g_A, g_S, g_T from Lattice QCD

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Discussions with

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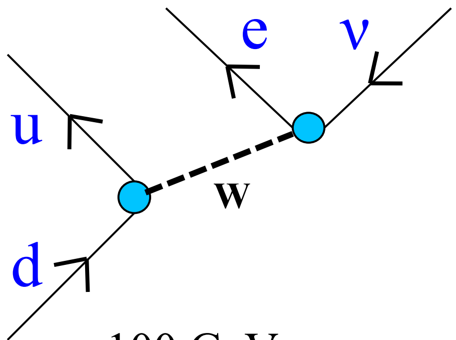
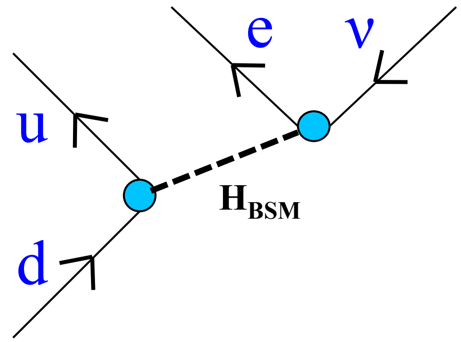
• Huey-Wen Lin, Saul Cohen	(U. Wash.)

Probing New Interactions: $M_{BSM} \gg M_W \gg 1$ GeV

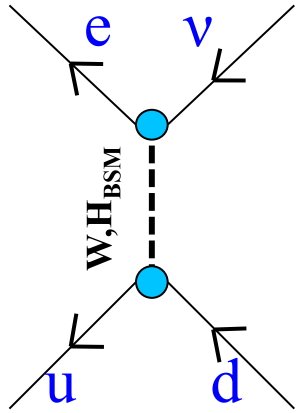
Many BSM possibilities for novel Scalar & Tensor interactions: Higgs-like, leptoquark, loop effects, …



100 GeV
V-A Weak decay
e_L with a ν_L

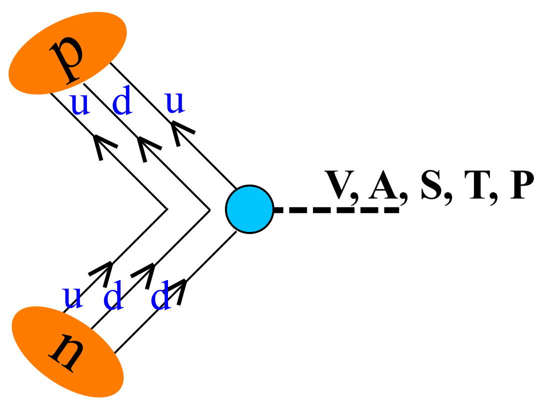


TeV
Novel S,T interactions
e_R with a ν_L



LHC

Neutron
Decay

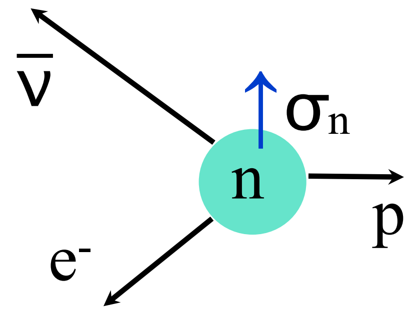


Effective Theory @ ~2 GeV
V-A (g_A, g_V) Weak interactions
S, T (g_S, g_T) New Interactions

[Ultra] Cold Neutron Decay:

Terms sensitive to new physics

Neutron decay can be parameterized as



$$d\Gamma \propto F(E_e)\left[1 + b\frac{m_e}{E_e} + \left(B_0 + B_1\frac{m_e}{E_e}\right)\frac{\vec{\sigma}_n \cdot \vec{p}_\nu}{E_\nu} + \cdots\right]$$

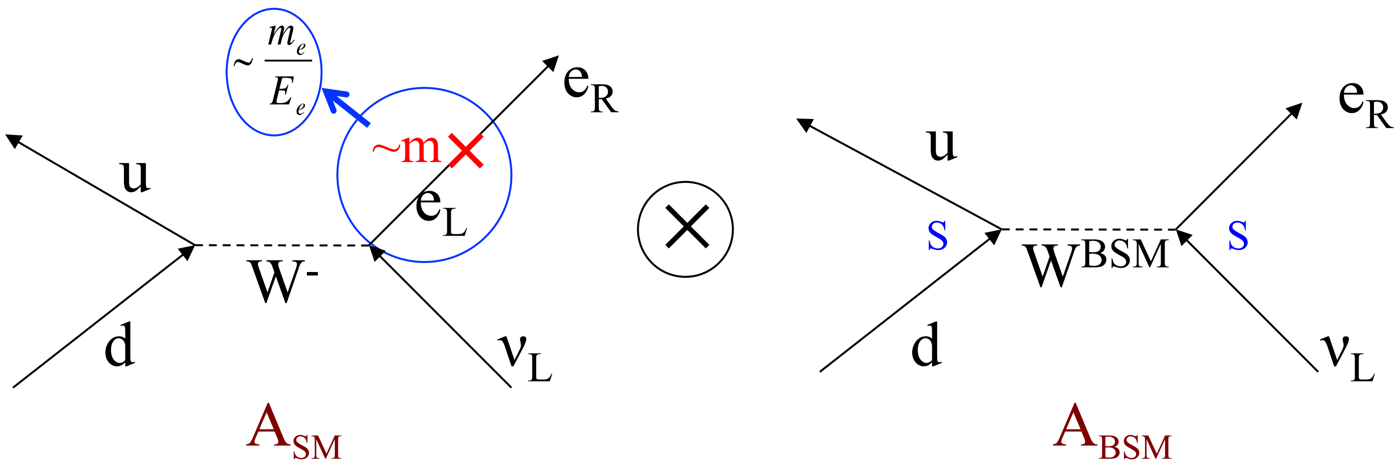
b: Deviations from the leading order electron spectrum: Fierz interference term

B_1: Energy dependent part of antineutrino correlation with neutron spin

Novel S and T interactions at TeV scale

→ Effective Theory at low energy

At leading order, contributions from BSM physics arise due to interference of A_{SM} and A_{BSM} and contribute to b and B_1 only through ε_S and ε_T



$$H_{\rm eff} = G_F \left[J^{\rm lept}_{V-A} \times J^{\rm quark}_{V-A} + \sum_{n=1,10} \epsilon^{\rm BSM}_n \hat{O}_n \right]$$

$\epsilon_S \;\; \bar{u} d \times \bar{e}(1-\gamma_5)\nu_e$

$\epsilon_T \;\; \bar{u}\sigma_{\mu\nu} d \times \bar{e}\sigma^{\mu\nu}(1-\gamma_5)\nu_e$

Physics Case: (BSM/SM) ~ O(1)

- Couplings $\epsilon_{P,S,T}$ $\sim (\Lambda_{BSM})^2/G_F \sim (v/\Lambda_{BSM})^2 \sim 10^{-3}$
- Recoil corrections: $q/M_N \sim 10^{-3}$
- Radiative corrections: $\alpha_{em}/\pi \sim 10^{-3}$
- Isospin-breaking: $(M_N\text{-}M_P)/M_N \sim q/M_N \sim 10^{-3}$
- UCN: small Doppler broadening of *e* spectrum
- SM contribution is $O(10^{-3})$ and known to $(\sim 10^{-5})$: It is controlled by 2 small parameters $(M_n\text{-}M_p)/M_n$ and α_{em}/π
- Unique: scalar and tensor BSM interactions involve helicity-flip (m_e/E_e suppression) and are hard to detect in high energy experiments

