The Entropy of Hawking Radiation

Netta Engelhardt

MIT

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The Role of Quantum Information in Quantum Gravity

- Connection between QI and quantum gravity: QI appears to be an important ingredient in gravitational physics.

- E.g. the AdS/CFT correspondence: von Neumann entropy, complexity, quantum error correction, quantum secret sharing, etc. play an important role in the description of space and time in quantum gravity.

- Very useful in the context of the black hole information paradox and Hawking radiation.

Hawking Radiation and Evaporating Black Holes

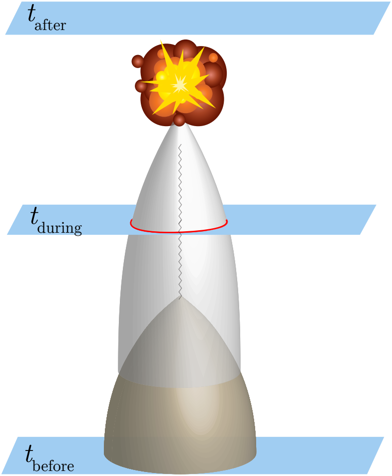
Black Hole Thermodynamics

Black holes are quantum mechanical systems; in particular they are thermal objects: they have a temperature, and they radiate.

$$T_{BH} \propto M_{BH}^{-1}$$

If $T_{BH} > T_{\text{surroundings}}$, the black hole shrinks as it radiates, eventually evaporating altogether.

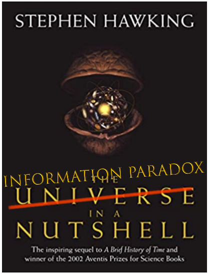
Black Hole Formation and Evaporation



Hawking Radiation In a Nutshell

1. Some pure state collapses into a black hole.
   Before the black hole forms:

$$S_{\mathrm{vN}}[\rho_{\text{outside BH}}] = S_{\mathrm{vN}}[\rho_{\text{universe}}] = 0$$



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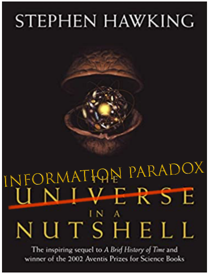

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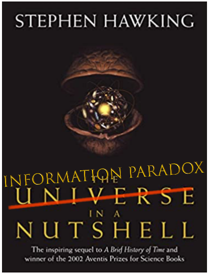

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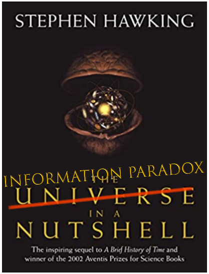
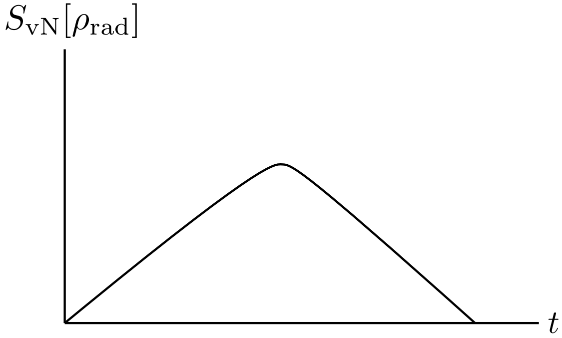
4. So we expect the Page curve for $S_{\mathrm{vN}}[\rho_{\mathrm{rad}}]$.





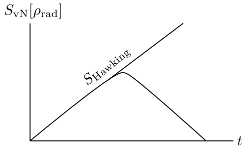
$S_{\mathrm{vN}}[\rho_{\mathrm{rad}}]$
t

The Paradox

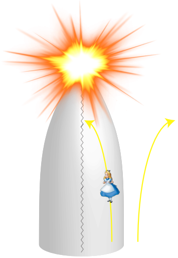
- To compute $S_{\mathrm{vN}}[\rho_{\mathrm{rad}}]$, we need a description of the physics at the black hole event horizon.

- *Semiclassical gravity*: the limit where large quantum gravity effects are suppressed; should be valid at the horizon.

- Semiclassical gravity $\overset{\text{Hawking}}{\Rightarrow}$ $-\mathrm{tr}\rho_{\mathrm{rad}} \ln \rho_{\mathrm{rad}}$ does *not* follow the Page curve:



Caricature of Information Loss



Quick argument for info loss:

$$|\psi\rangle = |+\rangle|-\rangle + |-\rangle|+\rangle$$

After evaporation, Alice disappears from the universe – even though the universe is a closed system.

State after evaporation is just $\rho_{\text{Bob}} = \frac{1}{2}\mathbb{I}$.

