Quantum information and spacetime I

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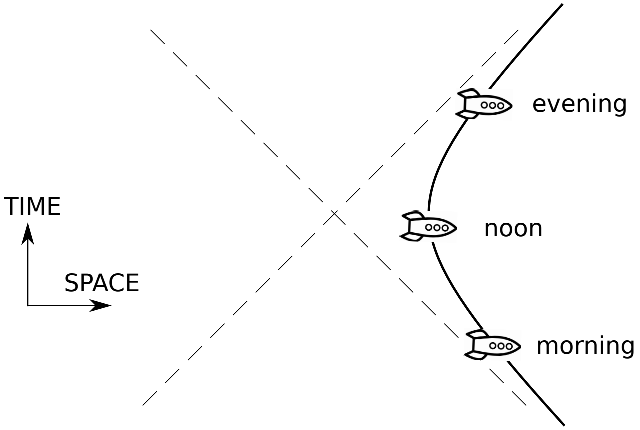
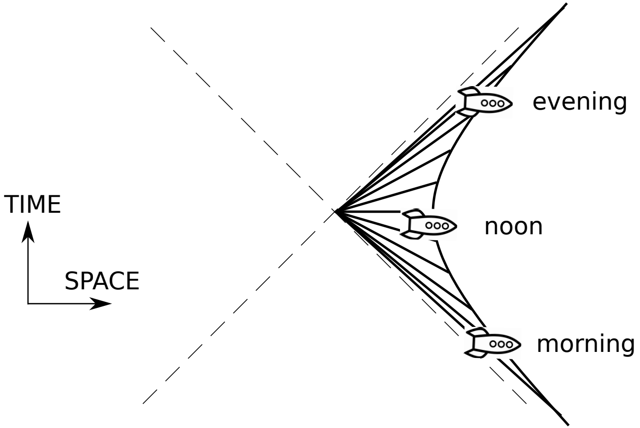
1. The "central dogma" of black hole physics
2. Simple examples
   - Thermalization
   - Chaos
3. Three slogans relating spacetime and...
   - ...entanglement
   - ...quantum error correction
   - ...complexity/tensor networks
4. A puzzle

Part 1: The "central dogma" of black hole physics

A uniformly accelerated observer in flat spacetime

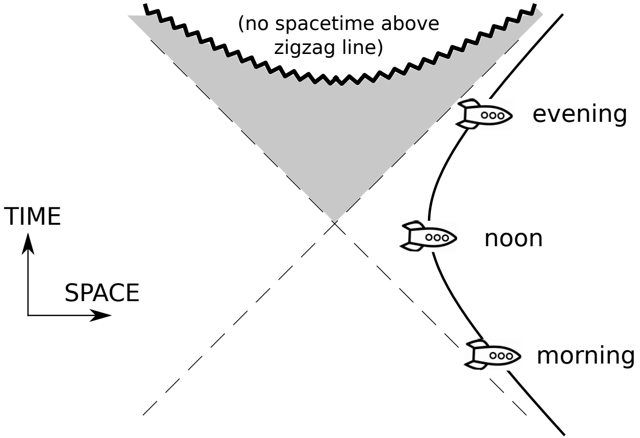


A uniformly accelerated observer in flat spacetime

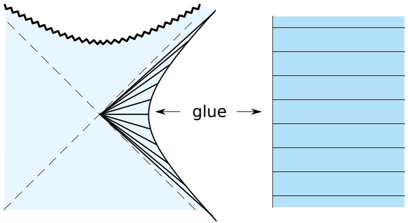


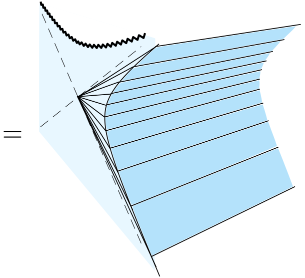
Time evolution corresponds to a Lorentz boost.

A uniformly accelerated observer just outside a BH



Slightly better model of BH geometry

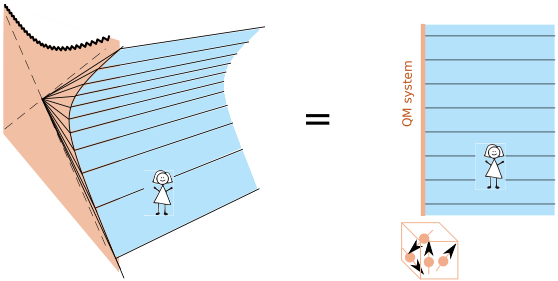




Symmetric under time evolution that is ordinary time evolution far away and a Lorentz boost near the horizon.