Relating b, B_1 to $g_{S,T}$ & BSM couplings $\varepsilon_{S,T}$

$$H_{eff} \supset G_F \left[\varepsilon_S \, \bar{u} d \times \bar{e}(1-\gamma_5)\nu_e + \varepsilon_T \, \bar{u}\sigma_{\mu\nu} d \times \bar{e}\sigma^{\mu\nu}(1-\gamma_5)\nu_e \right]$$

$$g_S = Z_S \langle p | \bar{u} d | n \rangle \quad g_T = Z_T \langle p | \bar{u}\sigma_{\mu\nu} d | n \rangle$$

Lattice QCD

Linear order relations from $n \rightarrow p\, e\, \bar{\nu}$ decay

$$b^{BSM} \approx 0.34\, g_S \varepsilon_S - 5.22\, g_T \varepsilon_T$$

$$b_\nu^{BSM} \equiv B_1^{BSM} = E_e \frac{\partial B^{BSM}(E_e)}{\partial m_e} \approx 0.44\, g_S \varepsilon_S - 4.85\, g_T \varepsilon_T$$

What we know

- Experiment
  - $g_A = 1.2701(25)$ Neutron decay
- Phenomenology: CVC

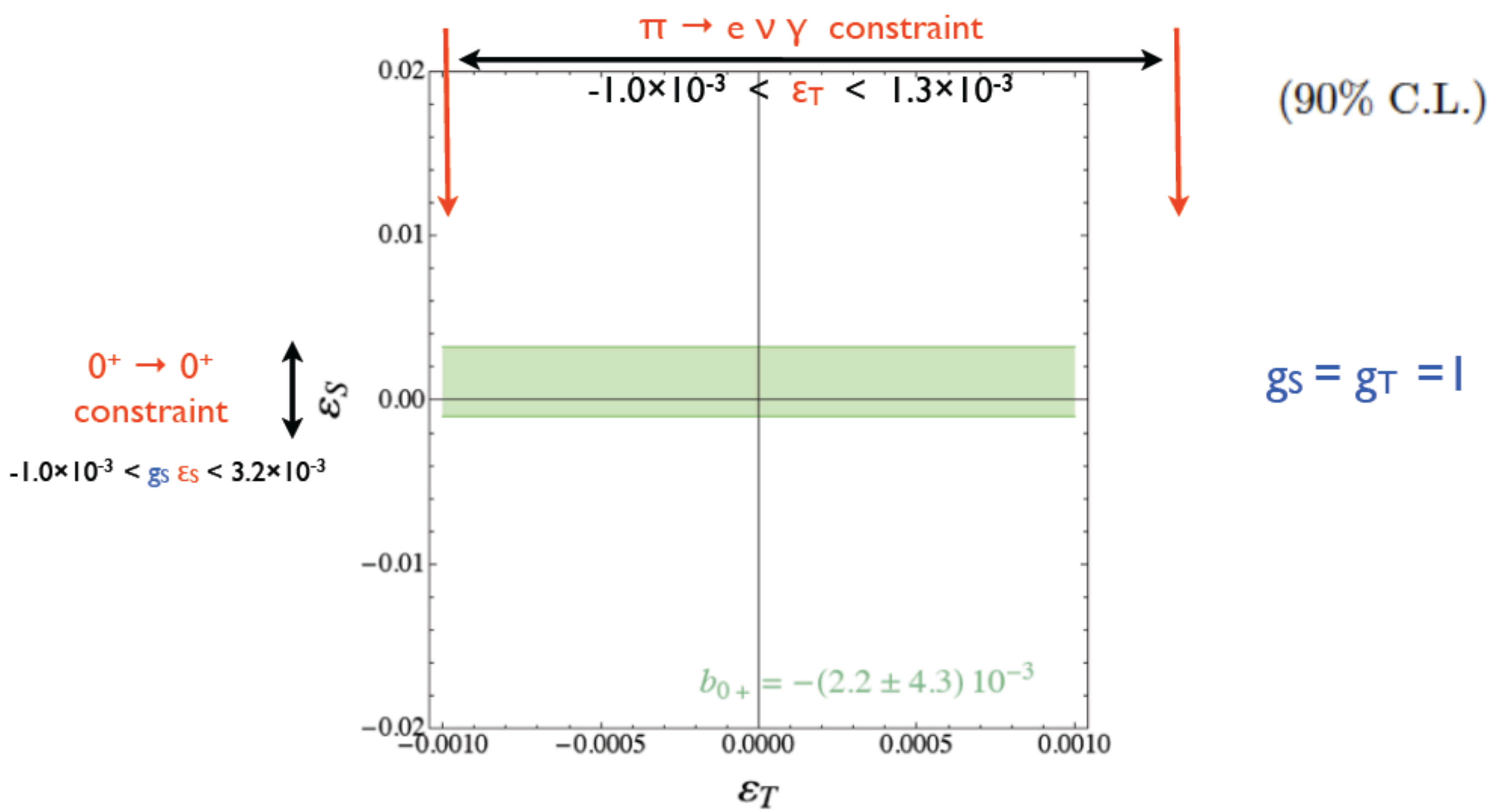
$$\frac{g_S}{g_V} = \frac{(M_N - M_P)^{QCD}}{(m_d - m_u)^{QCD}} = 1.02(8)(7)$$

Gonzalez-Alonso & Camalich
Phy. Rev. Lett. 112 (2014) 042501

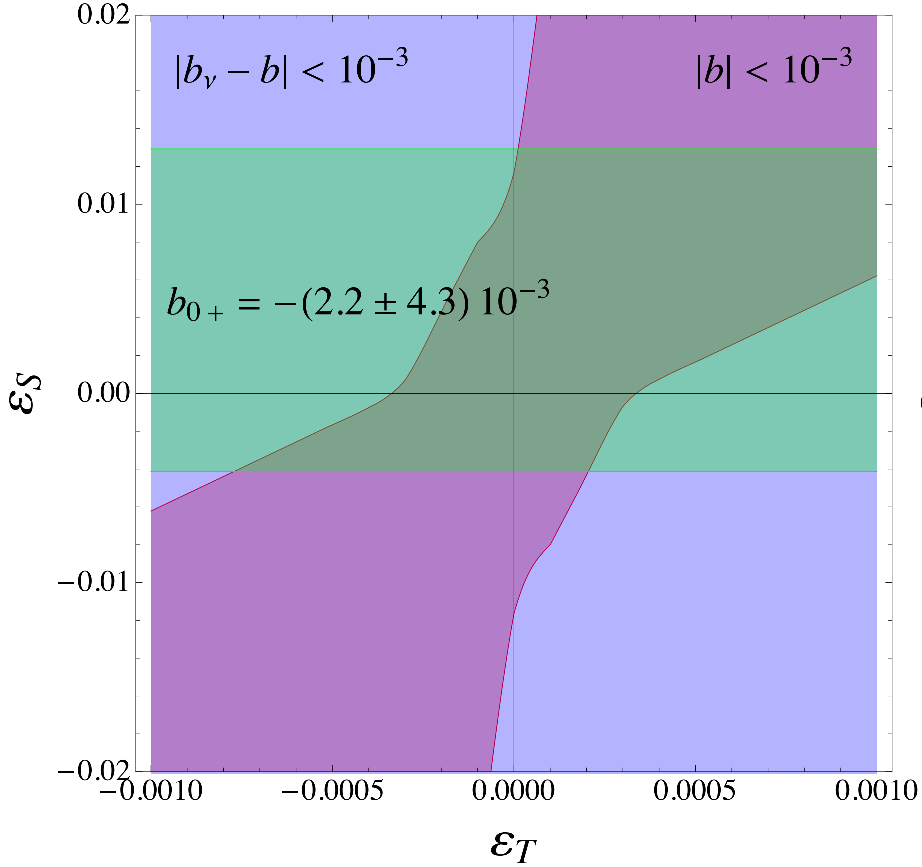
Lattice QCD can provide precise estimates of nucleon structure
- Charges (g_A, g_S, g_T)
- Vector and axial form factor
- Generalized Parton Distribution functions

