

# Quantum Simulations with Laser Assisted Tunneling

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2/11/2016

Brussels

Solvay Workshop on “Quantum Simulation  
with Cold Matter and Photons”



Massachusetts  
Institute of  
Technology



in magnetic fields, we have TWO momenta

$$\vec{p}_{\text{mech}} = \vec{p}_{\text{can}} - q\vec{A}$$

Similar, in spin-orbit coupling

$$P_{\text{mech},i} = P_{\text{can},i} - \vec{\sigma}_i \cdot \vec{A}$$

$$P_{\text{mech}}^2 \rightarrow P_i \sigma_j \text{ terms}$$

⇒ Spin-orbit coupling is equivalent to a spin-dependent vector potential

in magnetic fields, we have TWO momenta

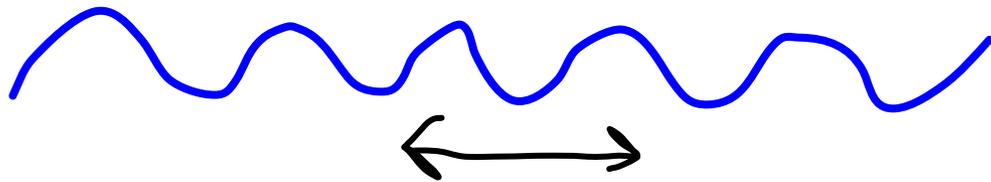
$$\vec{p}_{\text{mech}} = \vec{p}_{\text{can}} - \underbrace{q\vec{A}}_{n\hbar k}$$

photon momentum

Used in Raman coupling schemes and  
laser assisted tunneling

Photon  $\rightarrow e^{ikx}$

Shaking the Lattice



$t \rightarrow e^{i\phi} t$

$\phi(x) \hat{=} \vec{A}$

Peierl's  
Substitution

# Creation of effective magnetic fields in optical lattices: the Hofstadter butterfly for cold neutral atoms

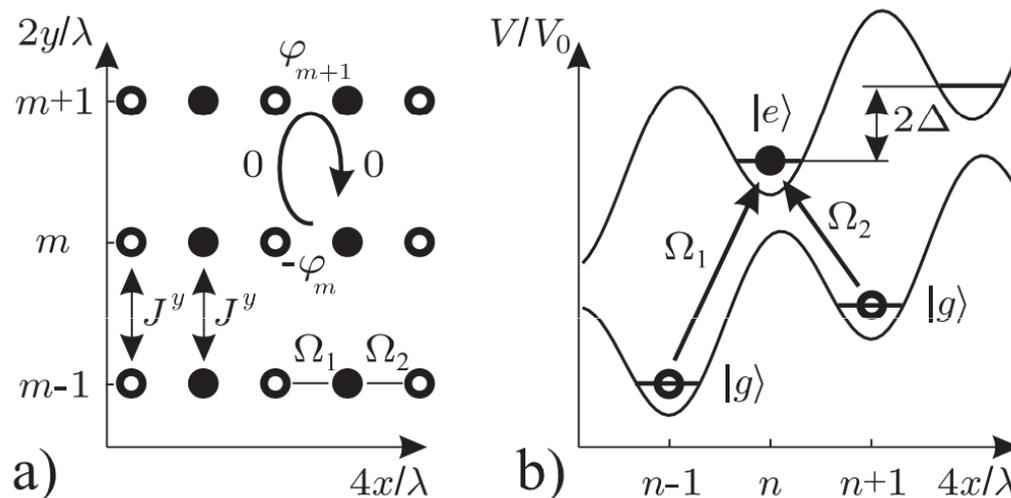
**D Jaksch**<sup>1,2</sup> and **P Zoller**<sup>2</sup>

<sup>1</sup> Clarendon Laboratory, Department of Physics, University of Oxford, Oxford OX1 3PU, UK

<sup>2</sup> Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck, Austria

E-mail: [dieter.jaksch@physics.oxford.ac.uk](mailto:dieter.jaksch@physics.oxford.ac.uk)

*New Journal of Physics* **5** (2003) 56.1–56.11 (<http://www.njp.org/>)



# How to engineer these phases?

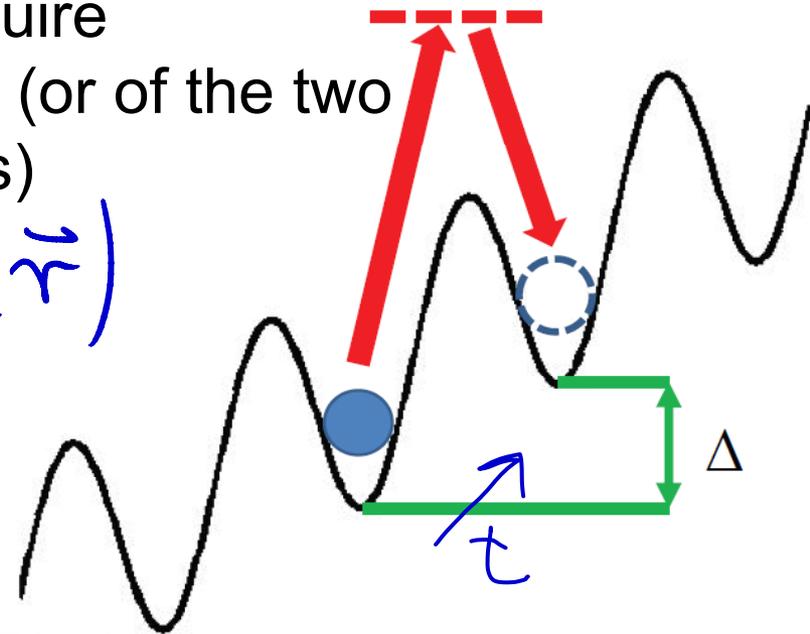
## Concept:

Create a situation where motion (tunneling) is only possible with the help of laser beams

## Result:

Tunneling matrix element will acquire the local phase of the laser beam (or of the two photon field for Raman processes)

$$t \rightarrow t e^{i\varphi_{\text{Laser}}(\vec{r})}$$
$$\varphi_{\text{Laser}} = \Delta \vec{k} \cdot \vec{r}$$



Realized: 2013 (MIT, Ketterle group; Munich, Bloch group)  
(without lattice: JQI 2009)

Miyake H, Siviloglou G A, Kennedy C J, Burton W C and Ketterle W,  
Phys. Rev. Lett. 111, 185302 (2013): **Realizing the Harper Hamiltonian with  
Laser-Assisted Tunneling in Optical Lattices**

Constant magnetic field

$$\vec{B} = B_0 \hat{z}$$

Vector potential MUST break translational invariance in x and y

Different gauges have different symmetries:

$$\text{For } \vec{B} = B \hat{z}$$

$\alpha = \frac{p}{q}$ , Flux per unit cell

Landau gauge

$$\vec{A} = -B y \hat{x}$$

$$\vec{A} = B x \hat{y}$$

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$\vec{A} = B x \hat{y}$

$1 \times q$

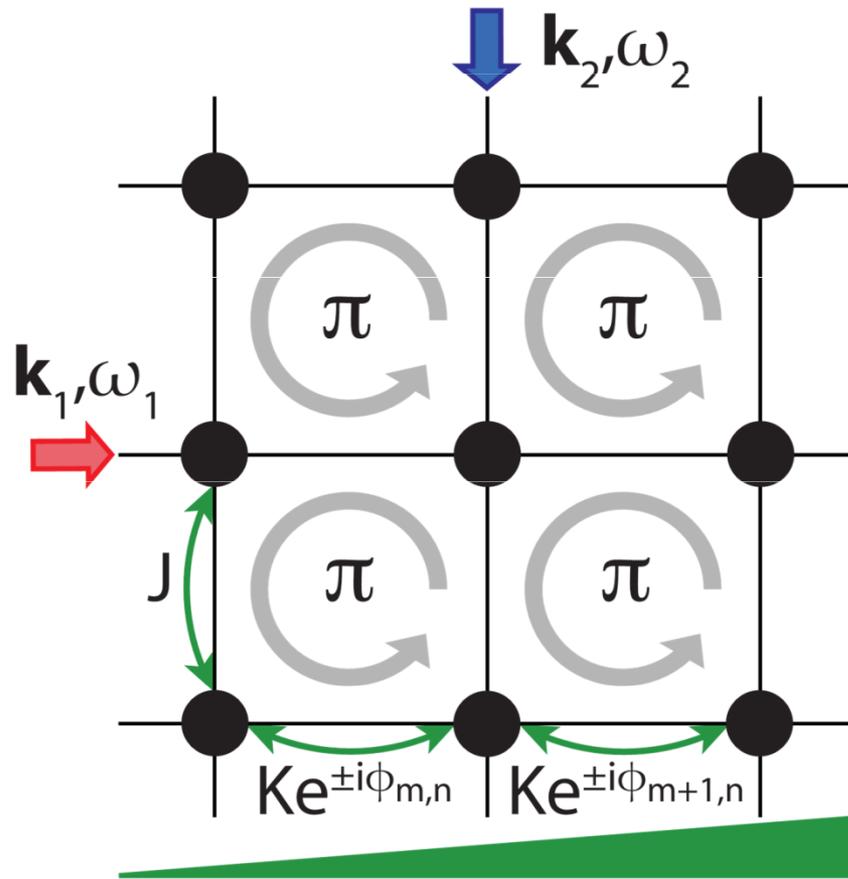
$q \times 1$



Wavefunction has different periodicity!

unit cell of Hamiltonian

# Experimental Setup for $\alpha = 1/2$



# Different gauges have different symmetries:

For  $\vec{B} = B \hat{z}$

$\alpha = \frac{p}{q}$ , Flux per unit cell

Landau gauge

$$\vec{A} = -B y \hat{x}$$

$$\vec{A} = B x \hat{y}$$

1 x q

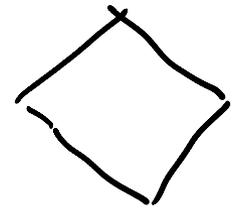


q x 1



Our implementation

$$\vec{A} = -B (y + x) \hat{x}$$



Wavefunction has different periodicity!