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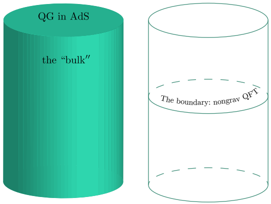
- It is hard to see where Hawking's analysis goes wrong.
- Are there large corrections from quantum gravity in a regime where there shouldn't be?
- Progress: a semiclassical calculation of the Page curve, but using a different – holographic – formula for S_{vN}.
- This formula is a bit of a black box: can be interpreted as indirect input from quantum gravity (or a saddle point analysis of Euclidean quantum gravity)

Holography

Holography

Quantum gravity "in a box" (with Anti-de Sitter boundary conditions) is dual to a lower-dimensional nongravitational QFT.



Colloquially, we call the gravitational theory the bulk, and the non-gravitational theory the boundary.

Holography: a Black Hole is just a Quantum System

This means that a black hole in AdS is just an ordinary, nongravitational quantum system.

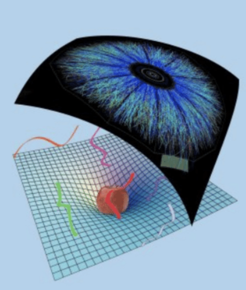


image credit: ESI Programme on AdS Holography and the Quark-Gluon Plasma

... nongravitational (closed) quantum systems evolve unitarily.

Computing the Page Curve

We know that $S_{\text{vN}}[\rho_{\text{rad}}]$ computed in the nongravitational “boundary” theory follows the Page curve.

AdS/CFT is a dictionary: it relates quantities in the boundary theory to the bulk theory.

What quantity in the bulk theory translates to $S_{\text{vN}}[\rho_{\text{rad}}]$ as computed by the boundary theory?

Holographic von Neumann Entropy

S_{vN} in AdS/CFT Ryu-Takayanagi, Hubeny-Rangamani-Takayanagi, Faulkner-Lewkowycz-Maldacena, Engelhardt-Wall

The von Neumann entropy of the radiation is given by:

$$S_{\mathrm{vN}}[\rho_{\mathrm{rad}}] = \frac{\mathrm{Area}[\chi]}{4} + S_{\mathrm{vN}}[\rho_{\mathrm{Out}[\chi]}]$$

where χ is a "quantum extremal surface": if you slightly perturb χ, the sum $\frac{\mathrm{Area}[\chi]}{4} + S_{\mathrm{vN}}[\rho_{\mathrm{Out}[\chi]}$ won't change to leading order in the perturbation.

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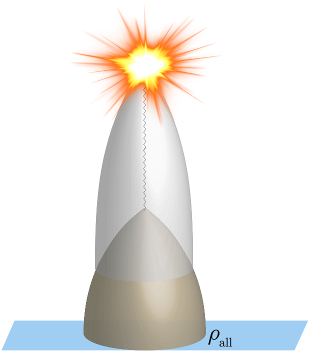
We get $\rho_{\mathrm{out}[\chi]}$ from a bipartition of the system into interior of χ and exterior.

Formula justified from euclidean gravity in evaporating black hole setup by Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini; Penington, Shenker, Stanford, Yang

Quantum Extremal Surfaces

Initially, the QES is the empty set.

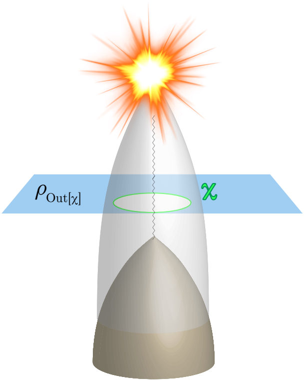
$$S_{vN}[\rho_{bdy}] = \frac{\text{Area}[\varnothing]}{4G\hbar} + S_{\text{vN}}[\rho_{\text{all}}] = S_{\text{vN}}[\rho_{\text{all}}]$$

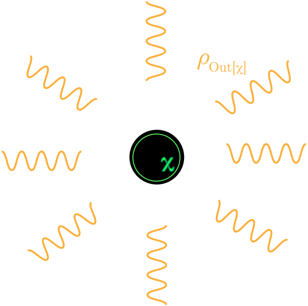


With time $S_{\text{vN}}[\rho_{\text{all}}]$ grows.

