbidge: I showed my reasons for accepting all the multiple-redshift systems as arising in the object. Thus one has, if the emission-line redshift is accepted as a cosmological redshift, to account for redshifts *in the object* differing from the cosmological value by amounts that, if they are Doppler shifts, give velocities of 1/3 to 1/2 the velocity of light. It is for this reason that I would like more effort to be put into making a model based on gravitational redshifts.

G. R. Burbidge: There has not been time to discuss all of the evidence bearing on the problem of the redshifts of the QSOs. A detailed discussion has been given by me in *(Nature Physical Science)* **246,** 17, (1973). There are a number of points and only one or two can be mentioned here. There is a considerable body of evidence both for galaxies and QSOs which suggests that objects with different redshifts are physically associated. Some of the best evidence is through the association of bright QSOs with galaxies. Also the redshift distribution of QSOs is most peculiar, and very different from that of galaxies. Not only are there obvious peaks whose statistical significance is hard to establish but a periodicity in the redshifts appears to the present (Burbidge and O'Dell, *Astrophysical Journal* **178,** 583 (1972)).

DISTRIBUTION OF QUASARS IN THE UNIVERSE

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We assume that the redshifts of quasars are of cosmological origin. Each quasar redshift then yields a distance and we will discuss how the distribution of quasars in space can be derived from well-defined quasar samples. Our aim is to find the luminosity function

$$\Phi\ (F_{opt}, F_{rad}, z)$$

representing the number of quasars per unit co-moving volume of space of optical luminosity F_{opt} and radio luminosity F_{rad} at redshift z. The small amount of statistically usable data allows only a rough derivation of the luminosity function and we shall review in this lecture the present situation.

We use a Friedman model of the universe with a deceleration parameter $q_o = 1$. In this model the bolometric flux f_{bol} depends on redshift z simply as $f_{bol} \sim F_{bol}\ z^{-2}$. Our results depend little on the value of q_o, reflecting the fact that the observable properties of quasars are mostly determined by their intrinsic properties and relatively little by the geometry of the universe.

Quasars are easiest found as *quasi-stellar radio sources* on the basis of coincidence of position of the radio source and an object of stellar appearance Confirmation is obtained if the optical spectrum shows. emission lines with a substantial redshift. All quasars so detected (except some of those with redshifts larger than 2.5) have an ultraviolet color U-B < 0.4. This property is used to find *quasi-stellar objects*, solely on the basis of their optical properties. Spectroscopy is required, again, to confirm the quasar nature of every candidate object.

The statistical discussion of the purely optically selected quasi-stellar objects is simpler, since only f_{opt} and z enter as observables. Of essential importance are the counts of quasars, n (f_{opt}), as a function of their optical flux. Braccesi, Formiggini, and Gandolfi (1970) have given a list of 175 objects with ultraviolet excess over an area of 37 square degrees. Redshifts observed by R. Lynds were given for a number of the objects. Inspection reveals that the spectroscopic work is essentially complete to the magnitude $b = 18.4$ (corresponding to $B \simeq 18.3$), except for objects that have colors unlike those of known quasars.

An estimate of the number of quasars among the 119 objects with $18.4 < b < 19.4$ may be made as follows (see Table 1). First we eliminate 16 objects that have (b-v) and (v-i) colors unlike those of quasars. Then we estimate the number of objects whose colors are acceptable yet are galactic stars. For $b < 18.4$ the complete spectroscopic work shows very few of such objects and the extrapolation to the next magnitude interval is not very sensitive. The expected number of quasars with $18.4 < b < 19.4$ is around 100. The increase per magnitude is by a factor of 5 or 6 as shown in Table 1.

TABLE 1

Statistics of Braccesi Objects with Ultraviolet Excess

	$b \leqq 16.4$	16.4 — 17.4	17.4 — 18.4	18.4 — 19.4
Total number	2	12	25	119
Non-quasar colors:	1	6	4	16
Quasar-like colors but star spectrum:	1	3-2	3-1	3-1 (extrapolated)
Quasars:	0	3-4	18-20	100-102 (inferred)

The above gradient of the counts is larger than that derived by Setti and Woltjer (1972) from the same material, but without regard of the color information. They limited their discussion to objects with $b < 19.1$ so as to avoid uncertainties in the magnitudes at the faint end. A re-derivation of the expected numbers, including color information as in Table 1, yields for $b < 17.1$, 17.1-18.1, 18.1-19.1: 0-1, 15-16 and 54-55 quasars. These counts show a smaller gradient than those of Table 1. If we use cumulative counts then we get $N\,(< 19.1)/N\,(< 18.1) = 4.4$ whereas Table 1 yields $N\,(< 19.4)/N\,(< 18.4) = 5.5$. The statistical uncertainty in the ratio is around 1, at least, and therefore the difference is not unexpected. We conclude that the gradient is a factor of around 5 per magnitude.

If quasars were local objects with a uniform distribution in space then their counts should increase by a factor of 4 per magnitude. This would not be in serious conflict with the above data and hence no statement about the nature of the redshifts can be made yet on the basis of quasar counts.

Next we discuss information concerning the statistics of redshifts. Such information is obtained from the study of samples of quasars. These samples must be complete in the sense that down to a given magnitude limit over a particular area of the sky the redshifts of (almost) all quasars have been obtained. We list in Table 2 the redshift distribution of quasars in the magnitude interval V = 17.4 — 18.4. For brighter quasars the samples are too small to yield useful results. The SLS entry in Table 2 refers to quasars in the Sandage-Luyten 0^h and 8^h

TABLE 2

Redshift Distribution of 18th Magnitude Quasars

log z	log F_{opt}	SLS	BL	3CR
0.2 — 0.4	23.6 — 24.0	5	4	3
0 — 0.2	23.2 — 23.6	7	2	7
— 0.2 — 0	22.8 — 23.2	2	2	6
— 0.4 — — 0.2	22.4 — 22.8	3	2	2
— 0.6 — — 0.4	22.0 — 22.4	0	3	1
< 0.6	< 22.0	2	4	0
unknown z		1	1	1

fields (Schmidt 1970). The BL column based on Lynds' redshifts of Braccesi quasars actually refers to the magnitude interval V = 17.2 — 18.2 which should make little difference. The third column gives data for a sample of 3CR quasi-stellar sources (Schmidt 1970) augmented with the quasar 3C 220.2 (Wills and Lynds 1972). We are leaving out of consideration quasars with log $F_{opt} < 22.0$ (F_{opt} is expressed in W m^{-2} Hz^{-1} at 2500 Å emitted wavelength), since these are not

TABLE 3

Redshift-Magnitude Table for Quasars
(Density law $\rho = 10^{5\tau}$; numbers refer to whole sky)

log z	ΔV (Gpc^3)	τ	m = 15	16	17	18 X1000	19 X1000
+ 0.5	25	0.899					12
+ .3	21	.832				5	25
+ .1	14	.738			1214	6	30
— .1	8.5	.622		202	1000	5	26
— .3	4.2	.496	23	116	580	3	10
— .5	1.75	.373	12	58	300	1	
— .7	0.63	.268	6	33	109		
— 0.9	0.21	.185	4	13			
— 1.1	0.061	0.124	2				
			47	422	3203	20,000	103,000

more luminous than the brightest elliptical galaxies. Considering the statistical uncertainties of the small numbers in Table 2 the different data seem to be in mutual agreement. (The fact that this applies to the radio-selected 3CR quasars, too, will be discussed presently). We adopt a fractional distribution (0.25, 0.35, 0.25, 0.15, 0.05) along a column of Table 2.

