Large fields in synthetic dimensions

I. B. Spielman

Synthetic fields: L. M. Aycock, D. Genkina, and H.-I Lu; departed: B. Stuh

Novel vortex formation: A. Putra, A. Valdés-Curiel, E. Marshall, and D. Trypogeorgos; departed: R. Price, D. Campbell

Chip: F. Salces-Carcoba, Y. Yue, A. Putra, and S. Sugawa; departed: A. R. Perry

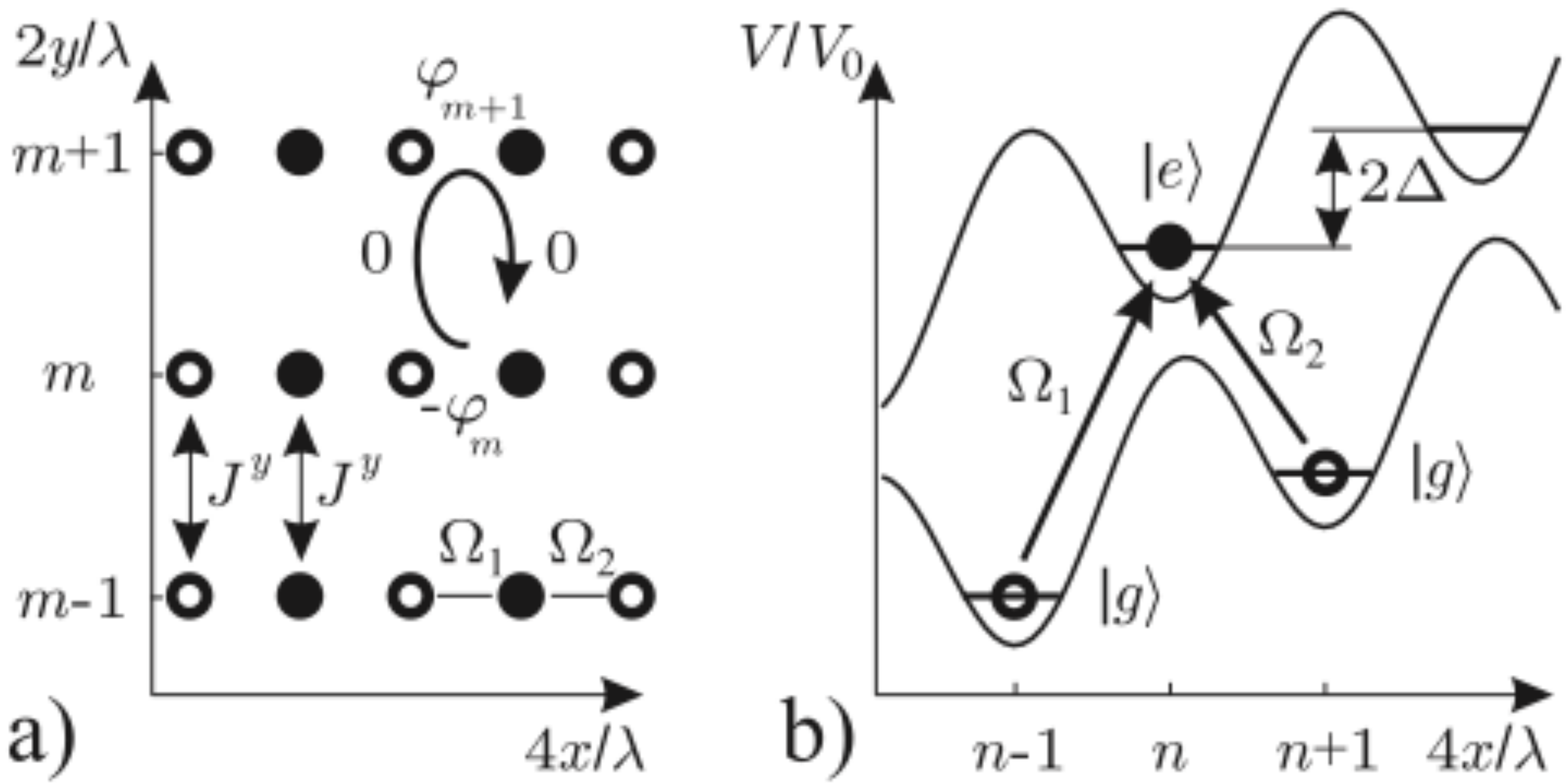
Support

AFOSR Quantum Phases MURI, ARO Atomtronics MURI, NIST, and NSF PFC @ JQI

Backdrop

Basic idea: laser induced hopping

$$H = -\sum_{j} \left[t^{(x)} e^{-i2\pi\Phi j_y} |\mathbf{j} + \mathbf{e}_x\rangle\langle\mathbf{j}| + t^{(y)} |\mathbf{j} + \mathbf{e}_y\rangle\langle\mathbf{j}| + \text{h.c.} \right]$$

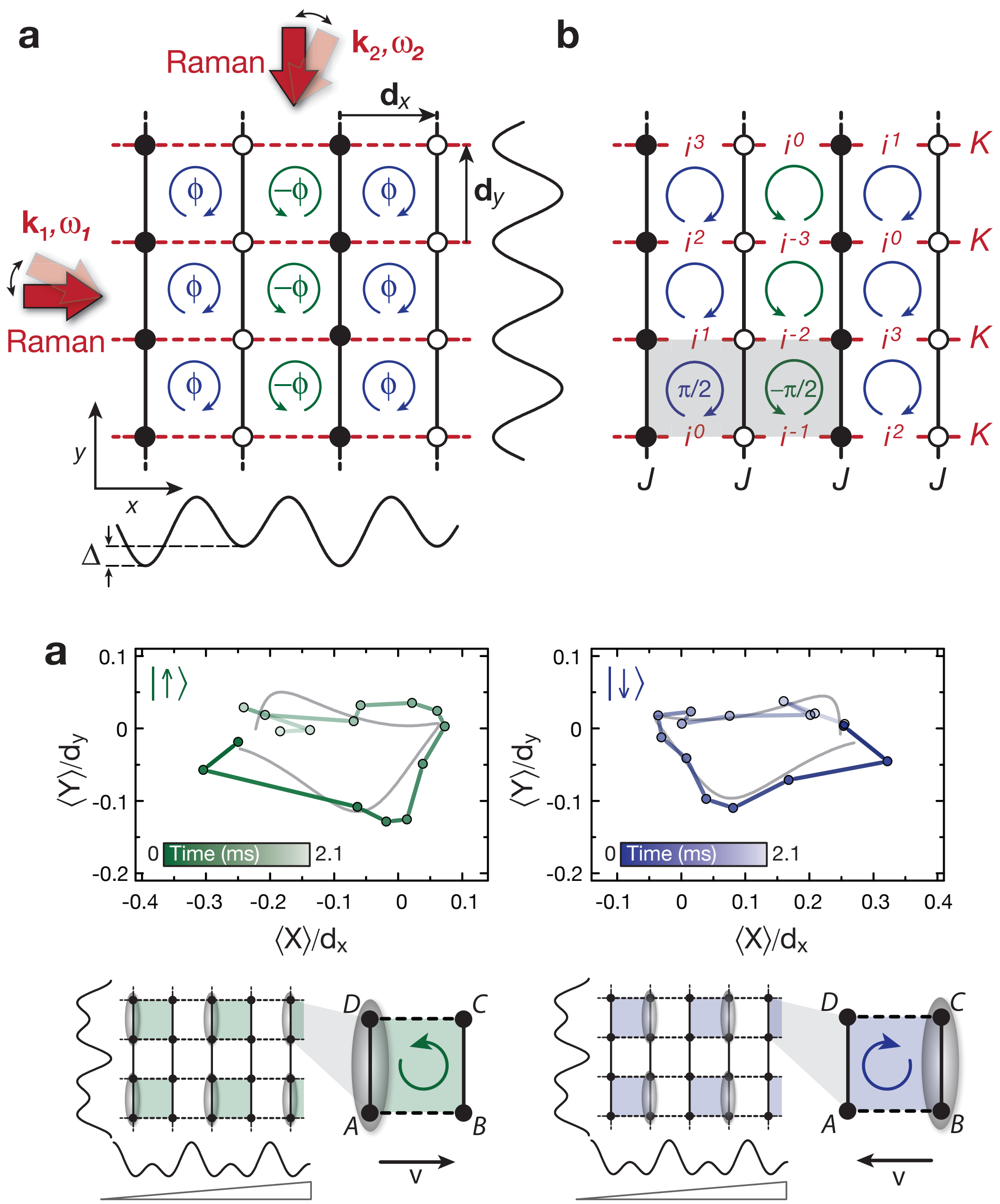


References

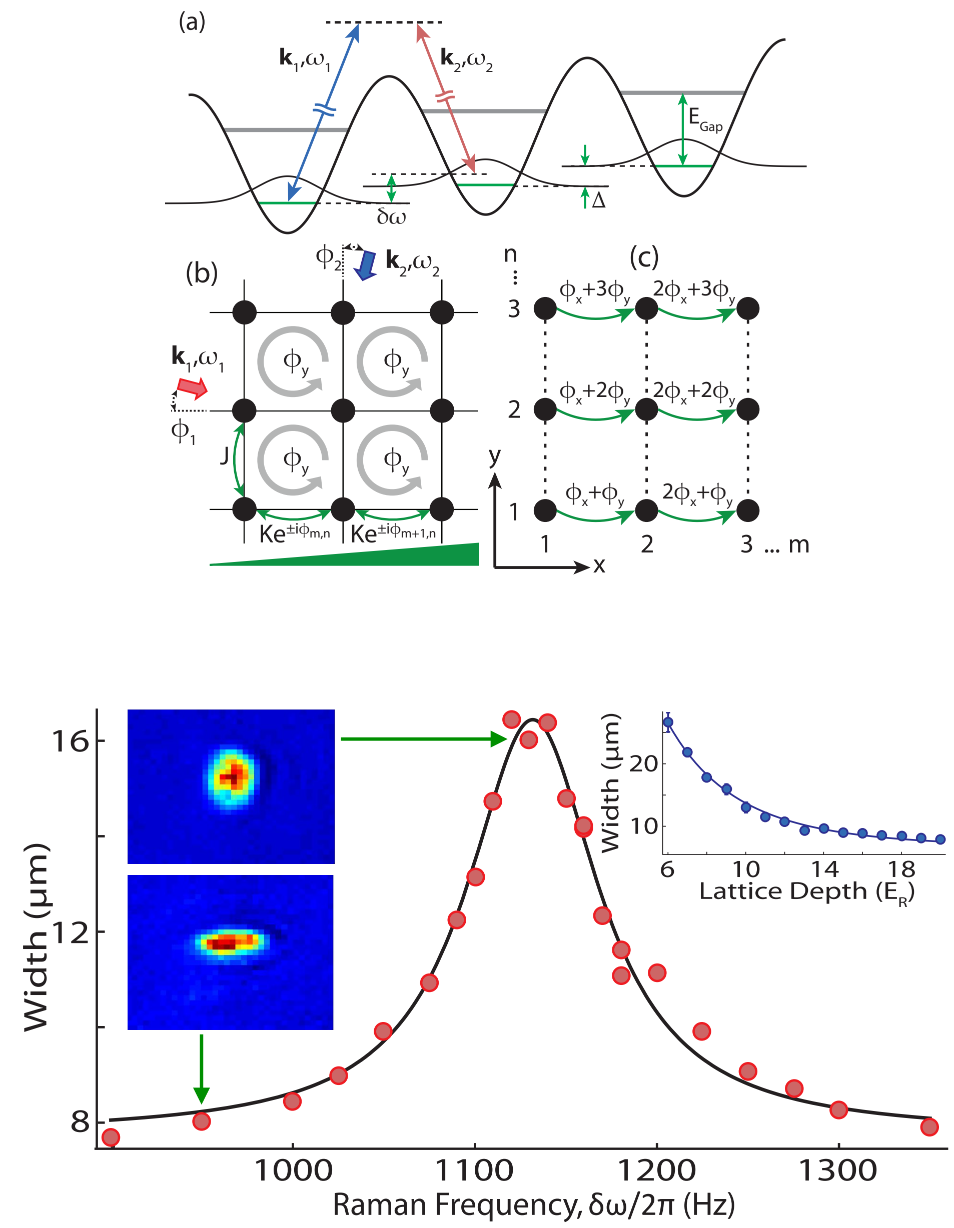
D. Jaksch and P. Zoller; New Journal of Physics (2003).

AMO: fields in lattices

Light assisted hopping I: Munich



Light assisted hopping II: MIT



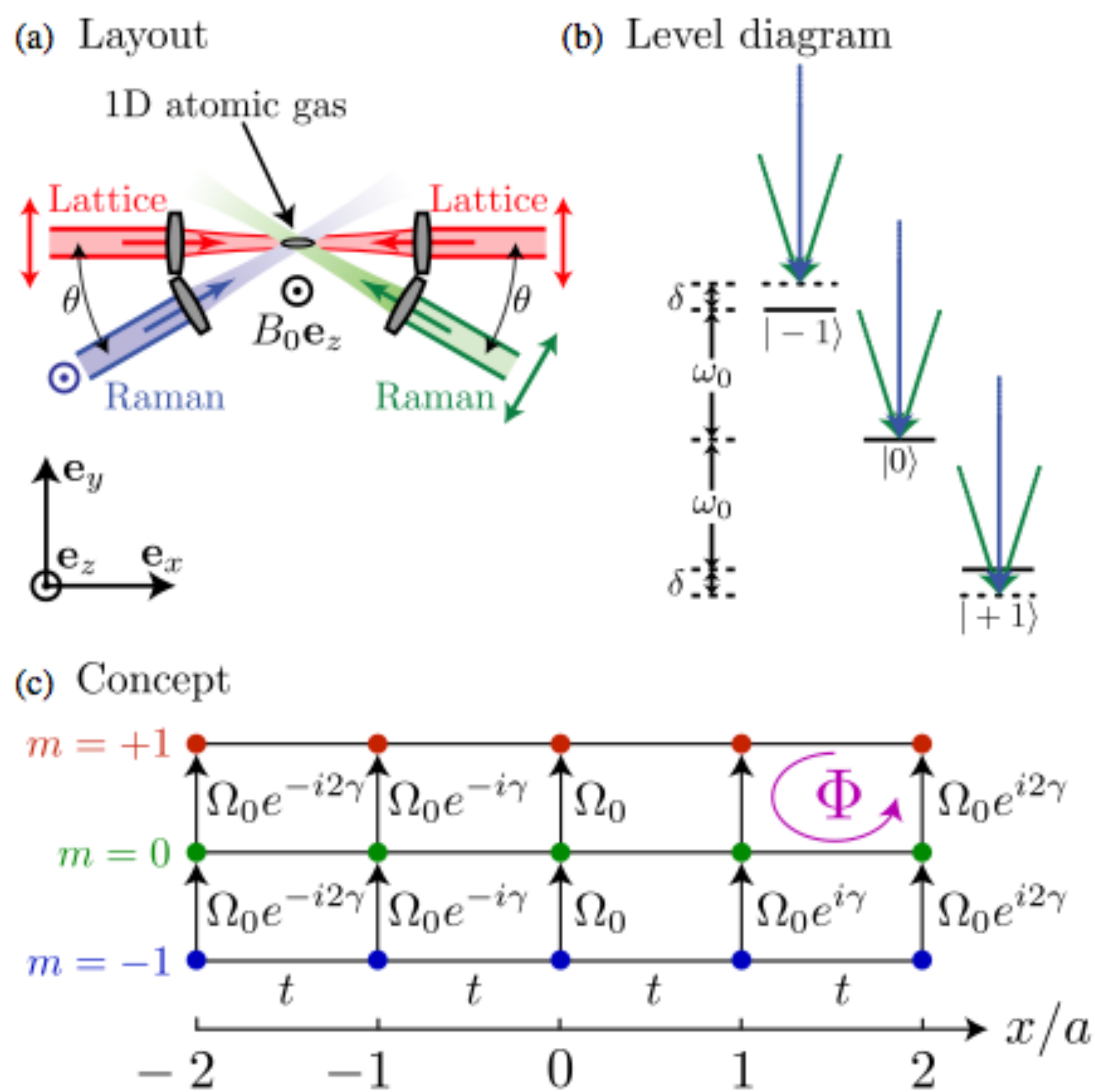
References

M. Aidelsburger *et al* PRL (2011), M. Aidelsburger et al PRL (2013)

References

C. J. Kennedy, et al Nat. Phys (2015), H. Miyake *et al* PRL (2013)

Synthetic dimensions

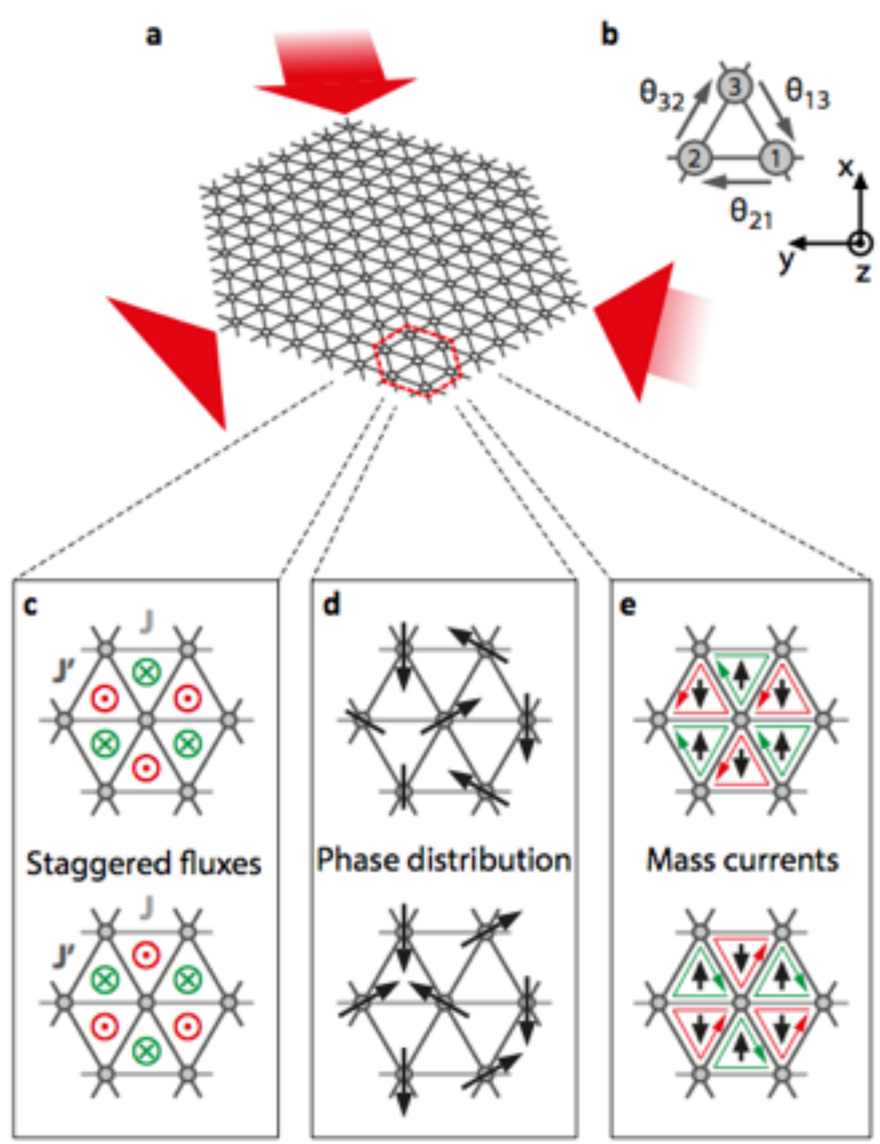


References

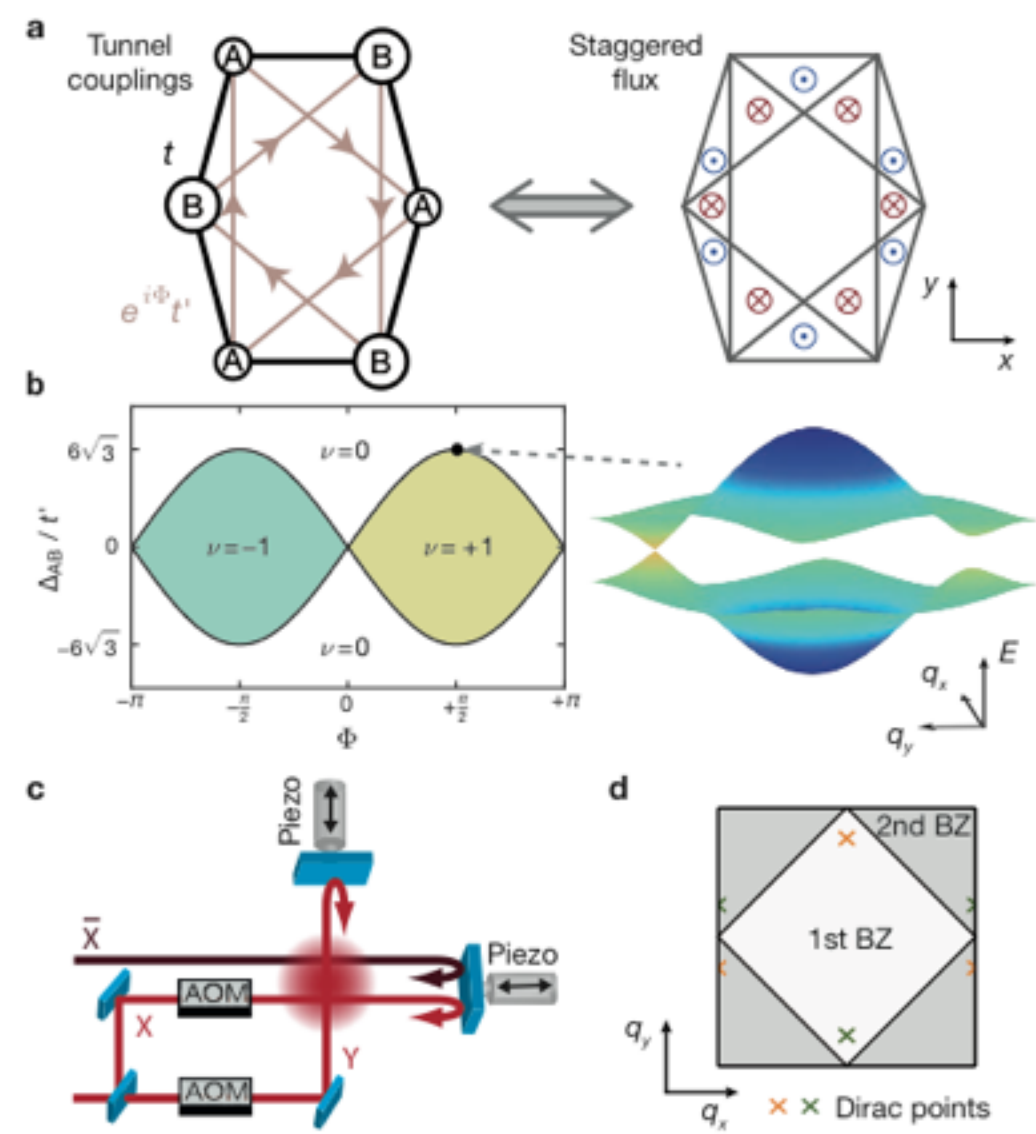
Celi, A. ...N. Goldman... et al. Phys. Rev. Lett. (2014)

AMO: exotic lattices

Modulation I: Hamburg



Modulation II: Zurich



References

J. Struck *et at* N. Physics (2013)

References

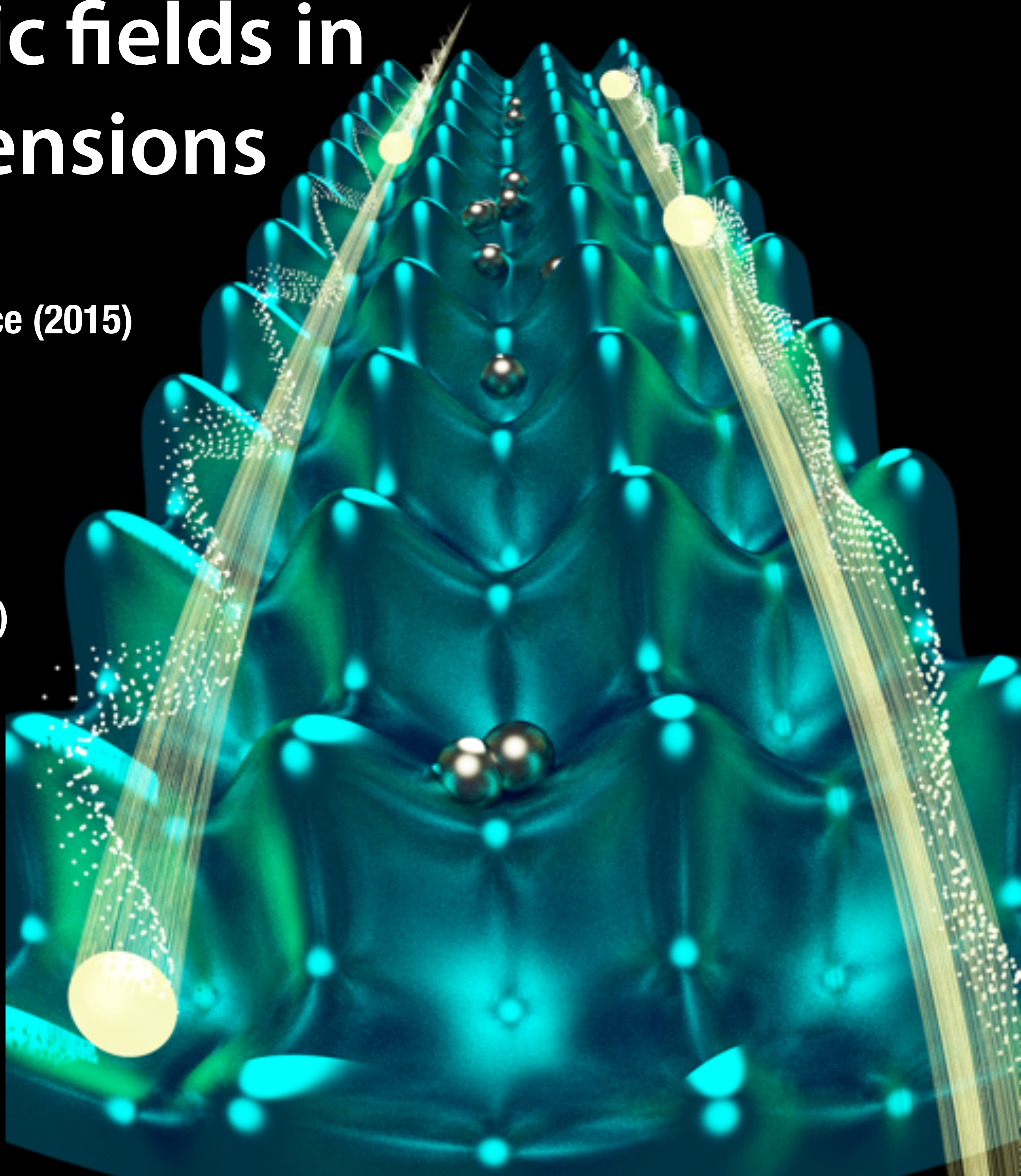
J. Gotzu *at al* Nature (2014)

Large magnetic fields in synthetic dimensions

<u>JQI:</u>
B. Stuhl and Hsin-I Lu, et al. Science (2015)

<u>Florence</u>
M. Mancini et al. Science (2015)

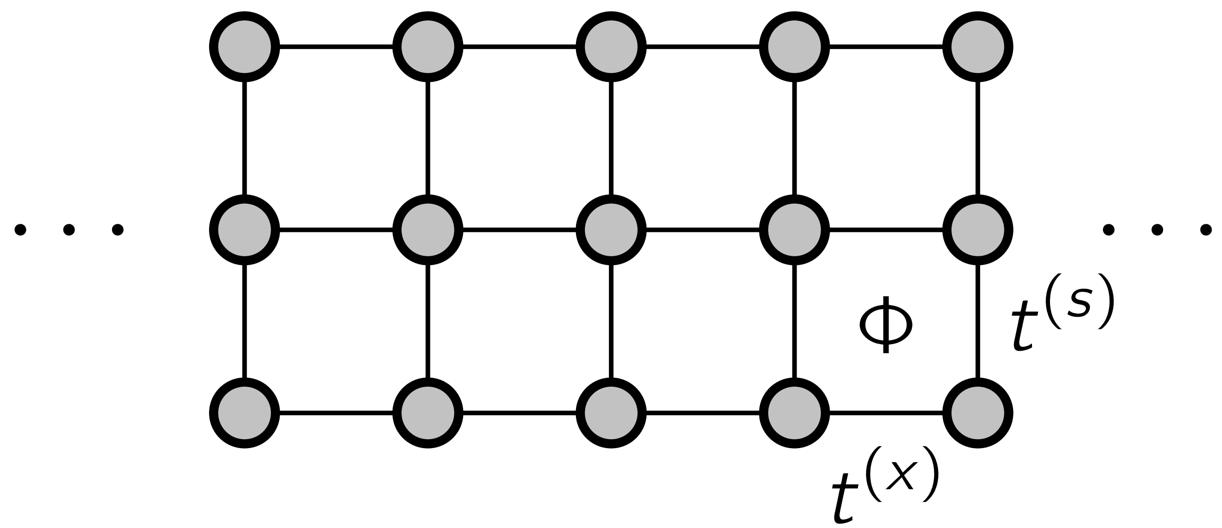
<u>THEORY PROPOSAL:</u>
Celi, A. et al. Phys. Rev. Lett. (2014)



Where we are going:

Large magnetic fields in a 2D lattice

$$\Phi \approx 1/3$$



We will engineer each term in the model individually

References

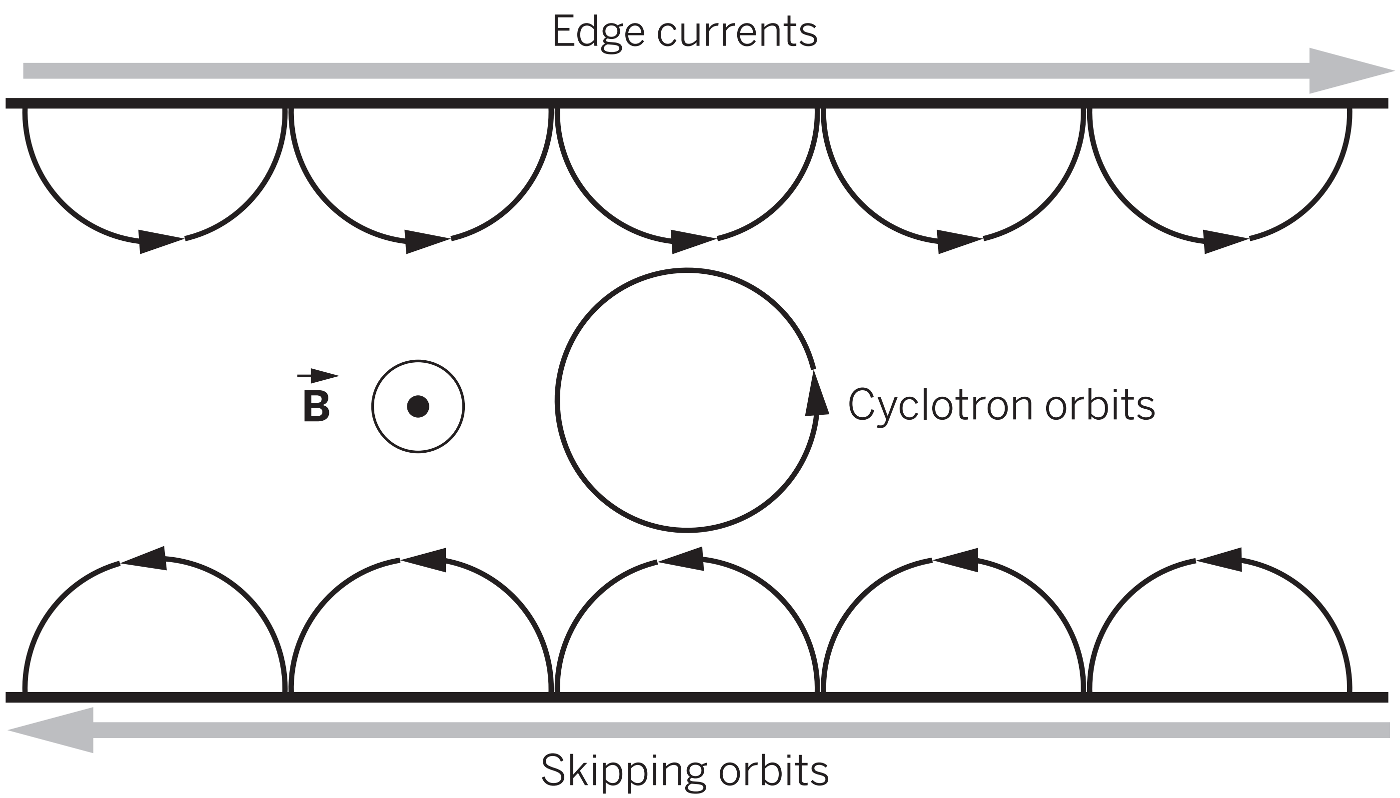
B. Stuhl, et al. arxiv (2015); Celi, A. et al. Phys. Rev. Lett. (2014);

