Topological states of matter with cold gases and Rydberg atoms

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Strongly interacting Rydberg slow light polaritons



SFB TRR21: Tailored quantum matter

Outline

Topological band structures with dipolar interactions

- realized with polar molecules and Rydberg atoms
- Chern number C=2





Majorana Edge modes

- exact solvable system with fixed particle number
- double wire system







Dipolar interactions

Dipole-dipole interaction

- anisotropic interaction

$$V_{dd} = \frac{\mathbf{d}_i \cdot \mathbf{d}_j - 3 \left(\mathbf{d}_i \cdot \mathbf{n}_{ij} \right) \left(\mathbf{d}_i \cdot \mathbf{n}_{ij} \right)}{|\mathbf{r}_i - \mathbf{r}_j|^3}$$



- coupling between orbital degree of freedom and internal degree of freedom



Observation in cold gases

- Einstein de Haas effect
 (Y. Kawaguchi et al, PRL 2006)
- demagnetization cooling (M. Fattori et al, Nature Phys 2006)
- pattern formation in spinor condensates (D. Stamper-Kurn, M. Ueda, RMP 2013).

Dipolar interactions

Exploit this spin orbit coupling for the generation of topological band structures



Requirements:

- particles in a 2D lattice



- internal "Spin" structure



- strong dipole-dipole interaction

Systems:

- polar molecules
- Rydberg atoms
- NV centers
- atoms with large magnetic dipole moments

Rydberg atoms in an optical lattice

Setup

- one atom per lattice site with quenched tunneling
- static external electric field and magnetic field
- select three internal states
 - $|-\rangle_i \ |+\rangle_i$: two excited states $|0\rangle_i$: ground state

Mapping onto two hard-core bosons:

- bosonic creation operators for excitations

$$\begin{split} |+\rangle_i &= b_{i,+}^{\dagger} |0\rangle \\ |-\rangle_i &= b_{i,-}^{\dagger} |0\rangle \end{split}$$



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Dipolar exchange interactions



Dipolar exchange interactions



hopping with spin flip

Single excitation Hamiltonian

 $-t^{-}$

Hamiltonian

- single particle hoping with spin orbit interaction

$$H = \sum_{i \neq j} \frac{a^3}{|\mathbf{r}_i - \mathbf{r}_j|^3} \psi_i^{\dagger} \begin{pmatrix} -t^+ \\ e^{-i2\phi_{ij}} \end{pmatrix}$$

$$\begin{aligned} \psi_i &= \left(\begin{array}{c} b_{+,i} \\ b_{-,i} \end{array}\right) \\ \psi_i \\ \psi_i \end{aligned}$$

Time-reversal symmetry breaking

- different hopping for excitations

$$t^- \neq t^+$$

- energy shift on levels

$$\Delta H = \sum_{i} \psi_{i}^{\dagger} \left(\begin{array}{cc} \mu & 0\\ 0 & -\mu \end{array} \right) \psi_{i}$$



Single excitation Hamiltonian

Hamiltonian

- single particle hoping with spin orbit interaction

$$\begin{aligned} \hat{\mathbf{c}}_{\mathbf{k}} = \sum_{i \neq 0} \frac{1}{|\mathbf{r}_i|^3} e^{i\mathbf{k} \cdot \mathbf{r}_i + i2\phi_i} \\ \hat{\mathbf{c}}_{\mathbf{k}} = \sum_{i \neq 0} \psi_{\mathbf{k}}^{\dagger} \left(\begin{array}{c} -t^+ \epsilon_{\mathbf{k}} + \mu \\ w \hat{\epsilon}_{\mathbf{k}} + u \\ w \hat{\epsilon}_{\mathbf{k}} \end{array} \right) \psi_{\mathbf{k}} \\ = \sum_{\mathbf{k}} \psi_{\mathbf{k}}^{\dagger} \left[n_{\mathbf{k}}^0 + \mathbf{n}_{\mathbf{k}} \sigma \right] \psi_{\mathbf{k}} \\ \epsilon_{\mathbf{k}} = \sum_{i \neq 0} \frac{1}{|\mathbf{r}_i|^3} e^{i\mathbf{k} \cdot \mathbf{r}_i} \end{aligned}$$

Topological character of band

 $= \sum \psi_{\mathbf{k}}^{\dagger} \left[n_{\mathbf{k}}^{0} + \mathbf{n}_{\mathbf{k}} \sigma \right] \psi_{\mathbf{k}}$

- characterized by a Chern number

$$C = \int \frac{d\mathbf{k}}{v_0} \mathbf{n}_{\mathbf{k}} \cdot \left(\frac{\partial \mathbf{n}_{\mathbf{k}}}{\partial k_x} \wedge \frac{\partial \mathbf{n}_{\mathbf{k}}}{\partial k_y} \right) \in \mathcal{Z}$$





see also Syzranov et al Nat. Comm. 2014



Without time reversal symmetry

- optimal experimental parameters
- Chern number C=2





Finite system in y-direction

- bulk edge correspondence (Hatsugai PRL 1993)



C= 2 implies two edge states

- exponential localization in presence of long-range hopping





Stability under disorder

Disorder

- missing molecules in the lattice
- stabilized by long-range hopping







Flat topological bands

Honeycomb lattice

- much flatter bands accessible
- very rich topological structure

 $C \in \{0, \pm 1, \pm 2, \pm 3, \pm 4\}$

- even richer for Kagame lattice













Outlook on topological bands

Dipolar interaction provides natural spin orbit coupling

- topological band structures with Chern number C=2

Robust to disorder

- topological nature is very robust to missing particles in the lattice

Towards bosonic fractional Chern insulators

- strongly interacting system
- is flatness high enough for topological phases
- candidate expected at 2/3 filling







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Majorana Edge modes

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Kitaev's Majorana chain

Kitaev's Majorana chain



Topological state

- robust ground state degeneracy
- non-local order parameter
- localized edge states



Why should we care

Topological invariant edge states

- ground state degeneracy is robust to local perturbations



robust quantum memory?