We are now in a position to construct a redshift-magnitude table, which represents the frequency distribution over the redshift-magnitude diagram of a complete sample of objects. Table 3 shows such a table, where columns correspond to intervals of one magnitude, and rows to $\Delta \log z = 0.2$. Objects of the same absolute luminosity F_{opt} will be situated along a diagonal rising from left to right. Their numbers along the diagonal will increase proportional to $\rho\,(z, F_{opt}) . \Delta v\,(z)$ where Δv is the volume (in co-moving coordinates) of the shell of thickness $\Delta \log z = 0.2$ and $\rho\,(z, F_{opt})$ is the space density of these objects.

Since we have from Table 2 information about the relative entries along the column under 18^{th} magnitude we can derive all the other entries in the table as soon as $\rho\,(z, F_{opt})$ is established. In particular, we can do so for a uniform space distribution, $\rho \equiv 1$. Summing the columns vertically will then yield the quasar counts, and we find easily $n\,(19^m)/n\,(18^m) = 1.6$. This result is inconsistent with the value of 5 derived above. We conclude that quasars cannot have a uniform distribution in space.

Let us assume that the density distribution of quasars of different luminosity F_{opt} is the same and that it has the form $\rho = (1 + z)^x$ or, alternatively, $\rho = 10^{y\tau}$ The first law is a power law of the radius of the universe (which is proportional to $(1 + z)^{-1}$), while the second is an exponential law of lighttravel time τ. Trial and error in a redshift-magnitude table such as Table 3 then shows that $x = 6$ or $y = 5$ so as to reproduce the observed $n\,(19^m)/n\,(18^m) = 5$. Although the two density laws are quite different in shape there is no way to discriminate between them for lack of further observational data. Both laws predict very high quasar densities at early cosmic epochs. At $z = 2$, corresponding to $\tau = 0.832$, the power law gives $\rho \sim 700$ and the exponential law 14,000, both expressed in terms of the local density $\rho\,(0) = 1$.

Next we turn to the statistics of quasars found in radio source catalogues such as the 3CR catalogue. These identifications can only be made, of course, if the object has a minimum optical brightness. Hence for radio detected quasars two selection effects, one radio and one optical, operate simultaneously and the situation is much more complex than that discussed above for the purely optically detected quasars. Yet, it has been possible to test in a straightforward manner whether or not the 3CR quasars have a uniform space distribution.

In the V/V_m-test (Schmidt 1968) each object in the 3CR sample (which is complete brighter than $18^m.4$ and above 9 flux units at 178 MHz) is hypothetically moved out until at a redshift z_m it drops out of the sample, either because it becomes fainter than $18^m.4$ or weaker than 9 flux units at 178 MHz. Now let the volume of the universe out to redshift z be V. Then the object has available to it the volume $V(z_m) = V_m$ within which it could be a member of the sample. The test for uniformity is that the sample members should show a uniform distribution of V/V_m between 0 and 1, and the average should be 0.50.

Application of the V/V_m-test to the 3CR quasars showed (Schmidt 1968) that only 6 of 33 quasars had $V/V_m < 0.50$, and the average V/V_m was 0.70 ± 0.05. Hence, for 3CR quasars too, the space density must increase with increasing redshift. Density laws can be tested by requiring that $<V'/V_m'> = 0.50$ where V' now stands for density-weighted volume. It turns out that the indicated density laws found above, $\rho = (1 + z)^6$ and $\rho = 10^{5\tau}$, can represent the distribution of 3CR quasars. Lynds and Wills (1972) in a similar study of 4C quasars found $<V/V_m> = 0.67 \pm 0.06$, but they preferred to use a density law $\rho = V^a$ which leads to zero density locally, i.e. at the present cosmic epoch.

We have seen that optically detected quasars (which usually have a very low radio flux if they are detectable at all) and 3CR radio quasars each have a density law $\rho = (1 + z)^6$ or $\rho = 10^{5\tau}$. Even though it is possible that the one law applies to the optically selected quasars and the other law to the radio quasars, this seems rather artificial and instead we assume that both kinds of quasars obey the same density law. This means that the density law is essentially independent of the radio luminosity F_{rad}. We have assumed, but this remains unchecked, that the density law also does not depend on F_{opt}. Hence our general luminosity function $\Phi(F_{opt}, F_{rad}, z)$ now takes the form $\Phi(F_{opt}, F_{rad}) . \rho(z)$.

The distribution of redshifts of 18^m 3CR quasars is not significantly different from that of optically selected quasars as seen in Table 2. This is inconsistent with the hypothesis that $\Phi(F_{opt}, F_{rad}) = \Phi(F_{opt}) \psi(F_{rad})$, i.e. that the radio luminosity function is independent of F_{opt}. Consider all the 18^m quasars with $\log z = 0.2 - 0.4$. These are very distant and only those with very large F_{rad} would be detected as a 3CR source above 9 flux units. Next take the 18^m quasars with $\log z = -0.6 - -0.4$. These are relatively nearby and F_{rad} need to be less large for inclusion in the 3CR catalogue. In brief, one would expect increasing discrimination against radio detection as one moves from low to high redshifts in Table 2, contrary to the evidence given in the table.

The solution proposed (Schmidt 1970) for this dilemma is that the radio luminosity function depends on optical luminosity F_{opt} in such a way that the discrimination discussed above is cancelled. It turns out that this is exactly the case if

$$\Phi\,(F_{opt}, F_{rad}) = \Phi\,(F_{opt})\,\psi\left(\frac{F_{rad}}{F_{opt}}\right),$$

i.e., if the distribution function of radio luminosities contains the ratio of radio and optical luminosities. Each column of the redshift-magnitude table of radio-selected quasars will then have the same relative distribution as that for optically selected quasars in Table 3. The total numbers in each column will be a fraction

$$G\,(R_{min}) = \int_{R_{min}}^{\infty} \psi\,(R)\,dR$$

of those given in Table 3.

Here $R = \dfrac{F_{rad}}{F_{opt}} = \dfrac{f_{rad}}{f_{opt}}$ and $R_{min} = \dfrac{f_{rad,\ min}}{f_{opt}}$ where $f_{rad,\ min}$ corresponds to the limiting flux density of the radio catalogue or survey. The function ψ (R) is, in fact, determined by comparing optical counts of radio quasars and optically detected quasars (see Schmidt 1970).

It appears, then, that the luminosity function of quasars may have the form

$$\Phi\,(F_{opt})\,\psi\,(F_{rad}/F_{opt})\,\rho\,(z).$$

For further discussion, see Schmidt (1972a, b) where Φ and ψ have been derived. The local total space density of quasars is 10 Gpc^{-3} for the density law $\rho = 10^{5\tau}$ and 180 Gpc^{-3} for $(1 + z)^6$. At a redshift of 2.5, $\tau = 0.867$, the density of quasars is around 2×10^5 Gpc^{-3} for either density law. It is of interest that at this early cosmic epoch quasars were about as numerous as giant elliptical galaxies (whose density is around 10^5 Gpc^{-3}) assuming that the density of the latter does not change with time.

