Radiative corrections for neutron and nuclear beta decays

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Photon bremsstrahlung

Virtual part of radiative correction

Model independent radiative correction results

Model dependent corrections

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Why are radiative corrections important?

They are small, but we are looking for small SM or non-SM effects:

- **Test of CKM unitarity (e.g. Cabibbo model in early sixties)**
- **Right-handed, scalar, tensor couplings**
- **Weak magnetism, second class currents, TRV**

keV mass sterile neutrino searching (warm dark matter) in tritium beta decay: electron energy spectrum, 10^{-7} accuracy needed

S. Mertens et al., to be published

Photon bremsstrahlung

Bloch-Nordsieck theorem (1937)

Charged particle processes: bremsstrahlung (BR) photons are always present; probability(no BR photons)=0

Finite energy resolution: only $K>K_{min}$ BR events can be distinguished from processes without any photons

neutron decay: K_{max}=780 keV

with K_{min}=1 keV:

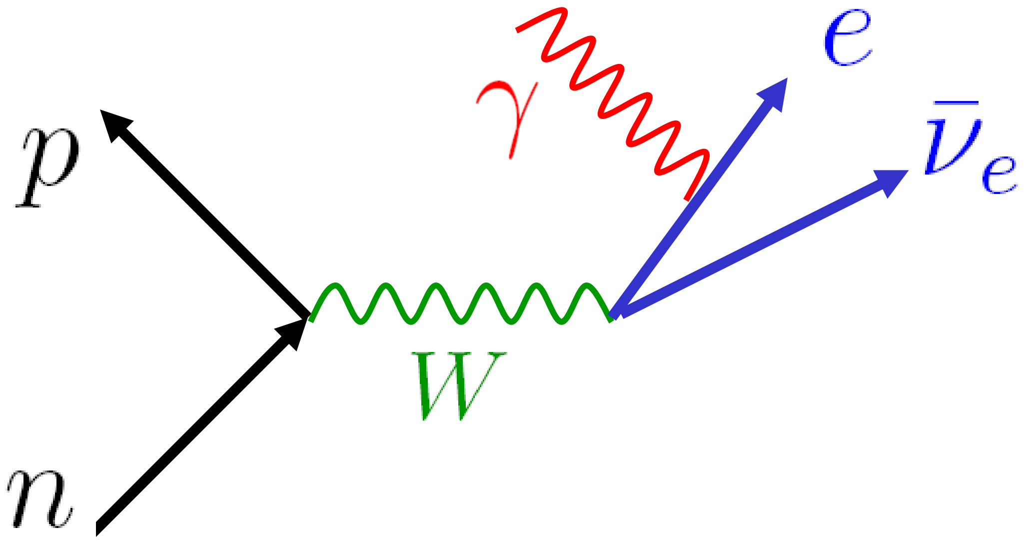
$P(1\ \gamma)=0.5\ \%$

$P(2\ \gamma)=0.001\ \%$

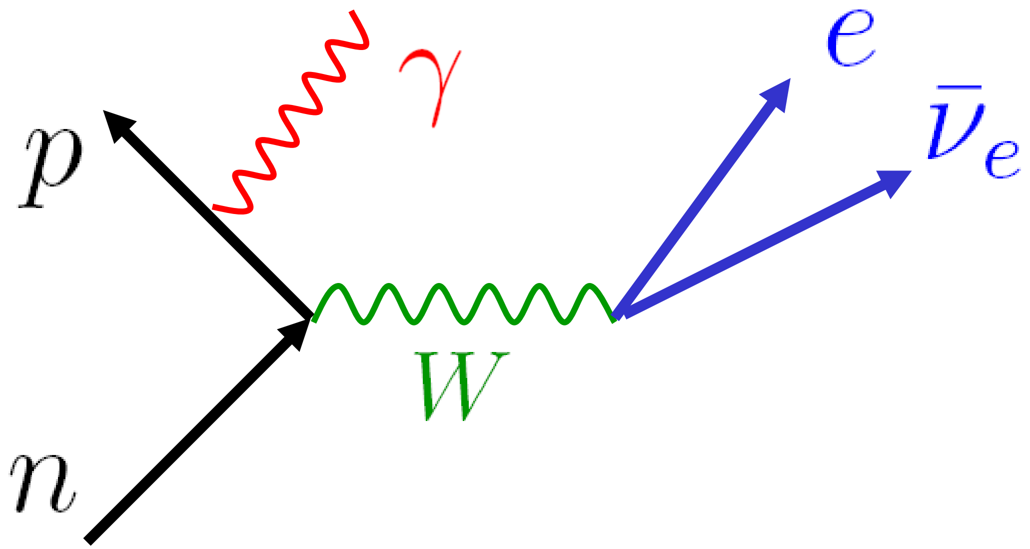
internal photon bremsstrahlung in neutron decay:

$$n \longrightarrow p e \bar{\nu}_e \gamma$$

$\mathcal{M}_l$

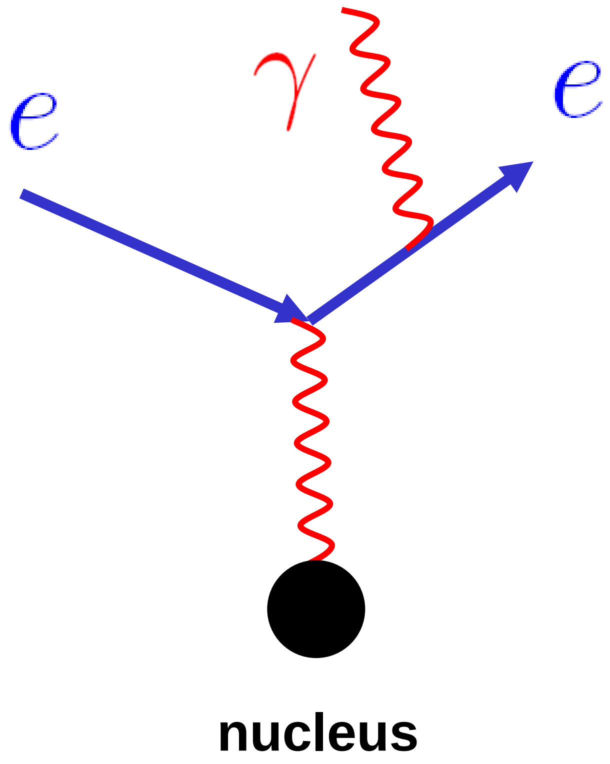


$\mathcal{M}_h$



Internal (inner) bremsstrahlung completely different from external BR. External BR is independent of the decay, internal BR occurs during the decay.

external BR of electron:



Possible confusion: inner and outer radiative correction.
The inner radiative correction (completely virtual process) has nothing to do with the inner bremsstrahlung !