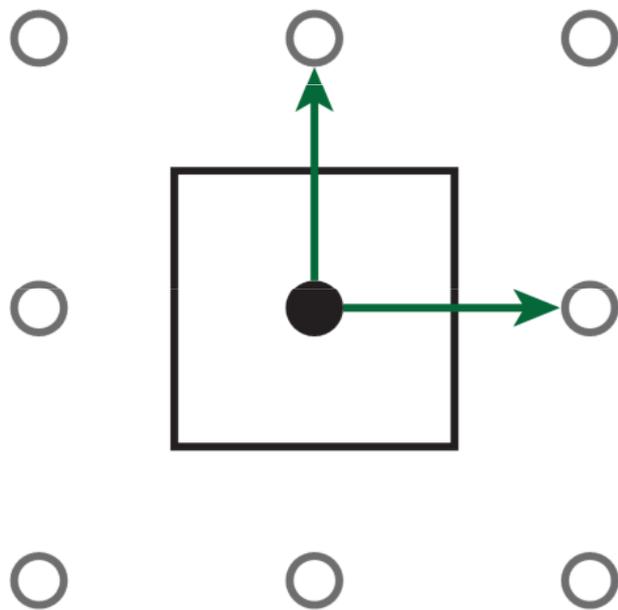
unit cell of Hamiltonian

Switch off lattice – wavefunction is not changed.  
“Self diffraction of coherent matter wave”

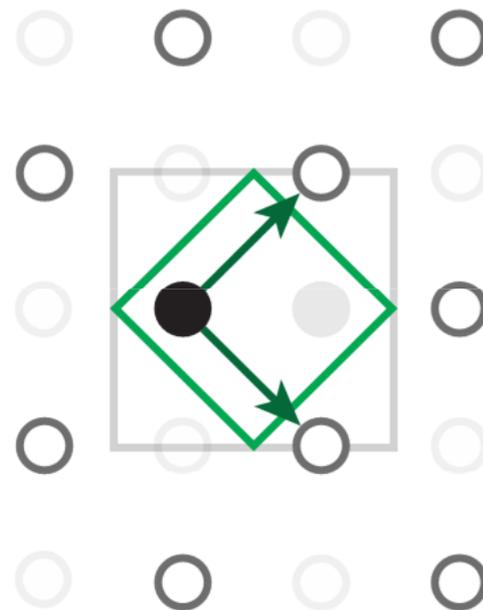
Canonical momentum becomes mechanical momentum.  
Observed after ballistic expansion.

Time-of-flight images show the momentum distribution of the wavefunction (which is NOT gauge invariant).

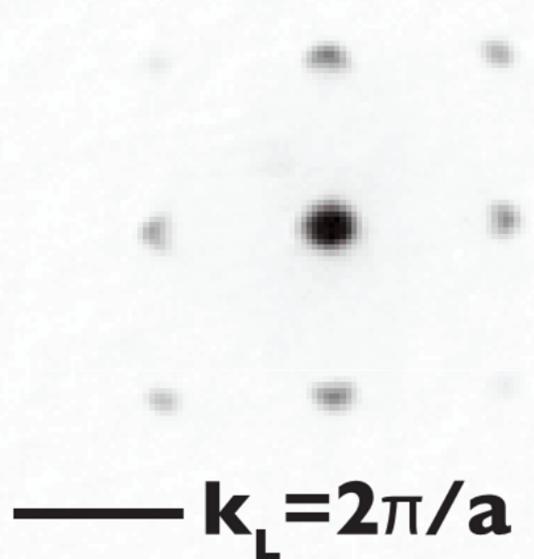
c) Trivial Superfluid



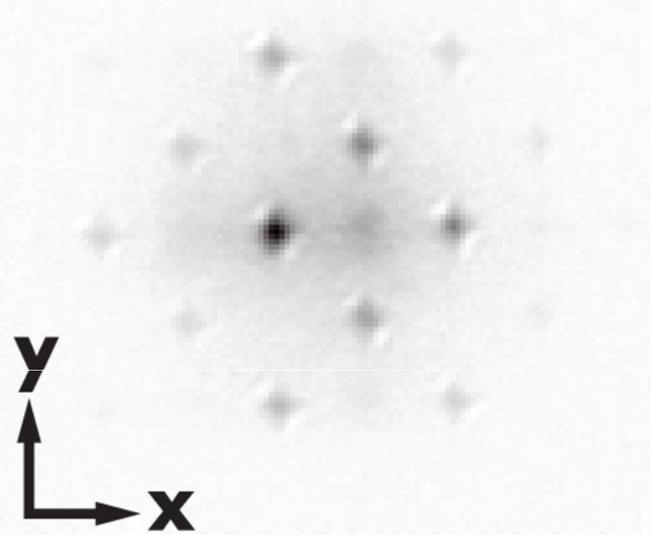
d) Single Minimum

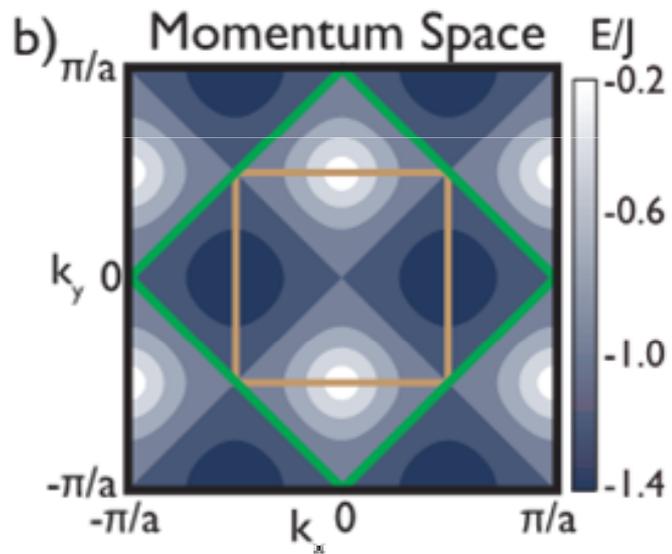
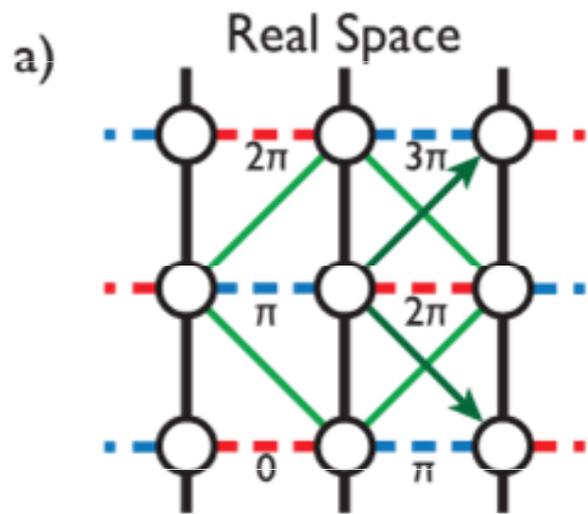


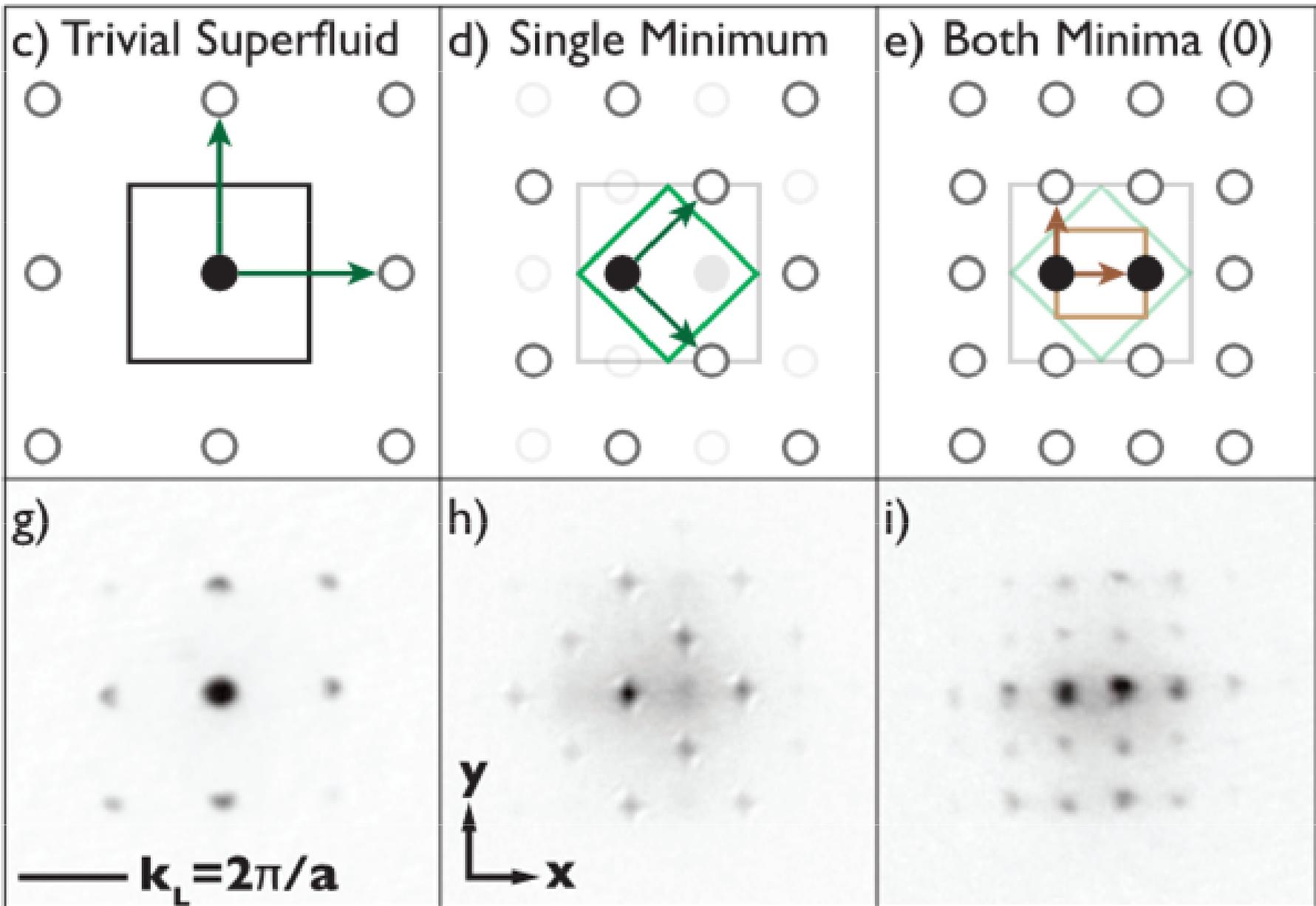
g)