Large fields in extended systems: MIT and Munich

Continuum magnetic fields

Single-particle eigenstate structure: Landau gauge

$$H = \frac{\hbar^2}{2m}\left[\left(k_x + \frac{qBy}{\hbar}\right)^2 + k_y^2\right], \quad \text{from} \quad \mathbf{A} = -By\mathbf{e}_x \quad \text{with} \quad \ell_B = \sqrt{\frac{\hbar}{qB}}$$



References

Figure From: A. Cili and L. Tarruell, Science Perspective (2015)

Continuum magnetic fields

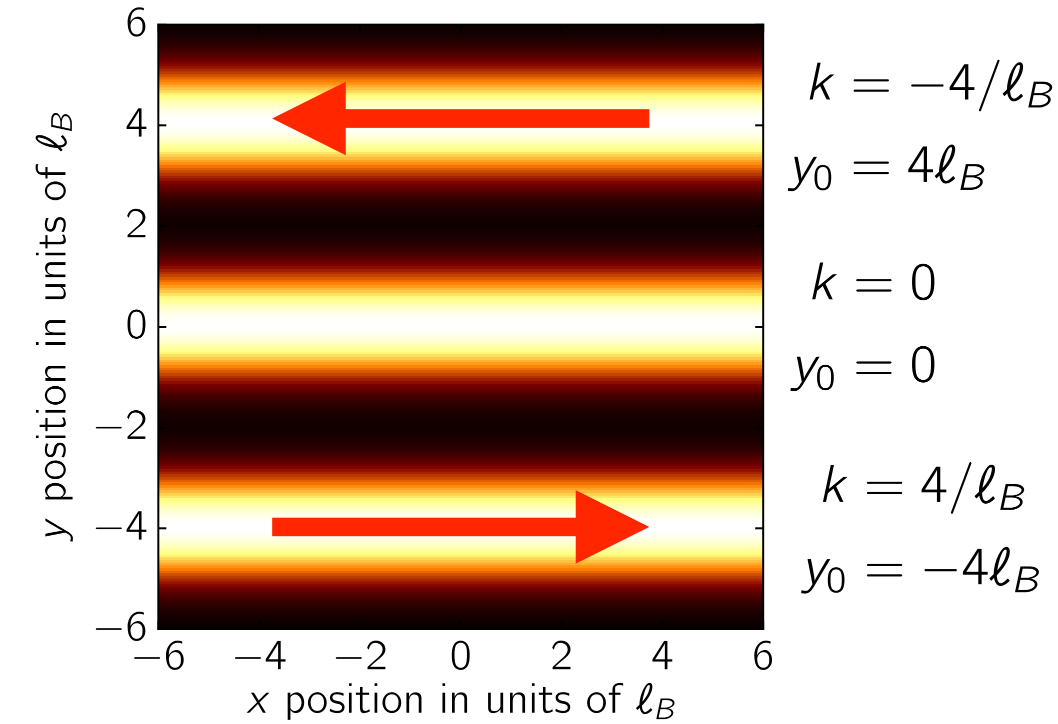
Single-particle eigenstate structure: Landau gauge

$$H = \frac{\hbar^2}{2m}\left[\left(k_x + \frac{qBy}{\hbar}\right)^2 + k_y^2\right], \quad \text{from} \quad \mathbf{A} = -By\mathbf{e}_x \quad \text{with} \quad \ell_B = \sqrt{\frac{\hbar}{qB}}$$

Has lowest Landau level eigenstates with **one** quantum number k

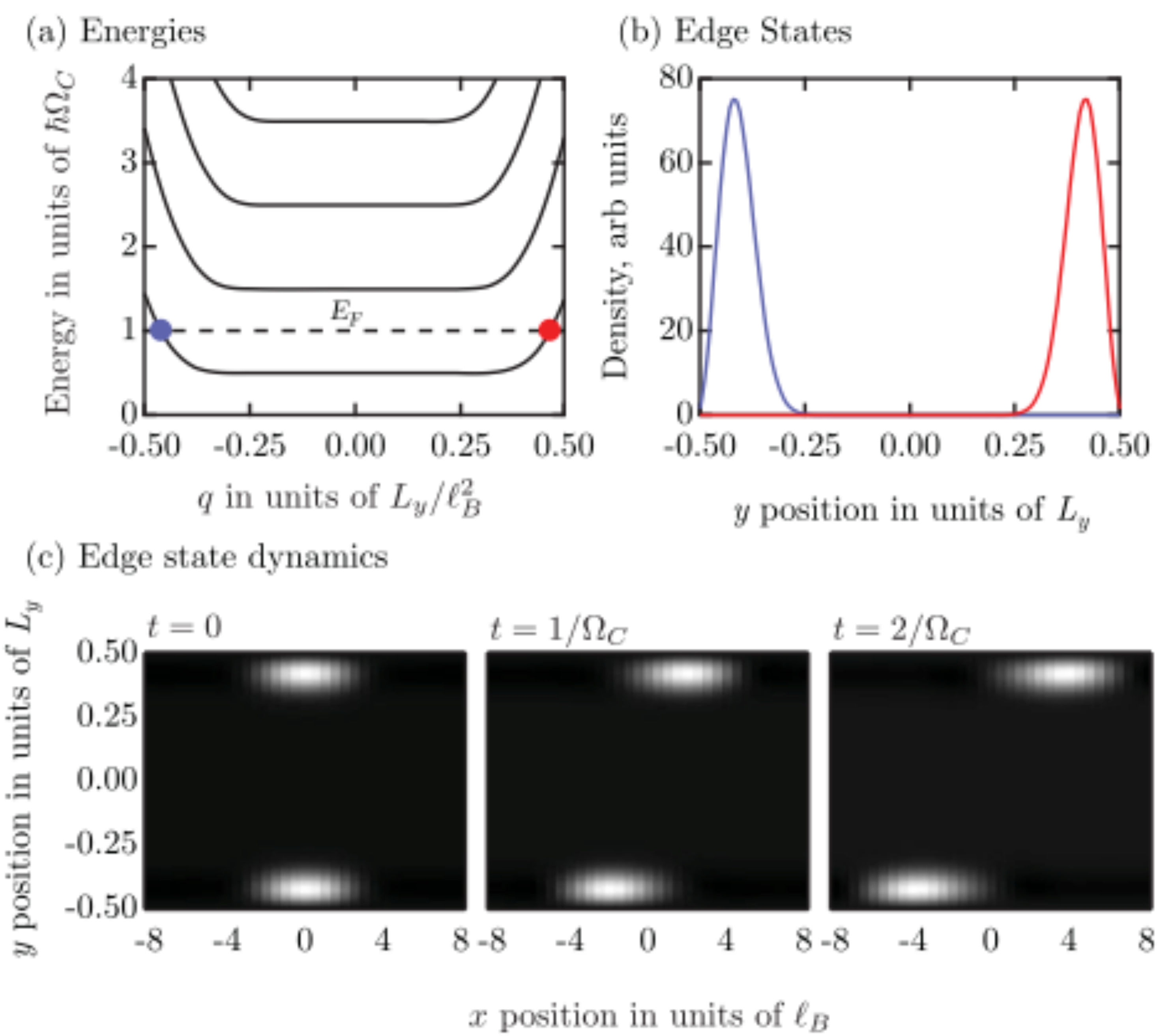
$$\psi(x,y) \propto e^{ikx} e^{-(y-y_0)^2/2\ell_B^2}$$

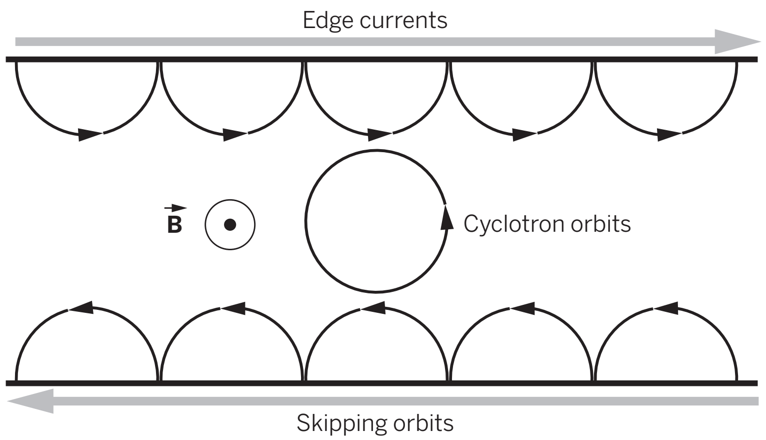
with guiding center $y_0 = -k\ell_b^2$



QHE systems: Edge states

Edge excitations in QHE systems





References

I. B. Spielman; Ann. der Phy. (2013).

Lattice magnetic fields

Single-particle eigenstate structure: Landau gauge

$$H = -\sum_{\mathrm{j}} \left[t^{(x)} e^{-i2\pi\Phi j_y} |\mathbf{j}+\mathbf{e}_x\rangle\langle\mathbf{j}| + t^{(y)} |\mathbf{j}+\mathbf{e}_y\rangle\langle\mathbf{j}| + \mathrm{h.c.} \right]$$

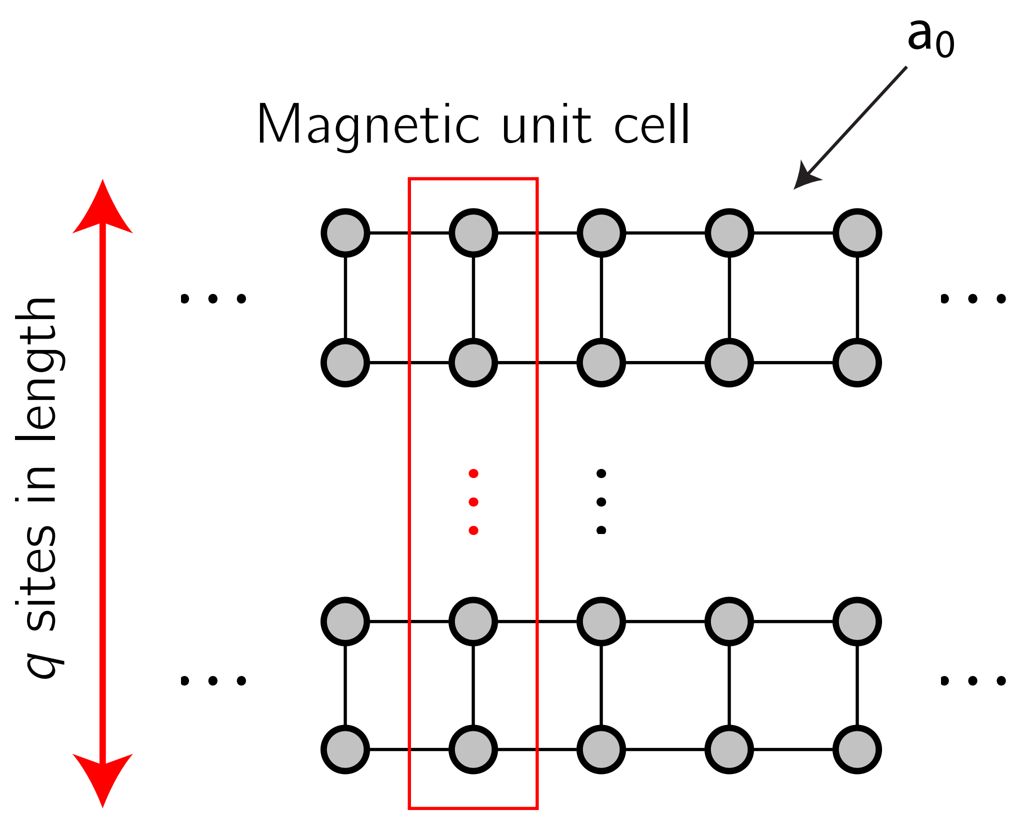
Has lowest Landau *band* eigenstates with **two** quantum numbers: k_x and k_y

With $\Phi = \dfrac{p}{q}$ in simplest form,

$$\psi(\mathbf{j}) = u_{\mathrm{k}}(j_y) e^{i\mathrm{j}\cdot\mathrm{k}}$$

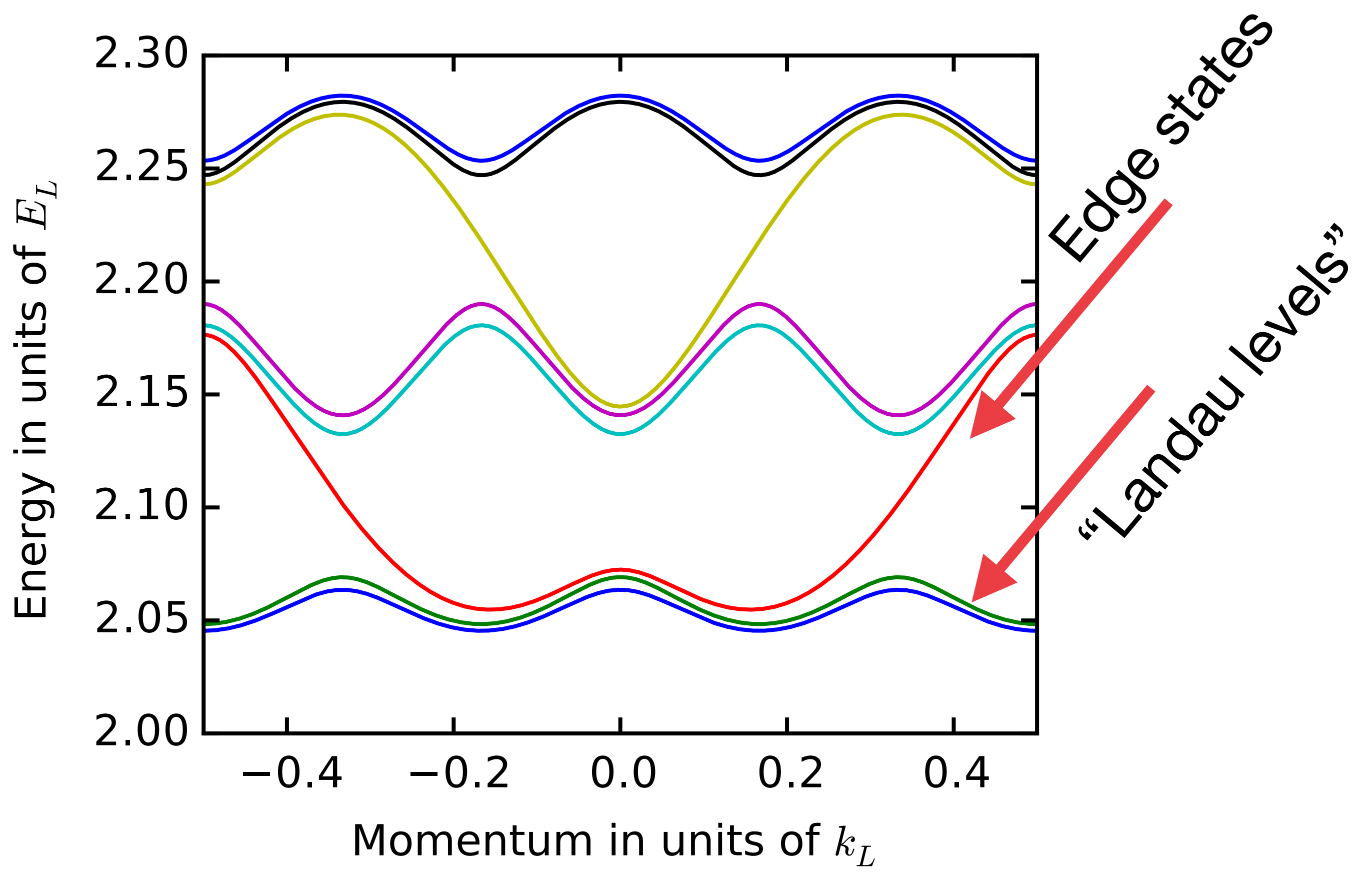
Magnetic B.Z. sized: $k_0 \times k_0/q$

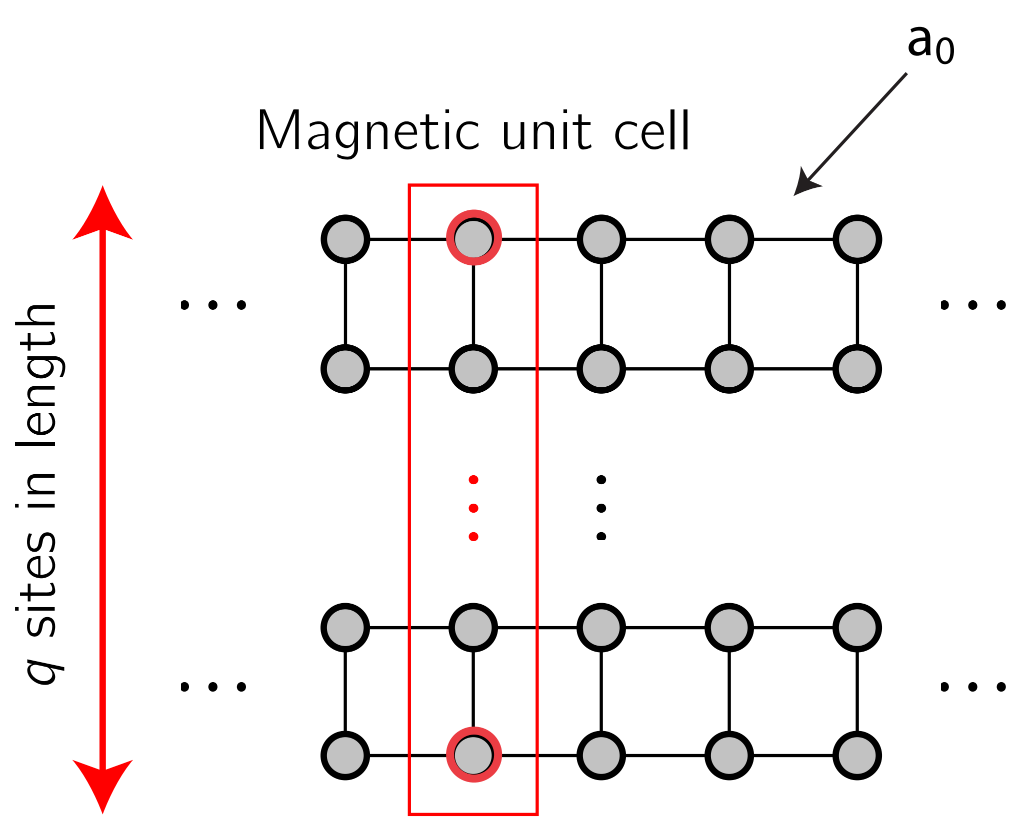
Unit cell sized: $a_0 \times q a_0$



Lattice magnetic fields

Single-particle eigenstate structure: Landau gauge



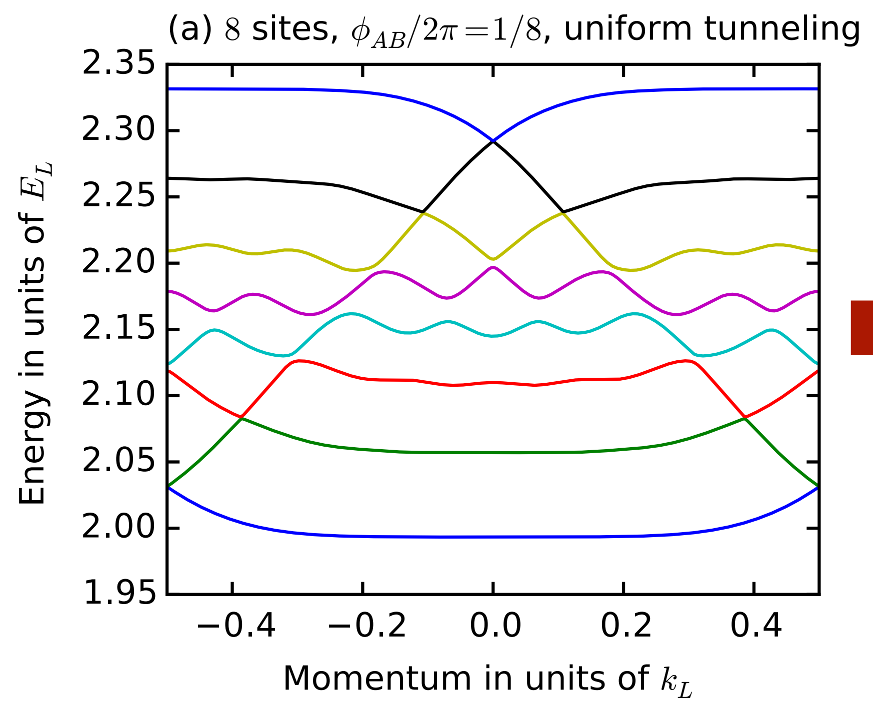


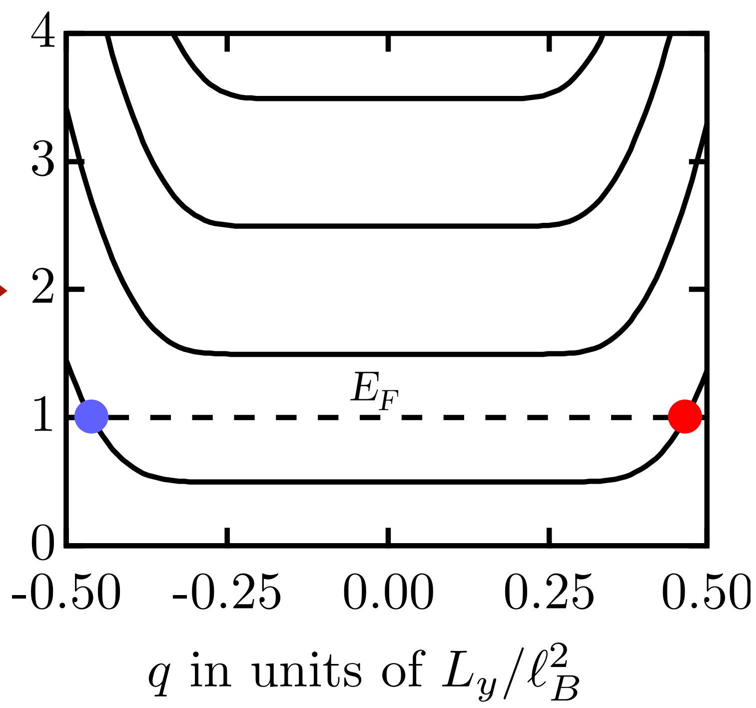
q degenerate ground states each with with a distinct "guiding center" *within* each magnetic unit cell

Finite width strips

Continuum limit

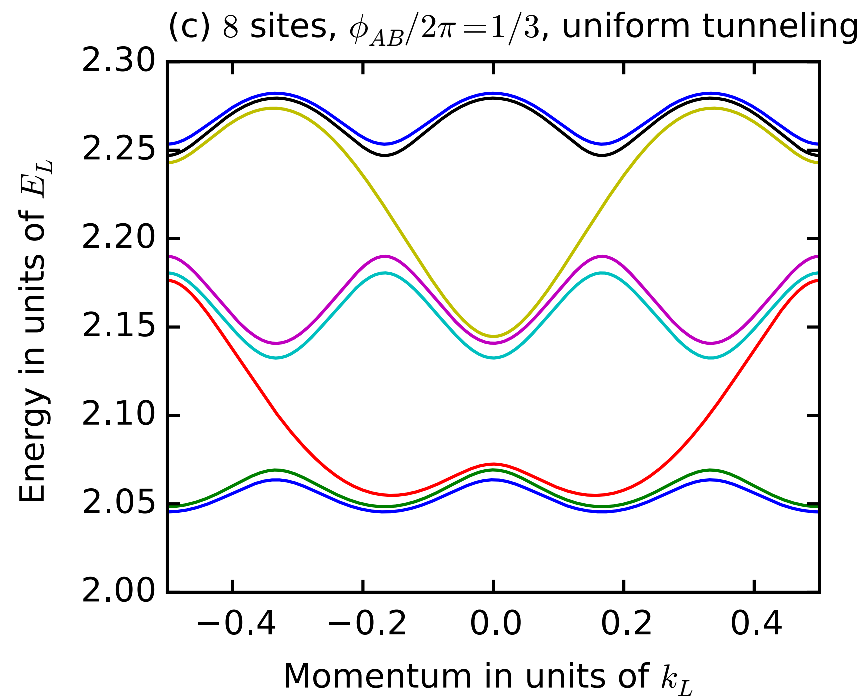
One unit cell in size (or less)



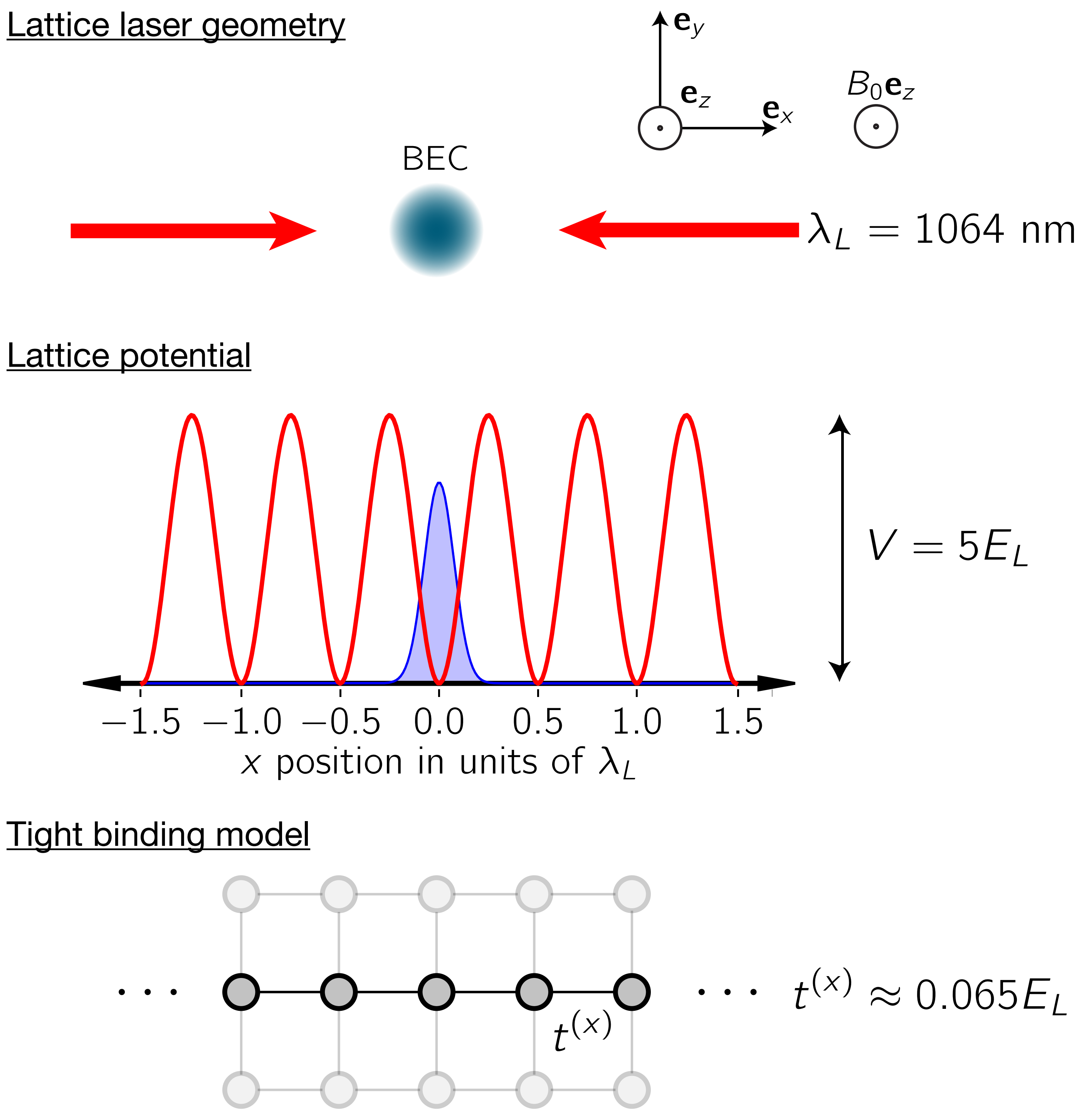


"Hofstadter" limit

More than one unit cell

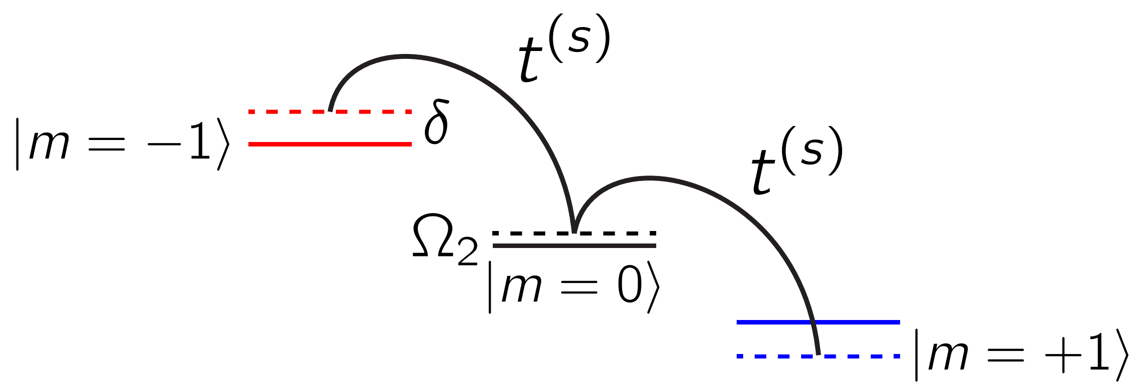


Building a magnetic lattice

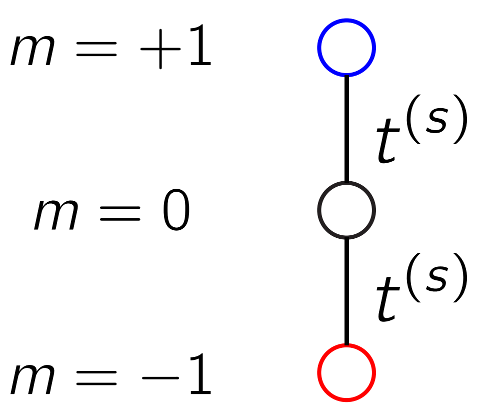


Building a magnetic lattice

Usual level diagram



Lattice picture



RF or Raman Hamiltonian

$$H = \Omega \cdot \hat{\mathbf{F}}$$

Simple "tunneling"

$$t^{(s)} = -\frac{\hbar}{\sqrt{2}}(\Omega_x - i\Omega_y)$$

Complete harmonic "potential"

$$E_m = (\hbar\delta)m + (\hbar\Omega_2)m^2$$

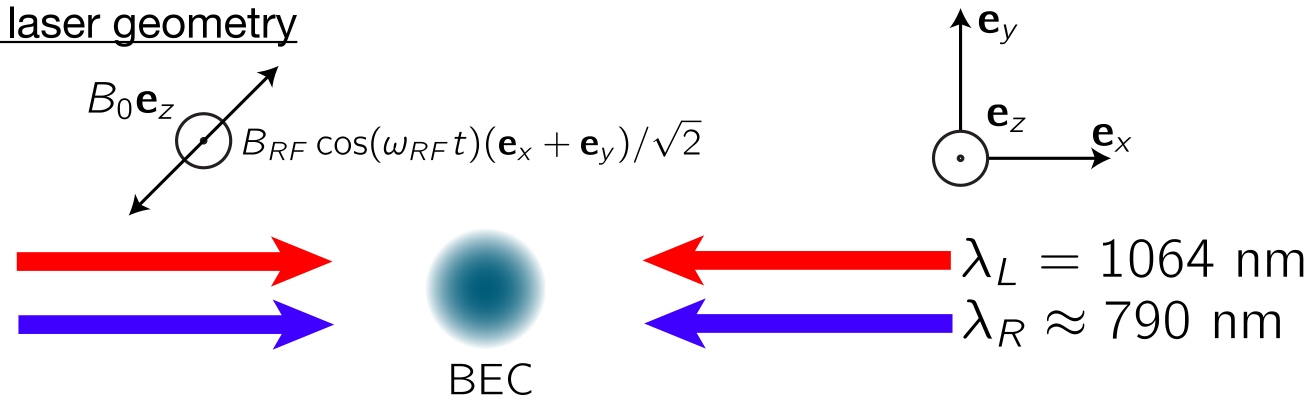
In todays experiments the m^2 term may be neglected.