Photon bremsstrahlung amplitude (gauge invariant):

$$\mathcal{M}_{BR} = \mathcal{M}_l + \mathcal{M}_h$$

$\mathcal{M}_l \longrightarrow$ QED (accurate,reliable calc.)

$\mathcal{M}_h \longrightarrow$ generally model (strong int.) dependent

BUT !

BR photon energy in neutron decay < 0.78 MeV

$\longrightarrow$ BR photon wavelength > 1000 fm

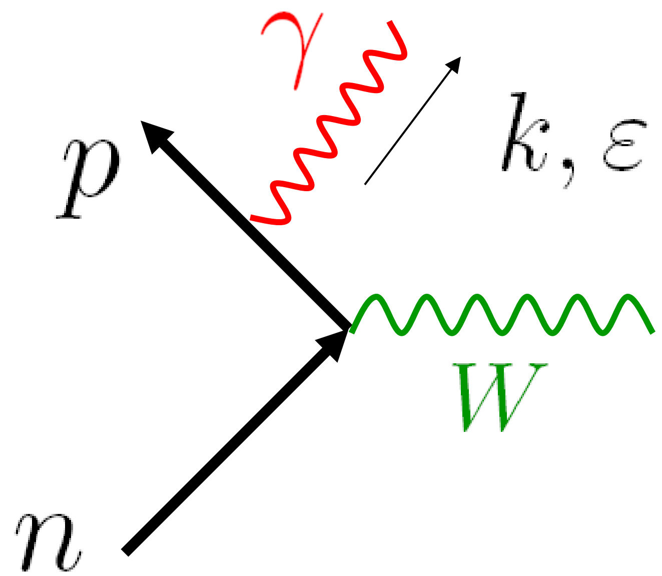
BR photons in neutron decay can see only the proton charge (and slightly the nucleon magnetic moment), but not the inner structure of the nucleons !

Order-K^{-1} part of the hadronic BR amplitude:

$$\mathcal{M}_h[K^{-1}] = e\frac{(p\varepsilon)}{(pk)}\mathcal{M}_0$$

$$k = (K, \mathbf{k}), \quad K = |\mathbf{k}|$$

$\mathcal{M}_0$ **: zeroth-order amplitude (without radiative corr.)**



→ 1/K behaviour of low energy BR photon spectrum

Low theorem **(F. E. Low, Phys. Rev. 110 (1958) 974)**

From EM current conservation (gauge invariance) the order-K^0 part (next order, subleading) of the hadronic BR amplitude can also be reliably (model independently) computed

(depends on magnetic moments of the nucleons)

Many experimental tests of Low theorem in high energy decay and scattering processes

From Low theorem: only the order-K part of the BR photon amplitude is model dependent

$$O(K^0) \sim \frac{K}{m_n} O(K^{-1}) \sim 10^{-3} \cdot O(K^{-1})$$ **(K=1 MeV)**

$$O(K) \sim \left(\frac{K}{m_n}\right)^2 O(K^{-1}) \sim 10^{-6} \cdot O(K^{-1})$$ **(K=1 MeV)**

→ **10^{-6} accuracy of photon BR calc. in neutron decay (for K=100 keV: 10^{-8} accuracy)**

No information about strong interaction dynamics from photon bremsstrahlung in neutron decay !

Photon BR measurement in neutron decay: test of QED and Low theorem in a low energy weak decay process

Photon bremsstrahlung: **part of radiative correction, calc. in neutron and nuclear beta decays is accurate and reliable**

BR calculation: **- theoretically simple**
- technically complicated

Integration in many dimensional phase space:

$$\int \frac{d^3\mathbf{p}_p}{E_p}\frac{d^3\mathbf{p}_e}{E_e}\frac{d^3\mathbf{p}_\nu}{E_\nu}\frac{d^3\mathbf{k}}{K}\delta^4(p_n - p_p - p_e - p_\nu - k)\cdot\sum_{spin}|\mathcal{M}_{BR}|^2$$

$\sum_{spin}|\mathcal{M}_{BR}|^2$ **: computation by hand, or by computer algebra code**

(Dirac matrix algebra, Lorentz-indices)

Phase space integration

i, analytical, semianalytical:

F. Glück, T. Toth, Phys. Rev. D 41, (1990) 2160,
Phys. Rev. 46 (1992) 2090;
F. Glück, Phys. Rev. D 47 (1993) 2840.

ii, Monte Carlo:

F. Glück, Comp. Phys. Comm. 101 (1997) 223.

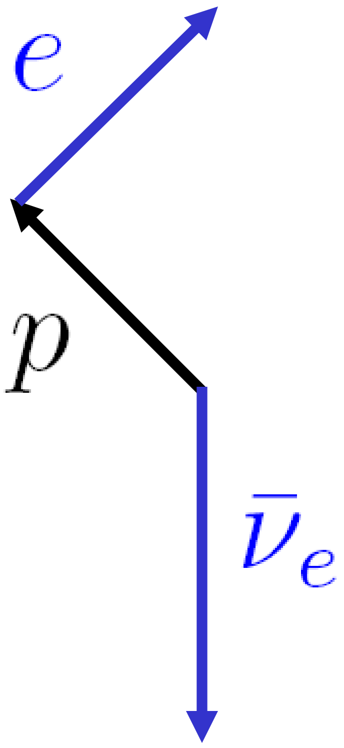
Advantages of MC : easier, flexible for experimental details, any kind of quantity can be computed; few hundred million events can be generated within 1 hour computation time (Poisson error<0.1 %)

Many comparisons among various computation methods.
Good agreement between semianalytical and MC results.