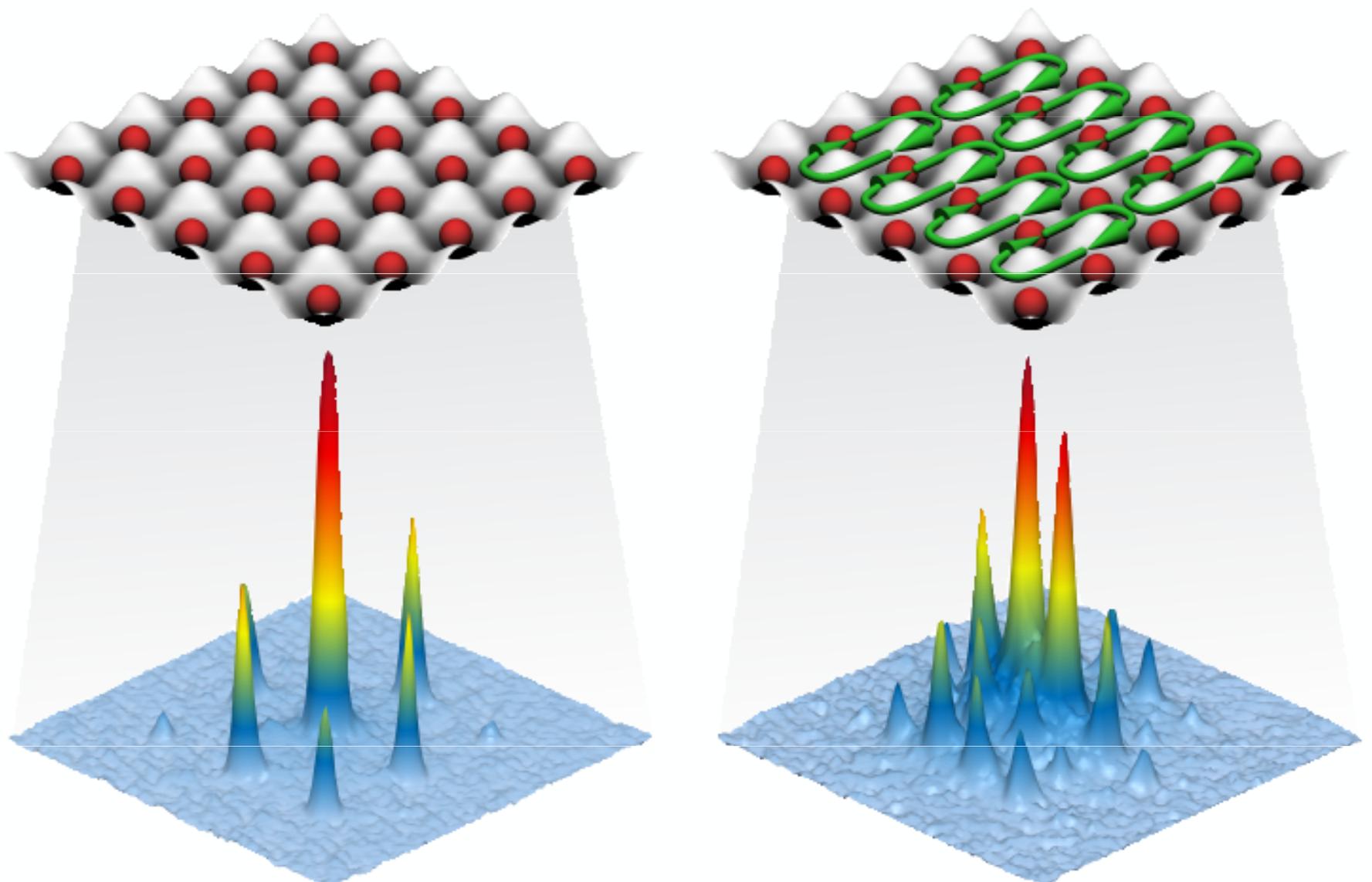
h)







C.J. Kennedy, W.C. Burton, W.C. Chung, and W. Ketterle, *Observation of Bose-Einstein Condensation in a Strong Synthetic Magnetic Field*, Nature Physics **11**, 859–864 (2015).



## Simplest explanation for time of flight pictures:

The wavefunction is unchanged, TOF pictures show momentum distribution (i.e.  $\langle \psi | -i\hbar \nabla | \psi \rangle$  )

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However, this cannot be done with electrons and real magnetic fields since the wavefunction is NOT gauge invariant.

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## Alternative description:

Canonical momentum  $\hat{p} = -i\hbar \nabla$  becomes mechanical momentum

Mechanical momentum changes from  $p - A$  to  $p$

Momentum change by  $A$  can be described by synthetic electric field

$$E = -\partial_t A$$

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$$E = -\partial_t A \quad \text{This is not gauge invariant!}$$

## For (real) electromagnetic fields:

$$E = -\partial_t A - \nabla \phi$$

and momentum distributions after switch-off (time-of-flight images) are independent of gauge

(but in general, don't show the momentum distribution of the wavefunction)

Now: Spin degree of freedom

How to engineer spin-orbit coupling for neutral atoms?

Raman spinflip scheme

Flip the spin in a momentum-dependent way (using the Doppler shift)

Our scheme

Affect the motion of spin up and down differently WITHOUT flipping the spin

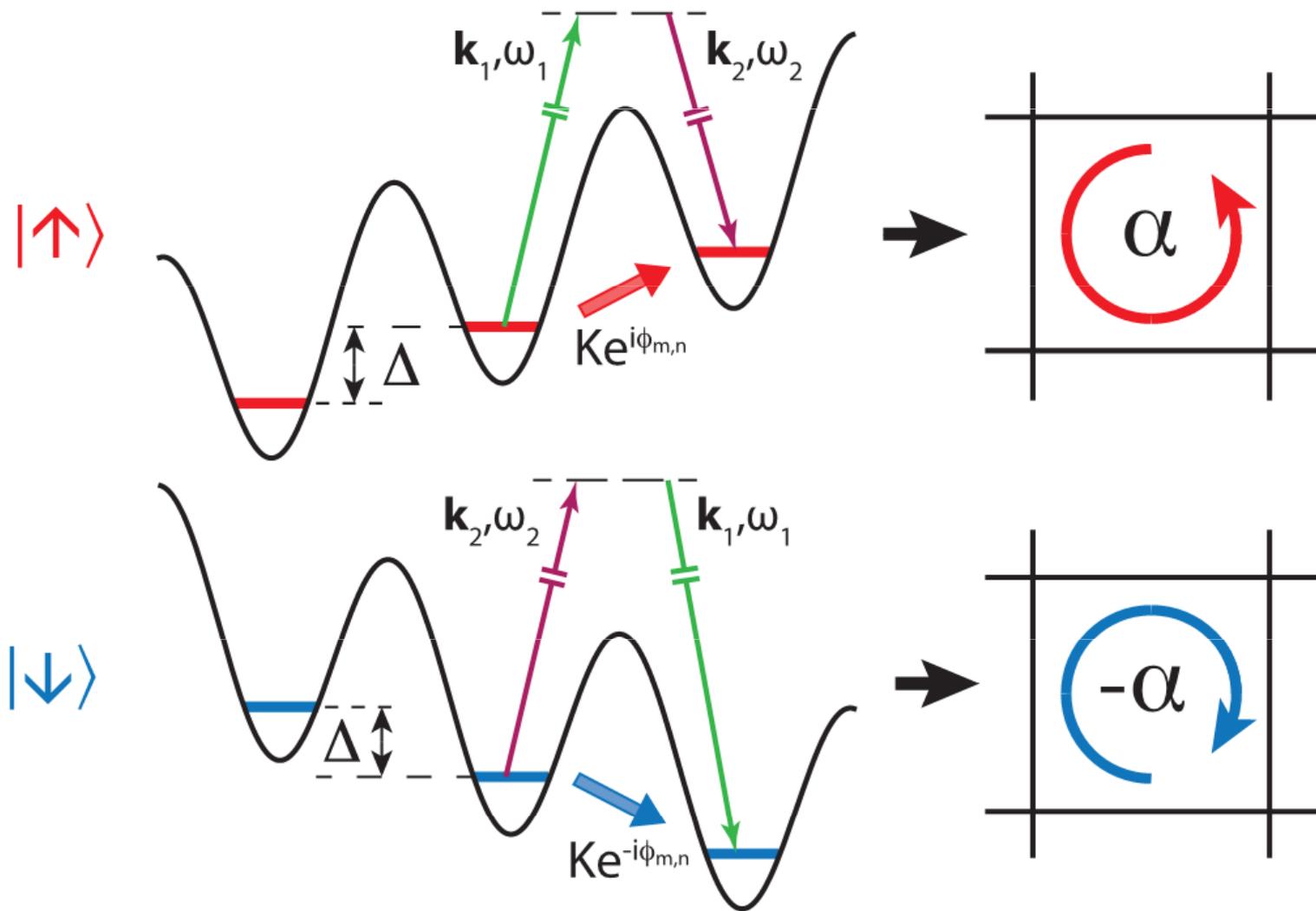
(using Zeeman shifts to address spin up and down differently)

Kennedy C J, Siviloglou G A, Miyake H, Burton W C and Ketterle W, Phys. Rev. Lett. **111**, 225301 (2013). Spin-orbit coupling and spin Hall effect for neutral atoms without spin-flips

In our scheme:

signs of  $B$ ,  $A$ , phase of tunneling matrix elements  
reflect the momentum transfer by Raman beams

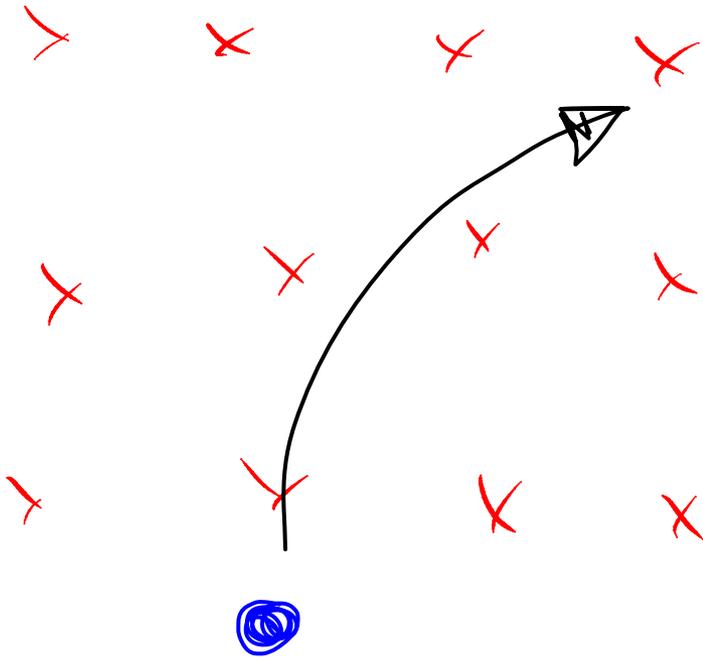
also: Munich



$$\phi_{m,n} = (mk_x a + nk_y a) \sigma_z \quad \mathbf{A} = \frac{\hbar}{a} (k_x x + k_y y) \hat{\mathbf{x}} \sigma_z$$

