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Quantum Information and Many-Body Systems

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Solvay 19/05/22

Quantum information and physics: some future directions*

John Preskill,†
Lauritsen Laboratory of High Energy Physics
California Institute of Technology
Pasadena, CA 91125, USA
(April, 1999)
arXiv:quant-ph/9904022

In the future, I expect quantum information to solidify its central position at the foundations of computer science, and also to erect bridges that connect with precision measurement, condensed matter physics, quantum field theory, quantum gravity, and other fields that we can only guess at today. I have identified two general areas in which I feel such connections may prove to be particularly enlightening. Progress in quantum information processing may guide the development of new ideas for improving the information-gathering capabilities of physics experiments. And a richer classification of the phases exhibited by highly entangled many-body systems may deepen our appreciation of the wealth of phenomena that can be realized by strongly-coupled quantum systems.

Overview

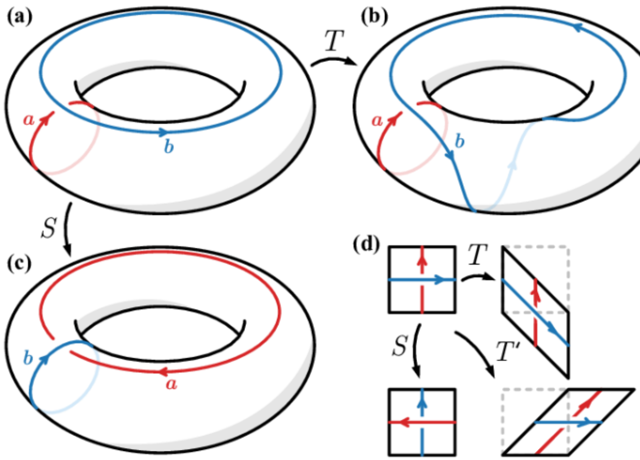
1. Topological Order & Quantum Error Correction
2. The entanglement structure of equilibrium many-body systems
3. Entanglement in non-equilibrium systems

Topological order & quantum error correction

- Knill-Laflamme conditions for Quantum Error-Correcting codes (‘96):

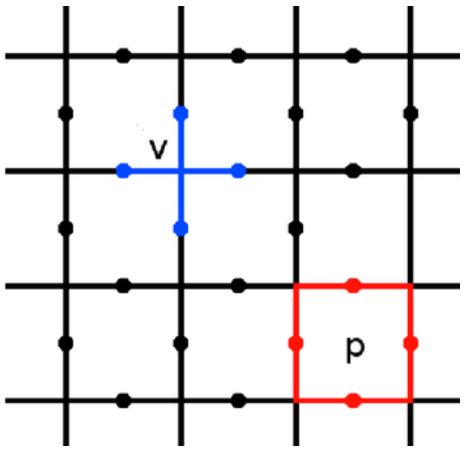
$$\langle\psi_i|E_\alpha^\dagger E_\beta|\psi_j\rangle = \delta_{ij} X_{\alpha\beta}$$

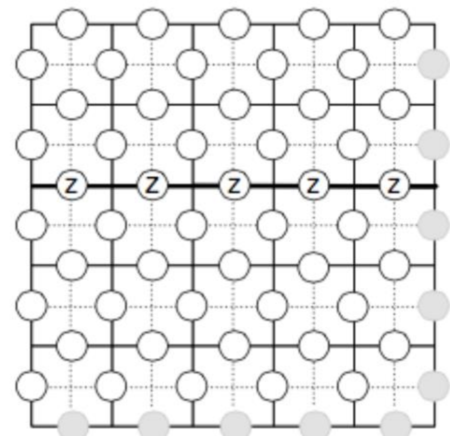
- Xiao-Gang Wen’s notion of topological order (’89):
  - defying Landau’s paradigm of local order parameters

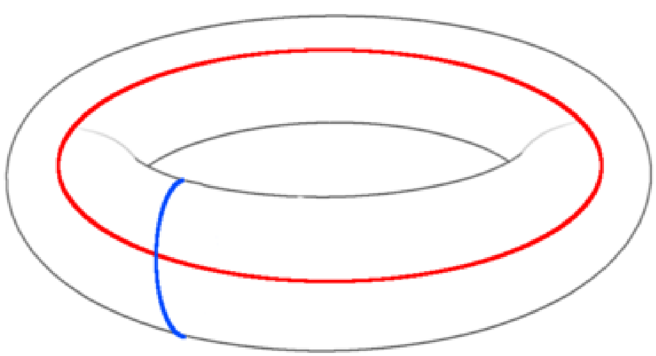


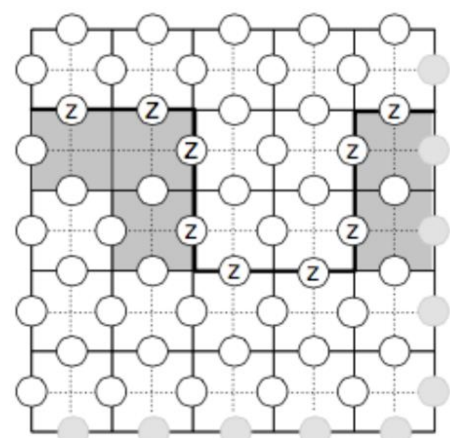
Kitaev: QEC=TO

- Simplest model: toric code / Z_2 lattice gauge theory ('97)
  - Code distance: L ; # qubits: 2

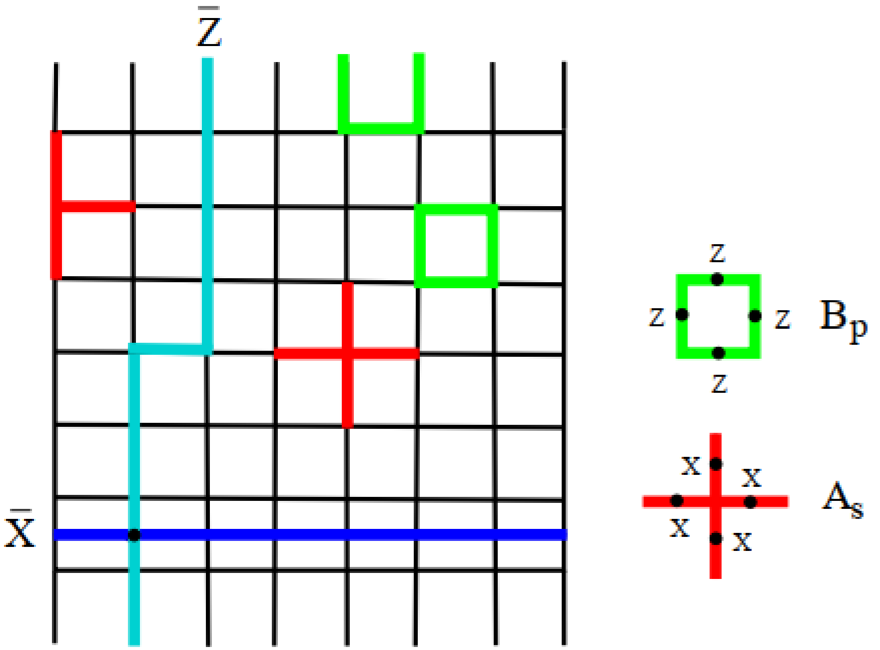




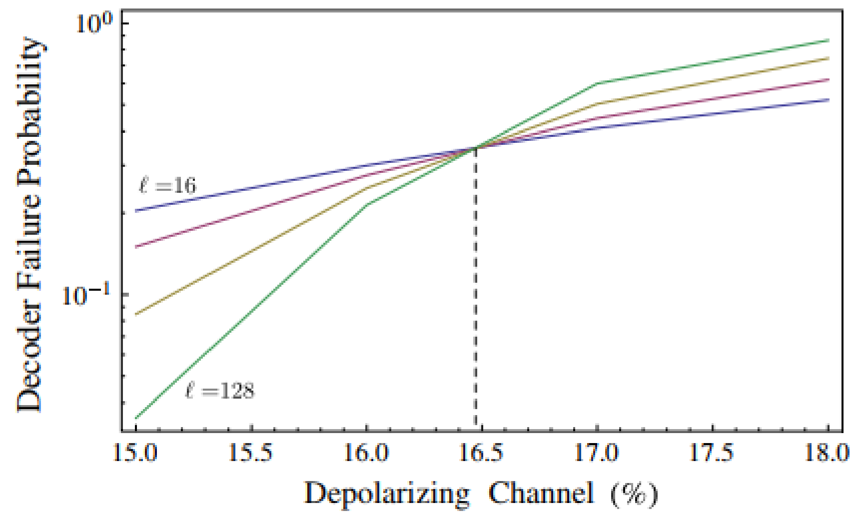




- Surface code / planar code: boundary excitations, magic state distillation, Nishimori line, …