Low energy constraints

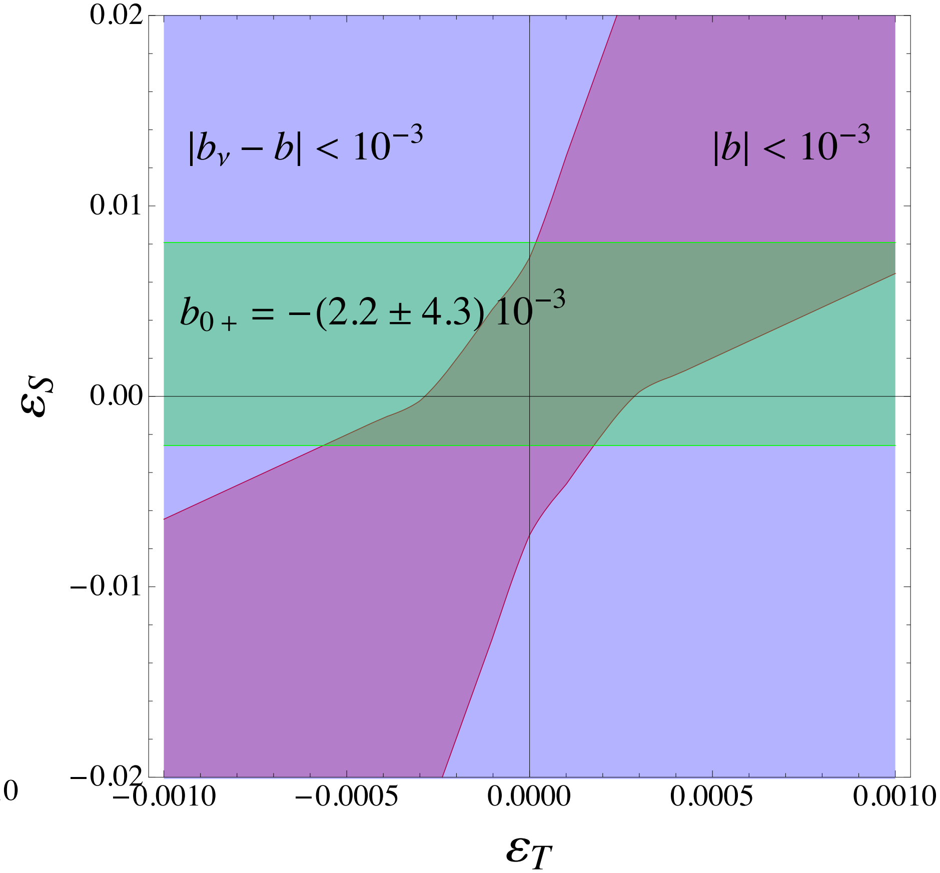
- Current: $0^+ \to 0^+$ and $\pi \to e \nu \gamma$



Allowed region in ε_S and ε_T are being constrained as estimates of g_S and g_T improve