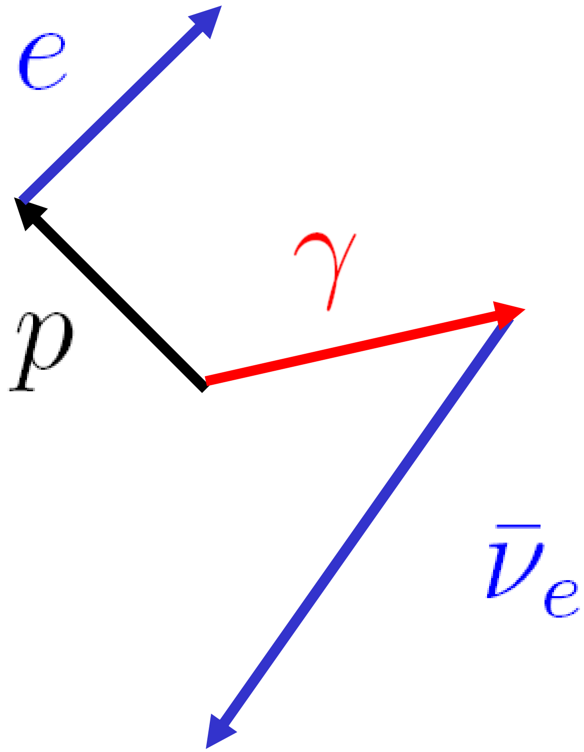
MC generator FORTRAN codes for unpolarized nuclear beta decay and for polarized neutron decay are available. Used e.g. in analysis of $^{38}K_m$ Fermi-type beta decay electron-neutrino correlation experiment (A. Gorelov et al., Phys. Rev. Lett. 94 (2005) 142501)

BR photon changes the decay kinematics !

no BR photon



with BR photon



Kinematics important for experimental details !!!

Radiative correction calculations to electron-neutrino correlation of Y. Yokoo and M. Morita (1976), K. Fujikawa and M.Igarashi (1976) Augusto Garcia and M. Maya (1978), Augusto Garcia (1982):

p_e, p_ν fixed, integration over photon momentum k:,

analytical integration possible

Problem: proton momentum changes with photon momentum *k* (momentum conservation), and no information about neutrino momentum (neutrino is usually not detected)

⟶ these radiative correction calculations to electron-neutrino correlation are not suitable for experimental analyses

(K. Toth, KFKI-1984-52, K. Toth et al., Phys. Rev. D33 (1986) 3306, Phys. Rev. D 40 (1989) 119)

Experiments: electron (positron) and proton (recoil nucleus) is detected, usually no information about neutrino and BR photon.

Radiative correction calculations should integrate over the BR photon with fixed charged lepton and recoil particle momenta.

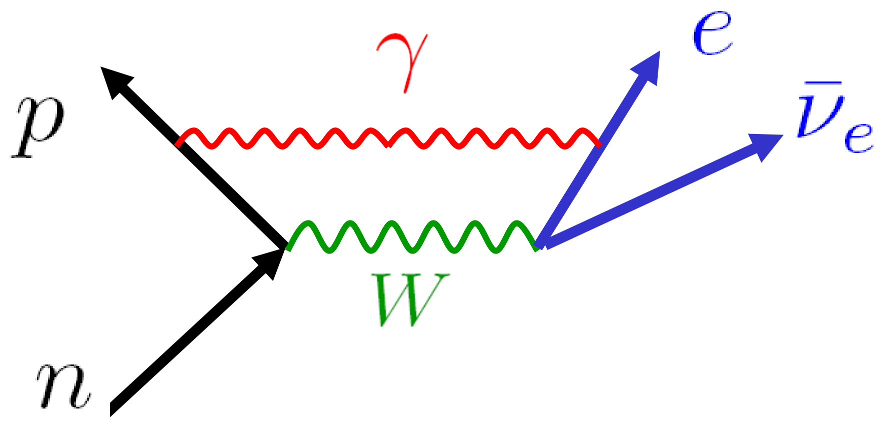
F.e.: proton spectrum in neutron decay → integration over electron and BR photon with fixed proton energy

Analytical integration in this case is difficult, but no extra problem with Monte Carlo method!

Experimental details (particle kinematics, cuts, energy resolution, etc.) could be important for the radiative correction calculation results !!!

Virtual corrections

Photon exchange between charged particles:



BR photon is on-shell:

$$k_{BR} = (K, \mathbf{k}), \quad k_{BR}^2 = K^2 - \mathbf{k}^2 = 0$$

Virtual photon is off-shell: $k_{VIRT}^2 \neq 0$

Energy (K) and momentum (k) of virtual photon are independent !

Order-α virtual amplitude by 4-dimensional integral:

$$\mathcal{M}_{VIRT} \sim \int d^4k \left\{ \begin{array}{l} \text{wave} \quad \text{functions} \\ \text{propagators} \\ \text{vertices} \end{array} \right\}$$

Interference between zeroth-order amplitude $\mathcal{M}_0$ and virtual correction amplitude $\mathcal{M}_{VIRT}$

(virtual process indistinguishable from zeroth-order process)

Photon bremsstrahlung: no interference with zeroth-order amplitude (BR photon is in principle detectable)