Terhal, RMP '15

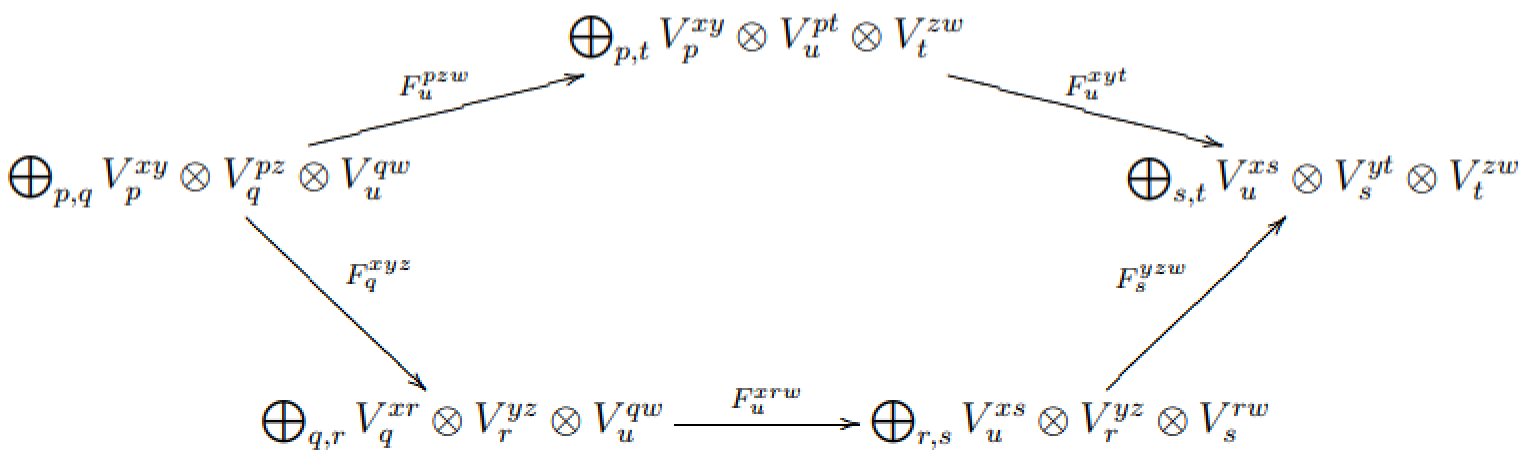


Duclos-Cianci, Poulin PRL '10

- More advanced techniques: code deformation & lattice surgery (Bombin, Kitaev), gauge fixing, … : involve deep abstract mathematical ideas

Appendix E: Algebraic theory of anyons

This appendix is an attempt to present an existing but difficult and somewhat obscure theory in an accessible form, especially for the reader without extensive field theory knowledge.



Appendix E: Algebraic theory of anyons

This appendix is an attempt to present an existing but difficult and somewhat obscure theory in an accessible form, especially for the reader without extensive field theory knowledge.

String-net condensation: A physical mechanism for topological phases

Michael A. Levin and Xiao-Gang Wen
Phys. Rev. B **71**, 045110 – Published 12 January 2005

An article within the collection: *Physical Review B* 50th Anniversary Milestones

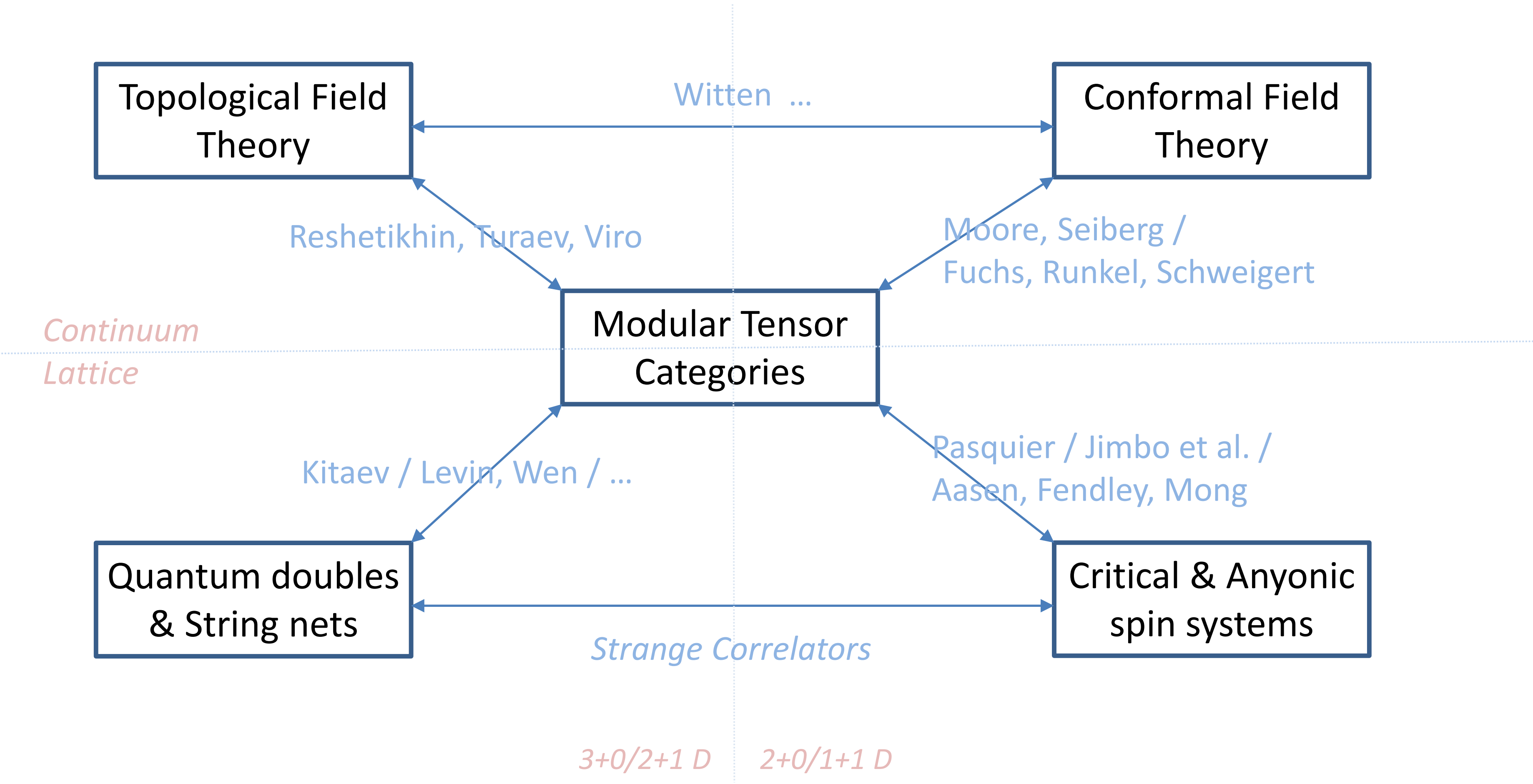
Commun. Math. Phys. 313, 351–373 (2012)
Digital Object Identifier (DOI) 10.1007/s00220-012-1500-5

Communications in **Mathematical Physics**

Models for Gapped Boundaries and Domain Walls

Alexei Kitaev[1], **Liang Kong**[2]

Topological order/ Category Theory: TFT & CFT



TOPOLOGICAL QUANTUM COMPUTATION

MICHAEL H. FREEDMAN, ALEXEI KITAEV, MICHAEL J. LARSEN, AND ZHENGHAN WANG

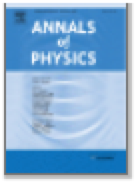
Bull. Amer. Math. Soc. **40** (2003), 31-38

Non-Abelian anyons and topological quantum computation

Chetan Nayak, Steven H. Simon, Ady Stern, Michael Freedman, and Sankar Das Sarma
Rev. Mod. Phys. **80**, 1083 – Published 12 September 2008

ELSEVIER

Annals of Physics
Volume 325, Issue 12, December 2010, Pages 2707-2749



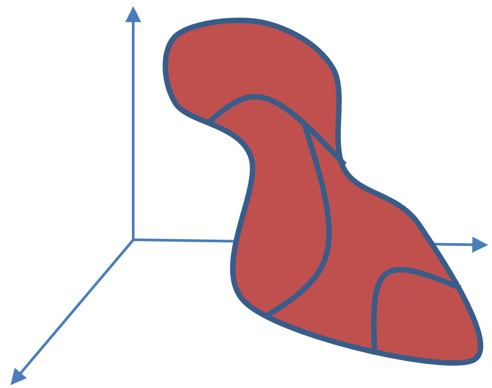
Quantum computation with Turaev–Viro codes