Reference

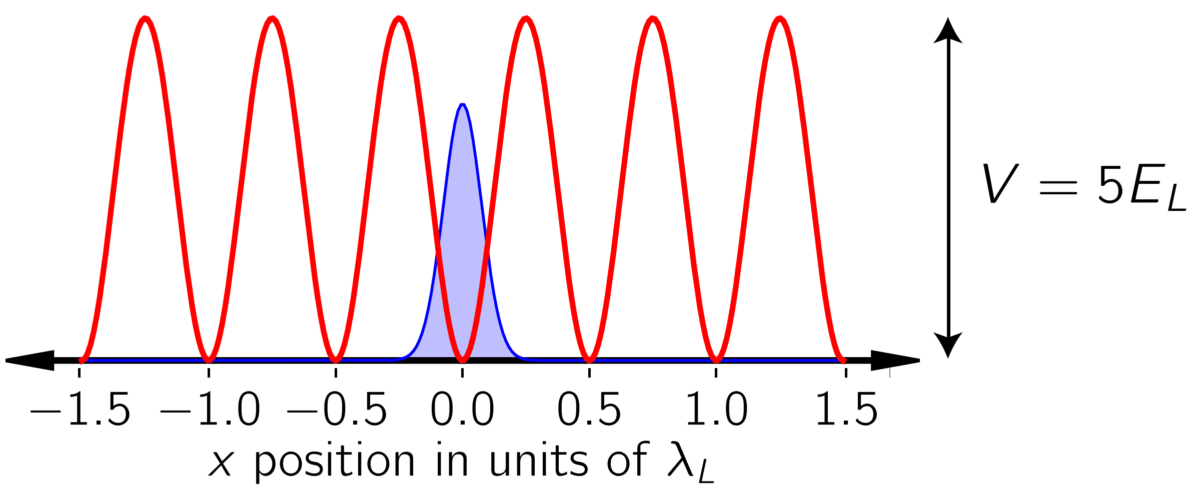
Celi, A. et al. Phys. Rev. Lett. (2014).

Full lattice

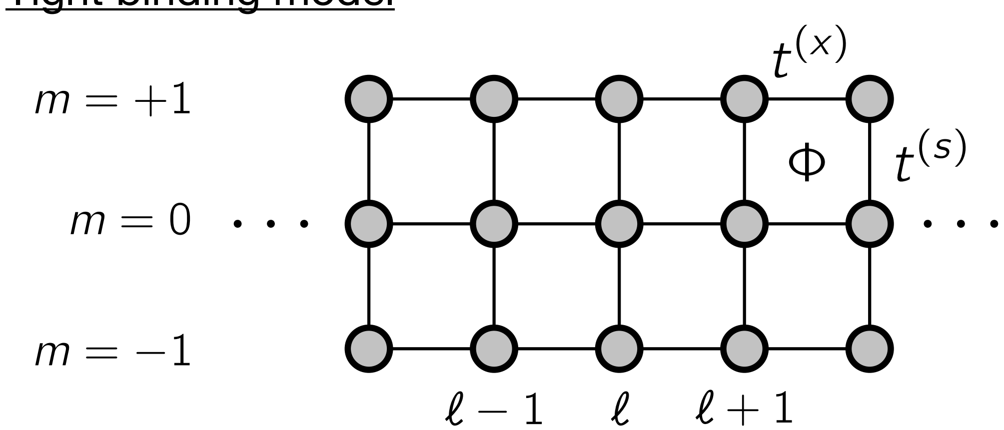
Lattice laser geometry



Lattice potential



Tight binding model



$t^{(x)} \approx 0.065E_L$

$|t^{(s)}| = 0.5t^{(x)}$ to $5t^{(x)}$

Synthetic hopping

RF

$$t^{(s)} = \frac{\Omega_{RF}}{\sqrt{2}}$$

Raman

$$t^{(s)} = \frac{\Omega_R}{\sqrt{2}}e^{i2k_R x} = \frac{\Omega_R}{\sqrt{2}}e^{i(2\pi\Phi)\ell}$$

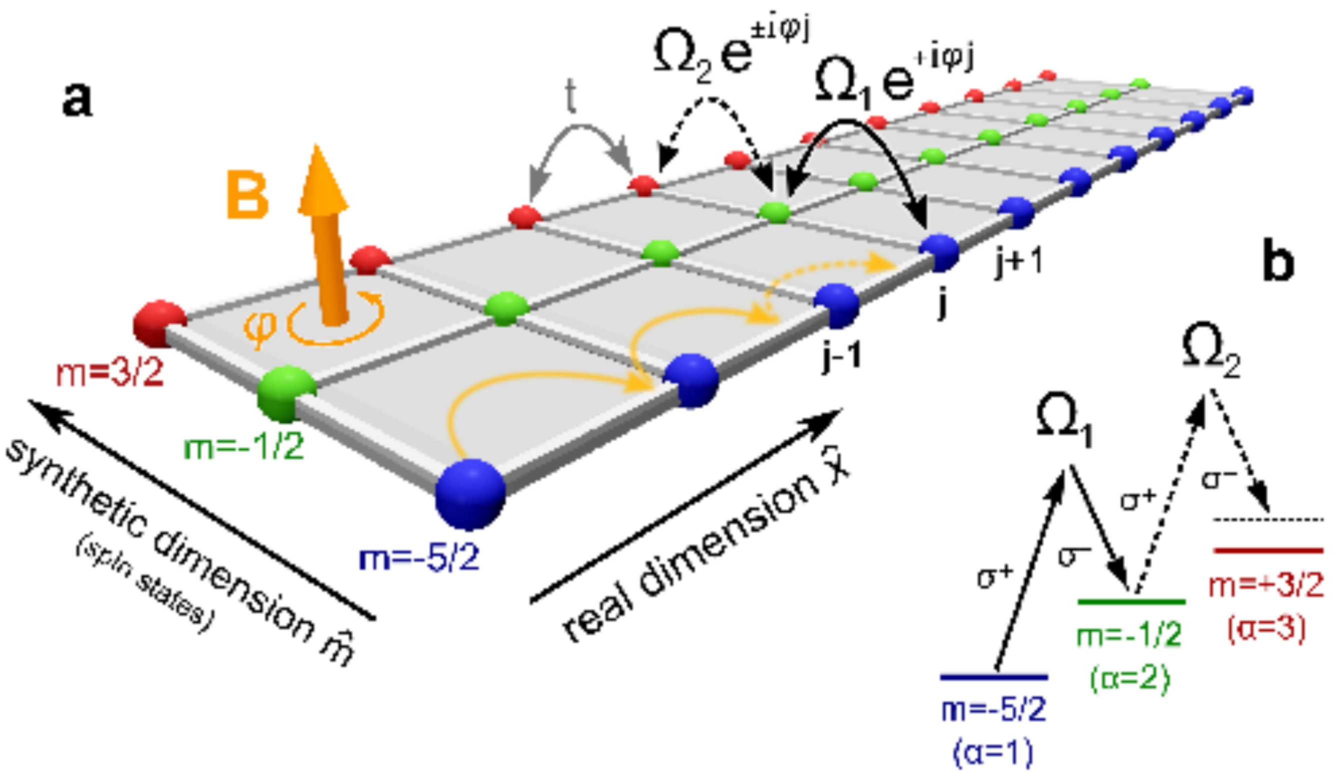
with $\Phi = \dfrac{\lambda_L}{\lambda_R} \approx 1.32$

References

Celi, A. et al. Phys. Rev. Lett. (2014); 3-site dual to: M. Atala et al. Nat. Phys. (2014)

In Yb, with spins

Related experiment in Florence, with Yb

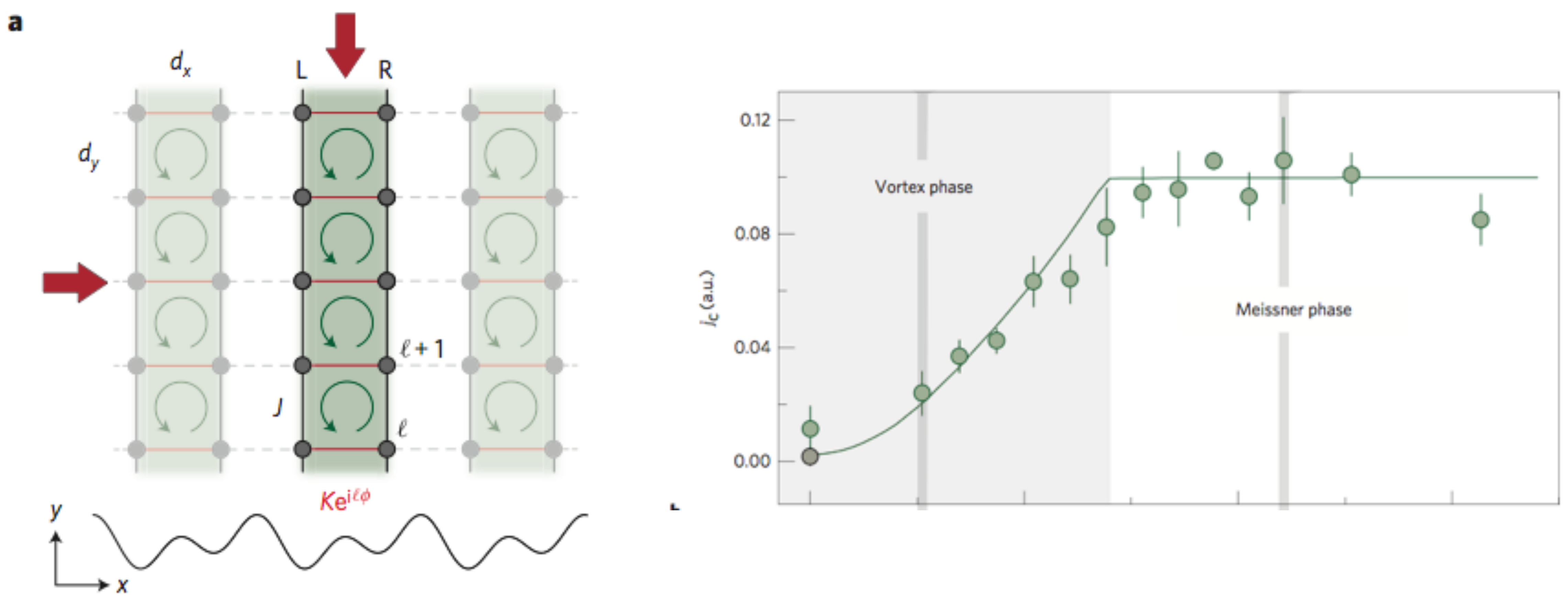


References

M. Mancini ... L. Fallani, Science (2015)

2x leg ladder experiments

Related experiment in Munich with a 2x leg ladder

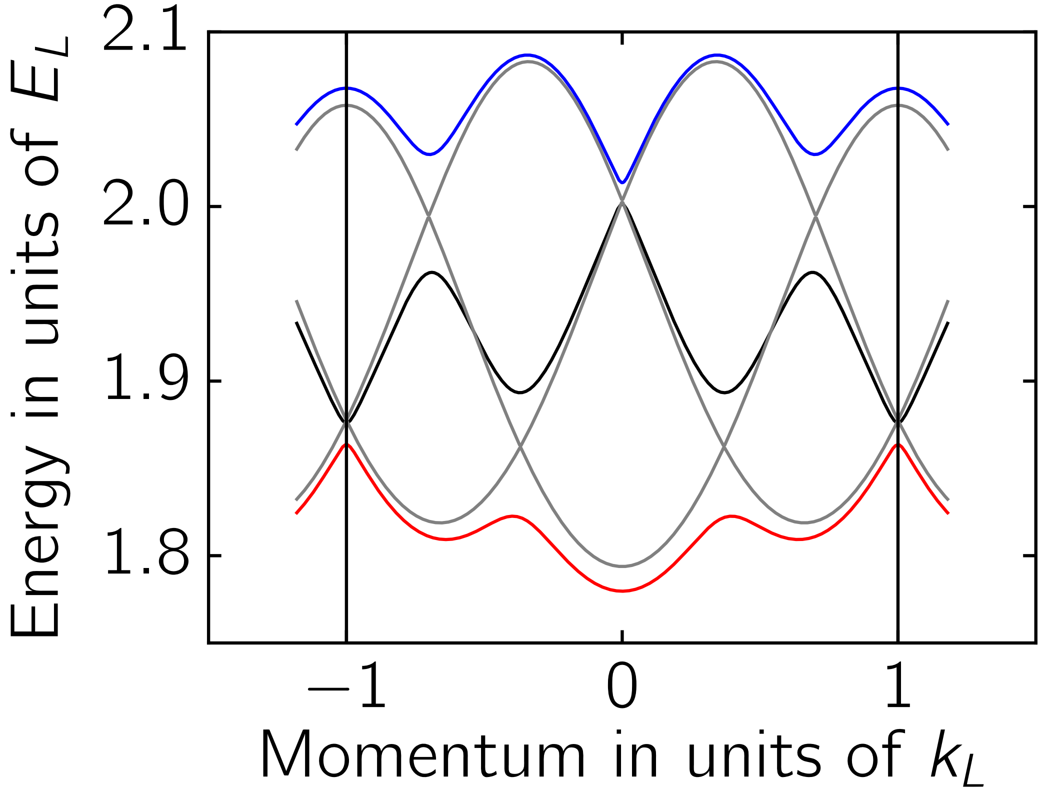


References

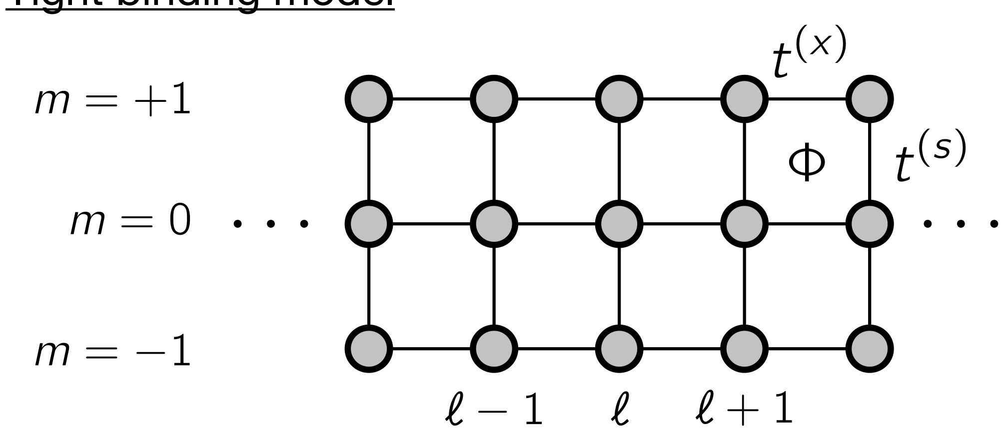
M. Atala, … I. Bloch, Nat Phys 10, 588–593 (2014)

Full lattice

Band structure



Tight binding model



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References

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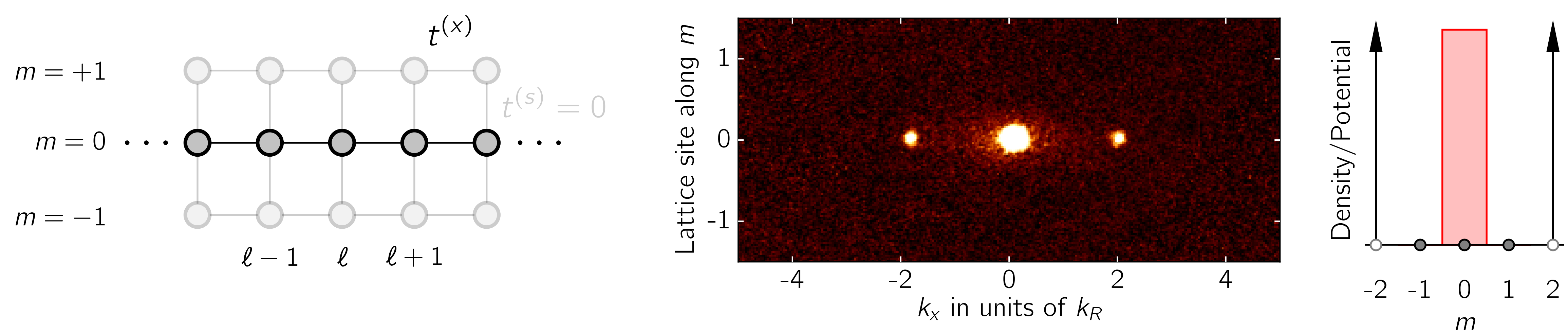
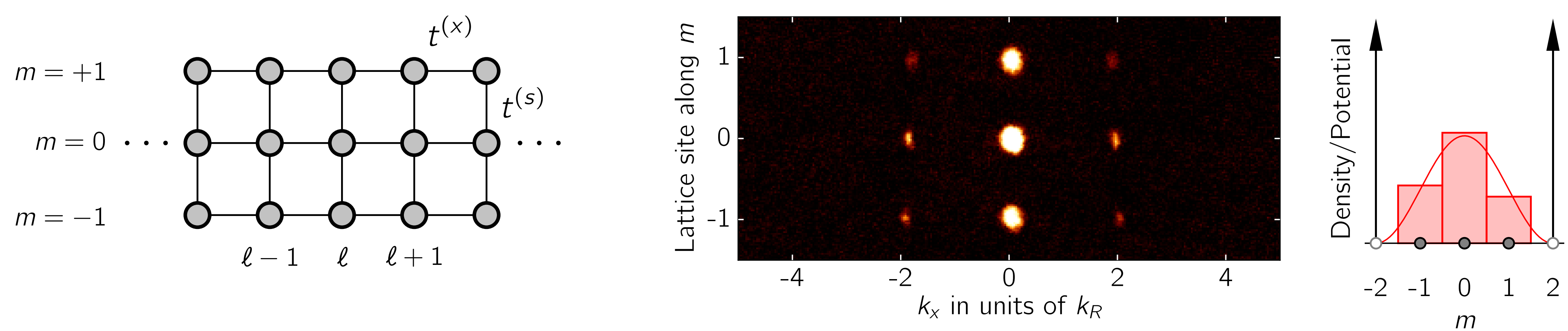
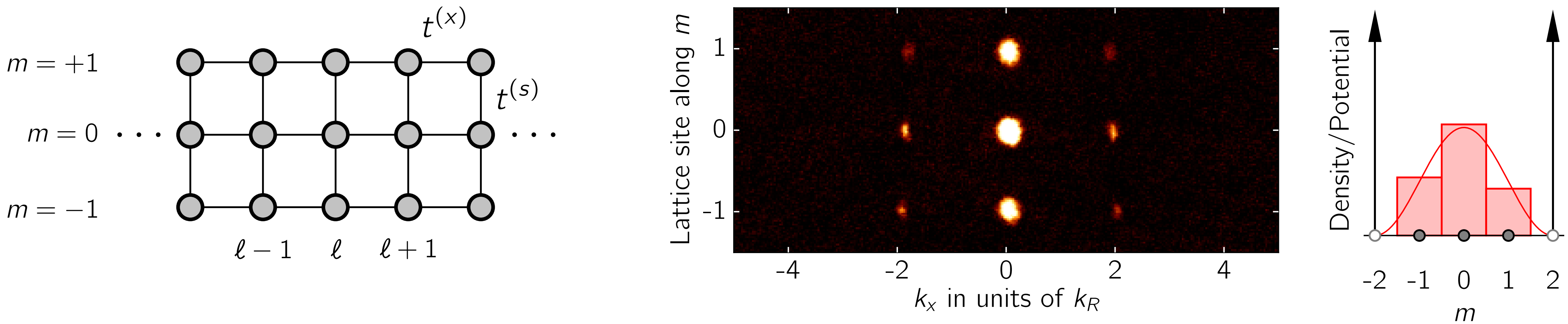
Interpreting data