We turn now to the important question of the frequency of large redshifts among quasars. For a number of years the largest observed redshift was 2.2 and the question arose whether there existed perhaps a cut-off in quasar redshifts at a value around 2.5. Subsequently, a redshift of 2.88 was found and this year two larger redshifts have been obtained, 3.40 for OH 471 (Carswell and Strittmatter 1972) and 3.53 for OQ 172 (Wampler, Robinson, Baldwin, and Burbidge 1972). These two cases show that quasars with larger redshifts are observable but the question remains whether or not their relative scarcity is significant.

Table 3 predicts that 12 per cent of the quasars of magnitude 19 should have redshifts in the range 2.5 — 4. At magnitude 20 the expected percentage is 14. Since no substantial complete sample of 19th magnitude quasars exists, we have to rely on quasars in non-complete samples. Large redshifts are probably well represented in these since they are determined from strong emission lines such as Lyman-alpha. Medium redshifts are probably under-represented in incomplete samples as they are determined from weaker lines. We will assume that the fraction of high redshifts among faint quasars is relatively unbiased.

Unfortunately, the number of redshifts of optically selected quasars fainter than $18^m.5$ is less than ten. Moreover, there are reasons to believe that the optical selection process, through the ultraviolet excess, discriminates against large redshifts: some of the known large-redshift quasars show the ultraviolet excess, others do not.

Hence, we turn to radio quasars fainter than $18^m.5$. For these, discrimination against large redshift will be much less since especially with accurate modern radio positions the color of the optical candidate is given little weight. Also, the color would usually be judged from the *Palomar Sky Atlas* blue and red charts (at effective wavelengths around 4000 and 6500 Angstroms). Only at a redshift around 4 would Lyman-alpha register on the red plates and make the object look red. I am aware of some 25 published and unpublished redshifts of radio quasars fainter than $18^m.5$. Among these, two have redshifts exceeding 2.5, one of them being OH 471 with $z = 3.40$. From our estimate based on Table 3 of 13 per cent we would have expected 3 redshifts larger than 2.5. We conclude that there is no significant deficiency of observed redshifts in the range 2.5 — 4.

It is worth emphasizing at this stage that we have relied heavily on the proposition that the redshift distribution of 19th magnitude quasars is the same for radio selected and optically selected quasars. As discussed above, this is indeed observed for 18th magnitude quasars and it will be the case at 19th magnitude if our interpretation in terms of a radio luminosity function of the form $\psi\ (F_{rad}/F_{opt})$ is correct.

Our discussion if the statistics of quasars with $z = 2.5 - 4$ was based on Table 3 which corresponds to the density law $\rho = 10^{5\tau}$. For the alternative density law $\rho = (1 + z)^6$ the expected number of $z = 2.5 - 4$ quasars would have been 9 (out of 25), quite inconsistent with the observations.

Several other quasars are known to have redshifts larger than 2.5 but they are brighter than $18^m_{\cdot}5$. They would fall above the entries in Table 3 in which they are not present since the luminous quasars in question are too rare to be well represented in the few complete samples. Much further statistical work will be needed to elucidate the position of these superluminous quasars.

Finally, I wish to mention the discussion of redshiftmagnitude diagrams of quasars by Setti and Woltjer (1973). They presented separately diagrams for radio quasars with steep radio spectra, radio quasars with flat spectra, and optically detected quasars. The diagram for steep-spectrum radio quasars conforms in shape to Table 3. However, the diagram for flat-spectrum quasars and optically detected quasars shows a steeper slope of the upper envelope due to an excess of relatively bright quasars (around 17^{m}) with large redshifts (around 2). Setti and Woltjer suggest that the optical luminosity function of flat-radio spectrum and optically selected quasars may be broader than that of steep-spectrum quasars. This might be related to a difference in the optical variability of the two kinds. It should be remembered that Setti and Woltjer used all quasar redshifts available in the literature, hence the sample may well suffer observational selection effects. Spectroscopic work on a complete sample of quasars selected from a high-frequency radio catalogue (most of which exhibit flat radio spectra) is in progress and work on further samples of optically selected quasars is nearing completion.

Much further observational work is needed to further elucidate quasar statistics. It is especially necessary that a substantial sample of bright quasars be isolated so that the relevant entries in Table 3 can be determined to check upon or to improve the density law. For this purpose a search is being undertaken based on blue-ultraviolet double exposures with the 18-inch Schmidt telescope at Palomar covering about one-quarter of the sky. Most of the objects with ultraviolet excess to be found will unavoidably be white dwarfs and hot stars belonging to our Galaxy. The extensive photometric and spectroscopic work is expected to take several years.

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Discussion of the report of M. Schmidt

E. P. J. Van Den Heuvel: Do I understand it right from your talk that it is no longer terribly important to keep studying quasars and looking for quasars with radiotelescopes, but that the main burden for investigating quasars and their space distribution is now on the optical astronomers as F_{opt}/F_{radio} seems to be independent of Z?

M. Schmidt: We badly need confirmation that F_{opt}/F_{radio} is really independent of Z. This is best done by checking whether the redshift distribution of radioselected quasars is the same as that of optically selected quasars for optical magnitudes other than 18, at which the effect was originally found. This does require large numbers of radio sources with accurate positions that allow optical identification.

Y. Ne'Eman: In the lagging-core model which Novikov and I suggested in 1964-5 (and assuming the arguments given by Zelkovich at Crakow have not excluded it - I do not know what these arguments were) one gets such an exponential law. The rate of quasars popping-up out of the " closed " post-singularity state is proportional to the available material, and one gets a simple exponential decrease. The $\tau = 10^9$ years seems appropriate. (see Y. Ne'eman, Proceedings of the IAU Boulder Symposium, 1972, pub. by Reidel). Of course, the existence of many such lagging cores with supergalactic mass would also help in making the universe's energy density larger, closer to the critical value.

THE MASSES OF THE GALAXIES AND THE MASS-ENERGY IN THE UNIVERSE

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THE MASSES OF GALAXIES

The only direct method of determining total masses or mass distributions within galaxies is to consider the acceleration of stars, gas, or whole galaxies under the action of the gravitational field exerted by the object to be studied. Many galaxies occur as binary systems; for these, a sample of average total masses can be determined. Galaxies also tend to occur in clusters ranging in size from small groups with only a few members, to large clusters like the Coma Cluster, and if a cluster is assumed to be in a stationary state, the virial theorem can be used to estimate average masses of the galaxies in it.

The method that has had the widest application for external galaxies is that in which the rotational velocity of one constituent subsystem of the galaxy is measured (e.g. the stars or the neutral or ionized gas component), as a function of distance from the center of the galaxy. This has the great advantage of giving mass distributions, which, upon being integrated, yield total masses, so one obtains more information than by those methods that yield only total masses.