QESs in Evaporating BHs Almheiri, Engelhardt, Marolf, Maxfield; Penington

About halfway through evaporation, a new quantum extremal surface appears:





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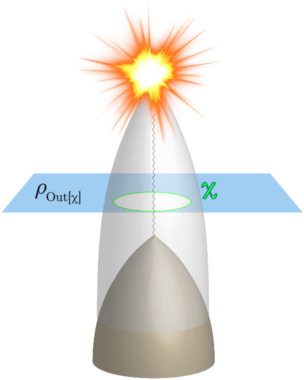

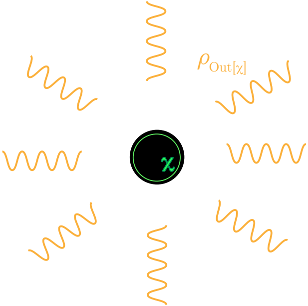

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This alternative computation of S_{vN} in gravity *does* yield a unitary Page curve.

The Entanglement Wedge

What is the significance of the region Out[χ]?

Reconstruction in AdS/CFT?

As Douglas discussed, AdS/CFT is a quantum error correcting code see work beginning with Almheiri, Dong, Harlow. How much of the bulk can we reconstruct from the state ρ_{bdy} of the nongravitational dual?

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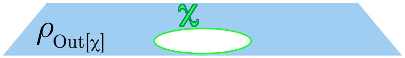
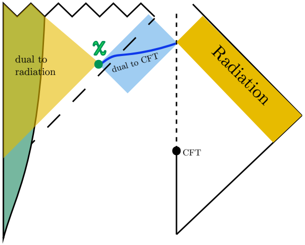
Everything in Out[χ].*

The Entanglement Wedge of the Radiation

After the Page time, this is most of the black hole interior: information that fell into the black hole is now available in the radiation.





Dependence on the Code Subspace

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The Code Subspace

Less naively: that depends on the code subspace. Given a choice of code subspace, we can reconstruct everything in Out[χ_M], where χ_M is the QES for the maximally mixed state in our code subspace Hayden, Penington.

The larger your code subspace, the less of the bulk you can reconstruct!

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- **Hayden-Preskill:** Assume BH dynamics drawn from unitary 2-design. Take an old black hole B and its radiation E. Throw $|i\rangle$ into it. The info is recoverable from the radiation in a time logarithmic in S_{BH}:

$$V_{EL} \left|\Psi\right\rangle_{B'EL} = |i\rangle_1 \otimes |\phi\rangle_{B',2}$$

  where E is the early radiation, L is the late radiation (after $|i\rangle$ is thrown in) and B' is the black hole after L has radiated (and 1 and 2 are factors of $\mathcal{H}_{LE}$). But:

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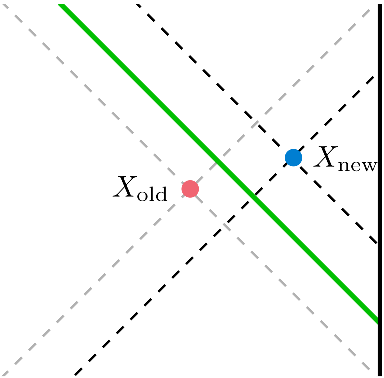
- **Harlow-Hayden**: decoding the radiation is exponentially complex in the entropy of the black hole. **Aaronson:** decoding the radiation is polynomially hard $\Rightarrow$ there are no injective one-way functions.

Holographic Realization of Hayden-Preskill

If you throw something into an evaporating black hole, it will cross into the entanglement wedge of the radiation (with some delay) as the QES moves outwards.



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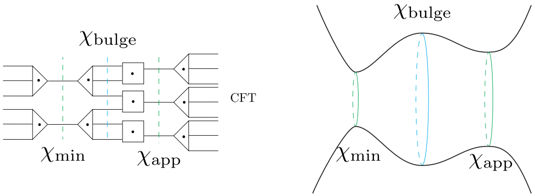

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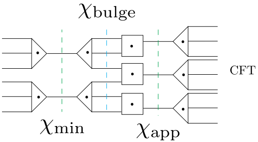

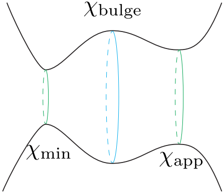

Python's Lunch
Brown, Gharibyan, Susskind, Penington; Engelhardt, Penington Shahbazi-Moghaddam

Reconstruction of a bulk operator is exponentially hard if and only if it lies behind a nonminimal quantum extremal surface. In that case the complexity of reconstruction is proportional to:

$$Exp\left[\frac{1}{2}\left(\frac{\text{Area}[\chi_{\text{bulge}}]}{4}+S[\rho_{\text{Out}[\chi_{\text{bulge}}]}]-\frac{\text{Area}[\chi_{\text{app}}]}{4}-S[\rho_{\text{Out}[\chi_{\text{app}}]}]\right)\right]$$