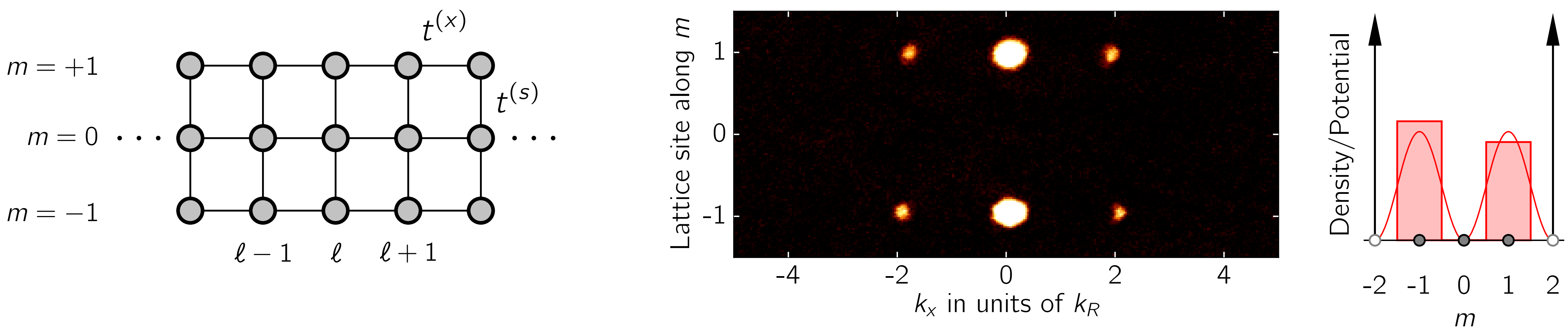


Interpreting data: zero flux

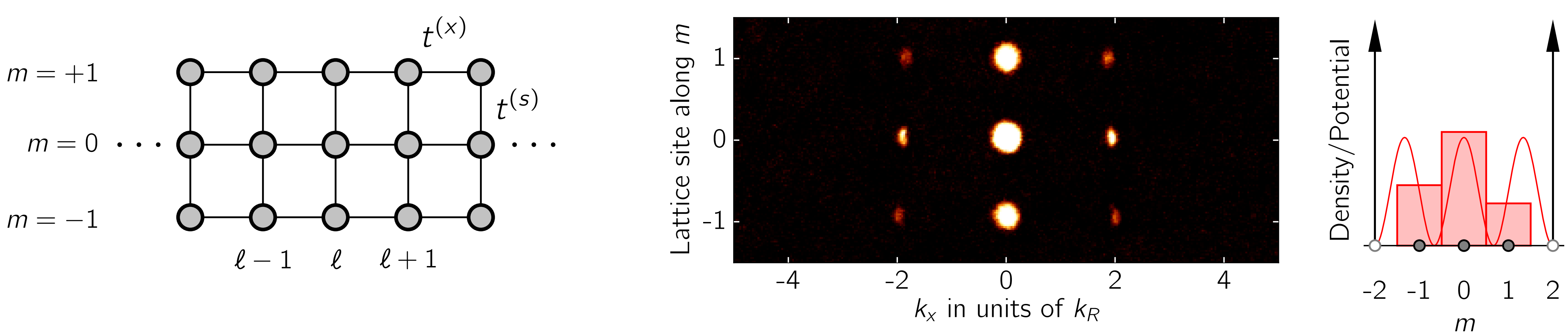
Ground state



First excited state

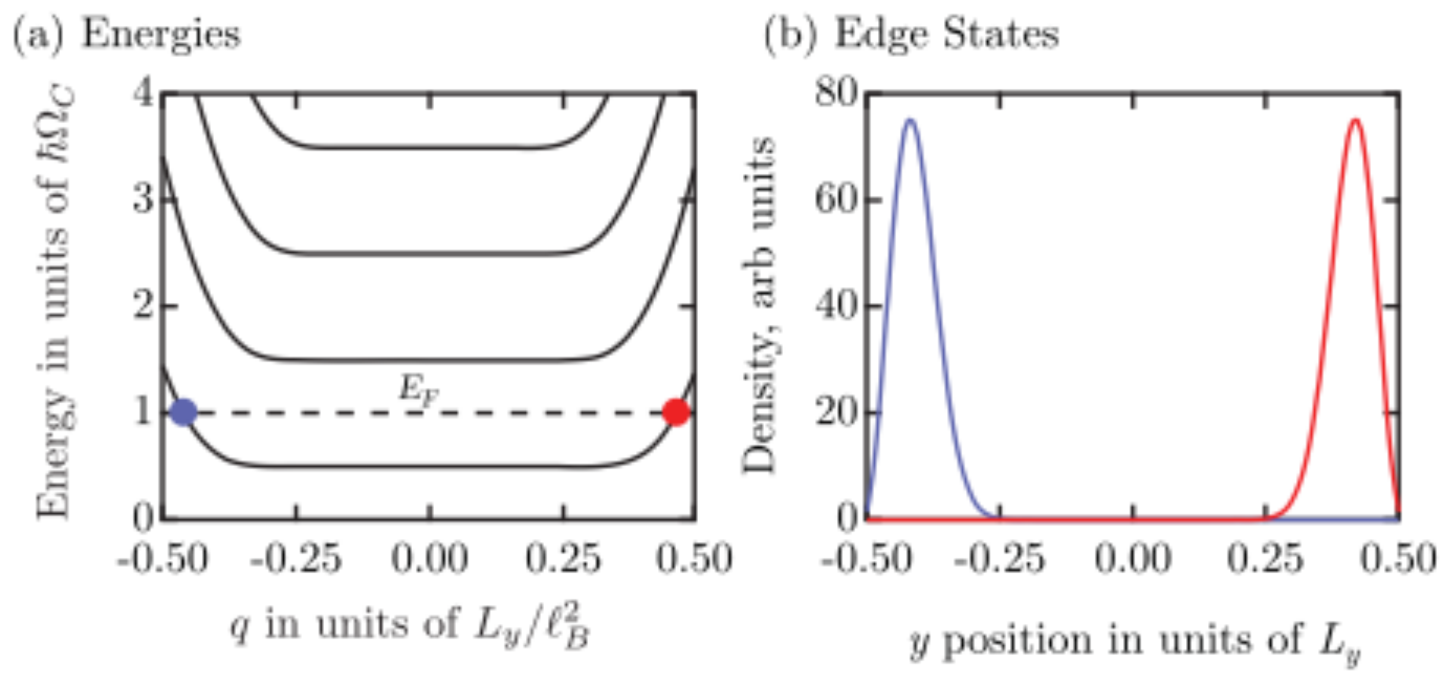


Second excited state



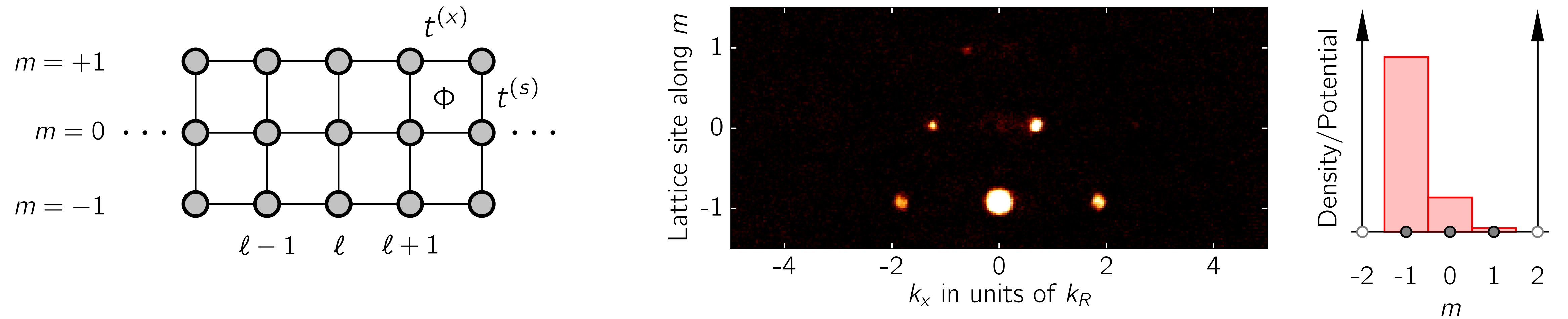
QHE systems: Edge states

Edge states in QHE systems

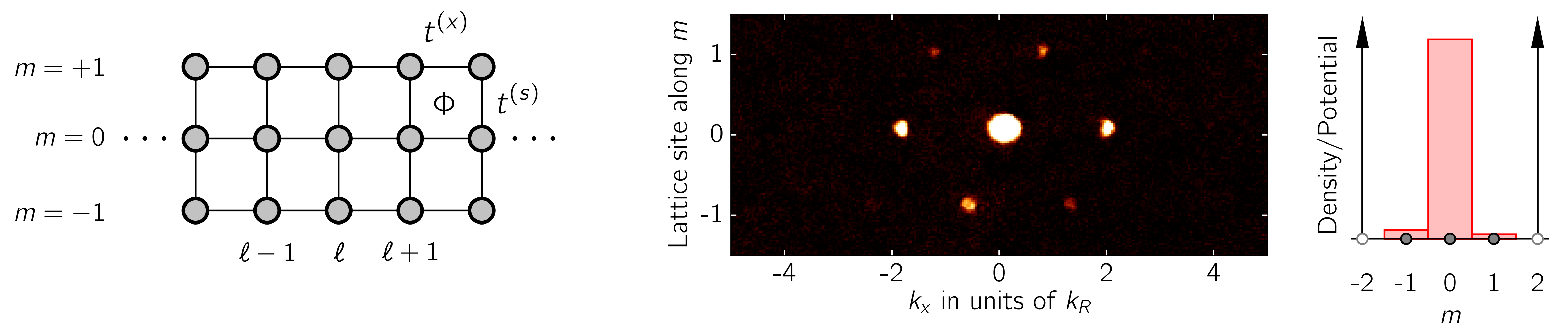


Interpreting data: +1/3 flux quantum

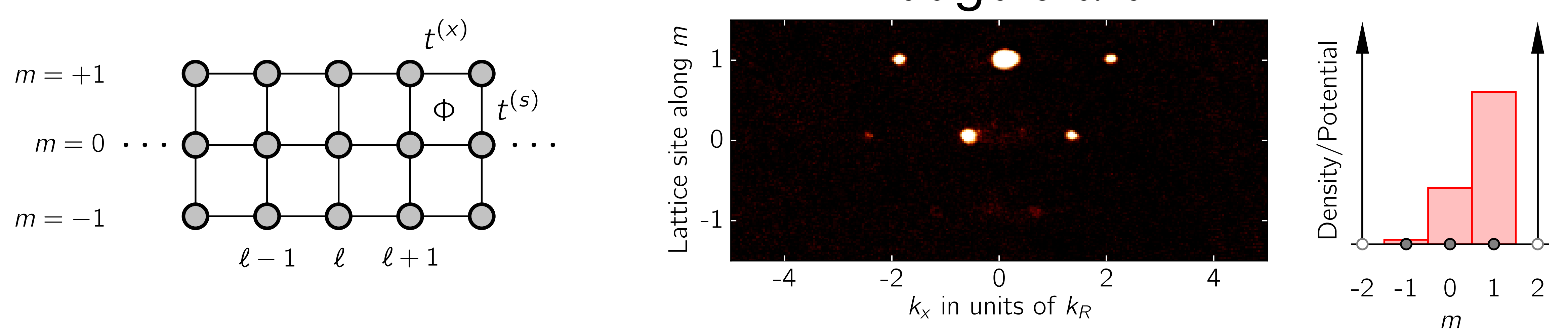
-1 edge state



Center

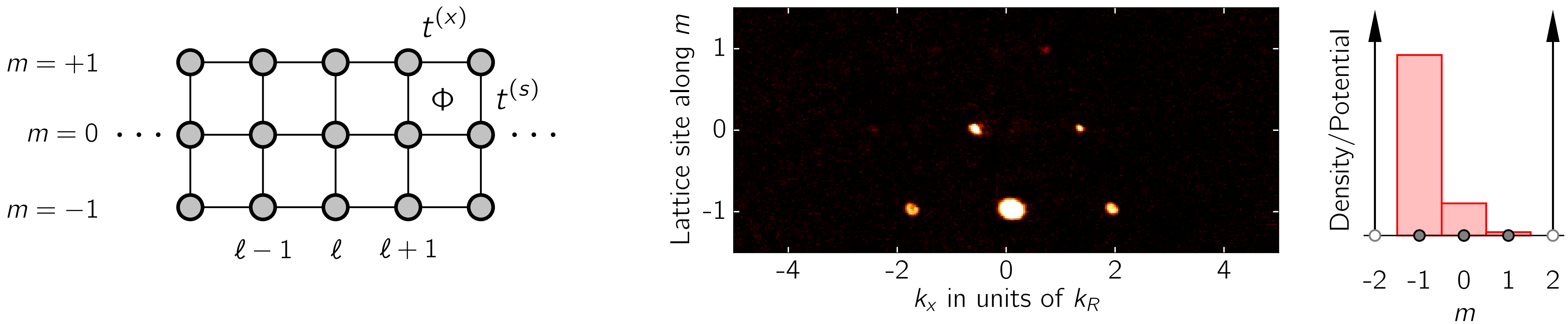


+1 edge state

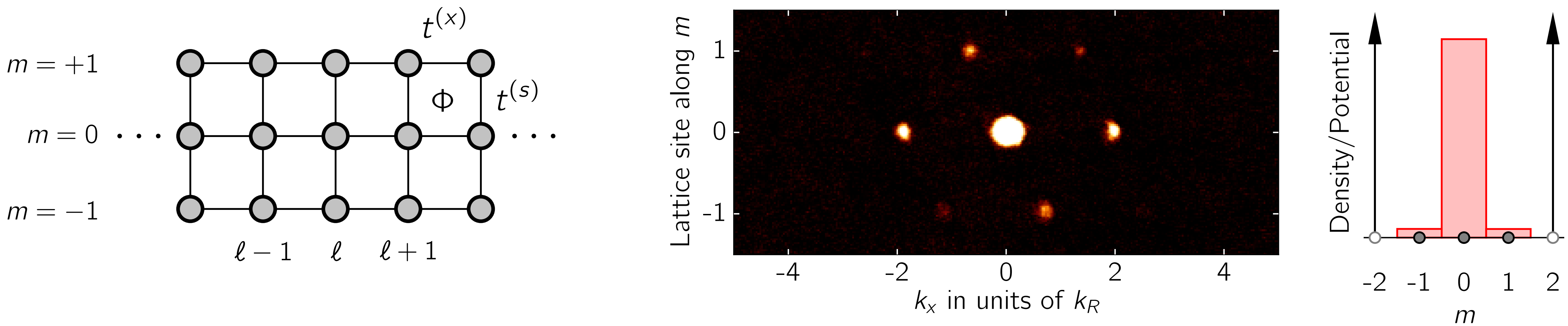


Interpreting data: -1/3 flux quantum

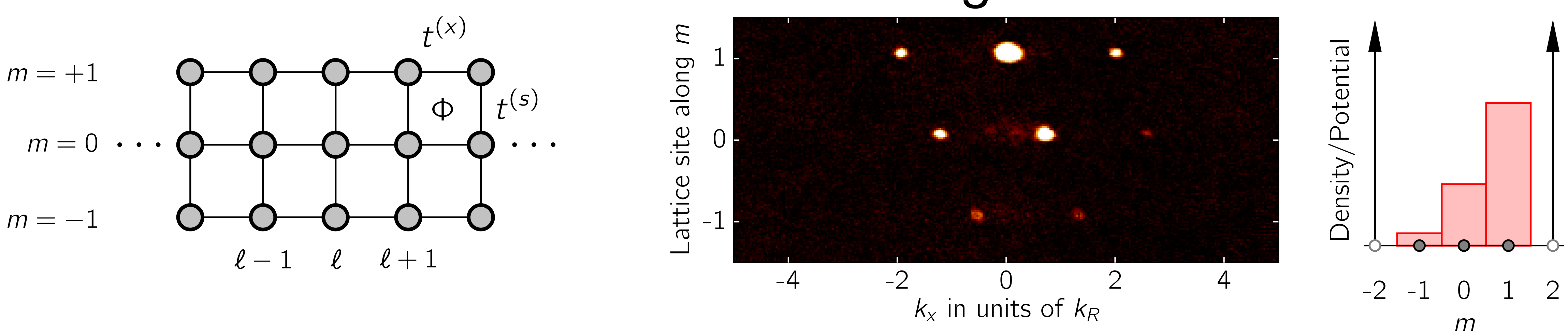
-1 edge state



Center



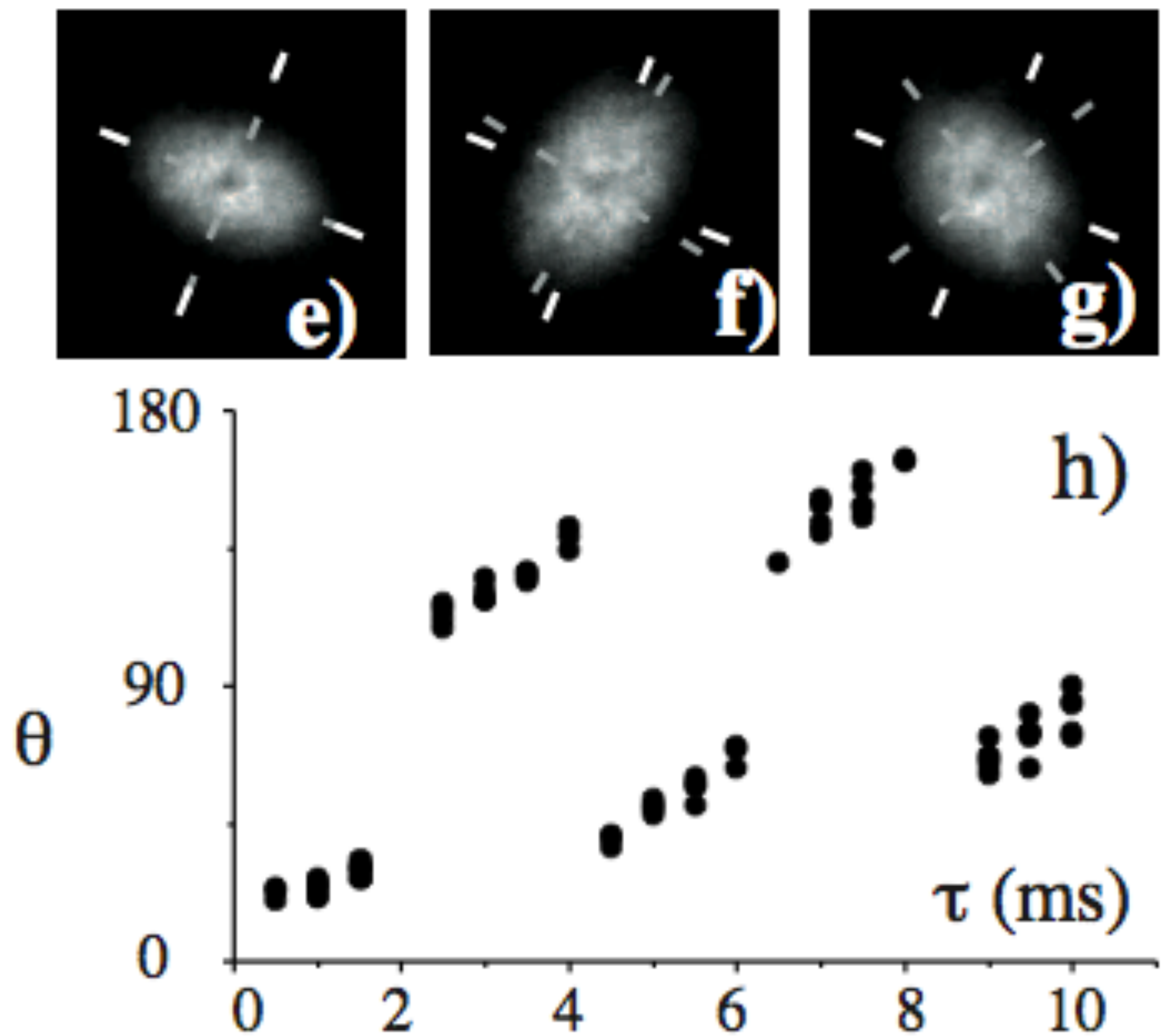
+1 edge state



Dynamics: collective modes

Rotating BEC

Quadrupole mode



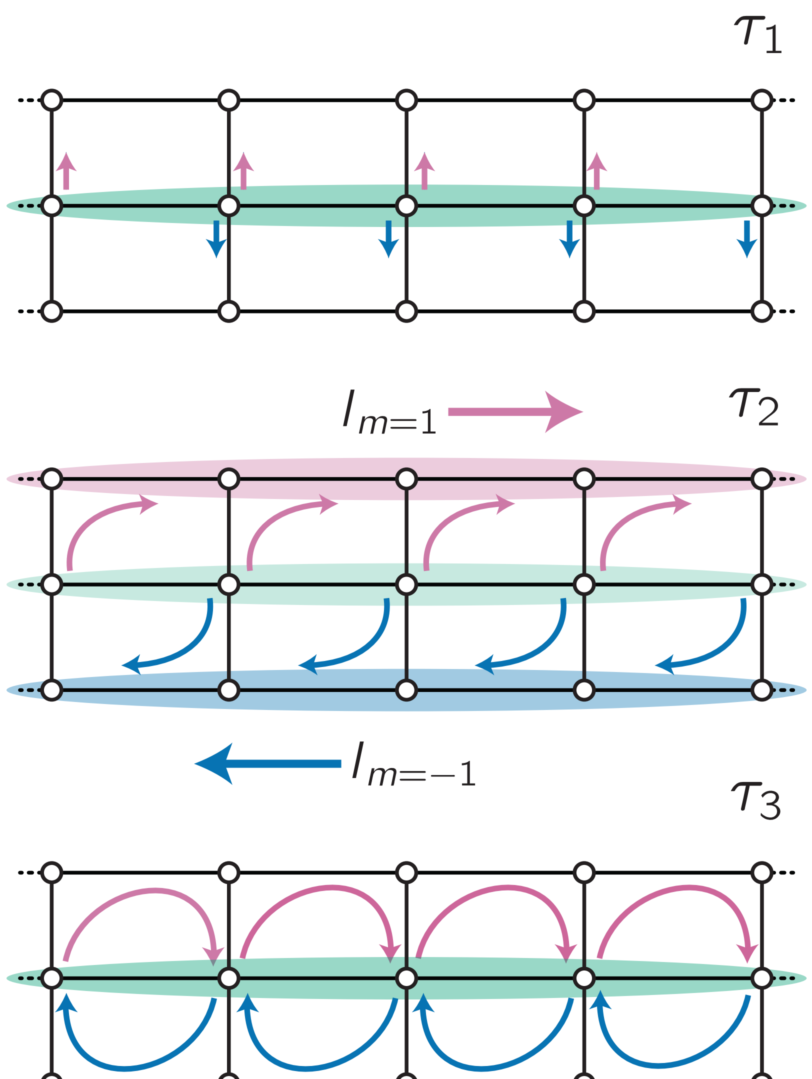
References

Chevy, F., Madison, K. W. & Dalibard, Phys. Rev. Lett. (2000).

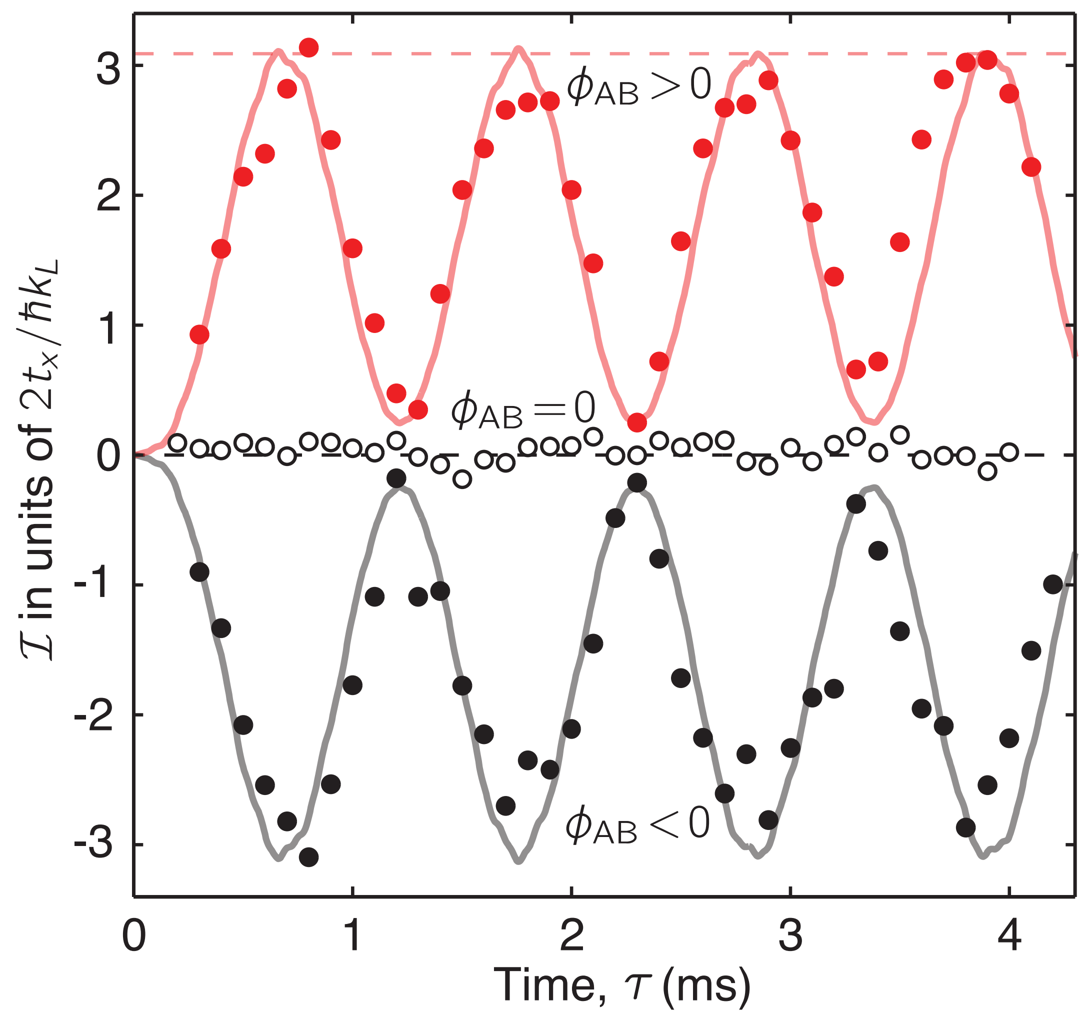
Dynamics: collective modes

Quadrupole mode

In the extreme large field limit



Chiral current



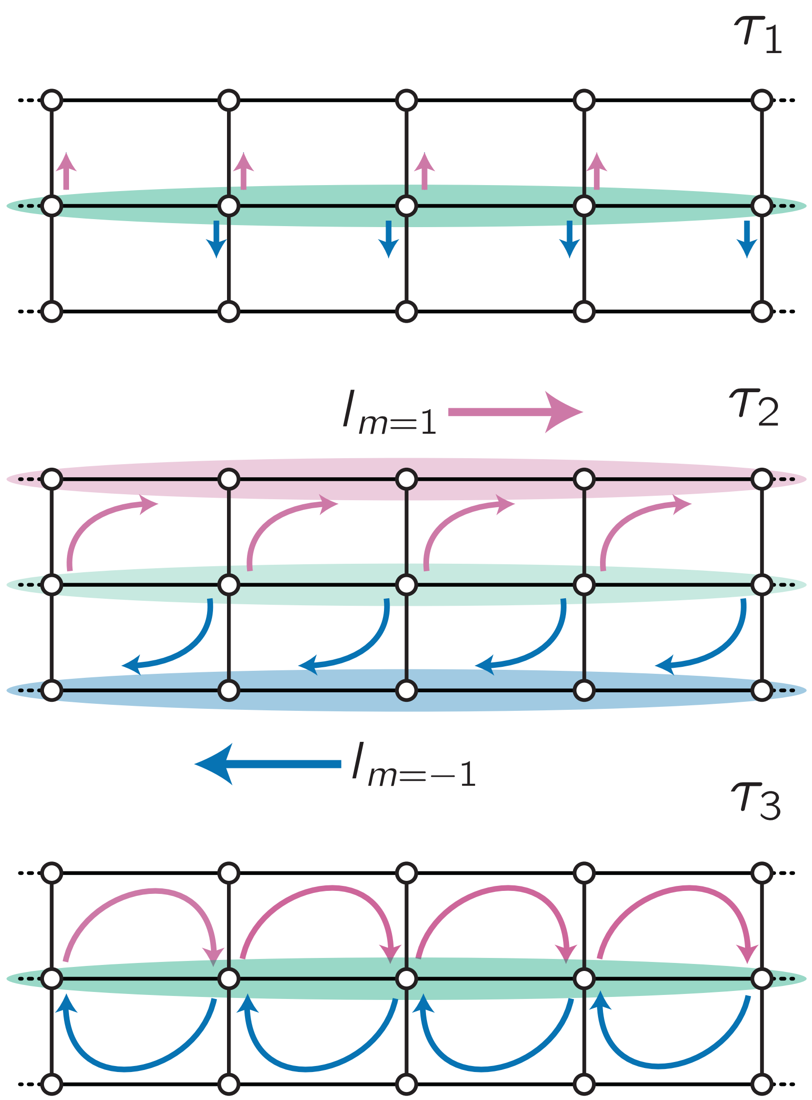
References

Celi, A. et al. Phys. Rev. Lett. (2014).

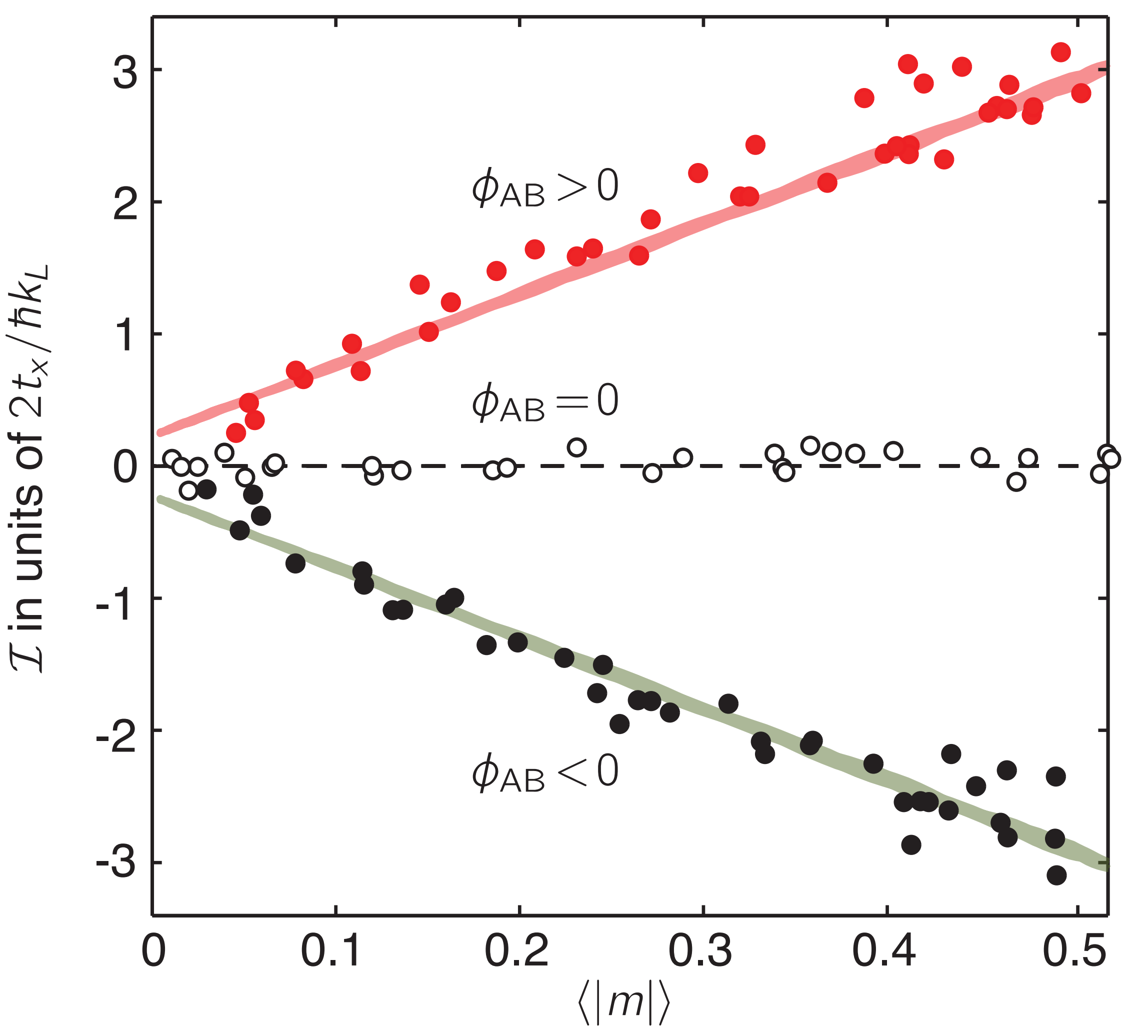
Dynamics: collective modes

Effective box potential

Release from central site



Chiral current



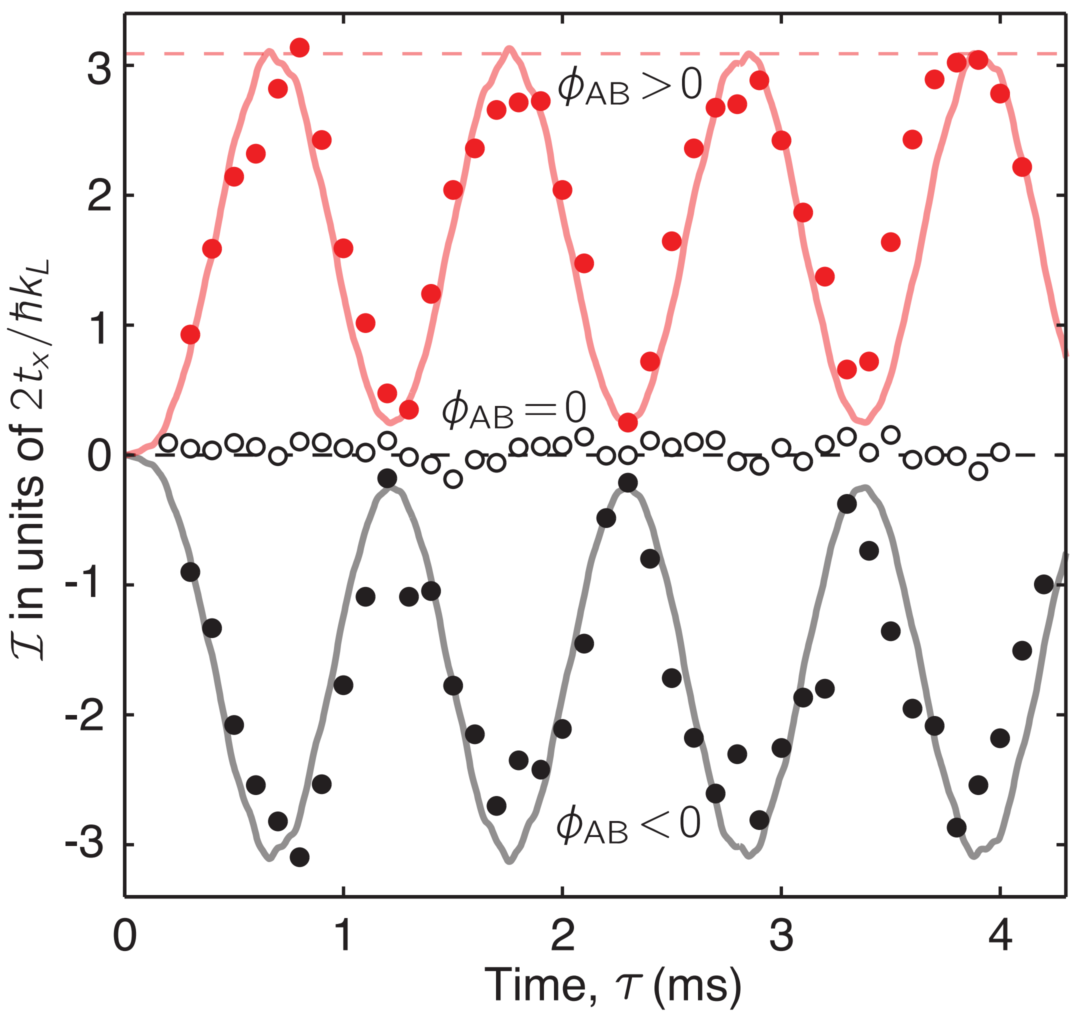
References

Celi, A. et al. Phys. Rev. Lett. (2014).

Dynamics: collective modes

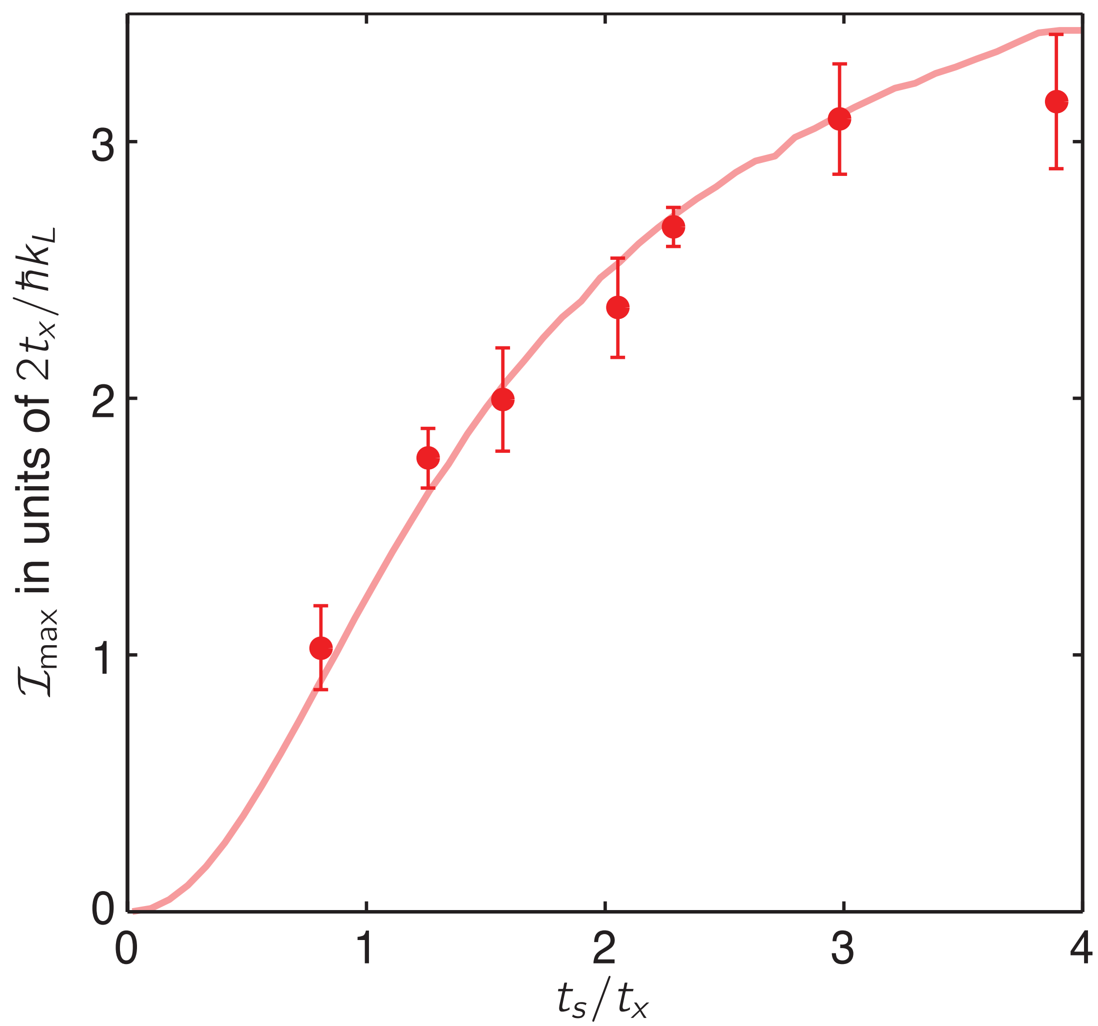
Chiral current

Peak current



Chiral current

Tunneling anisotropy



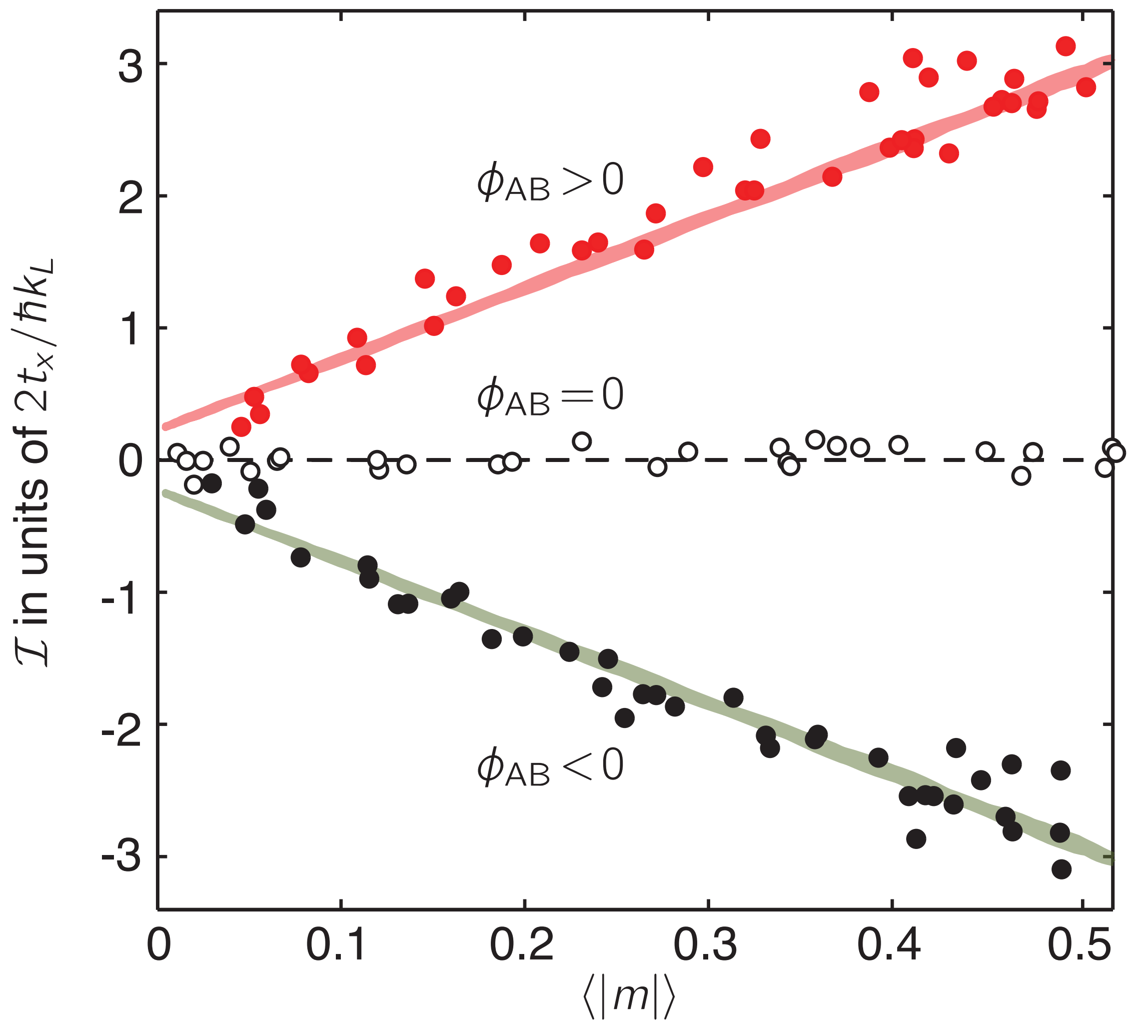
References

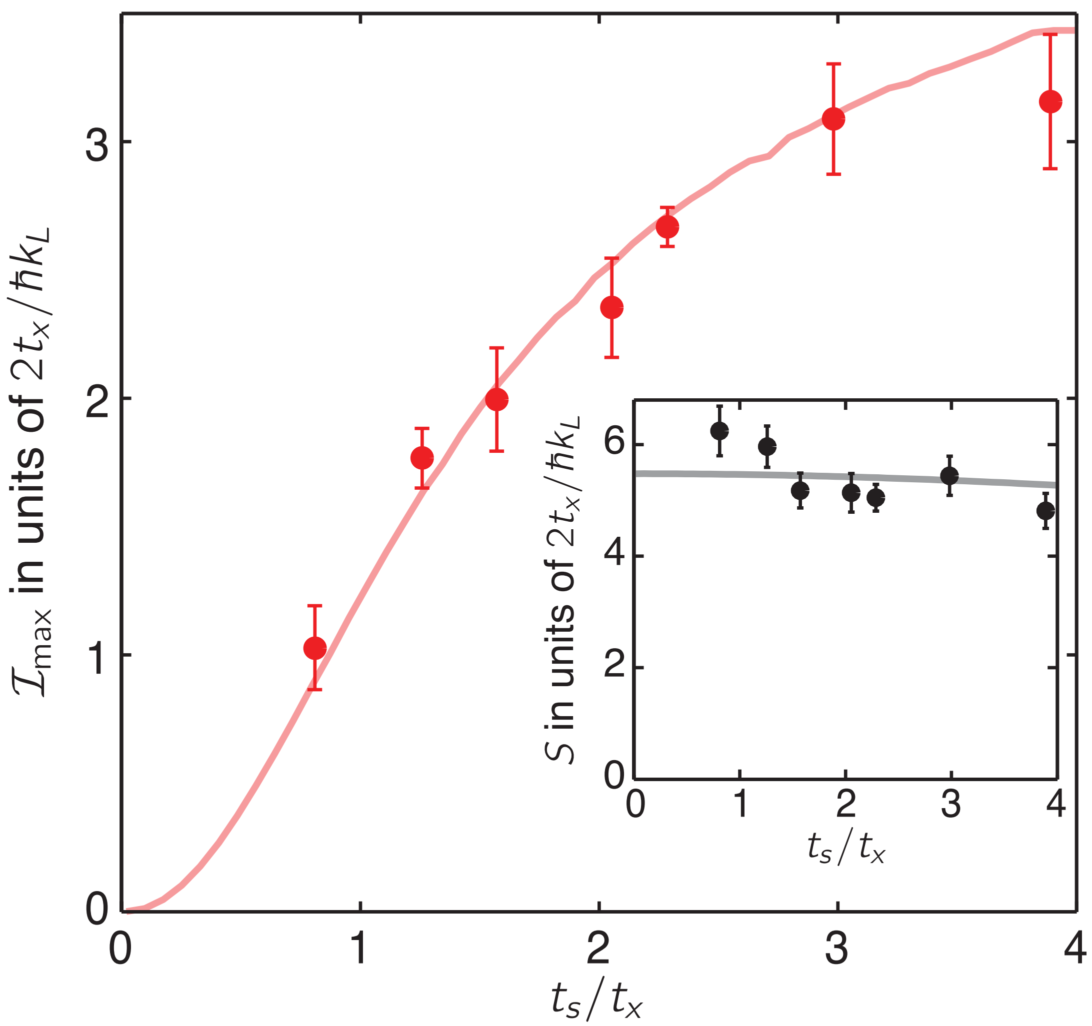
Celi, A. et al. Phys. Rev. Lett. (2014).