Robert Koenig [a], Greg Kuperberg [b], Ben W. Reichardt [c]

Quantum Error Correction Thresholds for the Universal Fibonacci Turaev-Viro Code

Alexis Schotte, Guanyu Zhu, Lander Burgelman, and Frank Verstraete
Phys. Rev. X **12**, 021012 – Published 15 April 2022

Topological Phases of Matter



- Landau: phases == group theory == symmetry breaking
- Wegner, Wen, …: topological order => no local order parameter
    - *Hidden (categorical) symmetries, cohomology theory*
    - Stability under perturbations: Hastings et al. (*Lieb-Robinson bounds*)

- Quantum Information point of view:
    - Two (ground) states are in the same phase iff there is a constant-depth (independent of system size) quantum circuit mapping the states into each other
    - No-go theorem: *constant-depth quantum circuits* cannot create states exhibiting TO nor states exhibiting long-range order (GHZ) from states in a trivial / different phase
    - Technical tool: Lieb-Robinson bounds

Bravyi, Hastings et al. '06

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Overview

1. Topological Order & Quantum Error Correction

2. The enta

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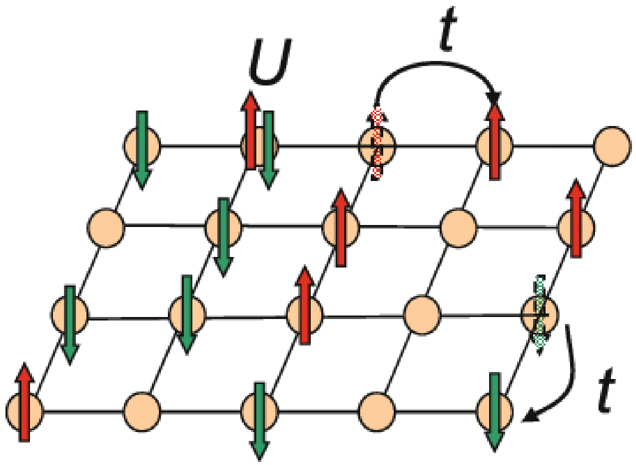
3. Entangle

The quantum many-body problem

- Large part of theoretical physics /chemistry in last 90 years has focused on describing many-body systems with polynomial instead of exponential complexity, starting from a good fiducial noninteracting state
  - Hartree-Fock, Perturbation theory / Feynman diagrams, Coupled cluster theory, Density Functional Theory, …

- Many strongly interacting quantum many body systems of interest do not have such a fiducial state
  - E.g. Hubbard model:

$$\hat{H} = -t \sum_{\langle i,j \rangle,\sigma} (\hat{c}^\dagger_{i,\sigma}\hat{c}_{j,\sigma} + \hat{c}^\dagger_{j,\sigma}\hat{c}_{i,\sigma}) + U \sum_{i=1}^{N} \hat{n}_{i\uparrow}\hat{n}_{i\downarrow}$$

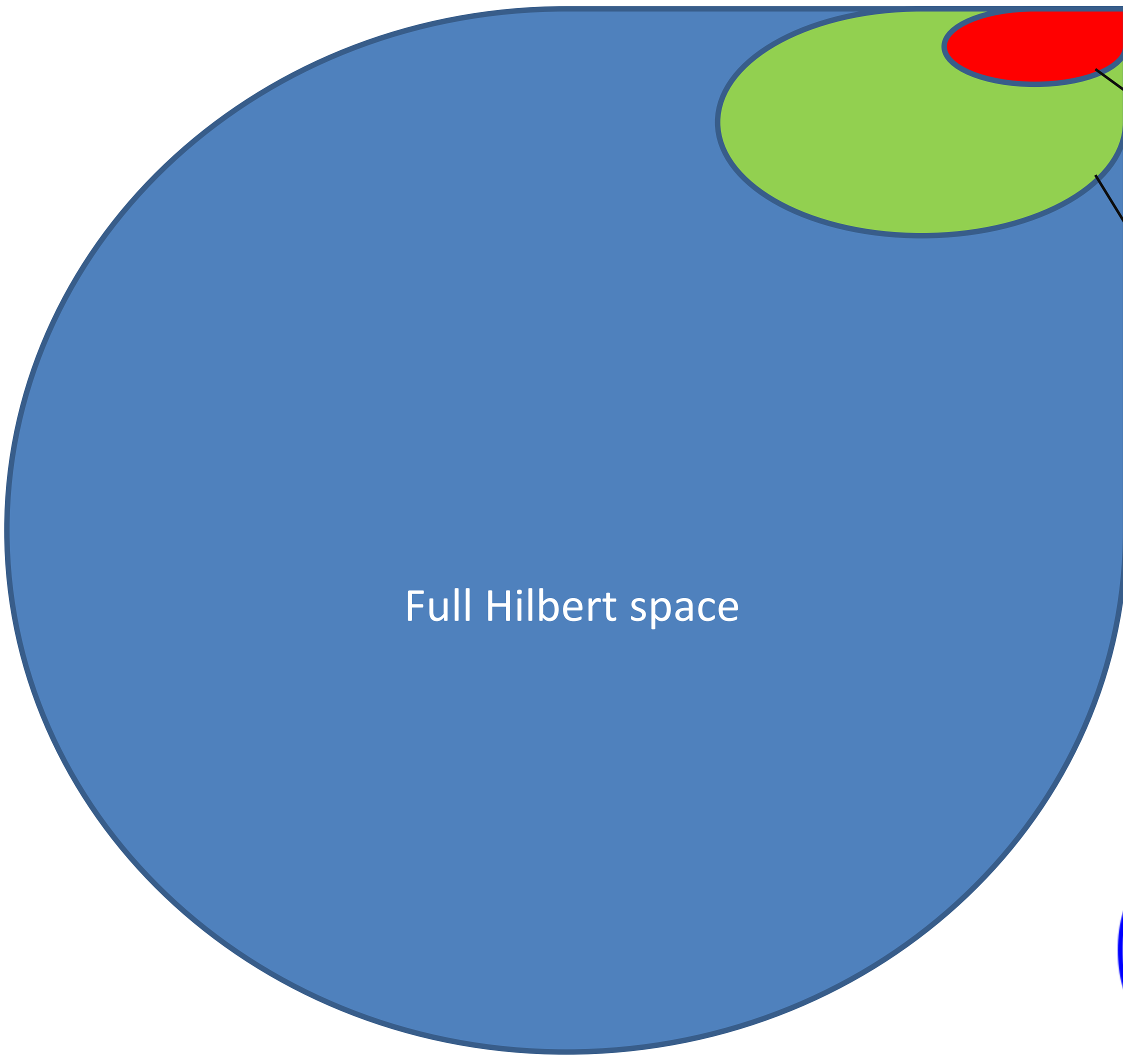


  - Quantum Monte Carlo can sometimes help, but in many cases of interest sign problem

- ***Problem is solved by the universal Quantum Computer : it speaks the language of nature (Feynman!)***

  - ***Premise: Complete solution of sign problem [-> quantum algorithms session]***