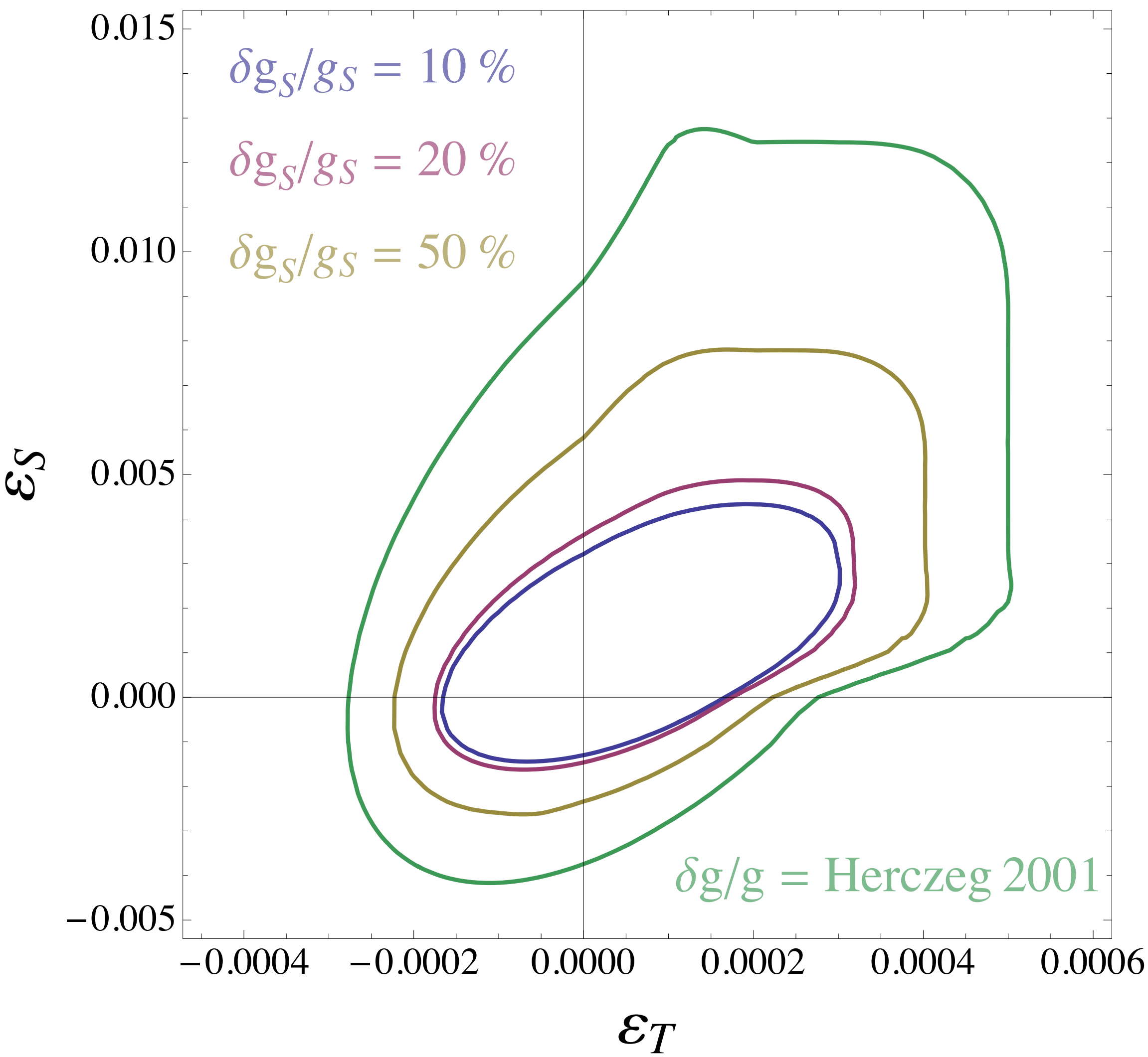


Herczeg–2001
$0.25 < g_S < 1.0$
$0.6 < g_T < 2.3$



Lattice–2011
$g_S = 0.8(4)$
$g_T = 1.05(35)$

Target Precision for g_S, g_T: **10-20%**

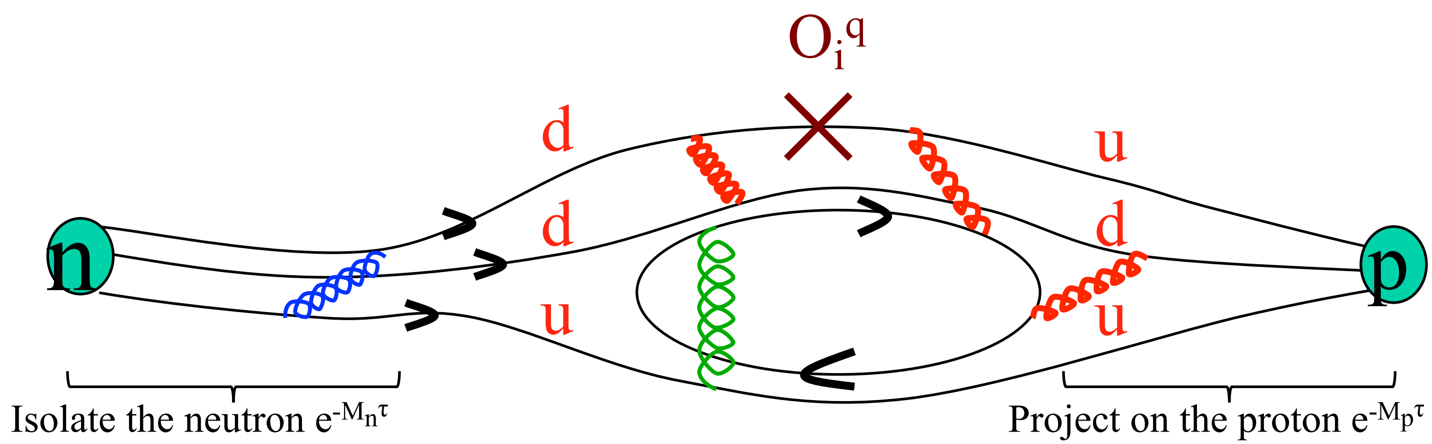


Allowed region in [ε_S , ε_T] (90% contours)

Expt. input

$|B_1\text{-}b| < 10^{-3}$
$|b| < 10^{-3}$
$b_{0+} = 2.2\ (4.3)*10^{-3}$

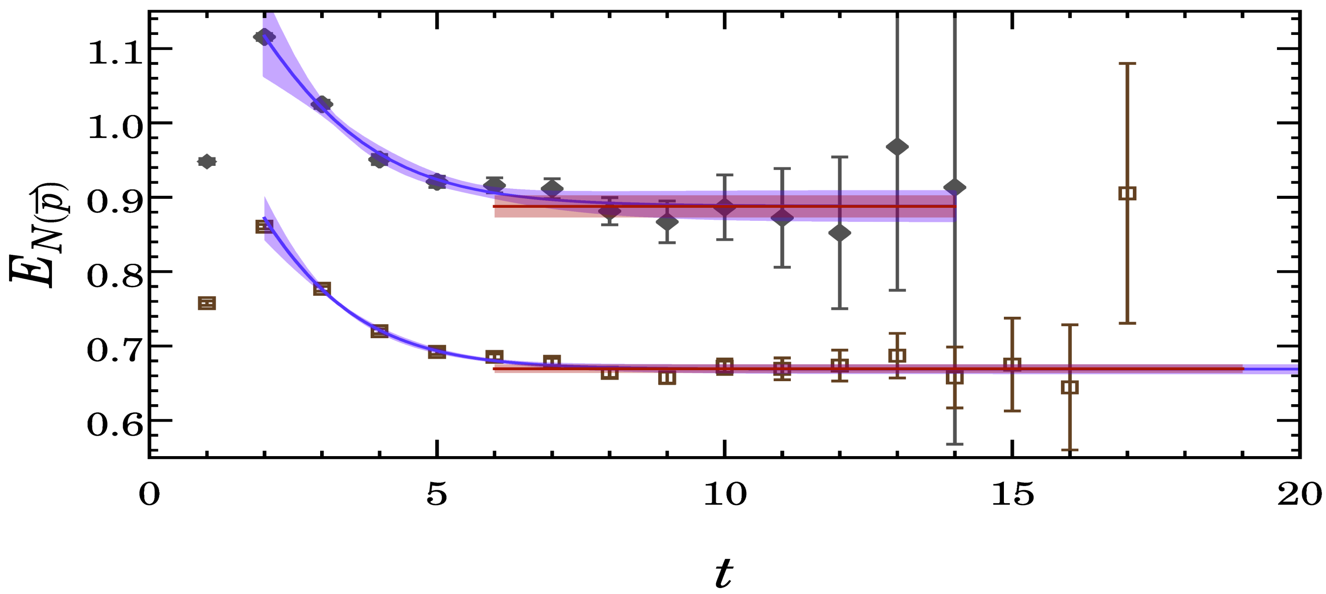
Precision Lattice QCD calculations: $\langle p|\bar{u}\,\Gamma d|n\rangle$



- **Achieving 10-20% uncertainty is a realistic goal but requires:**
  - High Statistics (O(15000) measurements)
  - Controlling all Systematic Errors:
    - Contamination from excited states
    - Non-perturbative renormalization of bilinears (RI_{smom} scheme)
    - Finite volume effects
    - Chiral extrapolation to physical m_u and m_d (simulate at physical point)
    - Extrapolation to the continuum limit (lattice spacing $a \rightarrow 0$)

Reducing excited state contamination
2-point correlation function $\rightarrow M_N$

$$\Gamma^2(t_f, t_i) = A_0\, e^{-M_0 \Delta t} + A_1\, e^{-M_1 \Delta t} + \ldots$$



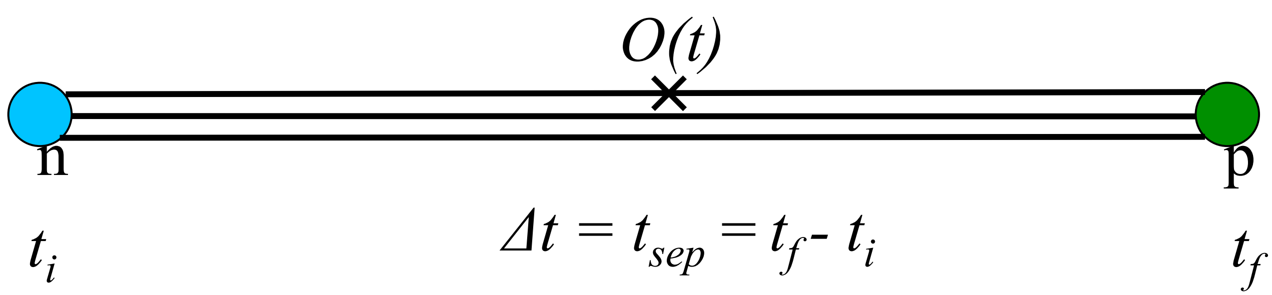
Current data are fit by including 1 “excited” state

Reducing excited state contamination: 3-pt fn.