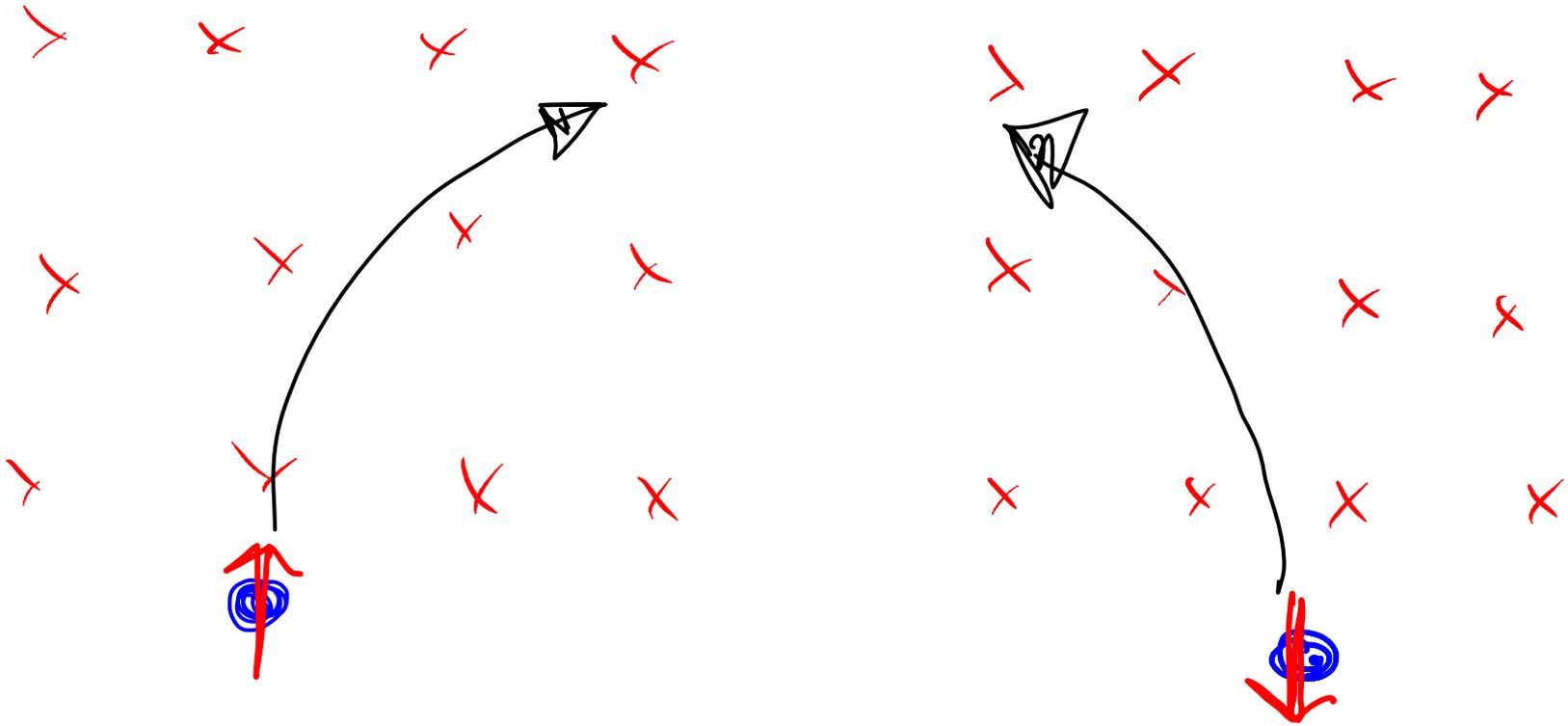
# Hall effect

B field separates charge



# Spin Hall effect

B field separates spin



Means that effective B field is different for two spins

# Time reversal symmetry

- Quantized spin Hall effect (two opposite quantum Hall phases)
- Z topological index (due to conservation of  $\sigma_z$ )
- Topological insulator

PRL **96**, 106802 (2006)

PHYSICAL REVIEW LETTERS

week ending  
17 MARCH 2006

## Quantum Spin Hall Effect

B. Andrei Bernevig and Shou-Cheng Zhang

*Department of Physics, Stanford University, Stanford, California 94305, USA*

Exact realization of this idealized proposal

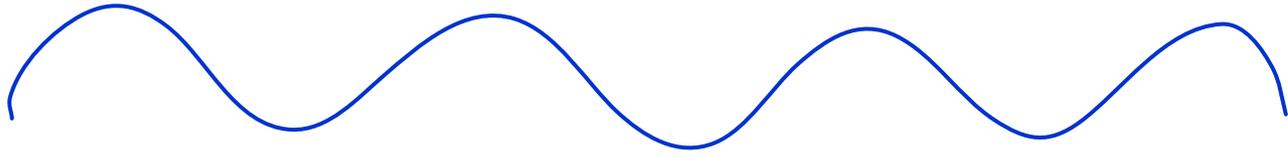
Diagonal in  $\sigma_z$   
Abelian SU(2) gauge  
field

How to do spinflips without transitions between spin states?

$\sigma^+$ ,  $\sigma^-$  terms

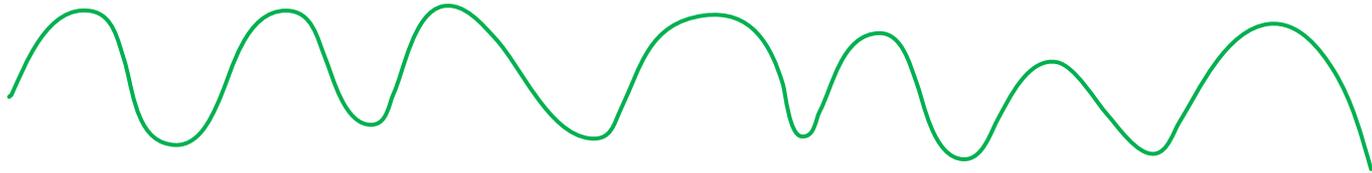
A new spin-orbit coupling scheme

# Superlattice



IR

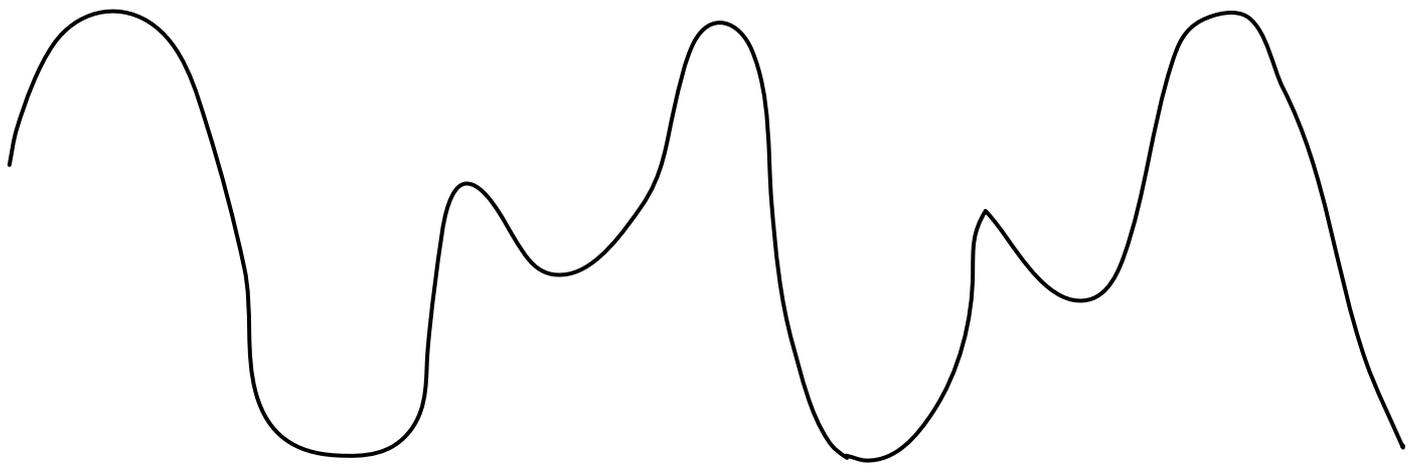
+



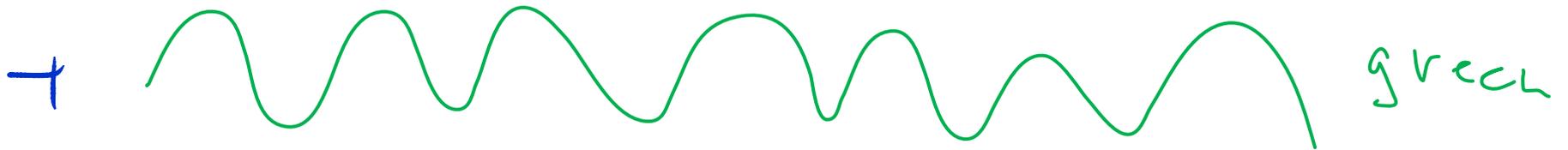
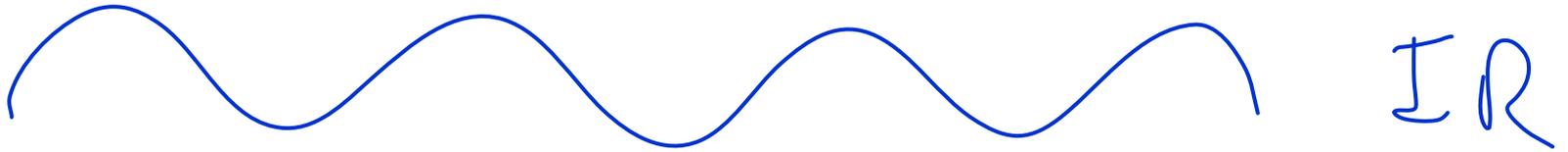
green