The convenient illusion of Hilbert space



Ground and thermal states of local n-body quantum Hamiltonians

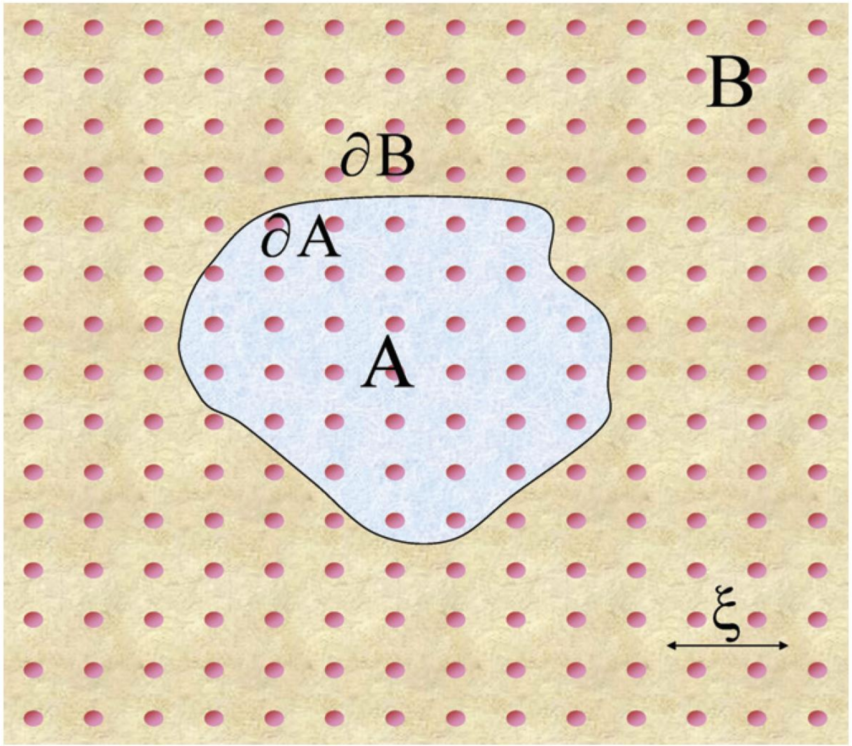
Manifold of States reachable in polynomial time with time dependent evolution with 2-body Hamiltonians starting from fully polarized state

$$\left(\frac{1}{\epsilon}\right)^{2^N} \gg N^{N^\alpha} \gg N^2$$

Poulin et al. '11

Area Laws for the entanglement entropy

- Ground and Gibbs states of interacting quantum many body Hamiltonians with ***local interactions*** have very peculiar properties
  - Area law for the entanglement entropy (ground states) or for mutual information (Gibbs states)
  - "explains" why physics is possible at all



Ground states:

$$S(\rho_A) = c.\partial A$$

Srednicki '93; Hastings '07; …
Landau, Vazirani, Vidick '15

$$S(\rho_A) = \frac{c}{6}.\log(A/\epsilon)$$

Holzhey, Larsen, Wilczek '94
Cardy, Calabrese '04

Gibbs states:

$$\begin{aligned} I(A,B) &= S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \\ &= c.\partial A \end{aligned}$$

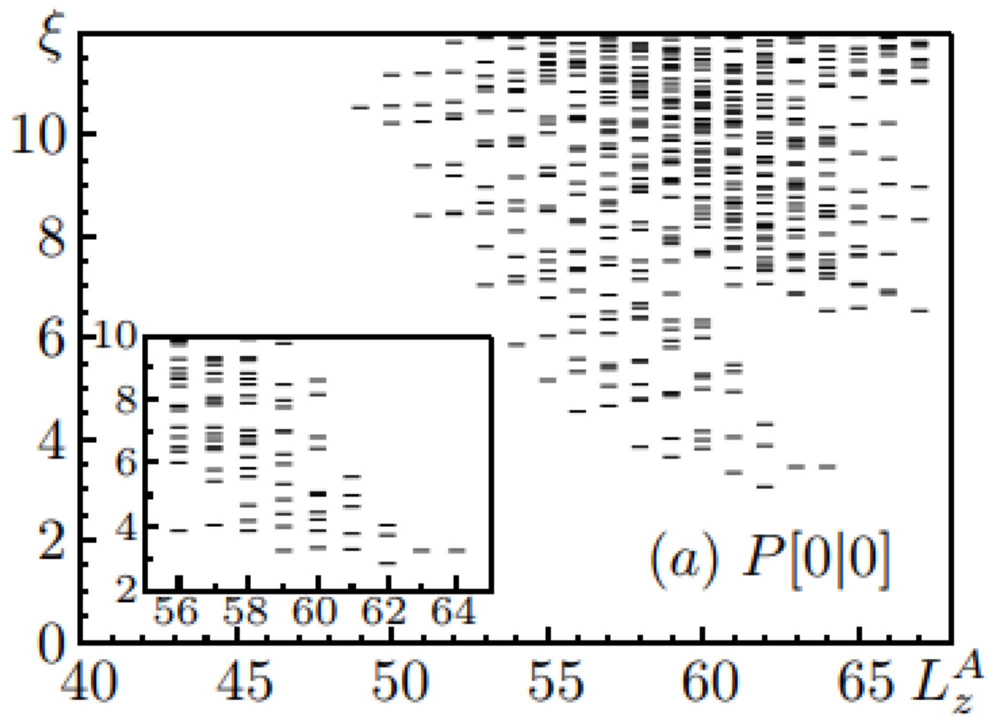
Wolf, …, Cirac '08

Entanglement spectrum

Entanglement Spectrum as a Generalization of Entanglement Entropy: Identification of Topological Order in Non-Abelian Fractional Quantum Hall Effect States

Hui Li and F. D. M. Haldane

Phys. Rev. Lett. **101**, 010504 – Published 3 July 2008



Entanglement spectrum of Moore-Read state

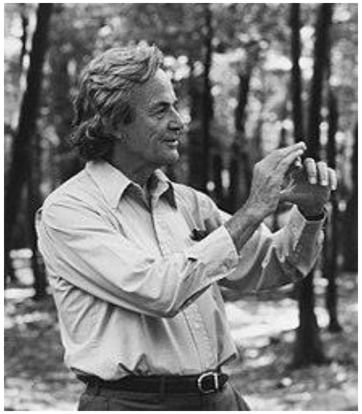
Entanglement

Entanglement gives a new perspective on and alternative solutions to fundamental problems plaguing traditional approaches to the quantum many-body problem:

- Computational methods: Lanczos, DFT, Monte Carlo, DMFT, DMRG
  - area laws, tensor networks -> Exponential wall, sign problem, dynamics
- Perturbation theory, renormalization group, effective field theories
  - disentangling & real space -> Strong coupling, frustration
- Exact methods: Conformal field theory, Bethe ansatz, Luttinger liquids
  - entanglement structure & symmetries -> Realistic systems and higher D

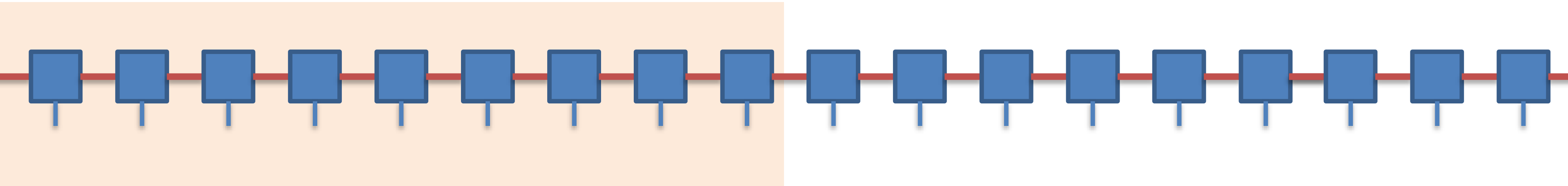
Feynman’s dream

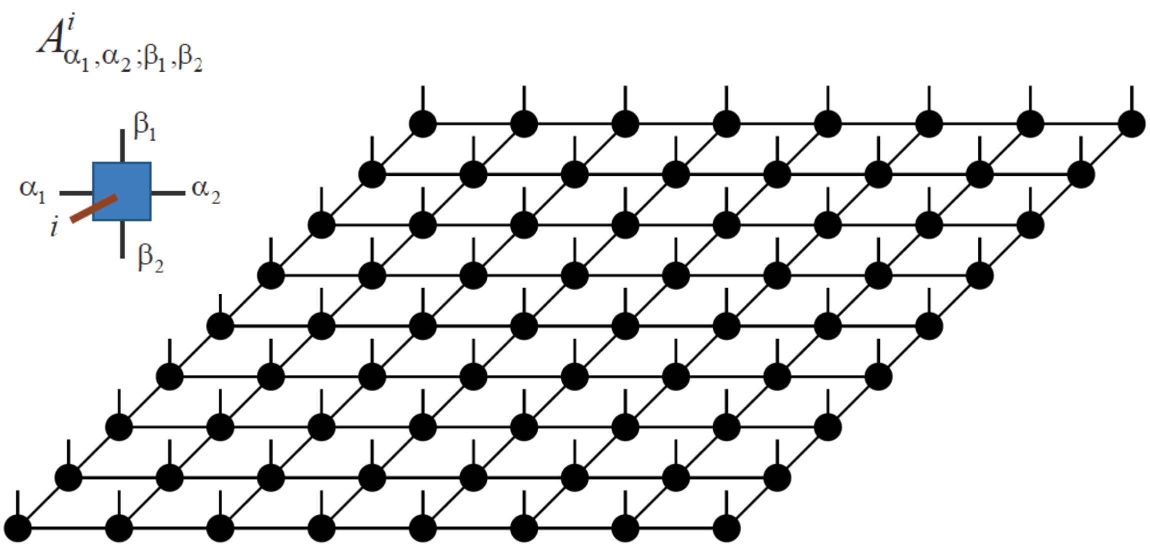
“Now, in field theory, what’s going on over here and what’s going on over there and all over space is more or less the same. Why do we have to keep track in our functional of all things going on over there while we are looking at the things that are going on over here? …. It’s really quite insane actually: we are trying to find the energy by taking the expectation of an operator which is located here and we present ourselves with a functional which is dependent on everything all over the map. That’s something wrong. Maybe there is some way to surround the object, or the region where we want to calculate things, by a surface and describe what things are coming in across the surface. It tells us everything that’s going on outside.
I’m talking about a new kind of idea but that’s the kind of stuff we shouldn’t talk about at a talk, that’s the kind of stuff you should actually do!”