Assuming 1 excited state, the 3-point function is given by

$$\Gamma^3(t_f, t, t_i) = |A_0|^2 \langle 0|O|0\rangle e^{-M_0 \Delta t} + |A_1|^2 \langle 1|O|1\rangle e^{-M_1 \Delta t} + A_0 A_1^* \langle 0|O|1\rangle e^{-M_0 \Delta t} e^{-M_1(\Delta t - t)} + A_0^* A_1 \langle 1|O|0\rangle e^{-M_1 \Delta t} e^{-M_0(\Delta t - t)}$$

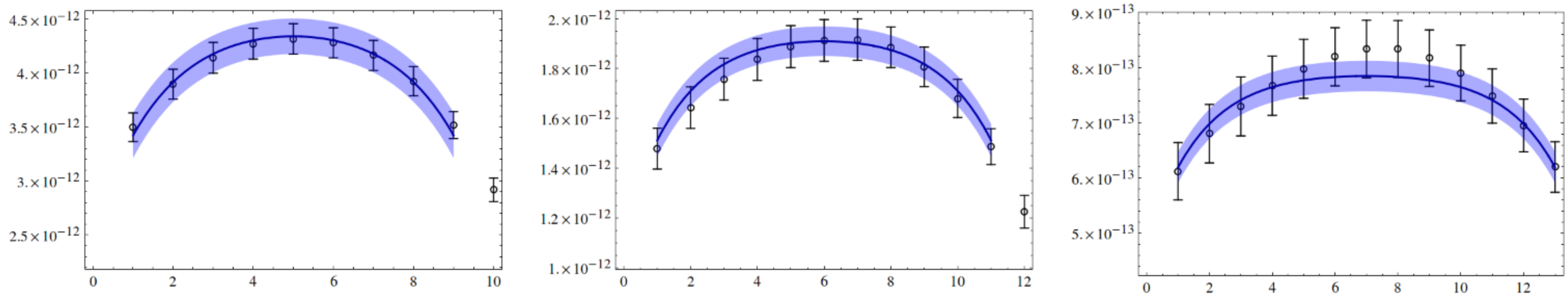
Where M_0 and M_1 are the masses of the ground & excited state and A_0 and A_1 are the corresponding amplitudes.



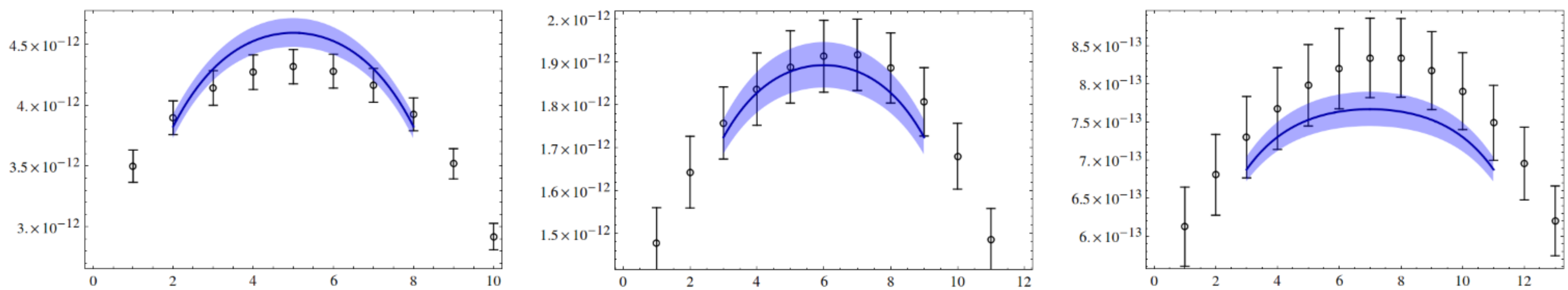
Need simultaneous fit to data at multiple $\Delta t = t_{sep} = t_f - t_i$

Simultaneous fit to multiple t_{sep}

Data for g_S on the M_π=220 MeV ensemble at a=0.09fm



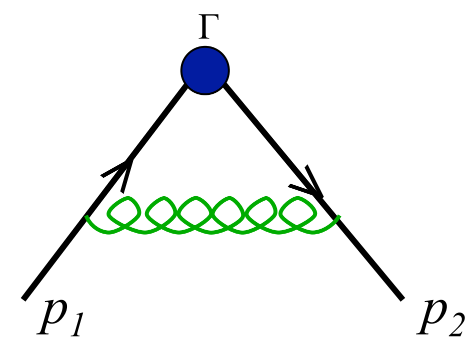
Excluding $\langle 1|O|1\rangle$ Term



t_{sep}=10 t_{sep}=12 t_{sep}=14

Renormalization of bilinear operators

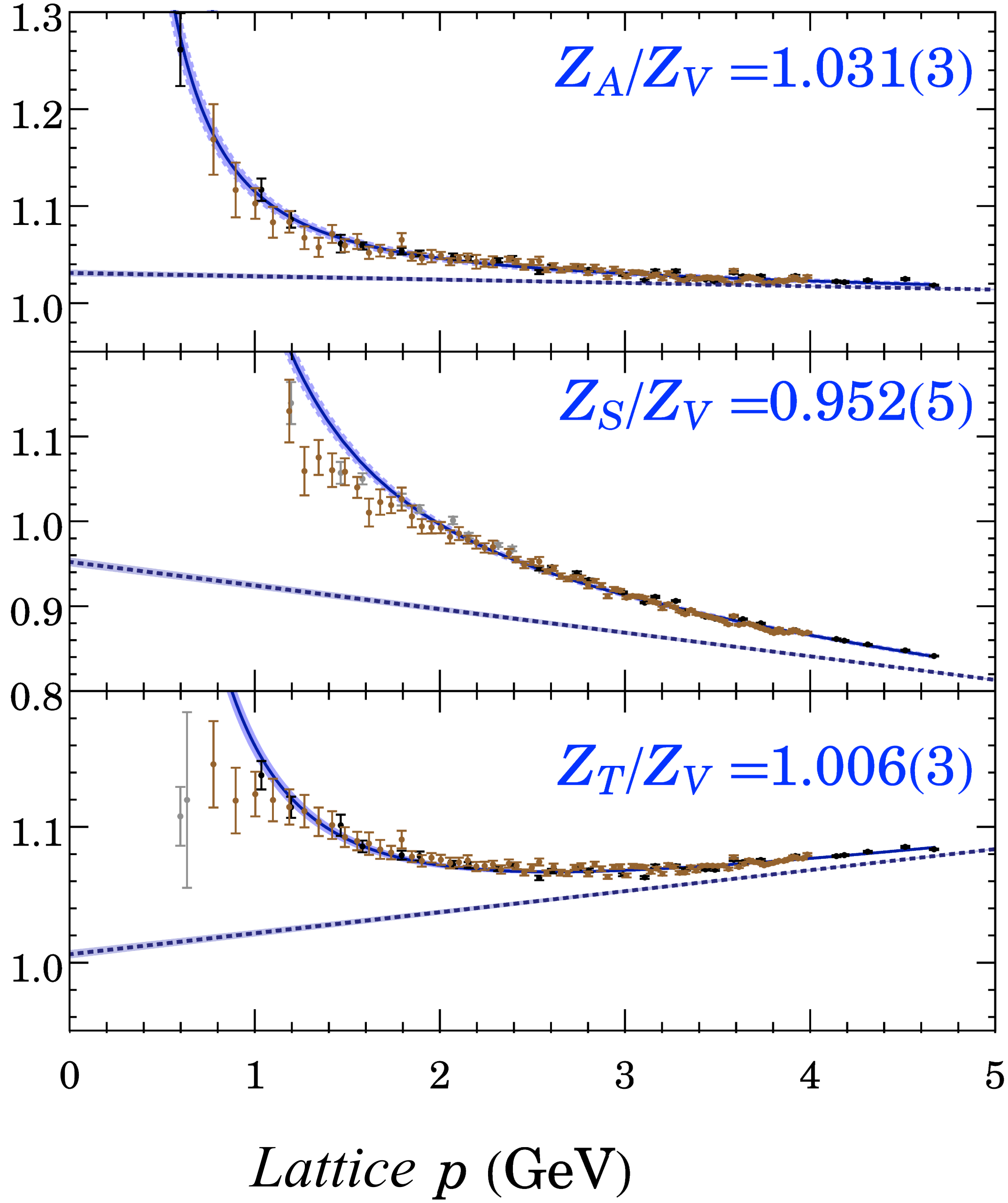
- Non-perturbative renormalization factors Z_Γ using the RI-sMOM scheme ($p_1^2 = p_2^2 = q^2$)
  - Need quark propagator in momentum space