The “central dogma” of black holes

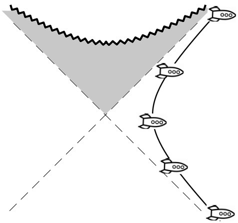
As seen from the outside, a black hole can be described as an ordinary* QM system with Area$/(4G_N)$ degrees of freedom, evolving unitarily under time evolution:



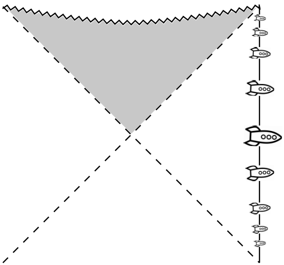
['t Hooft '93] [Susskind '93] [Maldacena '97] . . . [10^4 more papers]

Re-drawing using a Penrose diagram

Focus on the region near the black hole, redraw this

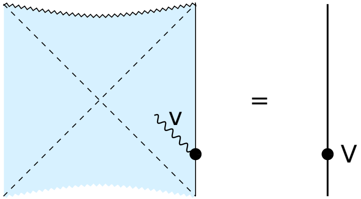


with a conformal transformation (light rays still move at 45 degrees) as



Central dogma (again)

Then

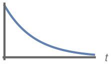


Part 2: Examples with simple correlators

Example 1: thermalization of perturbations

Let V be a few-body operator and $|\Psi\rangle$ be an equilibrated high-energy state. We can probe thermalization of small perturbations using

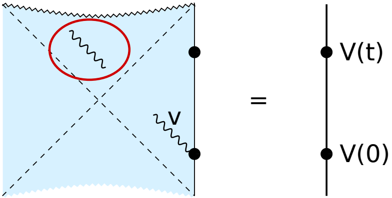
$$G(t) = \langle\Psi|V(t)V(0)|\Psi\rangle =$$



This decays, representing thermalization of the V perturbation.

It's hard to compute for an actual QM system.

In the black hole picture, the V perturbation creates a particle that falls into the black hole with some amplitude per unit time, so $G(t) \sim e^{-\gamma t}$

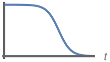


[Horowitz, Hubeny '99]

Example 2: chaos

The butterfly effect in quantum mechanics can be diagnosed by the out-of-time-order correlator

$$F(t) \;=\; \langle\Psi|W(t)V(0)\;\;W(t)V(0)|\Psi\rangle =$$



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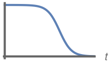
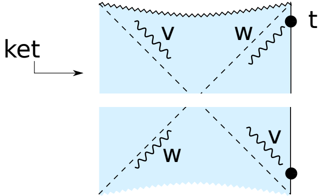
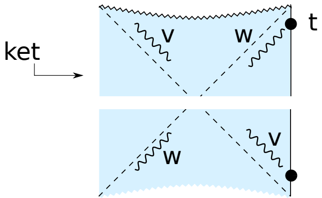


In the bulk, we can view it as an inner product of two states



[Larkin, Ovchinnikov '69] [Dray, 't Hooft '85] [DS, Shenker '13] [Kitaev '14]



Because time evolution corresponds to a boost, if t is large then the V and W particles collide with very high energy.

The OTOC is an interesting target for experimental setups and NISQ devices – requires enough control to evolve forwards and backwards.

[Swingle, Bensen, Schlier-Smith, Hayden '16] [Zhu, Hafezi, Grover '16] [Garttner, Bohnet, Safavi-Naini, Wall, Bollinger, Rey '16] [Li, Fan, Wang, Ye, Zeng, Zhai, Peng, Du '16] [Yao, Grusdt, Swingle, Lukin, Stamper-Kurn, Moore, Demler '16]

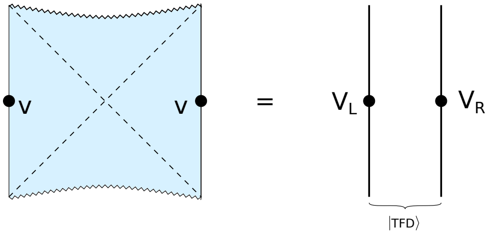
Part 3: Three slogans connecting spacetime to QI

Slogan 1: the bulk is the geometrization of entanglement

There is a natural entangled state of two quantum systems

$$|TFD\rangle = \frac{1}{Z(\beta)^{1/2}} \sum_n e^{-\beta E_n/2} |n\rangle_L |\bar{n}\rangle_R$$