⇒

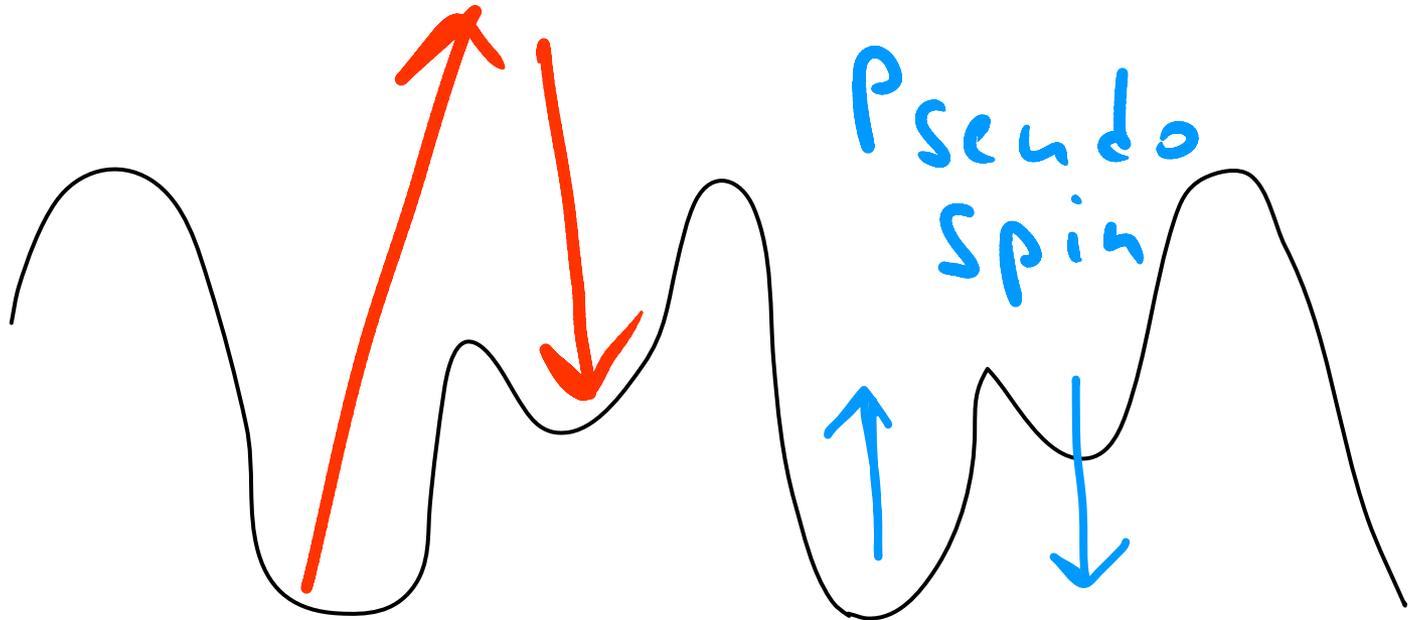
Double Well



# Superlattice

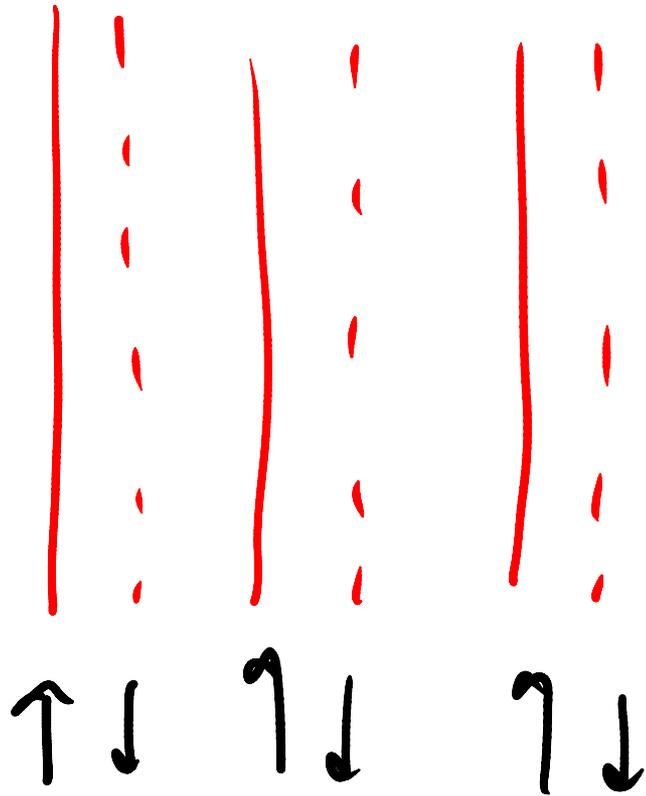


⇒  
Double  
Well

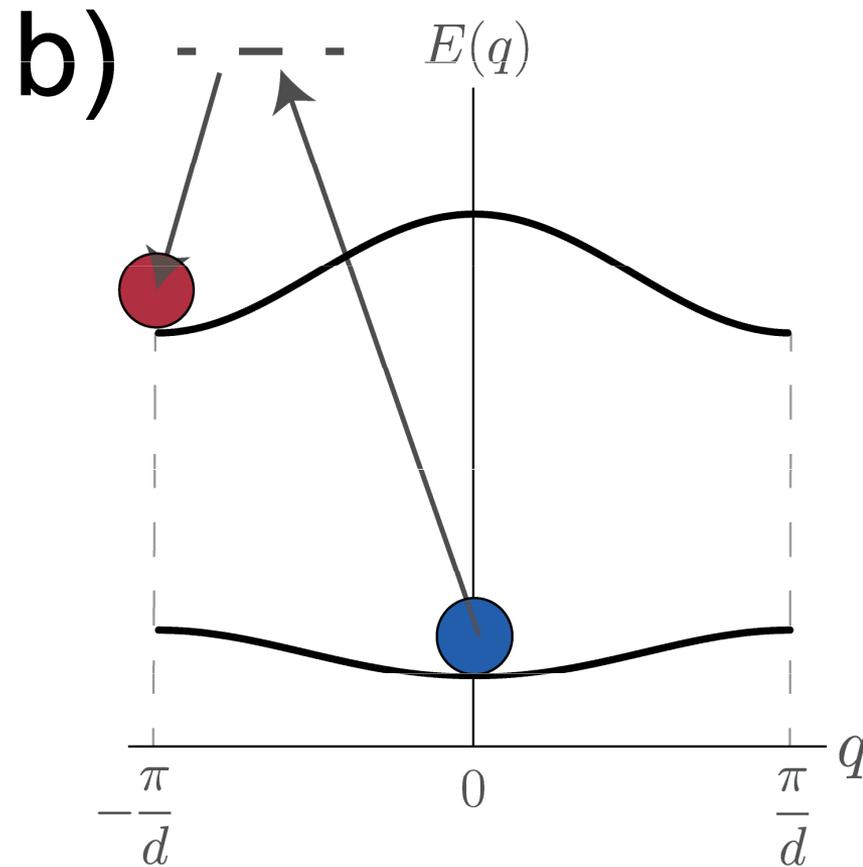
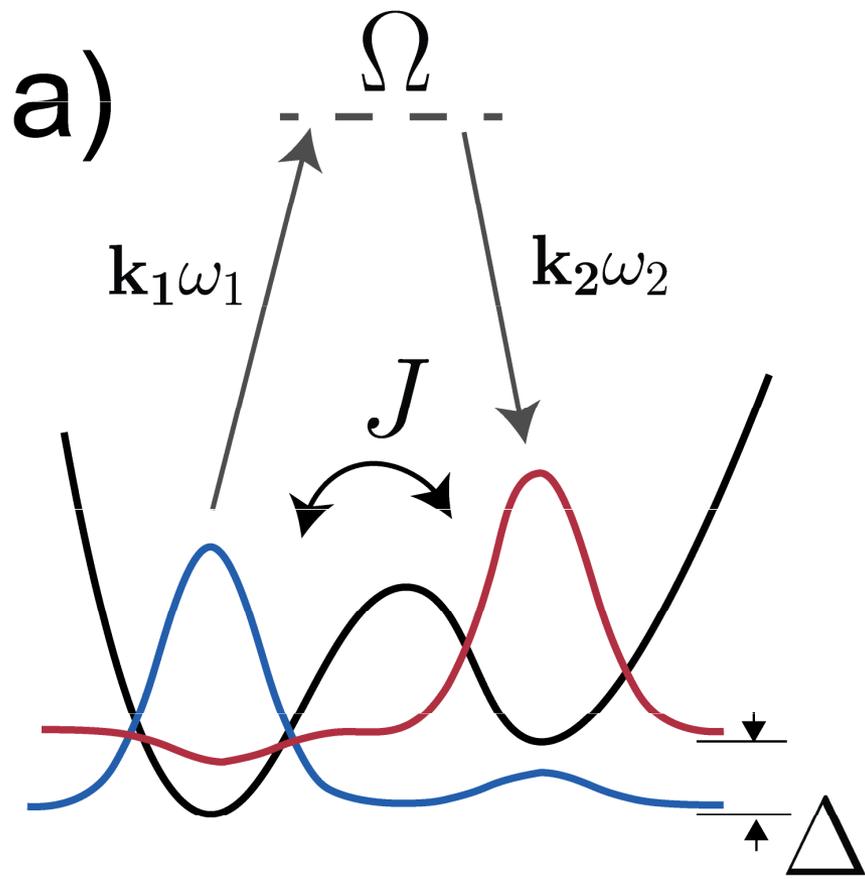


- Use orbital degree of freedom (lowest and first excited band) as pseudo-spin
- Double-well potential leads to long lifetimes (**see also: Hemmerich**) and adjustable interactions between the two spin states

# Stack of double pancakes

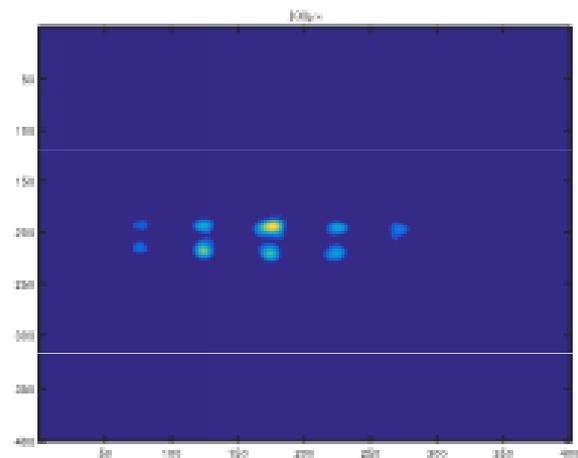


↑  $\hbar h$   
momentum  
transfer

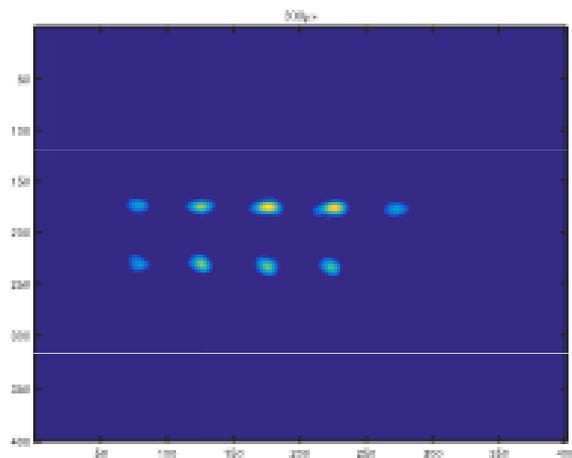


tunneling for  $\downarrow$  and  $\uparrow$  is **positive** / **negative**  
 band minimum at  **$k=0$**  /  **$k=\pm\pi$**