- Basic Assumption: there exists a window

$$\Lambda_{QCD} << p << \pi/a$$

- HYP Smearing introduces artifacts
  - Gluon momentum above (~$1/a$) are averaged out
  - $\Lambda_{QCD} << p << \pi/a$ window may not exist on coarse lattices
- No detectable dependence of Z's on m_q



$$\frac{Z_{A,S,T}}{Z_V}\left(\overline{MS}, 2\,GeV\right)$$

Fit data to: $A/p + Z + Cp$
in the range $\{1<p<4$ GeV$\}$
Or
Choose Z at $p^2 = 5GeV^2$
& errors from $\{4<p^2<6GeV^2\}$

Renormalized Charges

$$\frac{Z_{A,S,T}}{Z_V} \times \frac{g_{A,S,T}}{g_V}$$

Ward identity: $Z_V g_V = 1$

Observations and Lessons Learned

- For given statistics: $\sigma(g_S) \sim 5\ \sigma(g_A)$ [or $5\ \sigma(g_T)$]

  Need O(15000) independent measurement (Configs ✕ Sources)

- Excited state contamination is significant but controlled

  Need: data at multiple t_{sep} with good signal for $t_{sep} > 1.2$fm

  fits including at least one excited state to data $t_{sep} > 1.0$fm

- Renormalization (RI-sMOM): Smearing introduces artifacts

  Impose a prescription with a well-defined continuum limit

Analyzing lattice data:

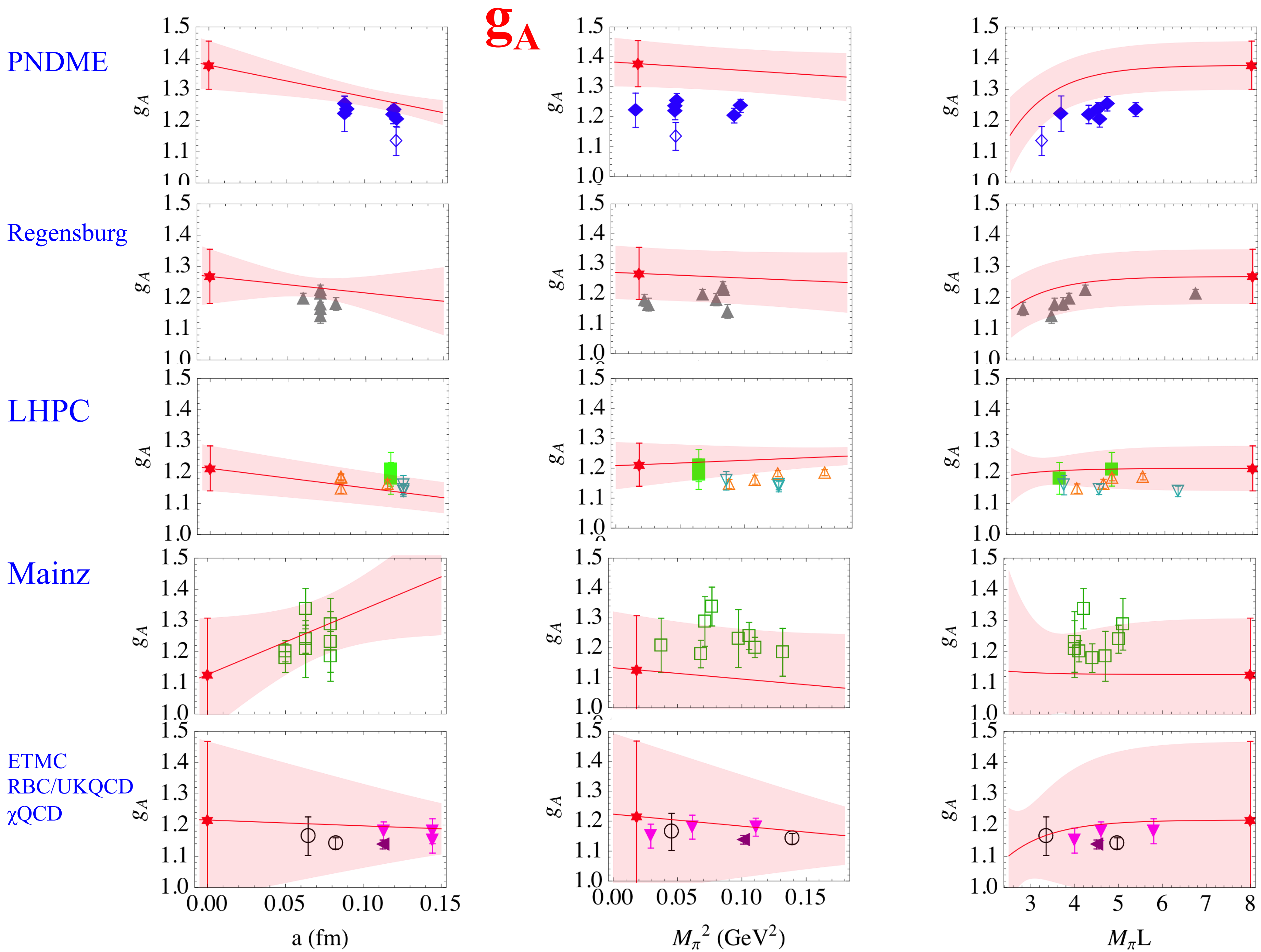
Extrapolations in a, M_π^2, L

Using lowest order corrections when fitting lattice data w.r.t.

- Lattice spacing a
- Dependence on quark mass $m_q \sim M_\pi^2$
- Finite volume $M_\pi L$