Non-abelian anyons

- localized edge modes obey non-abelian braiding statistics



topological quantum computation

- Novel state of matter

- Toy model of a topological phase



(Alicea et al Nat. Phys. 2011)

Beyond mean-field

$$H = -\sum_{i=1}^{L-1} \left[wa_i^{\dagger} a_{i+1} + \Delta a_i a_{i+1} + h.c. \right]$$

- mean-field coupling
- violates particle conservation

exist a particle conserving theory with Majorana modes in one-dimension?



Beyond mean-field



N. Lang and H. P. Büchler, Phys. Rev. B 92, 041118 (2015). F.lemini,et al., Phys. Rev. Lett. 115, 156402 (2015).

Microscopic model

Hamiltonian

- double wire system

$$H = H_a + H_b + H_{ab}$$

- intra-chain contribution

$$H_a = \sum_i A_i^a \left(1 + A_i^a\right)$$

- inter-chain contribution

$$H_{ab} = \sum_{i} B_i \left(1 + B_i \right)$$

Symmetries

- total number of particles $\,$ N
- time reversal symmetry T
- sub-chain parity P



Microscopic model

Inter-chain Hamiltonian



Inter-chain Hamiltonian (expanded)

$$H_i^a = a_i a_{i+1}^{\dagger} + a_{i+1} a_i^{\dagger} + n_i^a \left(1 - n_{i+1}^a\right) + n_{i+1}^a \left(1 - n_i^a\right)$$

Microscopic model

Intra-chain Hamiltonian

$$H_{ab} = \sum_{i} B_{i} (1 + B_{i})$$
$$B_{i} = a_{i}^{\dagger} a_{i+1}^{\dagger} b_{i} b_{i+1} + b_{i}^{\dagger} b_{i+1}^{\dagger} a_{i} a_{i+1}$$

pair-hopping between chains



positive Hamiltonian
zero-energy state
is ground state
fixed total number
of particles



$$|\psi\rangle = \sum_{n} |n\rangle |N - n\rangle$$

equal weight superposition of all possible distribution of N fermions between the two wires

Intra-chain Hamiltonian (expanded)

 $H_{ab}^{i} = a_{i}^{\dagger}a_{i+1}^{\dagger}b_{i}b_{i+1} + b_{i}^{\dagger}b_{i+1}^{\dagger}a_{i}a_{i+1} + n_{i}^{a}n_{i+1}^{a}\left(1 - n_{i}^{b}\right)\left(1 - n_{i+1}^{b}\right) + n_{i}^{b}n_{i+1}^{b}\left(1 - n_{i}^{a}\right)\left(1 - n_{i+1}^{a}\right)$

Ground state degeneracy

Two-open chains

- two-fold ground state degeneracy

$$\begin{split} |\psi_{\rm even}\rangle &= \sum_{n\in {\rm even}} |n\rangle |N-n\rangle \\ |\psi_{\rm odd}\rangle &= \sum_{n\in {\rm odd}} |n\rangle |N-n\rangle \\ \end{split}$$
 Two- closed chains

- only one zero energy state for total even number of particles



Even total number of particles





Wire networks

Networks of wires

- exact ground states for arbitrary networks
- degeneracy consistent with majorana modes at edges



 $2^{E/2-1}$

number of edges



Ground state properties

2

Density-density correlations

- independent on ground state

$$\langle n_i^{\sigma} n_j^{\sigma'} \rangle = \rho^2 \qquad i \neq j$$

Superfluid correlations

$$\langle a_i^{\dagger} a_{i+1}^{\dagger} a_j a_{j+1} \rangle = \rho (1-\rho)$$

- long-range superfluid p-wave pairing





Ground state properties

Stability of ground state degeneracy of edge states

- stable under all local perturbations
- splitting decays exponentially



Stability of ground state degeneracy for open wires

- Protected by either time-reversal symmetry or subchain parity

 $a_i^{\dagger}b_i + b_i^{\dagger}a_i$

: stable under time reversal hopping

 $ia_i^{\dagger}b_i - ib_i^{\dagger}a_i$

: finite overlap between two ground states



Excitation spectrum

Low-energy excitations

- Goldstone mode due to broken U(1) symmetry
- exact wave function for single phase kink excitation

$$|k,\psi\rangle = \sum_{j} e^{ikj} \left[(-1)^{n_{j}^{a}} + (-1)^{n_{j}^{b}} \right] |\psi\rangle$$

- quadratic excitation spectrum

$$\epsilon_k = 4\sin^2 k/2 \sim k^2$$

System is in a critical state

- vanishing compressibility
- Goldstone mode with quadratic dispersion

Non-abelian Braiding statistics

Setup for braiding of two edge states

- wire network with two edges
- restriction to the low energy sector
- very weak coupling terms: adiabatic switching between them
- 8 relevant states
- characterized by subchain parity





Non-abelian Braiding statistics

Adiabatic switching of coupling

 transformation of the ground state according to the non-abelian statistic of Majorana modes







Conclusion

Topological band structure with dipolar exchange interactions

- spin-orbit coupling natural present in dipolar system
- existence of topological band structures





Majorana Edge modes

- exact solvable system with fixed particle number
- analytical demonstration of Majorana edge modes