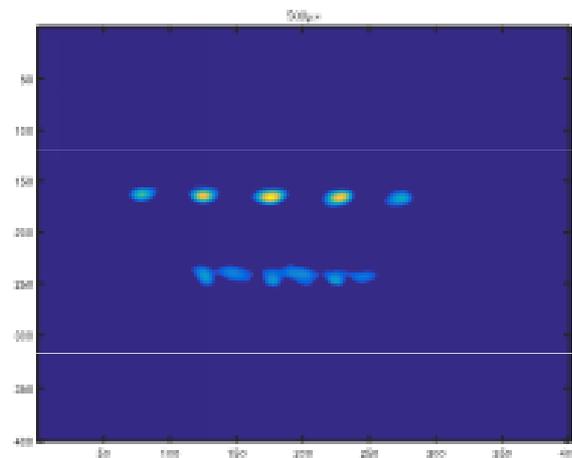
# separate pseudo-spins by optical “Stern Gerlach” effect



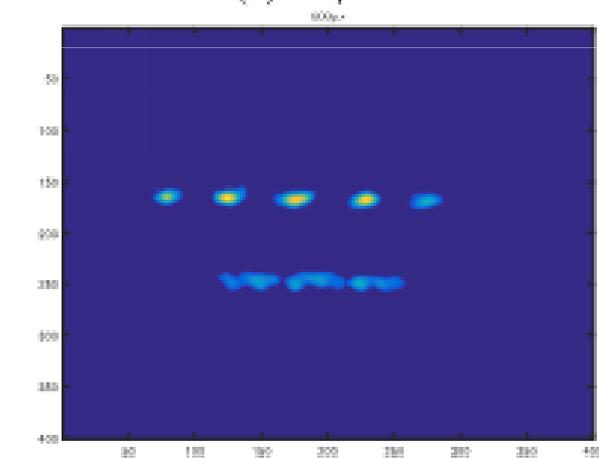
(a) 100  $\mu\text{s}$



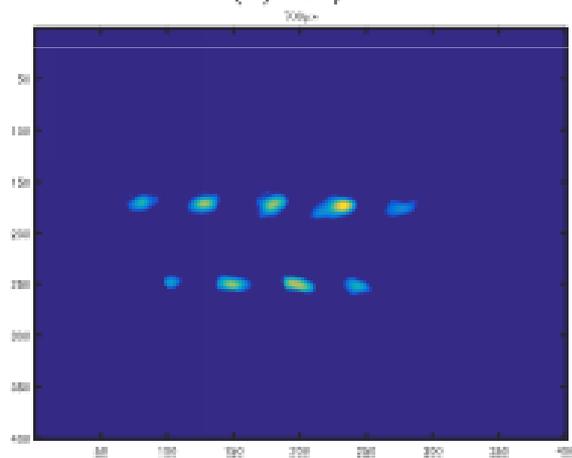
(b) 300  $\mu\text{s}$



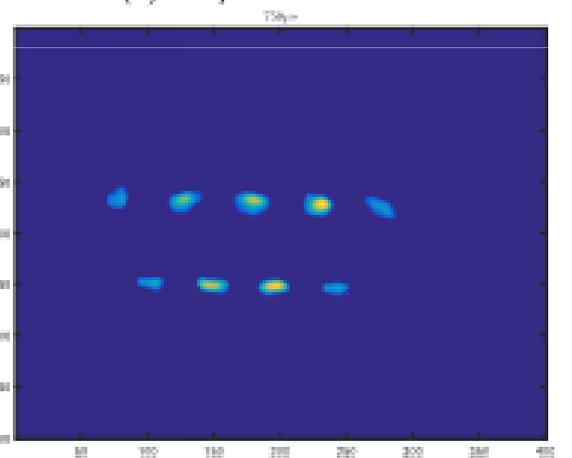
(c) 500  $\mu\text{s}$



(d) 600  $\mu\text{s}$

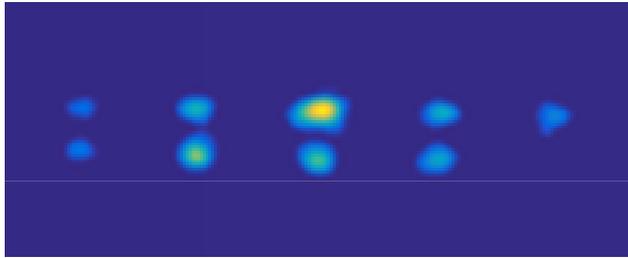


(e) 700  $\mu\text{s}$

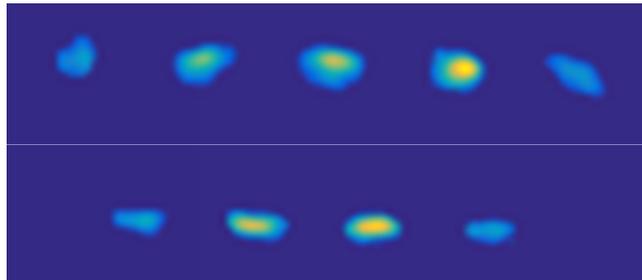
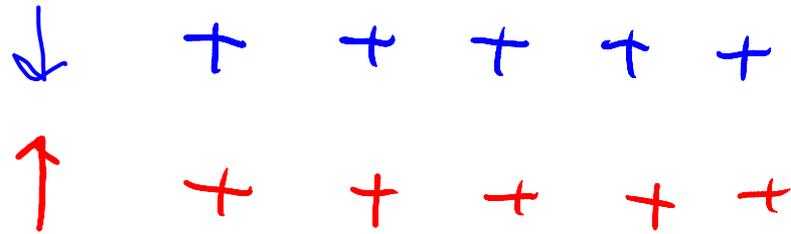


(f) 750  $\mu\text{s}$

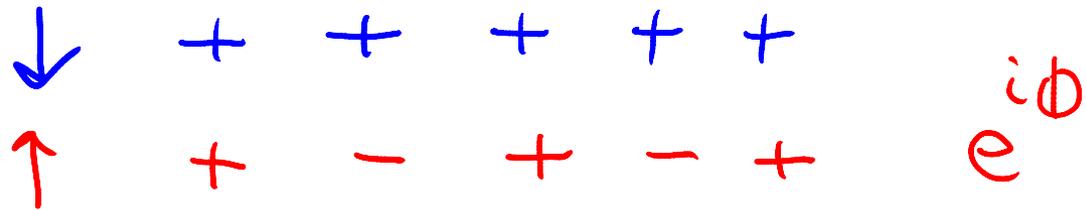
# Spontaneous formation of xy anti-ferromagnetic spin texture



xy ferromagnetic



xy anti-ferromagnetic



Spontaneous breaking of lattice symmetry (doubling of unit cell)  
 Spontaneous U(1) phase for angle of spin texture

trivial example for supersolid !! (?)

Notes:

.... a simple way to create spin dependent lattices for engineering new Hamiltonians ....

PRL 115, 073002 (2015)

PHYSICAL REVIEW LETTERS

week ending  
14 AUGUST 2015

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see also:

**Creating State-Dependent Lattices for Ultracold Fermions  
by Magnetic Gradient Modulation**

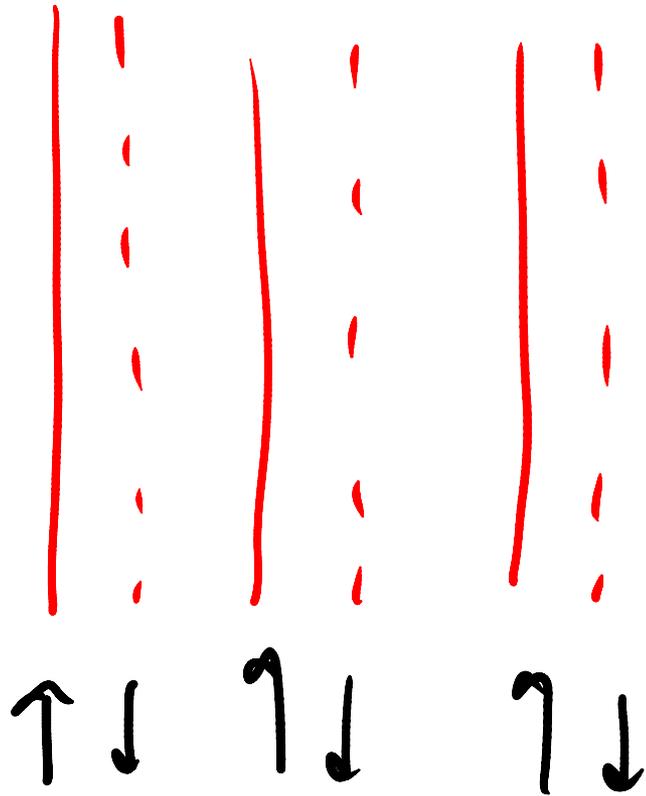
Gregor Jotzu, Michael Messer, Frederik Görg, Daniel Greif, Rémi Desbuquois, and Tilman Esslinger

*Institute for Quantum Electronics, ETH Zurich, 8093 Zurich, Switzerland*

(Received 21 April 2015; published 13 August 2015)

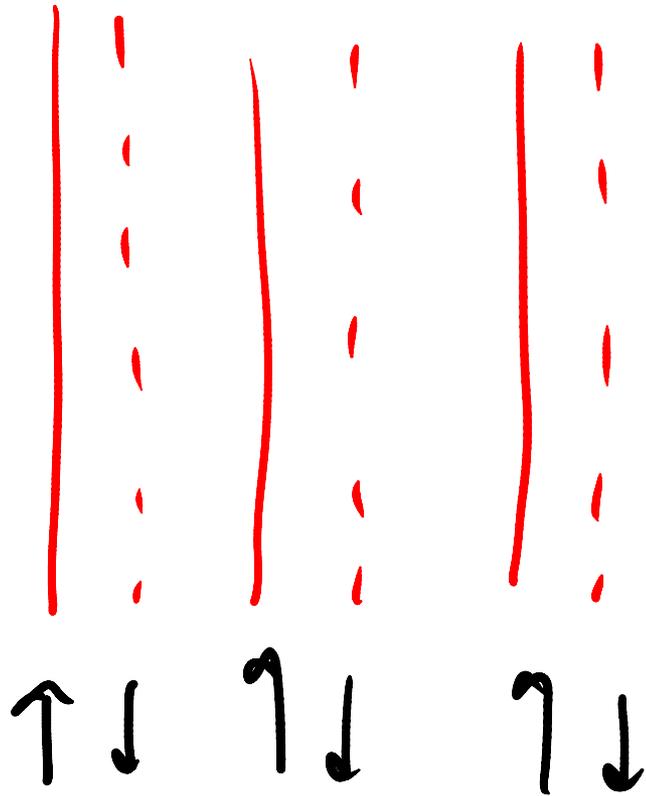
.... see population in left/right well without band mapping ....