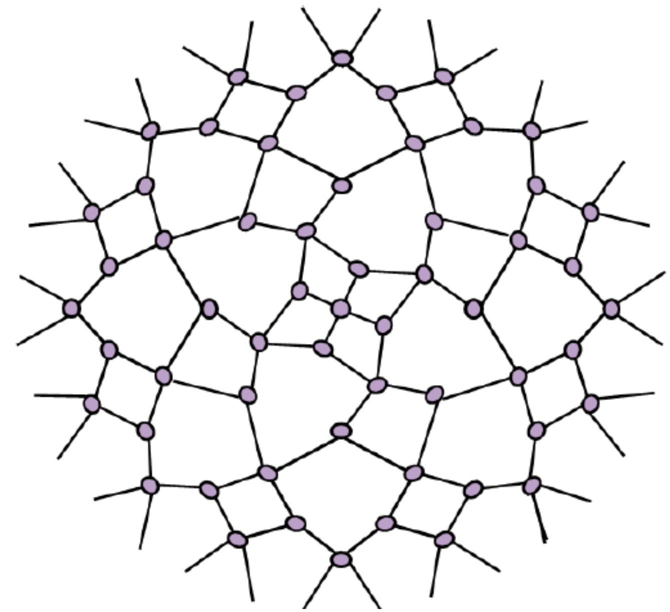


“Difficulties in Applying the Variational Principle to Quantum Field Theories”, Wangerooge 1987, Proceedings,
Variational calculations in quantum field theory

Entanglement as building block of matter: Quantum Tensor Networks





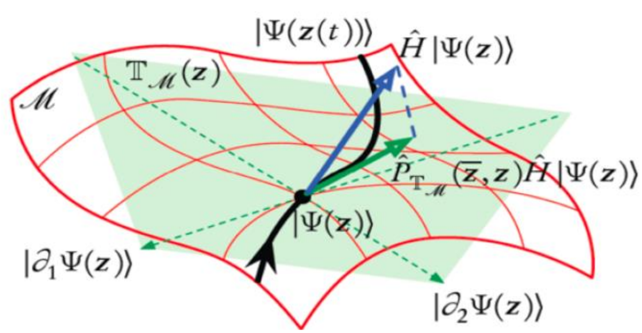


- ***tensor networks : crucial concepts***

  – Tensors model the entanglement structure in many-body wavefunctions: modelling correlations makes much more sense than modelling wavefunction directly
    - Tensors dictate the ***entanglement patterns***
    - ***Entanglement-based*** ansatz: as long as entanglement entropy satisfies area law, ok; not OK otherwise (such as time-evolution after quench)

  – Tensor networks can be efficiently contracted due to ***holographic property***: map quantum 3D -> 2D -> 1D -> 0D problems, and this can be done efficiently due to area laws; states can be defined in thermodynamic limit, with *finite size scaling* replaced by ***finite entanglement scaling***

  – Tensor network algorithms: TDVP on manifold of MPS/PEPS/MERA
    - State of the art simulations of strongly correlated systems

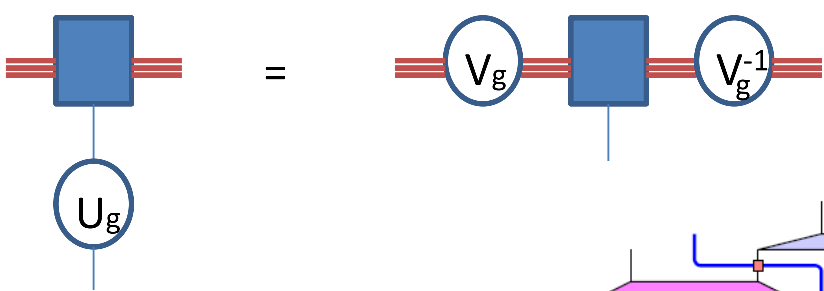


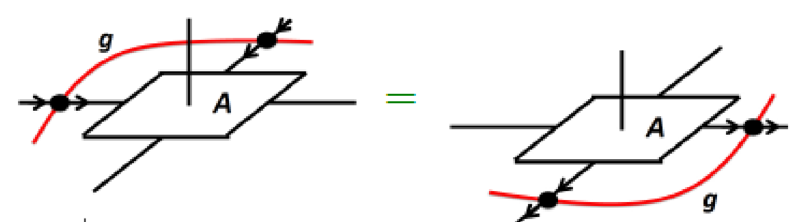
$$(i)\frac{d}{dt}|\psi\rangle = -\mathcal{P}_{\mathcal{T}(A)}\hat{H}|\psi\rangle$$

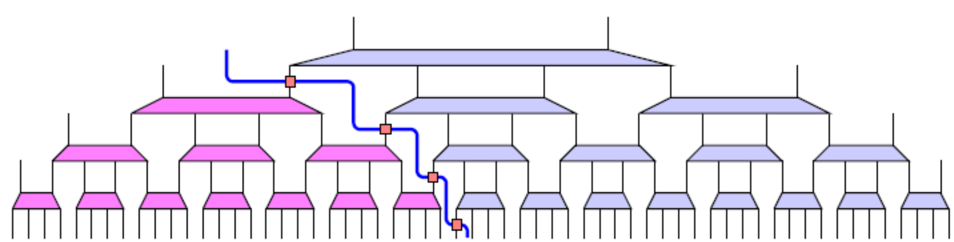
Haegeman, Vanderstraeten, Corboz et al.

It’s all about symmetries

- Anderson ‘72: *“physics is an applied form of group theory”*
- Essential paradigm: detect the ***global*** features of a system through its entanglement degrees of freedom / ***local*** tensor network description
  - The symmetries of the local tensor in the tensor network reveal the emerging properties of the system
  - Even for the case of topological order: *local* order parameters arise in the form of different representations of groups / fusion algebras





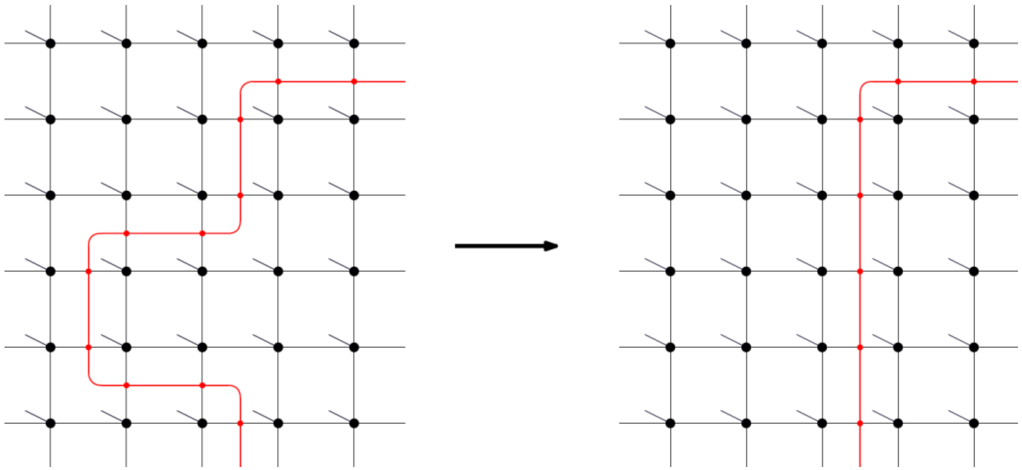


Topological Entanglement Entropy

- *Kitaev, Preskill '06; Levin, Wen '06*: additive correction to area law for entanglement entropy in case of topological order

$$S(A) = c.\partial A - \log\sqrt{\sum_i d_i^2}$$

- Categorical symmetries on the entanglement degrees of freedom



$$O_a.O_b = \sum_c N_{ab}^c O_c$$

  – Form basis for description of anyons, TFT-CFT correspondence, dualities, ...: tensor networks form the representation theory of the representation theory (bimodules) of fusion categories!