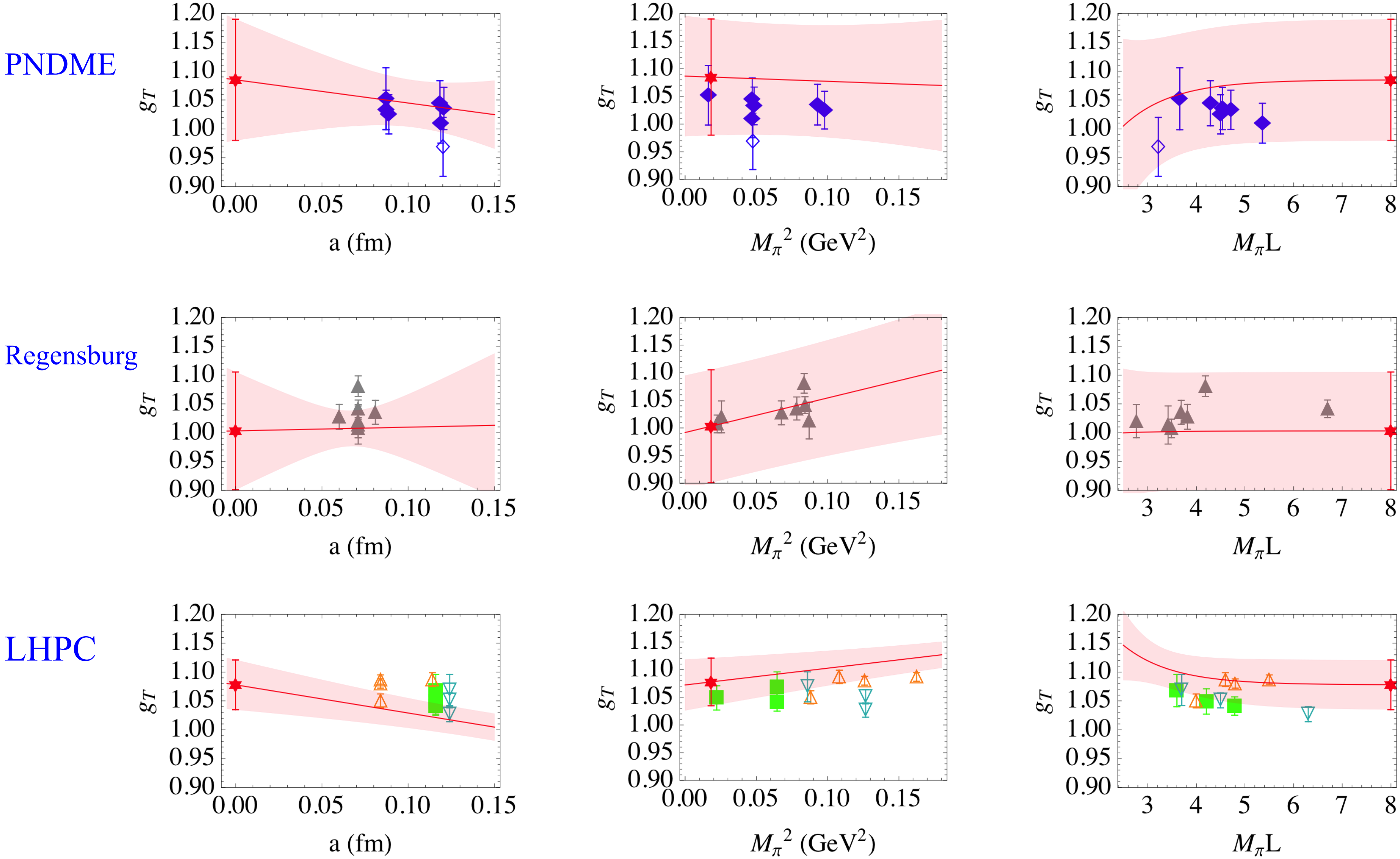
$$g(a, M_\pi, L) = g + A\, a + B\, M_\pi^2 + C\, e^{-M_\pi L} + \ldots$$

Lattice QCD Calculations are ongoing and collaborations are addressing all sources of systematic errors

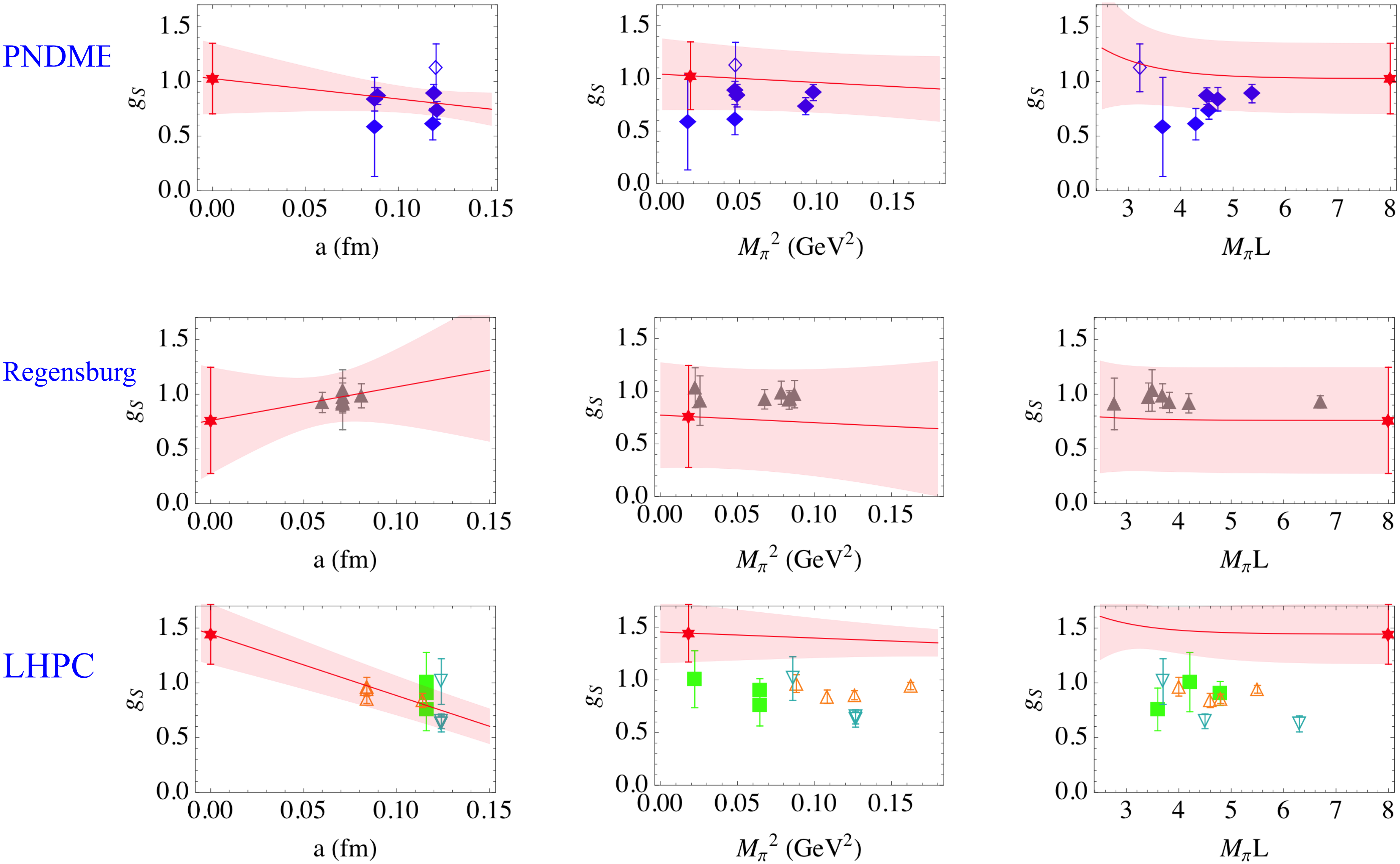


g_A
PNDME
Regensburg
LHPC
Mainz
ETMC
RBC/UKQCD
χQCD
g_A
a (fm)
M_π^2 (GeV^2)
M_πL