Order-α radiative correction calculation of observable quantities:

$$\int |\mathcal{M}_0 + \mathcal{M}_{VIRT}|^2 + \int |\mathcal{M}_{BR}|^2$$

Order-α terms:

$$\int |\mathcal{M}_{BR}|^2 = -C_1 \ln m_\gamma + C_2$$

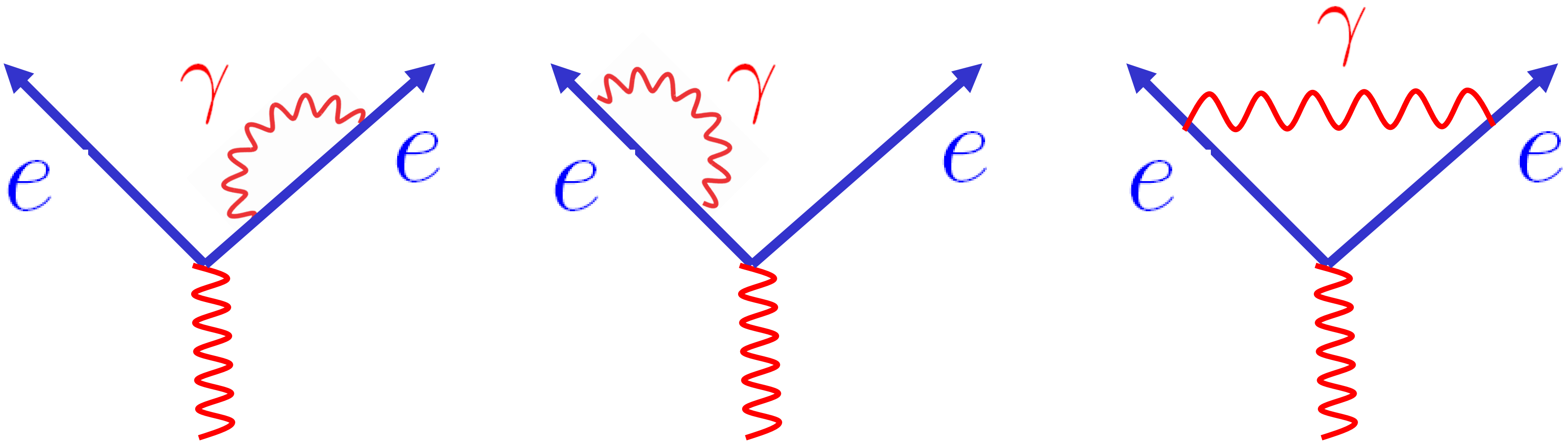
$$2 \int \Re(\mathcal{M}_0 \mathcal{M}^*_{VIRT}) = C_1 \ln m_\gamma + C_3$$

Infrared divergent terms cancel in the VIRTUAL+BR sum

Since both the virtual and the bremsstrahlung correction is IR divergent: it is not meaningful to give quantitative results only for the virtual, or only for the BR correction: only their sum is quantitatively meaningful

UV divergence of virtual correction

QED:



self-energy diagrams **vertex diagram**

UV divergence in each graph, but with mass and charge renormalization:

sum of virtual amplitudes is finite

Similar cancelation of order-α UV divergent terms in muon decay with V-A (4-fermion) theory

Neutron decay

UV divergence is present in 4-fermion and in intermediate vector boson theories

Conjecture in 60´s: perhaps strong interaction can help to solve the UV divergence problem?

$$(F_{EM}(q^2) \rightarrow 0 \quad \text{with} \quad q^2 \rightarrow \infty)$$

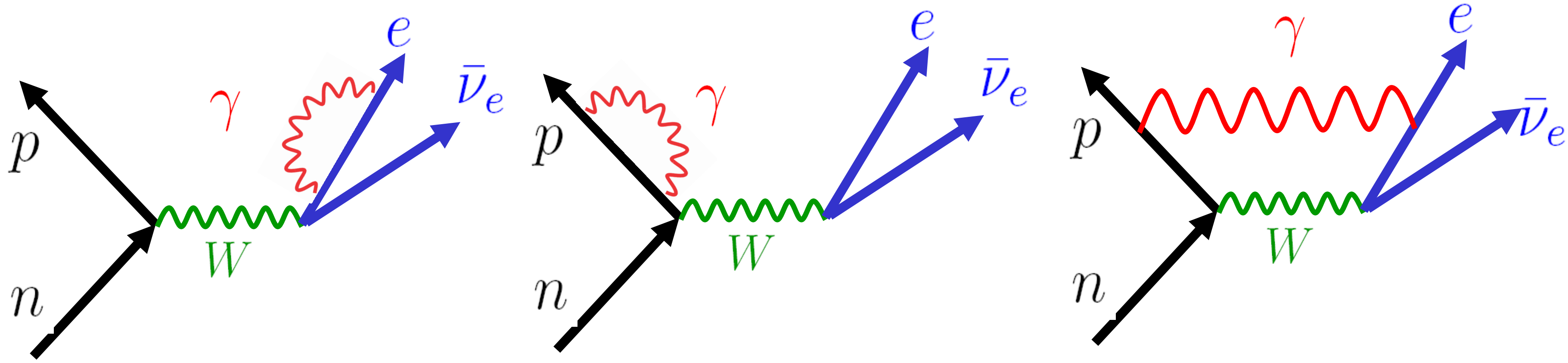
(Feynman, Källen, Berman, Sirlin)

Current algebra (middle 60´s): the strong interaction cannot solve the UV-divergence problem !

Solution by the $SU(3)_c$ x $SU(2)_L$ x U(1) non-Abelian gauge theory (Standard Model)

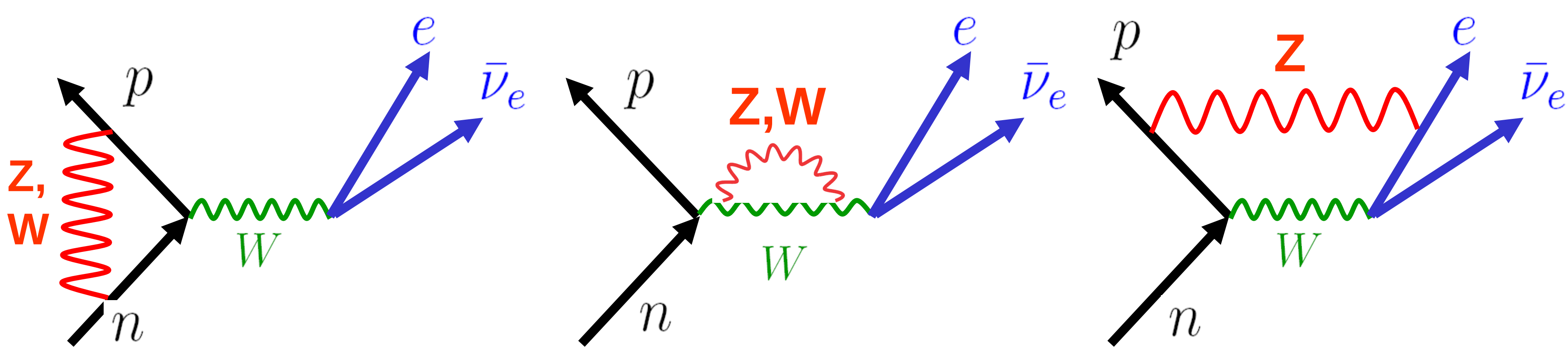
Sirlin (1974,1978)