# Stack of double pancakes



↑  $\hbar h$   
momentum  
transfer

# Stack of double pancakes



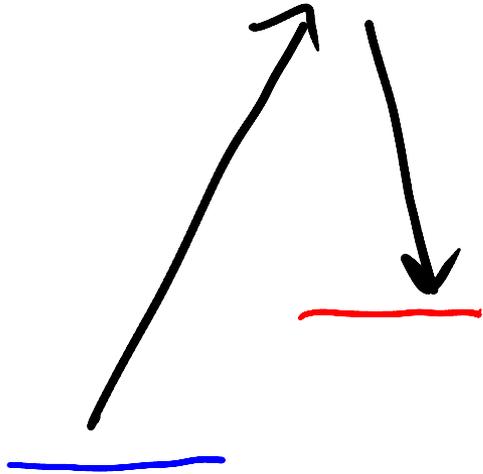
$\uparrow \hbar k$

Momentum transfer

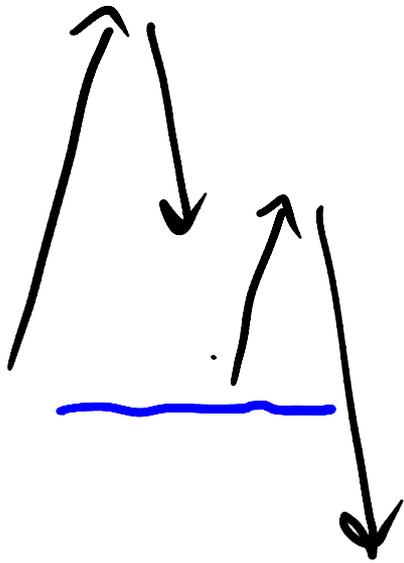
For SO coupling

$\xrightarrow{\hbar k}$

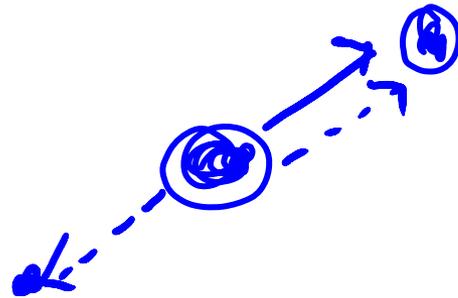
For coupling Wannier Functions

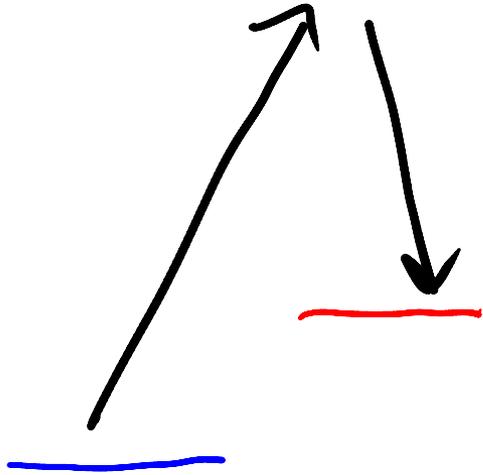


SO coupling  
resonant

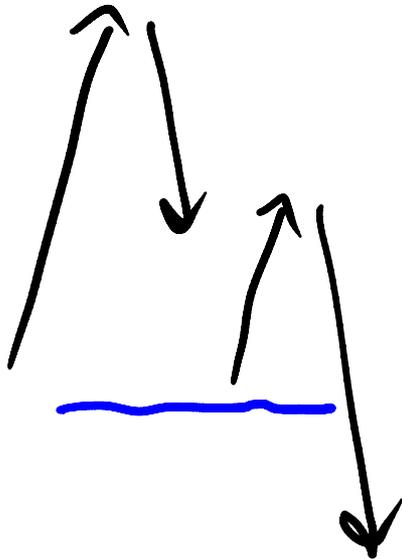


"on site" coupling  
non-resonant

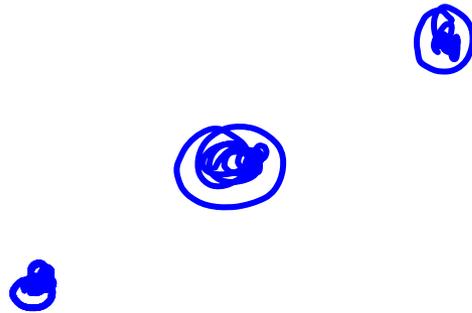


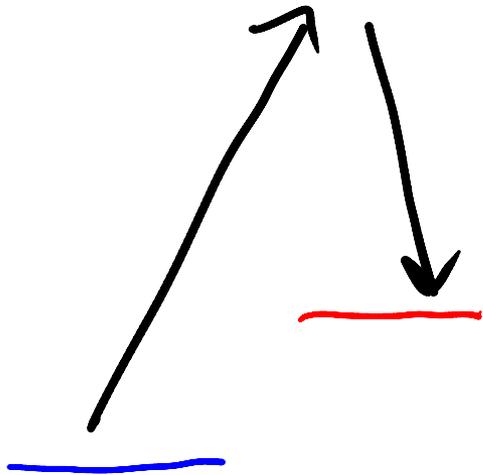


SO coupling  
resonant

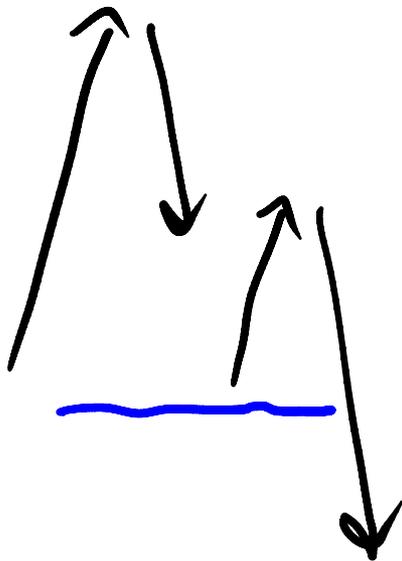


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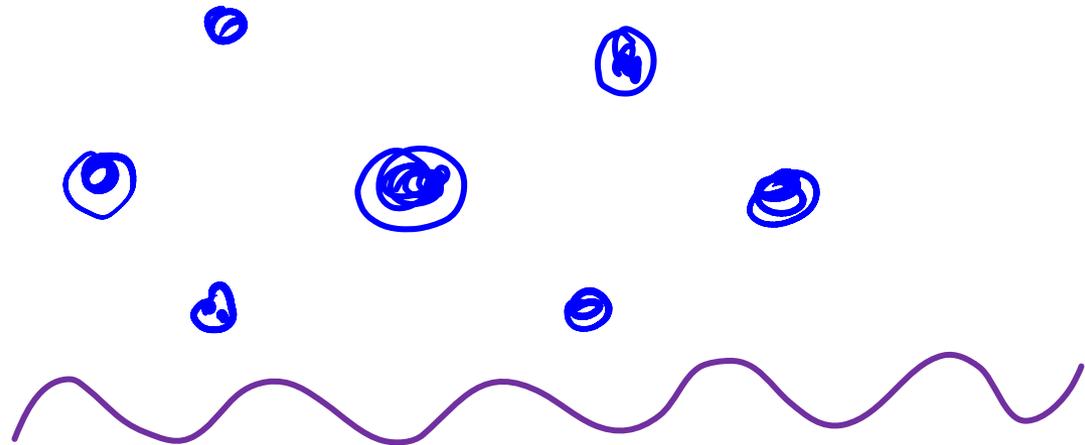