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Lieb-Robinson bounds

$$\| [A(t), B] \| \leq c \exp\{-a(d(X,Y) - v|t|)\}$$

- There is an effective light cone for the ***spreading of quantum information*** in quantum spin systems
- Matthew Hastings (+ co-authors) realized that this bound can be used to prove a wealth of open problems in mathematical physics by connecting it to ideas such as quasi-adiabatic evolution:
  - Exponential decay in gapped spin systems
  - Higher dimensional Lieb-Schultz-Mattis
  - Proof of Hall Conductance Quantization
  - Area law for entanglement entropy in 1D quantum spin systems
  - Robustness of topological phases of matter
  - Robustness of area law in quantum phases
  - Correctness of Quasi-particle (excitation) ansatz as plane waves on tangent space of tensor network manifold
  - …

Floquet phases

- Floquet: periodic driving of quantum spin systems, resulting in effective Hamiltonians

$$F_{\mathrm{MBL}} = T\{e^{-i\int_0^T H(t)dt}\} = \prod_{\{n_\alpha\}} e^{if(\Pi_{n_1},\Pi_{n_2},\ldots)}$$

- Classification of Floquet SPTs: group cohomology with group enhanced by time-translation symmetry

$$\tilde{G} = G\times\mathbb{Z} \text{ for unitary } G \text{ or } G\rtimes\mathbb{Z} \text{ for antiunitary } G,$$

$$\mathcal{H}^{d+1}(\tilde{G}, U(1))$$

- 2D: Chiral Floquet Phases due to absence of energy conservation

Potter, Morimoto, Vishwanath PRX ’16
Po, Fidkowski, Morimoto, Potter, Vishwanath PRX ‘16
Else, Nayak PRB ’16
von Keyserlingk, SL Sondhi PRB ‘16

Thermalization

- Deutsch (‘91), Srednicki (’94): local observables of eigenstates versus thermal states => eigenvector thermalization hypothesis (ETH)

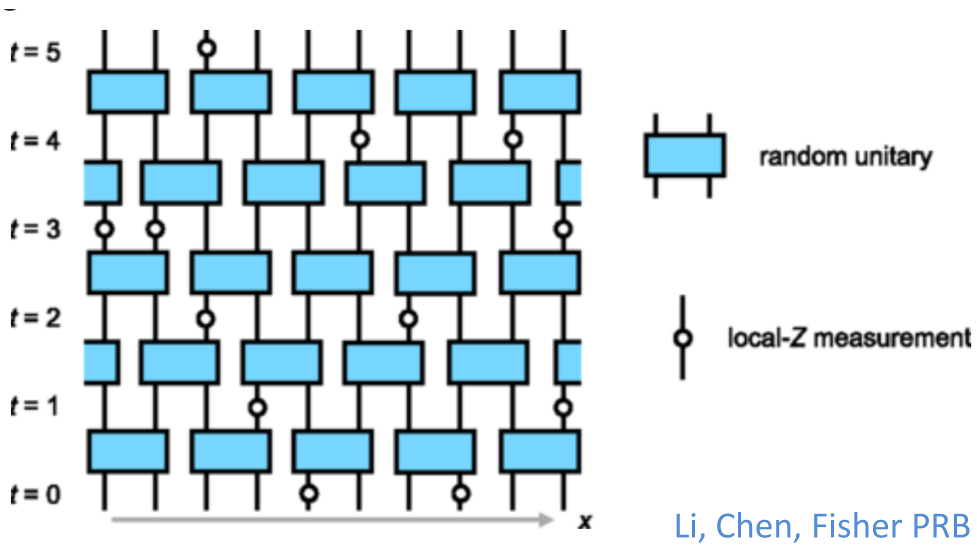
- “Entanglement and the foundations of Statistical Mechanics” [Popescu, Short, Winter Nat. Phys. ’06]: stat mech is about entanglement of subsystems with larger systems

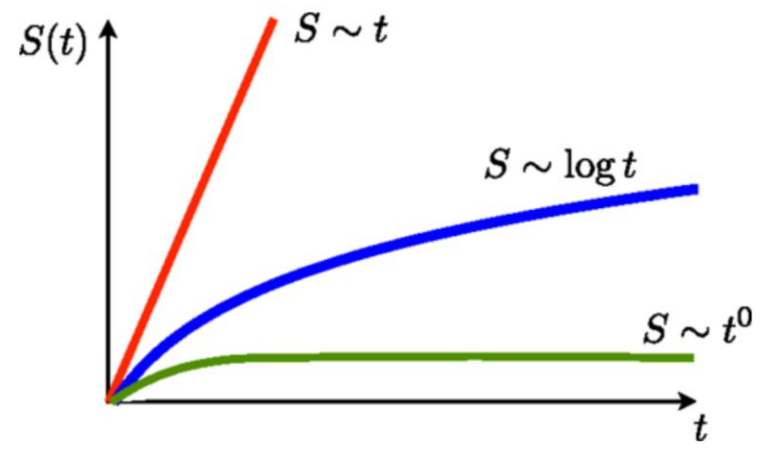
- Quenches: isolated systems thermalize (von Neumann ’29)
  - Prime example of where entanglement-based simulation methods fail
  - Fast information scrambling: ultimate limits, Sachdev-Ye-Kitaev, …

- Can a completely isolated, non-integrable systems ever fail to thermalize?

  - By adding static disorder and in 1D: Many-body localization (MBL)
    Eigenstates all look like (MPS) ground states, satisfy area laws, …
    [Altshuler, Huse, Abanin, Altman, Imbrie, …]

Measurement-induced phase transitions in quantum circuits





Li, Chen, Fisher PRB '18, PRB '19
Skinner, Rummer, Nahum PRX '19

- Volume vs area law entanglement as a function of measurement strength; at criticality: logarithmic scaling, critical exponents, …

- Related to the "phase transition" in quantum complexity: if the error rate is too high, then a quantum computation can be simulated efficiently

Quantum Supremacy:
simulating Sycamore with tensor networks

arXiv > quant-ph > arXiv:2111.03011

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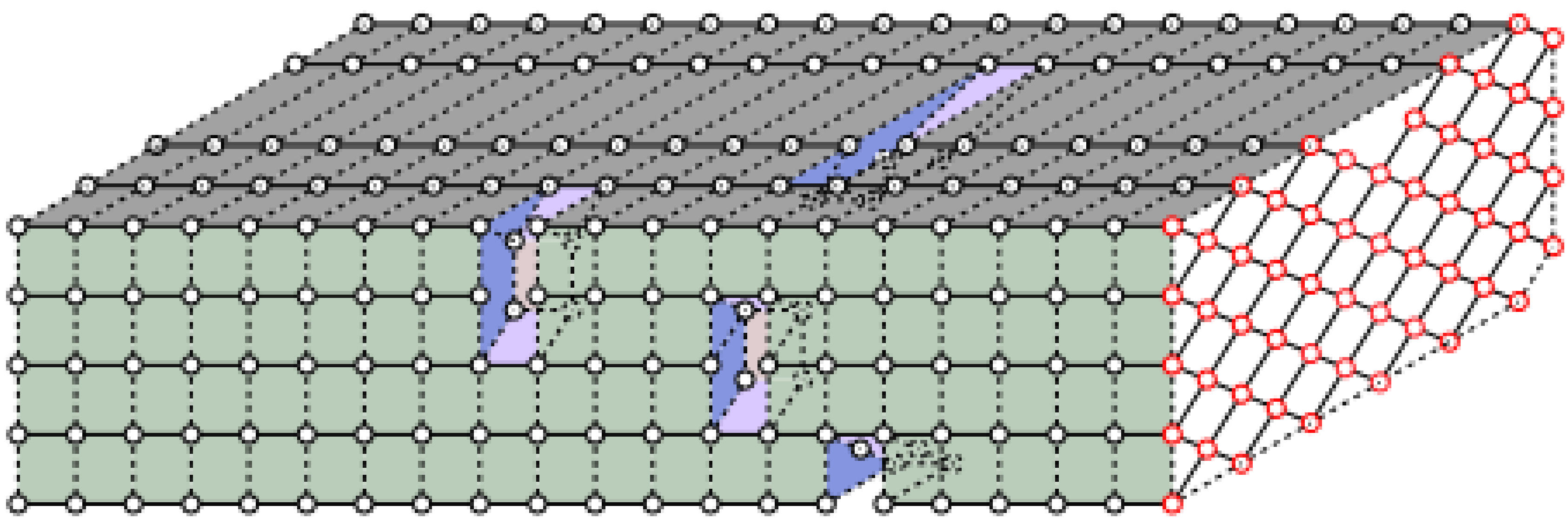
Quantum Physics