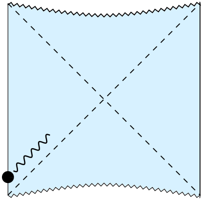
this is dual to a "two-sided black hole": two black holes connected by an Einstein-Rosen bridge (shared interior)



The correlation $\langle TFD|V_L V_R|TFD\rangle$ is large. Gravity explains this by saying the two sides are close to each other. The Einstein-Rosen bridge *geometrizes* the Einstein-Podolsky-Rosen entanglement

[Maldacena '01], [van Raamsdonk '10], [Maldacena, Susskind '13]

Some familiar features of entanglement are visible in the gravity description of this entangled state. For example, you can't (quite!) use it to signal from L to R:



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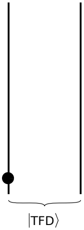
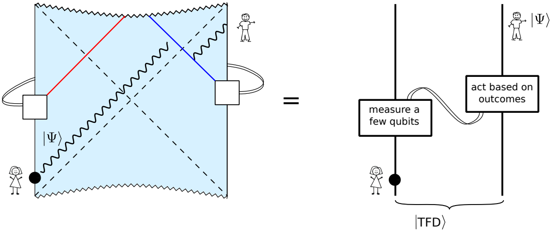


The ER bridge is *non-traversable*. But it is almost traversable...

... and if classical communication is allowed between the two boundaries, then the ER bridge can be made traversable:



This is a variant of quantum teleportation that allows you to understand what is happening during the teleportation.

It has been proposed to study this as a targer for NISQ devices and experiments: "quantum gravity in the lab."

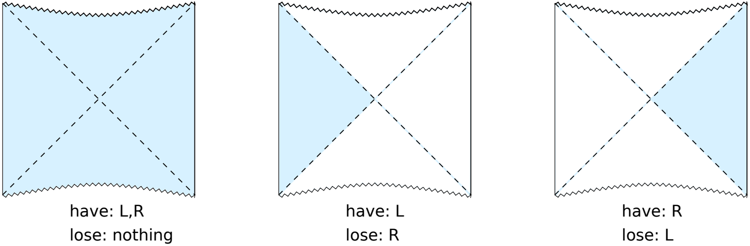
[Gao, Jafferis, Wall '16] [Maldacena, DS, Yang '17] [Maldacena, Milekhin, Fopov '18] [Landsman, Figgatt, Schuster, Linke, Yoshida, Yao, Monroe '19] [Brown, Gharibyan, Leichenauer, Lin, Nezami, Salton, Susskind, Swingle, Walter '19]

Slogan 2: the bulk is an error-correcting code

Low-energy bulk fields = logical qubits

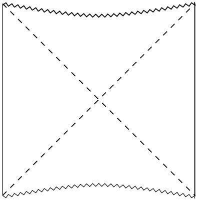
Boundary QM system = physical qubits

Consider an erasure error, where we lose access to part of the L and/or R QM systems. How much of the bulk can we still reconstruct?

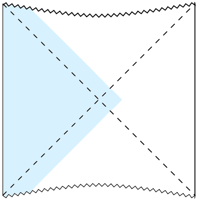


[Czech, Karczmarek, Nogueira, Van Raamsdonk '12] [Qi '13] [Almheiri, Dong, Harlow '14] [Pastawski, Yoshida, Harlow, Preskill '15] [Dong, Harlow, Wall '16] [Harlow '17] [Akers Penington '21]

What if we have access to the L system plus *part* of the R system?



have: small bit of R
lose: L, most of R



have: L, small bit of R
lose: most of R

The traversable ER bridge discussion is one way of understanding why this is possible, but Netta will discuss a more powerful method of determining what part of the bulk can be reconstructed from a given part of the boundary.

This gets more interesting for an evaporating BH! (see Netta's talk)

What are the physical qubits from the spacetime perspective?

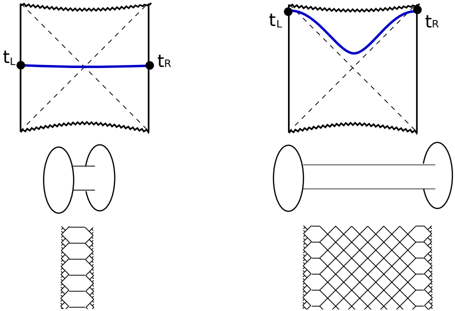
[Levine, Shahbazi-Moghaddam, Soni '21]

Slogan 3: the bulk is a tensor network

Bulk geometry ≈ geometry of tensor network that prepares bdy state

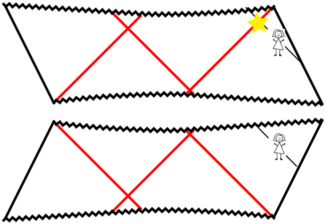
size of bulk geometry ≈ complexity of bdy state