Certain basic assumptions are made in the study of mass distributions from rotations of galaxies. Accelerations can never be measured directly in a galaxy; they must be deduced from observations of the velocity field at one instant of time. Velocities must therefore be assumed to be independent of time, or at least, only slowly changing with time, i.e., the galaxy is assumed to be in a steady state. Further, in all galaxies but our own, three-dimensional velocities cannot be determined for any of the constituents, because only the line-of-sight velocity component can be measured. Therefore it is necessary to assume axial symmetry of the galaxy. Fortunately, in regular normal spirals and SO systems this appears to be a valid assumption. It is clearly not valid for barred spirals and irregulars, which present a difficult problem. Barred spirals can be assumed to have two-fold symmetry about an axis perpendicular to their equatorial planes, while all spirals have complete rotational symmetry with respect to their axes.

Finally, it is assumed that Newtonian gravitation is the dominant force, so that, specifically, even in the case of the ionized gas component, magnetic forces are neglected.

For galaxies which do not appear to have much angular momentum, mass determinations can be made by measuring the velocity dispersion of the stars in the central region and estimating the mass from the virial theorem. Most of the mass determinations for elliptical galaxies have been made by this method.

A detailed account of the way in which the various methods of mass determination have been applied, and the results which have been obtained, has been given by Burbidge and Burbidge (1969).

The results are that the average masses of spiral galaxies are about 3×10^{10} $M_\odot$, and those of ellipticals are more than an order of magnitude higher, namely about 4×10^{11} $M_\odot$. Very recently Ostriker and Peebles (1973) have discussed the problem of understanding the stability of the disk of our Galaxy and have argued that this may require the existence of a fairly massive halo component. By analogy with this, it has been suggested that other galaxies may also have considerable masses in their halos, which have escaped detection, so that the total masses might be considerably greater than those given above. If the mass is distributed in an extended halo about a spiral galaxy, it is possible that it would not be detected from a study of the rotation. However, studies of double galaxies lead to the measurement of average masses which must include all of the mass lying within the diameter of the orbit of the double system. Since there is no significant difference within a factor of 2 between the average masses obtained using the double galaxy method and those obtained from averaging individual mass determination, it appears unlikely that a large amount of halo mass has so far remained undetected in galaxies in general.

Thus, these average masses can be used to make realistic estimates of the total mass density in the universe in the form of luminous galaxies. Such estimates have been made over many years by Oort (1958), Burbidge and Burbidge (1959) and others, and most recently by Noonan (1972). Following the work of Sandage (1972) and of Abell and Eastman (1968) we put the Hubble constant $H_o = 50$ km sec^{-1} Mpc^{-1}, and following Noonan and allowing for some uncertainties, we find that the mass density in the form of visible galaxies ρ_g lies in the range 7-13 $\times$ 10^{-32} gm cm^{-3}, corresponding to a particle density of 4-8 $\times$ 10^{-8} cm^{-3}.

G.R. BURBIDGE

THE MISSING MASS

In the popular Friedmann cosmological models, the mean density of mass energy in the universe, ρ, is related to the other cosmological parameters by the relation

$$\rho = \frac{3H_o{}^2 q_o}{8\pi\, G} \,.\, (\Lambda = 0)$$

The critical density corresponding to closure of the universe is obtained when $q_o = 1/2$. For $H_o = 50$ km sec^{-1} Mpc^{-1}, $q_o = 1/2$, $\rho_c = 4.7 \times 10^{-30}$ gm $cm^{-3} = 3 \times 10^{-6}$ particles cm^{-3}. This is also the mass energy density required in Hoyle's formulation of the steady-state cosmology.

It is the discrepancy between this value and the density in the galaxies (a factor of 30-60) which has given rise to much speculation concerning the possible existence of mass in forms other than that in the galaxies.

In what follows I shall discuss the various components which may contribute to the total mass energy. However, it should be emphasized that if we do not live in a steady-state universe of the type described by Hoyle, or if we live in a Friedmann universe with $0 \leq q_o \leq 1/2$, there may not be a large difference between the mass in galaxies and that in the universe.

In an earlier paper prepared for I.A.U. Symposium No. 44 (Burbidge 1972) I gave an extensive discussion of the evidence for intergalactic matter and radiation as it appeared in 1970. More recently Field (1972) has also reviewed the evidence for the presence of intergalactic matter. The first part of this paper is therefore concerned with the up-dating of these earlier discussions.

It is convenient to discuss the evidence under several headings:

(1) Neutrinos
(2) Relativistic Particles
(3) Gravitational Radiation
(4) Fields of Electromagnetic Radiation
(5) Diffuse Gas
(6) Mass Condensations.

NEUTRINOS

As was pointed out in the earlier review (Burbidge 1972) the experimental limits placed on the energy density of low-energy neutrinos from the cut-off in the β-decay spectrum, are not severe. They are such that the neutrino energy density could still be many orders of magnitude above the critical density and not have been detected.

RELATIVISTIC PARTICLES

We really have little idea as to what the energy density of relativistic particles amounts to. If cosmic rays are largely confined to galaxies, the mean energy density in the universe is only about 10^{-16} erg cm^{-3} $\simeq$ 10^{-37} gm cm^{-3}. If, on the other hand, the cosmic rays are universal with an energy density of 10^{-12} erg cm^{-3}, then the equivalent mass density $\simeq 10^{-33}$ erg cm^{-3}.

If the bulk of the cosmic rays has an extragalactic origin, it is likely that they are confined to clusters and superclusters, so that the universal energy density will be about 10^{-14} erg cm^{-3} $= 10^{-35}$ gm cm^{-3} (Brecher and Burbidge 1972a). Even the higher estimates only correspond to very small mass densities which do not have any appreciable effect on the evolution of the universe.

The only point of some importance is that, if diffuse matter is present at a density $\gtrsim 10^{-30}$ gm cm^{-3} in an evolving universe, its interaction with a cosmic-ray flux with an energy density $\geq 10^{-12}$ erg cm^{-3} would give rise to a γ-ray flux greater than that detected. Thus we cannot have universal cosmic rays and a gas filled universe.

GRAVITATIONAL RADIATION

Over the last several years Weber's results suggesting that large amounts of mass $\geq 10^3$ M$_\odot$ year^{-1} are being radiated in the form of gravitational waves from the galactic center, have led to speculations that during their evolution galaxies might radiate a large amount of mass. However, it is proving difficult to confirm Weber's results, so that there is a serious question as to whether or not gravitational radiation has been detected.

However, Rees (1971) has argued that early in an evolving universe there may have been a large amount of energy converted into long-wavelength gravitational waves, and that this may be present at critical density.

FIELDS OF ELECTROMAGNETIC RADIATION

There are four distinguishable components of electromagnetic radiation in the universe. They are:

(a) starlight;
(b) nonthermal radio emission;
(c) diffuse X-ray and γ-ray background;
(d) microwave background radiation.

The best estimate of the energy density of starlight is still that given by Felten (cf. Burbidge 1972) of about 1.5×10^{-14} erg cm^{-3} (1.5×10^{-31} gm cm^{-3}). The energy densities of nonthermal radio emission and of the X-ray background are exceedingly small, but the X-ray background may provide evidence for the existence of hot gas. We shall discuss this later.