Dynamics: collective modes

Chiral slope

Independent of anisotropy (tight binding)



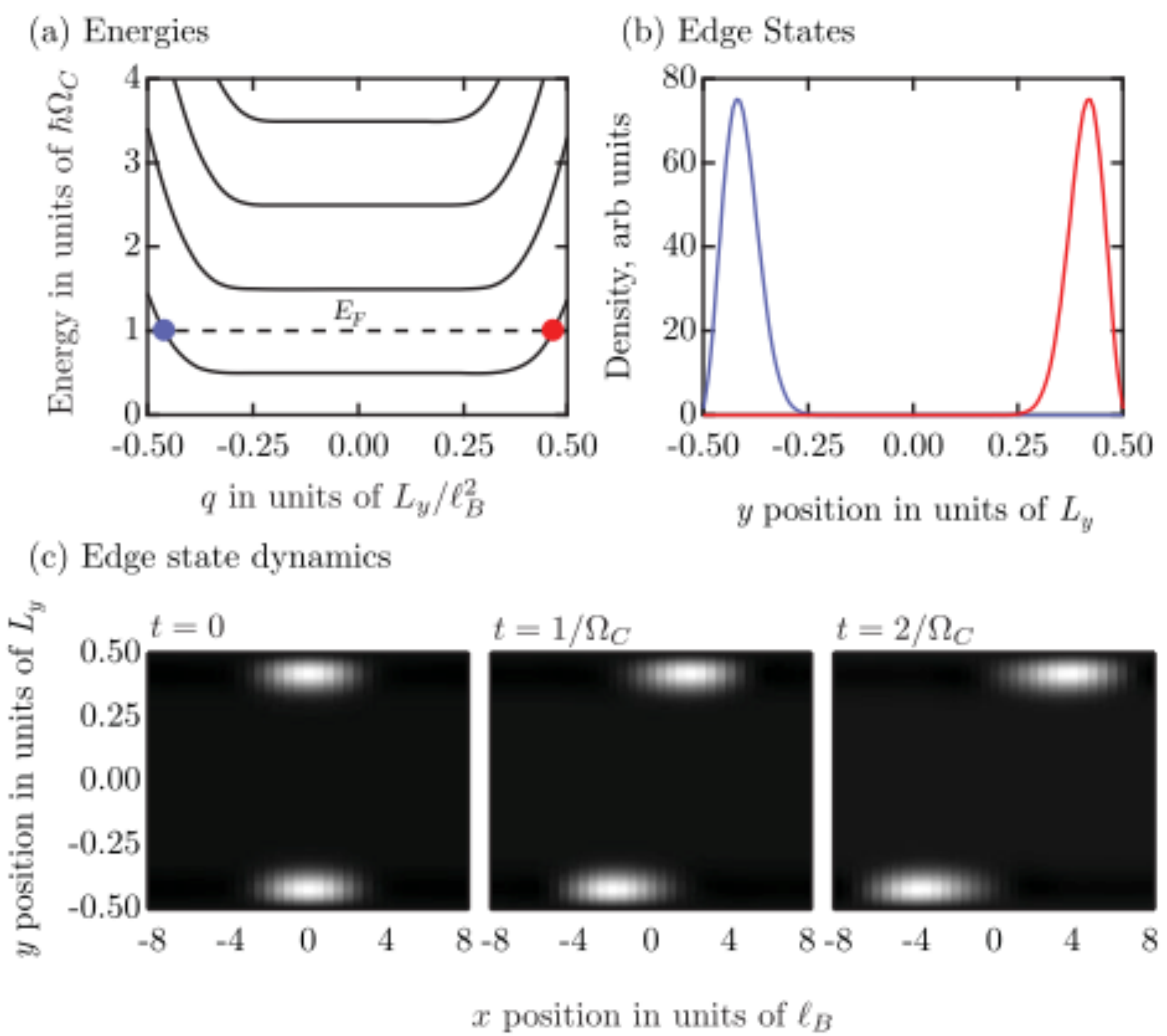


References

Celi, A. et al. Phys. Rev. Lett. (2014).

QHE systems: edge states, skipping?

Edge excitations in QHE systems



Dynamics: "edge magnetoplasmons"

In a single Landau level, the radius is set by

$$\sqrt{\frac{N\hbar}{qB}}$$

N is the Landau level index

In these data

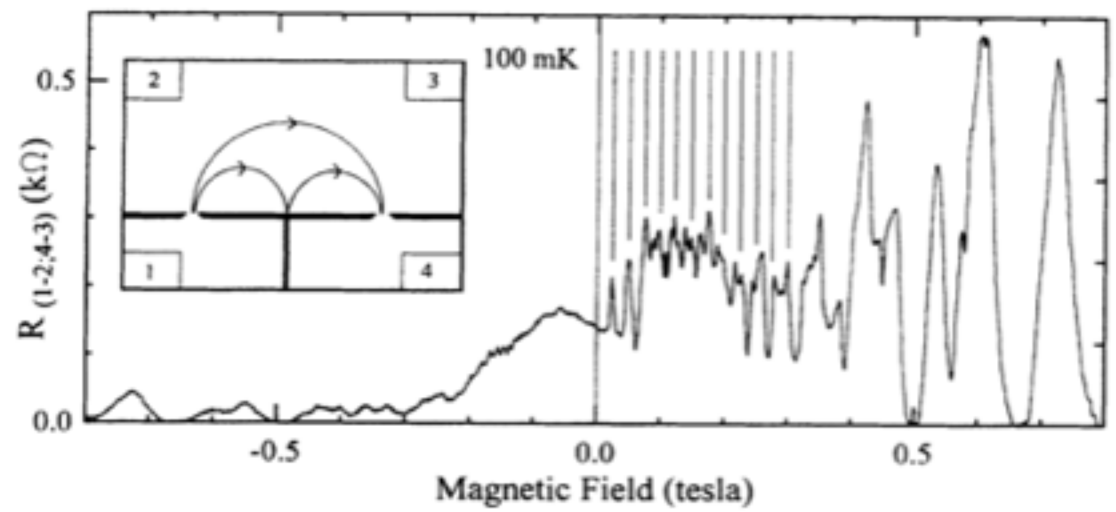
$$E_F \gg \Omega_C$$

So cyclotron radius at Fermi surface matters

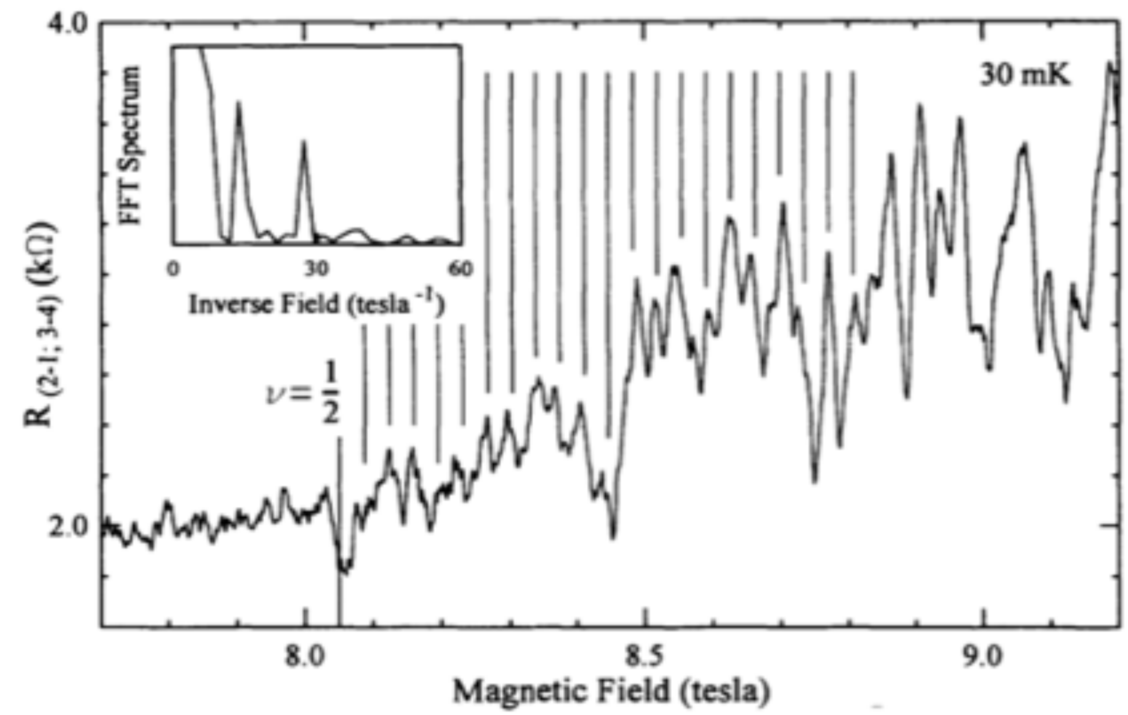
$$\frac{mv}{qB} = \frac{\hbar k_F}{qB}$$

<u>Edge magnetoplasmons</u>
Involve many "edge modes" in many Landau levels

<u>Magnetic focusing of electrons</u>



<u>Magnetic focusing of composite fermions</u>



<u>References</u>

van Houten, H. et al. Phys. Rev. B 39, 8556–8575 (1989).
Goldman, V. J., Su, B. & Jain, J. K. Physical Review Letters 72, 2065–2068 (1994).

QHE systems: edge states, skipping?

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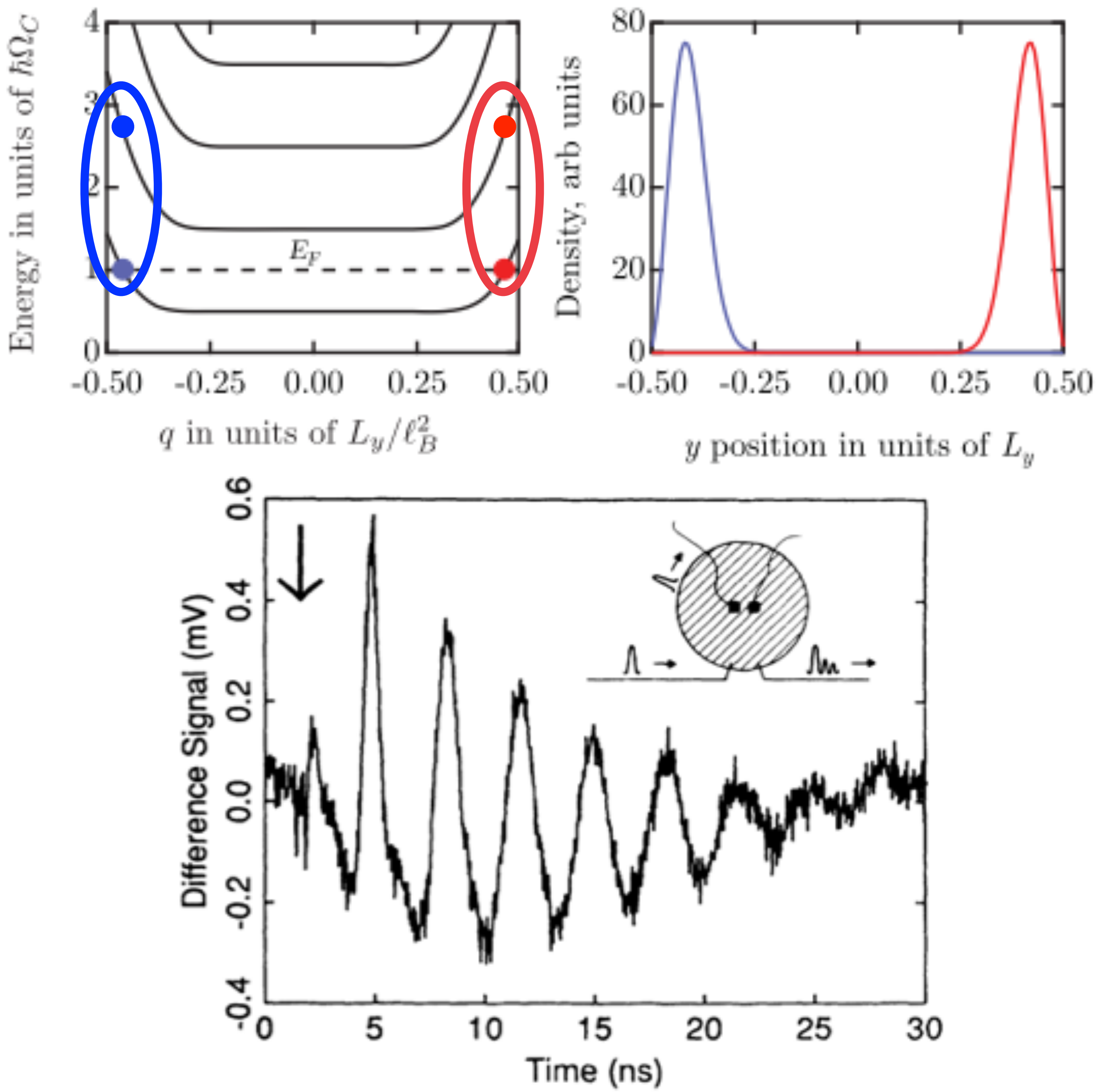
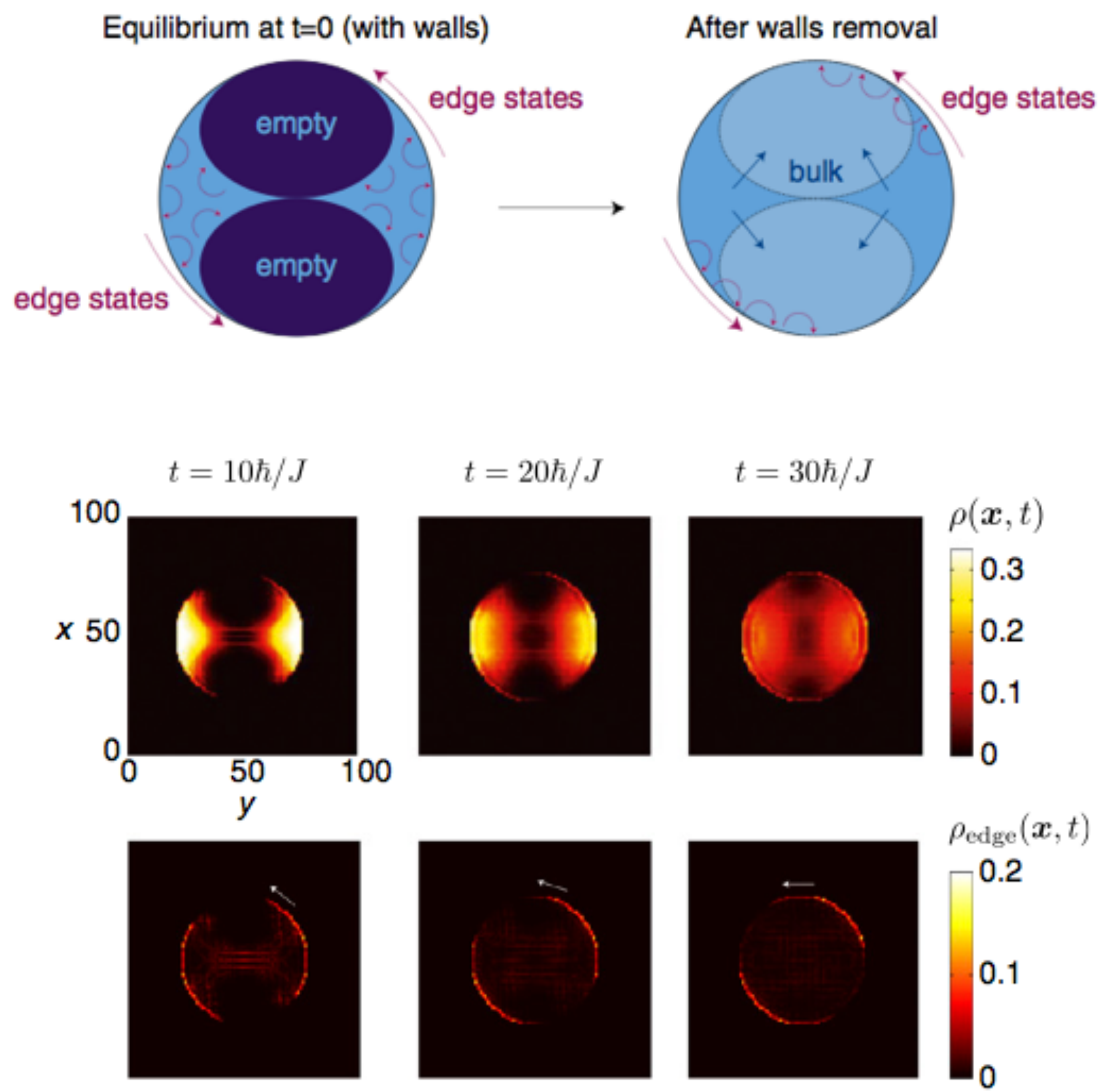


References

R. Ashoori, et al., Phys. Rev. B 45, 3894–3897 (1992).

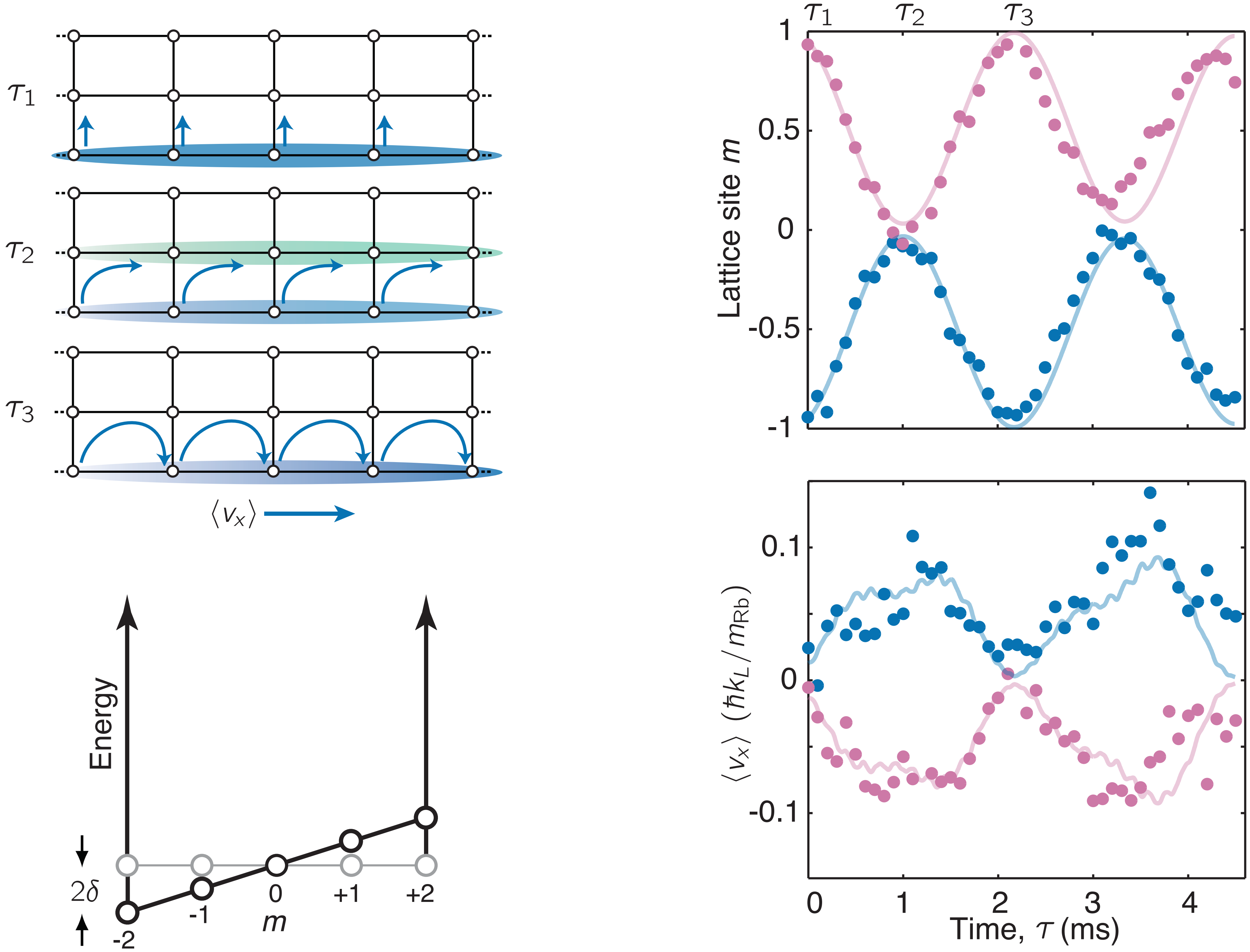
Cold-atom theory connection



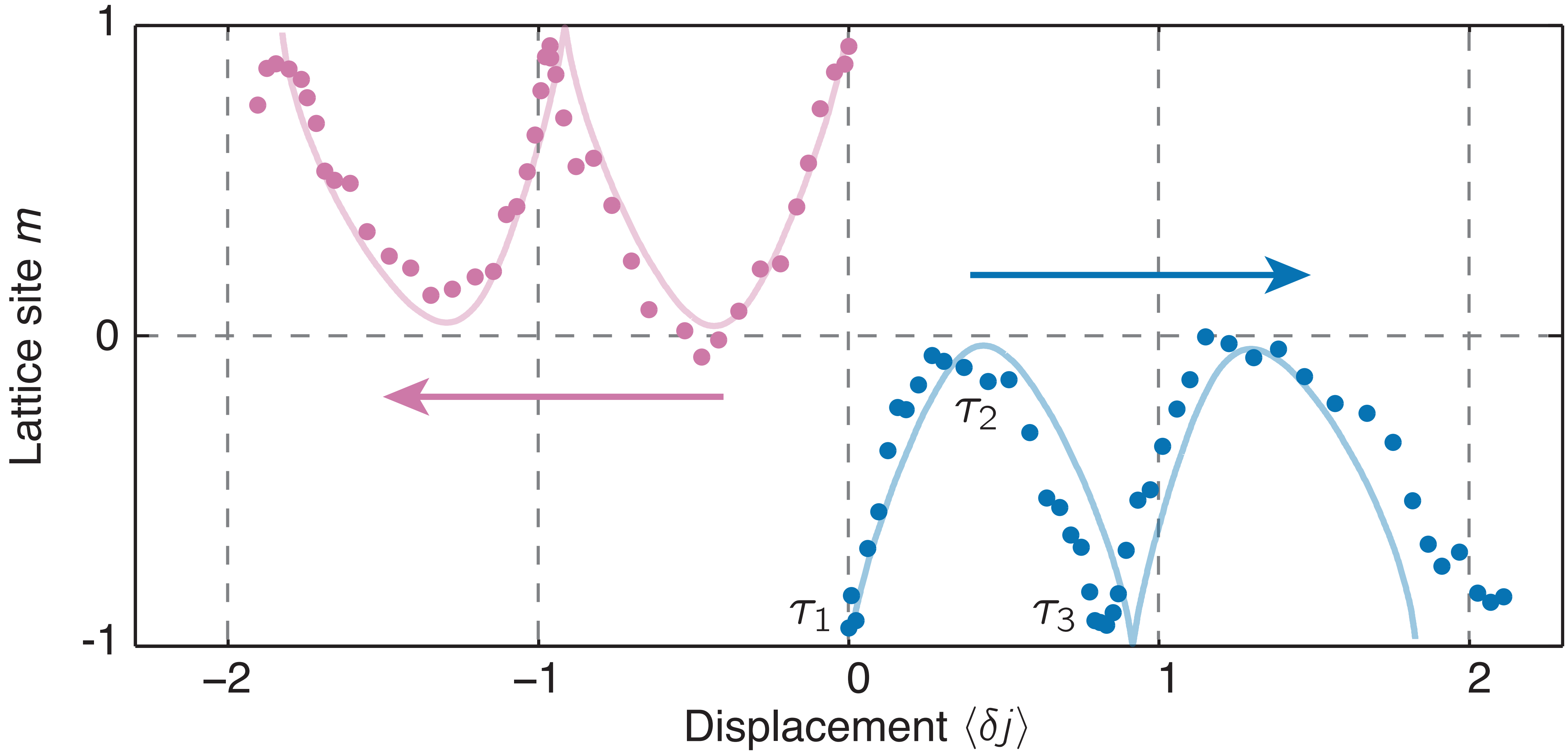
References

N. Goldman ... P. Zoller ... et al; PNAS (2013)

Dynamics: edge magnetoplasmons



Dynamics: edge magnetoplasmons

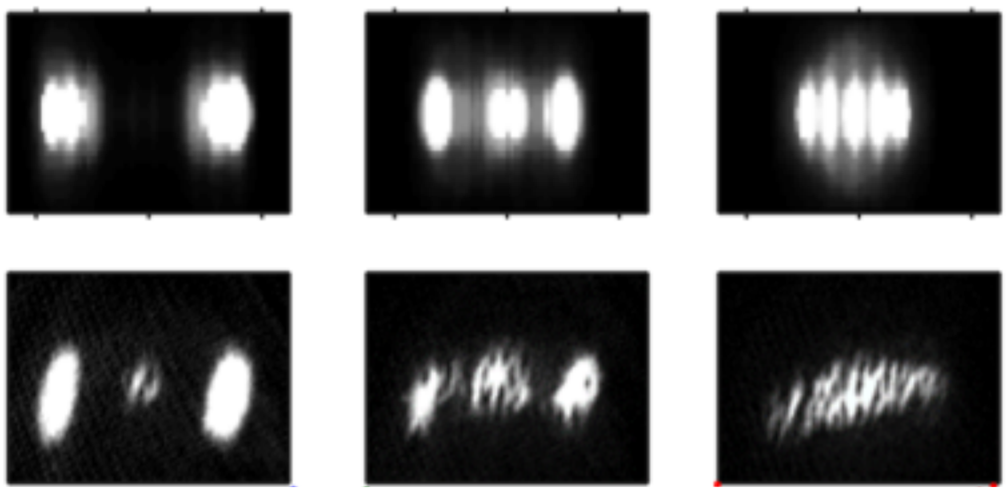


References

B. Stuhl and H.-I Lu, et al., Science (2015); M. Mancini, et al., Science (2015)

Other completed work

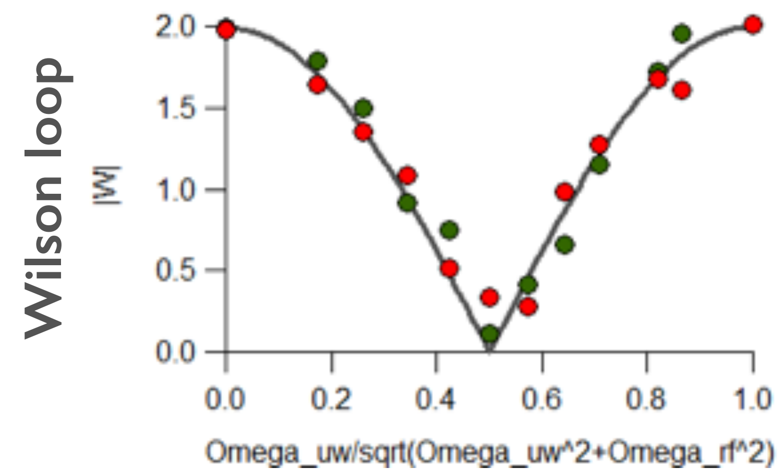
Adiabatic vortex generation



R. Price, et al. (in preparation)

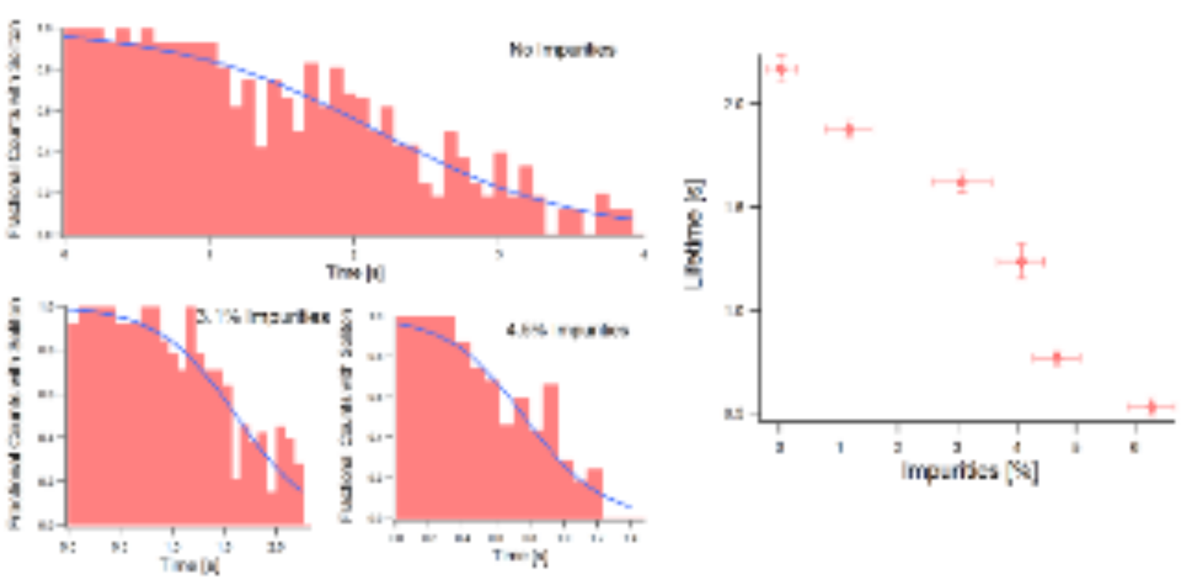
Topological transition

Invariants in parameter space for nonabelian systems



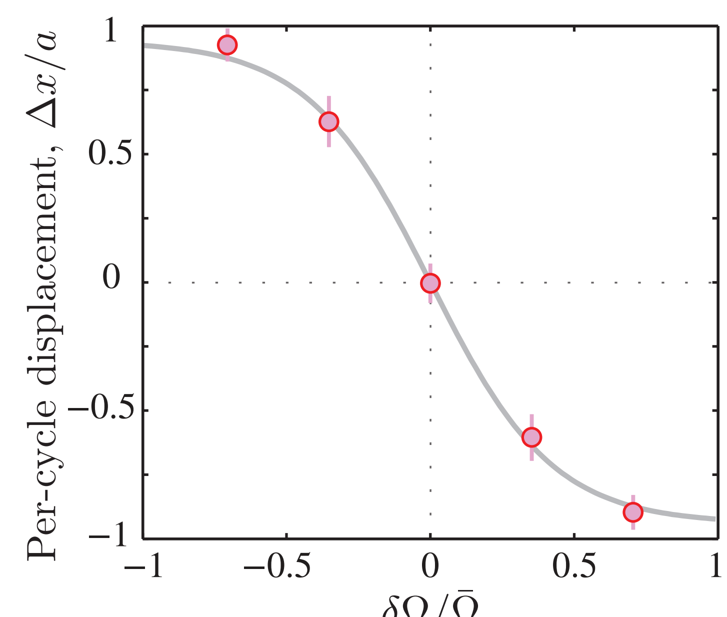
S. Sugawa, et al. (in preparation)