Example: time-evolved TFD state



[Swingle '09] [Hartman, Maldacena '13] [Maldacena '13] [Susskind '14] [Susskind, DS '14] [Susskind, DS, Roberts '14] [Brown, Roberts, Swingle, Susskind, Zhao '15] [Hayden, Nezami, Qi, Thomas, Walter, Yang '16] [Aaronson, Susskind] [Iliesiu, Mezei, Sarosi '21]

Puzzle: bulk is easy to measure, but TN is hard to "measure." E.g.:

$|1\rangle =$  $= W(t)W(-t)W(t)|TFD\rangle$

$|2\rangle =$ $= W(-t)W(t)W(-t)|TFD\rangle$

The complexity of distinguishing these states is believed to be exponential in the number of $W(\pm t)$ operators.

But it is easy for a bulk observer to distinguish them... jump in and see if you hit a firewall!

Does this violate a variant of the extended Church-Turing thesis? Can falling across the horizon help with other computational tasks?

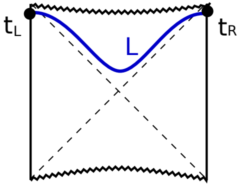
[Bouland, Fefferman, Vazirani '19] [Susskind]

Part 4: A puzzle

Let's return to a two-point correlation function. Consider

$$G(t_L, t_R) = \langle TFD | V_L(t_L) V_R(t_R) | TFD \rangle.$$

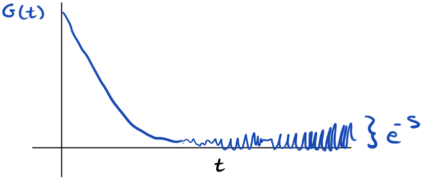
In gravity, this is given by $e^{-\gamma L}$ where L is the distance across the Einstein-Rosen bridge



$$L \approx t_L + t_R$$

This leads to exponential decay of $G(t, t)$. As the black hole becomes old, this correlator goes to zero.

But in finite-entropy quantum mechanics, $G(t, t)$ is a discrete sum of oscillating exponentials and it cannot exponentially decay forever. It rattles around a floor value of e^{-S}, with periodic recurrences to larger values.

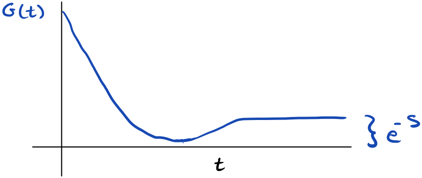


Where is this “pseudorandom quantum noise” in the bulk geometry?

[Maldacena '01] [Dyson, Kleban, Susskind '02] [Barbon, Rabinovici '03] [Polchinski]

... nobody knows! This is an important open problem that many people are working on.

But one thing that bulk spacetime geometry does seem to be able to do is to explain the smooth value you get by somehow averaging over this noisy curve

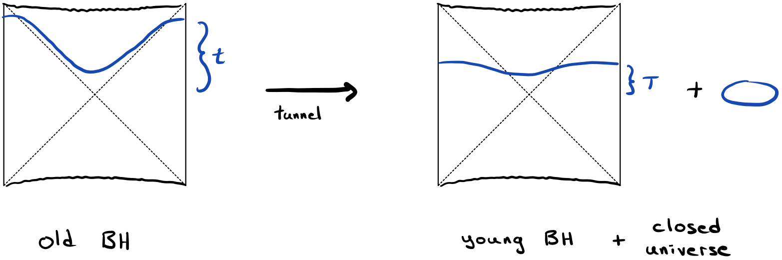


[Cotler, Gur-Ari, Hanada, Polchinski, Saad, Shenker, DS, Streicher, Tezuka '16]

Spacetime explains the nonzero late-time value because there is a small amplitude for an old black to tunnel to a young black hole

old (age t) black hole $\longrightarrow$ young (age T) black hole + closed universe

Pictorially

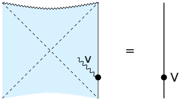


But where are the erratic wiggles? Why does gravity give us a simple picture of this averaged function?

[Saad, Shenker, DS '19] [Blommaert, Mertens, Verschelde '19] [Saad '19] [Yan '22]

Summary

- The central dogma of quantum black hole physics



- Examples with simple correlators

- Three related slogans connecting spacetime to QI
    1. the bulk as geometrization of entanglement
    2. the bulk as an error-correcting code
    3. the bulk as a tensor network/complexity-ometer
- A puzzle

Thank you!