As far as the microwave background is concerned, there have been some recent developments. While since 1965 most people have thought it likely that the microwave background radiation was generated in a hot big bang, the evidence that this is the case has not been unambiguous. This is due to the fact that while the measurements in the centimeter range fall very well on the Rayleigh-Jeans part of a black-body curve with a temperature of about 2.8 °K, there has been considerable confusion as far as the measurements close to the peak of such a black-body curve near 1 mm are concerned. The indirect measurements using CN, CH and CH^+ all were compatible, or indicated a maximum near 1 mm, but the balloon and rocket measurements were in conflict with these, and in some cases with each other, indicating the existence of a much higher flux of radiation than that from a black-body radiation field (for references see Burbidge 1971). It was these high measurements that led to the distinct possibility the radiation might not have arisen in a big bang[1], but that very large numbers of weak discrete sources were involved.

However, in the last six months rocket results from Los Alamos (Williamson, Blair, Catlin, Hiebert, Loyd and Romero 1973) and from Cornell (Houck, Soifer, Harwit and Pipher 1972) and balloon results from M.I.T. (Muehlner and Weiss 1973), the last two groups being those who had earlier reported high fluxes, do not indicate the presence of any flux higher than that expected from a black-body radiation field at a temperature of about 2.8 °K. If some of the earlier high measurements were correct at all, this flux must be variable and hence very local and not cosmological in origin.

DIFFUSE GAS

The most interest continues to center on the possible existence of low density gas both in the outer parts of galaxies, in clusters of galaxies, and possibly even between clusters of galaxies. I last reviewed this situation some three years ago, and at that time stressed that there

[1] It should be stressed that the only really strong evidence for an evolving universe is the existence of a background flux which appears to have a black-body energy distribution.

was no really compelling evidence for the existence of any appreciable amounts of gas anywhere outside galaxies. Field (1972) in his recent review has covered much of the same ground taking into account the most recent X-ray data (up to 1971) and has given a slightly more optimistic discussion of the problem. I shall now further update the discussion.

Detection of Neutral Atomic Hydrogen

The methods that have been used are to attempt to detect 21-cm absorption or emission in the spectra of radio galaxies or QSOs, or to detect Lyα absorption in the spectra of QSOs. As far as the 21-cm results are concerned, there is nothing new to report. The most reliable results are those of Penzias and Scott (1968) $n_H/T_E < 1.8 \times 10^{-8}$. Field (1972) concludes that $T_E \simeq 18^o$, so that $n_H < 3.2 \times 10^{-7}$ cm^{-3}, compared with a critical value of 2.8×10^{-6} cm^{-3}. The limit based on the absence of a step longward of 1420 MHz which would be due to intergalactic emission is a weaker one and is about 4.7×10^{-6} cm^{-3}. These results suggest that the neutral gas, smoothly distributed, can at most be about 10 times the mass in galaxies. A value close to this would imply a value of $q_o \simeq 0.2$, which is certainly not ruled out when the real uncertainties associated with the attempts to determine q_o by the direct method are evaluated.

I stress this result because it is the most certain upper limit to the density of either neutral or ionized gas in the cosmos.

We turn next to the method based on the attempts to detect Lyα absorption in the spectra of QSOs with redshifts $\gtrsim 1.8$. I shall not repeat a discussion of the history of these attempts which have been described elsewhere (Burbidge and Burbidge 1967; Burbidge and Burbidge 1969; Bahcall 1971). No evidence of an absorption trough in a QSO attributable to Lyα extending from the blue wing of Lyα emission to the atmospheric cutoff has ever been found, though a very large number of QSOs with large redshifts have by now been studied. As was stated in my previous review, either this means that the QSOs are not at cosmological distances, and this question is discussed elsewhere in these Conference Proceedings, or the gas which is smoothly distributed is highly ionized. There is also the very remote possibility, discounted by almost everyone, that the space between the galaxies is essentially devoid of gaseous matter, $n\,(z \simeq 2) \leq 1.5 \times 10^{-11}$ cm^{-3}, so that all of the matter is condensed into discrete objects.

I consider it very likely that the QSOs are comparatively close by, and if this is true we can get no information about the presence or absence of intergalactic gas from studying them. However, the majority has not yet accepted this view, and the absence of the Lyα trough marked the beginning of the many studies in which it was argued that the intergalactic gas must be very hot. We shall describe the evidence bearing on this possibility later.

What if the gas in intergalactic space is condensed into clouds? In this case, there is the possibility that the absorption spectra of typical clouds could be detected in the spectra of cosmologically distant QSOs.

It is well known that many QSOs with very large emission-line redshifts have rich absorption-line spectra. Since 1966 the question has been asked whether the absorption features arise in, or very close to the objects themselves, or whether they arise in the intervening medium. In addition to the optical observations, very recently Brown and Roberts (1973) have detected the first example of 21-cm absorption in the radio spectrum of a QSO. The object is 3C 286 which has an optical emission-line redshift $z \simeq 0.85$ while the absorption feature, it it has a rest wavelength of 21 cm, has a redshift of 0.69.

The general characteristics of the optical absorption spectra are:

(1) The majority of the absorption redshifts are very close to but less than the emission redshifts. However, some are very different.

(2) The QSOs which have absorption lines tend to have multiple absorption-line redshifts.

(3) Many of the absorption lines are exceedingly sharp; in the case of PHL 957 the line widths are $\sim$ 30 km sec^{-1}, and the width of the 21-cm line in 3C 286 is less than 8.2 km sec^{-1}.

(4) In some objects, notably PHL 5200 and PHL 957, there are very broad absorption features. In them line widths of $\sim$ 200 Å are found.

(5) There is considerable evidence that different redshift systems in some QSOs are connected by a line-locking mechanism suggesting that shells of gas are ejected from the QSOs by radiation pressure.

In her contribution M. Burbidge (p. 451) has discussed the question of whether or not any of the absorption features can be attributed to intergalactic clouds, and she has concluded that this is most unlikely. If this is the case, then independent of the arguments surrounding the problem of the distances of the QSOs, no information can be obtained about the intergalactic medium from these studies.

Detection of Highly Ionized Gas

The best chance of detecting ionized intergalactic gas would occur if it had a temperature high enough so that it emits thermal bremsstrahlung, with photon energies high enough that they are not appreciably absorbed by gas in our Galaxy. This means photons with energies $\geq$ 0.2 keV, or gas temperature $\geq 2 \times 10^6$ degrees.

Over the last few years a diffuse X-ray flux has been detected over an energy range from $\sim$ 0.25 keV to a few MeV. Whether any of this flux is thermal bremsstrahlung from hot intergalactic gas is still a matter of dispute. I shall discuss first the soft X-ray flux with energy near 0.25 keV and then the possible origin of the harder X-rays. Both Field (1972), Felten (1973), and most recently Silk (1973), have surveyed all but the most recent data.

(i) *Possible Origins of the Soft X-Ray Flux* — It seems likely that there are at least two components of the soft X-ray flux, one arising in the Galaxy and the other generated outside. Much attention has been paid to various kinds of discrete source models which could account for the Galactic component, but this will not concern us here.