Damping and diffusion of solitons



L. M. Aycock, et al. (in preparation)

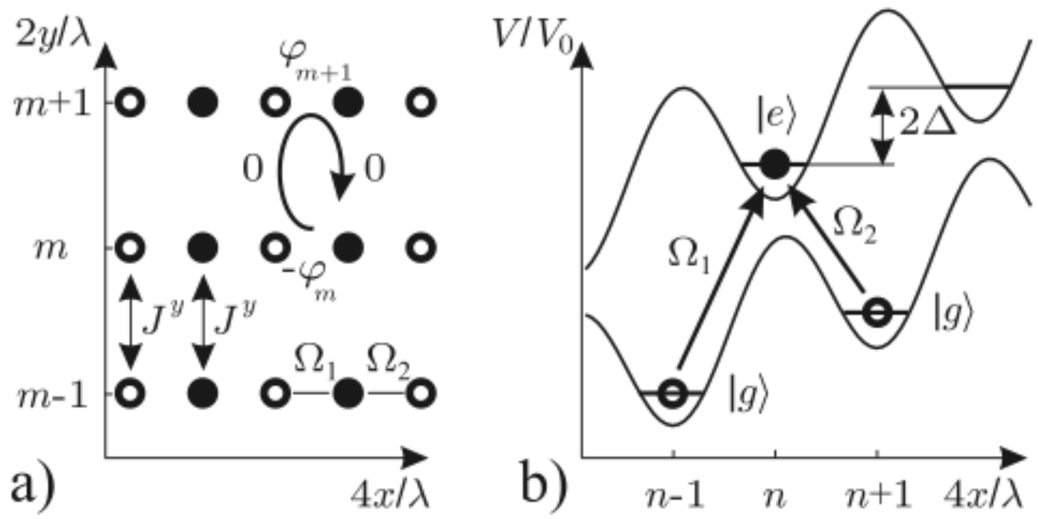
Geometric charge pumping



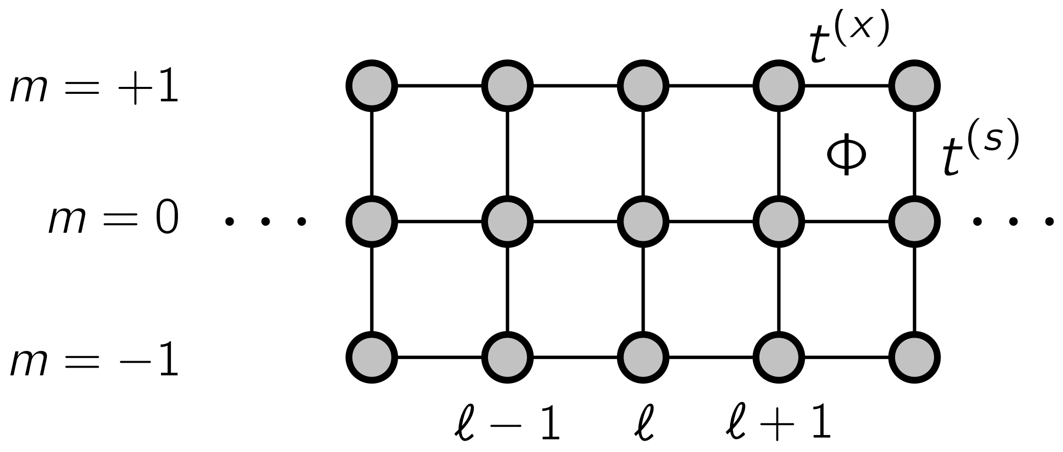
Hsin-I Lu, et al. arXiv: 1508.04480

Wrap-up

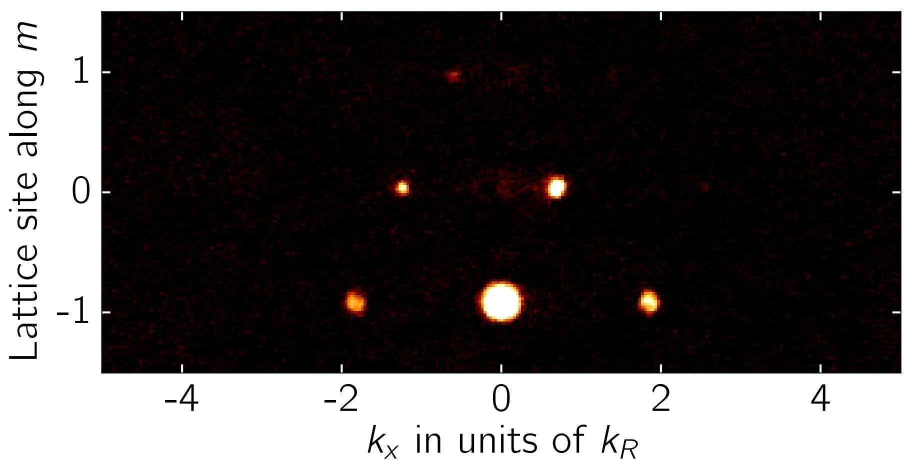
Using mechanism from



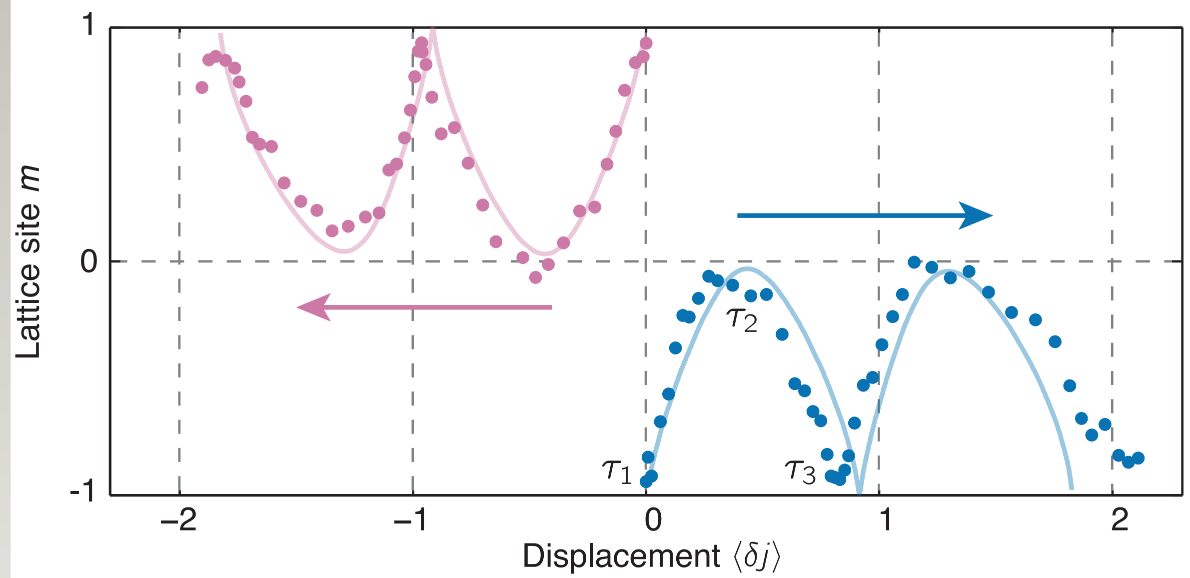
Synthetic dimensions lattice



Edge states

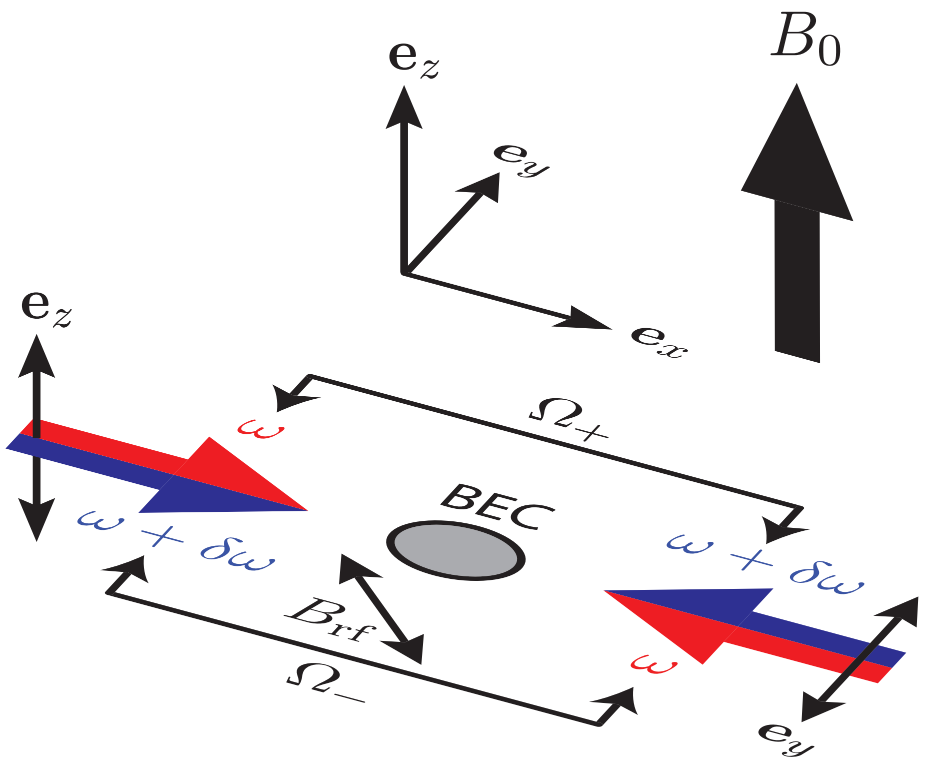


Edge excitations

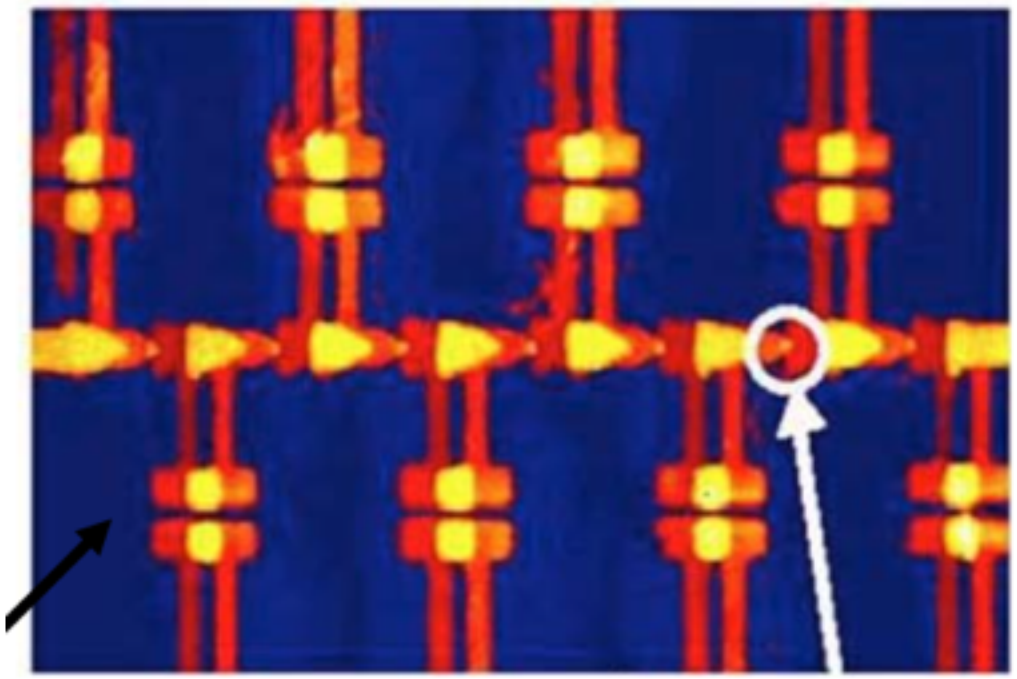


Pumps throughout history

Geometric/topological pumps



NIST single electron pump



Sometimes quantized pumping,
quantum mechanical (left) or not quantum mechanical (right_.

References

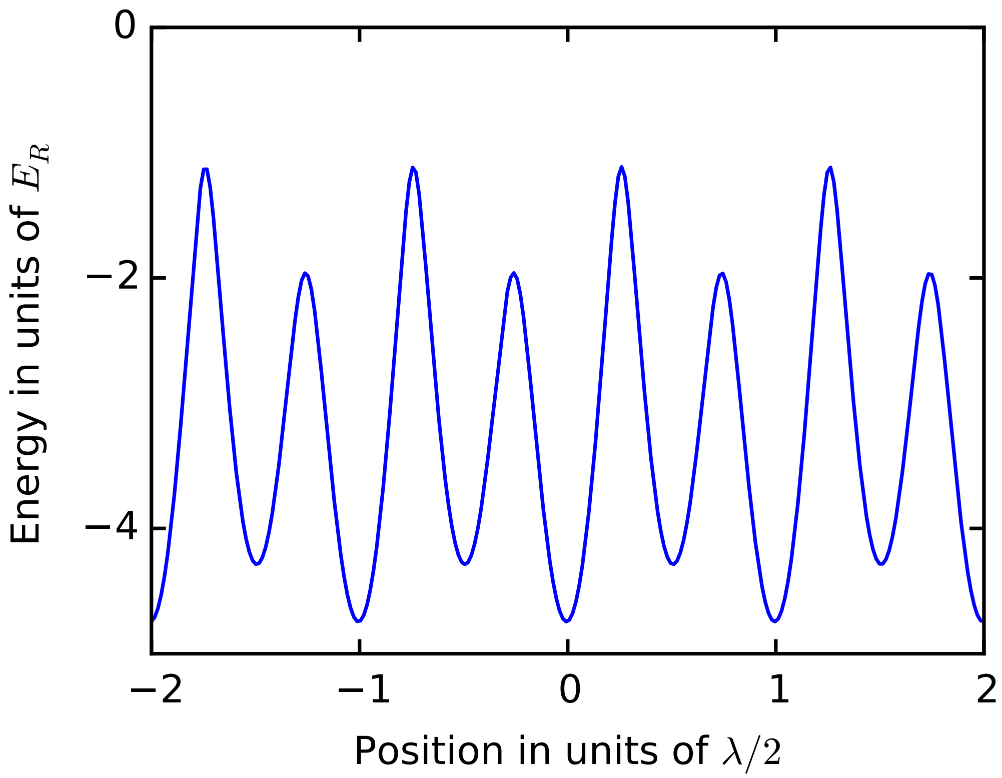
(1) Hsin-I Lu, Maximilian Schemmer, et al. arXiv: 1508.04480v1

(2) M. A. Keller et al Science (1999)

Basics

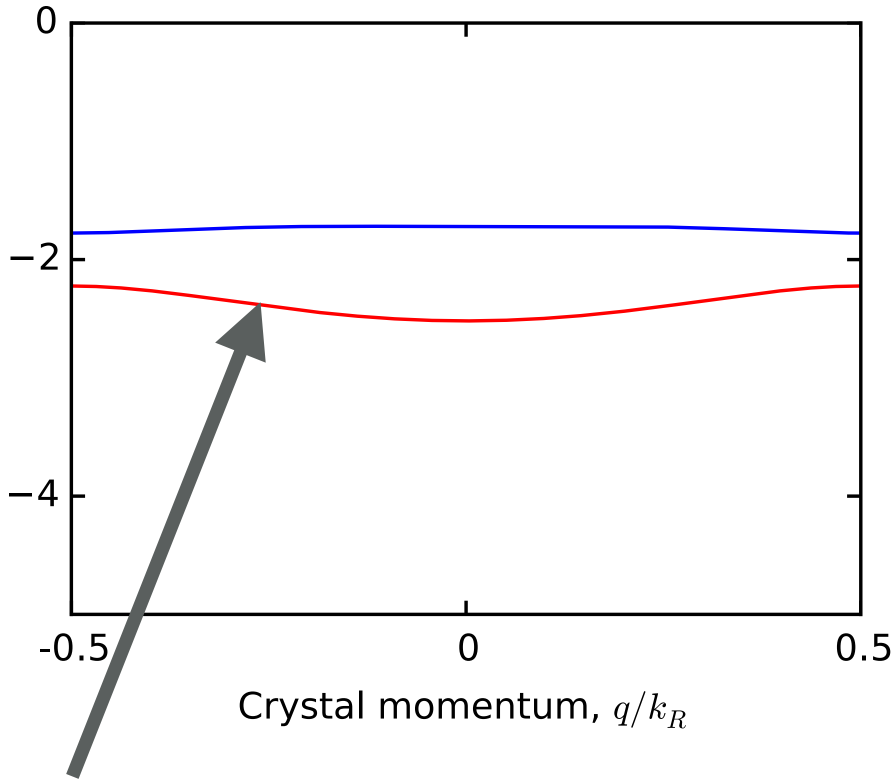
Lattices and Bloch bands

Spatial lattice potential



Unit cell with size $a = \lambda/2$,
and two sub-lattice sites

Band structure



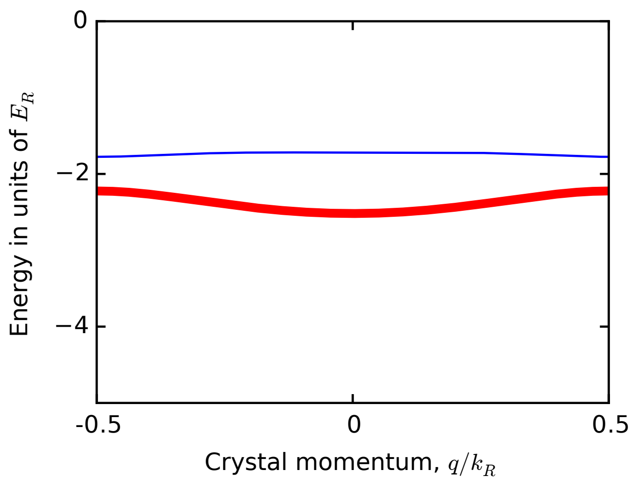
$|\psi_n(q)\rangle$

Each eigenstate has quantum numbers:
crystal momentum *q* and band index *n*.
e.g.
$n = 0, q = -0.6\ k_R$

Quantum charge pumps

Topological or "Thouless" charge pumps

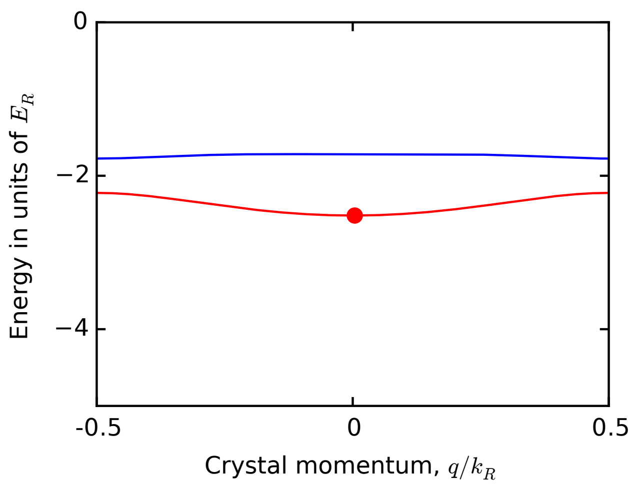
In condensed matter **topological** pumps focus on filled Bloch bands



Cold atoms can have:

- Thermally filled bands
- Interaction-filled bands (Mott insulator)
- Statistically filled bands (Fermions)

In contrast **geometric** pumps operate with a single crystal momentum state



Cold atoms can have:

- A very cold thermal cloud
- BECs

References

M. Lohse et al Nature Physics (2015), S. Nakajima et al Nature Physics (2015), Hsin-I Lu, et al. arXiv: 1508.04480v1 (NIST/JQI)

Physical mechanism

Charge pump basics

In a charge pump we adiabatically and periodically vary a Hamiltonian

$$\hat{H}(\hat{q}, t) = \hat{H}(\hat{q}, t + T)$$

with period ***T***, and where ***q*** is the crystal momentum.

Since we assume the pumping process is adiabatic, any initial crystal momentum state can at most acquire a phase after each pump cycle:

$$|\psi(q)\rangle \to e^{i\phi(q)} |\psi(q)\rangle$$

The phase *can* be different for different crystal momentum states. This allows us to define the single-cycle evolution operator

$$\hat{U}_{\text{single}} = e^{i\phi(\hat{q})}$$

Physical mechanism

Charge pump basics

Independent of its origin this phase leads to motion after each pump cycle

$$U^\dagger(\hat{q})\hat{x}U(\hat{q}) = \hat{x} - \frac{\partial\phi(\hat{q})}{\partial\hat{q}}$$

from the non-commutation of ***x*** and ***q***.

For any *q* the phase has two contributions. The first is:

dynamical (kinetic energy) $-\frac{1}{\hbar}\int_0^T dt E(q,t)$

Giving the displacement: $\delta x = \frac{T}{\hbar}\frac{\partial\bar{E}(q)}{\partial q}$

from mean group velocity.

Zero for: filled bands or particles at the dispersion minimum

Physical mechanism

Charge pump basics

The second phase is the geometric (Berry phase):

$$\phi_B(q) = \frac{1}{\hbar}\int_0^T dt\, \langle \psi(q,t)|\, i\hbar\partial_t\, |\psi(q,t)\rangle = \frac{1}{\hbar}\int_0^T dt\, \langle u(q,t)|\, i\hbar\partial_t\, |u(q,t)\rangle$$

In terms of the Bloch functions

$$|\psi(q,t)\rangle = e^{iq\hat{x}}|u(q,t)\rangle$$

Derived from the time component of the Berry connection

$$\mathbf{A}(q,t) = \langle \psi(q,t)|\, i\hbar\partial_q\, |\psi(q,t)\rangle\, \mathbf{e}_q + \langle \psi(q,t)|\, i\hbar\partial_t\, |\psi(q,t)\rangle\, \mathbf{e}_t$$

Giving

$$\delta x = -\frac{1}{\hbar}\int_0^T dt\ \partial_q A_t(q,t)$$

Physical mechanism

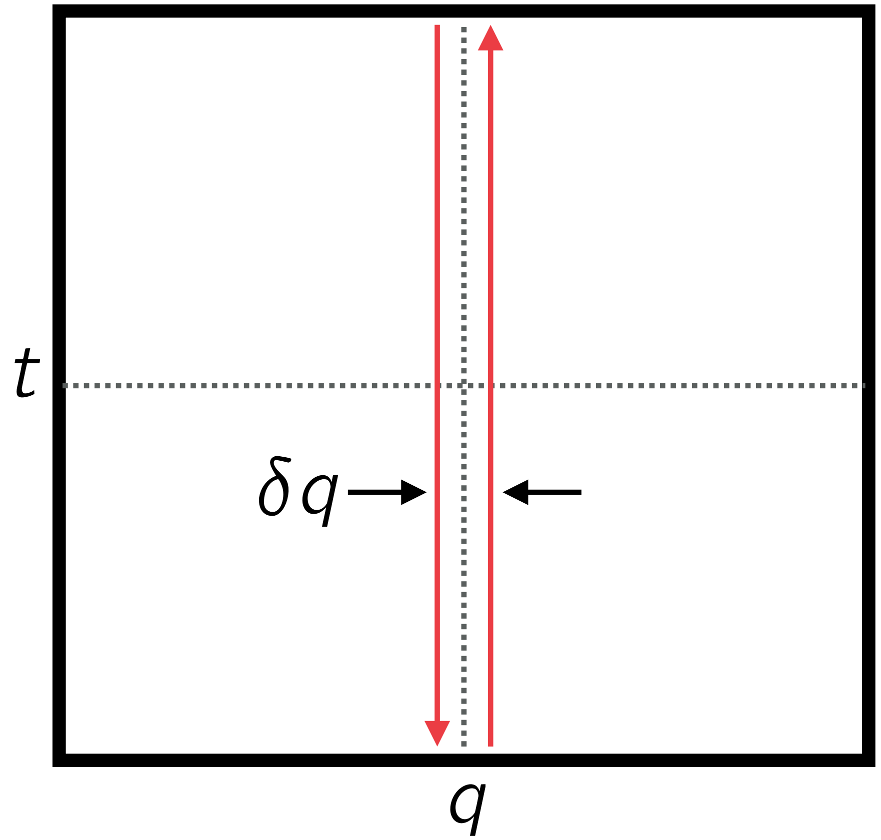
Charge pump basics

Next consider the Berry curvature

$$\begin{aligned} F(q,t) &= [\nabla_{qt} \times \mathbf{A}(q,t)]_z \\ &= i\hbar\left(\langle \partial_q u(q,t) | \partial_t u(q,t)\rangle - \langle \partial_t u(q,t) | \partial_q u(q,t)\rangle\right) \end{aligned}$$

And think about the displacement as a loop integral around the path

This is torus



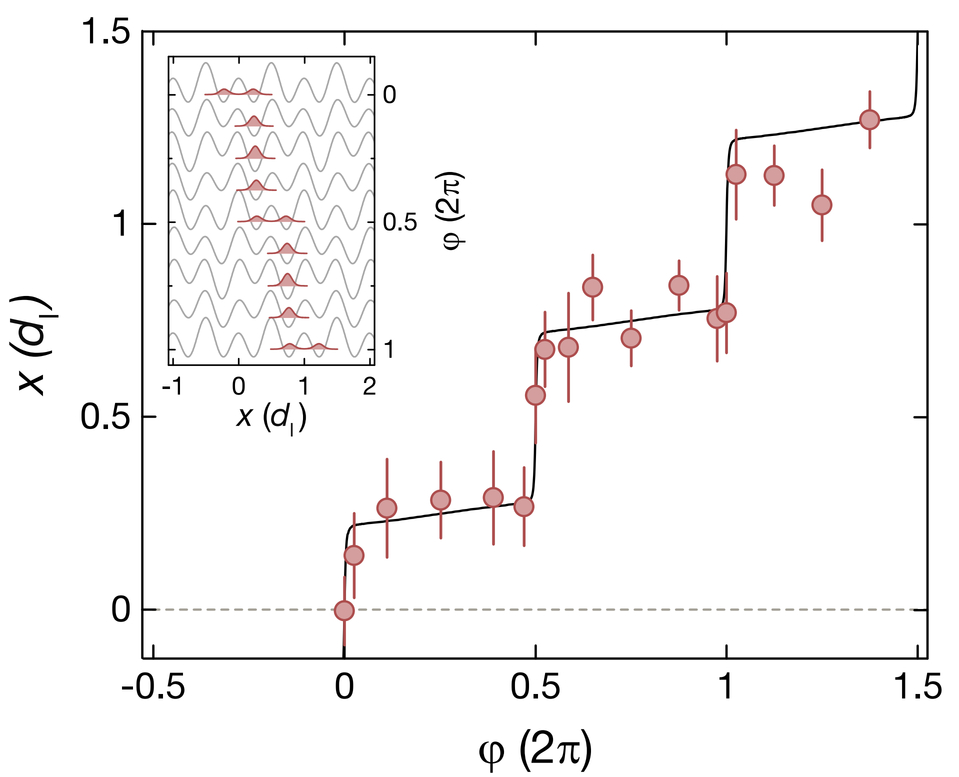
$$\begin{aligned} \delta x &= -\frac{1}{\hbar} \lim_{\delta q \to 0} \left[\frac{1}{\delta q} \oint d\boldsymbol{\ell} \cdot \mathbf{A}(\boldsymbol{\ell}) \right] \\ &= -\frac{1}{\hbar} \int_0^T dt\, \Omega(q,t) \end{aligned}$$

Quantum charge pumps

Every state occupied

topological, quantized

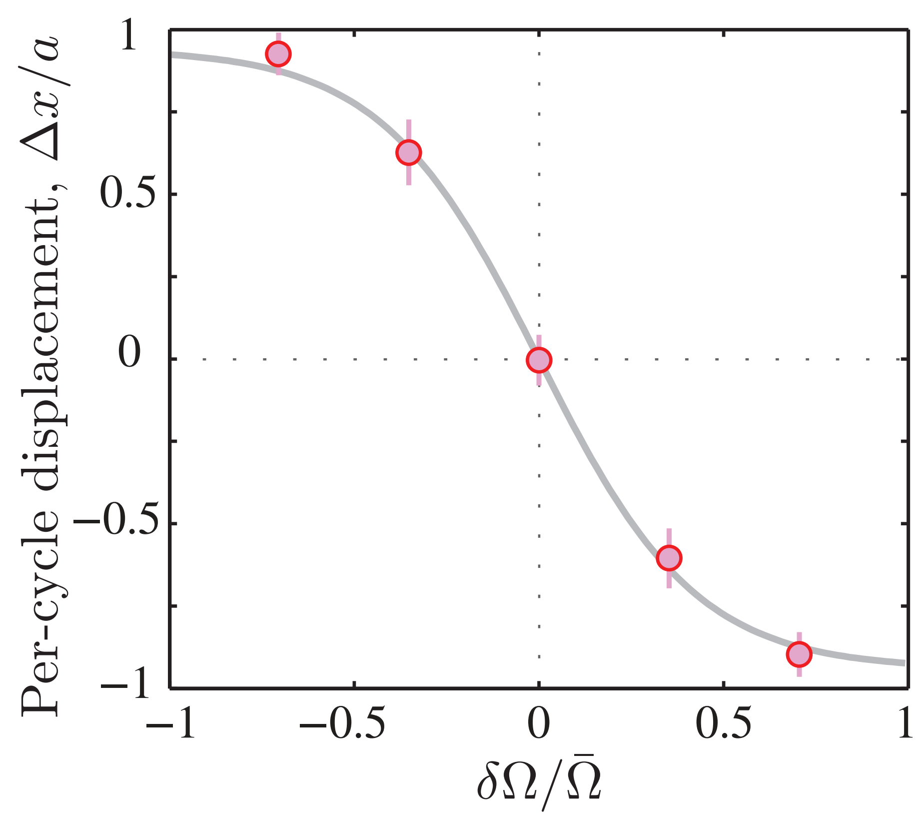
$$\delta x = -\frac{1}{2\pi\hbar}\int dq\,dt\,\Omega(q,t)$$



BEC at band minimum

geometric, not quantized

$$\delta x = -\frac{1}{\hbar}\int_0^T dt\,\Omega(q,t)$$



References

D. J. Thouless, Phys. Rev. B 27, 6083 (1983)

M. Lohse et al Nature Physics (2015)

S. Nakajima et al Nature Physics (2015)

References

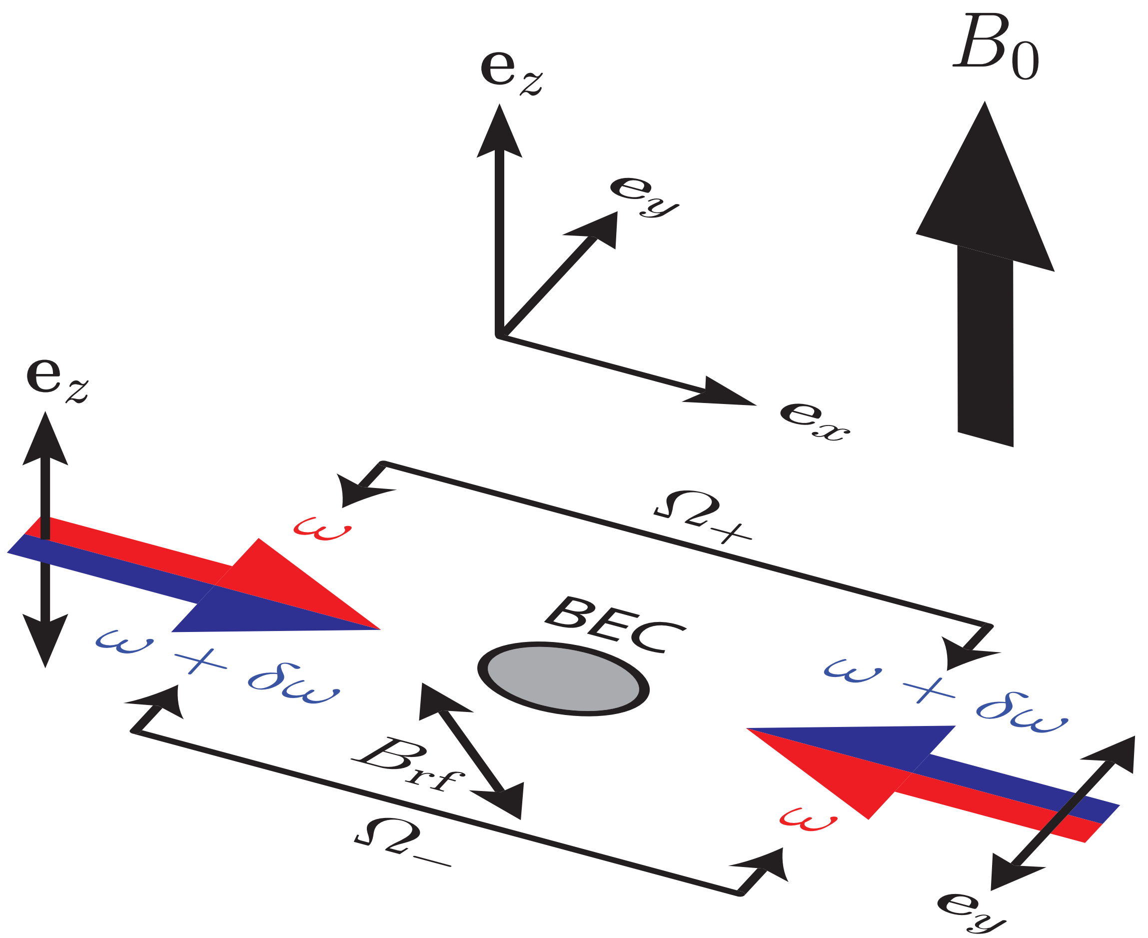
Hsin-I Lu, et al. arXiv: 1508.04480v1 (JQI)

In a phase-number basis for a single superconducting dot:

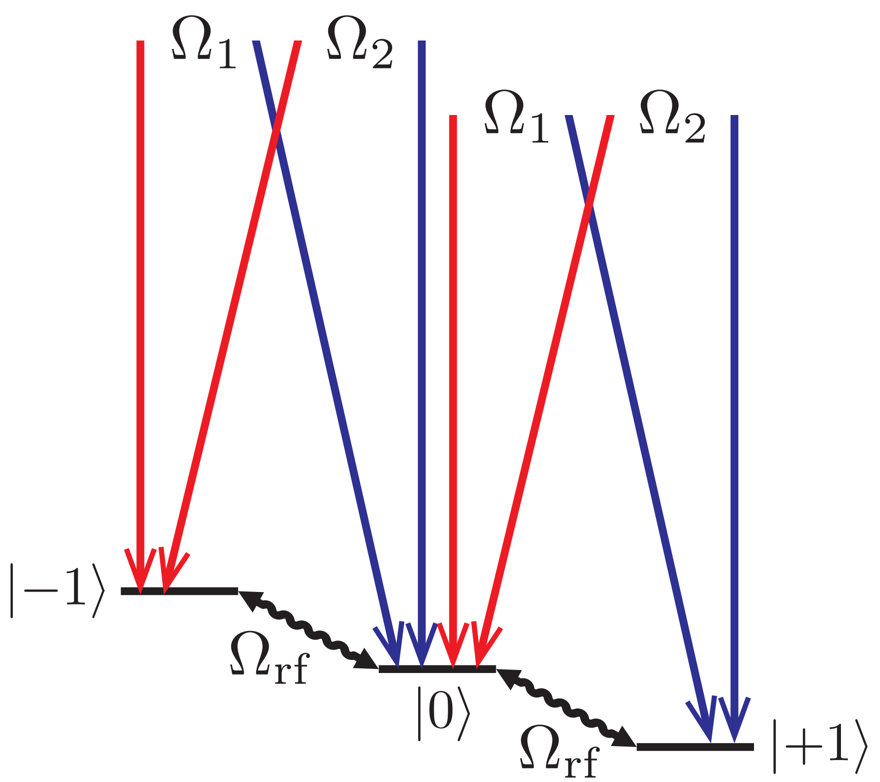
M. Mottonen, J. J. Vartiainen, J. P. Pekola; PRL (2008)

Technique

Geometry



Levels



References

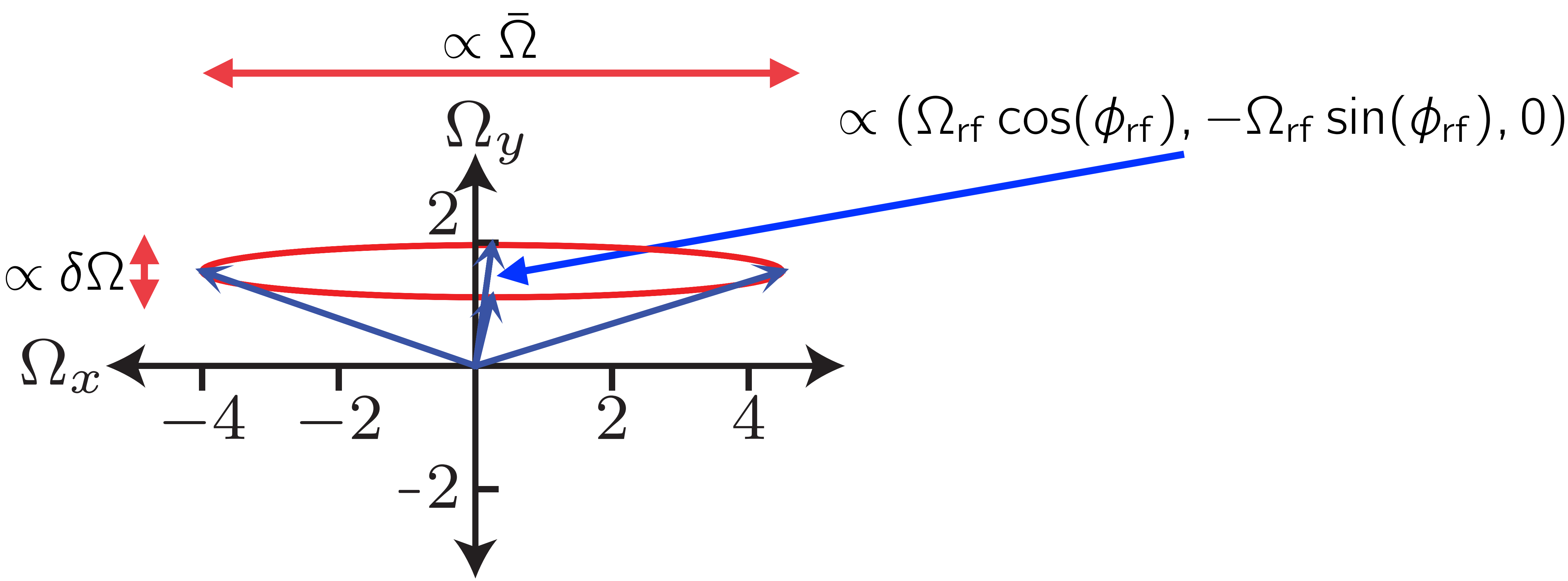
Extending: K. Jiménez-García, et al, PRL (2012), and L. W. Cheuk, et al PRL (2012)

Technique

Magnetic interpretation

$$\hat{H}_L = \Omega(\hat{\mathbf{r}}) \cdot \hat{\mathbf{F}}$$

$$\Omega(\hat{\mathbf{r}}) = \frac{1}{\sqrt{2}}(\bar{\Omega}\cos(2k_R x) + \Omega_{rf}\cos(\phi_{rf}), -\delta\Omega\sin(2k_R x) - \Omega_{rf}\sin(\phi_{rf}), 0)$$



References

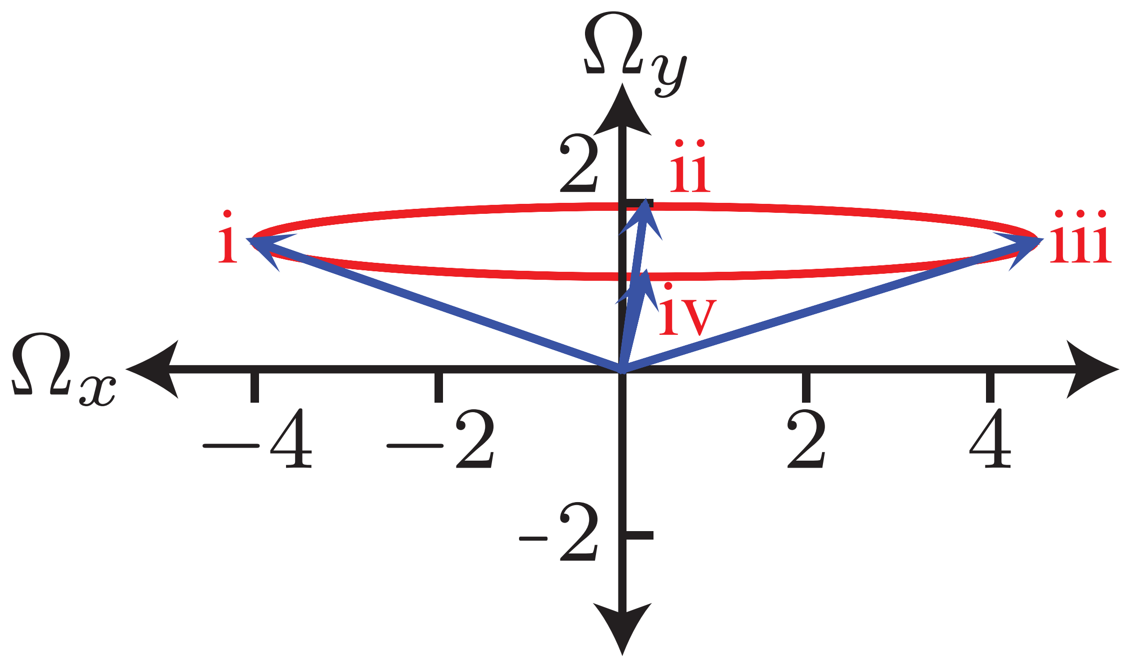
Morally similar in outcome to proposal of D.-W. Zhang, et al, PRA 92, 013612 (2015).

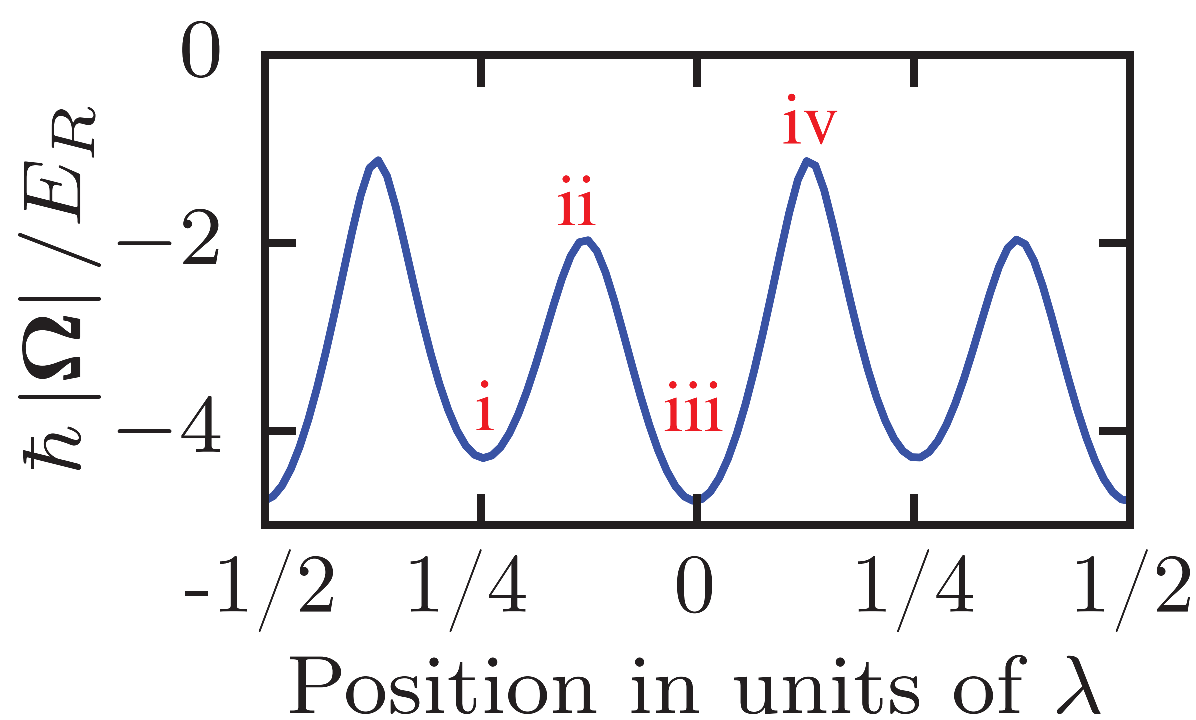
Technique

Magnetic interpretation

Joins the family of double-well lattices in for quantum gases

With “well spacing” of lambda/4





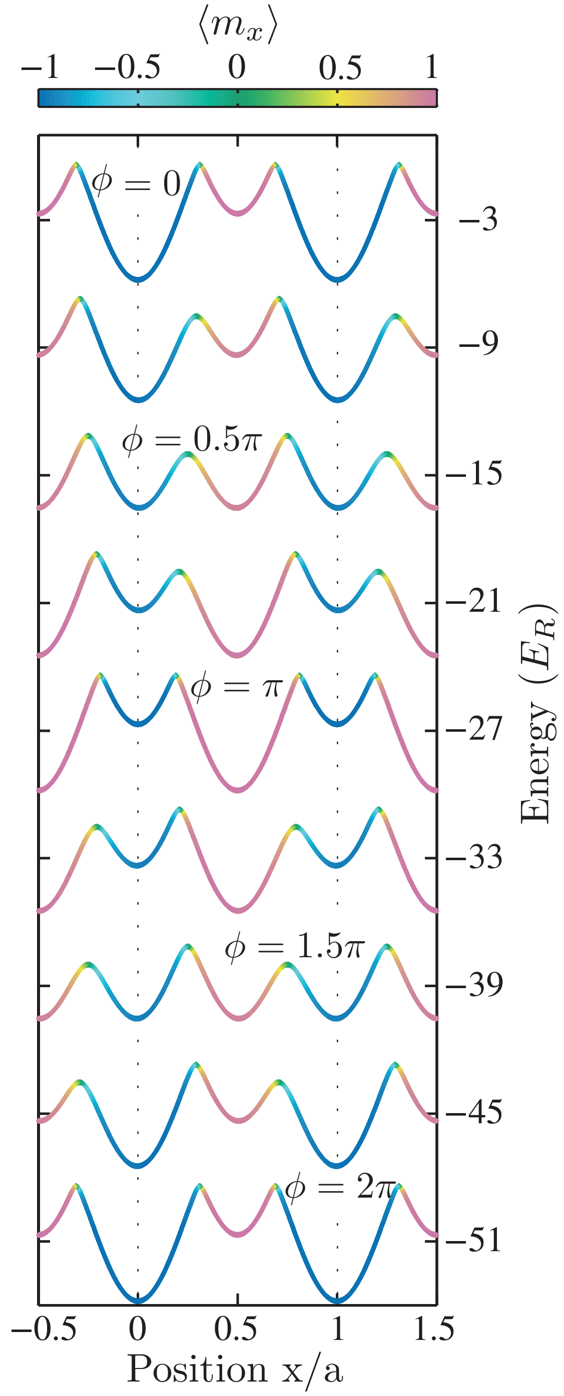
References

Higher spatial frequency components, related to work in Martin Weitz’s group.

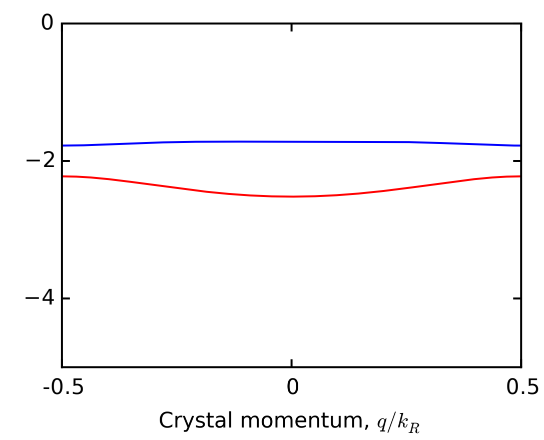
W. P. Su, J. R. Schrieffer, and A. J. Heeger, PRL (1979), and M. J. Rice and E. J. Mele, PRL (1982)

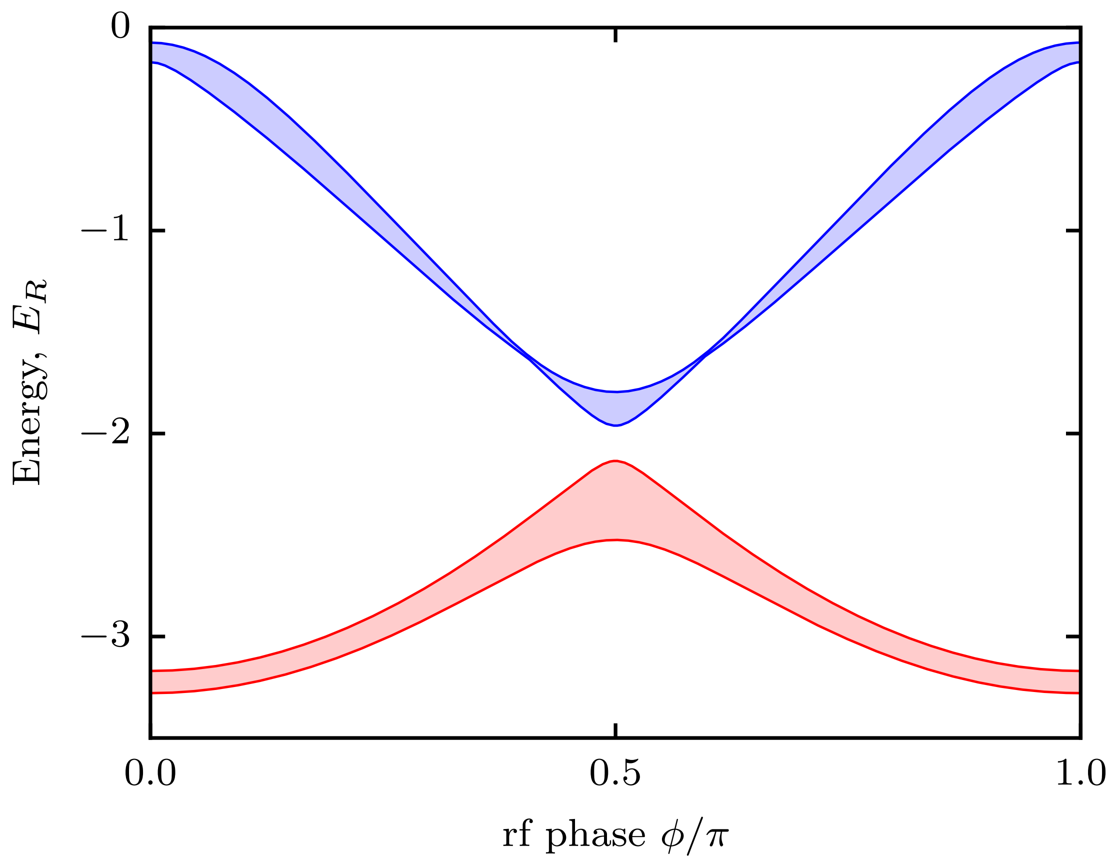
Essential properties

Spatial structure



Momentum structure

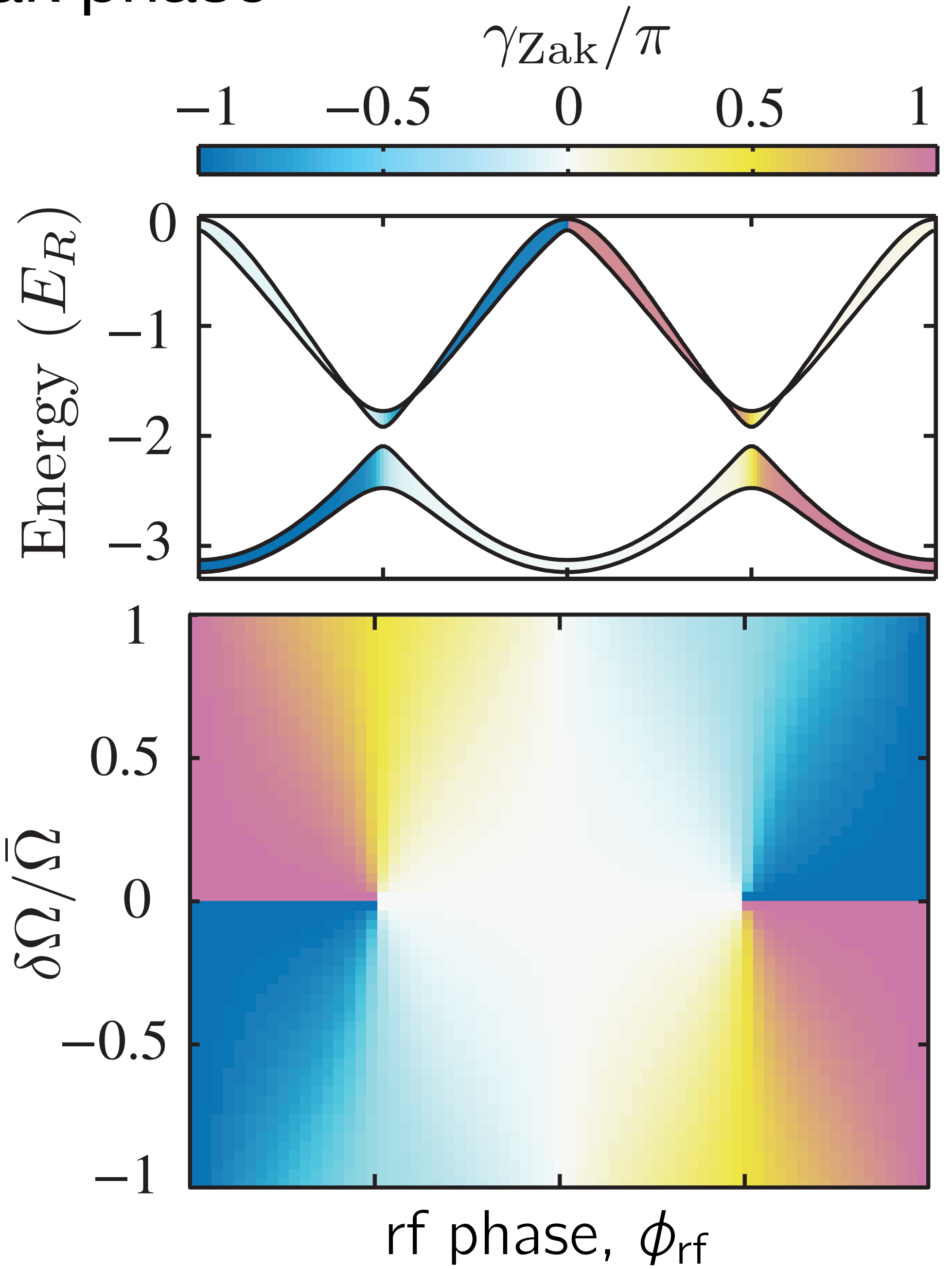




Topology and local geometry

Full-band topology

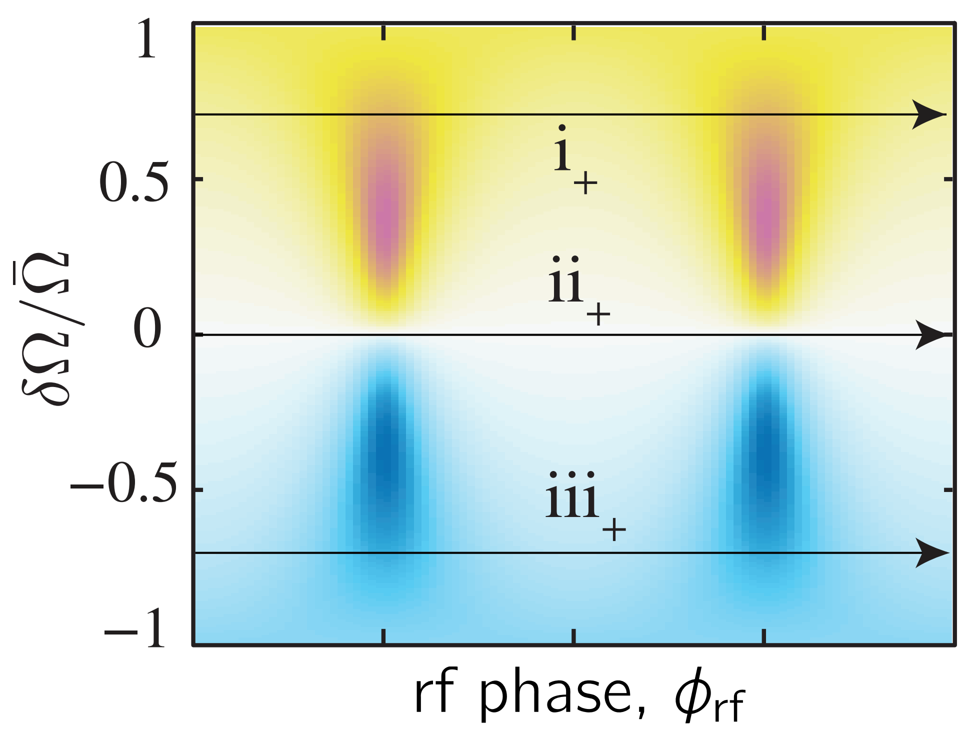
Zak phase



Local geometry

Berry curvature at q = 0

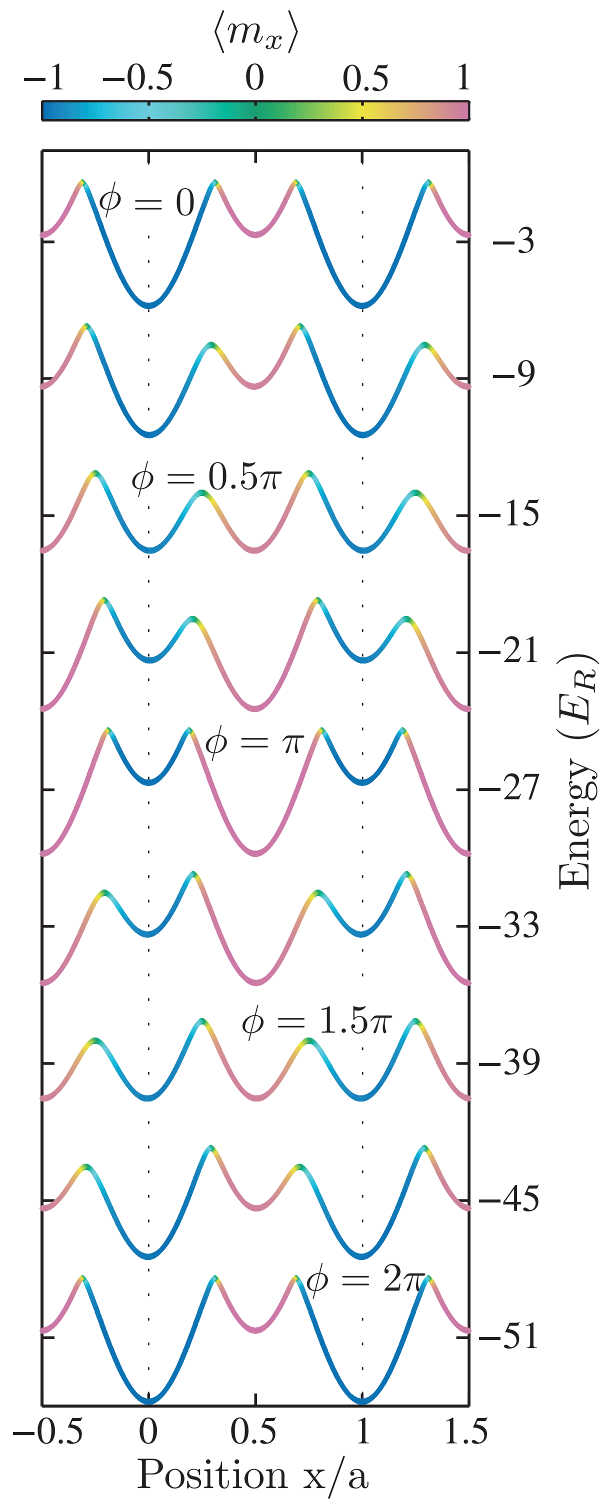
Nothing special at Topological singular points