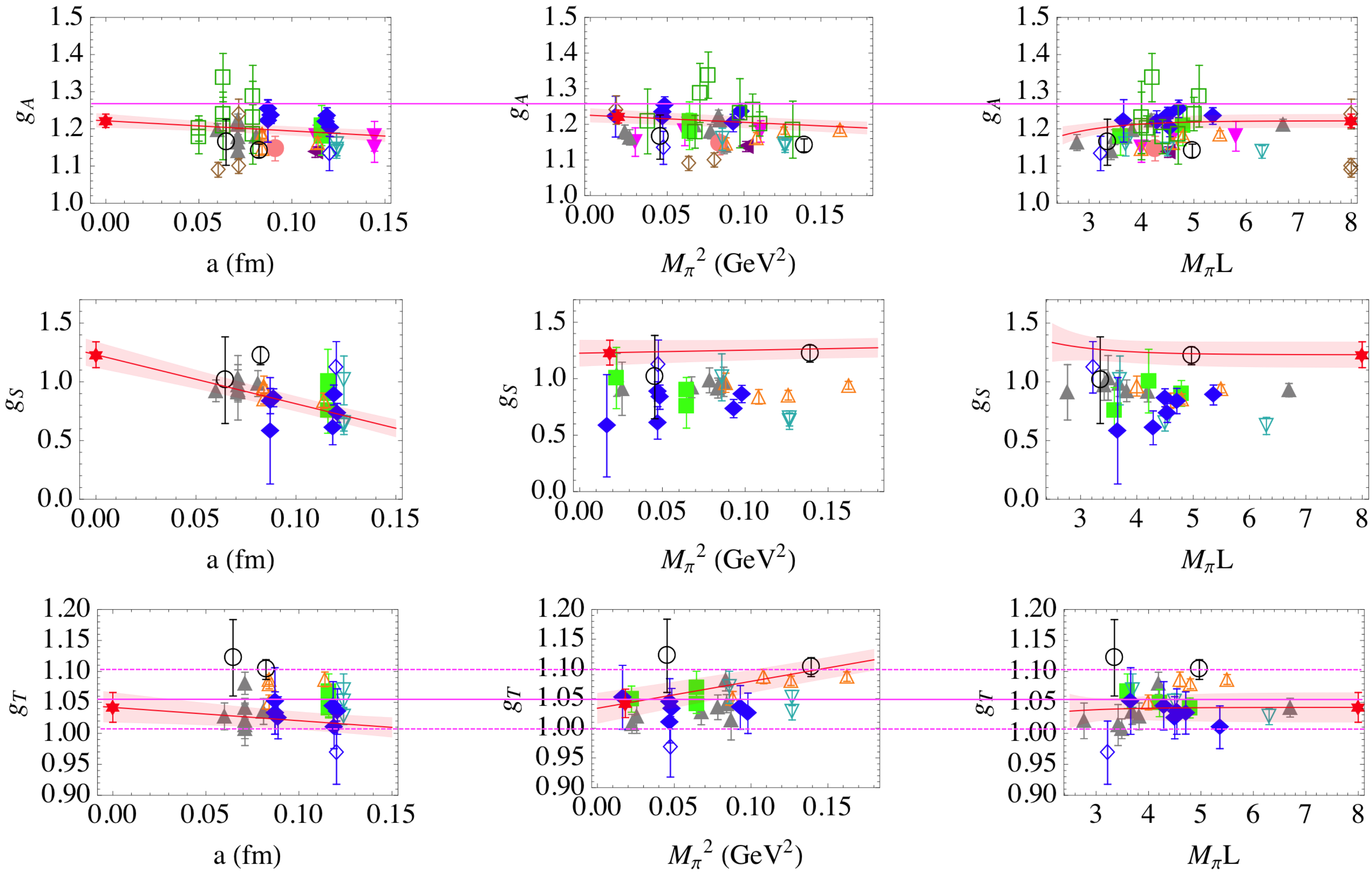
g_T


PNDME
Regensburg
LHPC
gT
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.00
0.05
0.10
0.15
a (fm)
Mπ² (GeV²)
MπL
3
4
5
6
7
8

g_S


PNDME
Regensburg
LHPC
g_S
a (fm)
M_π^2 (GeV^2)
M_πL
0.0
0.5
1.0
1.5
0.00
0.05
0.10
0.15
3
4
5
6
7
8

lattice data: combined plots



Towards Physical Estimates

- g_T: 1.05(5)
  - Estimate of Z_T is reliable
  - Small dependence on a, M_π^2, $M_\pi L$ (Caution: see g_A)
- g_A:
  - Estimate of Z_A is reliable
  - Extrapolations in a, M_π^2, $M_\pi L$ are not yet resolved
- g_S:
  - Statistical errors are large
  - Z_S is not well-determined
  - Extrapolations in a, M_π^2, $M_\pi L$ are not stable

Constraining ε_S and ε_T

- Measurements of b and B_1 at 10^{-3} to 10^{-4} will probe multi-TeV scale and place stringent constraints on novel scalar and tensor interactions

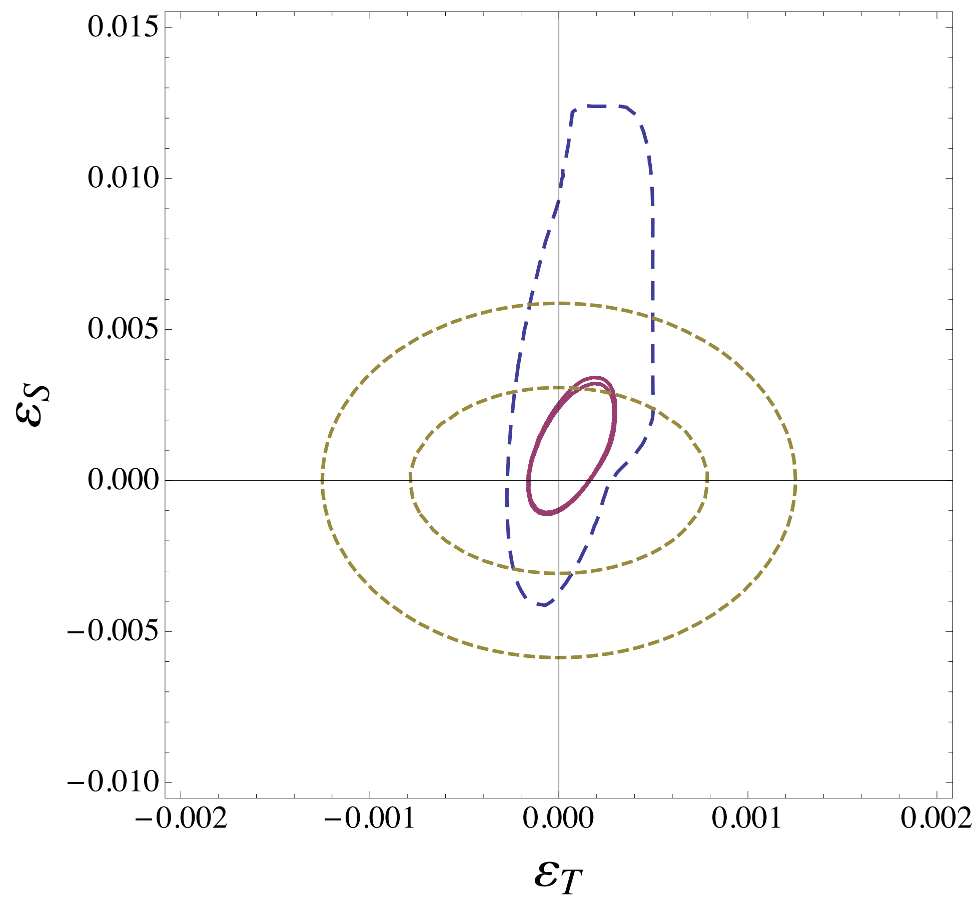
  - $G_F * \varepsilon_{S,T} = (1/\Lambda_{S,T})^2$
  - $\varepsilon_{S,T} = v^2/\Lambda^2_{S,T} \sim 10^{-3}$

  $\Lambda_{S,T} \sim 5$ TeV

- Constraints on ε_S and ε_T from [U]CN experiments combined with improved g_S and g_T will be more stringent than existing probes ($0^+ \rightarrow 0^+$; $\pi \rightarrow e\nu\gamma$).

- Collider experiments are not competitive until $\sqrt{s}$=14 TeV & 100 fb^{-1}

Constraints from β-decay versus LHC



LHC bounds:

- 10 fb^{-1} at 14TeV
- 300 fb^{-1} at 14TeV

Nuclear exp + Theory bounds:

- $b = B_1 = 10^{-3}$
  - g_S and g_T from Herczeg
  - g_S and g_T from Lattice QCD

Improving low energy bounds further depend on improvements in nuclear experiments for b and B_1