Felten (1973) and Hayakawa (1973) have concluded from a survey of the observations (some of which are in conflict), that the flux arising outside the galaxy in the direction of the poles $\sim$ 500 photons (cm^2 sec sr keV)$^{-1}$. It may be a real extragalactic flux coming from great distances. The strongest objection to this hypothesis is the fact that the Wisconsin group (McCammon *et al.* 1971) have failed to observe absorption of this flux by the Small Magellanic Cloud. Their result suggests either that the flux arises between us and the Clouds, perhaps suggesting a hot halo model, but no extensive extragalactic component, or that sources in the SMC fill up the hole caused by absorption of extragalactic flux coming from that direction. This latter proposal does not appear likely, but on the other hand it is difficult to explain all of the other observations without invoking the presence of some extragalactic component. At the same time Kraushaar (1973) has pointed out that if we accept this explanation, it implies that normal galaxies could explain the whole of the observed background if they radiate at the level required of the Small Magellanic Cloud. Consequently, the extragalactic component would arise in normal galaxies and not in the extragalactic medium. Thus, even if it is present, it does not necessarily imply the existence of hot intergalactic gas. Flux at this level of intensity would be radiation by a hot intergalactic gas distributed uniformly, with $T \sim 4 \times 10^6$ degrees and a density approximately 0.3 of the critical density, i.e., $n_e = 10^{-6}$ cm^{-3}. Of course, if the sources arise in a superposition of discrete components involving hot gas, perhaps gas in clusters of galaxies (for later discussion) the mean density of the gas will be correspondingly lower. (If the gas only fills 1 % of the total volume, the density in this volume must be $\sim$ 3 times the critical density but the mean density will be only 3 % of the critical density). Local models in which it is supposed that the flux arises in a hot halo, or hot gas in the Local Group ($R \sim 1$ Mpc, $n \approx 3 \times 10^{-4}$ cm^3, $T \simeq 10^6$ degrees) (Hunt and Sciama 1972) are also tenable.

With all of the present uncertainties, we see that the soft X-ray flux cannot be taken as a very reliable indicator of the presence of hot intergalactic gas.

(ii) *Possible Origins of the X-Ray Background with Photon Energies* ≥ 1 *keV* — There is no question but that a highly isotropic X-ray flux with photon energy 1 keV $\leq$ E $\leq$ 100 MeV is present and has a truly extragalactic origin. The only mechanisms which are likely to be responsible for generating this flux are Compton scattering of relativistic electrons on the microwave background radiation, or thermal bremsstrahlung from a very hot gas, and at the high energy, end π^0 decay. There are several important questions that need to be answered before we can decide how important the X-ray background is from the point of view of studies of intergalactic matter. Not only must we identify the production mechanism, but we must also ask whether the background radiation is likely to be made up of discrete sources of known types.

As far as the mechanism generating the hard photons is concerned, the most important point is the shape of the spectrum. The proposals of Felten and Morrison (1966) and of Brecher and Morrison (1969) were that the Compton scattering of relativistic electrons on the microwave background was likely to be responsible. However, as was pointed out by Cowsik and Kobetich (1972), if there is a sharp enough bend in the spectrum, and there is some indication of a change of slope near 40 keV, there may be some difficulty, if the relativistic electron spectrum does not have a bend as an intrinsic property. At the same time Brecher (1973) has correctly pointed out that on the basis of what is presently known about synchrotron radio spectra and their parent electron spectra the existence of bent electron spectra is not excluded.

On the observational side, it is clear that nothing can be concluded for certain until a very well determined spectrum of the background flux is measured. It still appears possible that there is very little change in the slope between $\sim$ 10 keV and $\sim$ 10 MeV (Pal 1973; Pinkau 1973), and if this is correct, the Compton explanation is adequate and very plausible. However, despite the uncertainties in the data, many theoreticians and observers seem to have largely excluded the Compton explanation and have turned to the thermal bremsstrahlung model:

Field (1972) recently concluded that the present observational data is not too far in disagreement either with a hot gas model (uniform density) in a Friedmann universe (big bang model) with an intergalactic gas temperature of 2×10^8 degrees (putting $\gamma = 5/3$ and adiabatic expansion since $z \simeq 1$ and closure density) or a Gold-Hoyle steady-state model ($T \simeq 6 \times 10^8$ degrees).

More recently, the results summarized by Schwartz and Gursky (1973) modify Field's conclusion and in terms of a hot gas model in a Friedmann universe, they argue that the density is less than the closure density by a factor $\sim \sqrt{2}$, or that the temperature $\leq 3 \times 10^7$ degrees, or that the Hubble constant is only about 50 km sec^{-1} Mpc^{-1}.

To summarize, with considerable uncertainty, the existence of the diffuse X-ray background can be adduced to be evidence in favor of the existence of very hot intergalactic gas, but in no sense do the observations prove that such a gas exists.

Even if the radiation is thermal bremsstrahlung, it is possible that the diffuse background is simply a sum of the radiation from discrete sources. We know that various types of extragalactic objects are discrete X-ray sources, though at present only a small sample of different types has been identified. The most prominent discrete sources are rich clusters of galaxies, but an attempt to add up the contributions from normal galaxies, radio galaxies, rich clusters, Seyfert nuclei, and QSOs led Schwartz and Gursky (1973) to the conclusion that they can only explain about 22 % of the background from discrete sources, and the bulk of this is due to Seyfert galaxies based on a sample of one, NGC 4151.

Other Observational Evidence for Gas in Clusters of Galaxies

It has just been pointed out that a number of rich clusters of galaxies have been identified as extended X-ray sources (cf. Kellogg *et al.* 1973). They may either be radiating X-rays which are due to Compton scattering, or they may be due to thermal bremsstrahlung, thus indicating the presence of gas in the clusters. Practically all of the clusters which have so far been identified also contain active radio galaxies so that, as was proposed by Brecher and Burbidge (1972b) [see also Burbidge (1973)], Bridle and Feldman (1972) and others, it is natural to suppose that X-rays will be generated through the Compton effect, since both relativistic electrons and the microwave background radiation are present. However, many workers have chosen to argue that the detection of extended X-ray sources indicates the presence of hot gas (cf. Silk 1973). Attempts have been made to show that there is a correlation between the random motions of the galaxies in a cluster and the X-ray luminosity (Solinger and Tucker 1972). While much has been made of the correlation $L_x \propto (\text{velocity dispersion})^4$, I do not find it very convincing since only a very small number of clusters (3 in the first instance) with good velocity dispersions are involved. These clusters are of different physical and dynamical types, while the velocity dispersions are not very different. This is why a correlation can only be found with a high power of Δv. Also theoretical work has been carried out based on the idea that the X-ray sources definitely show that hot gas is present (Yahil and Ostriker 1973).