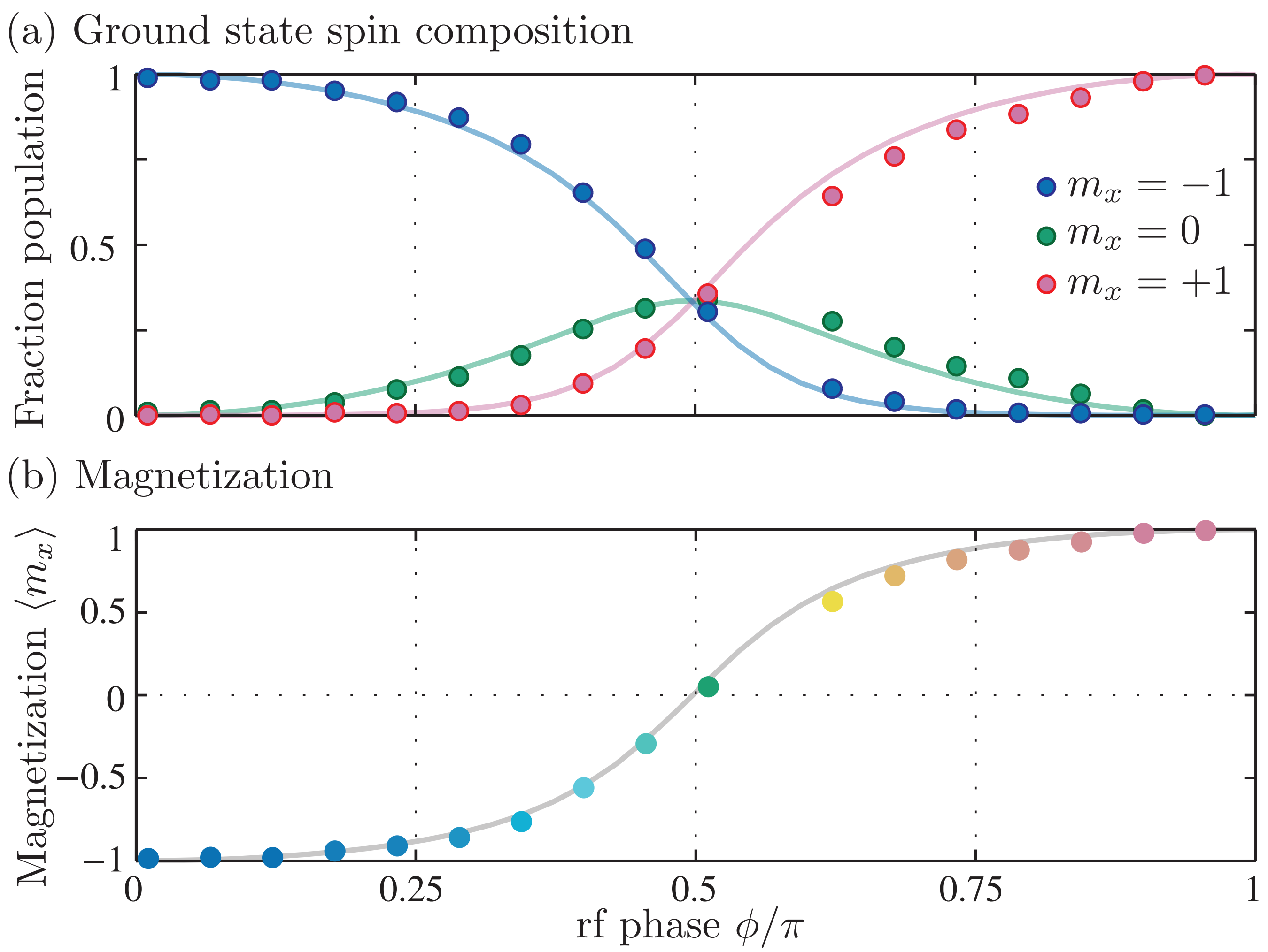
References

M. Atala, et al, Nat. Phys (2013)

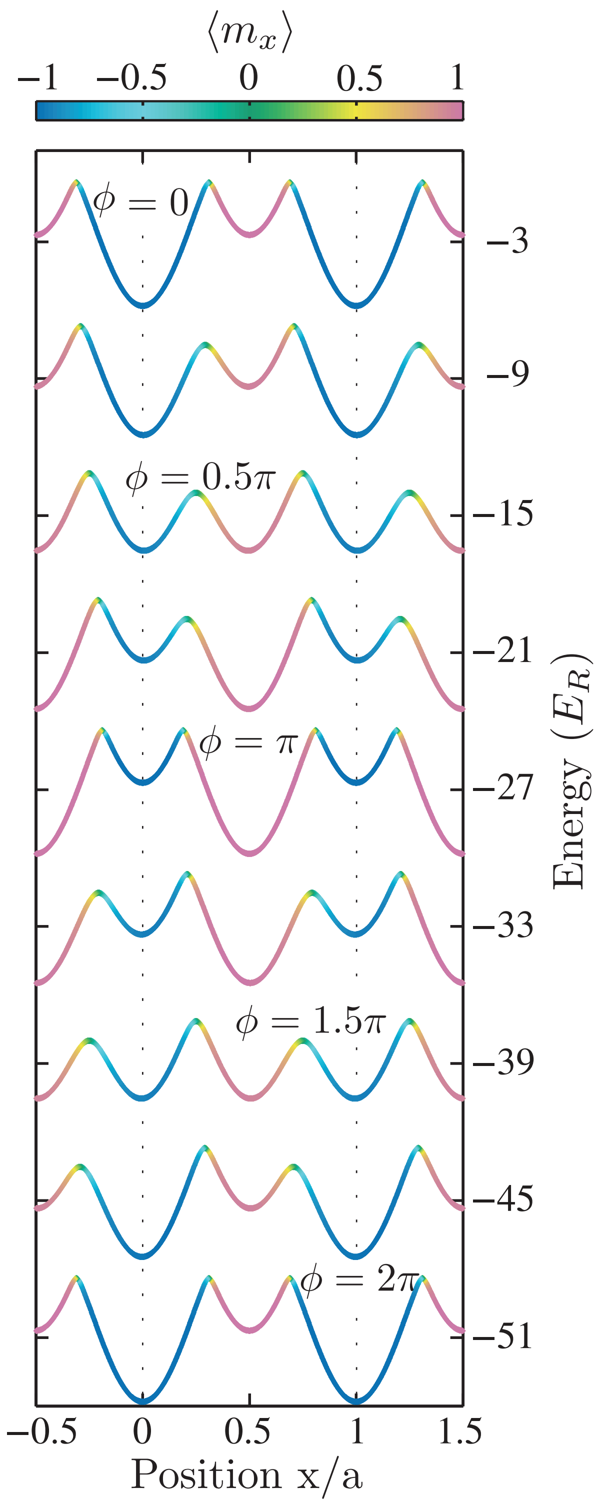
Magnetization



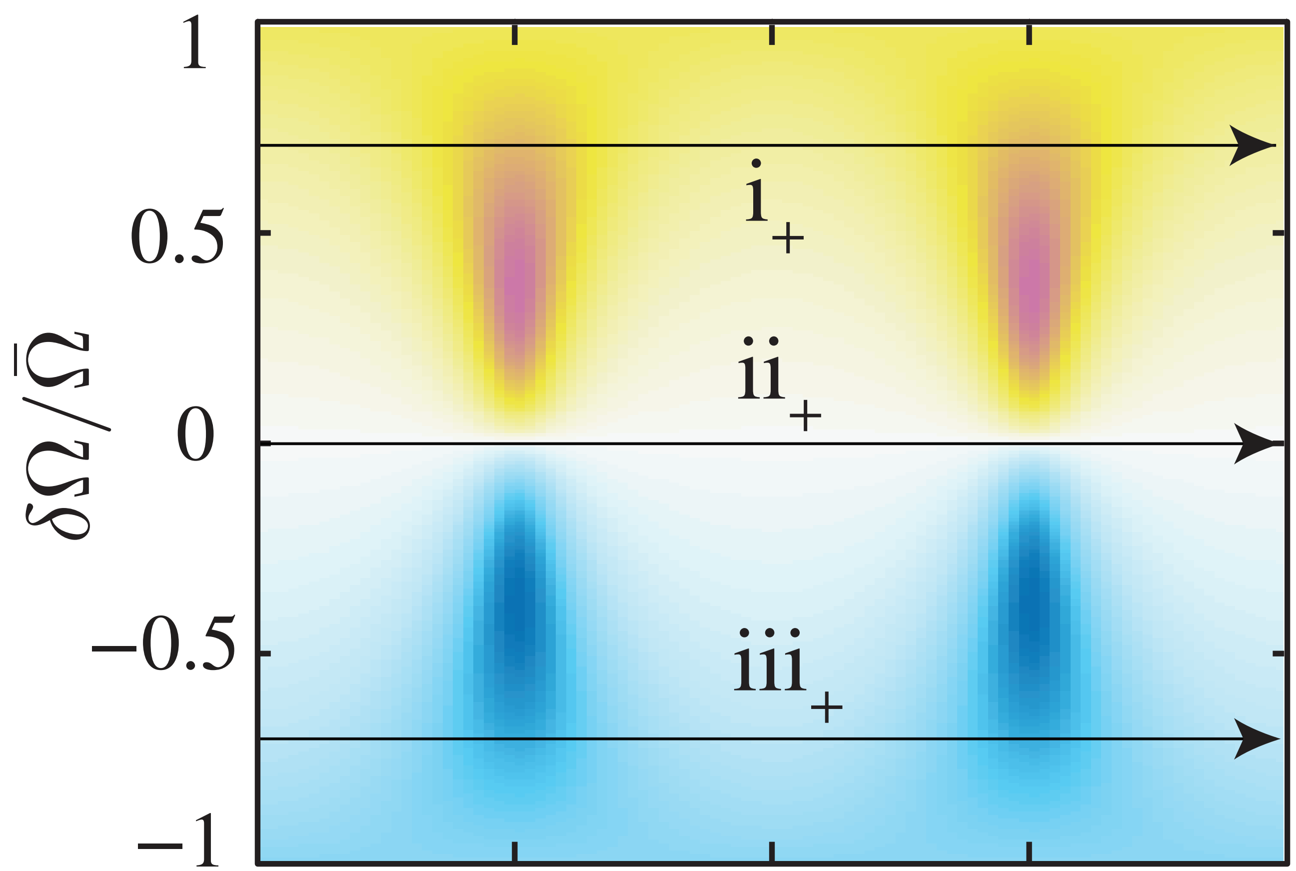
Direct sub-lattice readout



Trajectories

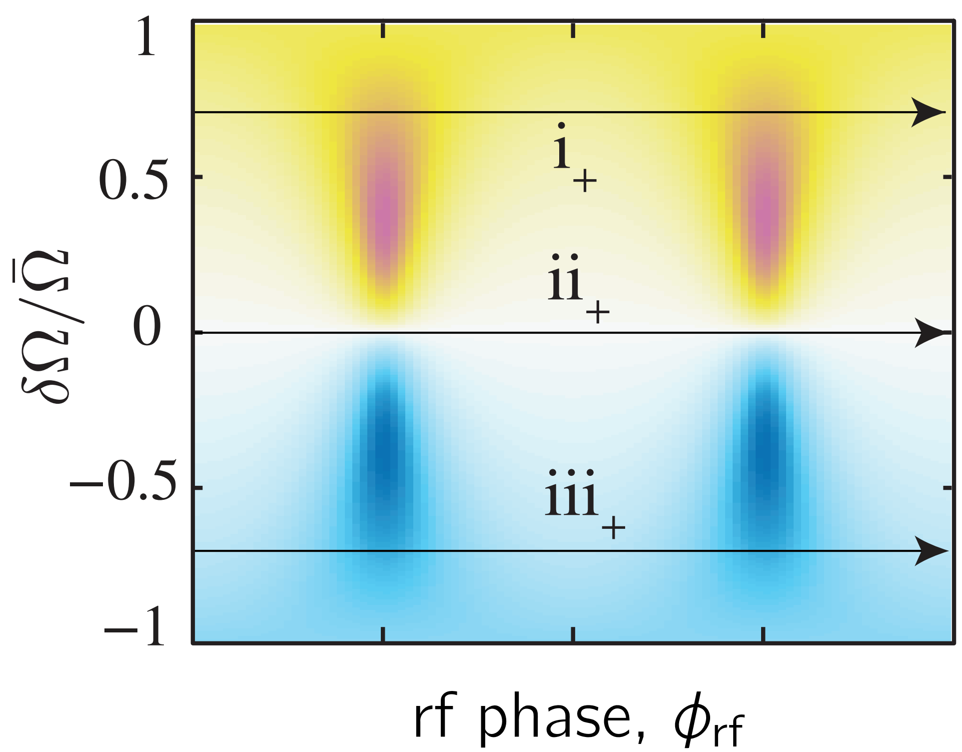


Trajectory



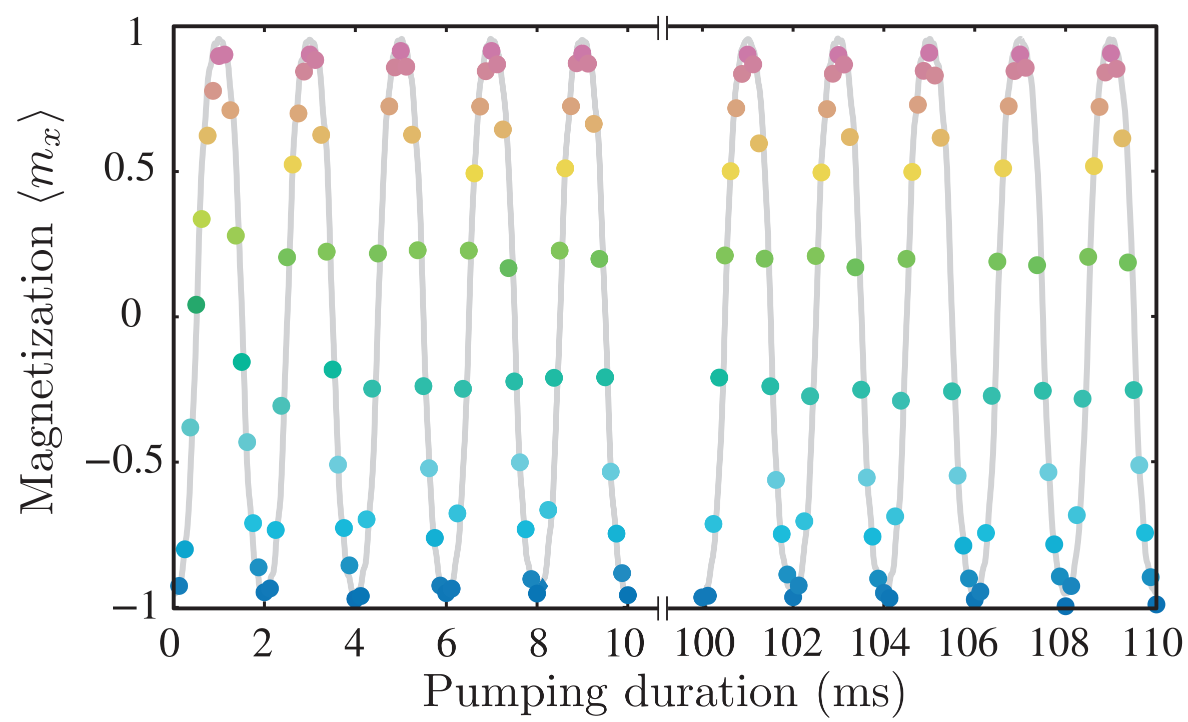
Pumping

Trajectory (i)



Oscillating local magnetization

"Polarization" effect in conventional charge pump theory



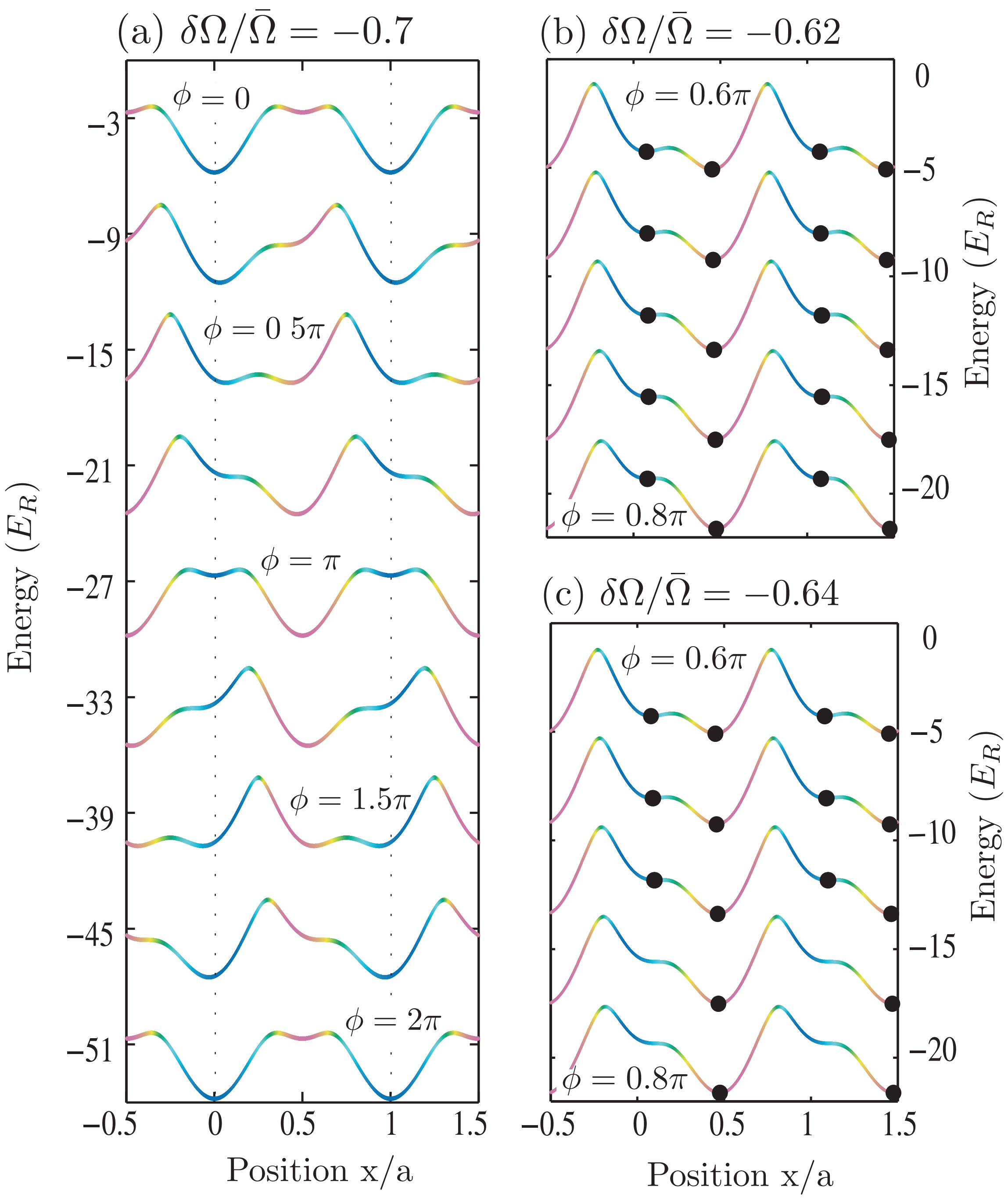
Conclusions?

- Atoms are oscillating back and forth in the same well?
- Atoms move by ± one cell

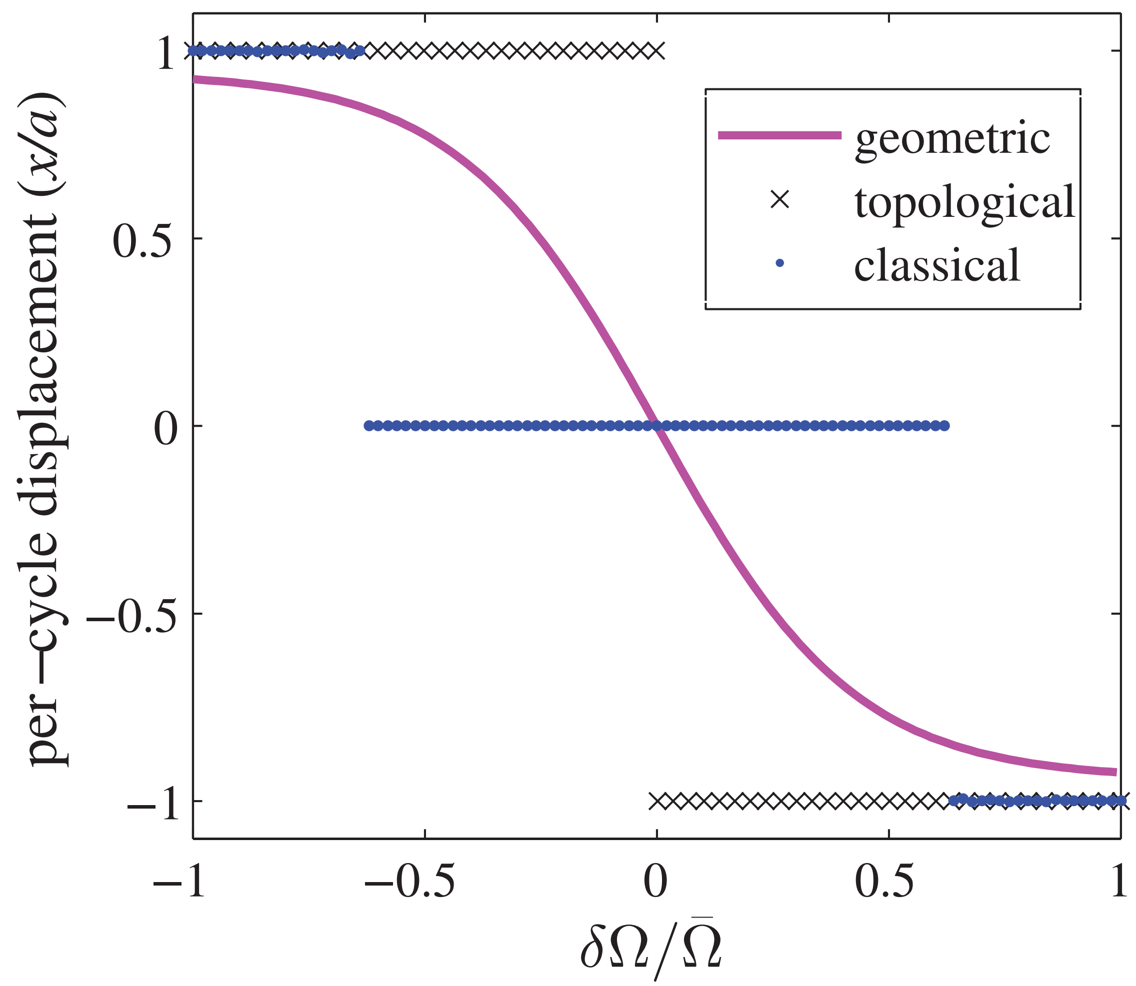
Correct for: a classical particle and a filled bands → quantized motion

Essential properties: Classical

Spatial structure

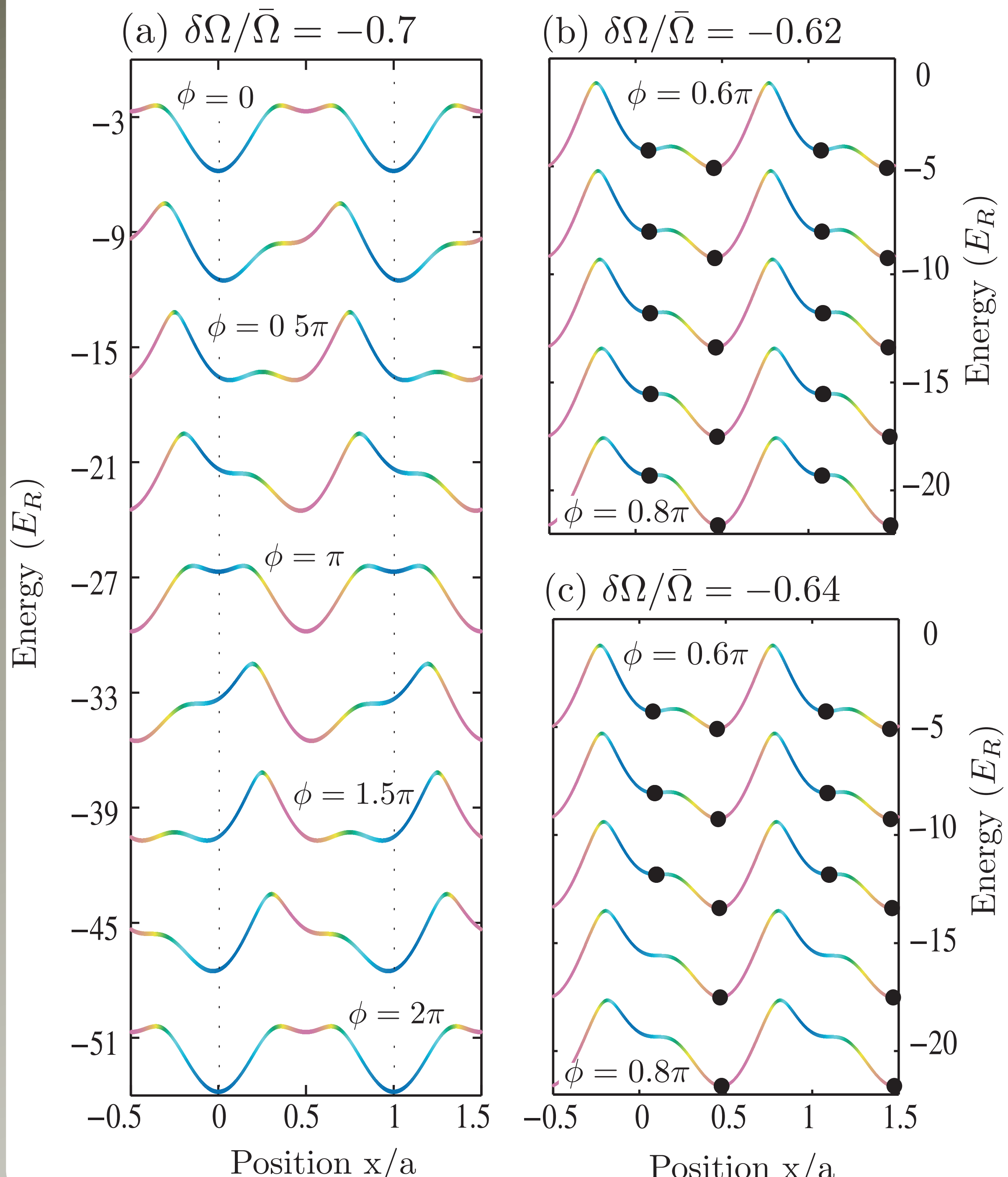


Classical/Quantum pumps

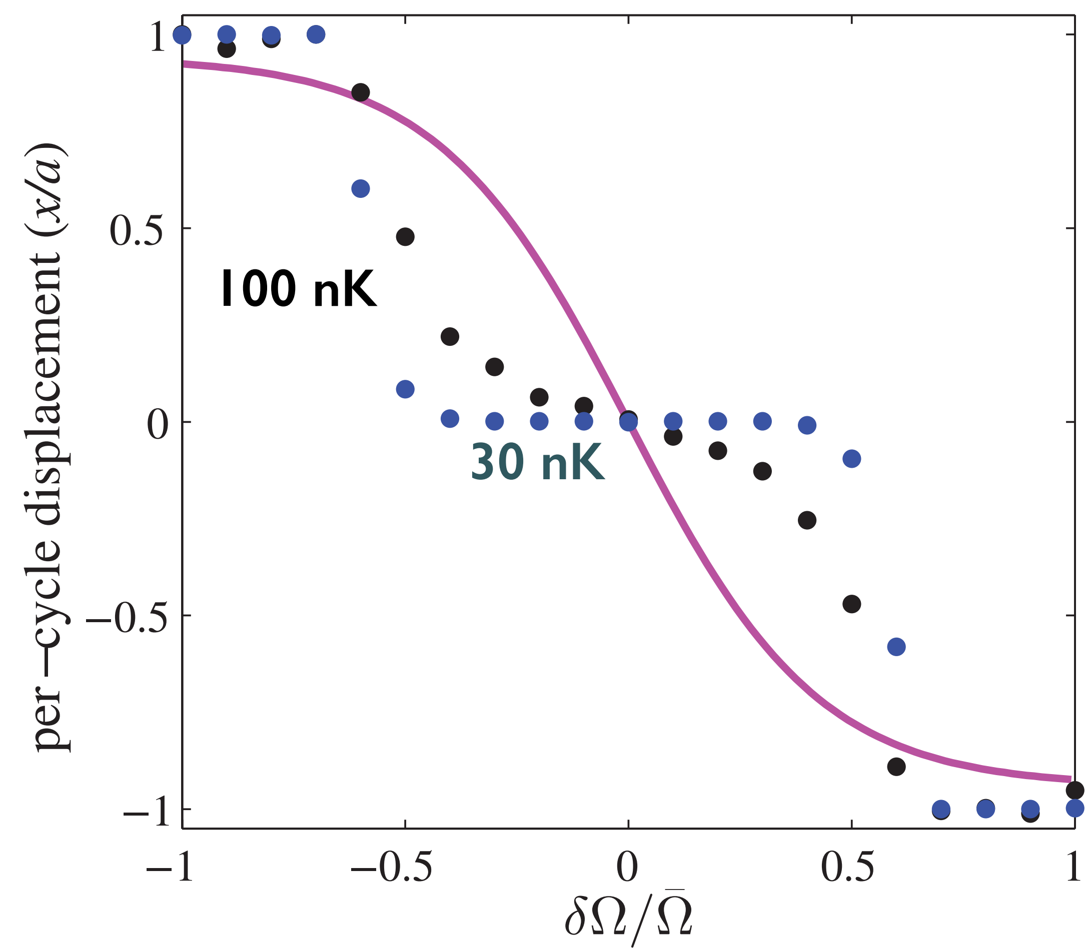


Essential properties: Classical

Spatial structure



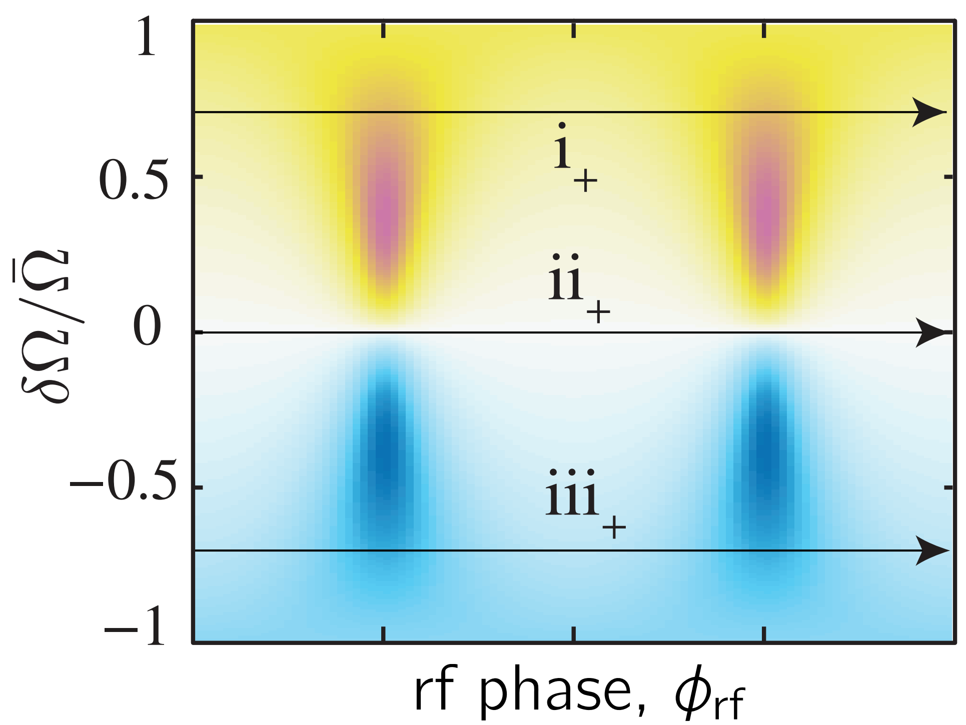
Classical pumps: $T > 0$



Pumping

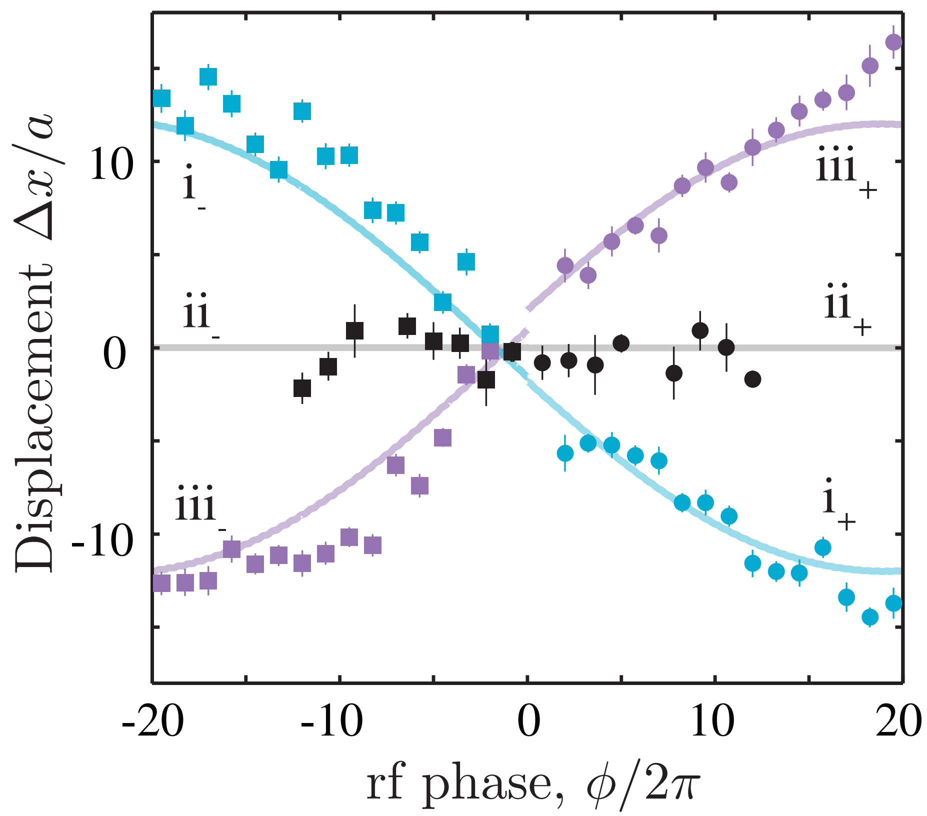
Trajectory

All three trajectories



Direct observation pumping

20 pump cycles



What else?

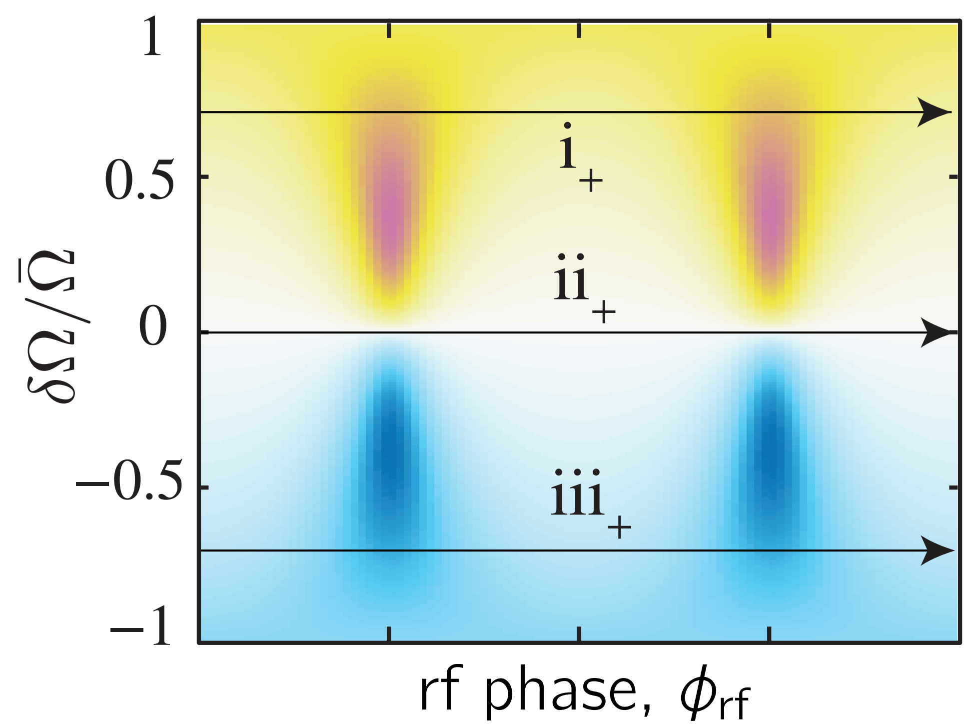
Harmonic trapping: 2 ms pump period vs (m*/m) x 70 ms → 240 ms)

Look at short time to see initial force

Pumping: short time response

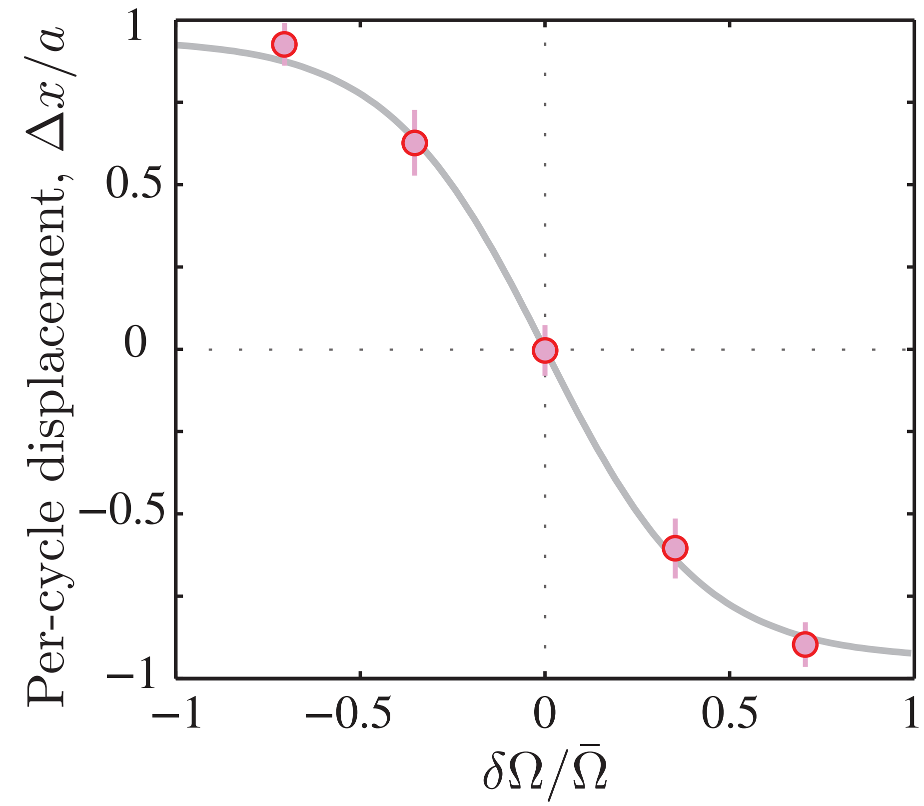
Trajectory

All three trajectories



Direct observation pumping

Agreement with Berry’s phase



Conclusions

Cold atom systems are

- the first systems to show the topological charge pumps
- can show new kinds of quantum charge pumping, e.g., geometric

M. Lohse et al Nature Physics (2015); S. Nakajima et al Nature Physics (2015)
Hsin-I Lu et al. arXiv: 1508.04480v1 (JQI)