[Submitted on 4 Nov 2021]

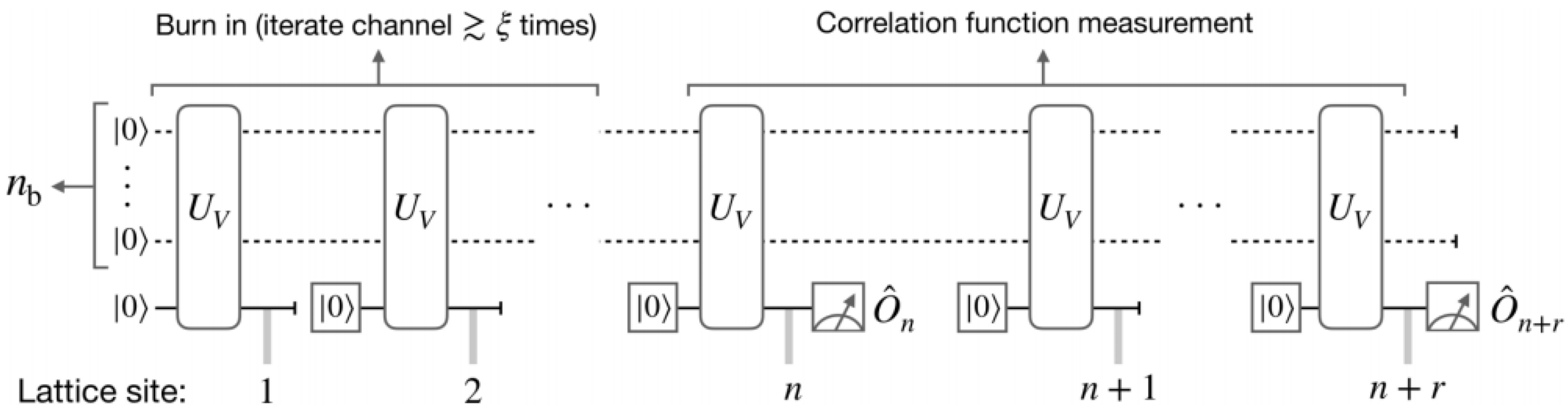
Solving the sampling problem of the Sycamore quantum supremacy circuits

Feng Pan, Keyang Chen, Pan Zhang



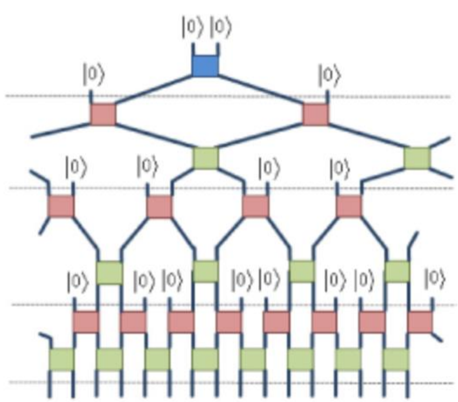
Hybrid tensor network – quantum computing platforms

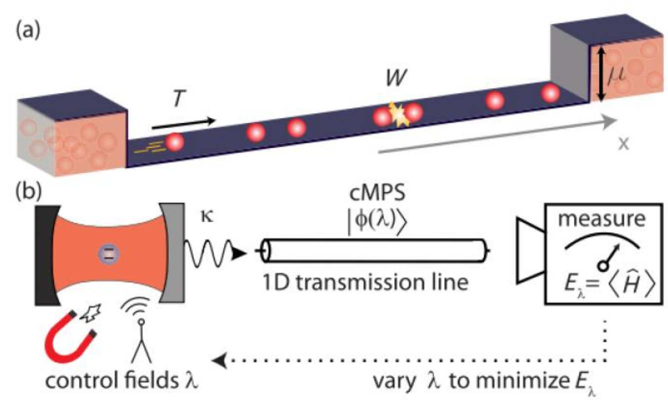
- Exchange circuit depth and space: Holographic quantum algorithms for simulating correlated spin systems



Foss-Feig, ..., Potter '21
Barratt, ..., Pollmann, Green '21

- Implementation of MERA as quantum circuits (Vidal), QAOA (Farhi), creating cMPS in superconducting circuits (Walraff et al.), ...





Outlook

- Entanglement provides a new vocabulary for describing the ubiquitous quantum many-body problem

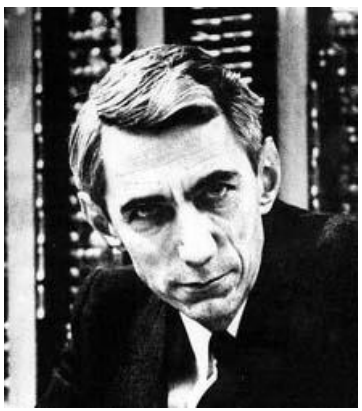
  - *Where is the quantum complexity frontier dividing classical from quantum, it is for what (useful) problems do we need quantum computers?*
    - *Classical algorithmic development is also going fast*
    - *What about hybrid tensor network – quantum computing platforms?*
    - *What fidelity do we need to beat state of the art classical algorithms?*
    - *What about noisy quantum simulators (cfr. Lukin's talk)?*
    - *Are many-problems problems which are hard for quantum computers physically relevant?*

  - tensor networks provide the rules for a new unifying language and the means to tackle it, but how to provide the *semantics?*
    - *A big challenge is to develop better / different algorithms and scale them up to be useful for real material science, for problems like QCD, for experiments with continuous degrees of freedom, …*
    - *Development of representation theory of fusion categories, 2-categories, fractons as defects in TFTs, quantum groups, … : make all this incredibly abstract but useful mathematics tangible!*
    - *Use Tensor networks to analyze quantum error-correcting codes based on categories / Levin-Wen*

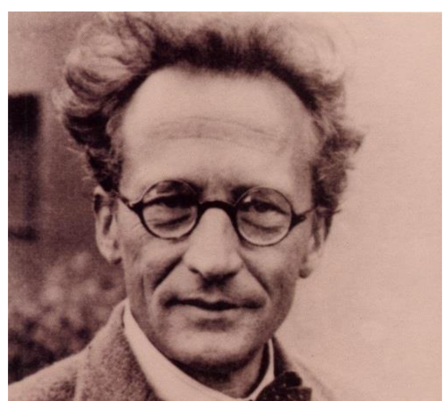
  - *How do you put fields on the lattice? What about boundary conditions, dualities, …?*

    - *How to Construct real-space renormalization group flows (respecting all symmetries, continuum limits, …) both using tensor networks and quantum computers?*
    - *Variational methods for relativistic quantum field theories*
    - *How to rigorously set up a formalism for entanglement & finite quantum circuit depth scaling?*
    - *How are tensor networks useful for the "*it from qubit*" programme?*
    - *How do you measure entanglement in experiments (cfr. proposals of Zoller et al.)*

Theoretical Quantum Physics of the 21st century



Entanglement



Quantum information and physics: some future directions*

John Preskill,†
Lauritsen Laboratory of High Energy Physics
California Institute of Technology
Pasadena, CA 91125, USA
(April, 1999)