Photonic diagrams (+3 WWγ graphs):



self-energy e　　　　self-energy p　　　　box

Non-photonic diagrams (examples):



Duplication of photonic self-energy integrals by photon propagator decomposition:

$$\frac{1}{k^2} = \underbrace{\frac{1}{k^2 - M_W^2}}_{\text{1. part}} - \underbrace{\frac{M_W^2}{k^2 - M_W^2}\frac{1}{k^2}}_{\text{2. part}}$$

weak correction: all non-photonic + WWγ graphs + 1. part ph. self-energy

photonic correction: photonic box + 2. part photonic self-energy

photonic corrections are UV finite

weak correction: asymptotoc freedom of QCD and electroweak renormalization → cancelation of UV divergences, finite rad. corr.; also IR finite

Weak correction to total beta decay rate:

$r_{WEAK}=0.02\ \%$ **(A. Sirlin, Rev. Mod. Phys. 50 (1978) 573)**

Alberto Sirlin

The model independent (MI, outer) correction

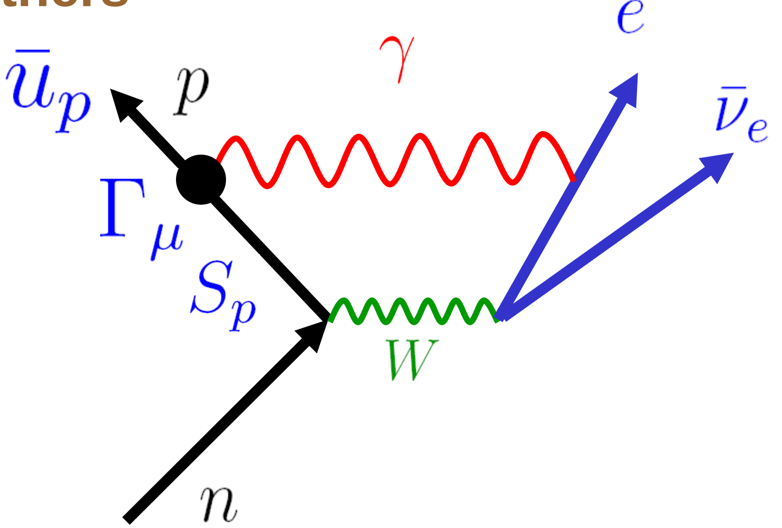
Photonic virtual correction: **- IR divergent**
- strong interaction dependent

(1 GeV photons disturb the nucleon inner structure)

Radiative correction contribution with small photon energy (BR + virtual): IR divergent, no strong interaction dependence, depends on particle momenta (changes the spectrum shapes)
→ should be separated from the others

Sirlin, 1967:

$$Z_\mu = \bar{u}_p \Gamma_\mu S_p$$



Point-like hadron model:

$$Z_\mu = \bar{u}_p \gamma_\mu \frac{\not{p} + \not{k} + m}{(p+k)^2 - m^2}$$

Convective term – spin term separation (Yennie, Frautschi, Suura,1961; Meister, Yennie,1962):

$$\bar{u}_p \gamma_\mu (\not{p} + \not{k} + m) = \bar{u}_p \left\{ \underbrace{2p_\mu + k_\mu}_{\text{convective term}} \quad + \quad \underbrace{\frac{1}{2}[\gamma_\mu, \not{k}]}_{\text{spin term}} \right\}$$

Model independent (MI) virtual correction:

photonic virtual integrals with convective term

$$Z_\mu = \bar{u}_p \Gamma_\mu S_p = Z_\mu^{\mathrm{MI}} + Z_\mu^{\mathrm{MD}}$$

$$Z_\mu^{\mathrm{MI}} = \bar{u}_p \frac{2p_\mu + k_\mu}{(p+k)^2 - m^2}$$

Z_μ^{MD} : **precise calculation difficult, but its general properties are similar to spin term**

(see later)

MI radiative correction= MI virtual + bremsstrahlung

Properties of model independent correction:

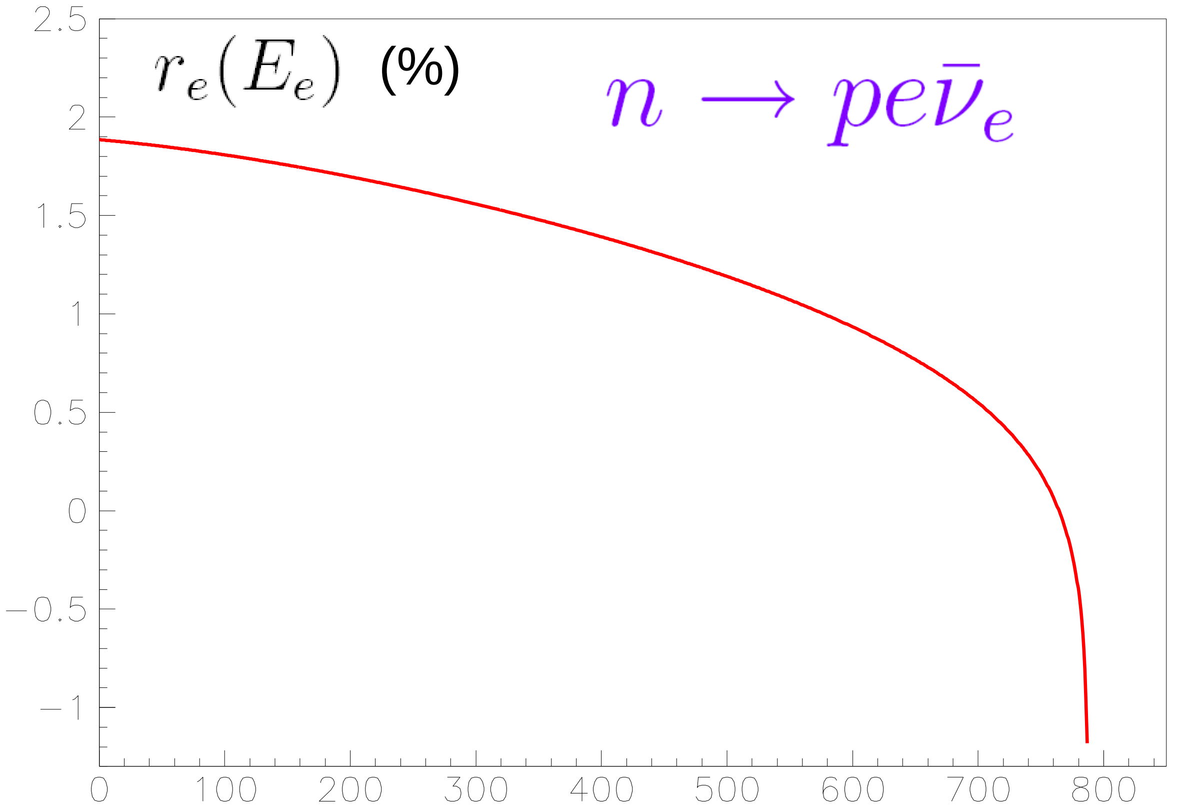
i, no strong interaction dependence, reliable

ii, sensitive to experimental details (f.e.: photon bremsstrahlung changes the kinematics)

iii, changes the spectrum shapes and asymmetries

Model independent radiative correction is important in the experimental analyses !

MI radiative correction to electron energy spectrum in neutron decay



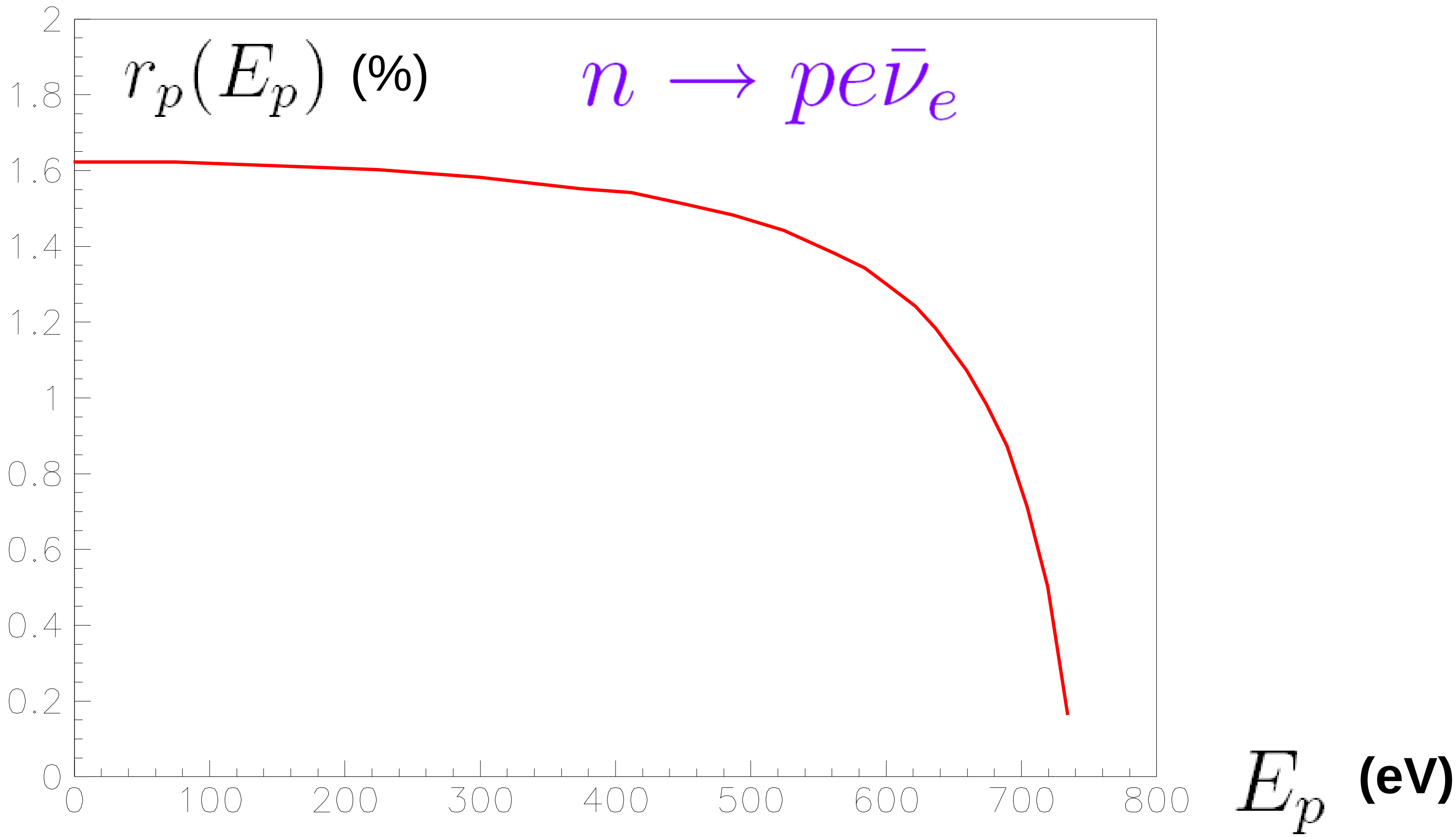
Using analytical formula of A. Sirlin, Phys. Rev. 164 (1967) 1767

$$E_e \longrightarrow E_{emax} \quad : \quad r_e \sim \ln(E_{emax} - E_e)$$

(electron energy goes to maximum → BR phase space decreases → IR divergence of virtual correction starts to appear)