In fact, in my view, for the X-ray sources in clusters in general, no conclusion as to which mechanism is operating can yet be drawn. The best way of deciding this question is through studies of the X-ray spectra and measurements of the angular sizes and intensity profiles of the sources. In the simplest models we might expect that the Compton X-rays would have a spectrum of the form $I(E) \propto E^{-\alpha}$, while an exponential form with a single temperature would be expected if the X-rays are thermal bremsstrahlung. In more realistic models we expect considerable departures from these conditions. Compounding the difficulties in interpretation has been the problem encountered by some groups in handling the Gaunt factor (cf. Margon 1973). The majority of the clusters have been detected by the UHURU satellite and in a recent paper, Kellogg *et al.* (1973) have shown that there is a wide range of X-ray luminosities and that either mechanism can be accommodated. The most closely studied cluster is the Coma cluster. In its central part a soft X-ray source ($E_x \leq 2$ keV) has been discovered (Gorenstein *et al.* 1973). The spectral data from these measurements together with the higher energy measurements suggest an exponential spectrum rather than an inverse power-law form. Gorenstein *et al.* therefore argue that the radiation most likely arises in a hot gas $T \simeq 10^8$ degrees, $\rho \simeq 10^{-27}$ gm cm^{-3}. Even assuming a uniform density, the total mass of gas is far less than that required to bind the cluster.

The general conclusion from the data available on X-ray emission from clusters is that evidence for the presence of gas is ambiguous, but even if hot gas is present, it only contributes a small fraction of the total mass if the clusters are gravitationally stable.

Another approach to the problem of the existence of intergalactic or intracluster gas is through the study of extragalactic radio sources. Sources which have been identified with optical galaxies both inside and outside clusters frequently have two major components (which are structured) centered about the optical system. They have clearly been ejected from the parent galaxy and, since they have not freely expanded, they have either been contained by the pressure of an external medium or the components must contain enough mass to hold themselves together or only to expand. A critical account of the various schemes which have been proposed has recently been given (De Young and Burbidge 1973). Many authors who favor the ram-jet mechanism feel that the fact that it can explain some features of the sources is evidence in favor of the existence of intergalactic gas, and with a judicious choice of parameters it has sometimes been claimed that the gas present is adequate to bind clusters, or equal to the critical cosmological density. In fact, there is great uncertainty in these quantities, and, of course, if one goes to a model in which it is argued that the sources are generated from a number of coherent objects which are ejected from the galaxy, very little, if any, gas is required.

Further arguments along somewhat similar lines have been made following the studies of radio sources which appear to consist of active galaxies with radio trails (Miley *et al.* 1972; Miley 1973; Wellington *et al.* 1973). Since these observations have only so far been interpreted by those who have used the observations on the *assumption* that an intergalactic medium is causing the effects observed, it is difficult to decide at this stage whether or not unambiguous evidence for the presence of gas is given by these results.

Another line of evidence which also remains somewhat ambiguous is concerned with the high latitude 21-cm observations which have been interpreted by Oort (1971) as evidence that intergalactic gas is being accreted by the galaxy. Other interpretations have been proposed, the most plausible of which is that this gas has been ejected from the galaxy and is now falling back in.

Finally, we describe briefly evidence that gas is likely to be present following the evolution of galaxies.

If galaxies are formed by a process of condensation from a lower density medium, it is to be expected that some intergalactic gas is left behind. For those who are quite certain that galaxies formed by condensation, this provides the primary reason for believing in the existence of intergalactic gas. Oort (1970) has argued that only a small fraction ($\sim 1/16$) of the mass will condense into galaxies, leading to the view that 15/16, or approximately the closure density, is present in intergalactic gas. For the minority, including Ambartsumian, or Hoyle and Narlikar who believe that galaxies have evolved from very high density states, this cosmogonical argument for the presence of much intergalactic gas is hardly convincing.

Given that galaxies are indeed present, there are a number of processes which we know are operating which will tend to expel gas from them into the outside medium. These include:

(1) Large-scale explosions in the nuclear regions which are able to eject large masses at velocities considerably in excess of the escape velocity (cf. Burbidge 1970).

(2) Processes involving galactic winds (Mathews and Baker 1971) which will drive gas from galaxies and eventually from clusters. Within clusters it is therefore reasonable to suppose that gas is present which has been ejected from galaxies and heated by the passage of other galaxies (Ruderman and Spiegel 1971). If gas were present outside clusters, the converse process of accretion might occur (Gott and Gunn 1972).

However, if the gas which is present in the intergalactic medium is only that which has been ejected from galaxies, it follows that the total mass involved is likely to be very much less than that presently condensed into galaxies, so that it will not approach the critical density.

DISCRETE OBJECTS

It appears likely that there could still be much mass in the universe in the form of dark discrete objects which have so far remained undetectable. This possibility has been discussed before by me and by others, and there is very little that is new that one can add. If some clusters of galaxies (like the Coma cluster) are bound, it is likely that this mass is in the form of evolved galaxies, dwarf systems, black holes or white holes.

Press and Gunn (1973) have recently discussed the possibility that if the number of such objects is high enough (close to the critical density) it may be possible to detect them by the method of gravitational imaging.

CONCLUSIONS

There is still no very good direct evidence for the presence of significant amounts of mass-energy in forms other than that of discrete luminous objects-galaxies. Some intergalactic diffuse matter is likely to be present, but no strong evidence is available that its density approaches the critical value. My own view is that it is most likely that a significant amount of cold intergalactic hydrogen is present. As was stated earlier, if the quasi-stellar objects are comparatively close by, no limits are placed on the density of intergalactic gas from their absorption spectra, and the limit set by Penzias and Scott (1968) allows us to assume that quite a large mass—much greater than that in the galaxies—is present in the form of cold gas. It is also possible that we live in a universe in which $\rho << \rho_c$. Alternatively, as is well known, we cannot exclude the possibility that the bulk of the mass-energy is present in such exotic forms as neutrinos or black holes.

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GENERAL DISCUSSION
BURST ASTRONOMY

Communication of M.J. Rees

I would first like to make one comment concerning the possible occurrence of very long wavelength gravitational waves, since Professor Burbidge referred to this question in his report. Such waves could exist as a vestige of primordial inhomogeneities, and one might expect their predominant wavelength to be comparable with the dominant lengthscale for inhomogeneities in the universe—perhaps 1—10 Mpc. Conventional ideas on galaxy formation (in which one invokes small amplitude initial fluctuations) suggest that these irregularities would generate only a very low energy density in the form of associated gravitational waves. It is, however quite conceivable that the early universe was highly " chaotic ", in which case the energy density ρ_G of these gravitational waves could be comparable with the " critical density " ρ_C needed to close the universe. Such waves would induce a velocity gradient $\sim \left(\frac{\rho_G}{\rho_C}\right)^{1/2}$ times that due to the Hubble recession on all scales smaller than a wavelength. These velocities would have to be taken into account in applications of the virial theorem to cluster of galaxies - even if $\rho_G \simeq \rho_C$, however, the effect is unimportant for " rich " clusters, though it may be more significant for small groups of galaxies such as those studied by de Vaucouleurs. From the precision with which the Hubble law is known to be obeyed, one can already exclude the possibility that the so-called " missing mass " is all contributed by gravitational waves with wavelengths $>>$ 10 Mpc (This is the only wavelength range for which cosmological considerations provide a significant upper limit).