arXiv:quant-ph/9904022

A. Some signposts in Hilbert space

B. Quantum error-correcting codes

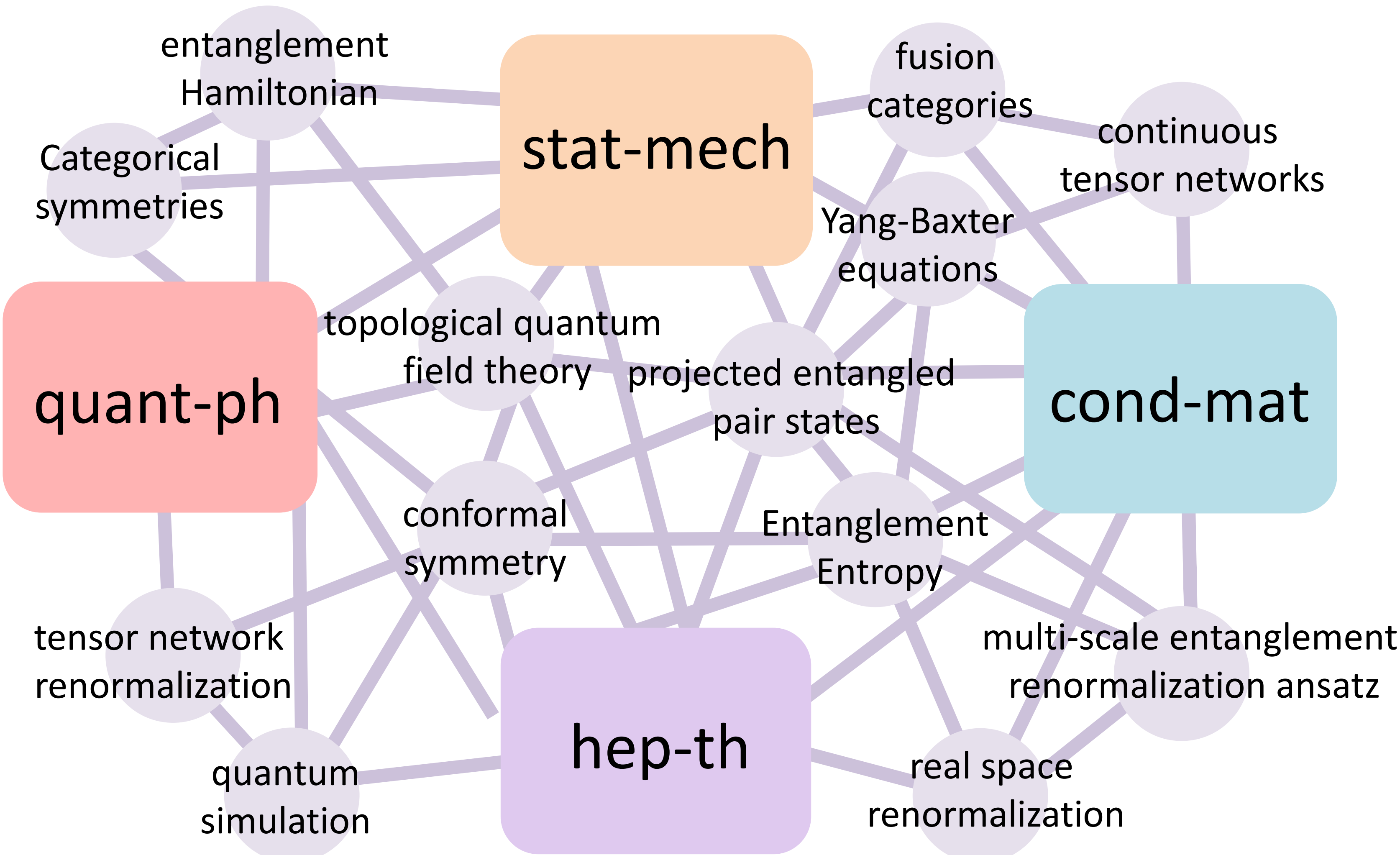
C. Classes of entangled states

D. Information and renormalization group flow

E. Bulk-boundary interactions

F. Holographic universe

In the future, I expect quantum information to solidify its central position at the foundations of computer science, and also to erect bridges that connect with precision measurement, condensed matter physics, quantum field theory, quantum gravity, and other fields that we can only guess at today. I have identified two general areas in which I feel such connections may prove to be particularly enlightening. Progress in quantum information processing may guide the development of new ideas for improving the information-gathering capabilities of physics experiments. And a richer classification of the phases exhibited by highly entangled many-body systems may deepen our appreciation of the wealth of phenomena that can be realized by strongly-coupled quantum systems.



entanglement Hamiltonian
stat-mech
fusion categories
continuous tensor networks
Categorical symmetries
Yang-Baxter equations
quant-ph
topological quantum field theory
projected entangled pair states
cond-mat
conformal symmetry
Entanglement Entropy
tensor network renormalization
multi-scale entanglement renormalization ansatz
hep-th
quantum simulation
real space renormalization

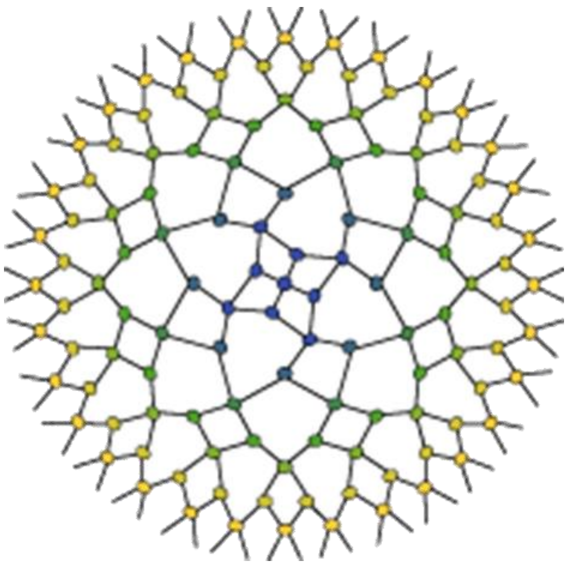
“the unreasonable effectiveness of tensor networks in physics”

Real-Space Renormalization Group

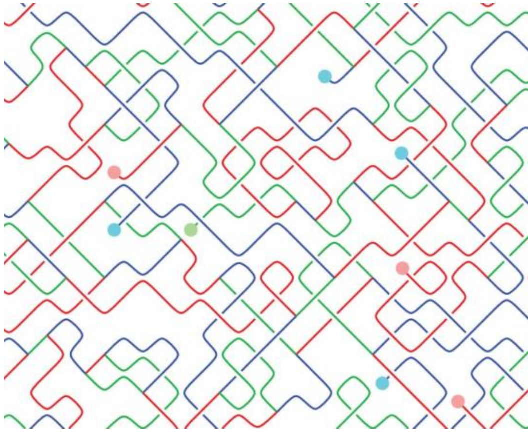
Symmetries and dualities

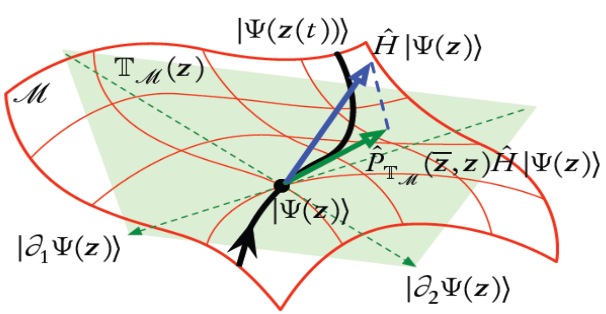
Tensor Networks

Quantum Field Theory
Statistical Mechanics



- High-Energy Physics
- Condensed Matter
- Cold Atoms

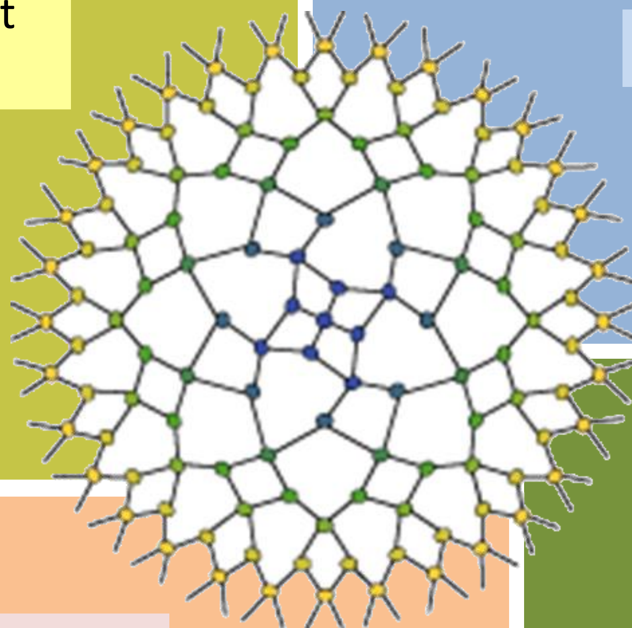




Entanglement Matters

- Quantum Computation
- Projected Entangled Pair States
- Multiscale Entanglement Renormalization Ansatz
- Matrix Product States
- Lieb-Robinson bounds
- Entanglement

- Lattice Gauge Theories
- Anyon Condensation
- Holographic Principle
- Quantum Topological Order
- Renormalization Group



- Quantum Quenches
- Cold Atomic Gases
- Quasi-Particles
- Quantum Phase Transitions
- Non-Commutative Gross-Pitaevskii

- Fractional Quantum Hall
- Bosonic SPT phases
- Hubbard Model
- (Virtual) Order Parameter
- Quantum Spin Liquids