Including higher orders: logarithmic singularity disappears

MI radiative correction to proton energy spectrum in neutron decay

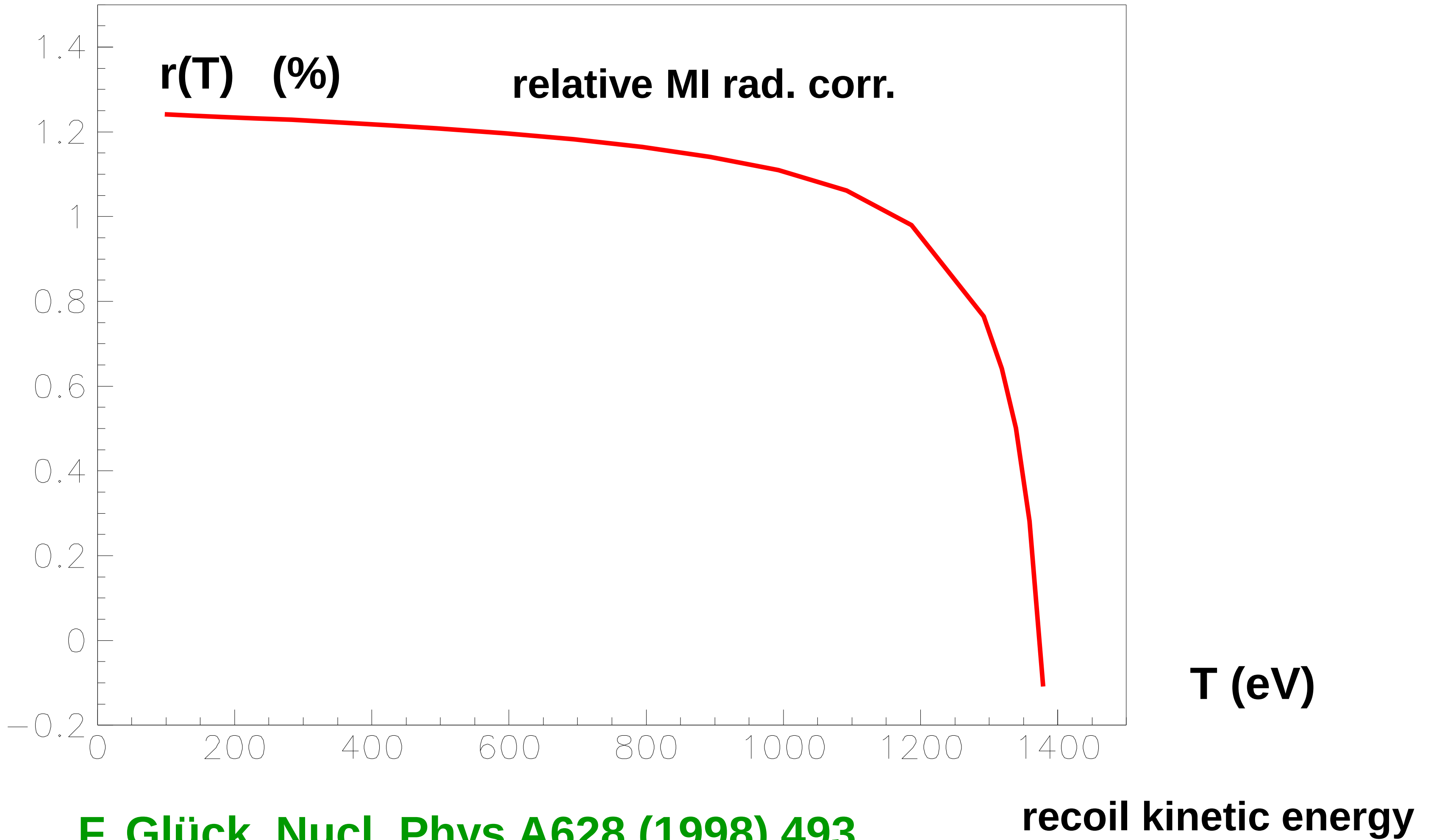


R. Christian, H. Kühnelt, Acta Phys. Austriaca, 49 (1978) 229;
F. Glück, Phys. Rev. D47 (1993) 2840

Change of fitted axialvector-to-vector coupling constant ratio:

$$\delta\lambda \approx 0.01$$

MI radiative correction to recoil energy spectrum in ^{6}He decay



F. Glück, Nucl. Phys A628 (1998) 493

Allowed nuclear beta decay similar to neutron decay
(see f.e.: B. R. Holstein and S. B. Treiman, Phys. Rev. C3 (1971) 1921)

Experimental electron-neutrino correlation result of
C. Johnson et al., Phys. Rev. 132 (1963) 1149:

$$a^0_{e\nu}[^6\mathrm{He},\ \mathrm{Johnson}\ 1963] = -0.3343 \pm 0.0030$$

With radiative correction:

$$a^\gamma_{e\nu}[^6\mathrm{He},\ \mathrm{Glück}\ 1998] = -0.3308 \pm 0.0030$$

0.0035 (>1 σ) shift due to radiative correction !

Using the rad. corr. result of Y. Yokoo and M. Morita, Suppl. Prog. Theor. Phys. 60 (1976) 37, discussed in W. Kleppinger et al. , Nucl. Phys. A293 (1977) 46:

shift is only 0.0015

The kinematical change of the decay due to BR photons was not taken into account in the calculation of Yokoo and Morita !

Model independent radiative correction results for polarization asymmetries in polarized neutron decay:

R. T. Shann, Nuovo Cimento 5A (1971) 591,
F. Glück, K. Toth, Phys. Rev. D46 (1992) 2090,
F. Glück, Phys. Lett. B376 (1996) 25,
F. Glück, Phys. Lett. B436 (1998) 25.

Relative MI radiative corrections:

electron asymmetry: $\delta\alpha_e \approx -0.01\%$

proton asymmetry: $\delta\alpha_p \approx 0.04\%$

electron-proton asymmetry (PERKEO experiment): $\delta\alpha_{ep} \approx -0.05\%$

Model dependent (MD, inner) correction

Radiative corr.= BR + virtual = MI + MD

MD = weak + MD part of photonic virtual corr.