GAMMA RAY BURSTS OF COSMIC ORIGIN

Robert Hofstadter
(Stanford University)

I have been asked to report on the recently reported observation of gamma ray bursts found by a Los Alamos group(1). It is important to mention this matter because the origin of the gamma ray bursts are as yet undetermined and may be of fundamental significance to astrophysics. The reason why I believe I have been chosen to give this report is that for the last several years I have been trying, through the High Energy Astronomy Laboratory (HEAO) program of NASA in the U.S., to enter the field of satellite gamma ray astronomy in the high energy range (30MeV - 10,000 MeV). At first our program was selected for the HEAO-B flight, but the entire program was " suspended " by NASA early this year. Subsequently we have made a proposal to fly on the substituted HEAO-C which will be one of three mini-versions of the old program.

Our collaborators are C. Fichtel (Goddard), A. Favale (Grumman) and K. Pinkau (Max-Planck Institute-Munich) A decision will be made early next year and we shall see whether or not we are placed on HEAO-C.

My involvement in the gamma ray program followed from Stanford work by E. B. Hughes, myself and others, on total absorption gamma ray counters, a subject we have worked on steadily since 1968 with great success.

No one knows what to expect from space in the high energy gamma ray range but I feel confident that there will be much to measure from point sources.

The Klebesadel report deals with the accidental discovery made on four Vela Spacecraft flying in circular orbits at 10 earth diameters from the center of the earth. The spacecraft were uniformly situated around the circle. Small (10 cm^3) CsI scintillation counters were used to detect gamma rays in roughly the 0.2-1.5 MeV range. Shields prevented low energy electrons (0.75 MeV) and protons (threshold $\sim$ 20 MeV) from being recorded. There is much evidence that the detected events were not of particulate origin, but were truly gamma rays. Only a rapid rise in count rate was the criterion which allowed discrete counting

(1) R. W. Klebesadel, I. B. Strong, R. A. Olson, *Astrophysics J.* **182**, L85-L88 (1973).

GAMMA-RAY BURSTS OF COSMIC ORIGIN

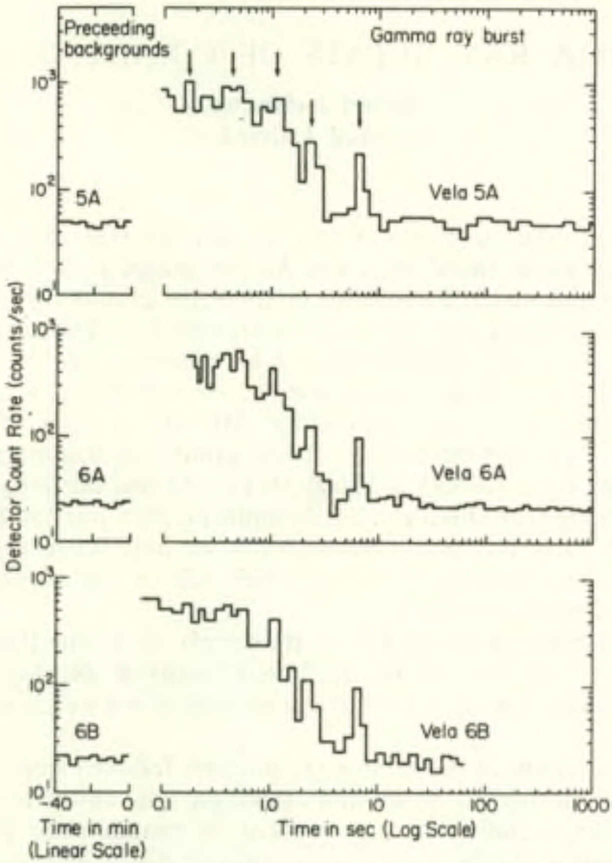

Fig. 1. Count rate as a function of time for the gamma-ray burst of 1970 August 22 as recorded at three Vela spacecraft. Arrows indicate some of the common structure. Background count rates immediately preceding the thurst are also shown. *Vela 5A* count rates have been reduced by 100 counts per second (a major fraction of the background) to emphasize structure.

in the events to be recorded. Thus only relatively large events were selected. Background counting rates were carefully measured and were low enough for definite events to be observed.

The criterion that at least two spacecrafts must record the burst reduces the number of possible events over a two year period to 16 events. Two of these events were recorded by all four spacecrafts.

The bursts are widely varied but generally last from less than a second to 30 seconds. Some show clearly resolved peaks. The time integrated flux density varies from 10^{-5} ergs/cm^2 to more than 2×10^{-4} ergs/cm^2. Instantaneous flux densities exceed 4×10^{-4} ergs/cm^2 sec.

Measurements of flux and use of the inverse square law show that the sources are at least 3×10^6 km. distant from the earth. The earth and sun are excluded as sources by various but definite criteria. A single solar outburst was identified as a source of one burst, but of course was not included in the 16.

Figure (1) shows the time variation of a representative burst observed on August 22, 1970.

The discussion in the Klebesadel paper shows that the events could not be correlated with any known active novae or supernovae in the recording period. It is still possible that supernovae, not necessarily bright in the optical region, could be responsible for the events. The authors say " A source at a distance of 1 Mpc would need to emit 10^{46} ergs in the form of electromagnetic radiation between 0.2 and 1.5 MeV in order to produce the level of response observed here. Since this represents only a small fraction (10^{-8}) of the energy usually associated with supernovae, the energy observed is not inconsistent with a supernova as a source ".

I believe that Dr. Giacconi and his colleagues have also observed these bursts in the X-ray region. I also believe that other scientists at this conference have some additional material on directionality observed by the Los Alamos group and also some possible source distance information.

I believe these events are of great interest and I would like to see them with our gamma ray instruments in HEAO-C. Not withstanding whether these events are to be observed or not, I believe that elementary particle physics is fundamental to many parts of the subject of astrophysics. I would be greatly surprised if energetic gamma ray events would not be associated with stellar, pulsar, black hole, or supernovae phenomena.

It would be most interesting to look at the rotating double star situations in which matter is being transferred between the stars.

Discussion of the communication of R. Hofstadter

R. Giacconi: There appears to be no doubt about the reality of such events. They have been observed on all satellite instruments in orbit at the time. The spectrum in one case has been extended by 0S0-7 to 10 kev showing a flat spectrum in the 10 to 100 kev region and a steeper in the 100 kev to 1 Mev region. Since the energy at Earth appears to be extremely large, it will be quite straightforward to extend by X-ray detectors their observation by 10^7 smaller fluxes and to measure their position to 1′. This will allow possible identifications and number-luminosity relationship.

M. J. Rees: The most straightforward inference that can be drawn from these results concerns the energy involved. If the events occur at distances $\sim$ 10 Mpc, then each emits $\sim 10^{48}$ ergs in the form of Mev γ-rays; if they are at " cosmological distances ", then each involves $\sim 10^{53}$ ergs; but if they originate within our galaxy, at distances $\leq$ 100 parsecs, the energy is $\leq 10^{38}$ ergs. The high isotropy is of course equally consistent with *either* an extragalactic origin, *or* with a *very* local origin where the events occur at distances smaller than the scale height of the galactic disc. At the moment, the available data are so limited, and place so little constraint on speculative theories, that it is all too easy to conjecture numerous explanations for these phenomena. Let us hope that further work soon pins down the situation further: to speculate at greater length today would merely be to indulge in the kind of pursuit which brings astrophysics into disrepute among some " serious " physicists!