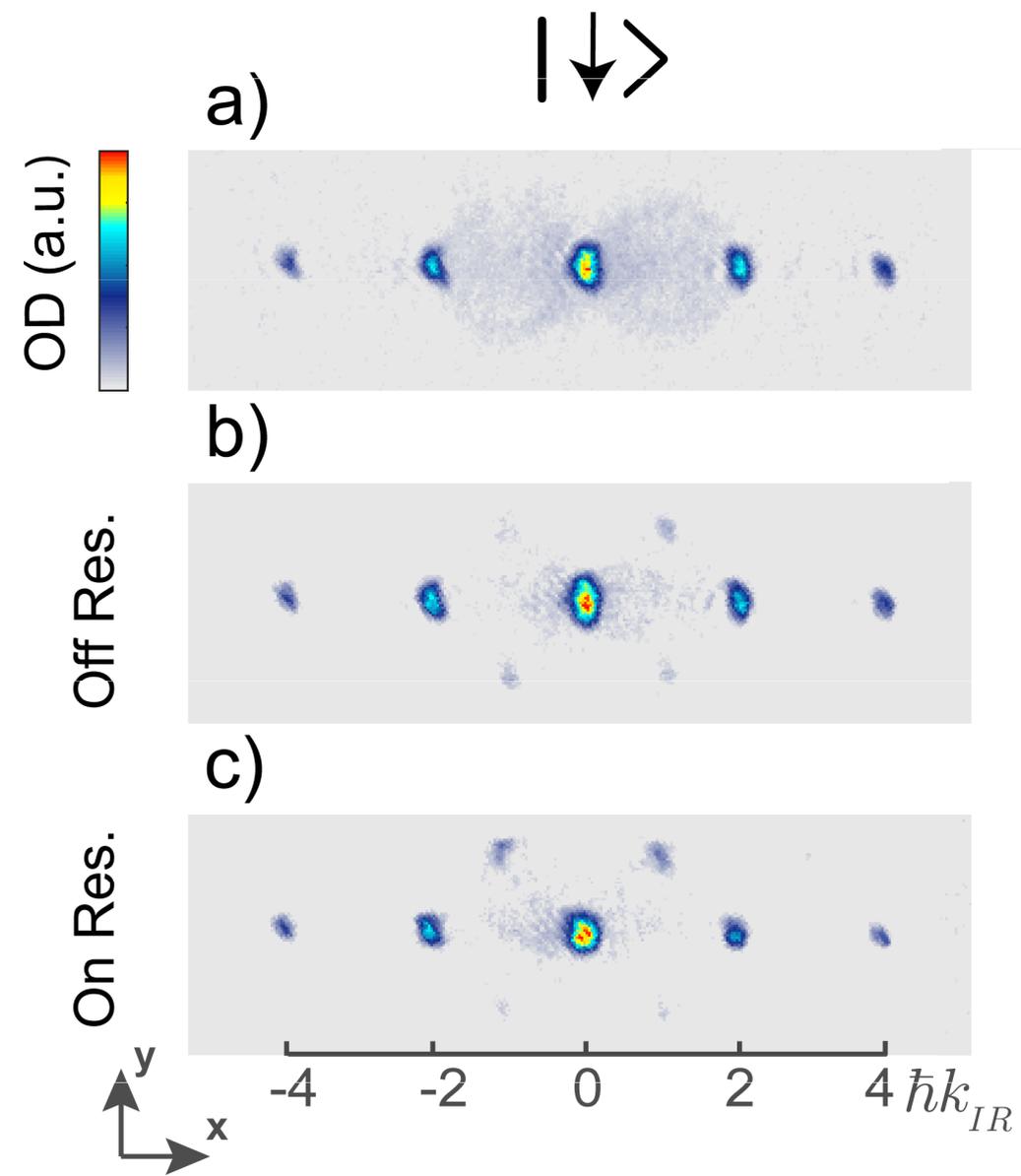


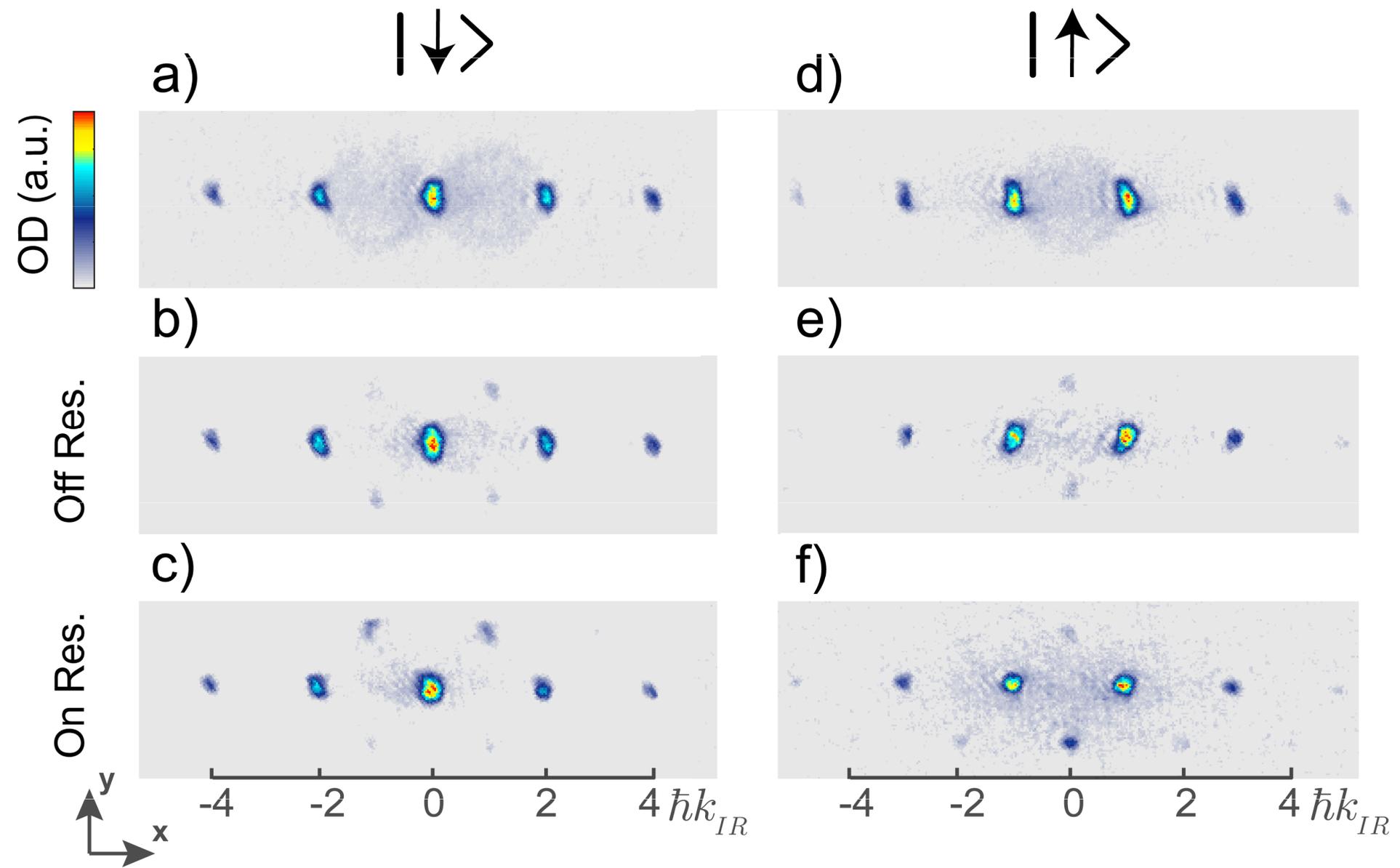
SO coupling  
resonant

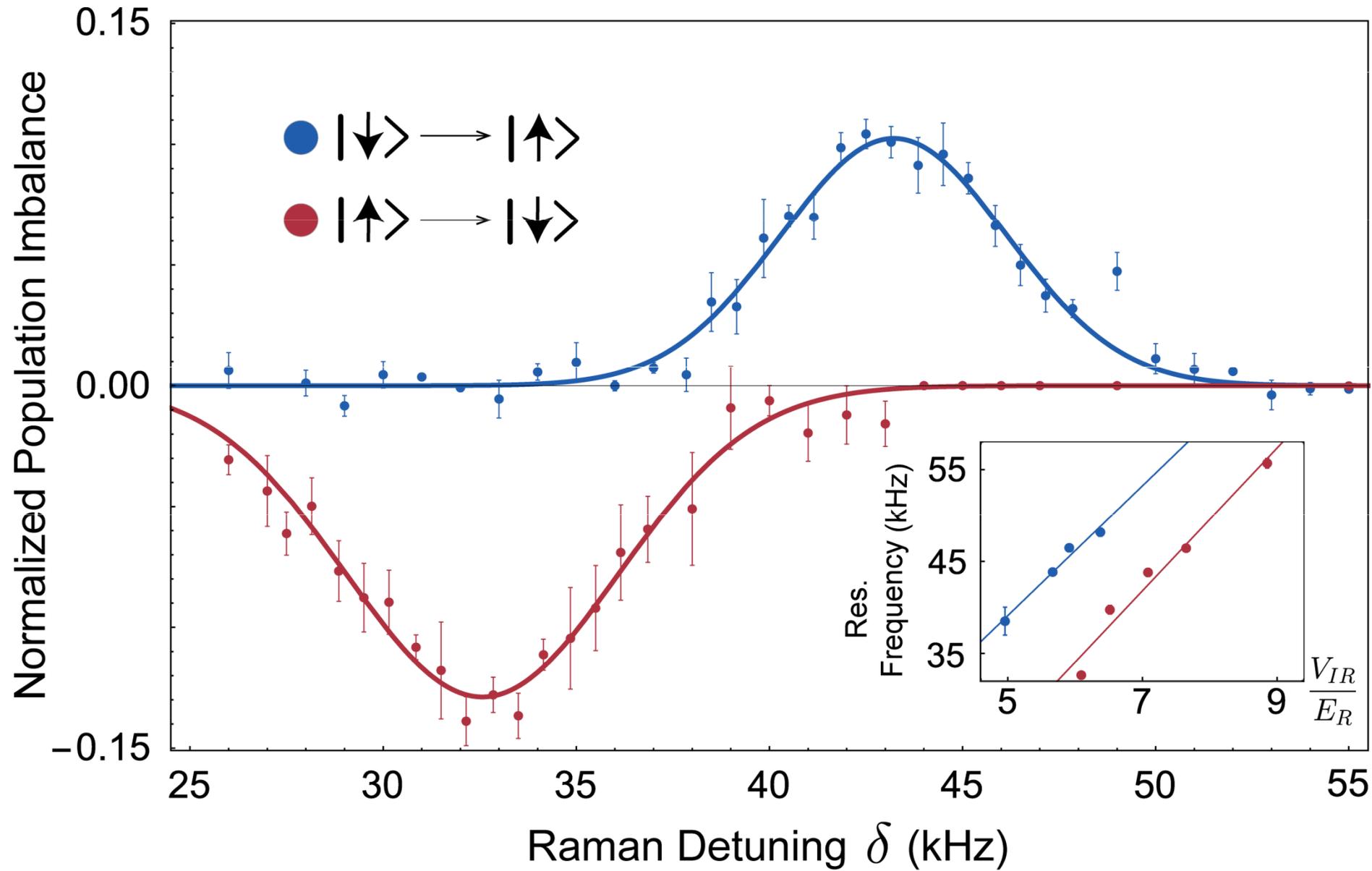


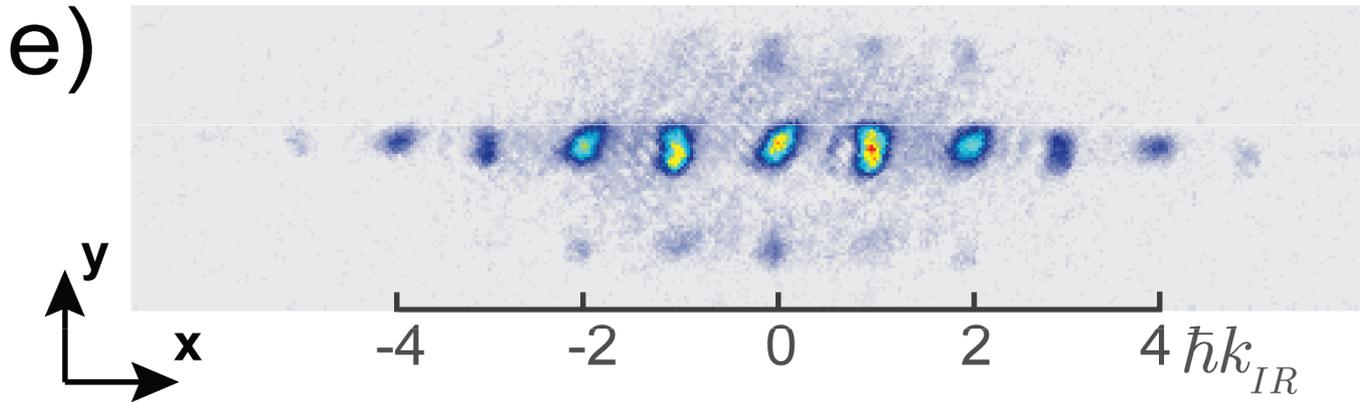
"on site" coupling  
non-resonant











## 50/50 mixture with spin-orbit coupling

Should be BEC with stripe phase (density modulation)  
breaks two continuous symmetries (gauge and translation)  
(Zhai, Ho, Stringari, Baym, Santos, Liu, Paramekanti and more)

see: PHYSICAL REVIEW A **90**, 041604(R) (2014)

**Approach for making visible and stable stripes in a spin-orbit-coupled Bose-Einstein superfluid**

Giovanni I. Martone,<sup>1</sup> Yun Li,<sup>1,2</sup> and Sandro Stringari<sup>1</sup>

our scheme was inspired by their suggestion to use spin-dependent lattices

Credits:

## BEC 4

Colin Kennedy

Cody Burton

Woo Chang Chung

former members:

Hiro Miyake

Georgios Siviloglou

Rb BEC in optical  
lattices

synthetic magnetic  
fields

## BEC 2

Wujie Huang

Junru Li

Boris Shteynas

Furkan Top

Sean Burchensky

Alan Jamison

Superlattices

Spin orbit coupling

\$\$

NSF

NSF-CUA

MURI-AFOSR

MURI-ARO