→ MD correction is pure virtual (no IR divergence)

MI virtual: main contribution from small energy virtual photons (small energy = much smaller than nucleon mass)

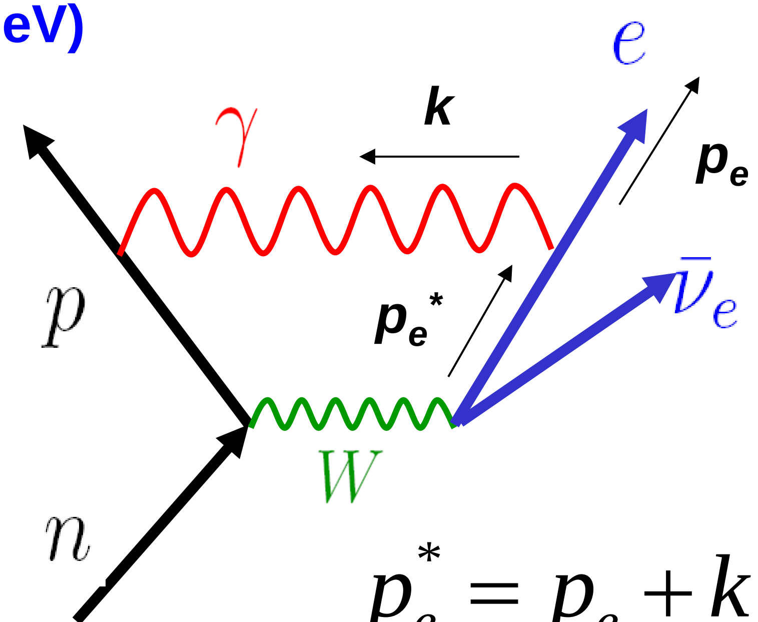
MD: main contribution from intermediate and high energy virtual photons (intermediate energy: not far from 1 GeV; high energy: much larger than 1 GeV)

Small photon energy (momentum):

Propagator momenta are sensitive to external momenta

Large photon energy (momentum):

Propagator momenta depend mainly on virtual photon momentum, they are not sensitive to the external momenta



$$p_e^* = p_e + k$$

→ **no change of spectrum shapes and angular distributions due to the model dependent correction**

A. Sirlin, Phys. Rev. 164 (1967) 1767

Neglecting terms of order $\sim \alpha \frac{E_e}{m_n} \ln\left(\frac{m_n}{E_e}\right) \sim 10^{-5},$

the MD correction can be absorbed into the dominant form factors f_1 and g_1

Effective form factors:

$$f_1' := f_1 \left(1 + \frac{\alpha}{2\pi} c\right), \quad g_1' := g_1 \left(1 + \frac{\alpha}{2\pi} d\right)$$

MD corr.: 2 numbers (c, d)

Redefinition of G_V and λ :

$$G_V := G_\mu V_{ud} f_1', \quad \lambda := g_1' / f_1'$$

All measureable quantities in neutron decay depend on these effective parameters (c and d are the same for all quantities)

SM tests by comparison of λ from different types of experiments (like electron asymmetry and electron-neutrino correlation) are independent of the MD correction !

Model dependent correction to the vector coupling constant is important for V_{ud} determination and for CKM unitarity test !

Model dependent correction of the decay rate:

$$\rho_0 \to \rho_0(1 + \delta_{MD})$$

$$\delta_{\rm MD} = \delta_{\rm as} + \delta_{\rm QCD} + \delta_{\rm med}$$

$\delta_{\rm as}$: high energy (>>1 GeV) virtual photons

Asymptotic part: **reliable calculation is possible due to the non-Abelian feature of QCD (asymptotic freedom)**

Sirlin, Rev. Mod. Phys. 50 (1978) 573:

$$\delta_{\mathrm{as}} = \frac{3\alpha}{2\pi}(1+2\bar{Q})\ln\left(\frac{M_Z}{m_p}\right) \qquad \bar{Q} = \frac{1}{2}(Q_u + Q_d) = \frac{1}{6}$$

$$\delta_{\mathrm{as}} = \frac{2\alpha}{\pi}\ln\left(\frac{M_Z}{m_p}\right) = 2.13\ \%$$

δ_{QCD} **: perturbative QCD correction to the asymptotic part**

$$\delta_{\mathrm{QCD}} = -0.04\ \%$$

δ_{med} **: intermediate energy (near 1 GeV) virtual photons**

Reliable, precise calculation of the intermediate correction is difficult !

W. Marciano, A. Sirlin, Phys. Rev. Lett. 96 (2006) 032002 :

$$\delta_{\mathrm{med}} = 0.10\,\% \pm 0.04\,\%$$

Total decay rate of neutron decay with order-α rad. corr. :

$$\tau^{-1} \sim G_\mu^2 V_{ud}^2 (1 + 3\lambda^2)(1 + \delta_{\mathrm{CB}} + \delta_{\mathrm{MI}} + \delta_{\mathrm{MD}})$$

$$\left.\begin{array}{l} \delta_{\mathrm{CB}} = 3.5\ \% \\ \delta_{\mathrm{MI}} = 1.5\ \% \\ \delta_{\mathrm{MD}} = 2.2\ \% \end{array}\right\} \Rightarrow \begin{array}{l} \delta\tau = -70\ \ \mathrm{s} \\ (\tau \approx 885\ \ \mathrm{s}) \end{array}$$

Future of radiative corrections simulations

Order-α radiative corrections

- less analytical, more Monte Carlo!
- MC simpler than analytical
- MC more general:
  - arbitrary observables; experimental effects (e.g.: cuts, detection efficiency etc.)
- improved efficiency: importance sampling
- comparisons with analytical results
- experience from simulations in high energy physics needed

Order-α^2 radiative correction simulations

- much more difficult than order-α
- order-$Z\alpha^2$: results in literature
- computer algebra software useful for complicated matrix element calculations
- integrations: Monte Carlo
- experience from simulations in high energy physics needed

Model dependent radiative correction simulations

- good knowledge of strong interaction effects
- lattice QCD computations ?
- reliability of the claimed uncertainty?